Demodulation Techniques

- Demodulation: process by which the carrier is removed from the received signal (inverse of modulation process)

- Demodulation process can be coherent or non-coherent

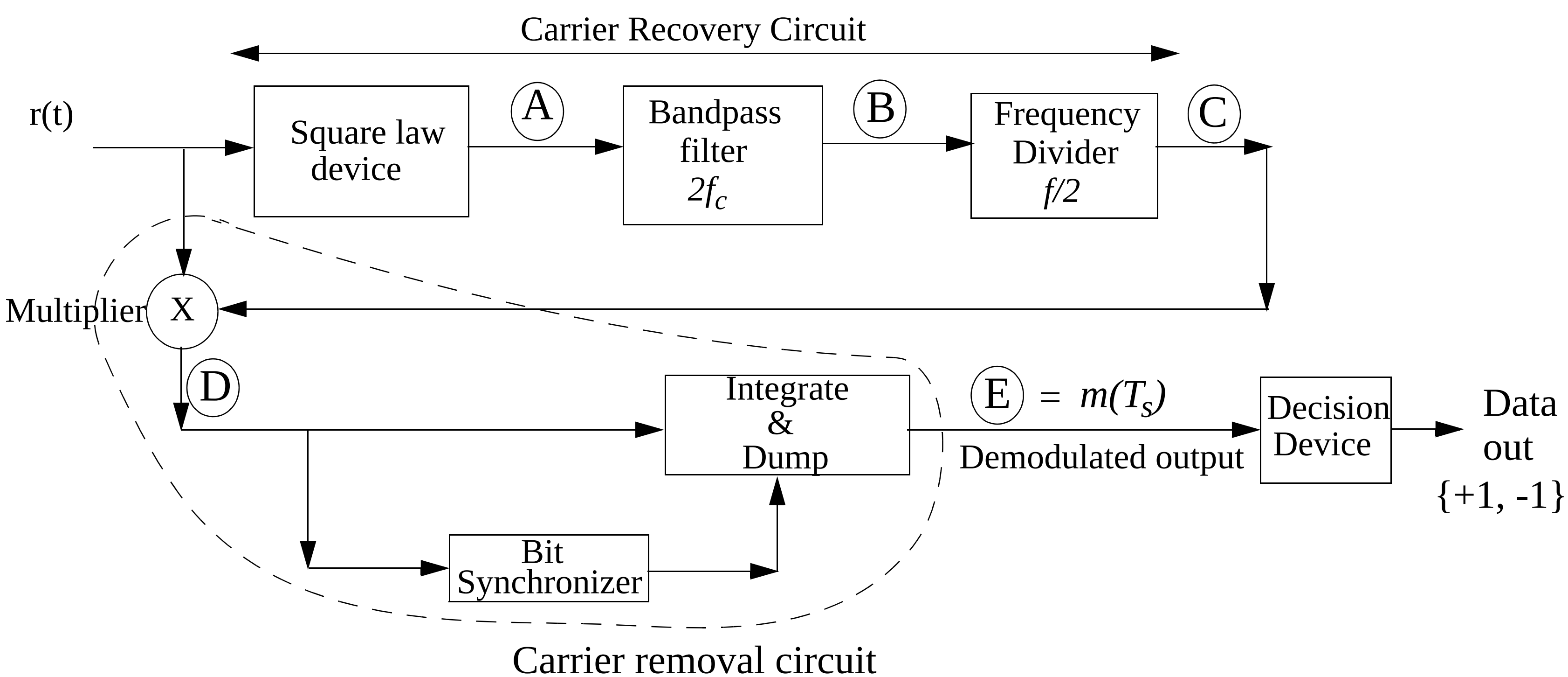
- Coherent Demodulation:
  - receiver requires knowledge of the carrier's phase/frequency to detect the transmitted signal
  - complex receiver due to need to estimate carrier phase/frequency via a carrier recovery circuit
  - provides optimal performance

- Noncoherent Demodulation:
  - receiver does not require carrier's phase/frequency to detect the transmitted signal
  - less complex receiver as carrier phase/frequency estimation is not required
  - provides less-optimal performance

Coherent Receiver Example: Coherent BPSK Receiver

<u>Receiver Operation:</u> Demodulation + Detection

- Demodulation: carrier recovery + removal of carrier from received signal via correlation process
- Detection: process of deciding the transmitted symbol

