

**Problem 1.**

(a) Set 2.1

13.  $\neg(p \wedge q) \vee (p \vee q)$

$p$	$q$	$\neg(p \wedge q)$	$(p \vee q)$	$\neg(p \wedge q) \vee (p \vee q)$
$F$	$F$	$T$	$F$	$T$
$F$	$T$	$T$	$T$	$T$
$T$	$F$	$T$	$T$	$T$
$T$	$T$	$F$	$T$	$T$

17.  $\neg(p \wedge q) \stackrel{?}{\equiv} \neg p \wedge \neg q$

$p$	$q$	$\neg(p \wedge q)$	$\neg p \wedge \neg q$
$F$	$F$	$T$	$T$
$F$	$T$	$T$	$F$
$T$	$F$	$T$	$F$
$T$	$T$	$F$	$F$

$\neg(p \wedge q) \not\equiv \neg p \wedge \neg q$  because they do not have identical truth values for all possible substitutions.

(b) Set 2.1

22.  $p \wedge (q \vee r) \stackrel{?}{\equiv} (p \wedge q) \vee (p \wedge r)$

$p$	$q$	$r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
$F$	$F$	$F$	$F$	$F$
$F$	$F$	$T$	$F$	$F$
$F$	$T$	$F$	$F$	$F$
$F$	$T$	$T$	$F$	$F$
$T$	$F$	$F$	$F$	$F$
$T$	$F$	$T$	$T$	$T$
$T$	$T$	$F$	$T$	$T$
$T$	$T$	$T$	$T$	$T$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$  because they have identical truth values for all possible substitutions.

24.  $(p \vee q) \vee (p \wedge r) \stackrel{?}{\equiv} (p \vee q) \wedge r$

$p$	$q$	$r$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
$F$	$F$	$F$	$F$	$F$
$F$	$F$	$T$	$F$	$F$
$F$	$T$	$F$	$T$	$F$
$F$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$	$F$
$T$	$F$	$T$	$T$	$T$
$T$	$T$	$F$	$T$	$F$
$T$	$T$	$T$	$T$	$T$

$(p \vee q) \vee (p \wedge r) \not\equiv (p \vee q) \wedge r$  because they do not have identical truth values for all possible substitutions.

(c) Set 2.1

42.  $((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$

$p$	$q$	$r$	$\neg p \wedge q$	$q \wedge r$	$(\neg p \wedge q) \wedge (q \wedge r)$	$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$
$F$	$F$	$F$	$F$	$F$	$F$	$F$
$F$	$F$	$T$	$F$	$F$	$F$	$F$
$F$	$T$	$F$	$T$	$F$	$F$	$F$
$F$	$T$	$T$	$T$	$T$	$T$	$F$
$T$	$F$	$F$	$F$	$F$	$F$	$F$
$T$	$F$	$T$	$F$	$F$	$F$	$F$
$T$	$T$	$F$	$F$	$F$	$F$	$F$
$T$	$T$	$T$	$F$	$T$	$F$	$F$

Contradiction.

43.  $(\neg p \vee q) \vee (p \wedge \neg q)$

$p$	$q$	$r$	$\neg p \vee q$	$p \wedge \neg q$	$(\neg p \vee q) \vee (p \wedge \neg q)$
$F$	$F$	$F$	$T$	$F$	$T$
$F$	$F$	$T$	$T$	$F$	$T$
$F$	$T$	$F$	$T$	$F$	$T$
$F$	$T$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$F$	$T$	$T$
$T$	$F$	$T$	$F$	$T$	$T$
$T$	$T$	$F$	$T$	$F$	$T$
$T$	$T$	$T$	$T$	$T$	$T$

Tautology.

