ENEL 529 - Lab 2

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Abstract

The purpose of this lab is explore the relationship between measured signal data corresponding to a fading channel and the theoretical Rayleigh distribution as well as look at the statistical measures of the Rayleigh distribution.

Exercise 1 - Analyzing Measured Signal Data of a Fading Channel

Exercise Objective:

The objective of this exercise is to import measured signal data collected from a fading channel, and analyze its distribution with that of the Rayleigh Distribution. This exercise will consist of 3 parts:

1. Importing and normalizing the measured data.
2. Calculating the measured and theoretical probability density functions (pdfs).
3. Testing both quantitatively and qualitatively the relationship between the measured distribution and the theoretical Rayleigh distribution.

Figures and corresponding MATLAB code will be presented to show the procedural formulation of the steps and their respective results.

Procedure: Part 1 - Importing and Normalizing Data

1. Step 1 - Import the Measured Signal Data from the file *series11.mat* into the workspace and store into a matrix, DATA. Then plot the measured data, as Power(dBm) in respect to Time(sec):

```
%% Clear Variable Data
clear all;

%% Import Measured Data into Time and Power
Data = importdata('series11.mat');

%% Plot Received Signal Data
figure(1);
plot(Data(:,1),Data(:,2), 'r-');
xlabel('Time: (sec)');
ylabel('Received Power: (dBm)');
title('Measured Signal Data');

%% What Type of Fading is Shown in Figure 1?
display('What type of Fading is Displayed in Figure 1?');
display('Answer: Rayliegh Fading');
```

The above code, when executed creates the following graph seen on the next page (See Figure 1):

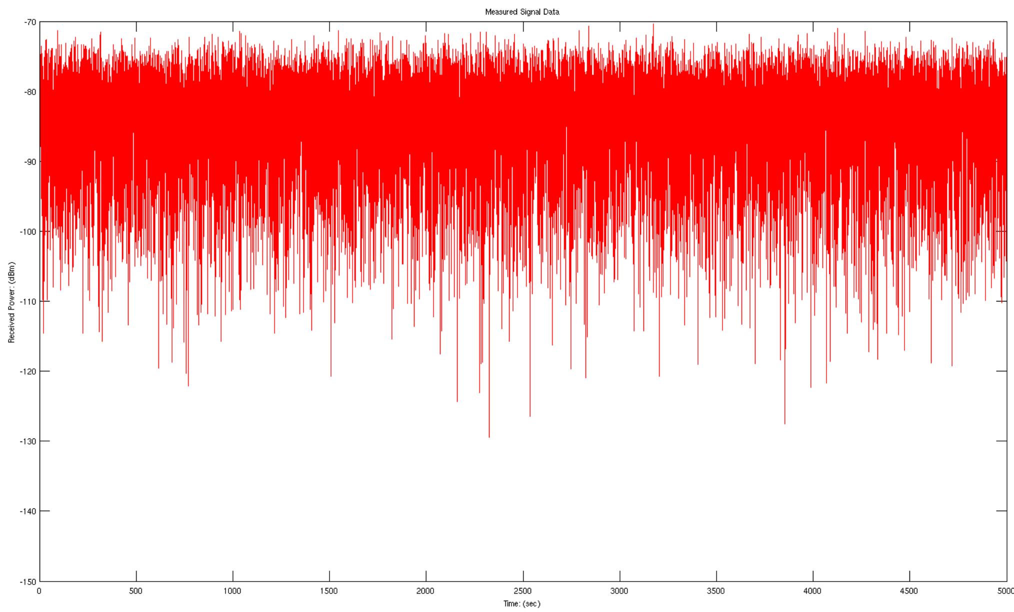

Figure 1: The Measured Signal Data, Plotted as Power as a function of Time.

As one can see from the plot, the signal is of the form corresponding to that of a signal undergoing Rayleigh Fading.

