# The University of Calgary
# Department of Electrical & Computer Engineering

## ENEL 529 Wireless Communications Systems
## Lab 3 – Characteristics and Simulation of Rayleigh Fading Channel: Part II

Lab Day & Date: Monday, October 27, 2014

Lab Report due date: Monday, November 3, 2014 @ 12:00 noon

## 1. Objectives

The objectives of this Lab are to:

- simulate the Rayleigh fading channel using the inverse discrete Fourier transform (IDFT) approach, and
- calculate the outage probability due to the Rayleigh fading signal

## 2. Overview

The received signal over a non-line-of-sight (NLOS) wireless channel can be generated using either of two approaches: first, vector addition of multiple arriving paths [1] and second, inverse discrete Fourier transform (IDFT) of two independently filtered Gaussian signals [2]. In this Lab you will use the IDFT approach to develop a MATLAB-based simulator to generate multipath faded signals.

### 2.1 The Inverse Discrete Fourier Transform (IDFT) Approach

According to the central limit theorem, the in-phase $X(t)$ and quadrature $Y(t)$ components derived using the first approach (Topic 2 Lecture Notes) are approximately Gaussian random variables at any time $t$ if $N$ is sufficiently large. This is used as the starting point for the second approach, known as the inverse discrete Fourier transform (IDFT), whose block diagram is shown in Fig. 1 [2].

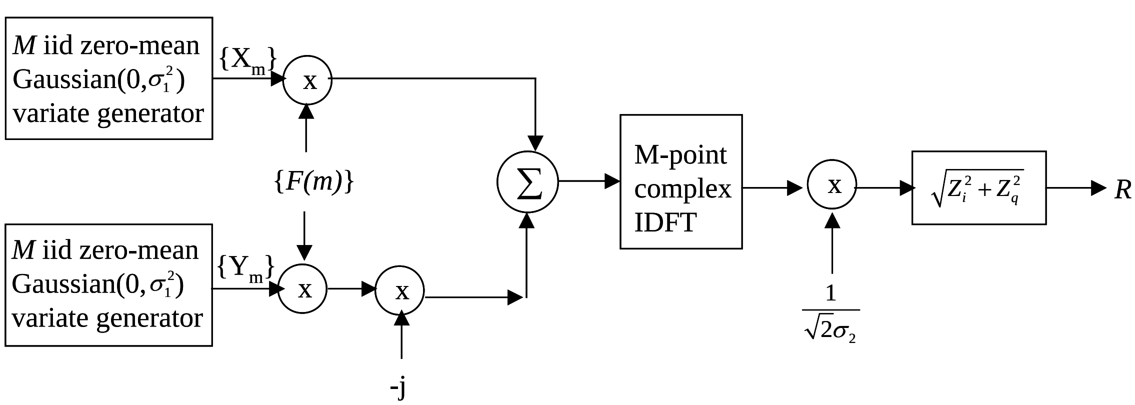


Fig. 1. Block diagram of the IDFT approach for simulating Rayleigh fading sequence

## 3. Procedure

*Step 1:* Select the system operating frequency $f_c$ = 900 *MHz*. Assume that sampling interval (i.e., time between 2 Rayleigh variates in the time domain), $T_s$ = 1 msec.. Set the vehicle speed, $V$ = *90 km/hr (25 m/sec)*, and set the variance of each zero-mean Gaussian generator, $\sigma_1^2$ = *1.0*.
*Step 2:* Calculate the normalized maximum Doppler frequency, $f_{d,norm} = f_{d_{\max}} T_s$ ( $f_{d_{\max}} = Vf_c/c$ , where $c$ = 3x10$^8$ m/sec, the velocity of light)
*Step 3:* Calculate the number of points in the frequency domain for which the shaping filter magnitude is nonzero: $k_m = \lfloor f_{d,norm} M \rfloor$, where $M$ is the number of points in the time domain and $\lfloor x \rfloor$ = *floor* of $x$ is the largest integer smaller than or equal to $x$. Take $M$ = 1000.
*Step 4:* Calculate the sampling interval in the frequency domain, $\Delta f = 1/(MT_s)$.
*Step 5:* Generate two vectors (***X*** and ***Y***) each of length $M$ where, for each vector, the samples are taken from a *Gaussian* distribution with (mean = 0, variance = $\sigma_1^2$). ***Hint:*** In MATLAB, samples of a *Gaussian(0,* $\sigma_1^2$*)*-distributed random variable can be generated using the function *randn(N,M)* where $N$ and $M$ are the dimensions of an ($N$ *by* $M$) matrix with random elements. Take $\sigma_1^2$ = 1 and generate vectors ***X*** and ***Y*** as follows: ***X*** = $\sigma_1$**randn*(1, $M$) and ***Y*** = $\sigma_1$**randn*(1, $M$) where $M$ = 1000. The $m^{th}$ element of vectors ***X*** and ***Y*** are denoted by $X_m$ and $Y_m$, respectively, $m$ = 1, …, 1000.
*Step 6:* Generate the magnitude response of the shaping filter, *F(m)*, $m$ = 1, 2, …, $M$. The sequence of filter coefficients {*F(m)*} is given by [2] (re-written using MATLAB implementation format):

