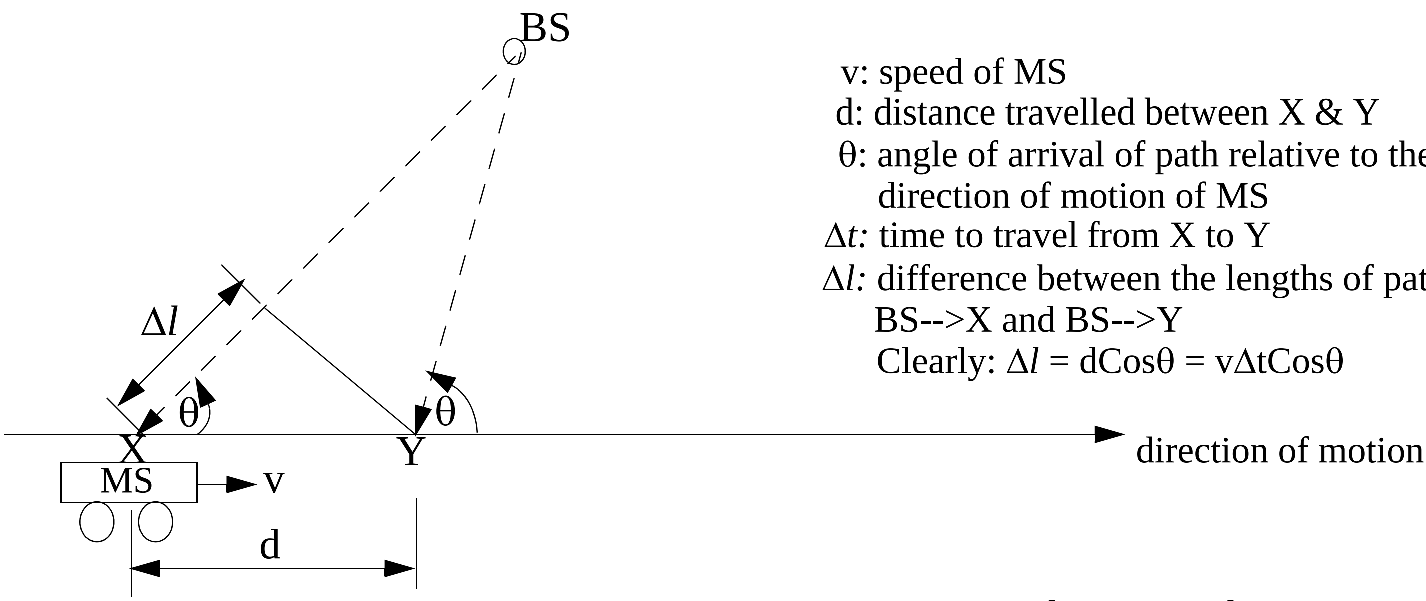
Frequency Dimension Characterization of Wireless Channels

Doppler Shift:

- shift in the operating (carrier) frequency due to relative motion between the transmitter and the receiver

Case 1: Doppler shift caused by motion in free space (i.e. only one path exists)



v: speed of MS
d: distance travelled between X & Y
θ: angle of arrival of path relative to the direction of motion of MS
Δ*t:* time to travel from X to Y
Δ*l:* difference between the lengths of paths BS-->X and BS-->Y

Clearly: $\Delta l = d\cos\theta = v\Delta t\cos\theta$

Assumption: BS is very far away from X and Y

Case 1: Analysis of Doppler Shift in Free Space

<u>Assumption:</u> BS is very far away from the MS, so that θ is approximately the same at points X and Y (on Page 51)

Let

$\Delta\phi$: phase change in the received signal due to path length change Δl

Now, $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta l = \frac{2\pi v \Delta t}{\lambda} Cos\theta$

Hence, the apparent change in frequency of the received signal (or Doppler Shift), f_d, is given by:

$$f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}Cos\theta = v\frac{f_o}{c}Cos\theta = f_{d_{max}}Cos\theta$$

where f_o is the operating frequency and $f_{d_{max}} = v\frac{f_o}{c} = \frac{v}{\lambda}$ = maximum Doppler shift

f_{in}, the instantaneous frequency of the received signal is then given by:

$$f_{in} = f_o + f_d = f_o + f_{d_{max}}Cos\theta \qquad \text{or} \qquad (f_o - f_{d_{max}}) \le f_{in} \le (f_o + f_{d_{max}})$$

Case 2: Analysis of Doppler Shift in a Multipath Channel

In a multipath channel, the transmitted signal arrives at the receiving antenna via

