ENEL 573 - Assignment #2

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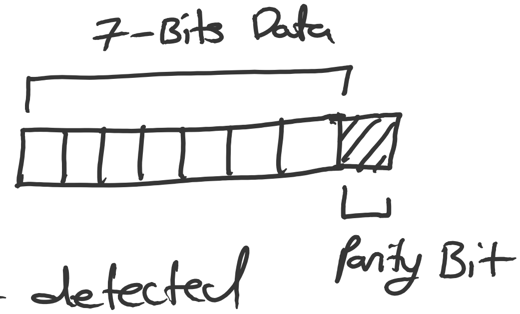
Q1:

7-Bit Packets:



- Errors that aren't detected are going to be even # of bits

$\rightarrow$ 2, 4, 6 Bit errors $\qquad \underline{P = 0.0001}$

$$P_+ = \binom{7}{2} p^2 (1-p)^5 + \binom{7}{4} p^4 (1-p)^3 + \binom{7}{6} p^6 (1-p)$$

$$= \binom{7}{2}(0.0001)^2(0.9999)^5 + \binom{7}{4}(0.0001)^4(0.9999)^3$$

$$+ \binom{7}{6}(0.0001)^6(0.9999)$$

$$P_+ = (21)(0.0001)^2(0.9999)^5 + (35)(0.0001)^4(0.9999)^3 +$$

$2.099 \times 10^{-7} +$

$3.499 \times 10^{-15} +$

$\ldots \times 10^{-24}$

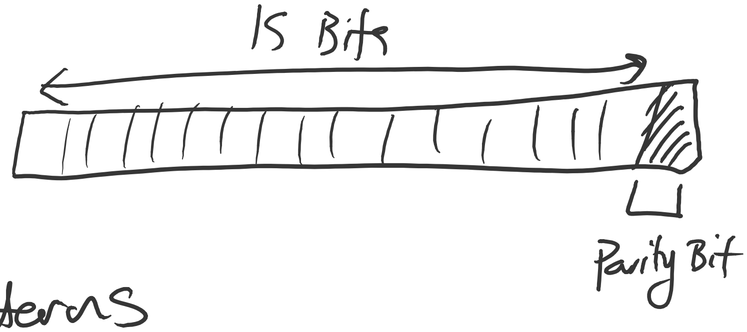
$$(35)(0.0001)(0.9999) +$$

$$(7)(0.0001)^6(0.9999)$$

$$6.999 \times 10^{-24}$$

$$= \boxed{2.099 \times 10^{-7}}$$

15-Bit Packet:



- only even bit patterns
  - → 2, 4, 6, 8, 10, 12, 14

$$P_T = \sum_{i=(2,4,6,8\ldots 14)} \binom{15}{i} p^i (1-p)^{15-i}$$

$$= (105)(0.0001)^2(0.9999)^{13} + (1365)(0.0001)^4(0.9999)^{11}$$

$$+ (5005)(0.0001)^6(0.9999)^9 + (6435)(0.0001)^8(0.9999)^7$$

$$+ (3003)(0.0001)^{10}(0.9999)^5 + (455)(0.0001)^{12}(0.9999)^3$$

$$+ (15)(0.0001)^{14}(0.9999)$$

$= 1.049\times10^{-6} + 1.363\times10^{-13} + 5.000\times10^{-21} + \ldots$

$= \boxed{1.049\times10^{-6}}$

The longer the packet, the worse the performance

↳ Albeit, most error is due to <u>two bits</u> being wrong out of the packet.

