The University of Calgary
Department of Electrical & Computer Engineering

ENEL 529 Wireless Communications Systems
Lab 2 – Characteristics of Rayleigh Fading Channel – Part I

Lab Day & Date: Monday, October 20, 2014

Lab Report due date: Monday, October 27, 2014 @ 12:00 noon.

1. Objectives

The objectives of this Lab are to study and understand:

- how well the probability density function of measured fading channel data fits to the Rayleigh distribution
- the statistical measures of the Rayleigh fading channel and their inter-relationships

2. Pre-Requisites

This Lab assumes that you have completed Lab 1.

3. Overview

The power in the received signal over a non-line-of-sight (NLOS) fast fading wireless channel is measured in an urban environment and stored in file "*series11.mat*" (referred to as measured data, stored in Lab 2 folder on Desire2Learn). The data in file "*series11.mat*" is arranged in two columns: the first column is the time axis in seconds and the second column is the measured power in the received signal (after demodulation) in dBm. Recall that dBm is the unit of power in dB relative to 1 milliWatt.

It is generally assumed that signal variations caused by multipath can be modeled by a Rayleigh distribution when the received signal (i.e. magnitude of the complex envelope) is expressed in units of voltage. The probability density function (pdf) of the Rayleigh distribution is given by

$$f(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right), \qquad r \geq 0 \tag{1}$$

where r is a voltage which actually represents $|r|$, the magnitude of the complex envelope. The magnitude operator is dropped in (1) for notational simplicity. The Rayleigh distribution has a single parameter denoted by σ, the mode or modal value of the distribution. In this Lab you will analyze the measured data (stored in file "*series11.mat*") by computing its probability density function and determine whether or not the data fits with a Rayleigh distribution.

4. Procedure

Step 1: Plot the received power P (in dBm) vs. time in seconds (see Fig. 1) (1 pt). What conclusion can you draw from the plot? I.e., what type of fading is exhibited in Fig. 1? (1 pt)