- a random number N paths, each with random amplitude a_i and random phase ϕ_i
- each path i arrives at a random angle θ_i, uniformly distributed between 0 and 2π

With relative motion, each path i experiences its own Doppler shift $f_{d,i} = f_{d_{max}} cos(\theta_i)$ (from Page 52)

Let $r(t) = \sum_{i=1}^{N} a_i exp[j(\phi_i + 2\pi f_{d,i} t)]$ = low-pass complex equivalent of received signal envelope

Let $R(\Delta t)$ = autocorrelation function of $r(t)$ and $S_d(f)$ = power spectrum

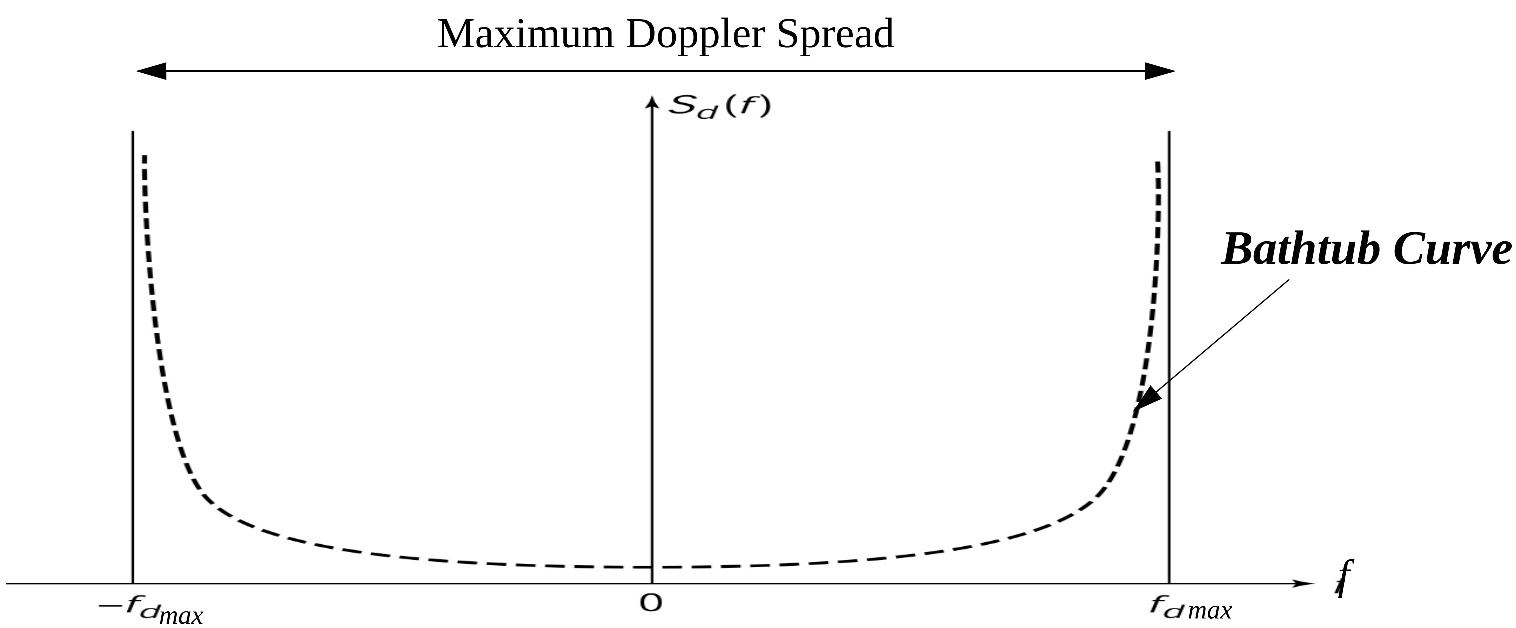
By definition: $R(\Delta t) = E[r(t)r^*(t + \Delta t)]$ and $S_d(f) = F\{R(\Delta t)\}$ = Doppler spectrum

where $r^*(t)$ is the complex conjugate of $r(t)$, $E[.]$ is the expectation operator, and $F\{\}$ is the Fourier transform

<u>Conclusion:</u> Relative motion causes an impulse transmitted at a single frequency to spread out in frequency at the receiver and, in the presence of multipath, each path experiences a different Doppler shift. This phenomenon is referred to as <u>Doppler Spread</u> or <u>Frequency Dispersion</u>

Example Doppler Spectrum: Jake's Spectrum

$$S_d(f) = \frac{s_o}{\pi f_{d_{max}} \sqrt{1 - \left(\frac{f}{f_{d_{max}}}\right)^2}}, \quad |f \leq f_{d_{max}}| \quad s_o = \text{a constant}$$



