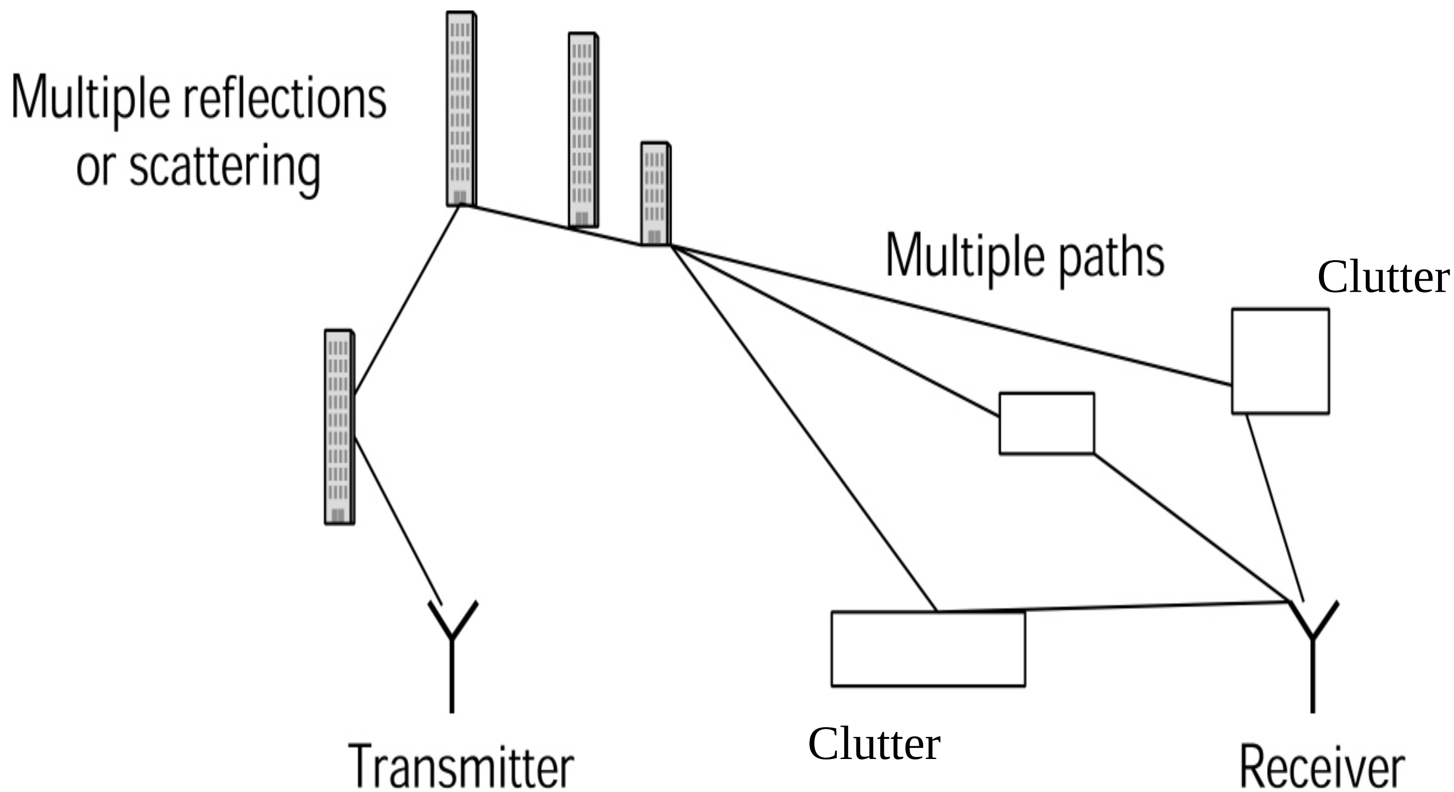
Shadowing Loss Model

- Random distribution of clutter (e.g. trees, buildings, moving vehicles, etc.) move in the way and out of the way of the signal path (see diagram on page 27)
- Transmitted signal passes through or reflects off the random number of objects, resulting in occasional extra rises and falls in power loss
- The final received signal power is the product of transmission or reflection factors of all these objects with the transmit power
- The logarithm of the received signal equals to the sum of a large number of logarithms of transmission/reflection factors, each of which is in dB
- As the number of transmission/reflection factors becomes large, the distribution of the sum approaches a zero-mean Gaussian distribution, by the Central Limit Theorem
- ***Known Fact:*** If a random variable X_{dB} (expressed in dB) is Gaussian, its linear counterpart $X = 10^{X_{dB}/10}$ has a log-normal distribution
- Shadowing loss in linear units (dB units) is modeled by the log-normal (Gaussian) distribution

Physical Causes of Shadowing Loss



The random distribution of objects around the receiver results in shadowing loss

Fig. 2.34 of Shankar: Wireless Systems

Shadowing Loss Model

Let:

P_{dBx} = received power in dBx units (e.g. dBW or dBm)

- Clearly, if shadowing loss ~ Gaussian() ==> P_{dBx} ~ Gaussian() or Normal()

- The probability density function (pdf) of P_{dBx} is then given by:

$$f(p_{dBx}) = \frac{1}{\sqrt{2\pi}\sigma_{dB}} exp\left[-\frac{(p_{dBx} - P_{av,\,dBx})^2}{2\sigma_{dB}^2}\right] \quad \text{(Gaussian pdf)}$$

where

$P_{av,\,dBx}$ = average received power in dBx units

σ_{dB} = standard deviation of the shadowing loss (in dB)

Shadowing Loss Model

Explanation of the parameters in the Gaussian prob. density function on Page 28:

- P_{dBx} = random received power in dBx units, calculated by:

$$P_{dBx} = P_{t,\,dBx} - PL(d)_{dB}$$

where $P_{t,\,dBx}$ is the transmit power in dBx units, and $PL(d)_{dB}$ is the random path loss in dB.