2. Step 2 - Convert the Measured Signal Power from dBM to Normalized Voltage Values, and plot the result. This is done via the following steps:
   (a) Extract the Power from the Data Matrix
   (b) Convert from Power into Voltage using the following relationships:
   $$P = 10^{\frac{P_{dBm}}{10}}$$
   $$V = \sqrt{2RP}, \text{where } R = 50\Omega$$
   (c) Calculate the Average Voltage of Voltage Data (100000 Samples).
   (d) Assume Rayleigh Distributed, Estimate σ using the relationship:
   $$\bar{V} = \sigma\sqrt{\pi/2}$$
   (e) Normalize the Vector V in respect to σ_V and denote as Vector, V_{norm}.
   (f) Plot the Normalized Voltage Data.
   (g) After plotting the Data, Calculate the Mean of the Normalized Voltage, $\bar{V}_{norm}$, and use it to estimate the standard deviation,σ_{norm}, assuming Rayleigh Conditions. using the following relationship:
   $$\sigma_{norm} = \frac{\bar{V}_{norm}}{\sqrt{\pi/2}}$$

The following MATLAB code shows the above mentioned procedures as executed in the lab (Code Continues onto next page):

```
%% Convert Power to Normalized Voltage Vector
% Get Length of Vector
M = length(Data);

% Create Empty Vector for Power
P = zeros(M,1);

% Create Empty Vector for Voltage and Normalized Voltage
V = zeros(M,1);
Vnorm = zeros(M,1);

% Extract Power and Convert to Watts
for i=1:M
    P(i,1) = 10^(Data(i,2)/10)/1000;
end

% Assume Resistance is 50 ohm
R = 50;

% Calculate Voltage from power
for i=1:M
    V(i,1) = sqrt(2*R*P(i,1));
end

% Calculate the Average Voltage
uV = mean(V);

% Estimate the Standard Distribution of Rayleigh Signal Data
sigma_V = uV/sqrt(pi/2);

% Display Mean and Sigma Parameter
display(uV); display(sigma_V);

% Normalize The Voltage Vector
for i=1:M
    Vnorm(i,1) = V(i,1)/sigma_V;
end

%% Plot Normalized Voltage Vector
figure(2);
plot(Data(:,1),Vnorm(:,1),'r-');
xlabel('Time: (sec)');
ylabel('Normalized Voltage: (V)');
title('Normalized Voltage Values for received Signal');

%% Calculate Mean and Sigma of Normalized Vector
uVnorm = mean(Vnorm);
sigma_Vnorm = uVnorm/sqrt(pi/2);

% Display the Normalized Mean and Sigma
display(uVnorm); display(sigma_Vnorm);
```

After running the code, the Voltage mean $\bar{V}$ and parameter σ were calculated as:

$$\bar{V} = 2.6619 * 10^{-5}$$

$$\sigma = 2.1239 * 10^{-5}$$

and the mean $\bar{V}_{norm}$ and the parameter σ_{norm} were calculated. The values respectively were:

$$\bar{V}_{norm} = 1.2533$$

$$\sigma_{norm} = 1.0$$

The following figure was plotted on the graph and shows the normalized Voltage Data(V) in respect to Time(sec) (see Figure 2):

