

## **Topic 4: Fading Mitigation Techniques in Wireless Systems**

### Content:

- 1) The Diversity Concept
- 2) Microdiversity Techniques (independent signal creation)
  - Space or Antenna Diversity
  - Angle Diversity
  - Polarization Diversity
  - Frequency Diversity
  - Time Diversity
  - Multipath Diversity
- 3) Diversity Combining Techniques (processing)
  - Selection Combining
  - Maximal Ratio Combining
  - Equal Gain Combining
- 4) MODEM Performance over Wireless Channels with Microdiversity
- 5) Macroscopic Diversity

## Topic 4 Learning Outcomes

At the end of this Topic, you will be able to:

- Define the diversity techniques for mitigating the fading effects of wireless propagation channel
- Quantify the improvement in bit error rate performance provided by the diversity combining techniques

# The Diversity Concept

## Definition:

- involves the creation of multiple independent versions of the received signal and processing them

## Forms of Diversity: Microdiversity and Macrodiversity

- Microdiversity:
  - used to mitigate small-scale fading
  - independent signal creation mechanisms are co-located
- Macrodiversity:
  - used to combat large-scale fading
  - independent signal creation mechanisms are much further apart

# Microdiversity Techniques

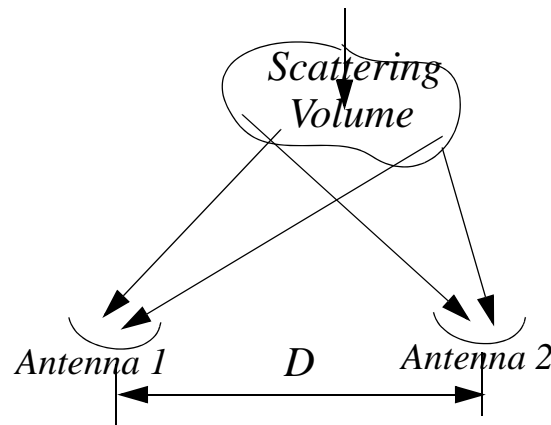
Microdiversity Techniques: Classification based on the mechanism used for creating multiple independent signals

1. Space (or antenna) diversity
2. Angle diversity
3. Polarization diversity
4. Frequency diversity
5. Time diversity
6. Multipath diversity

# Space (or Antenna) Diversity

## Description:

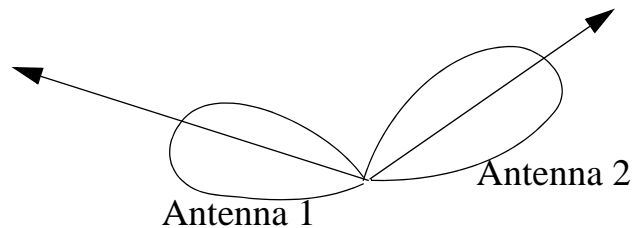
- Multiple signals are created by multiple antennae at the receiver
- The antennae are separated such that the individual received signals are uncorrelated  
(In practice: distance  $D$  between antennas at the MS  $\geq \lambda/2$   
distance  $D$  between antennas at the BS: several tens of wavelengths)
- The diversity array can be located either at the MS, BS or both
- In principle, there is no limitation on the number of antennae  $M$  but the benefit decreases as  $M$  grows larger



# Angle (or direction) Diversity

## Description:

- Multiple signals are created by multiple antennae that are physically collocated but each antenna has different directional properties
- Each antenna receives signals from a specific direction to produce uncorrelated signals with those of other antennae
- The directional antennae can be located either at the MS, BS or both



# Polarization Diversity

## Description:

- Scatterers in the transmission medium can split a transmitted signal of a given polarization into orthogonal signals of vertical and horizontal polarizations
- The fading of signals with different polarizations is statistically independent
- Receiving both polarizations using a dual-polarized antenna offers polarization diversity
- Polarization diversity does not require a minimum distance between antenna elements
- Polarization diversity can be implemented either at the MS, BS or both

Limitation: Typically, polarization diversity can generate a maximum of 2 independent signals

