ENEL 469 Assignment #1

Kyle Derby MacInnis
Lab Section: B03
Professor: Dr. Mintchev

October 1, 2012

Assignment Objective

The purpose of this this assignment is to design and analyze a full-wave rectifier circuit which will then become the focus of the upcoming lab. The use of CAD software will be allowed for the purpose of this lab and any-and-all use of the software will be recorded appropriately. The rectifier will have the following specifications:

Transformer:
Primary winding $V_{1_{rms}} = 110V, 60Hz$
Secondary winding $V_{2_{rms}} = 18V, 60Hz$

Load Resistance:
$R_L = 1.0k\Omega$

The diodes that will be used in this assignment will all be of the 1N4005 variety.

Original Circuit Design

The following is the basic filtered full-wave bridge rectifier circuit which will be explored in this assignment:

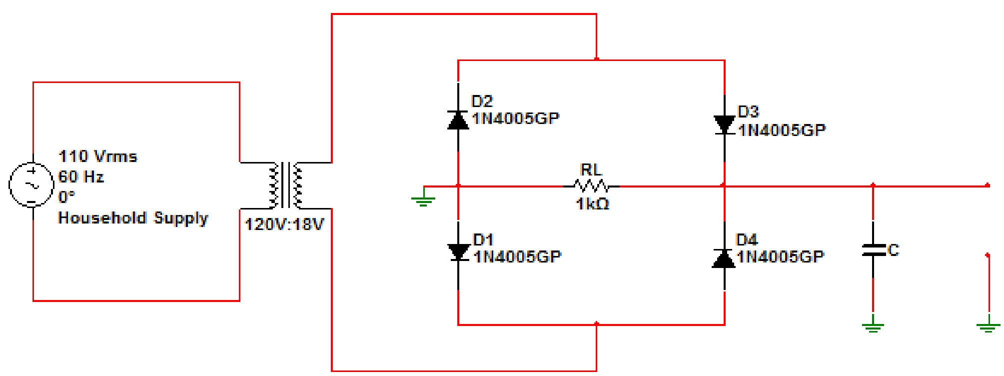


Figure 1: Full-Wave Bridge Rectifier with Filtering Capacitor

Assignment Questions/Solutions[1]

1 Calculate the peak value of the supply voltage V_s.

Given that the input voltage is a standard household supply, and assuming an ideal transformer is present in the circuit, we can make the following statements:

$$V_{s_{rms}} = \frac{V_{s_{max}}}{\sqrt{2}} = 18\text{V} \tag{1}$$

$$V_{s_{max}} = V_{s_{rms}}\sqrt{2} = 18\sqrt{2}V \approx \mathbf{25.46V} \tag{2}$$

The result for the approximate peak voltage at the supply terminals is 25.46V.
{

2 Determine the load current I_L. For the diodes used, identify from a relevant data-sheet the voltage drop on them and their internal resistance r_d for the 1.0kΩ load and thus calculate the peak rectified voltage V_P.

AC Supply Voltage:

$$\nu_s = V_{s_{max}} \sin(\omega t + \theta_0), \qquad \theta_0 = 0^\circ \tag{3}$$

Current Relationships using KCL:

$i_D =$

$$\begin{gathered} i_L + i_C \quad when(i_D > 0) \\ -(i_L + i_C) \quad when(i_D < 0) \\ 0 \quad when(i_L = -i_C) \end{gathered}$$

Output Voltage using KVL:

$$\nu_o = \nu_s - 2\nu_D = \nu_s - 2(i_D r_d + V_0) \tag{4}$$

The load current can then be ascertained via use of Ohm's Law:

$$i_L = \frac{\nu_o}{R_L} = \frac{V_{s_{max}} \sin(\omega t) - 2(i_C r_d + V_0)}{R_L + 2r_d} \tag{5}$$

Since we know that the current across the load is directly proportional to ν_o we can state that:

$$i_L \propto \nu_o \quad \rightarrow \quad i_{L_{max}} \propto \nu_{o_{max}} = V_P \quad \& \quad i_{L_{min}} \propto \nu_{o_{min}} = V_{min} \tag{6}$$

[1] Unless otherwise noted, all work assumes the piece-wise constant voltage-drop model of the diode.