- $P_{av,\,dBx}$ = average received power in dBx units, calculated by:

$$P_{av,\,dBx} = P_{t,\,dBx} - \overline{PL}(d)_{dB}$$

where $\overline{PL}(d)_{dB}$ is the average path loss in dB (e.g., calculated by log-distance path loss model on Page 18)

Now, with shadowing loss, the random path loss (in dB) is given by:

$$PL(d)_{dB} = \overline{PL}(d)_{dB} + X_{dB}$$

where X_{dB} is the shadowing loss in dB, a random variable distributed as zero-mean Gaussian random variable with standard deviation σ_{dB} also in dB

- *Fact:* The probability density function (pdf) of the random path loss $PL(d)_{dB}$ is Gaussian with mean $\overline{PL}(d)_{dB}$ and standard deviation σ_{dB}.

Received Power: with and without Lognormal Fading

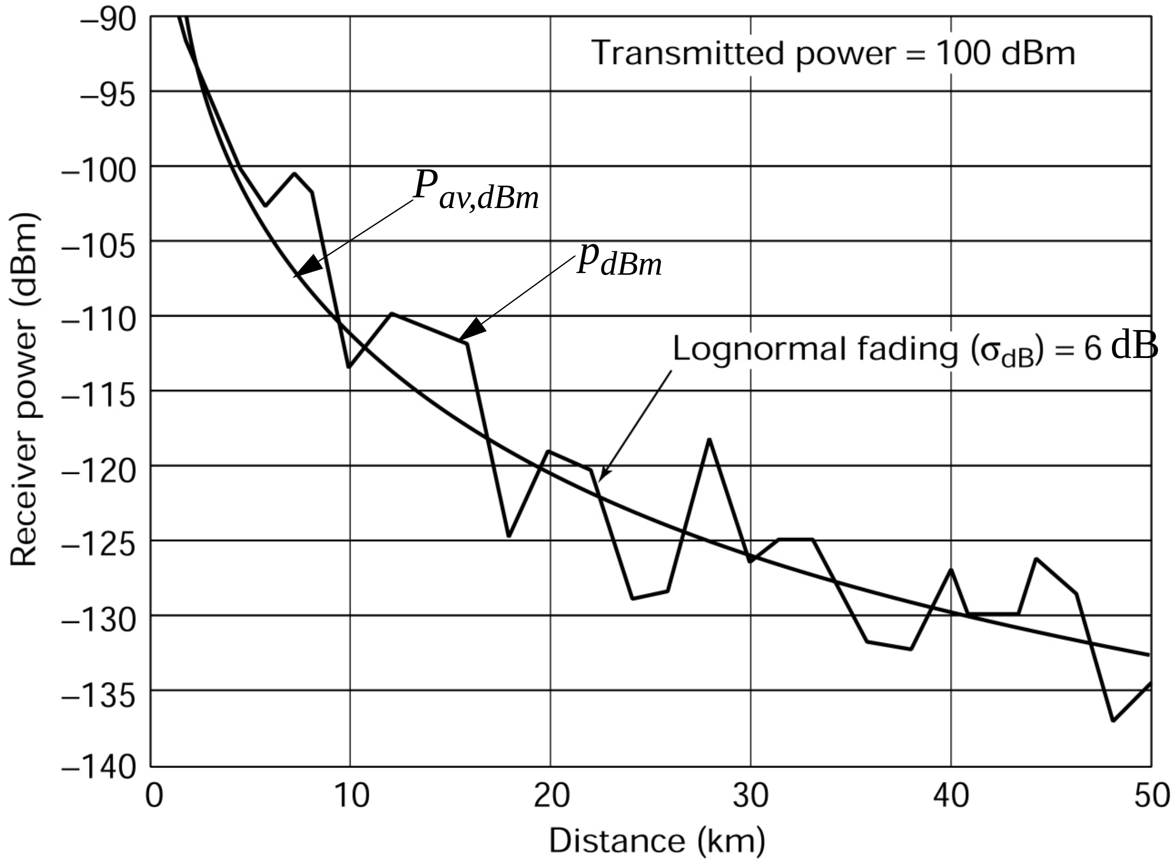


Fig. 2.36 of Shankar: Wireless Systems

Other Jargons for describing Shadowing Loss

- *Slow fading*: The number of random objects are correlated over short distances, hence resulting in gradual variation of the received signal with distance (see Page 30)