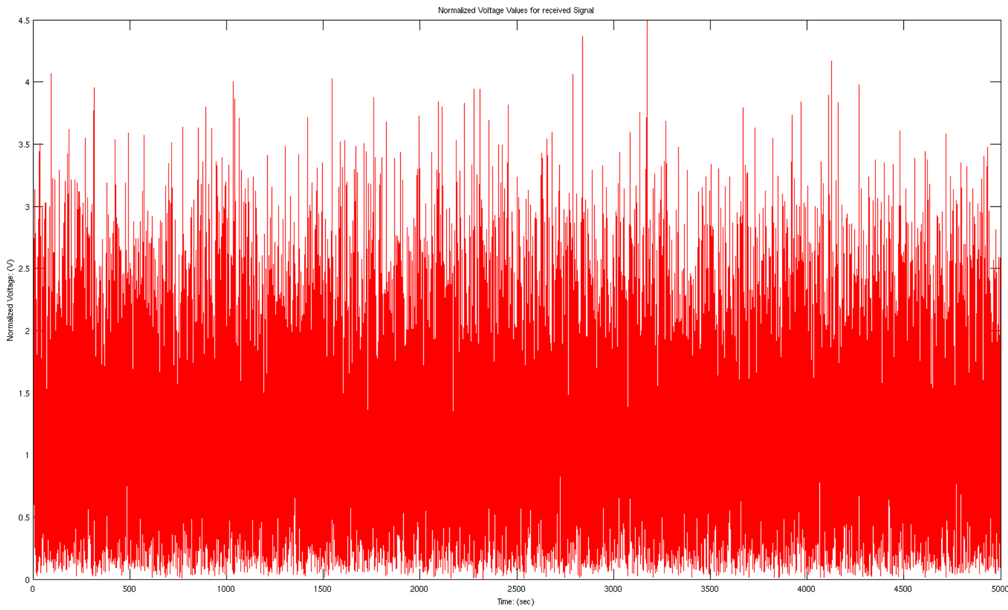

Figure 2: The Normalized Voltage Data as Calculated from the Measured Signal Data.

Procedure: Part 2 - Calculating the Measured/Theoretical PDF of $\bar{V}_{norm}$

1. Step 3 - Calculate the Measured PDF of the Normalized Voltage Data found in Part 1 of the lab.

   (a) Divide samples into K disjoint intervals:

```
%% Calculate Measured PDF of Normalized Voltage Vector
% Split into K disjoint intervals
K = 10;
[m_j, C_j] = hist(Vnorm,K);
```

   (b) Calculate Δ, the length of each bin_j, assuming equal lengths:

```
% Calculate Delta, assuming equal lengths
delta_Vnorm = C_j(2)-C_j(1);
```

   (c) Calculate the pdf of the simulated samples calculated by:

$$measured\_pdf(C_j) = \frac{m_j}{M * \Delta}$$

```
% Create Empty PDF
measured_pdf = zeros(K,1);
% Calculate Measured pdf
for i=1:K
    measured_pdf(i,1) = m_j(i)/(M*delta_Vnorm);
end
```

2. Step 4 - Calculate the Theoretical Rayleigh pdf using the previously calculated parameter, σ_{norm}:

$$theoretical\_pdf(C_j) = \frac{C_j}{\sigma_{norm}^2} \text{EXP} \left(-\frac{C_j^2}{2\sigma_{norm}^2} \right)$$

```
%% Calculate Theoretical PDF of Normalized Voltage Vector

% Create Empty PDF Container
theoretical_pdf = zeros(K,1);
% Calculate Theoretical PDF
for i=1:K
    theoretical_pdf(i,1) = (C_j(i)/(sigma_Vnorm^2))*exp(-(C_j(i)^2)/(2*sigma_Vnorm^2));
end
```

Procedure: Part 3 - Perform Qualitative & Quantitative Test

1. Step 5 - Qualitative Test: plot the two pdfs onto the same graph and do a visual comparison between the measured theoretical distributions:

```

% TEST 1 - VISUAL
figure(3);
plot(C_j,measured_pdf,'r--o', C_j,theoretical_pdf, 'g--x');
title('Measured PDF vs Theoretical PDF for Normalized Voltage Signal Data');
xlabel('Centre of Bin, C_j');
ylabel('Normalized PDF, F_X(C_j)');
```

