ENEL 573: Assignment 1

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Question 1

Assume that two nodes are transmitting over a noisy link. The probability of bit error is p. You can assume that the bit errors are independent from bit to bit. Data is organized into frames of length N.

a) In a 4 bit frame, write an expression in terms of p for the probability that the first and last bits are good but the middle two bits are bad.

b) In an N bit frame, write an expression in terms of p for the probability that there will be k bad bits somewhere in the frame. **Hint:** Remember to account for all the different patterns.

c) A *frame error* occurs when one or more bits are bad anywhere in a frame. Write an expression in terms of p for the probability of frame error in an N bit frame.

d) If $p = 0.0001$, calculate the probability of frame error for $N = 500$ bits and $N = 1000$ bits.

e) Most systems need to operate with a probability of frame error that is no higher than 0.01. If $p = 0.0001$, determine the maximum value of N that you can have and still meet this performance requirement.

Question 2

In 2009, an article made the following claim:

> At 487 billion gigabytes (GB), if the world's rapidly expanding digital content were printed and bound into books it would form a stack that would stretch from Earth to Pluto 10 times.

You can assume:

- Your home Internet connection is 8 Mbit/s.
- The average book contains 100,000 words.
- The average length of a word is 5 characters.

- Each character requires a byte of storage (ASCII encoding).
- The average thickness of a book is 3.4 cm.
- The smallest separation between Earth and Pluto (accounting for orbital variation) is 4.2 billion km.

a) How many exabytes is 487 billion gigabytes?

b) How many years would it take to download all this content on your home Internet connection?

c) Using the assumptions above, how high would the stack of books actually be? Does it make it to Pluto?

Question 3

Later on in class, we will look at schemes where frames with errors in them are retransmitted. Let P_F denote the probability of frame error that you derived in Question 1. Let p be the probability of bit error and N be the number of bits in the frame.

a) Find an equation for the average number retransmissions that are caused by frame errors. This equation should be expressed in terms of P_F. Remember that it's possible that a frame might have to be retransmitted more than just once. You'll need to use the identity

$$\sum_{k=-\infty}^{\infty} kp^k = p/(1-p)^2 \text{ for } |p| < 1$$

b) For $p = 0.0001$ and $N = 1000$, calculate the average number of retransmissions.

c) If you are sending the frames over a 1 Mbit/s link, what is the effective throughput experienced by the user once you take the average number of retransmissions into account.

Question 4

The following bit sequence represents a PPP packet encoded using the byte stuffing scheme described in class. Recover the header, payload and CRC fields from this pattern. Write your result in hexadecimal numbers.

`0x7e 0xa4 0x33 0x7d 0x5e 0x7d 0x5e 0x11 0x7d 0x5d 0x7e`

Question 5

Consider a 5 bit frame payload where 0's and 1's are equally likely and independent. A bit stuffing scheme is used where a 0 stuffing bit is added after 3 consecutive 1's. What's the probability that you need to use bit stuffing in the frame?