- *Large-scale* or *long-term fading*: the distance between two successive fades (a fade is a big fall in the received power) is large or long e.g. on the order of tens of meters (see Page 30)

- *Lognormal fading*: the variation in shadowing loss (in linear units) is modeled by the lognormal distribution (from Page 26)

Effect of Shadowing Loss: Signal Outage

- An MS (at distance d from the BS) is said to be in outage if the received power $P_r(d)$ (= P_r) is less than the minimum power P_{min} required for acceptable communication (i.e., receiver sensitivity)

- Mathematically, the probability that an MS is in outage, P_{out} is given by:

$$P_{out}(d) = Prob\{P_r(d) < P_{min}\}$$

- From Probability Theory: [<u>Note:</u> 2nd equality below assumes that P_r and P_{av} are in dBx units]

$$Prob\{P_r(d) < P_{min}\} = \int_{-\infty}^{P_{min}} f(p_r)dp_r = \int_{-\infty}^{P_{min}} \frac{1}{\sqrt{2\pi\sigma_{dB}^2}} exp\left[-\frac{(p_r - P_{av})^2}{2\sigma_{dB}^2}\right]dp_r$$

where $f(p_r)$ is the *pdf* for $P_r = P_r(d) = P_{dBx}$ ($f(p_r)$ is given on Page 28 if P_r is in dBx unit, e.g., dBm)

Integrating the term to the right of the 2nd equality of the preceding equation gives:

$$P_{out}(d) = Prob\{P_r(d) < P_{min}\} = Q\left(\frac{P_{av} - P_{min}}{\sigma_{dB}}\right)$$

where $Q(.)$ is the Q function defined by: $Q(u) = \frac{1}{\sqrt{2\pi}}\int_u^{\infty} exp(-x^2/2)dx$

Methods for Calculating the value of *Q(u)*

1) Use MATLAB, by calling the "*qfunc()*" built-in function.

2) Extract values of *Q(u)* using the *Q(.)* function Table (posted on ENEL 529 Desire2Learn site, under Topic 2 folder)

Class Example

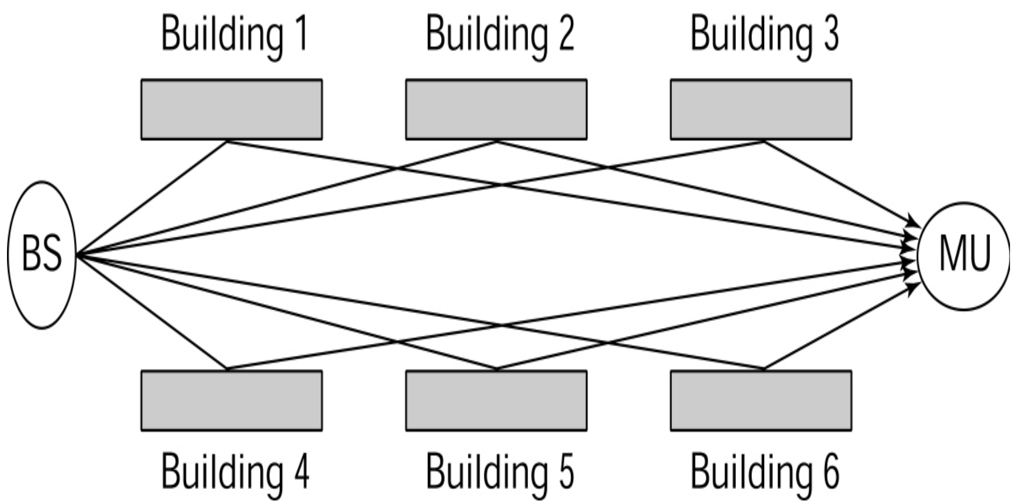
Problem Statement:

Suppose the power received at the MS is lognormal (i.e., normal in dBm unit) with an average of -95 dBm and a standard deviation of 8 dB. What is the outage probability if the minimum received power required for acceptable communication is -98 dBm?

Multipath Fading

The Multipath Concept:

- transmitted signal arrives at the receiver via a number of random and independent paths
- paths have different characteristics (amplitude, signal phase, arrival time, angle of arrival)
- paths combine vectorially: can sum destructively (i.e., fade) or constructively



Effects of Multipath:

- variation in received signal strength. Variations are very rapid or *fast* (due to vector addition) and distance between successive fades occur over very small distances (on the order of a wavelength), hence it is called *small scale* or *short-term* fading.
- time dispersion, caused by multipath delays (i.e. paths arrive at different times)

Multipath Fading Model

- Multipath fading model describes the fast variation of the channel or in the received signal strength (i.e. envelope)

- Two multipath fading models (ENEL 529 scope):
  - Rayleigh distribution: transmitted signal reaches the receiver via N multipath components and no line of sight (LOS) component
  - Rician distribution: transmitted signal reaches the receiver via one dominant path and N multipath components

Multipath Fading Model: Rayleigh distribution

- The received radio frequency (RF) signal at time t, $e_r(t)$ is given by:

$$e_r(t) = \sum_{i=1}^{N} a_i Re[exp(j2\pi f_o t + j\phi_i)] = \sum_{i=1}^{N} a_i cos(2\pi f_o t + \phi_i) = X cos(2\pi f_o t) - Y sin(2\pi f_o t)$$

where

f_o = carrier frequency; a_i = amplitude (random variable) of path i, $i = 1, 2, ..., N$