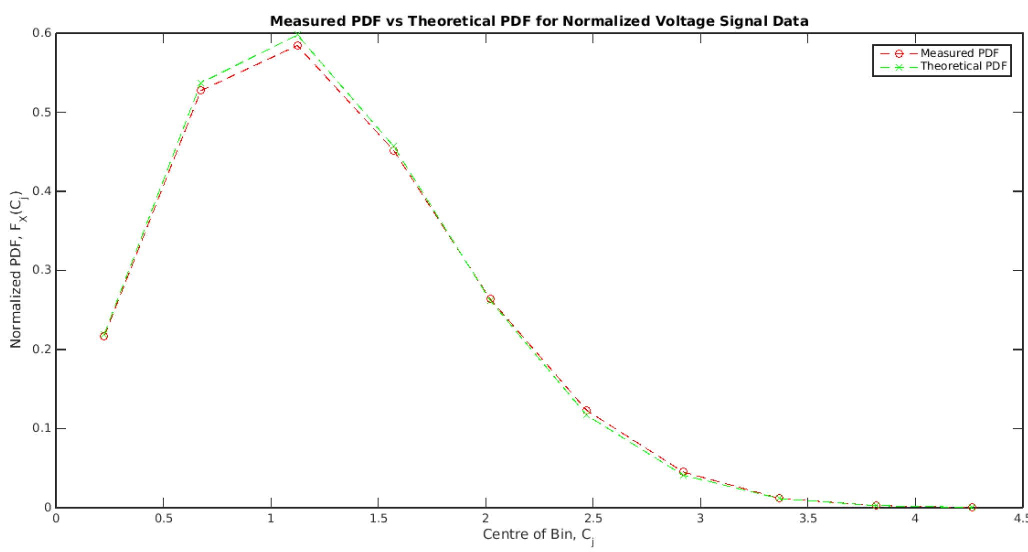

Figure 3: The Measured Signal Data, Plotted as Power as a function of Time.

2. Step 5 - Quantitative Test: Perform a Chi-Square Goodness-of-fit Test on the Measured pdf corresponding with 1% significance level, and compare it to the value taken from the tables:

   (a) Calculate the theoretical number of samples that fall into each of the K intervals as created previously in Part 2.

   - Calculate Δ. See Part 2 for details.
   - Calculate L_j, the Lower limit of the interval:

   $$L_j = C_j - \frac{\Delta}{2}$$

   - Calculate U_j, the Upper limit of the interval:

   $$U_j = C_j + \frac{\Delta}{2}$$

   - Calculate p_j, the probability that a sample falls into a particulate interval, assuming Rayleigh distribution with pdf, $f_X(x)$:

   $$f_X(x) = \frac{x}{\sigma_{norm}^2} \text{EXP}\left(-\frac{(x)^2}{2\sigma_{norm}^2}\right)$$

   where

   $$p_j = P[L_j < X < U_j] = \int_{L_j}^{U_j} f_X(x)dx$$

   $$p_j = \text{EXP}\left(-\frac{L_j^2}{2\sigma_{norm}^2}\right) - \text{EXP}\left(-\frac{U_j^2}{2\sigma_{norm}^2}\right)$$

   - Calculate $T_j = Mp_j$:

   The following code describes the above mentioned procedure as it was done in the lab:

```
% Create Empty Vector for Probability and Expected # of Samples
p_j = zeros(K,1);
T_j = zeros(K,1);
% Create Empty Vectors for Upper and Lower Limits
U_j = zeros(K,1);
L_j = zeros(K,1);
% Calculate Upper and Lower Limits of Intervals
for i=1:K
    U_j(i) = C_j(i) + delta_Vnorm/2;
    L_j(i) = C_j(i) - delta_Vnorm/2;
end
% Calculate Probability and Expected # of Samples
for i=1:K
    p_j(i) = exp(-L_j(i)^2/(2*sigma_Vnorm^2)) - exp(-U_j(i)^2/(2*sigma_Vnorm^2));
    T_j(i) = M*p_j(i);
end
```

   (b) Calculate the Chi-Square statistic Z_1:

   $$Z_1 = \sum_{j=1}^{K} \frac{(m_j - T_j)^2}{T_j}$$

```
% Calculate The Chi-Square Statistic , Z1
Z1 = 0;
for i=1:K
    Z1 = Z1 + (m_j(i) - T_j(i))^2/(T_j(i));
end
```