# Frequency Diversity

## Description:

- Multiple signals are created by transmitting the same information on more than one carrier frequency
- The transmit frequencies are separated such that the frequency separation is greater than the channel's coherence bandwidth  $B_c$ , so that the received signals are uncorrelated
- Requires one transmit antenna and one receive antenna

## Limitations:

- i) Requires a wider bandwidth spectrum to allow a number of carrier frequencies
- ii) Need to build multiple transmitters & receivers tuned to different frequencies

## Applications:

- i) Used in CDMA2000 due to the allocation of large bandwidth for 3G applications
- ii) used in Orthogonal Frequency Division Multiple (OFDM) access scheme



# Time Diversity

## Description:

- Multiple signals are created by transmitting the same information repeatedly at different times
- The times of transmission are separated such that the time separation is greater than the channel's coherence time  $T_c$ , so that the received signals are uncorrelated
- Requires one transmit antenna and one receive antenna

## Limitation:

- i) Requires memory space to buffer received data prior to processing of the multiple versions of received data at different times

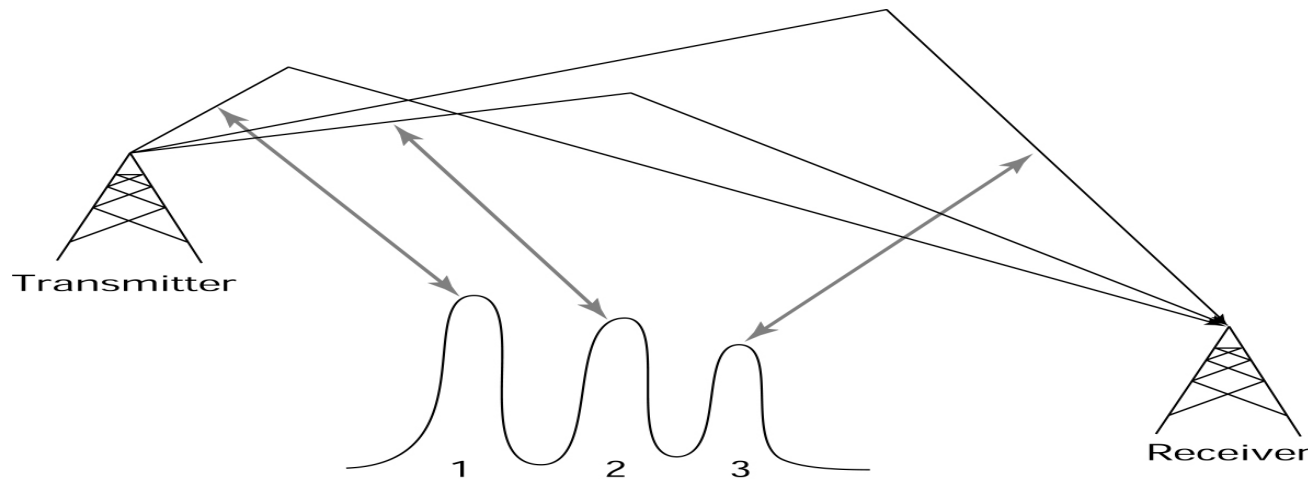
## Application:

- i) Error control coding and time interleaving can be regarded as forms of time diversity

# Multipath Diversity

## Description:

- Multiple signals are created by the multipath signals present in the wireless channel
- Each multipath component gives rise to a pulse (i.e., signal) at the receiver
- If the pulse duration  $\ll$  rms delay spread, then the pulses of different paths will not overlap and each path can be resolved at the receiver.
- The multipath signals corresponding to the resolved paths are uncorrelated because the multipath components are independent.



## Diversity Combining Methods

*How should the multiple independent versions of the received signal be combined to improve performance?*