ϕ_i = phase of path i, ϕ_i is uniformly distributed between 0 and 2π

- After demodulation, the carrier is removed, the demodulated signal comprises X and Y

- By the Central Limit Theorem (as N ---> ∞):

$$X = \sum_{i=1}^{N} a_i cos(\phi_i) \sim \text{Gaussian(mean = 0, variance} = \sigma^2\text{);}$$

$$Y = \sum_{i=1}^{N} a_i sin(\phi_i) \sim \text{Gaussian(mean = 0, variance} = \sigma^2\text{)}$$

- By definition, the signal envelope, A, is given by: $A = \sqrt{X^2 + Y^2}$

- The pdf for A, $f_A(a)$ is Rayleigh: $f_A(a) = \frac{a}{\sigma^2} exp\left[-\frac{a^2}{2\sigma^2}\right]$, $0 \le a \le \infty$, σ = parameter = mode

Multipath Fading Model: Rician distribution

- The received signal at time t, $e_r(t)$ is given by:

$$e_r(t) = A_0 + \sum_{i=1}^{N} a_i Re[exp(j2\pi f_o t + j\phi_i)]$$

where A_0 denotes the peak amplitude of the dominant component

- The pdf for the received signal envelope A, $f_A(a)$ is Rician:

$$f_A(a) = \frac{a}{\sigma^2} exp\left[-\frac{a^2 + A_0^2}{2\sigma^2}\right] I_0\left(\frac{aA_0}{\sigma^2}\right), \quad 0 \le a \le \infty$$

- Rician distribution is characterized by the Rician factor, K

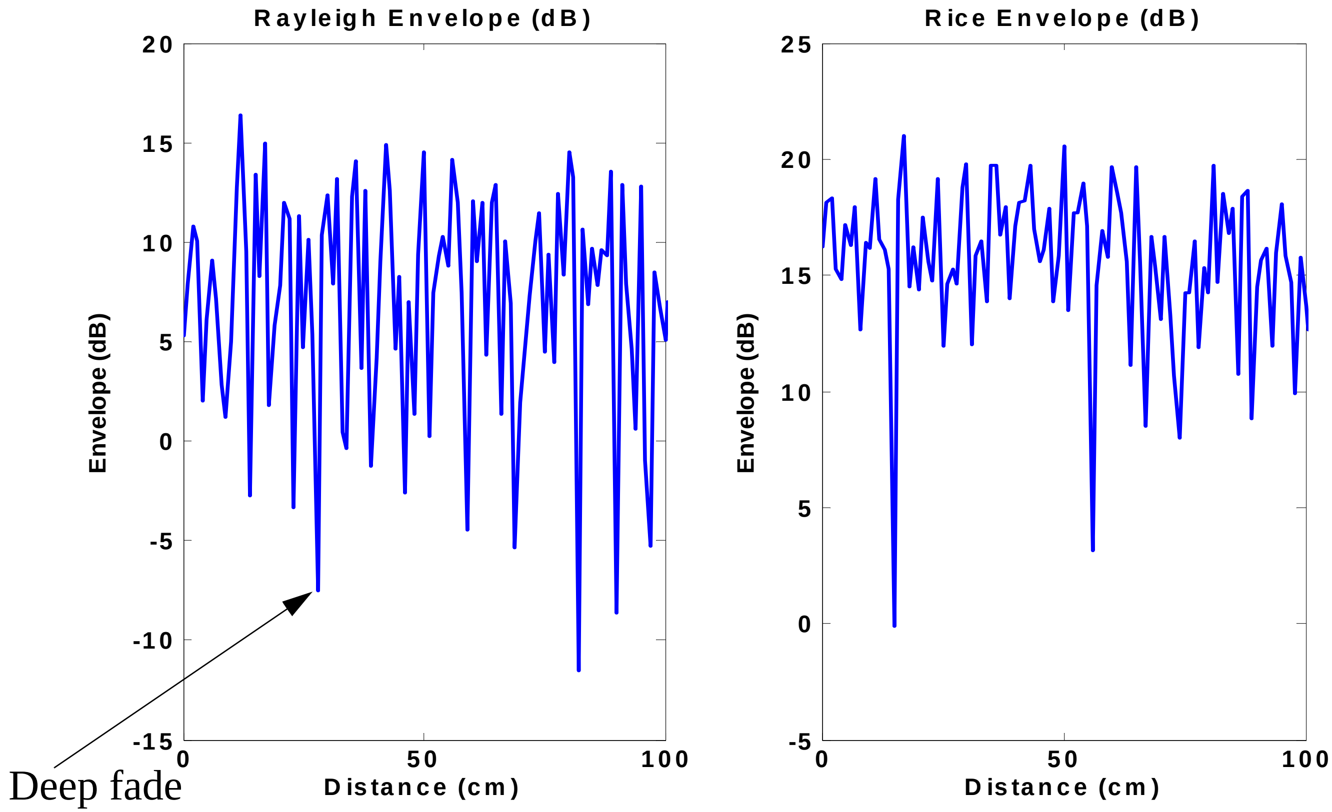
$$K = \frac{A_0^2}{2\sigma^2}$$

<u>*Notes:*</u>

1) As $A_0^2 \ll 2\sigma^2$ ==> $K << 1$ ==> $K_{dB} = 10log_{10}(K)$ ---> $-\infty$ dB, and Rician ---> Rayleigh

