The initial model for this system is given as:

$$mc\ddot{\theta}(t) = -bc\dot{\theta}(t) + K_m u(t) - Mg \tag{1}$$

$$\ddot{\theta}(t) = \frac{1}{mc}[K_m u(t) - bc\dot{\theta}(t) - Mg] \tag{2}$$

We can state some substitutions:

$$x(t) = c\theta(t) \tag{3}$$

$$v(t) = c\dot{\theta}(t) = \dot{x}(t) \tag{4}$$

$$\dot{v}(t) = c\ddot{\theta}(t) = \ddot{x}(t) \tag{5}$$

so we can rewrite (2) in two different ways:

$$\dot{v}(t) = \frac{1}{m}[K_m u(t) - bv(t) - Mg] \tag{6}$$

$$\ddot{x}(t) = \frac{1}{m}[K_m u(t) - b\dot{x}(t) - Mg] \tag{7}$$

Now we can focus on simplifying this even more by looking at our control signal and imaging it has DC and AC parts:

$$u(t) = C_u + \mu(t)$$

Where we set this equal to our base required torque:

$$K_m C_u = Mg$$

Since we can cancel, this now gives:

$$\dot{v} = \frac{1}{m}[K_m \mu(t) - bv(t)] \tag{8}$$

$$\ddot{x} = \frac{1}{m}[K_m \mu(t) - b\dot{x}(t)] \tag{9}$$

This can be converted via Laplace to:

$$sV(s) = \frac{1}{m}[K_m \mu(s) - bV(s)] \tag{10}$$

$$s^2 X(s) = \frac{1}{m}[K_m \mu(s) - bsX(s)] \tag{11}$$

To find the Transfer functions for the Motor:

$$V(s)[s+\frac{b}{m}] = \frac{K_m}{m}\mu(s) \tag{12}$$

$$H(s) = \frac{V(s)}{\mu(s)} = \frac{K_m}{ms+b} \tag{13}$$

$$X(s)[ms^2+bs] = K_m\mu(s) \tag{14}$$

$$F(s) = \frac{X(s)}{\mu(s)} = \frac{K_m}{s(ms+b)} \tag{15}$$

However, we also need the compensator control transfer function:

$$C(s) = \frac{\mu(s)}{E(s)} = ?$$

Where:

$$e(t) = X_{des} - x(t)$$

The laplace transform is:

$$E(s) = \frac{X_{des}}{s} - X(s)$$

We must find the compensator transfer function. It must be a PID controller (Type-2 System) as it is required to bring both the error in position and speed down to zero. The control signal function will result as:

$$\mu = K_p e(t) + K_i \int e(t) + K_d \frac{de(t)}{dt}$$

$$\mu(s) = K_p E(s) + K_i \frac{E(s)}{s} + K_d s E(s)$$

$$\frac{\mu(s)}{E(s)} = C(s) = K_P + \frac{K_i}{s} + K_d s$$