We can assume that our output current will be relatively constant (DC output) so we say that the DC load current, I_L is:

$$I_L \equiv i_{L_{max}} \approx i_{L_{min}} \qquad \rightarrow \qquad I_L \approx \frac{V_P}{R_L} \tag{7}$$

$$I_L = \frac{V_{s_{max}} - 2V_0}{R_L + r_d} \tag{8}$$

With the aide of an appropriate data-sheet[1] for the 1N4005 diode, the following values were identified at 25°C:

$$InternalVoltageDrop: \qquad V_0 = 0.6V \tag{9}$$

$$InternalResistance: \qquad r_d = \frac{\Delta I_F}{\Delta V_F} = \frac{1000 - 87.8}{1470 - 740} = 1.25\Omega \tag{10}$$

The peak rectified voltage, V_P, occurs when i_L is at its max (ie. $i_L = I_L$). This happens to coincide when $i_C = 0$:

$$i_C = 0 \qquad \rightarrow \qquad i_D = i_L + i_C = i_{L_{max}} = I_L \tag{11}$$

$$V_P = I_L R_L = \frac{V_{s_{max}} - 2V_0}{R_L + r_d} R_L = \frac{18\sqrt{2} - 1.2}{1002.5}(1000) = 24.195V \tag{12}$$

The peak rectified voltage for this circuit is 24.195V.

3 Calculate the filtering capacitor C that will result in a ripple voltage of $V_r <= 1V$, and the fraction of the 360-degree wave cycle during which the diodes are conducting. Calculate the minimal and maximal output voltage V_o.

The ripple voltage is the difference between the minimum and maximum output voltages:

$$1 \geq V_r = V_P - V_{min} \tag{13}$$

$$V_{min} = V_P - V_r \geq 23.20V \tag{14}$$

$$I_{L_{min}} = \frac{V_{min}}{R_L} = \frac{23.20}{1000} = 23.20mA \tag{15}$$

The capacitor is placed in parallel to the load to help regulate a constant V_o. The Capacitor has two modes, Charge and Discharge, which can be modelled roughly as:

$$\nu_o = \Big\{ V_P e^{\frac{-t}{R_L C}} \quad (Discharging)\ V_P \cos(\omega(t - \Delta t)) \quad (Charging) \tag{16}$$

The charge that is discharged(lost) must be gained once again by the capacitor on the charging phase, also because the voltage cannot change instantaneously through a capacitor:

$$\{\, \nu_o(0^-) = \nu_o(0^+) = V_P \quad (t = 0 is the end of the conduction phase)\ \nu_o(-\Delta t^-) = \nu_o(-\Delta t^+ = V_{min} \quad (t = -\Delta ti \tag{17}$$

Since $\Delta t << T$ we can say that $T_D C \approx T$, where $T_D C$ is the discharge time, and T is the cycle period:

$$V_{min} = V_P e^{\frac{-T_D C}{R_L C}} \approx V_P e^{\frac{-T}{R_L C}} \quad \rightarrow \quad V_{min} \approx V_P (1 - \frac{T}{R_L C})^2 \tag{18}$$

This leads to:

$$V_r \approx V_P \frac{T}{R_L C} \quad \rightarrow \quad C \approx \frac{V_P T}{R_L V_r} = \frac{24.2(0.0083)}{1000(1)} = 202\mu F \tag{19}$$

During the Charging Phase:

$$\{\, V_P \cos(\omega \Delta t) = V_{min} \quad \rightarrow \quad V_P(1 - \tfrac{1}{2}(\omega \Delta t)^2) = V_{min}\ \theta_{cond} = \omega \Delta t = \frac{\sqrt{\frac{2V_r}{V_P}}}{=} .287 rads = 16.47^\circ \tag{20}$$

The Capacitance that should be used is around $200\mu F$ and conduction occurs for 16.5° out of every rectified cycle(half of the supply cycle). The minimum and maximum voltages that occur are respectively: 24.2V and 23.2V.

