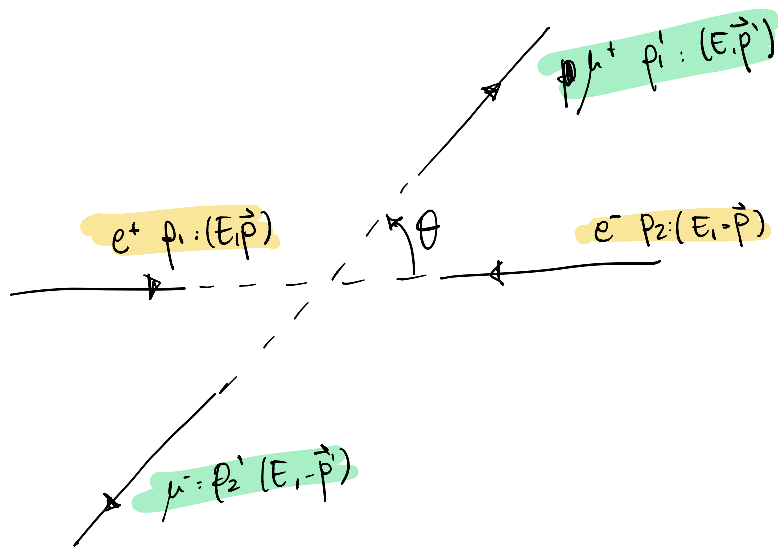
$e^+ + e^- \longrightarrow \mu^+ + \mu^-$ CROSS SECTION



$$\left.\frac{d\sigma}{d\Omega}\right|_{CoM} = \frac{e^4}{16\pi^2}\,\frac{1}{2E^2}\,\frac{p'}{E}\,\frac{1}{16E^4}\,2E^2\left(E^2 + p'^2\cos^2\theta + m_\mu^2\right)$$

$$= \frac{\alpha^2}{16E^4}\left(\frac{p'}{E}\right)\left(E^2 + m_\mu^2 + p'^2\cos^2\theta\right)$$

From QED: $\mathcal{M} = (ie)^2 \cdot \bar{v}_{r_1}(p_1)\gamma_\alpha u_{r_2}(p_2) \dfrac{-ig^{\alpha\rho}}{(p_1+p_2)^2} \bar{u}_{s_2}(p_2')\gamma_\rho v_{s_1}(p_1')$

Scattering Amplitude

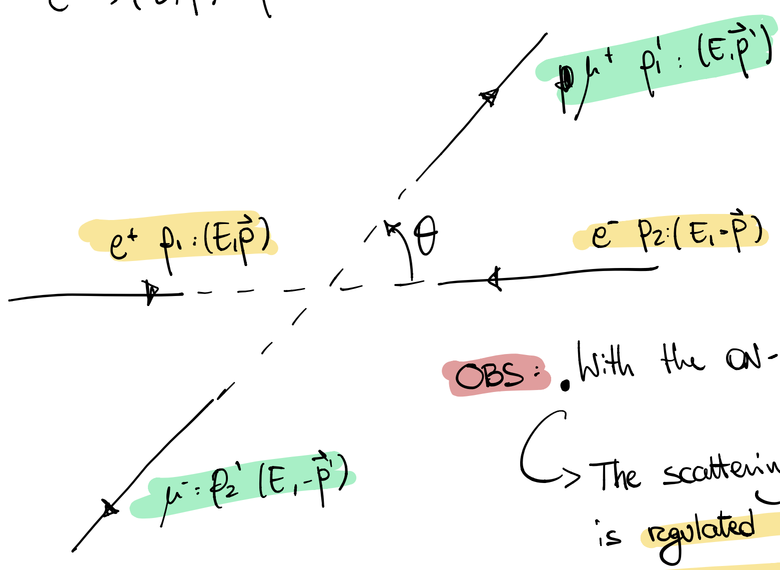
=> Unpolarized Amplitude: $|\bar{\mathcal{M}}| := \dfrac{1}{4}\sum_{r_1,r_2,s_1,s_2} |\mathcal{M}|^2$

$$|\bar{\mathcal{M}}| = \dots \text{conti} \dots = \frac{e^4}{2m_e^2 m_\mu^2((p_1+p_2)^2)^2}\underbrace{\Big((p_1\cdot p_1')(p_2\cdot p_2') + (p_1\cdot p_2')(p_2\cdot p_1') + m_e^2(p_1'\cdot p_2') + m_\mu^2(p_1\cdot p_2) + 2m_e^2 m_\mu^2\Big)}_{:= X}$$

$$= \frac{e^4}{2m_e^2 m_\mu^2((p_1+p_2)^2)^2} \cdot X \quad \text{(Arbitrary Frame)}$$

Let's pass now to the CoM frame.