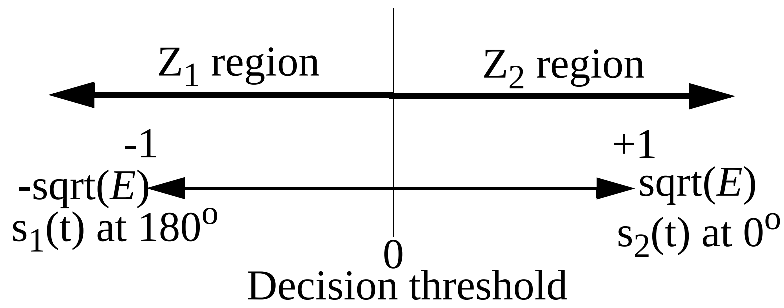


Coherent BPSK Receiver Operation

- Square law Device:
  - squares the received signal *r(t)* to produce a dc signal and a sinusoid at 2X the carrier freq.
  - Square law device output is: $cos^2[2\pi f_c t + \theta] = \left[\frac{1}{2} + \frac{1}{2}cos(4\pi f_c t + 2\theta)\right]$
- Bandpass Filter:
  - Filters out the dc signal to produce: $cos(4\pi f_c t + 2\theta)$
- Frequency Divider:
  - Recreates the carrier waveform: $cos(2\pi f_c t + \theta)$
- Multiplier: multiplies the frequency divider output with the received signal
  - Multiplier output is: $m(t)cos^2[2\pi f_c t + \theta]$
- Correlator (integrate & dump):
  - integrates multiplier output over a symbol period T_S and "dumps" the result to the decision device at time $t = T_S$. Correlator or sampled output at $t = T_S$ is: $m(T_S)$ = *decision metric*
- Bit Synchronizer: facilitates the sampling of the integrator output to occur "exactly" at the end of each symbol period
- Decision Device: If decision metric $m(T_S) > 0$, output data is a "1";

  If decision metric $m(T_S) < 0$, output data is a "0" or "-1"

Performance of Coherent BPSK Receiver in AWGN Channel



<u>*Decision Rule:*</u> If $m(T_s)$ falls in Z_1 region (i.e., $m(T_s) < 0$), receiver decides "-1" was transmitted else if $m(T_s)$ falls in Z_2 region (i.e., $m(T_s) > 0$), receiver decides "1" was transmitte

<u>*Correct Decisions:*</u> I.e., ideal channel ==> no errors introduced by the channel

1) Symbol "-1" was transmitted and $m(T_s)$ lies in the Z_1 region, i.e., $-\infty < m(T_s) < 0$

2) Symbol "1" was transmitted and $m(T_s)$ lies in the Z_2 region, i.e., $0 < m(T_s) < \infty$

<u>*Erroneous Decisions:*</u> due to additive white Gaussian noise (AWGN) channel

Type 1 error: Symbol "-1" was transmitted but AWGN causes $m(T_s)$ to fall in Z_2 region hence receiver decides on symbol "1"

Type 2 error: Symbol "1" was transmitted but AWGN causes $m(T_s)$ to fall in Z_1 region hence receiver decides on symbol "-1"

Performance of Coherent BPSK Receiver in AWGN Channel

Performance Metric: Probability of bit error

Definition: number of bit errors divided by the number of bits transmitted when infinitely many bits are transmitted

<u>*Note:*</u> In the literature, the term bit error rate (BER) is often interpreted as bit error probability. Strictly speaking, BER = number of bit errors per unit time

Let:

$P_e(1|-1)$ = probability that receiver detects bit "1" given that bit "-1" was transmitted

$P_e(-1|1)$ = probability that receiver detects bit "-1" given that bit "1" was transmitted

- The average probability of bit error for coherent BPSK in an AWGN channel is given as:

$$P_{e,BPSK} = \frac{P_e(1|-1) + P_e(-1|1)}{2} = \frac{1}{2} erfc\left(\sqrt{\frac{E}{N_o}}\right) = Q\left(\sqrt{\frac{2E}{N_o}}\right)$$

where *erfc(.)* is the complementary error function, E is the energy per bit, N_0 is the noise power spectral density, and the factor $\frac{1}{2}$ means that the symbol "-1" and symbol "1" can be transmitted with equal probability.