<u>*Notes:*</u> 1. Doppler spectrum $S_d(f)$ = power spectrum of the received signal at different frequencies

2. Maximum Doppler Spread = $2f_{d_{max}}$, where $f_{d_{max}}$ = maximum Doppler shift

Frequency-Dimension Metric: Doppler Spread or Frequency Dispersion

Doppler Spread:

- defined as the range of frequency over which the channel transfer function *H(f)* is spread due to relative motion between the transmitter and the receiver
- occurs in the frequency domain
- Mathematically, Doppler Spread $\zeta = 2f_d$

<u>*Statistic:*</u> maximum Doppler Spread $\zeta_{max} = 2f_{d_{max}}$

<u>*Effect of Doppler Spread:*</u> Frequency distortion (a.k.a "random" FM)

Physical Manifestation of Doppler Spread in the Time Domain: Coherence Time

Coherence Time, T_c: characterizes frequency-dispersiveness of the channel in the *time domain*

- T_c is defined as the period of time over which the impulse response *h(t)* of the channel is approximately the same

$$h(t) \approx h(t+\Delta t), \quad |\Delta t| \le T_c$$

$$h(t) \ne h(t+\Delta t), \quad |\Delta t| > T_c$$

- Alternatively, T_c is the period of time over which the channel amplitudes are correlated
- T_c is a derived quantity from the max Doppler spread (T_c is inversely proportional to ζ_{max})

NOTE: $T_c \ne \dfrac{1}{B_c}$

Example Relationships between T_c and ζ_{max}:

$$T_c \approx \frac{9}{16\pi\zeta_{max}}$$ = time interval for which the envelope correlation function > 50%

$$T_c \approx \left(\frac{9}{16\pi\zeta_{max}^2}\right)^{\frac{1}{2}} = \frac{0.423}{\zeta_{max}}$$ = geometric mean of $\{T_c = 1/\zeta_{max}\}$ and $\{T_c = 9/(16\pi\zeta_{max})\}$

Physical Manifestation of Doppler Spread in the Time Domain: Time Selective vs. Time Flat Channel

Time Selectivity of the Channel: occurs at non-zero Doppler spread (i.e. $0 < \zeta_{max} < \infty$) ==> $T_c < \infty$

- different components of the impulse response experience different attenuation (i.e. channel is time-varying)
- the magnitude of the impulse response at different times are uncorrelated

Time Flatness of the Channel: occurs at zero Doppler spread (i.e. $\zeta_{max} = 0$) ==> $T_c = \infty$

- different components of the impulse response experience the same attenuation (i.e. channel is time-invariant or time-flat)
- the magnitude of the impulse response at different times are correlated

<u>Notes:</u>

1) zero Doppler spread means the absence of relative motion (e.g. MS is stationary)
2) For a given value of $T_c < \infty$, time-selective channels can be classified into slow and fast channels, depending on the relative value of symbol duration T_S.
3) A time-selective channel is *<u>slow</u>* when the symbol duration T_S is much less than T_C the coherence time. I.e. $T_s \ll T_c$. Conversely, the channel is *<u>fast</u>* when $T_s \gg T_c$.

Summary of Channel Types in both Time and Frequency Dimensions

<u>*Time Dimension Model:*</u> Zero delay spread vs. non-zero delay spread (i.e. no multipath vs. multipath)

<u>Channel types in freq domain:</u> Frequency-flat vs. Frequency-selective channels

<u>*Freq. Dimension Model:*</u> zero Doppler spread vs. non-zero Doppler spread (i.e. no motion vs. motion)

<u>Channel types in time domain:</u> Time-flat vs. Time-selective channels

	Flat	Selective
Frequency	**Frequency-Flat** - Zero delay spread - No multipath - Channel constant w/ freq	**Frequency-Selective** - Non-zero delay spread - Multipath exists - Channel changing with freq @ rate B_c
Time	**Time-Flat** - Zero Doppler spread - No motion - Channel constant w/ time	**Time-Selective** - Non-zero Doppler spread - Motion exists - Channel changing with time @ rate T_c

Summary of Channel Models in Distance, Time and Frequency Dimensions & User Impacts

Dimension	Channel Model	User Impact
Distance	Path loss	Quality
	Lognormal shadowing	Quality
	Multipath (Rayleigh & Rician)	Quality
Time	Zero delay spread vs. non-zero delay spread (Channel types: Frequency-flat vs. Frequency-selective)	Data rate & quality
Frequency	Zero Doppler spread vs. Non-zero Doppler spread (Channel types: Time-flat vs. Time-selective)	Data rate & quality