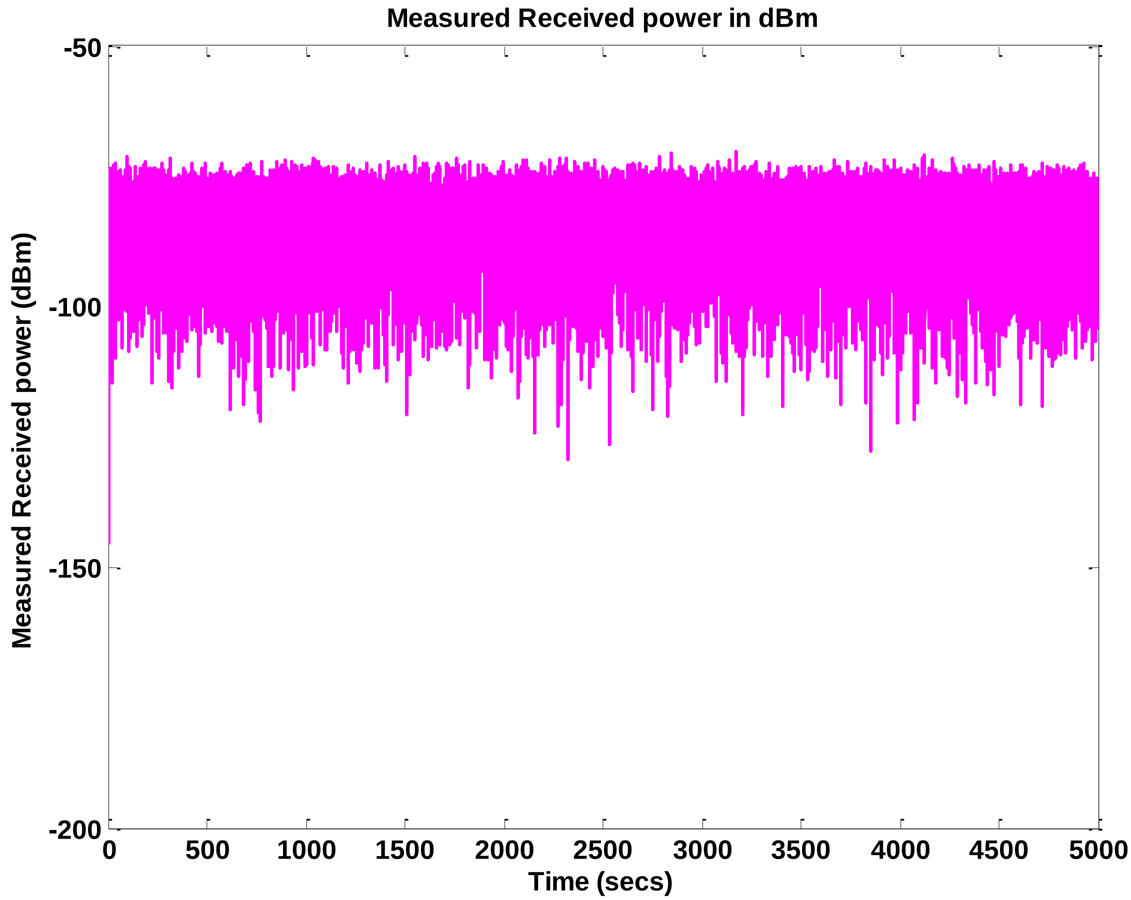

Fig. 1. Measured received power (in dBm) v. time (in seconds).

Step 2: Convert the measured power values (in dBm) in file "*series11.mat*" to normalized voltage values. To achieve this, proceed as follows:

i) Extract the measured power values (i.e. 2nd column of the file "*series11.mat*") to form a vector consisting of M = 100,000 elements.

ii) Convert each power value (in dBm) to a voltage value v in volts. ***Hint:*** The relationship between voltage v (in volts) and power p (in Watts) is: $v = \sqrt{2Rp}$ where R is the load resistance. Assume that $R = 50\ \Omega$ Denote the new vector of voltage values by V.

iii) Calculate $\overline{V}$ the mean of the M = 100,000 voltage values. (1 pt)

iv) Assuming that the values in V are Rayleigh distributed, estimate its parameter σ, the mode of the postulated Rayleigh distribution. (1 pt) ***Hint:*** For the Rayleigh distribution, the mode σ can be determined from knowledge of its mean $\overline{V}$ by the relationship: $\overline{V} = \sigma\sqrt{\pi/2}$.

v) Normalize the vector V by σ, denote the normalized voltage vector by V_{norm}.

vi) Plot the normalized voltage values vs. time (see Fig. 2) (1 pt). What conclusions can you draw from Fig. 2? (1 pt) Calculate $\overline{V}_{norm}$, the mean of V_{norm}. (1 pt) Also assuming that V_{norm} is Rayleigh distributed and, with knowledge of its mean, estimate the parameter σ_{norm}. (1 pt)