Q2:   89 − 103 → 1's complement

↳ $89_{10} \rightarrow 01011001_2 \rightarrow 01011001_{\text{1's comp}}$

$\begin{pmatrix} 64 \\ +16 \\ +8 \\ +1 \end{pmatrix}$

$-103 \rightarrow 1100111_2 \rightarrow 0011000_{\text{1's comp}}$

$\begin{pmatrix} 64 \\ +32 \\ +4 \\ +2 \\ +1 \end{pmatrix}$

$$\begin{array}{r} 01011001 \\ +\ 00011000 \\ \hline 01110001 \end{array}$$

1's comp

$\boxed{10001110_2 \rightarrow -14_{10}}$

Q3:

8-Bit checksum

a) 3-Bytes: 0x13 0xaa 0xf2

$$\begin{array}{lcr} 0x13 & \rightarrow & 0001\ 0011 \\ 0xaa & \rightarrow & 1010\ 1010 \\ 0xf2 & \rightarrow & 1111\ 0010 \\ \hline & & +\ 1010\ 1111 \end{array} + + \bmod 2^8$$

$\downarrow$

$x \rightarrow \text{checksum} = -x$

$\downarrow$

$\boxed{01010000}$ ← one's complement

b) 3-Bytes w/ Errors:

error detection

0x13	:	0001 0011
0xac	:	1010 1100
0xf0	:	1111 0000
checksum	:	0101 0000

$$+\ \underbrace{1111\ 1111}$$

$$\downarrow$$

$$x \rightarrow (-x)$$

$$\downarrow$$

$$\underbrace{0000\ 0000}$$

The check failed to catch the errors.

c) Based on the data it would suggest it cannot detect errors that are off by bits of even numbers on a bit-by-bit basis.

Q4:

$$\underbrace{0011}\,\underbrace{0101}\,\underbrace{1100}\,\underbrace{0101}$$

$$\boxed{x^{13} + x^{12} + x^{10} + x^{8} + x^{7} + x^{6} + x^{2} + 1}$$

The polynomial is of degree 13.

Q5:

$$P(x) = x^4 + x^2 + x$$

a)

$$g(x) = x^3 + x + 1$$

$$\begin{array}{lr} P(x): & 0001\ 0110 \\ g(x): & 0000\ 1011 \\ \hline & \boxed{0010\ 0001} \end{array} \quad +$$

b)

$$\begin{array}{r} 0001\ 0110 \\ 0000\ 1011 \\ \hline 0001\ 0110 \\ 0010\ 1100 \\ 1011\ 0000 \end{array} \quad \begin{array}{l} \times \\ \\ \\ + \\ + \\ \end{array}$$

$$\boxed{1111\ 0010}$$

c)

$$
\begin{array}{r|l}
 & \boxed{10} \rightarrow x \\
\hline
1011 & 10110 \\
 & 1011 \\
\hline
 & 0000
\end{array}
$$

$$x^4 + x^2 + x = x(x^3 + x + 1)$$

d) $q(x) = x \rightarrow 10$

$r(x) = 0 \rightarrow 00$

$$p(x) = g(x)q(x) + r(x)$$

$$= x(x^3 + x + 1) + 0$$

Q6:

a) 11-Bit Packet

(x^5+x^2+1)

```
                101110100010
100101 ) 11010101011 00000
         100101
          100001011
          100101000
          011000011
            10010100
            00101011
              100101
              000010000000
                   1010100
                   0101000
```

Remainder

← 10100

$\boxed{\text{checksum is } 10100}$

b)

```
  1101 0101 0110 0000
              + 1 0100
  ---------------------
  1101 0101 0111 0100
```

```
                 10111011
100101 ) 1101 0101 0111 0100
         1001 01
```

100101) 1101 0101 011...
1001 01
1 ~~1~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ 010111 0100
10 0101
01 1 ~~0~~ ~~0~~ ~~0~~ ~~0~~ 1110100
1 00101
0 1 1 0 0 0 1110100
100101
0 ~~1~~ ~~0~~ ~~0~~ ~~0~~ ~~0~~ 110100
1 00101
0 1 ~~1~~ ~~0~~ ~~1~~ ~~0~~ ~~0~~ 100
100101
00100 ~~1~~ ~~0~~ ~~1~~ ~~0~~ 0
100101
$\boxed{00000}$ ✓

c)

$$\frac{00010101010011010 0}{100101}$$

$\boxed{11010}$ ← Error detected

remainder