The University of Calgary
Department of Electrical & Computer Engineering

ENEL 529 Wireless Communications Systems
Lab 4 – Digital Modulation and Demodulation Schemes for Wireless Systems

Lab Day & Date: Monday, November 3, 2014

Lab Report due date: Wednesday, November 13, 2014 @ 12:00 noon

1. Objectives

The objectives of this Lab are to:

- understand the effect of imperfections in the demodulation process on the transmitted data
- study the performance of MODulation and DEModulation (MODEM) techniques in additive white Gaussian noise (AWGN) and Rayleigh fading channels.

2. Overview

2.1 Transmitted Signal over AWGN Channel with Coherent BPSK Receiver

The probability of error for coherent BPSK is given by [1]:

$$P_{BPSK}(e) = \frac{1}{2} erfc\left(\sqrt{\frac{E}{N_o}}\right) \tag{1}$$

Coherent receiver assumes perfect knowledge of carrier information: carrier frequency and phase. However, in a practical system, estimation of the carrier information can be erroneous due to non-idealities of receiver components. With imperfect carrier phase recovery of constant error $\Delta\phi$, the probability of error becomes [1]:

$$P_{BPSK}(e) = \frac{1}{2} erfc\left(\sqrt{\frac{E}{N_o}}\cos(\Delta\phi)\right) \tag{2}$$

With imperfect carrier phase recovery of random error, assumed to be Gaussian with zero mean and standard deviation σ_ϕ radians, the average probability of error is [1]:

$$P_{av,BPSK} = \frac{1}{2\sqrt{2\pi}\sigma_\phi}\int_{-\infty}^{\infty} erfc\left(\sqrt{\frac{E}{N_o}}\cos(\Delta\phi)\right)\exp\left(-\frac{\Delta\phi^2}{2\sigma_\phi^2}\right)d\Delta\phi \tag{3}$$

3. Procedure

Case 1: BER Simulation for BPSK over AWGN Channel with Perfect Coherent Receiver

Step 0. Preambles:

Step 0.1: A bit stream consisting of 1000 bits {b_1, b_2, …, b_{999}, b_{1000}} where $b_i \in \{-1,1\}$ is to be transmitted using the BPSK modulation scheme over AWGN channel and coherent receiver with perfect carrier phase estimation.

Step 0.2: Assume that each data bit is of duration T_b seconds. The data waveform for bit b_i, $d_i(t)$ = b_i, $(i-1)T_b \le t \le iT_b$, i = 1, 2, …, 1000. Take T_b = 0.125 milliseconds. Note that, for BPSK modulation scheme, the duration of each channel symbol T is equal to the bit duration, i.e., $T = T_b$.

Step 0.3: Select f_c = 10 *KHz*. Let N denote the number of samples in one bit interval. Assume N = 15 samples so that the sampling interval, $\Delta t = T_b / N$ = 8.333 microseconds.

Step 1: Generate 1000 random data bits where bit i $b_i \in \{-1,1\}$. A sample MATLAB code for generating the bit pattern is given as follows:

```
binary_data = rand(1, 1000) > 0.5; % generates 1000 bits each having value 0 or 1
b  =  (2.*binary_data) – 1; % converts binary data 0 and 1 to –1 and 1 digital data, respectively
```

Step 2: For the generated data bit stream in step 1, generate the BPSK transmitted waveform:

$$s_i(n\Delta t) = b_i * \sqrt{\frac{2E}{T}} \cos(2\pi f_c n\Delta t), \quad b_i \in \{+1, \ -1\} \tag{4}$$

where $s_i(n\Delta t)$ is the BPSK modulated waveform for symbol i, and n is the time index where n = 1, …, 1000N. Take the energy per symbol, $E = T/2$. Values of N, f_c and Δt are as given or calculated in steps 0.2 and 0.3. Keep track of the time element, i.e., time interval for bit b_1 is: $0 \le t \le T$ (n = 1, …, 15), …, and time interval for bit b_{1000} is: $999T \le t \le 1000T$ (n = 14986, …, 15000).

Step 3: At each time index n (where n = 1, 2, 3, …, 1000N), generate the received signal waveform at the input of the receiver:

$$r(n\Delta t) \ = \ s(n\Delta t) \ + \ AWGN(n\Delta t) \tag{5}$$

where *AWGN(n∆t)* is the additive white Gaussian noise variate at time index n. *AWGN(n∆t)* is generated using the MATLAB function *normrnd(mean, standard deviation)*. Assume that: *mean* = 0 and *standard deviation* = 0.1. (*Note:* A standard deviation of 0.1 implies a noise variance of 0.01).

Step 4: Normalize the received signal vector r (calculated using eqn. (5)) by the maximum of its absolute value to obtain the normalized received waveform r_N:

$$r_N = r / \max(abs(r)) \tag{6}$$

Step 5: Perform coherent demodulation of the normalized received waveform, $r_N(.)$, by multiplying it with the carrier wave:

$$x(n\Delta t) = r_N(n\Delta t) * (2 * \cos(2\pi f_c n\Delta t)) \tag{7}$$

Step 6: Perform low-pass filtering, using MATLAB functions *butter(ω, κ)* and *filtfilt(α, β, γ)*, where *ω, κ, α, β, and γ* are the parameters of the filter functions. Enter the following two lines in your MATLAB code after the coherent demodulation step:

```
[bb  aa] = butter(10, 0.2); % calculate the filter coefficients bb and aa
filter_output = filtfilt(bb, aa, x);   % perform filtering of the sequence x(.)
```

generated in Step 5 above.