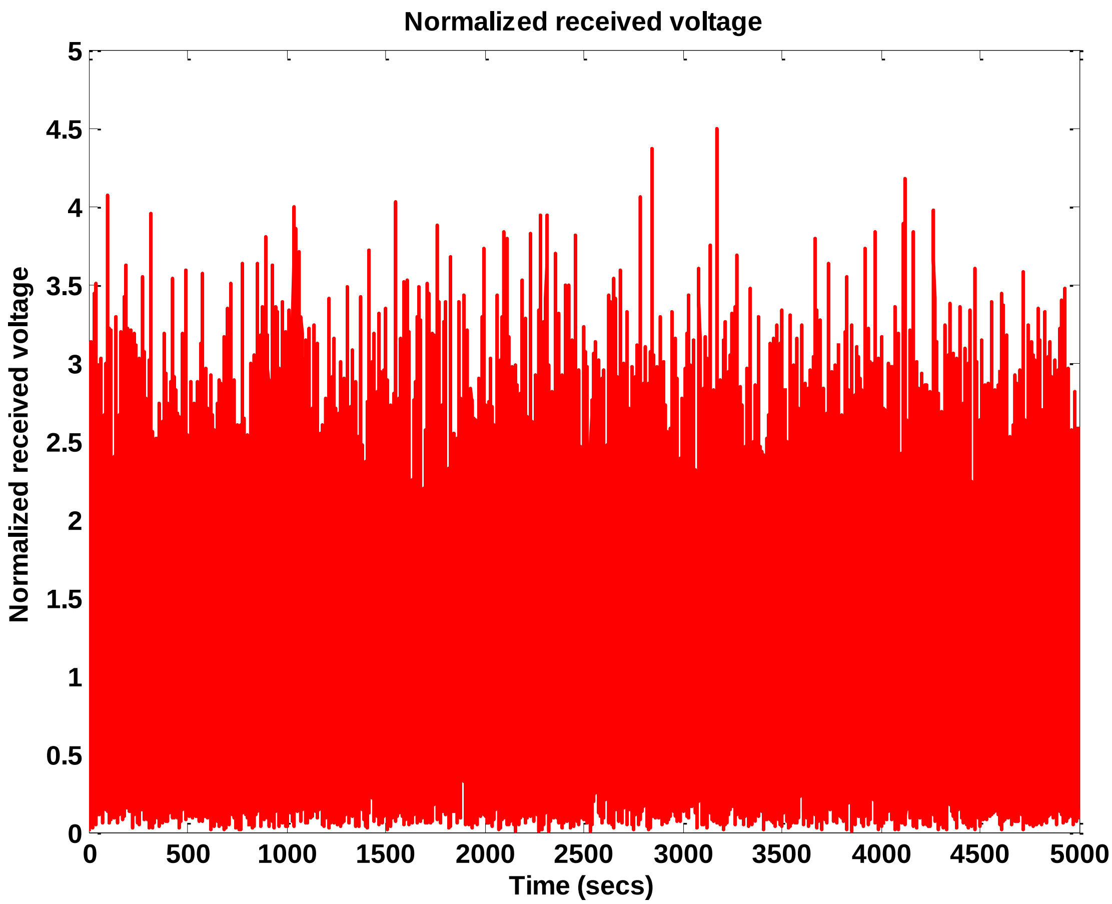

Fig. 2. Normalized received voltage.

__Step 3:__ Compute the probability density function (pdf) of the normalized voltage values in vector V_{norm}. This pdf is referred to as the *measured* pdf because vector V_{norm} is derived from the given measured power values (see Step 2 above). The calculation of the measured pdf proceeds as follows:

i) Divide the normalized voltage values (vector V_{norm}) into K disjoint intervals (i.e. K bins). Assume K = 10. ***__Hint:__*** Use the MATLAB built-in "hist()" function. ***__Note:__*** The outputs of calling the "hist()" function are: i) the actual number of samples in each bin, and ii) the center of each bin. Mathematically, denote the actual number of samples in bin *j* by m_j, j = 1, 2, 3, …, K. Similarly, let C_j denote the center of bin *j*, j = 1, 2, 3, …, K.

ii) Calculate Δ, the length of each bin. Assume equal length for all the K bins so that $\Delta = C_2 - C_1 = C_3 - C_2 = \cdots = C_K - C_{K-1}$.

iii) Let $measured\_pdf\,(C_j)$ denote the value of measured pdf at C_j, calculated by:

$$measured\_pdf\,(C_j) = \frac{m_j}{M \text{ x } \Delta} \tag{2}$$

where j = 1, 2, 3, …, K and M is the number of samples in vector V_{norm}.

<u>*Step 4:*</u> Calculate the *theoretical* Rayleigh pdf with parameter σ_{norm} (determined in Step 2) using the formula:

$$theoretical\_pdf\,(C_j) = \frac{C_j}{\sigma_{norm}^2} \exp\left(-\frac{C_j^2}{2\sigma_{norm}^2}\right) \tag{3}$$

where j = 1, 2, 3, …, K.

<u>*Step 5:*</u> Conduct 2 tests to check the fitness of the *measured* pdf (result of Step 3) to the Rayleigh distribution:

<u>*Test 1:*</u> Visual Comparison or Subjective Test.

