ENEL 441 Lab Demo #1

Group: GC7

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System Model Analysis

The initial model for the system is given as:

$$mc\ddot{\theta}(t) = -bc\dot{\theta}(t) + K_m u(t) - Mg$$

$$\ddot{\theta}(t) = \frac{1}{mc}[K_m u(t) - bc\dot{\theta}(t) - Mg]$$

We can state some substitutions for simplification:

$$x(t) = c\theta(t)$$

$$v(t) = c\dot{\theta}(t) = \dot{x}(t)$$

$$a(t) = \dot{v}(t) = c\ddot{\theta}(t) = \ddot{x}(t)$$

so we can rewrite (2) in two different ways:

$$\dot{v}(t) = \frac{1}{m}[K_m u(t) - bv(t) - Mg]$$

$$\ddot{x}(t) = \frac{1}{m}[K_m u(t) - b\dot{x}(t) - Mg]$$

Now we can focus on simplifying this even more by looking at our control signal and imaging it has DC (Steady-State) and AC (Transient) parts:

$$u(t) = C_u + \mu(t)$$

We can set the DC portion equal to the value necessary to cancel out the force of the attached weight:

$$K_m C_u = Mg$$

Since these cancel, this now gives the system as:

$$\dot{v} = \frac{1}{m}[K_m \mu(t) - bv(t)]$$

$$\ddot{x} = \frac{1}{m}[K_m \mu(t) - b\dot{x}(t)]$$

This can be converted via the Laplace Transform to:

$$sV(s) = \frac{1}{m}[K_m \mu(s) - bV(s)]$$

$$s^2X(s) = \frac{1}{m}[K_m\mu(s) - bsX(s)]$$

To find the Transfer function for the motor for Velocity and Position Respectively:

$$V(s)[s + \frac{b}{m}] = \frac{K_m}{m}\mu(s) \qquad \rightarrow \qquad H(s) = \frac{V(s)}{\mu(s)} = \frac{K_m}{ms + b}$$

$$X(s)[ms^2 + bs] = K_m\mu(s) \qquad \rightarrow \qquad G(s) = \frac{X(s)}{\mu(s)} = \frac{K_m}{s(ms + b)}$$

However, we also need the controller (ie. compensator) for our system such that our car will be as efficient as possible:

$$C(s) = \frac{\mu(s)}{E(s)} = ?$$

Where $E(s)$ is the Laplace Transform of the positional error $e(t)$:

$$e(t) = X_{des} - x(t)$$

$$E(s) = \frac{X_{des}}{s} - X(s)$$

We must find the compensator transfer function. It must be a PID controller (Type-2 System Controller) as it is required to bring both the error in position and speed down to zero. The control signal function will result as:

$$\mu = K_pe(t) + K_i\int e(t) + K_d\frac{de(t)}{dt}$$

Via Laplace Transform:

$$\mu(s) = K_pE(s) + K_i\frac{E(s)}{s} + K_dsE(s)$$

Which gives us:

