ENEL 573

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Assignment #1:

Q.1:

a) 4-Bit frame

| ✓ | x | x | ✓ |

$err = p \quad N = 4$

$$P_{\checkmark x x \checkmark} = (1-p)(p)(p)(1-p)$$

$$\boxed{= p^2(1-p)^2}$$

b) $$P_k = \binom{N}{k}(p)^k(1-p)^{N-k}$$

c) All possible combinations of different errors:

$$P_F = \sum_{i=1}^{N}\binom{N}{i}p^i(1-p)^{N-i}$$

d) $N_1 = 500 \quad N_2 = 1000$

$p = 0.0001$

$$P_{F500} = \sum_{i=1}^{500} \binom{500}{i} p^i (1-p)^{500-i}$$

$$= \sum_{i=1}^{500} \binom{500}{i} (0.0001)^i (0.9999)^{(500-i)}$$

$\Rightarrow$ Binomial Theorem.

$$= 1 - (1-p)^N = 1 - (0.9999)^{500}$$

$$= 1 - (0.6064)$$

$$= \boxed{0.394}$$

$$P_{F1000} = \sum_{i=1}^{1000} \binom{1000}{i} p^i (1-p)^{1000-i}$$

$$= 1 - (1-p)^{1000}$$

$$= 1 - (0.9999)^{1000}$$

$$= \boxed{0.095}$$

$= 1 - (0.9\ldots)$

$= \boxed{0.095}$

e) $P_{Max} = 0.01 \quad p = 0.0001$

$N = ?$ (Maximize)

$0.01 = 1 - (1-p)^N$

$= 1 - (0.9999)^N$

$(0.9999)^N = 1 - 0.01$

$= 0.99$

$N \log(0.9999) = \log(0.99)$

$$N = \frac{\log(0.99)}{\log(0.9999)}$$

$= 100.49 \rightarrow 100$

$\boxed{\text{maximum } 100 \text{ bits}}$

Q2:

a) 1000000000 bytes

$\hookrightarrow$ GB

1000000000 GB

$\hookrightarrow$ EB

487 B GB

$\hookrightarrow$ $\boxed{487 \text{ EB}}$

b) $$\frac{487 \text{ EB}}{8 \text{ Mb}}$$

$\downarrow$

$$\frac{487 \times 10^{18} \text{ B}}{8 \times 10^{6} \text{ bits}}$$

$\downarrow$

$$\frac{487 \times 10^{18} \text{ B}}{1 \times 10^{6} \text{ B}}$$

$\downarrow$

$$\boxed{487 \times 10^{12} \text{ seconds}}$$

$$487 \times 10^{12}\,s / 31536000\,\tfrac{s}{yr}$$

$$= 15442668 \text{ years}$$

$$= \boxed{15.4 \text{ Million Years}}$$

c) $487 \times 10^{18} / 5$

$$\frac{97.4 \times 10^{18} \text{ words}}{100\,000}$$

97.4×10^{13} Books

$\times\ 3.4 \times 10^{-2}\,m$

$3.3116 \times 10^{13}\,M$

3.3×10^{9} KM

$3.3 < 4.2$

so No, it would not make it.

Q3:

a)

$$P_F = \sum_{i=1}^{N} \binom{N}{i} p^i (1-p)^{N-i}$$

$$\sum_{k=1}^{\infty} k p^k = p/(1-p)^2$$

$p \longrightarrow P_F$ which gives:

$$\sum_{k=1}^{\infty} k P_F^{\,k} = P_F / (1-P_F)^2$$

$$= \frac{\sum_{i=1}^{N} \binom{N}{i} p^i (1-p)^{N-i}}{\left(1 - \sum_{i=1}^{N} \binom{N}{i} p^i (1-p)^{N-i}\right)^2}$$

b) $p = 0.0001 \quad N = 1000$

$$\sum^{1000} \binom{1000}{\ } \ldots (\ldots)^{1000-i}$$

$$\text{Avg}\,\# = \frac{\sum_{i=1}^{1000} \binom{1000}{i} (0.0001)^i (0.9999)^{1000-i}}{\left(1 - \sum_{i=1}^{1000} \binom{1000}{i} (0.0001)^i (0.9999)^{1000-i}\right)^2}$$

$$= \frac{1 - (0.9999)^{1000}}{\left(1 - \left(1 - (0.9999)^{1000}\right)\right)^2}$$

$$= \frac{0.09517}{(1 - 0.09517)^2}$$

$$= \frac{0.09517}{0.9048^2}$$

$$= \boxed{0.116}$$

c)

$1\ \text{Mbit/s} \times (1 - 0.116)$

$1\ \text{Mbit/s} \times (0.884)$

$\hookrightarrow 1\,\text{Mbit/s} \times (0.884)$

$= \boxed{905\ \text{Kbit/s}}$

Q4:

$\underbrace{0\text{x}7\text{E}}_{\text{Header}}$

$\underbrace{0\text{x}7\text{d}}$ ← CRC escape character

→ 0x7d 0x5e

→ 0x7d 0x5d

$\underbrace{\begin{bmatrix} 0\text{xa}4 & 0\text{x}33 & 0\text{x}7\text{e} \\ 0\text{x}7\text{e} & 0\text{x}11 & 0\text{x}7\text{d} \end{bmatrix}}$

Payload w/out escape characters.

Q5:

# of times a 3-consecutive one pattern shows up.

$N = 5 \quad P = 0.5$

$2^5 = 32$ options

00000	01000
00001	01001
00010	01010
00011	01011
00100	01100
00101	01101
00110	01110 ✓
00111 ✓	01111 ✓
10000	11000
10001	11001
10010	11010
10011	11011
10100	11100 ✓
10101	11101 ✓
10110	11110 ✓
10111 ✓	11111 ✓

$\frac{8}{32}$

$\downarrow$

$\boxed{P = 0.25}$