(c) Compare Z_1 with Z_T. If $Z_1 < Z_T$ then test passes:

```
% Zt From Chi-Square Tables (K - r - 1) = 10 - 1 - 1 = 8 @ 1%
Zt = 20.09;

% Compare Z1 with Zt
if Z1 < Zt
    display(Z1);
    display(Zt);
    display('The Distribution is Rayleigh Distributed!');
else
    display(Z1);
    display(Zt);
    display('The Distribution is not Rayleigh Distributed.');
end
```

After running the code, and comparing the Chi-Square Statistic to that of the Table, the test passed, and the distribution was found to be Rayleigh Distributed.

Conclusions from the Exercise

The conclusions which can be extracted from the exercise was that the given data from the file *series11.mat* did in fact represent a signal having a Rayleigh distribution. This would indicate that the signal underwent Multipath(Rayleigh) Fading between the transmitting antenna and the receiving antenna, and one could assume from this information that the signal was under NLOS (Non-Line-of-Sight) conditions and that there was no direct path existing between them.

Additional Questions for the Lab

Task 1.1 - Calculating V_{rms} from V

$$V_{rms} = \frac{2}{\sqrt{\pi}}\bar{V} = \sqrt{2}\sigma$$

```
%% Task 1.1
% Calculate V_rms from Vector V
V_rms = (2/sqrt(pi))*uV;
display(V_rms);
```

The value of V_{rms} was found to be:

$$V_{rms} = 3.0036 * 10^{-5}$$

Task 1.2 - Normalize V in respect to V_{rms}

$$V_{normrms} = \frac{V}{\sqrt{2}\sigma}$$

```
%% Task 1.2
% Generate Empty Vector Vnormrms
Vnormrms = zeros(M,1);
% Normalize V by V_rms
for i=1:M
    Vnormrms(i,1) = V(i,1)/V_rms;
end
```

Task 1.3 - Convert $V_{normrms}$ to dB and Plot vs. Time

$$V_{normrms,dB} = 20\text{LOG}_{10}(V_{normrms})$$

```
%% Task 1.3
% Generate Empty Vector Vnormrms_db
Vnormrms_db = zeros(M,1);
% Convert to dB
for i=1:M
    Vnormrms_db(i,1) = 20*log10(Vnormrms(i,1));
end
% Plot Vnormrms_db against Time
figure(4);
plot(Data(:,1),Vnormrms_db(:,1),'r--');
title('RMS Normalized Voltage (dB) vs Time');
xlabel('Time: (sec)');
ylabel('V_(normrms,dB): (dB)');
```

The Following figure was plotted in response to the code above (Please see Figure 4):