$e^+ \to (E,\vec{p}) = p_1 \quad e^- \to (E,-\vec{p}) = p_2 \quad \mu^+ \to (E,\vec{p}') = p_1' \quad \mu^- \to (E,-\vec{p}') = p_2'$



OBS: • With the on-shell condition, $\begin{cases} m_e^2 = p_{1,2}^2 \\ m_\mu^2 = p_{1,2}'^2 \end{cases}$

↳ The scattering is always PLANAR, and it is regulated by an angle θ.

• Energy conservation implies that the particles MUST scatter the same energy (pairs of equal particles)

The cross-section formula yields the equation:

$$\left.\frac{d\sigma}{d\Omega}\right|_{CoM} = \frac{1}{64\pi^2}\frac{1}{(E_1+E_2)^2}\frac{|\vec{p}_1'|}{|\vec{p}_1|}\prod_{\ell}\left(2m_e\,|\bar{\mathcal{M}}|^2\right)$$

$$\begin{cases} \ell = e^+, e^-, \mu^+, \mu^- \\ (E_1+E_2)^2 = (2E^2) \\ |\vec{p}_1'| = p',\ |\vec{p}_1| = p \end{cases}$$

$$= \frac{1}{16\pi^2}\cdot\frac{1}{E^2}\frac{p'}{p}\,m_e^2 m_\mu^2\,|\bar{\mathcal{M}}|^2$$

OBS: $E \gg m_\mu \simeq 207\,m_e$. $\Rightarrow$ It's a good approximation to take $p = |\vec{p}| = E$, and to neglect terms proportional to m_e^2.

$$\Rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{CoM} = \frac{1}{16\pi^2}\cdot\frac{1}{E^2}\cdot\frac{p'}{E}\cdot m_e^2 m_\mu^2\cdot\frac{e^4}{2m_e^2 m_\mu^2}\cdot\frac{1}{(2E)^4}X$$

$$= \frac{e^4}{16\pi^2}\cdot\frac{1}{2E^2}\cdot\frac{p'}{E}\cdot\frac{1}{16E^4}\cdot X$$

Let's compute now X. $\Rightarrow$ The kinematic factors now take the following form:

$$\begin{cases} (p_1\cdot p_1') = E^2 - pp'\cos\theta = p_2\cdot p_2' \\ p_1\cdot p_2 = E^2 + p^2 \\ p_1\cdot p_2' = E^2 + pp'\cos\theta = p_2\cdot p_1' \\ p_1'\cdot p_2' = E^2 + p'^2 \\ (p_1+p_2)^2 = 4E^2 \end{cases}$$

$$\Rightarrow X \equiv \left((E^2 - pp'\cos\theta)^2 + (E^2+pp'\cos\theta)^2 + m_e^2(E^2+p'^2) + m_\mu^2(E^2+p^2) + 2m_e^2 m_\mu^2\right)$$

$\longrightarrow$ Then, assuming again $m_e \sim \phi$ $(p \to E)$

$$X = \left.\left((E^2 - pp'\cos\theta)^2 + (E^2+pp'\cos\theta)^2 + m_\mu^2(E^2+p^2)\right)\right|_{p=E} = \left.\left(2E^4 + 2p^2p'^2\cos^2\theta + m_\mu^2(E^2+p^2)\right)\right|_{p=E}$$

$$= 2E^2\left(E^2 + p'^2\cos^2\theta + m_\mu^2\right)$$

finally:

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{CoM} = \frac{e^4}{16\pi^2}\frac{1}{2E^2}\frac{p'}{E}\frac{1}{16E^4}2E^2\left(E^2+p'^2\cos^2\theta+m_\mu^2\right) = \frac{\alpha^2}{16E^4}\left(\frac{p'}{E}\right)\left(E^2+m_\mu^2+p'^2\cos^2\theta\right)}$$

$\left(\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}\right)$