$$C(s) = \frac{\mu(s)}{E(s)} = K_P + \frac{K_i}{s} + K_ds$$

Simulink Models

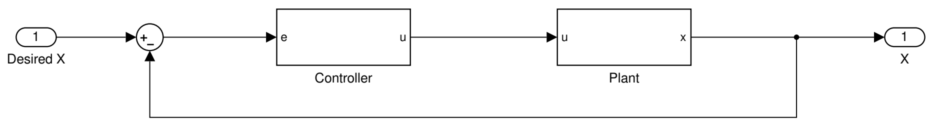

Figure 1: Simple Controller-Plant Model

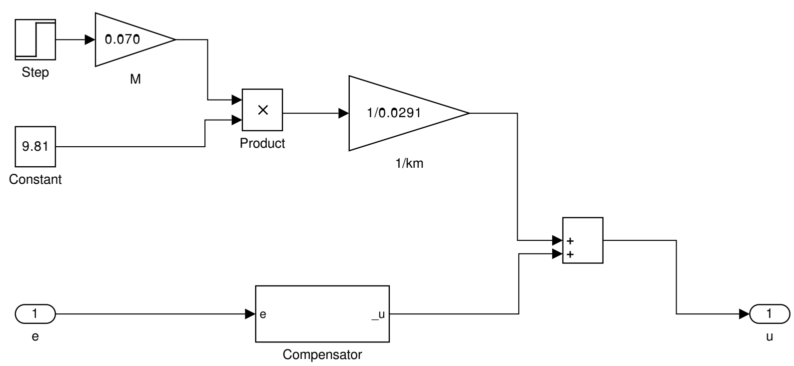

Figure 2: Controller Subsystem Model

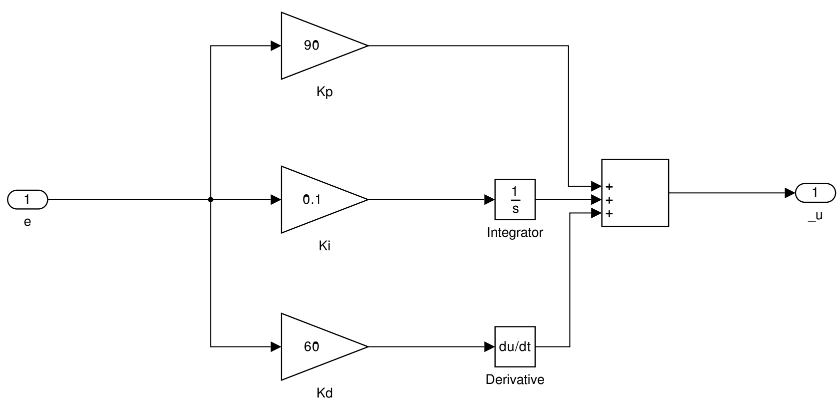

Figure 3: Compensator Subsystem Model, Transfer Function C(s)

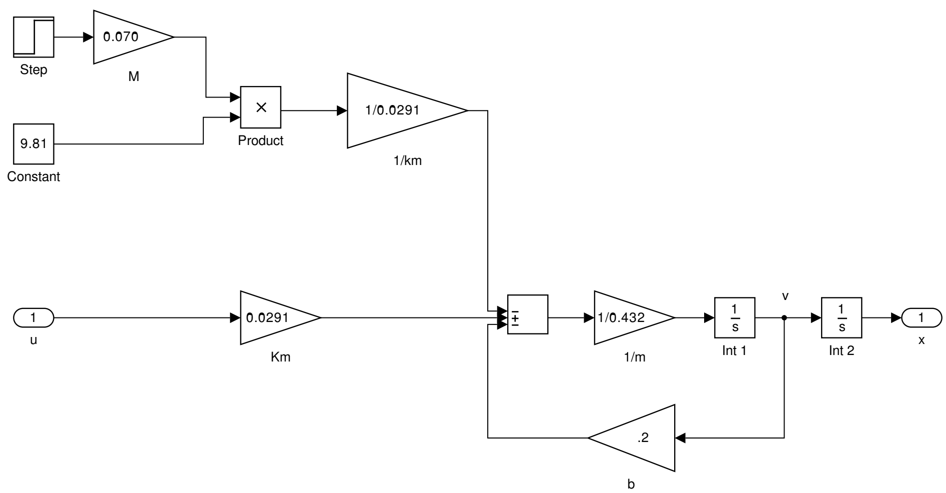

Figure 4: Plant Subsystem Model, Transfer Function G(s)

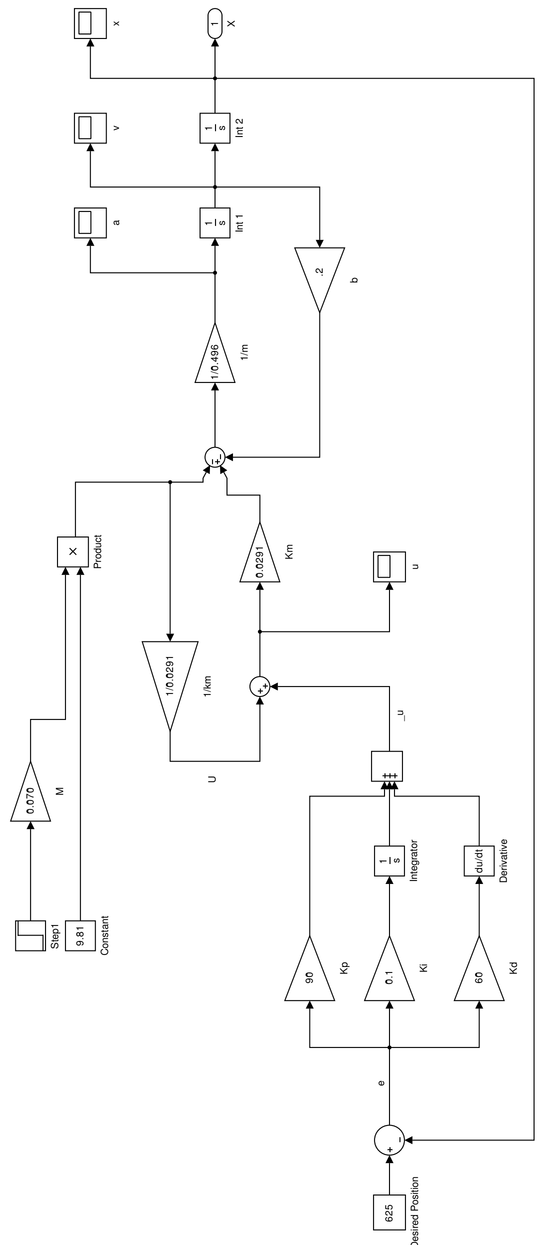

Figure 5: Fully expanded System Model