Problem 1.

Set 3.1

10. $\forall$ positive integers m and n, $m \times n \geq m + n$

$m = 0, n = 1$ $0 \times 1 \overset{?}{\geq} 0 + 1$

$0 \not\geq 1$

12. $\forall$ real numbers x and y, $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$

$x = 1, y = 1$

$m = 0, n = 1$

$\sqrt{1 + 1} \overset{?}{=} \sqrt{1} + \sqrt{1}$

$\sqrt{2} \neq 2$

Problem 2.

Set 3.1

29.

    (a) $\exists x$ such that $\text{Rect}(x) \wedge \text{Square}(x)$

    There are geometric figures that are both rectangles and squares.

    True; squares are both rectangles and squares.

    (b) $\exists x$ such that $\text{Rect}(x) \wedge \neg\text{Square}(x)$

    There are geometric figures that are rectangles but not squares.

    True; rectangles of unequal side lengths are rectangles but not squares.

    (c) $\forall x$, $\text{Square}(x) \rightarrow \text{Rect}(x)$

    If a geometric figure is a square, it is a rectangle.

    True; squares have all the criteria of rectangles but have the added criteria of equal side lengths.

Problem 3.

Set 3.1

33.

    (c) $ab = 0 \Rightarrow a = 0$ or $b = 0$

    True

    (d) $a < b$ and $c < d \Rightarrow ac < bd$

    $a = -1, b = 0, c = -1, d = 0$

    $-1 \times -1 \overset{?}{<} 0 \times 0$

        $1 \not< 0$

    False

Problem 4.

Set 3.2

10. $\forall$ computer programs P, if P compiles without error messages, then P is correct.

$\exists P$ such that P compiles without error messages and isn't correct.

17. $\forall$ integers d, if $6/d$ is an integer then $d = 3$.

$\exists d$ such that $6/d$ is an integer and $d \neq 3$.

19. $\forall n \in \mathbf{Z}$, if n is prime then n is odd or $n = 2$.

$\exists n \in \mathbf{Z}$ such that n is prime and n is even and $n = 2$.

21. $\forall$ integers n, if n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

$\exists n$ such that n is divisible by 6 and not divisible by 2 and not divisible by 3.

23. If a function is differentiable then it is continuous.

There exists a function that is differentiable and not continuous.

Problem 5.

Set 3.2

40. Being divisible by 8 is a sufficient condition or being divisible by 4.
    If n is divisible by 7, then n is divisible by 4.

42. Passing a comprehensive exam is a necessary condition for obtaining a master's degree.
    If one does not pass a comprehensive exam, then one cannot obtain a master's degree.

44. Having a large income is not a necessary condition for a person to be happy.
    $\neg(\forall x(\text{HighIncome}(x) \leftrightarrow \text{Happy}(x)))$
    There exists a happy person that doesn't have a large income.

46. Being a polynomial is not a sufficient condition for a function to have a real root.
    There exists a non polynomial function

47.

Problem 6.

Set 3.3

41.

(c)

(d)

(f)

(g)

(h)

Problem 7.

Set 3.4

13.

14.

15.

17.

18.

Problem 8.

Set 3.4

22.

23.

24.

26.

27.