(d) Set 2.1

46.

$$\begin{aligned}
\text{(a)} \quad p \oplus p &\equiv (p \vee p) \wedge \neg(p \wedge p) & (p \oplus p) \oplus p &\equiv F \oplus p \\
&\equiv T \wedge \neg T & &\equiv (F \vee p) \wedge \neg(F \wedge p) \\
&\equiv F & &\equiv p \wedge T \\
& & &\equiv p
\end{aligned}$$

(b)  $(p \oplus q) \oplus r \stackrel{?}{=} p \oplus (q \oplus r)$

$p$	$q$	$r$	$p \oplus q$	$(p \oplus q) \oplus r$	$q \oplus r$	$p \oplus (q \oplus r)$
$F$	$F$	$F$	$F$	$F$	$F$	$F$
$F$	$F$	$T$	$F$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$T$	$T$	$T$
$F$	$T$	$T$	$T$	$F$	$F$	$F$
$T$	$F$	$F$	$T$	$T$	$F$	$T$
$T$	$F$	$T$	$T$	$F$	$T$	$F$
$T$	$T$	$F$	$F$	$F$	$T$	$F$
$T$	$T$	$T$	$F$	$T$	$F$	$T$

$(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$  because they have identical truth values for all possible substitutions.

(e) Set 2.2

6.  $(p \vee q) \vee (\neg p \wedge q) \rightarrow q$

$p$	$q$	$p \vee q$	$\neg p \wedge q$	$(p \vee q) \vee (\neg p \wedge q)$	$(p \vee q) \vee (\neg p \wedge q) \rightarrow q$
$F$	$F$	$F$	$F$	$F$	$T$
$F$	$T$	$T$	$F$	$T$	$T$
$T$	$F$	$T$	$T$	$T$	$F$
$T$	$T$	$T$	$F$	$T$	$T$

8.  $\neg p \vee q \rightarrow r$

$p$	$q$	$r$	$\neg p \vee q$	$\neg p \vee q \rightarrow r$
$F$	$F$	$F$	$T$	$F$
$F$	$F$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$F$
$F$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$F$	$T$
$T$	$F$	$T$	$F$	$T$
$T$	$T$	$F$	$T$	$F$
$T$	$T$	$T$	$T$	$T$

13.

(a)  $p \rightarrow q \equiv \neg p \vee q$

$p$	$q$	$p \rightarrow q$	$\neg p \vee q$
$F$	$F$	$T$	$T$
$F$	$T$	$T$	$T$
$T$	$F$	$F$	$F$
$T$	$T$	$T$	$T$

They have identical truth values for all possible substitutions.

(b)  $\neg(p \rightarrow q) \equiv p \wedge \neg q$

$p$	$q$	$\neg(p \rightarrow q)$	$p \wedge \neg q$
$F$	$F$	$F$	$F$
$F$	$T$	$F$	$F$
$T$	$F$	$T$	$T$
$T$	$T$	$F$	$F$

They have identical truth values for all possible substitutions.

(f) Set 2.2

10.  $(p \rightarrow r) \leftrightarrow (q \rightarrow r)$

$p$	$q$	$r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \leftrightarrow (q \rightarrow r)$
$F$	$F$	$F$	$T$	$T$	$T$
$F$	$F$	$T$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$F$	$F$
$F$	$T$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$F$	$T$	$F$
$T$	$F$	$T$	$T$	$T$	$T$
$T$	$T$	$F$	$F$	$F$	$T$
$T$	$T$	$T$	$T$	$T$	$T$

11.  $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

$p$	$q$	$r$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$
$F$	$F$	$F$	$T$	$T$	$F$	$T$	$T$
$F$	$F$	$T$	$T$	$T$	$F$	$T$	$T$
$F$	$T$	$F$	$F$	$T$	$F$	$T$	$T$
$F$	$T$	$T$	$T$	$T$	$F$	$T$	$T$
$T$	$F$	$F$	$T$	$T$	$F$	$T$	$T$
$T$	$F$	$T$	$T$	$T$	$F$	$T$	$T$
$T$	$T$	$F$	$F$	$F$	$T$	$F$	$T$
$T$	$T$	$T$	$T$	$T$	$T$	$T$	$T$

(g) Set 2.2

30.  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

$p$	$q$	$r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
$F$	$F$	$F$	$F$	$F$	$T$
$F$	$F$	$T$	$F$	$F$	$T$
$F$	$T$	$F$	$F$	$F$	$T$
$F$	$T$	$T$	$F$	$F$	$T$
$T$	$F$	$F$	$F$	$F$	$T$
$T$	$F$	$T$	$T$	$T$	$T$
$T$	$T$	$F$	$T$	$T$	$T$
$T$	$T$	$T$	$T$	$T$	$T$