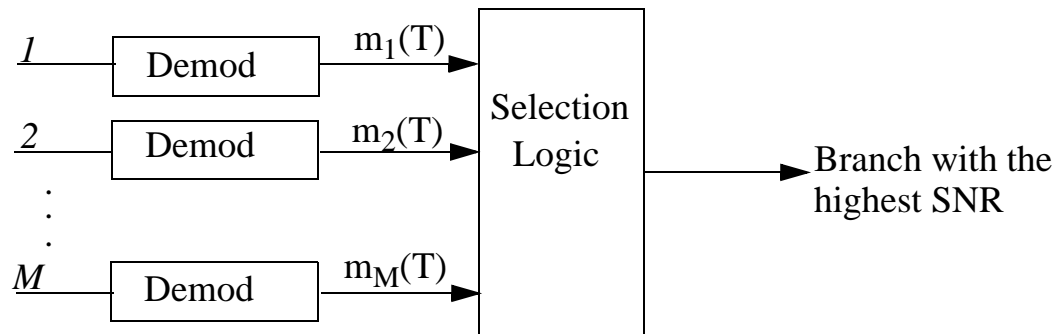
*Diversity Combining Approaches:*

- 1) Selection Combining (SC)
- 2) Maximal Ratio Combining (MRC)
- 3) Equal Gain Combining (EGC)

# Selection Combining (SC)

## Operating Principles:

- All the multiple independent signals will NOT fade simultaneously
- Selection logic selects the diversity branch with the strongest SNR for detection



## Average Received SNR under Selection Combining Scheme:

$$\bar{\Gamma}_{SC} = \int_0^{\infty} x f_{\Gamma_{SC}}(x) dx = \gamma_o \sum_{n=1}^M \frac{1}{n}$$

Improvement in Average Received SNR relative to No Diversity = Diversity Gain,  $G_{SC}$ :

$$G_{SC} = \frac{\bar{\Gamma}_{SC}}{\gamma_o} = \sum_{n=1}^M \frac{1}{n}$$

Implication:  $G_{SC}$  presents a “law of diminishing returns”

## Outage Probability for a Selection Combiner

The expression for the probability density function for  $\Gamma_{SC}$  is:

$$f_{\Gamma_{SC}}(x) = \frac{M}{\gamma_o} \left[ 1 - \exp\left(-\frac{x}{\gamma_o}\right) \right]^{M-1} \exp\left(-\frac{x}{\gamma_o}\right)$$

- The outage probability is the probability that  $\Gamma_{SC}$  is less than a specified threshold  $\Gamma_{th}$ :

$$P_{outage, SC} = Prob\{\Gamma_{SC} \leq \Gamma_{th}\} = \int_0^{\Gamma_{th}} f_{\Gamma_{SC}}(x) dx = \int_0^{\Gamma_{th}} \frac{M}{\gamma_o} \left[ 1 - \exp\left(-\frac{x}{\gamma_o}\right) \right]^{M-1} \exp\left(-\frac{x}{\gamma_o}\right) dx$$

Integrating gives:

$$P_{outage, SC} = \left[ 1 - \exp\left(-\frac{\Gamma_{th}}{\gamma_o}\right) \right]^M$$

## Class Example

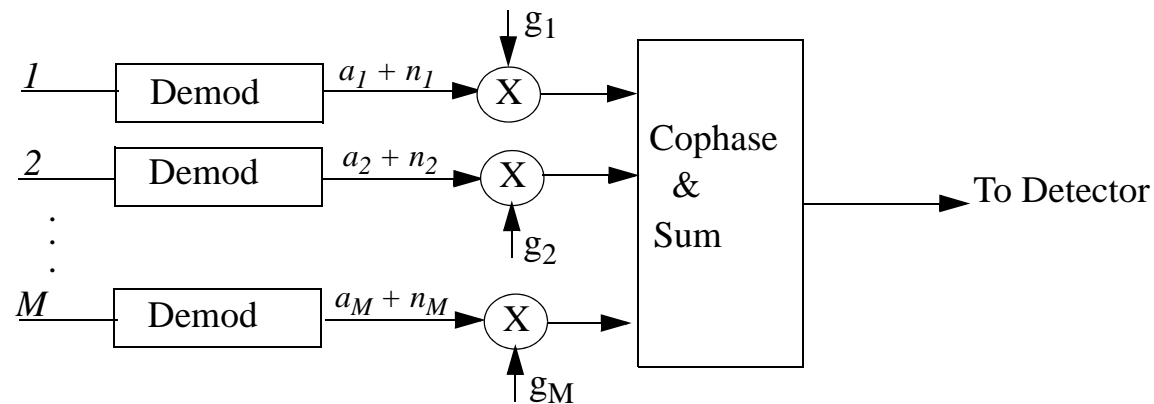
### Problem Statement:

Suppose two branch diversity is used in a receiver where each branch receives an independent Rayleigh fading signal. If the average SNR without diversity  $\gamma_0$  is 20 dB, determine the probability that the SNR will drop below a 14 dB SNR threshold, assuming a selection combiner. Repeat the calculation for the case of a single receiver (i.e., no diversity).