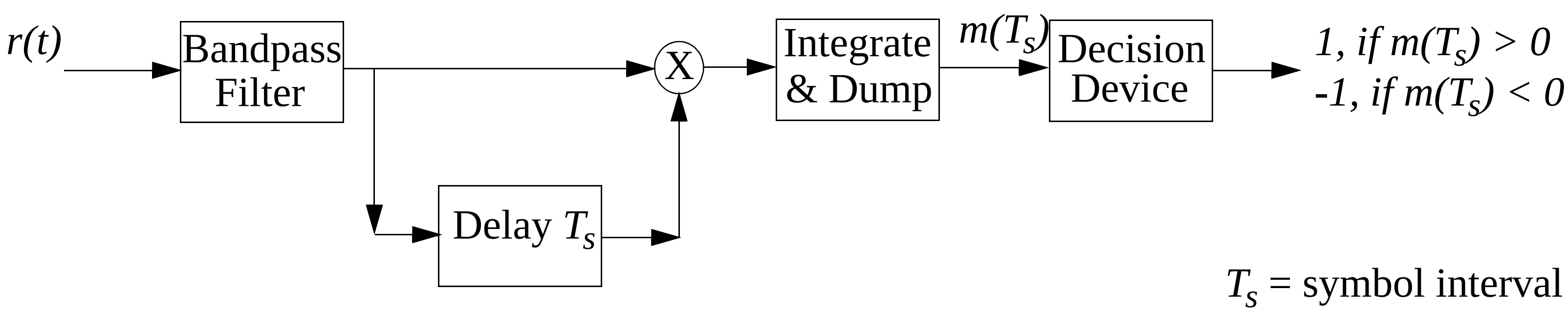
Differential Receiver Example: DPSK Receiver

<u>Receiver Operation:</u> Demodulation + Detection

- Demodulation: differential process + removal of carrier from received signal via correlation process (Note: compared to coherent BPSK, carrier recovery circuit is not required)

  - Differential process: signal used for demodulation of the signal in the current bit interval is based on the received signal during the current bit interval and that received in the previous interval

- Detection: process of deciding the transmitted symbol: correlator output is compared with a threshold to determine the transmitted symbol



Performance of DPSK Receiver in AWGN Channel

Let:

$P_e(1|-1)$ = probability that receiver detects bit "1" given that bit "-1" was transmitted

$P_e(-1|1)$ = probability that receiver detects bit "-1" given that bit "1" was transmitted

- The average probability of bit error for DPSK in an AWGN channel is given as:

$$P_{e,\,DPSK} = \frac{P_e(1|-1) + P_e(-1|1)}{2} = \frac{1}{2} exp\left(-\frac{E}{N_0}\right)$$