Plot on the same graph the *theoretical* Rayleigh pdf (i.e. result of Step 4) and the *measured* pdf (i.e. result of Step 3), as illustrated in Fig. 3 (1 pt). How well does the *measured* pdf compare with the *theoretical* Rayleigh pdf? (1 pt)

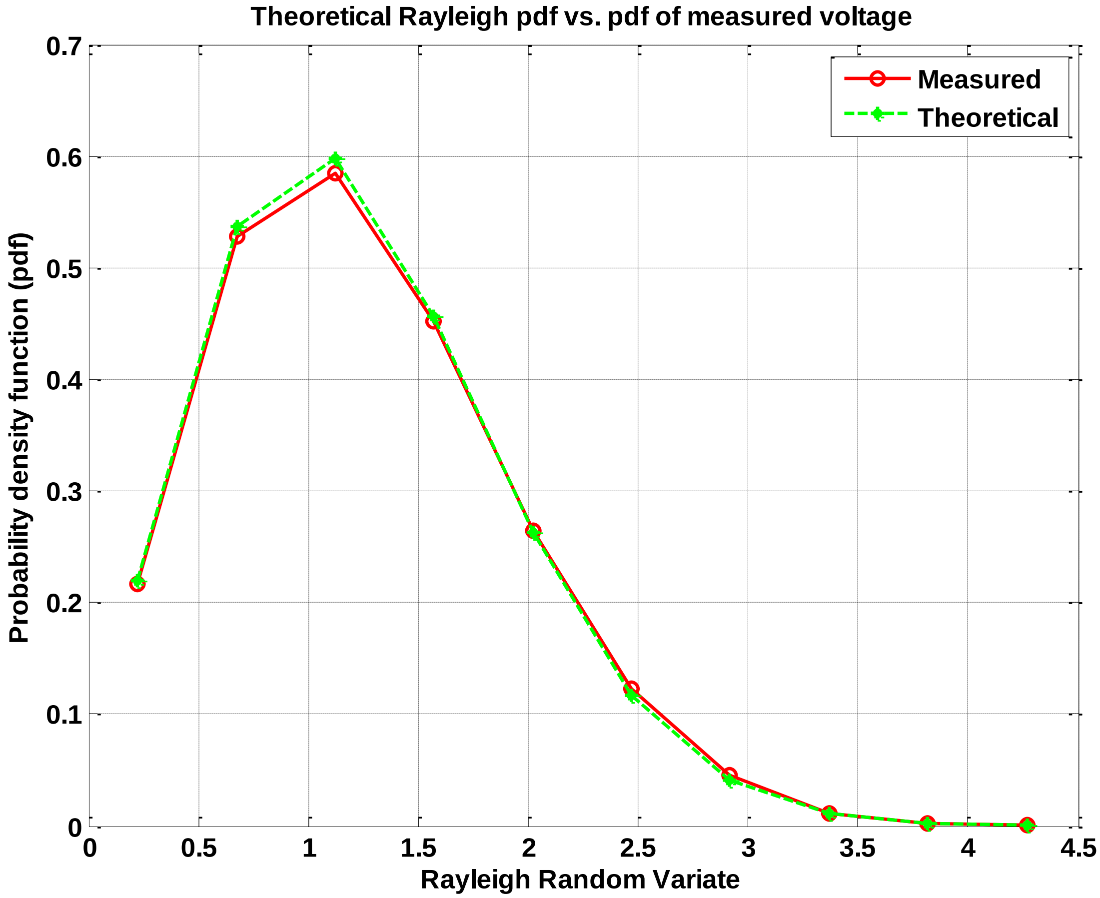

Fig. 3. Measured pdf vs. theoretical Rayleigh pdf.

<u>Test 2:</u> Quantitative test, i..e. quantify how good the fit is – this is a confirmatory test.
To do so, conduct a Chi-square goodness-of-fit test of the normalized voltage values for 1% significance. The approach for calculating the Chi-square statistic proceeds as follows:

i) Calculate L_j, the lower limit of bin *j*. Mathematically, $L_j = C_j - (\Delta/2)$. Recall that C_j and Δ are determined in Step 3.

ii) Calculate U_j, the upper limit of bin *j*. Mathematically, $U_j = C_j + (\Delta/2)$.