$$F(m) = \begin{cases} 0, & m = 1 \\ \sqrt{\dfrac{0.5}{\sqrt{1 - \left(\dfrac{m-1}{MT_s f_{d_{\max}}}\right)^2}}}, & m = 2, \dots, k_m \\ \sqrt{\dfrac{k_m}{2}\left[\dfrac{\pi}{2} - \arctan\left(\dfrac{k_m - 1}{\sqrt{2k_m - 1}}\right)\right]}, & m = k_m + 1 \\ 0, & m = k_m + 2, \dots, M - k_m \\ \sqrt{\dfrac{k_m}{2}\left[\dfrac{\pi}{2} - \arctan\left(\dfrac{k_m - 1}{\sqrt{2k_m - 1}}\right)\right]}, & m = M - k_m + 1 \\ \sqrt{\dfrac{0.5}{\sqrt{1 - \left(\dfrac{M-(m-1)}{MT_s f_{d_{\max}}}\right)^2}}}, & m = M - k_m + 2, \dots, M \end{cases} \tag{10}$$

An example plot of the Doppler fading magnitude spectrum is shown in Fig. 2.

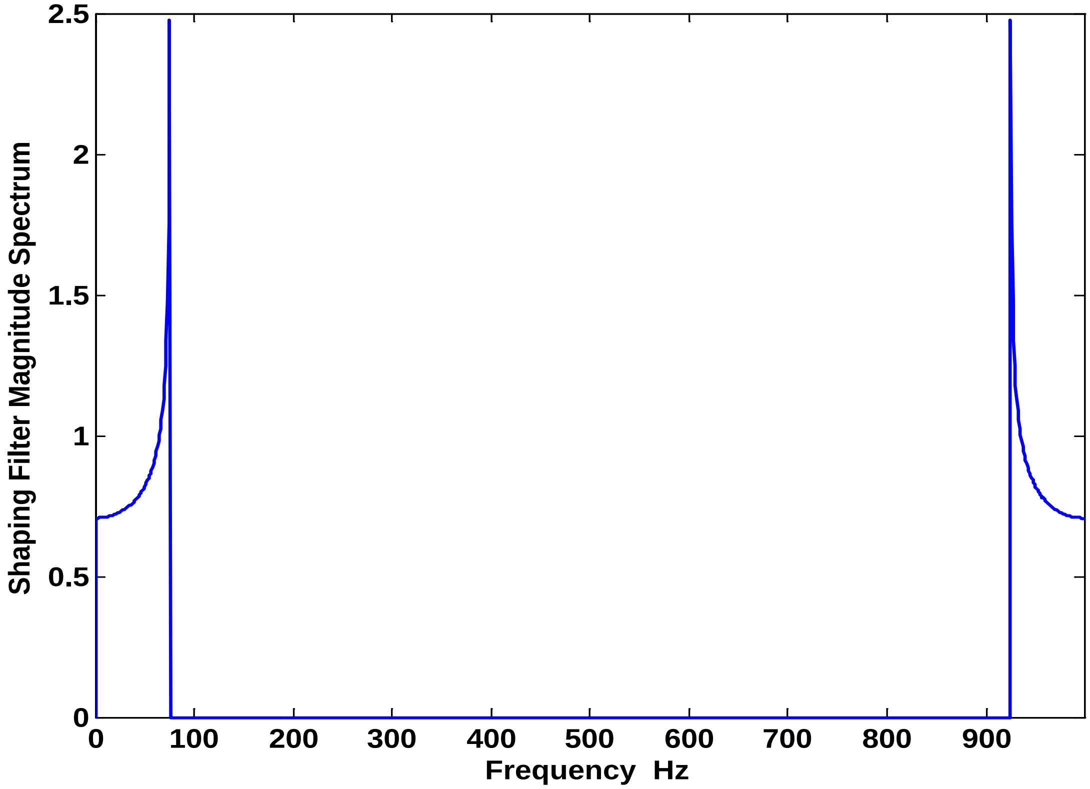


Fig. 2 Doppler fading magnitude spectrum

*Step 7:* Calculate the output of the shaping filter, denoted by *filt_oup(m)*, $m = 1, 2, \ldots, M$ where $M = 1000$. That is, calculate: $filt\_oup(m) = X_m * F(m) - j * Y_m * F(m)$, $m = 1, 2, \ldots, M$ where $M = 1000$.