where

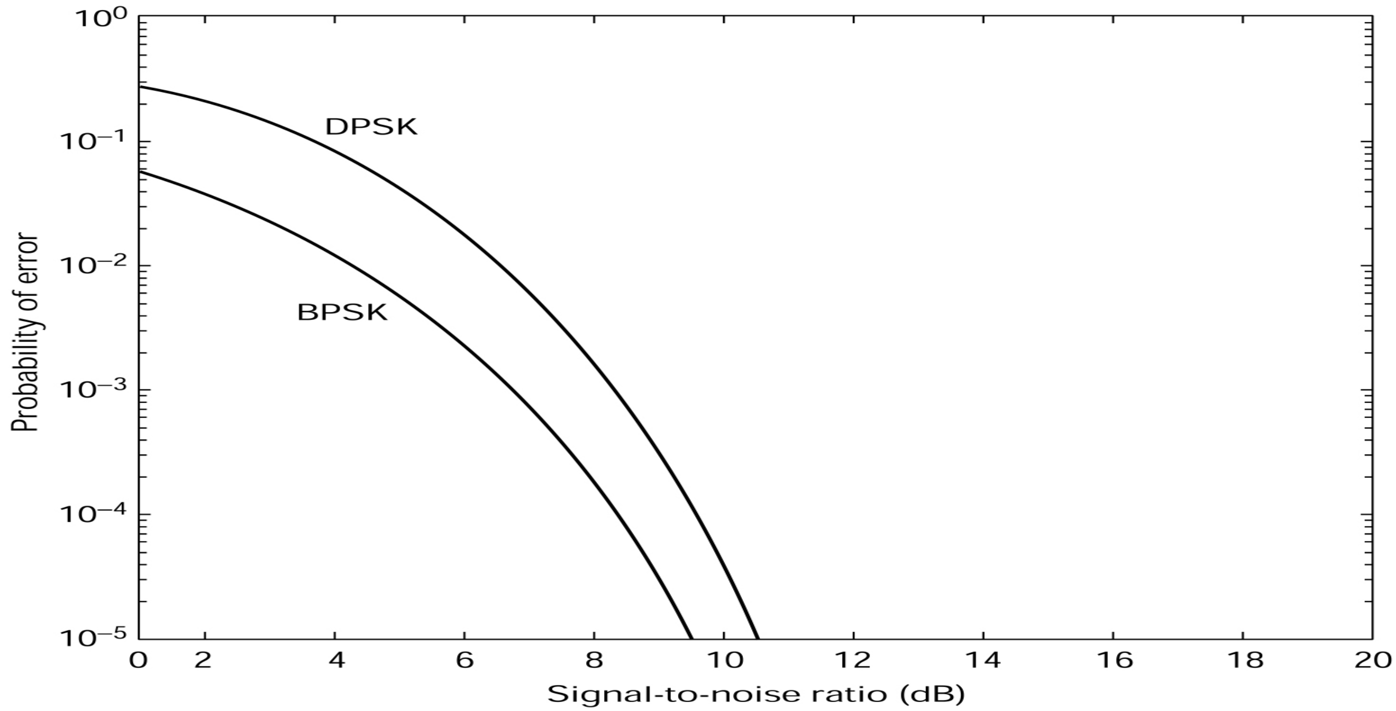
$exp(.)$ is the exponential function

E is the energy per bit

N_0 is the noise power spectral density, and

the factor $\frac{1}{2}$ means that the symbol "-1" and symbol "1" can be transmitted with equal probability.

Coherent BPSK vs. DPSK Receiver Performance Comparison



Conclusion:

- At practical probability of bit error (<= 10^{-3}), the signal to noise ratio (SNR) required by DPSK is about 1 dB higher than that required by coherent BPSK.

Question: What is the practical interpretation of 1 dB SNR increase?

Class Example

Problem Statement:

Calculate the average energy per bit to noise ratio required to maintain a probability of bit error of 2×10^{-4} in coherent BPSK and DPSK modems.

Probability of Error for Coherent GMSK in AWGN Channel

The probability that a bit is in error for coherent GMSK is given by

$$P_{e,\,GMSK} = \frac{1}{2} erfc\left(\sqrt{\frac{\zeta E}{N_o}}\right) = \frac{1}{2} erfc\left(\sqrt{\left(\frac{E}{N_o}\right)_{GMSK}}\right) = Q\left(\sqrt{2\left(\frac{E}{N_o}\right)_{GMSK}}\right)$$

where ζ ($\zeta < 1$) is a constant related to BT_S, the normalized *3 dB* bandwidth delay product. For example, at $BT_S = 0.25$, $\zeta = 0.68$

If $P_{e,\,GMSK} = P_{e,\,BPSK}$, this implies:

$$\left(\frac{E}{N_o}\right)_{GMSK} = \zeta\left(\frac{E}{N_o}\right)_{BPSK}$$

Since ($\zeta < 1$), $\left(\frac{E}{N_o}\right)_{GMSK} < \left(\frac{E}{N_o}\right)_{BPSK}$ to achieve the same probability of bit error objective

MODEM Performance over Wireless Channels

<u>Task:</u> Compute the average probability of bit error for a given MODEM over a specified wireless channel model (small scale fading model in most cases)

<u>Analysis Steps:</u>

<u>Step 1:</u> For the given MODEM and wireless channel, determine the probability of bit error for any arbitrary received SNR over the wireless channel

<u>Step 2:</u> Determine the distribution of the received SNR for the given wireless channel

<u>Step 3:</u> Average the probability of error over the distribution of the received SNR

<u>NOTE:</u> In practice, performance of a given MODEM in a mobile environment is determined by computer simulations that take into account the exact propagation conditions

Example: Coherent BPSK Performance over Rayleigh Fading Channel

Step 1: Determine the probability of bit error for any arbitrary received SNR

For coherent BPSK in AWGN channel, $P_{e,\,BPSK} = 0.5erfc(\sqrt{SNR})$ (Page 25) where SNR = E/N_0

For coherent BPSK in Rayleigh fading channel, $P_{e,\,BPSK,\,fad}(SNR_{fad}) = 0.5erfc(\sqrt{SNR_{fad}})$

where SNR_{fad} is the received SNR in a Rayleigh fading channel, $SNR_{fad} = \frac{A^2 E_b}{N_0}$

A = Rayleigh fading envelope. Denote SNR_{fad} by γ (for notational simplicity)