# Maximal Ratio Combining (MRC)

## Operating Principles:

- All the multiple ( $M > 1$ ) independent signals will NOT fade simultaneously
- Output signal for detection is a linear combination of ALL the multiple signals
- Linear combination operation includes: weighting, cophasing and summing of the multiple signals



Weighting: multiplication of the received signal and noise on branch  $i$  by the weighting factor  $g_i$

Cophasing: tracking the phases of the signals & correcting for differences between phases

Summing: add the weighted and cophased signals of the different branches

# Maximal Ratio Combining

- Fact: The maximum SNR of the maximal ratio combiner is the sum of the SNR's of all the  $M$  independent diversity branches. That is:

$$\Gamma_{MRC, max} = \sum_{i=1}^M \gamma_i \quad \text{where } \gamma_i \text{ is exponentially distributed}$$

- SPECIAL CASE: Assume that all the  $M$  independent signals are also identically distributed so that  $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_3 = \dots = \bar{\gamma}_M = \gamma_o$

Hence:  $\bar{\Gamma}_{MRC, max} = M\gamma_o$

Conclusion:  $\bar{\Gamma}_{MRC, max}$  varies linearly with the number of diversity branches  $M$ , for the special case where all the  $\gamma_i$ 's are identically distributed

Improvement in Average Received SNR relative to No Diversity = Diversity Gain,  $G_{MRC}$ :

$$G_{MRC} = \frac{\bar{\Gamma}_{MRC, max}}{\gamma_o} = M$$



## Outage Probability for the MRC

The expression for the probability density function for  $\Gamma_{MRC,max}$  is:

$$f_{\Gamma_{MRC,max}}(x) = \frac{1}{(M-1)!} \frac{x^{M-1}}{\gamma_o^M} \exp\left(-\frac{x}{\gamma_o}\right)$$

- The outage probability is the probability that  $\Gamma_{MRC,max}$  is less than a specified threshold  $\Gamma_{th}$ :

$$P_{outage,MRC} = Prob\{\Gamma_{MRC,max} \leq \Gamma_{th}\} = \int_0^{\Gamma_{th}} f_{\Gamma_{MRC,max}}(x) dx = 1 - \exp(-\Gamma_{th}/\gamma_o) \sum_{j=0}^{M-1} \frac{(\Gamma_{th}/\gamma_o)^j}{j!}$$

By changing the summation index “j” to “j-1”:

$$P_{outage,MRC} = 1 - \exp(-\Gamma_{th}/\gamma_o) \sum_{j=1}^M \frac{(\Gamma_{th}/\gamma_o)^{(j-1)}}{(j-1)!}$$