2) As $A_0^2 \gg 2\sigma^2$ ==> $K >> 1$ ==> K_{dB} ---> ∞ dB, and Rician ---> Gaussian distribution

Comparison of Rayleigh and Rician received signals



Assumed parameter values are: Rayleigh parameter σ = 2; Rician factor K_{dB} = 6 dB

<u>*Note:*</u> A deep fade = a severe drop in the received signal envelope

Effect of Variation in received signal due to Multipath: Signal Outage

Let P = power in the received signal = (received signal envelope)^2 = A^2

Assumed Multipath fading model: Rayleigh distribution

<u>*Fact:*</u> If A ~ Rayleigh ===> P ~ Exponential

- The Exponential pdf for P, $f_P(p)$, with parameter = mean = $2\sigma^2$ (= $E[P]$ = average power)

$$f_P(p) = \frac{1}{2\sigma^2} exp\left[-\frac{p}{2\sigma^2}\right]$$

- Outage Probability, P_{out}:

$$P_{out} = Pr\{P \le P_{min}\} = \int_o^{P_{min}} \frac{1}{2\sigma^2} exp\left[-\frac{x}{2\sigma^2}\right] dx = 1 - exp\left[-\frac{P_{min}}{2\sigma^2}\right]$$

Level Crossing Rate and Average Fade Duration

Level Crossing Rate, N_{Ath}:

- defined as the expected rate at which the received signal envelope A crosses a specified envelope level A_{th} in a positive going direction
- Mathematically:

$$N_{A_{th}} = \int_0^{\infty} \dot{a} f(A_{th}, \dot{a}) d\dot{a} = \sqrt{2\pi} f_{d_{max}} \alpha e^{-\alpha^2}$$

where $\dot{a} = \frac{da(t)}{dt}$, $f(A_{th}, \dot{a})$ is the joint pdf of a and $\dot{a}$ at $a = A_{th}$, and $\alpha = A_{th}/A_{rms}$

$f_{d_{max}}$ = maximum Doppler shift, defined as the ratio of speed to operating wavelength

A_{rms} = root mean square value of the received signal envelope $= \sqrt{E[A^2]}$

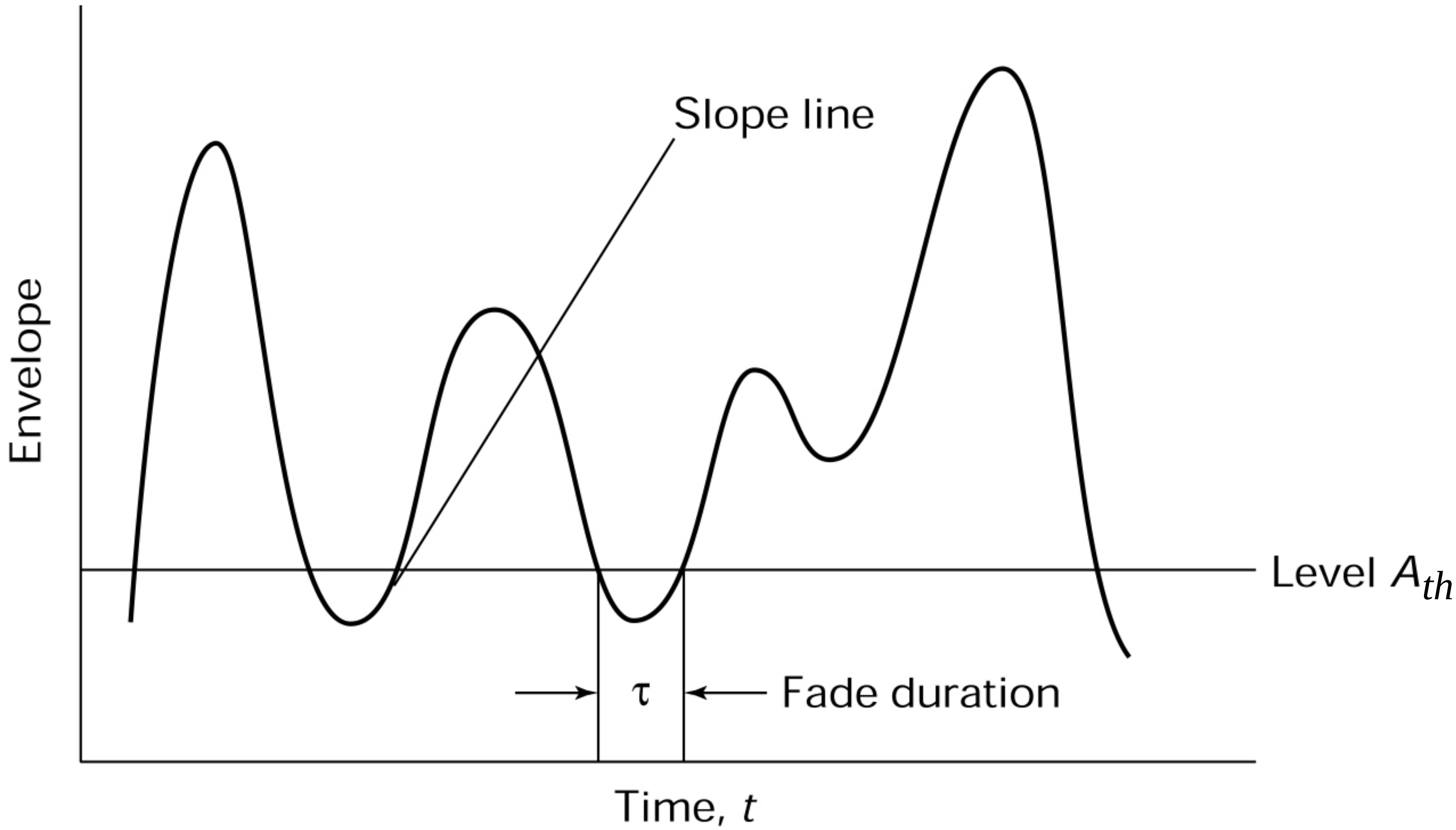
Average Fade Duration, τ_{av}:

- defined as the average period of time for which the received signal envelope stays below a certain level A_{th}
- Mathematically:

$$\tau_{av} = \frac{Pr\{a \le A_{th}\}}{N_{A_{th}}} = \frac{1 - e^{-\alpha^2}}{\sqrt{2\pi} f_{d_{max}} \alpha e^{-\alpha^2}} = \frac{e^{\alpha^2} - 1}{\sqrt{2\pi} f_{d_{max}} \alpha}$$

Note: Both N_{Ath} and τ_{av} are useful for estimating the average bit error rate (BER) and average probability that a bit is in error

Graphical Illustration of Level Crossing Rate



Class Example

Problem Statement:

A mobile station (MS) belongs to a cellular system operating at 900 MHz. Suppose the speed of the MS is 50 km/hr and the received signal by the MS is experiencing Rayleigh fading, determine:

a) the average number of times the received Rayleigh distributed signal envelope falls below the signal threshold $A_{th} = A_{rms}/(\sqrt{2})$ during 1 minute of test.

b) the average length of each fade, based on the level crossing rate determined in a).

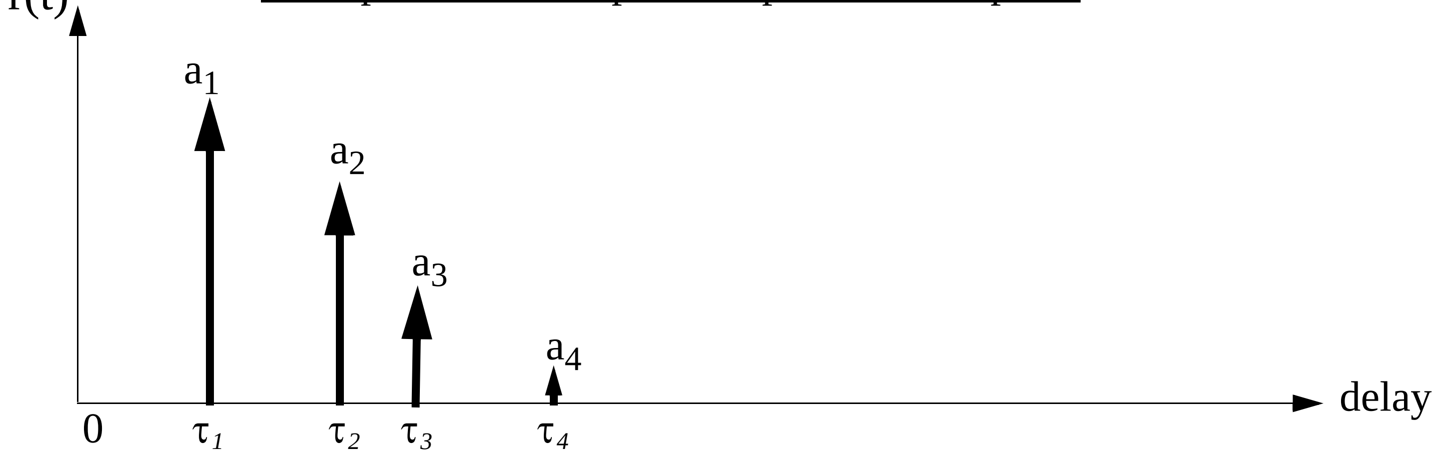
c) If the bit rate $R_b = 50$ bps, estimate the average probability that a bit is in error.