iii) Calculate p_j, the theoretical probability that a normalized voltage value of V_{norm} falls in bin *j*, under the assumption that the normalized voltage values follow the Rayleigh distribution with parameter σ_{norm} (determined in Step 2). Let $f_R(r)$ denote the pdf of a Rayleigh distributed random variable *R* given by:

$$f_R(r) = \frac{r}{\sigma_{norm}^2} \exp\left(-\frac{r^2}{2\sigma_{norm}^2}\right) \tag{4}$$

By definition:

$$p_j = \Pr\{L_j < R < U_j\} = \int_{L_j}^{U_j} f_R(r)dr \qquad (5)$$

Substituting the expression for $f_R(r)$ (eqn. (4)) in (5) and integrating gives:

$$p_j = \exp\left(-\frac{L_j^2}{2\sigma_{norm}^2}\right) - \exp\left(-\frac{U_j^2}{2\sigma_{norm}^2}\right) \qquad (6)$$

iv) Finally calculate T_j, the theoretical expected number of samples that fall in bin *j*. Mathematically, $T_j = Mp_j$, j = 1, 2, …, K. Recall that *M* = 100,000 in this Lab.

v) Calculate the Chi-square statistic Z_1 using the formula [1] (2 pts):

$$Z_1 = \sum_{j=1}^{K} \frac{(m_j - T_j)^2}{T_j} \qquad (7)$$

vi) From the Chi-square tables [1] (pdf file is in Lab 2 folder), what is Z_T, the Chi-square threshold statistic corresponding to 1% significance level of the test? (1 pt) Is the Rayleigh fit to the normalized voltage values quantitatively good? (2 pts)

Additional Questions and Exercises

1.1 Calculate v_{rms}, the root mean square (rms) value of vector V. (See Note i) below.)

1.2 Normalize V by v_{rms}. Denote the normalized vector by $V_{norm.rms}$.

1.3 Convert $V_{norm.rms}$ to dB (see Note ii) below) and plot $V_{norm,rms,dB}$ vs. time in seconds. (3 pts)

Notes: i) Assuming the values in vector V is Rayleigh, its *rms* value denoted by v_{rms} is given by:

$$v_{rms} = \sqrt{2}\sigma = (2/\sqrt{\pi})\bar{V}.$$

ii) If the vector V is normalized by its rms value, the normalized voltage (in dB unit) is: $V_{norm,rms,dB} = 20\log_{10}(V/v_{rms})$.

1.4 Calculate $E[V_{norm.rms}]/\sqrt{Var[V_{norm,rms}]}$ where $E[V_{norm,rms}]$ and $Var[V_{norm,rms}]$ are the mean and variance of $V_{norm.rms}$, respectively. (2 pts)

1.5 Calculate $V_{norm,rms}^2$ (note that: magnitude squared = power). Next, calculate $E[V_{norm,rms}^2]$ and $Var[V_{norm,rms}^2]$, the mean and variance of $V_{norm,rms}^2$, respectively (2 pts).

1.6 Compute $E[V_{norm,rms}^2]\Big/\sqrt{Var[V_{norm,rms}^2]}$ (2 pts).

1.7 Calculate the pdf of $V_{norm,rms}^2$. This pdf is referred to as the *measured* pdf for $V_{norm,rms}^2$. ***Hint:*** Use the same approach as in Step 3.

1.8 Compute the *theoretical* exponential probability density function with parameter $E[V^2_{norm,rms}]$.

Hint: The pdf of an exponentially distributed random variable X with parameter $E[X]$ is: $f_X(x) = \frac{1}{E[X]} \exp\left(-\frac{x}{E[X]}\right)$.

1.9 Plot on the same graph the *theoretical* exponential probability density function (i.e. result of 1.8) and the *measured* pdf (i.e. result of 1.7). What conclusion can you draw about the *pdf* for $V^2_{norm,rms}$ (i.e. power of a Rayleigh distributed signal)? (3 pts)

1.10 Prove Equation (6). (3 pts)

5. Further Readings

[1]. P.M. Shankar, Introduction to Wireless Systems. Chapter 2: Sections 2.3.1 and 2.5.

6. Lab Report

Prepare a Lab Report (one per group) using ENEL529 Lab Reporting Guidelines.