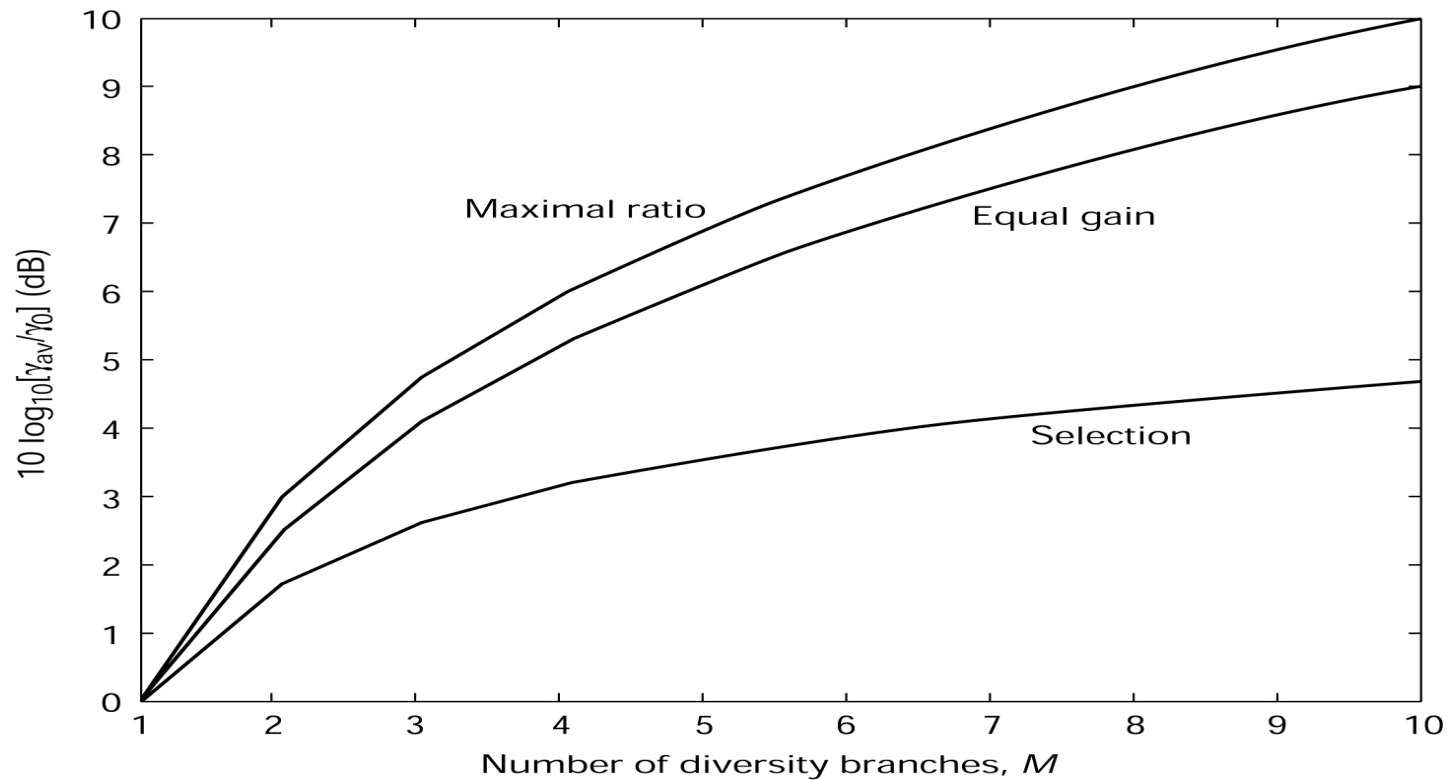
## Class Example

### Problem Statement:

Consider a two-branch MRC system. The outage occurs when the SNR  $\Gamma_{MRC, max}$  goes below one-fourth of the average signal to noise ratio.

- a) Show that the outage probability with a two-branch MRC is smaller than the outage probability with no diversity.
- b) Compare the outage probability with a 2-branch MRC with the result obtained for the 2-branch selection diversity (Class Example on Page 14)

# SNR Gain Comparison for MicroDiversity Combining Schemes



## Conclusions

- Maximal ratio Combiner exhibits the best performance, closely followed by the Equal Gain Combiner
- Selection Combiner has the least performance but is the easiest to implement

# MODEM Performance over Wireless Channels with Diversity

Task: Compute the average probability of error for a given MODEM over a specified wireless channel model and diversity reception

Analysis Steps:

Step 1: For the given MODEM and wireless channel, determine  $P_e(x)$ , the probability of bit error for any arbitrary received SNR  $x$  (same idea used on pages 31 & 32 of Topic 3 Notes)