31.  $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

$p \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$

$p$	$q$	$r$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$	$p \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$
$F$	$F$	$F$	$T$	$T$	$F$	$T$	$T$
$F$	$F$	$T$	$T$	$T$	$F$	$T$	$T$
$F$	$T$	$F$	$F$	$T$	$F$	$T$	$T$
$F$	$T$	$T$	$T$	$T$	$F$	$T$	$T$
$T$	$F$	$F$	$T$	$T$	$F$	$T$	$T$
$T$	$F$	$T$	$T$	$T$	$F$	$T$	$T$
$T$	$T$	$F$	$F$	$F$	$T$	$F$	$T$
$T$	$T$	$T$	$T$	$T$	$T$	$T$	$T$

(h) Set 2.2

25. A conditional statement is not logically equivalent to its inverse.

$a \rightarrow b \not\equiv \neg a \rightarrow \neg b$

$a$	$b$	$a \rightarrow b$	$\neg a \rightarrow \neg b$
$F$	$F$	$T$	$T$
$F$	$T$	$T$	$F$
$T$	$F$	$F$	$T$
$T$	$T$	$T$	$T$

27. The converse and inverse of a conditional statement are logically equivalent to each other.

$b \rightarrow a \equiv \neg a \rightarrow \neg b$

$a$	$b$	$b \rightarrow a$	$\neg a \rightarrow \neg b$
$F$	$F$	$T$	$T$
$F$	$T$	$F$	$F$
$T$	$F$	$T$	$T$
$T$	$T$	$T$	$T$

## Problem 2.

(a) Set 2.1

26. Sam isn't an orange belt nor is Kate a red belt.

28. The units digit of  $4^{67}$  is not 4 and not 6.

29. This computer program doesn't have a logical error in the first ten lines and it isn't being run with an incomplete data set.

30. The dollar isn't at an all-time high nor is the stock market at a record low.

31. The train isn't late and my watch isn't fast.

(b) Set 2.1

33.  $-10 \geq x \geq 2$

35.  $x < -1$  and  $x \leq 1$

37.  $0 \leq x < -7$

39.  $(\text{num\_orders} \geq 50 \text{ or } \text{num\_instock} \leq 300)$  and  
 $(50 > \text{num\_orders} \geq 75 \text{ or } \text{num\_instock} \leq 500)$

(c) Set 2.2

20.

- (a)  $P$  is a square and  $P$  isn't a rectangle.
- (b) Today is New Year's Eve and tomorrow isn't January.
- (c) The decimal expansion of  $r$  is terminating and  $r$  isn't rational.
- (d)  $n$  is prime and  $n$  isn't positive nor 2.
- (e)  $x$  is nonnegative and  $x$  isn't positive and not 0.
- (f) Tom is Ann's father and Jim isn't her Uncle nor is Sue her aunt.
- (g)  $n$  is divisible by 6 and  $n$  isn't divisible by 2 nor 3.

**Problem 3.**

(a) Set 2.2

- 33. If this integer is even, then it equals twice some integer; and if this integer equals twice some integer, then it is even.
- 35. If Sam is allowed on Signe's racing boat, then he is an expert sailor.  
If Sam isn't an expert sailor, then he isn't allowed on Signe's racing boat.
- 38. If it doesn't rain, then Ann will go.
- 39. If the security code isn't entered, then the door won't open.

(b) Set 2.2

- 41. If this triangle has two  $45^\circ$  angles, then it is a right triangle.
- 43. If Jim didn't do homework regularly, then he won't pass the course.  
If Jim passes the course, then he did his homework regularly.
- 45. If this computer program isn't correct, then it produces error messages during translation.  
If this computer program doesn't produce error messages during translation, then it is correct.
- 46.  $X$  is boiling  $\rightarrow$  temperature  $\geq 150^\circ\text{C}$ 
  - (a) temperature  $\geq 150^\circ\text{C} \not\rightarrow X$  is boiling  
Not necessarily true.
  - (b) temperature  $< 150^\circ\text{C} \rightarrow X$  isn't boiling  
Always true.
  - (c)  $X$  is boiling  $\not\rightarrow$  temperature  $\geq 150^\circ\text{C}$   
Not necessarily true.
  - (d)  $X$  isn't boiling  $\rightarrow$  temperature  $< 150^\circ\text{C}$   
Not necessarily true.
  - (e)  $X$  is boiling  $\not\rightarrow$  temperature  $\geq 150^\circ\text{C}$   
Not necessarily true.
  - (f)  $X$  is boiling  $\rightarrow$  temperature  $\geq 150^\circ\text{C}$   
Always true.