Step 7: Perform detection. In this experiment, a decision about the i^{th} transmitted symbol is made by sampling at the middle of symbol *i*. A sample MATLAB code to sample at the middle of symbol *i* and make a decision on the detected symbol is as follows:

```
for i = 1 : 1000
      k = ⌈N/2 + (i-1)N⌉;    % sample at middle of symbol i, denoted by index k
      if filter_output(k) > 0
   recovered_bit(i) = 1;
else
   recovered_bit(i) = -1;
      end;
end;
```

Step 8: Compare the transmitted bit stream $\{b_i\}$ with the recovered bit stream {*recovered_bit(i)*}, *i* = 1, 2, 3, …, 1000. Calculate the number of bit errors: a transmitted bit 1 (or –1) is corrupted if the recovered bit is –1 (or 1). Calculate the bit error rate (BER) using the formula:

$$\text{BER} = \text{number of incorrectly recovered bits} / 1000 \tag{8}$$

Note: Keep a record of the calculated BER for the assumed AWGN variance (i.e., standard deviation squared). (4 points)

Step 9: Repeat Steps 3 to 8, assuming AWGN variance (Step 3) is set to 0.25, 1.0, and 5.0.

Step 10: *Case 2: Effect of imperfect coherent receiver*: Suppose there exists a *constant* phase error during the carrier recovery process. To study the effect of this type of imperfection, repeat Steps 3 to 8 but with one modification: replace equation (7) for perfect coherent demodulation by the formula:

$$x(n\Delta t) = r_N(n\Delta t) * \left(2 * \cos(2\pi f_c n\Delta t + \Delta\phi)\right) \tag{9}$$

where $\Delta\phi$ is the *constant* phase error in the carrier recovery process. Assuming AWGN variance (Step 3) is set to 1.0, calculate the BER for $\Delta\phi$ = 5°, 30° and 85°.

<u>Note:</u> Keep a record of the calculated BER for the assumed constant phase errors. (4 points)

<u>Step 11:</u> *<u>Case 3: Effect of imperfect coherent receiver</u>*: Suppose there exists a *random* phase error during the carrier recovery process. To study the effect of this type of imperfection, repeat Steps 3 to 8 but with one modification: replace equation (7) for perfect coherent demodulation by the formula:

$$x(n\Delta t) = r_N(n\Delta t) * \left(2 * \cos(2\pi f_c n\Delta t + \Delta\phi(n\Delta t))\right) \qquad (10)$$

where $\Delta\phi(n\Delta t)$ is the *random* phase error in the carrier recovery process. Assuming AWGN variance (Step 3) is set to 1.0, calculate the BER if $\Delta\phi(n\Delta t)$ is Gaussian with (mean = 0°, standard deviation = 0.0°), (mean = 0°, standard deviation = 10°), (mean = 0°, standard deviation = 20°), and (mean = 0°, standard deviation = 30°).

<u>Note:</u> Keep a record of the calculated BER for the assumed Gaussian distributed random errors with the specified parameters (i.e., mean and standard deviation) given above. (4 points)

<u>Step 12:</u> *<u>Case 4: Effect of Rayleigh fading channel (perfect coherent detection):</u>* Suppose the transmitted bits are sent over a Rayleigh fading channel. To study the effect of Rayleigh fading, repeat Steps 3 to 8 but with one modification: replace equation (5) by the formula

$$r(n\Delta t) = \left(rayleigh\_variate(n\Delta t) * b_i * \cos(2\pi f_c n\Delta t + \theta(n\Delta t))\right) \; + \; AWGN(n\Delta t) \qquad (11)$$

where, at time index *n*, $rayleigh\_variate(n\Delta t)$ is generated by calling the MATLAB built-in function *raylrnd(parm), parm* is the parameter of the *raylrnd* function, and $\theta(n\Delta t)$ is the phase of the Rayleigh faded path at time index *n.* Assume that $\theta(n\Delta t)$ is random and uniformly distributed over [0, 2π]. A sample MATLAB code to generate $rayleigh\_variate(n\Delta t)$ is as follows:

$$rayleigh\_variate(n\Delta t) = raylrnd(1);$$

Calculate the BER over a (Rayleigh + AWGN) channel, assuming AWGN variance (Step 3) is set to 0.01, 0.25, 1.0 and 5.0.

<u>Note:</u> Keep a record of the calculated BER for the assumed AWGN variance. (4 points)

<u>Questions on Cases 1 to 4:</u>

1.1 In Case 1, what is the effect of the AWGN variance on the simulated bit error rate? (2 points)
1.2 In Case 2, what is the effect of constant phase error on the bit error rate? (2 points)
1.3 In Case 3, what is the effect of random phase error on the bit error rate? (2 points)
1.4 In Case 4, what is the effect of Rayleigh fading on bit error rate? (2 points)

1.5 Comment on the relative impacts of Rayleigh fading and imperfect coherent receiver on bit error rate performance. (2 points)

4. Further Readings

[1]. P.M. Shankar, Introduction to Wireless Systems. Chapter 3: Sections 3.3, Chapter 5, sections 5.4 and 5.5.

5. Lab Report

Prepare a Lab Report (one per group) using ENEL529 Lab Reporting Guidelines.