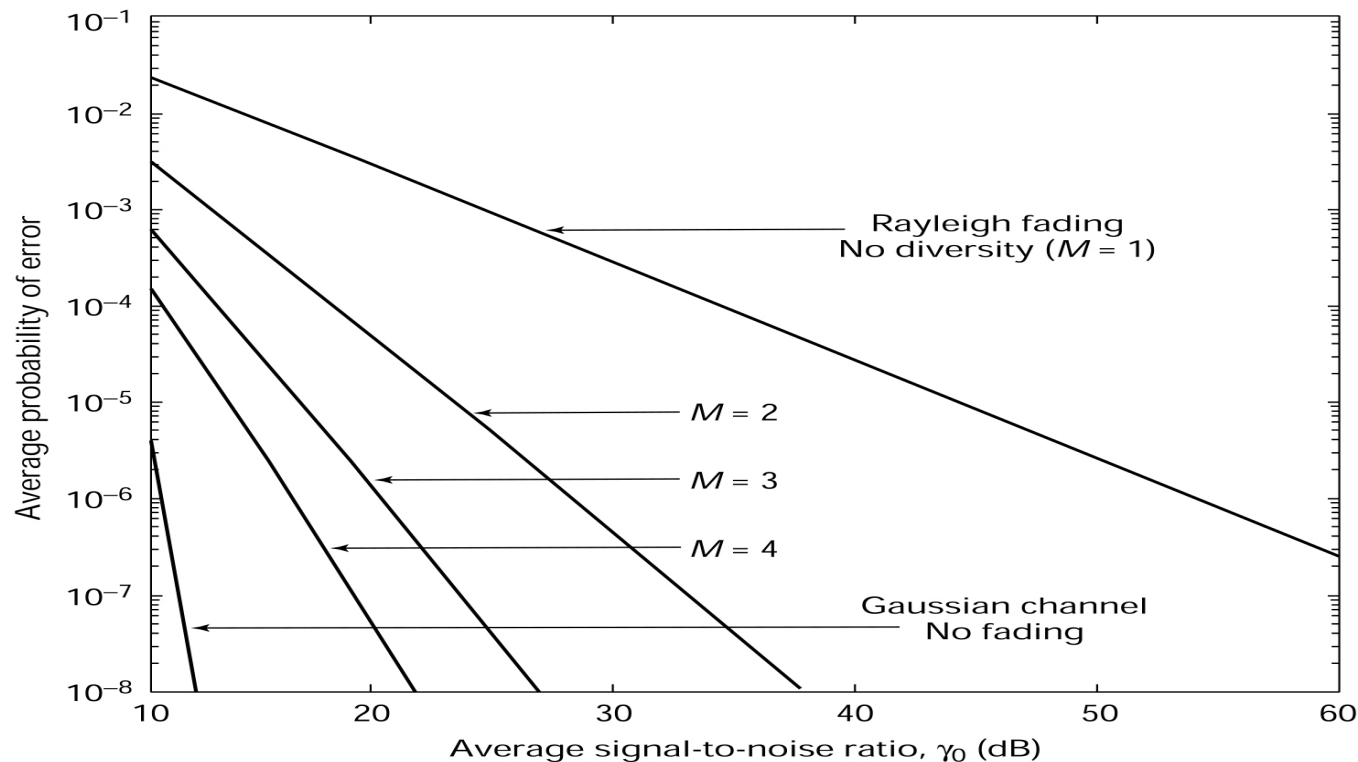
Note:  $x$  is the faded SNR at the diversity combiner output

Step 2: Determine  $f_{\Gamma_{div}}(x)$ , the probability density function of the SNR for the given wireless channel and diversity combining technique  $div = \{SC, MRC, EGC\}$ .

Step 3: Average the probability of error over the distribution of SNR to determine  $P_{e,av,div}$ , the average probability of bit error (a.k.a “bit error rate”) with diversity:  
$$P_{e,av,div} = \int_0^{\infty} P_e(x) f_{\Gamma}(x) (dx)$$