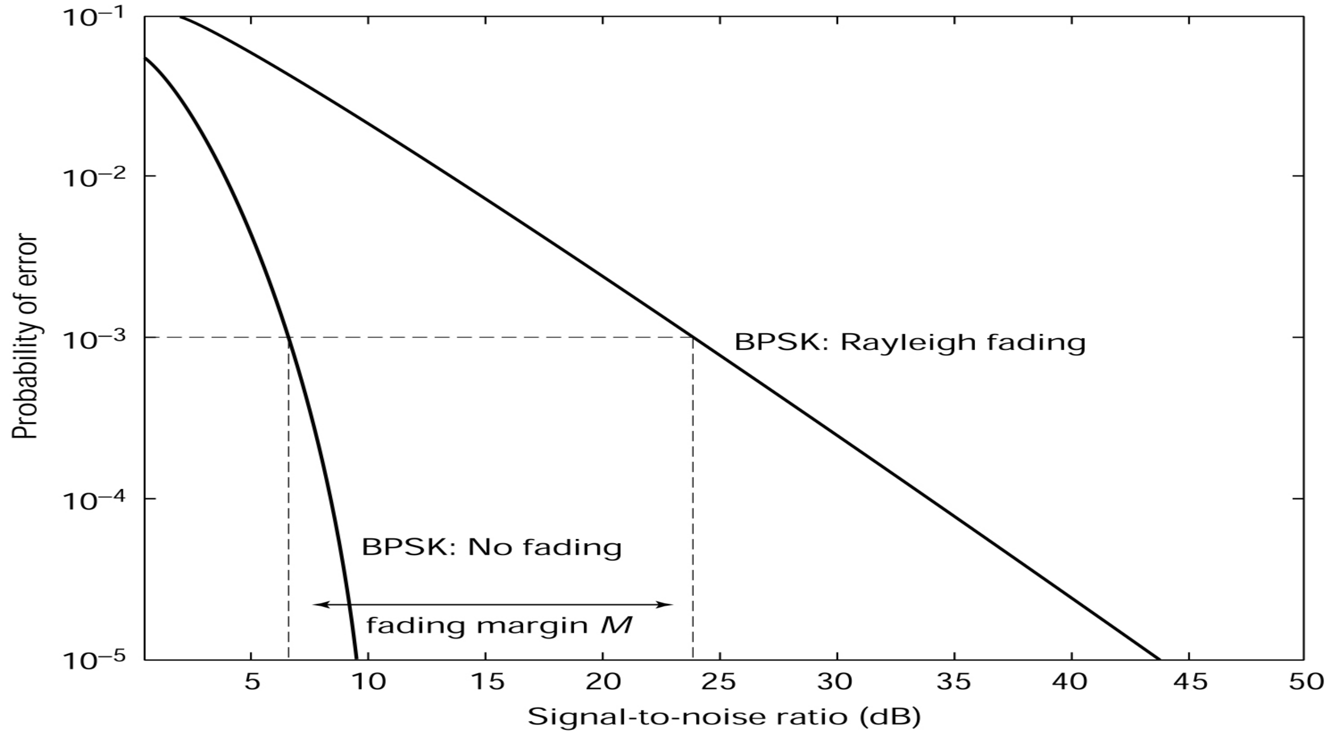
Step 2: Determine $p(\gamma)$, the pdf of γ over a Rayleigh fading channel: $p(\gamma) = (1/\gamma_0)exp(-\gamma/\gamma_0)$

where $\gamma_o = E[\gamma] = E[SNR_{fad}] = (E_b/N_o)E[A^2]$ is the average received SNR over the Rayleigh channel

Step 3: Determine $\bar{P}_{e,\,BPSK,\,fad}$, the average probability of bit error for coherent BPSK over Rayleigh fading:

$$\bar{P}_{e,\,BPSK,\,fad} = \int_0^{\infty} P_{e,\,BPSK,\,fa}(\gamma)p(\gamma)(d\gamma) = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma_o}{1+\gamma_o}}\right]$$

Coherent BPSK Performance over Rayleigh fading Channel



Comments:

1. For a given SNR, BER in a Rayleigh fading channel is higher than that in a fading-free channel

   Implication: To achieve the same BER, faded channel requires a higher SNR than fading-free channel to compensate the effect of fading

2. As SNR increases, BER reduction rate is slower in a faded channel than in a fading-free channel

Class Example

Consider the transmission of a BPSK signal over a fading channel. In addition to the additive white Gaussian noise (AWGN) of zero mean and single-sided noise power spectral density N_o, the transmitted BPSK signal is corrupted by a fading channel, taking on values $A \in \{A_1, A_2\}$, where A_1 = 1 and A_2 = 0.1 with probability 0.8 and 0.2, respectively.

a) Derive the expression for the average bit error rate (BER) under the assumption of coherent detection. Express your result in terms of the Q-function and E_b/N_o. *Hint:* The BER performance of BPSK over the AWGN channel with coherent detection is: $BER_{AWGN} = Q(\sqrt{2(E_b/N_o)})$.

b) If E_b/N_o = 5 dB, calculate the average bit error rate over the fading channel.