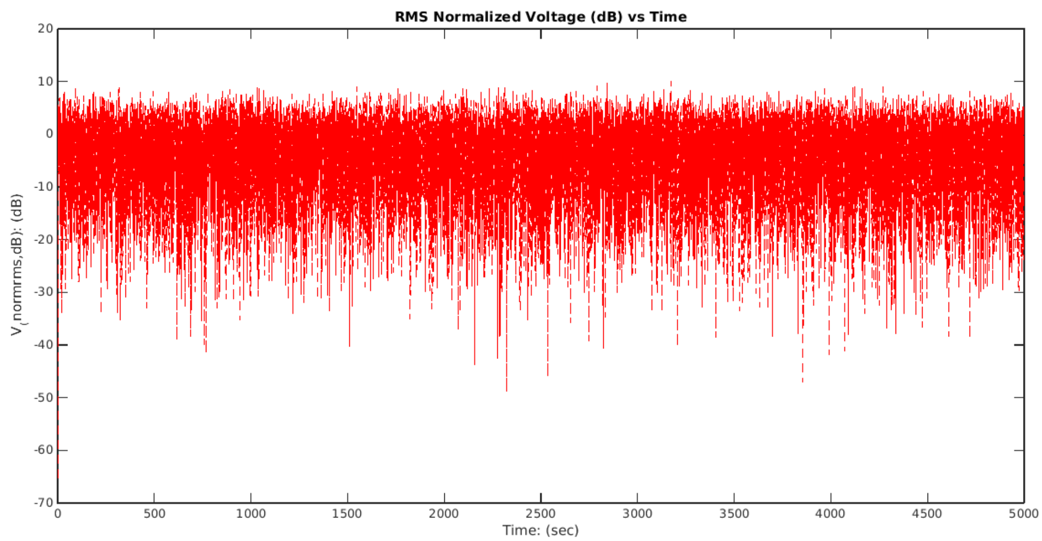

Figure 4: The RMS Normalized Voltage in dB plotted against Time.

Task 1.4 - Calculate the Mean and Variance of $V_{normrms}$

$$E[V_{normrms}] = \sigma\sqrt{\frac{\pi}{2}}$$

$$VAR[V_{normrms}] = E[V_{normrms}^2] - E[V_{normrms}]^2 = \frac{4-\pi}{2}\sigma^2$$

```
%% Task 1.4
% Calculate the Mean and Variance
uVnormrms = mean(Vnormrms);
var_Vnormrms = var(uVnormrms);
% Calculate the Ratio between Mean and Std Dev.
ratio_normrms = uVnormrms/sqrt(var_Vnormrms);
```

The Values calculated for the Mean and Variance of $V_{normrms}$ were:

$$E[V_{normrms}] = 0.8862$$

$$VAR[V_{normrms}] = 0.2161$$

$$\frac{E[V_{normrms}]}{VAR[V_{normrms}]} = 1.9066$$

Task 1.5 - Calculate $V^2_{normrms}$ and its Mean and Variance

$$E[V^2_{normrms}] = VAR[V_{normrms}] + E[V_{normrms}]^2 = 2\sigma^2$$
$$VAR[V^2_{normrms}] = 4\sigma^4$$

```
%% Task 1.5
% Generate empty Vector V2normrms
V2normrms = zeros(M,1);
% Calculate V2normrms
for i=1:M
    V2normrms(i,1) = Vnormrms(i,1)^2;
end
% Calculate the Mean and Variance of V2normrms
uV2normrms = mean(V2normrms);
var_V2normrms = var(V2normrms);
```

The Mean and Variance as found by the above code was:

$$E[V^2_{normrms}] = 1.0015$$

$$VAR[V^2_{normrms}] = 0.9964$$

Task 1.6 - Compute Ratio of Mean to Variance of $V^2_{normrms}$

```
%% Task 1.6
% Calculate the ratio between Mean and Variance of V2normrms
ratio_V2normrms = uV2normrms/sqrt(var_V2normrms);
```

The Ratio between the Mean and the Variance was found to be:

$$\frac{E[V^2_{normrms}]}{VAR[V^2_{normrms}]} = 1.0033$$

Task 1.7 - Calculate the measured PDF of $V^2_{normrms}$

```
%% Task 1.7
[RMSm_j, RMSC_j] = hist(V2normrms,K);
% Calculate Delta, assuming equal lengths
delta_V2normrms = RMSC_j(2)-RMSC_j(1);
% Create Empty PDF
measured_pdf_rms = zeros(K,1);
% Calculate Measured pdf
for i=1:K
    measured_pdf_rms(i,1) = RMSm_j(i)/(M*delta_V2normrms);
end
```

Task 1.8 - Calculate the Theoretical PDF of $V_{normrms}^2$

$$f_{2\sigma^2}(C_j) = \frac{1}{2\sigma^2}\text{EXP}\left(-\frac{C_j}{2\sigma^2}\right)$$

```
%% Task 1.8
% Create Empty PDF Container
theoretical_pdf_rms = zeros(K,1);
% Calculate Theoretical PDF
for i=1:K
    theoretical_pdf_rms(i,1) = (1/(uV2normrms))*exp(-RMSC_j(i,1)/(uV2normrms));
end
```