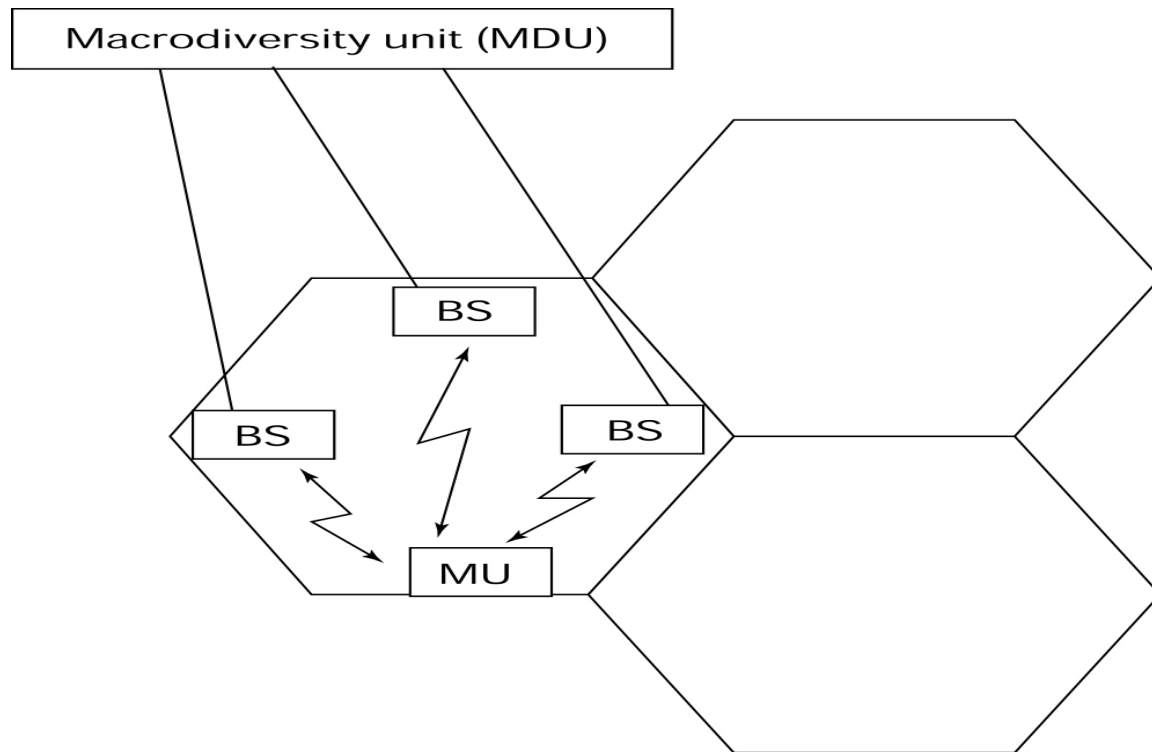
NOTE: In practice, closed-form analytic results for  $P_{e,av,div}$  are very difficult to derive, hence resort is made to approximate analytic expressions or numerical computations

## Average Probability of Error for Selection Diversity for Coherent BPSK



- Conclusions:
- 1) As  $M$  (the diversity order) increases, the average probability of error approaches that of an additive white Gaussian noise channel
  - 2) Diversity gain decreases with successive increase in diversity order

# Concept of Macrodiversity



## Macrodiversity Unit (MDU) Operation:

*Uplink:* Combines the signals from different BS's to create the data from the MS

*Downlink:* Sends signal to one of the multiple BS's or distributes the signal to all the multiple BS's

## Outage Probability Performance Improvement with Macrodiversity

### Without Macrodiversity:

Recall from Topic 2 Notes, outage probability due to shaddowing (Page 32):

$$P_{out} = Prob\{P_r < P_{th}\} = Q\left(\frac{P_{av} - P_{th}}{\sigma_{dB}}\right) = Q(SNR_{lgf, dB}), \quad \text{where } SNR_{lgf, dB} = \frac{P_{av} - P_{th}}{\sigma_{dB}}$$