*Step 8:* Take the inverse discrete Fourier transform of the output of the shaping filter. Use the function *ifft* in MATLAB. ***Hint:*** In MATLAB, the $M$-point inverse discrete Fourier transform of vector ***O*** is obtained using the function *ifft*(***O***, *M*). Hence, generate the inverse discrete Fourier transform of the output of the shaping filter as follows: ***inv_filt_out*** = *ifft*(***filt_oup***, *M*) where $M = 1000$.

*Step 9:* Multiply the result of Step 8 by a scaling (or calibration) factor $SF = 1 \big/ \left(\sigma_2 \sqrt{2}\right)$, where $\sigma_2^2 = \sigma_1^2 \sum_{m=1}^{M} \left( \frac{F_m}{M} \right)^2$ so that the average power in the received envelope is normalized to unity. Denote the filtered, IDFT and scaled versions of vector ***X*** (upper branch of Fig. 1) and vector ***Y*** (lower branch of Fig. 1) by $\boldsymbol{Z}_i$ (in-phase component) and $\boldsymbol{Z}_q$ (quadrature component), respectively. Calculate the power in each of the filtered signals $\boldsymbol{Z}_i$ and $\boldsymbol{Z}_q$ using the formulas $P_i = \sum_{m=1}^{M} Z_{i,m}^2$ and $P_q = \sum_{m=1}^{M} Z_{q,m}^2$, respectively. Note that $Z_{i,m}$ and $Z_{q,m}$ denote the $m^{th}$ sample of vectors $\boldsymbol{Z}_i$ and $\boldsymbol{Z}_q$, respectively, where $m = 1, 2, \ldots, M$ and $M = 1000$.

*Step 10:* Calculate the received signal envelope: $R_m = \sqrt{Z_{i,m}^2 + Z_{q,m}^2}$ , $m$ = 1, 2, …, $M$ where $M$ = 1000. A sample plot of the received signal envelope vs. time in milliseconds is shown in Fig. 3.

