

Problem 1.

(a) Set 2.1

13. $\neg(p \wedge q) \vee (p \vee q)$

p	q	$\neg(p \wedge q)$	$(p \vee q)$	$\neg(p \wedge q) \vee (p \vee q)$
F	F	T	F	T
F	T	T	T	T
T	F	T	T	T
T	T	F	T	T

17. $\neg(p \wedge q) \stackrel{?}{\equiv} \neg p \wedge \neg q$

p	q	$\neg(p \wedge q)$	$\neg p \wedge \neg q$
F	F	T	T
F	T	T	F
T	F	T	F
T	T	F	F

$\neg(p \wedge q) \not\equiv \neg p \wedge \neg q$ because they do not have identical truth values for all possible substitutions.

(b) Set 2.1

22. $p \wedge (q \vee r) \stackrel{?}{\equiv} (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ because they have identical truth values for all possible substitutions.

24. $(p \vee q) \vee (p \wedge r) \stackrel{?}{\equiv} (p \vee q) \wedge r$

p	q	r	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
F	F	F	F	F
F	F	T	F	F
F	T	F	T	F
F	T	T	T	T
T	F	F	T	F
T	F	T	T	T
T	T	F	T	F
T	T	T	T	T

$(p \vee q) \vee (p \wedge r) \not\equiv (p \vee q) \wedge r$ because they do not have identical truth values for all possible substitutions.

(c) Set 2.1

42. $((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$

p	q	r	$\neg p \wedge q$	$q \wedge r$	$(\neg p \wedge q) \wedge (q \wedge r)$	$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$
F	F	F	F	F	F	F
F	F	T	F	F	F	F
F	T	F	T	F	F	F
F	T	T	T	T	T	F
T	F	F	F	F	F	F
T	F	T	F	F	F	F
T	T	F	F	F	F	F
T	T	T	F	T	F	F

Contradiction.

43. $(\neg p \vee q) \vee (p \wedge \neg q)$

p	q	r	$\neg p \vee q$	$p \wedge \neg q$	$(\neg p \vee q) \vee (p \wedge \neg q)$
F	F	F	T	F	T
F	F	T	T	F	T
F	T	F	T	F	T
F	T	T	T	T	T
T	F	F	F	T	T
T	F	T	F	T	T
T	T	F	T	F	T
T	T	T	T	T	T

Tautology.

(d) Set 2.1

46.

$$\begin{aligned}
\text{(a) } p \oplus p &\equiv (p \vee p) \wedge \neg(p \wedge p) & (p \oplus p) \oplus p &\equiv F \oplus p \\
&\equiv T \wedge \neg T & &\equiv (F \vee p) \wedge \neg(F \wedge p) \\
&\equiv F & &\equiv p \wedge T \\
& & &\equiv p
\end{aligned}$$

(b) $(p \oplus q) \oplus r \stackrel{?}{=} p \oplus (q \oplus r)$

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r$	$q \oplus r$	$p \oplus (q \oplus r)$
F	F	F	F	F	F	F
F	F	T	F	T	T	T
F	T	F	T	T	T	T
F	T	T	T	F	T	F
T	F	F	T	T	F	T
T	F	T	T	F	T	F
T	T	F	F	F	T	F
T	T	T	F	T	F	T

$(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$ because they have identical truth values for all possible substitutions.

(e) Set 2.2

6. $(p \vee q) \vee (\neg p \wedge q) \rightarrow q$

p	q	$p \vee q$	$\neg p \wedge q$	$(p \vee q) \vee (\neg p \wedge q)$	$(p \vee q) \vee (\neg p \wedge q) \rightarrow q$
F	F	F	F	F	T
F	T	T	F	T	T
T	F	T	T	T	F
T	T	T	F	T	T

8. $\neg p \vee q \rightarrow r$

p	q	r	$\neg p \vee q$	$\neg p \vee q \rightarrow r$
F	F	F	T	F
F	F	T	T	T
F	T	F	T	F
F	T	T	T	T
T	F	F	F	T
T	F	T	F	T
T	T	F	T	F
T	T	T	T	T

13.

(a) $p \rightarrow q \equiv \neg p \vee q$

p	q	$p \rightarrow q$	$\neg p \vee q$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

They have identical truth values for all possible substitutions.