4 Calculate the Peak Inverse Voltage (PIV). Compare the calculated PIV with the maximal PIV value from the datasheet of this particular type of diodes. Verify that the calculated PIV (plus the necessary safety margin of 50%) is below the maximal PIV value for this diode.

Based on the reverse analysis of the circuit when it is conducting through D3 and D2, the PIV that is experience by D4 is:

$$\{\, PIV = \nu_{s_{max}} + \nu_{D_{max}} \approx V_{s_{max}} + V_0{}^3 = 18\sqrt{2} + 0.6 = 26.1V\ PIV + 50\% = 39.084V \tag{21}$$

---

[2] Since $R_L C >> T$ we can say $e^{\frac{-T}{R_L C}} \approx 1 - T/CR$

If we compare the values from the data-sheet[1] with the calculated value for our circuit:

$$PIV_{Data-Sheet} = 600V \quad \rightarrow \quad (PIV + 50\%) << PIV_{Data-Sheet} \tag{22}$$

Overall, it appears that there is no risk involved with the PIV caused by our circuit.

5 Calculate the average diode current i_{Dav}. Why is the calculated i_{Dav} is higher than the load current I_L?

If we refer to (Eqn. 4) we see that we can rewrite it as:

$$i_{D_{av}} = i_{C_{av}} + I_L \tag{23}$$

In addition if we relate the concept of conservation of charge, we get:

$$\{ Q_s upplied = i_{C_{av}} \Delta t Q_{lost} = CV_r Q_{lost} = Q_{supplied} \quad \rightarrow \quad i_{C_{av}} = \frac{CV_r}{\Delta t} = 0.525A \tag{24}$$

Finally, we solve for $i_{D_{av}}$:

$$i_{D_{av}} = 0.525 + 0.0242 = 0.549A \tag{25}$$

The average value for the diode current is 0.55A. The reason that this is on average larger than I_L is because the supply current on the system is alternating, while the output voltage is relatively constant. The constancy is caused by the presence of the filtering capacitor which discharges across the load and it is this constant charging and discharging that is the reason for the much larger average current.

6 Calculate the peak diode current i_{Dmax}. Make sure it does not exceed the maximal peak diode current for 1N4005. What is the safety margin between the two currents? Why i_{Dmax} is approximately twice the value of i_{Dav}?

Again, if we rewrite (Eqn. 4) we get:

$$i_{D_{max}} = i_{C_{max}} + I_L \tag{26}$$

Since I_L is assumed to be constant, the only factor that varies is the i_C. i_C is at its maximum voltage when $\nu_o = V_m in$:

$$i_{D_{max}} = I_L(1 + 2\pi\sqrt{\frac{2V_P}{V_r}}) = 0.0242(1 + 2\pi\sqrt{2V_p}) = 1.08A \tag{27}$$

Comparing the value found for our circuit, and the value from the data-sheet[1] we get:

$$i_{D_{max\,Data-sheet}} = 30A \quad \rightarrow \quad SafetyMargin = i_{D_{max\,Data-sheet}} - i_{D_{max}} = 28.92A \tag{28}$$

As we can see, there is a large margin of safety for the diodes used, and the reason that $i_{D_{max}} \approx 2i_{D_{av}}$ is because when the capacitor is charging, both I_L and i_C are quite large

7 Now change the capacitor determined in Section 3 to two smaller standard values and to two larger standard values, and document in a table the change in the ripples. Determine and document the PIV, i_{Dav}, I_L, and i_{Dmax} (Sections 4 to 6) in each case. Which is the best filtering capacitor to be used for the 1.0kΩ load that was able to maintain the average output voltage level unchanged and below the ripple level required in Section 3?