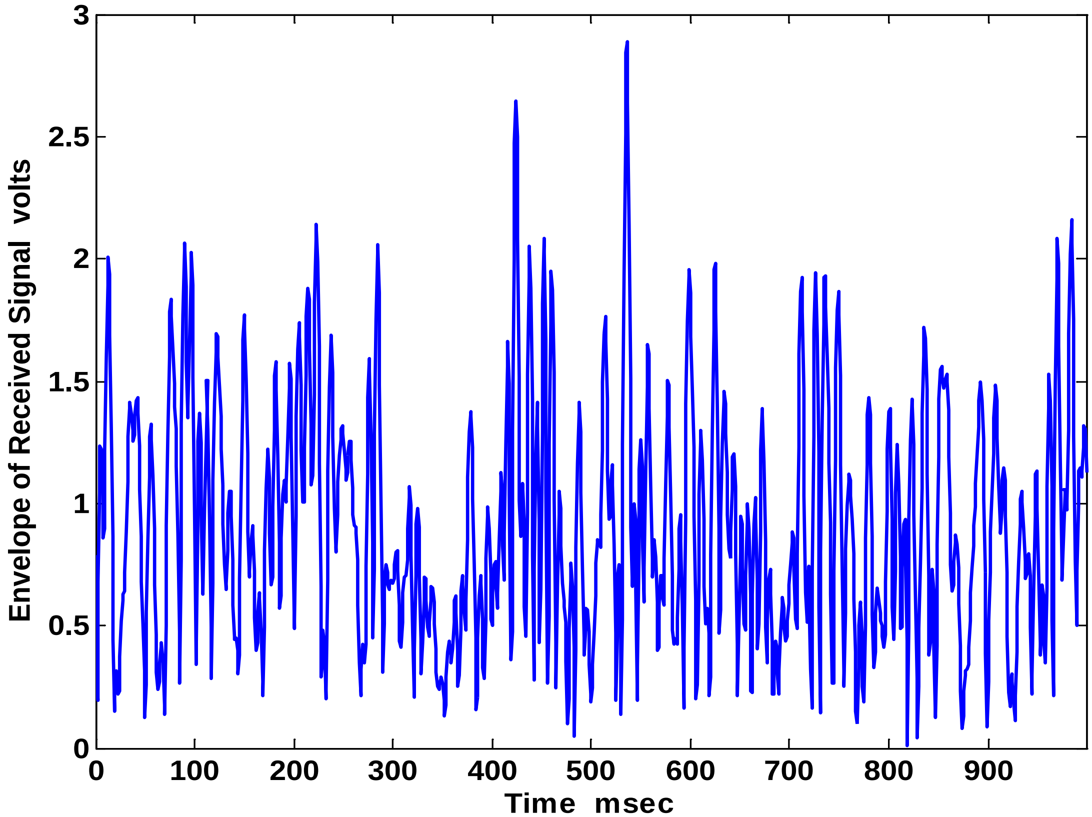


Fig. 3. Envelope of received signal.

*Step 11:* Calculate the average power of the received signal using the formula: $(P_i + P_q)/M$ . Store the result as $P_{av,k}$ , the average power at iteration $k$ where $k$ = 1 the first time (i.e., at the first iteration). Note that $k$ will assume values 2, …, $K$ in subsequent iterations provided $K$ > 1 in Step 13 below. $K$ is the total number of iterations to be performed in the simulation.

*Step 12:* Define outage as the probability that the received power falls below an acceptable threshold power $P_{th}$ . At iteration $k$, assume $P_{th}$ = $P_{av,k}$ and compute the simulated outage probability. To do the outage simulation, compare for each value of $m$, (where $m$ = 1, 2, …, $M$ and $M$ = 1000) the sum $\left(Z_{i,m}^2 + Z_{q,m}^2\right)$ from Step 9 with $P_{th}$ . Count the number of times the sums $\left(Z_{i,m}^2 + Z_{q,m}^2\right)$ falls below $P_{th}$ . Denote this number by $N_{out,k}$ . Then, the simulated outage probability at iteration $k$ is given by: $P_{out,sim,k} = N_{out,k}/M$ .

*Step 13:* Repeat Steps 5 to 12 forty nine times to generate $\{P_{av,2},\ P_{av,3},\ \cdots,\ P_{av,50}\}$ (i.e., set $K = 50$). Calculate the overall average power over $K$ iterations, $P_{av} = \frac{1}{K}\sum_{k=1}^{K} P_{av,k}$ .

*Step 14:* Calculate the average simulated outage probability over $K$ iterations, $P_{out,sim,av} = \frac{1}{K}\sum_{k=1}^{K} P_{out,sim,k}$ .

***NOTE:*** **Steps 1 to 10 are the steps required to generate the Rayleigh variates using the IDFT approach.**

## 4. Questions

1.1 Plot:

a) Doppler fading magnitude spectrum vs. frequency $f_m$ in Hz where $f_m = m\Delta f$ , and $m = 1, 2, \ldots, M$, $M = 1000$. (5 points)

b) Normalized received signal envelope (i.e., envelope normalized by its *rms* value, $R_{rms}$) in dB unit vs. time $t_m$ in milliseconds where $t_m = mT_s$ , and $m = 1, 2, \ldots, M$, $M = 1000$. (5 points)

Notes: 1. Normalized received signal envelope in dB unit = $20\log_{10}(R_m/R_{rms})$.

2. The *rms* value of the envelope $R$ is given by $R_{rms} = \sqrt{\frac{1}{M}\sum_{m=1}^{M} R_m^2}$ .

1.2 In Step 13, what is the simulated overall average power, $P_{av}$ ? (5 points)

1.3 In Step 14, what is the average simulated outage probability $P_{out,sim,av}$ over the 50 iterations? (5 points)

1.4 The theoretical formula for the outage probability in a Rayleigh fading channel is given by:

$$P_{out,theory} = 1 - exp\left[-\frac{P_{th}}{P_{av}}\right]$$

Using the theoretical formula and, assuming that $P_{th} = P_{av}$ (determined in Step 13), calculate the theoretical outage probability. (5 points)

1.5 Comment on the results obtained in questions 1.3 and 1.4. That is, compare simulated and theoretical outage probabilities – what is the percent difference between simulated and theoretical results? (5 points)

## 5. Further Readings

[1]. P.M. Shankar, Introduction to Wireless Systems. Chapter 2: Sections 2.3.1, 2.3.3, 2.3.4 and 2.5.

[2]. D.J. Young and N.C. Beaulieu, “The generation of correlated Rayleigh random variates by inverse discrete Fourier transform,” *IEEE Transactions on Communications*, pp. 1114 – 1127, July 2000.

## 6. Lab Report

Prepare a Lab Report (one per group) using ENEL529 Lab Reporting Guidelines.