Task 1.9 - Plot the PDFs on the Same Graph

```
%% Task 1.9
% Plot Theoretical and Measured PDFs
figure(5);
plot(RMSC_j,measured_pdf_rms,'r--o', RMSC_j, theoretical_pdf_rms, 'g--o');
title('Measured and Theoretical PDFs of V_(normrms)^2');
xlabel('Centre of Interval Bin_j, C_j');
ylabel('Probability Density Function, f(C_j)');
```

The Following Figure was Plotted in response to the above mentioned code (Please see Figure 5):

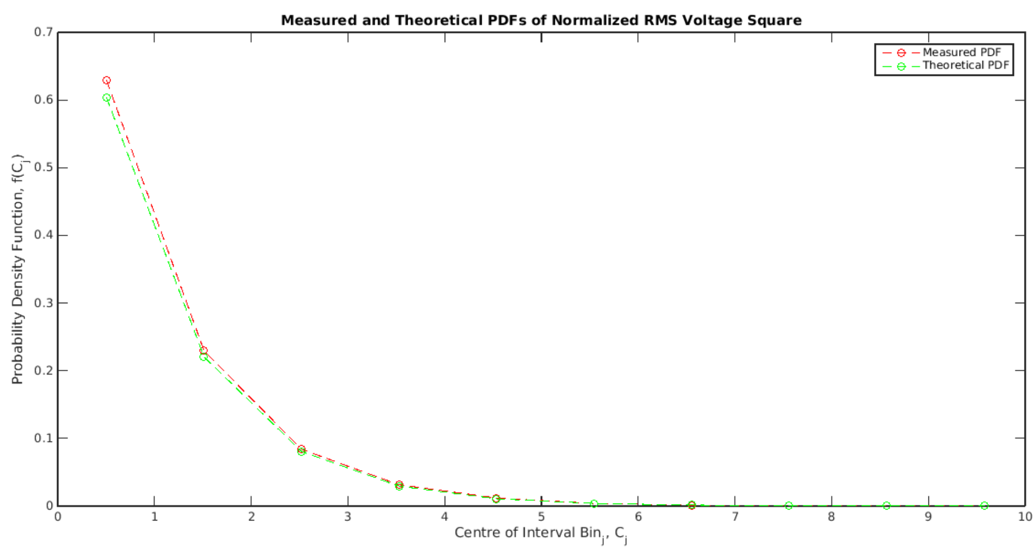

Figure 5: The Comparison between the Measured and Theoretical PDFs of the RMS Normalized Square Voltage.

Task 1.10 - Equation Proof

The following equation states the probability that a value will fall between the lower and upper boundaries.

$$p_j = \text{EXP}\left(-\frac{L_j^2}{2\sigma_{norm}^2}\right) - \text{EXP}\left(-\frac{U_j^2}{2\sigma_{norm}^2}\right)$$

1. Given that the Random Variable in question is Rayleigh Distributed, the PDF of the variable is:

$$f_A(a) = \frac{a}{\sigma_{norm}^2} \text{EXP}\left(-\frac{a^2}{2\sigma_{norm}^2}\right)$$

2. The CDF of this variable is then given:

$$F_A(a) = 1 - \text{EXP}\left(-\frac{a^2}{2\sigma_{norm}^2}\right)$$

3. Since, the CDF gives the probablity that a value is between $(0, a)$ it makes sense that the difference of cumulative probabilities at 2 points would be the cumulative probability of a value appearing between those points:

$$p_j = F_A(U_j) - F_A(L_j) = \left(1 - \text{EXP}\left(-\frac{U_j^2}{2\sigma_{norm}^2}\right)\right) - \left(1 - \text{EXP}\left(-\frac{L_j^2}{2\sigma_{norm}^2}\right)\right)$$

4. Thus we arrive at our initial equation:

$$p_j = \text{EXP}\left(-\frac{L_j^2}{2\sigma_{norm}^2}\right) - \text{EXP}\left(-\frac{U_j^2}{2\sigma_{norm}^2}\right)$$