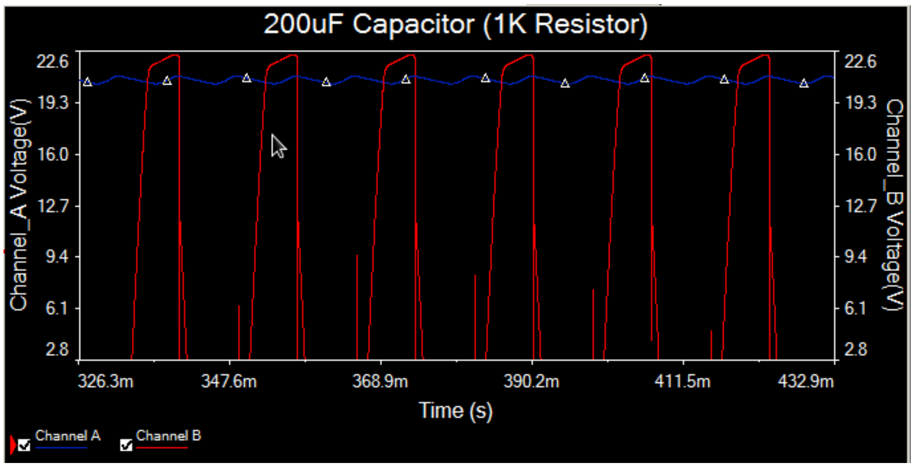
Capacitor (μF)	PIV	$i_{D_{av}}$	I_L	$i_{D_{max}}$

The optimal capacitor to use is .

8 With the optimal capacitor determined in Section 7, change the load resistor to two smaller standard values (e.g. 500Ω and 250Ω) and two standard larger values (e.g. 2kΩ and 10kΩ) and document in a table the changes in the ripples observed. Determine and document the PIV, i_{Dav}, I_L, and i_{Dmax} (Sections 4 to 6) in each case.

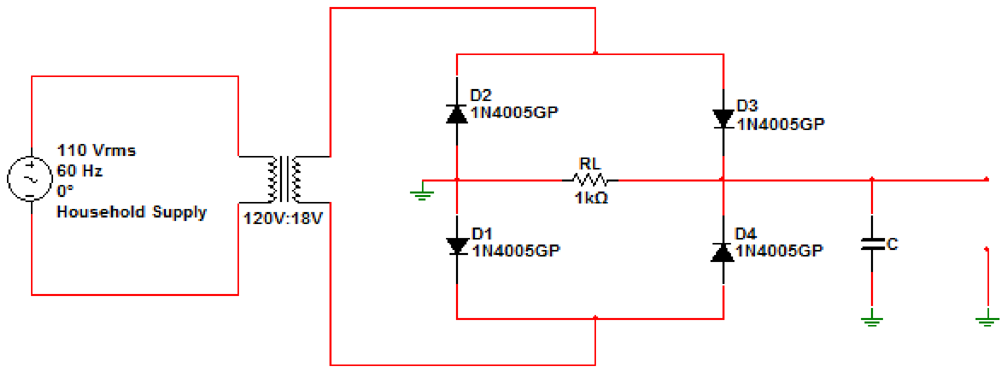
Resistor (Ω)	V_r	PIV	$i_{D_{av}}$	I_L	$i_{D_{max}}$

9 Present time-domain waveform plots of your final design and a Bode plot (with the optimal capacitor found in section 7 with a load resistor of 1kΩ. Explain why does the Bode plot look that way.



Final Design

The final and optimized circuit design resulting from the calculations presented:



References

[1] Philips Semiconductors. *1N4001G-1N4007G Rectifiers*. Philips.