**Problem 4.**

(a) Set 2.3

11.  $p \rightarrow q \vee r$

$\neg q \vee \neg r$

$\therefore \neg p \vee \neg r$

$p$	$q$	$r$	Premise 1 $p \rightarrow q \vee r$	Premise 2 $\neg q \vee \neg r$	Conclusion $\neg p \vee \neg r$
$F$	$F$	$F$	$T$	$T$	$T$
$F$	$F$	$T$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$T$	$T$
$F$	$T$	$T$	$T$	$F$	
$T$	$F$	$F$	$F$	$T$	
$T$	$F$	$T$	$T$	$T$	$T$
$T$	$T$	$F$	$T$	$T$	$F$
$T$	$T$	$T$	$T$	$F$	

Invalid argument because true premises can lead to a false conclusion.

**Problem 5.**

(a) Set 2.3

38.

(b) Another two natives  $C$  and  $D$  approach you but only  $C$  speaks.

$C$  says: Both of us are knaves.

What are  $C$  and  $D$ ?

**Suppose  $C$  is a knight.**

$\therefore$  What  $C$  says is true

definition of knight

$\therefore C$  and  $D$  are knaves

what  $C$  said

$\therefore C$  is a knave

specialization

$\therefore C$  is a knight and a knave

$\therefore$  The supposition is false

contradiction

**Suppose  $C$  is a knave.**

$\therefore$  What  $C$  says is false

definition of knave

$\therefore C$  is a knight or  $D$  is a knight

opposite of what  $C$  said

$\therefore D$  is a knight

by elimination

$\therefore C$  is a knave and  $D$  is a knight

(c) You then encounter natives  $E$  and  $F$ .

$E$  says:  $F$  is a knave.

$F$  says:  $E$  is a knave.

How many knaves are there?

**Suppose  $E$  and  $F$  are knaves.**

$\therefore F$  is a knight

opposite of what  $E$  said

$\therefore F$  is a knight and a knave

contradiction

**Suppose  $E$  is a knave and  $F$  is a knight.**

$\therefore F$  is a knight

opposite of what  $E$  said

$\therefore E$  is a knave

what  $F$  said

$\therefore$  1 knight and 1 knave

**Suppose  $E$  is a knight and  $F$  is a knave.**

$\therefore F$  is a knave

what  $E$  said

$\therefore E$  is a knight

opposite of what  $F$  said

$\therefore$  1 knight and 1 knave

**Suppose  $E$  and  $F$  are knights.**

$\therefore F$  is a knave

what  $E$  said

$\therefore F$  is a knight and a knave

contradiction

There is 1 knave.

# **Problem 6.**

(a) Set 2.3

44.

$p \rightarrow q$		(1)
$r \vee s$		(2)
$\neg s \rightarrow \neg t$		(3)
$\neg q \vee s$		(4)
$\neg s$		(5)
$\neg p \wedge r \rightarrow u$		(6)
$w \vee t$		(7)
$\therefore r$	elimination by (2) and (5)	(8)
$\therefore \neg t$	modus ponens by (3) and (5)	(9)
$\therefore \neg q$	elimination by (4) and (5)	(10)
$\therefore \neg p$	modus tollens by (1) and (10)	(11)
$\therefore u$	modus ponens by (11), (8) and (6)	(12)
$\therefore w$	elimination by (7) and (9)	(13)
$\therefore u \wedge w$	conjunction by (12) and (14)	(14)

**Problem 7.**

- (a) For the circuit corresponding to the following Boolean expression, there is an equivalent circuit with at most two logic gates. Find such a circuit.
- $$(\neg P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$$

Using DeMorgan's Laws to rewrite and using only nots and ors:

$$\neg(P \vee Q) \vee \neg(P \vee \neg Q) \vee \neg(\neg P \vee Q)$$