Time Dimension Characterization of Wireless Channels

$s(t) = \delta(t) \longrightarrow$ Multipath channel $h_c(t, \tau)$ $\longrightarrow r(t) = h_c(t, \tau)$ = channel impulse response

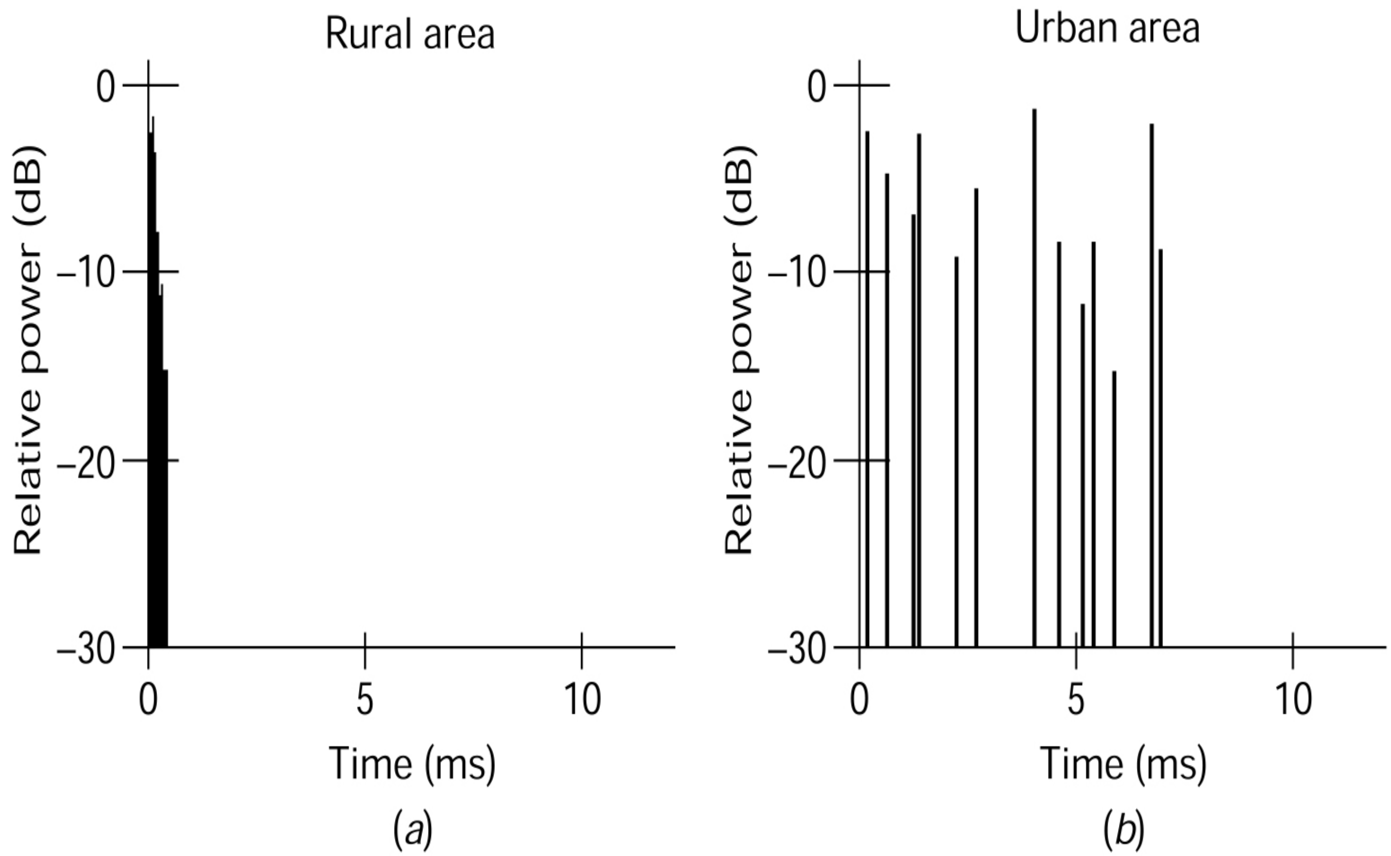
- a very narrow pulse (impulse = delta function) is transmitted over the wireless channel
- the impulse is sent via multiple paths (due to the multipath nature of channel)
- the received impulses arrive at different times with different power levels

<u>Example channel impulse response with 4 paths</u>



a_i = amplitude of path *i*; τ_i = time of arrival of path *i*

Impulse Response of Representative Radio Channels



Impulse Response Model of a Multipath Channel

The baseband impulse response of a multipath channel with *N* paths is modeled by:

$$h_c(t, \tau) = \sum_{i=0}^{N-1} a_i(t, \tau) exp(j\phi_i(t, \tau))\delta(\tau - \tau_i)$$

where:

$a_i(t, \tau)$: amplitude of received path *i* at time *t*

$\phi_i(t, \tau)$: phase shift of received path *i* at time *t*

τ_i: excess delay of received path *i* (relative to the first arriving path)

τ: multipath delay variable

For a time-invariant wireless channel:

$$h_c(\tau) = \sum_{i=0}^{N-1} a_i exp(j\phi_i)\delta(\tau - \tau_i)$$

<u>Note:</u> For a path *i* arriving at τ_i, its power $P(\tau_i)$ is given by: $P(\tau_i) = a_i^2$

Time-Dimension Metric: Delay Spread or Time Dispersion

Delay Spread: defined as the duration of time over which the impulse response $h_c(t, \tau)$ lasts