Class Example

Problem Statement:

Consider an antenna transmitting at 900 MHz. The receiver, an MS, is travelling at a speed of 30 km/hr and is receiving/transmitting data at 200 kbps. Examine whether the channel fading is slow or fast. Assume that the coherence time, $T_c = 9/(16\pi\zeta_{max})$

Simulation of Rayleigh Fading Signal Case 1: MS is stationary

<u>Inputs:</u>

- Operating frequency, f_o
- Average Power of the received signal, $E[P] = 2\sigma^2$ (from Page 40)

Implementation:

Step 1: Calculate the sampling rate, $f_s = M * f_o$, (typically M = 4). Hence, sampling interval, $\Delta t = 1/f_s$

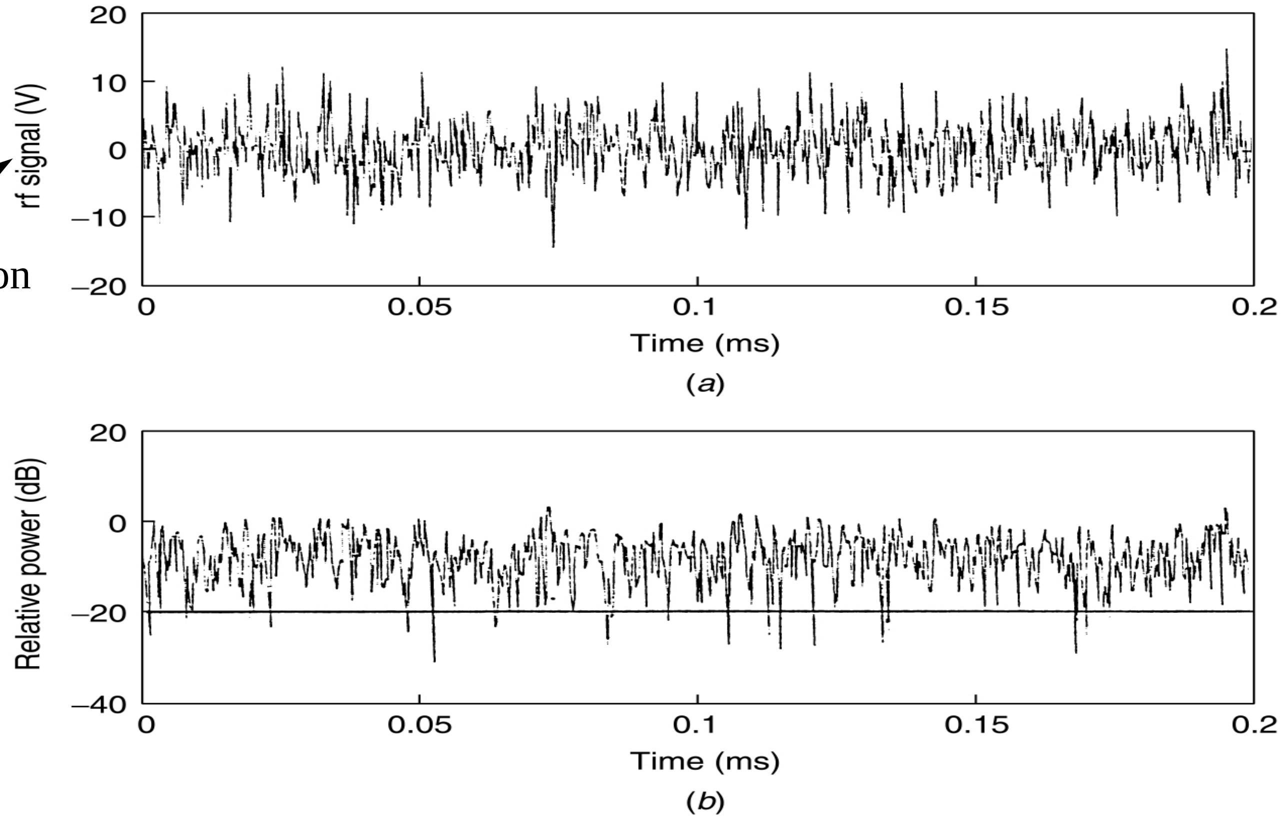
Step 2: At each time instant, generate two Gaussian random variables X and Y, where X ~ Gaussian(0, σ^2) and Y ~ Gaussian(0, σ^2) (Step 2 follows from 3rd bullet on page 37)

Step 3: Calculate the envelope $A = \sqrt{X^2 + Y^2}$ (Step 3 follows from 4th bullet on page 37)