### With Macrodiversity:

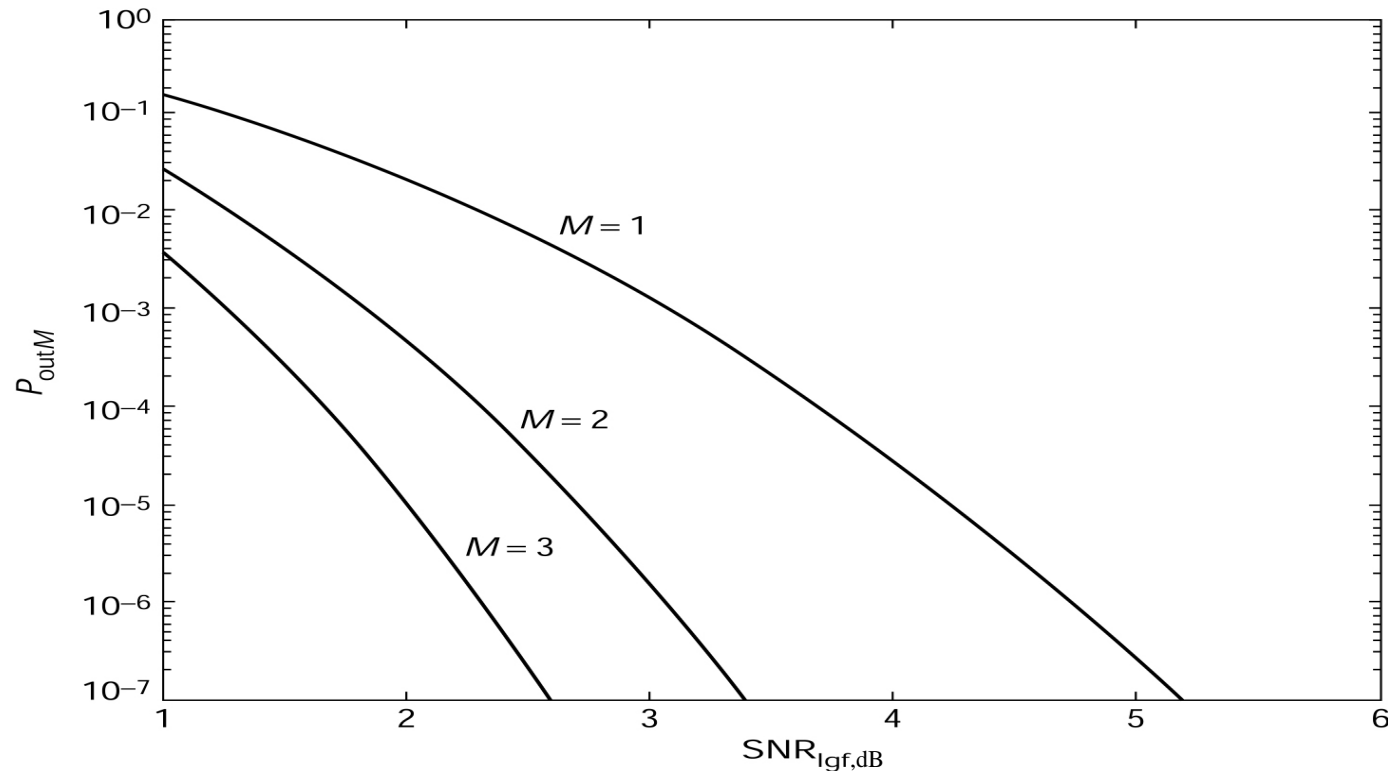
- assume  $M$  identical BS's, each operating independently of others
- assume all BS's receive the same average power and have same standard deviation of fading

### Analysis:

- Probability that all  $M$  BS's will be in outage simultaneously is given by the product of individual BS outage probabilities:

$$P_{out, macro} = [P_{out}]^M = [Q(SNR_{lgf, dB})]^M$$

## Outage Probability Performance Improvement with Macrodiversity



Conclusions: 1. The improvement increases with macro-diversity order  $M$   
2. For a given  $M$ , the outage probability improves with increasing  $\text{SNR}_{\text{lgf,dB}}$  (i.e. at low severity of fading  $\sigma_{\text{dB}}$ ).



## Class Example

### Problem Statement:

Suppose the power received at the MS from a base station is lognormal (i.e., normal in dBm unit) with an average of -95 dBm and a standard deviation of 8 dB. What is the outage probability if the minimum received power required for acceptable communication is -98 dBm? [Class Example on Page 34 of Topic 2 Notes]

If the MS now moves into the overlapping coverage area of three base stations (i.e., macrodiversity order  $M$  of 3), what is the improvement in outage probability.

# Combined Microdiversity and Macrodiversity System

## Characteristics:

- both microdiversity and macrodiversity are implemented simultaneously
- microdiversity is performed at each BS with diversity receivers to mitigate short-term fading
- macrodiversity is performed at the Macroscopic Diversity Unit (MDU) to mitigate the effects of long-term fading