<u>*Statistics:*</u> Mean delay spread, RMS delay spread and Maximum delay spread

- Mean Delay Spread (or mean excess delay), $\bar{\tau}$:

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

- Root Mean Square (rms) Delay Spread, σ_τ:

$$\sigma_\tau = \left[\sum_k \overline{\tau^2} - (\bar{\tau})^2\right]^{\frac{1}{2}} = \left[\frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)} - \left(\frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}\right)^2\right]^{0.5}$$

<u>*Note:*</u> σ_τ, rms delay spread is the most popular statistic

- Maximum Delay Spread, τ_{max}:

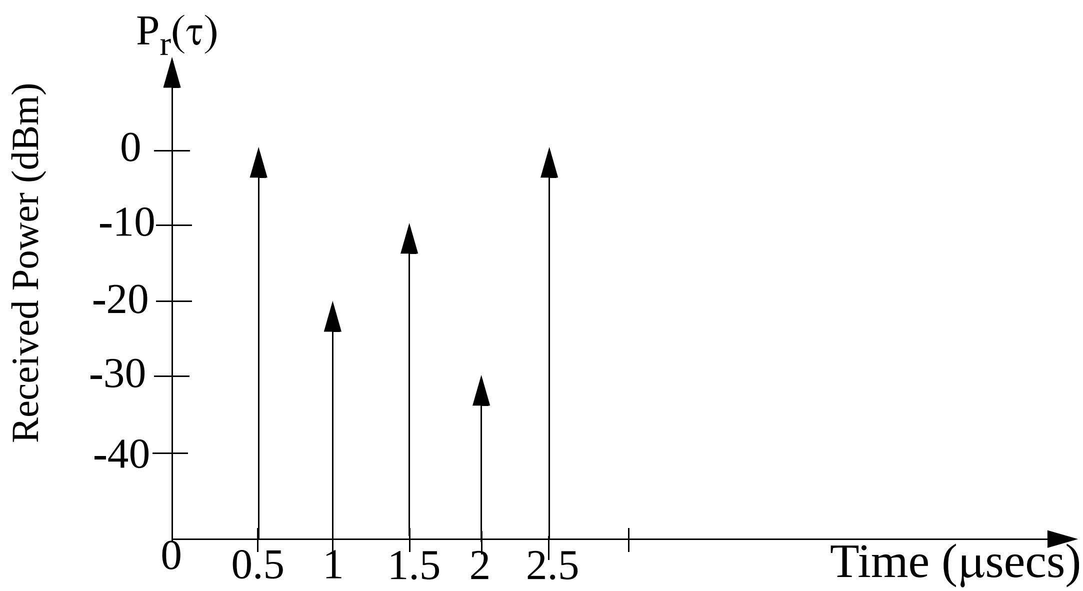
  - time difference between the earliest arriving path and the latest arriving path

<u>*Effect of Delay Spread:*</u> Intersymbol Interference (ISI), limits the data rate that can be supported

Class Example

Problem Statement:

A typical impulse response (i.e., power delay profile) of a wireless channel is given in the Figure below. Calculate the mean delay spread, rms delay spread and maximum delay spread for the wireless channel.



Physical Manifestation of Delay Spread in the Frequency Domain: Coherence Bandwidth

Coherence Bandwidth, B_c: characterizes time-dispersiveness of the channel in the *frequency domain*

- B_c is defined as the range of frequency over which the transfer function $H(f)$ of the channel is approximately the same

$$H(f) \approx H(f + \Delta f), \quad |\Delta f| \le B_c$$

$$H(f) \ne H(f + \Delta f), \quad |\Delta f| > B_c$$

- Alternatively, B_c is the range of frequency over which the channel is correlated

- B_c is a derived quantity from the rms delay spread (B_c is inversely proportional to σ_τ)

<u>*Example Relationships between B_c and σ_τ*</u>:

$$B_c \approx \frac{1}{50\sigma_\tau}$$ = bandwidth for which the envelope correlation function > 90%

$$B_c \approx \frac{1}{5\sigma_\tau}$$ = bandwidth for which the envelope correlation function > 50%

Physical Manifestation of Delay Spread in the Frequency Domain: Frequency Selective vs. Frequency Flat Channel

Frequency Selectivity of the Channel: occurs at non-zero delay spread (i.e. $0 < \sigma_\tau < \infty$) ==> $B_c < \infty$

- different components of the frequency response experience different attenuation
- the magnitude of the frequency response at different frequencies are uncorrelated

Frequency Flatness of the Channel: occurs at zero delay spread (i.e. $\sigma_\tau = 0$) ==> $B_c = \infty$

- different components of the frequency response experience the same attenuation
- the magnitude of the frequency response at different frequencies are correlated

Notes:

1) zero delay spread means the absence of multipath (e.g. free space channel)
2) In practice multipath always exist.
3) Hence, a frequency flat channel can be approximated by making the signal bandwidth B_{sig} to be much less than B_c the coherence bandwidth. I.e. $B_{sig} \ll B_c$. Conversely, the channel is frequency selective when $B_{sig} \gg B_c$.