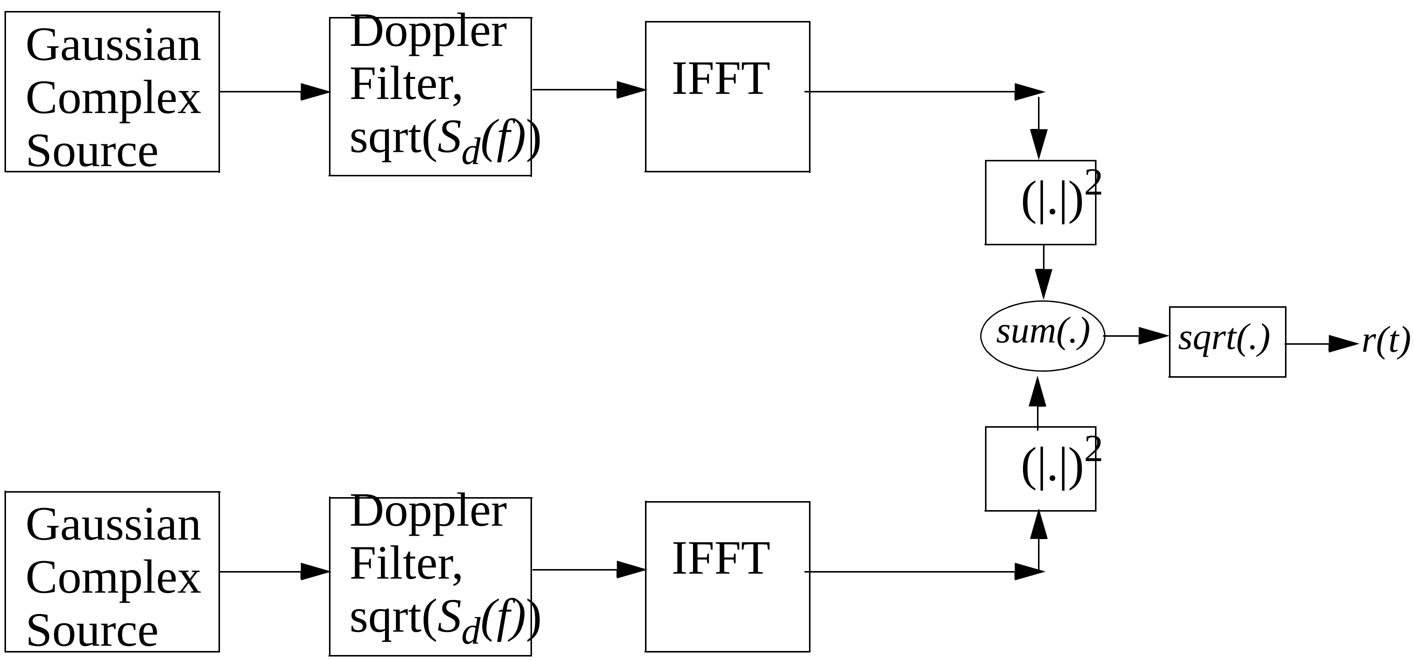
Step 4: Calculate the power $P = A^2$ (Step 4 follows from the definition of power on page 40)

Illustrative Rayleigh faded Signal

Before
Demodulation
(1st bullet,
pg. 37)



Simulation of Rayleigh fading signal, Case 2: MS is in motion (i.e., with Doppler)



IFFT: Inverse Fast Fourier Transform
$S_d(f)$: Doppler (power) spectrum

Simulation of Rayleigh fading signal: MS is in motion

Inputs:

- Operating frequency f_o and Speed of motion v. (Hence can calculate $f_{d_{max}} = (v f_o)/c$)
- Doppler Spectrum, $S_d(f)$ (see page 54 for an example $S_d(f)$)
- Number of sampling points in the frequency domain, M_f (M_f is usually a power of 2)

Implementation:

Step 1: Generate two sets (in-phase and quadrature) of complex Gaussian random variables for each sampling interval Δf (= $f_{d_{max}} / M_f$) in the frequency domain

Step 2: Generate shaped filter ($\sqrt{S_d(f)}$) samples at each Δf

Step 3: Multiply the in-phase and quadrature noise sources by the fading spectrum $\sqrt{S_d(f)}$

Step 4: Perform an Inverse Fast Fourier Transform (IFFT) on the resulting freq domain signals

Step 5: Add the squares of each signal point in time

Step 6: Take the square root of the signal obtained in step 5 to calculate the Rayleigh distributed envelope