(b) $\neg(p \rightarrow q) \equiv p \wedge \neg q$

p	q	$\neg(p \rightarrow q)$	$p \wedge \neg q$
F	F	F	F
F	T	F	F
T	F	T	T
T	T	F	F

They have identical truth values for all possible substitutions.

(f) Set 2.2

10. $(p \rightarrow r) \leftrightarrow (q \rightarrow r)$

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \leftrightarrow (q \rightarrow r)$
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	F	F
F	T	T	T	T	T
T	F	F	F	T	F
T	F	T	T	T	T
T	T	F	F	F	T
T	T	T	T	T	T

11. $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$
F	F	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	T	T	T	T	F	T	T
T	F	F	T	T	F	T	T
T	F	T	T	T	F	T	T
T	T	F	F	F	T	F	T
T	T	T	T	T	T	T	T

(g) Set 2.2

30. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
F	F	F	F	F	T
F	F	T	F	F	T
F	T	F	F	F	T
F	T	T	F	F	T
T	F	F	F	F	T
T	F	T	T	T	T
T	T	F	T	T	T
T	T	T	T	T	T

31. $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

$p \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$	$p \rightarrow (q \rightarrow r) \leftrightarrow (p \wedge q) \rightarrow r$
F	F	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	T	T	T	T	F	T	T
T	F	F	T	T	F	T	T
T	F	T	T	T	F	T	T
T	T	F	F	F	T	F	T
T	T	T	T	T	T	T	T

(h) Set 2.2

25. A conditional statement is not logically equivalent to its inverse.

$a \rightarrow b \not\equiv \neg a \rightarrow \neg b$

a	b	$a \rightarrow b$	$\neg a \rightarrow \neg b$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

27. The converse and inverse of a conditional statement are logically equivalent to each other.

$b \rightarrow a \equiv \neg a \rightarrow \neg b$

a	b	$b \rightarrow a$	$\neg a \rightarrow \neg b$
F	F	T	T
F	T	F	F
T	F	T	T
T	T	T	T

Problem 2.

(a) Set 2.1

26. Sam isn't an orange belt nor is Kate a red belt.

28. The units digit of 4^{67} is not 4 and not 6.

29. This computer program doesn't have a logical error in the first ten lines and it isn't being run with an incomplete data set.

30. The dollar isn't at an all-time high nor is the stock market at a record low.

31. The train isn't late and my watch isn't fast.

(b) Set 2.1

33. $-10 \geq x \geq 2$

35. $x < -1$ and $x \leq 1$

37. $0 \leq x < -7$

39. $(\text{num_orders} \geq 50 \text{ or } \text{num_instock} \leq 300)$ and
 $(50 > \text{num_orders} \geq 75 \text{ or } \text{num_instock} \leq 500)$

(c) Set 2.2

20.

- (a) P is a square and P isn't a rectangle.
- (b) Today is New Year's Eve and tomorrow isn't January.
- (c) The decimal expansion of r is terminating and r isn't rational.
- (d) n is prime and n isn't positive nor 2.
- (e) x is nonnegative and x isn't positive and not 0.
- (f) Tom is Ann's father and Jim isn't her Uncle nor is Sue her aunt.
- (g) n is divisible by 6 and n isn't divisible by 2 nor 3.

Problem 3.

(a) Set 2.2

- 33. If this integer is even, then it equals twice some integer; and if this integer equals twice some integer, then it is even.
- 35. If Sam is allowed on Signe's racing boat, then he is an expert sailor.
If Sam isn't an expert sailor, then he isn't allowed on Signe's racing boat.
- 38. If it doesn't rain, then Ann will go.
- 39. If the security code isn't entered, then the door won't open.

(b) Set 2.2

- 41. If this triangle has two 45° angles, then it is a right triangle.
- 43. If Jim didn't do homework regularly, then he won't pass the course.
If Jim passes the course, then he did his homework regularly.
- 45. If this computer program isn't correct, then it produces error messages during translation.
If this computer program doesn't produce error messages during translation, then it is correct.
- 46. X is boiling \rightarrow temperature $\geq 150^\circ\text{C}$
 - (a) temperature $\geq 150^\circ\text{C} \not\rightarrow X$ is boiling
Not necessarily true.
 - (b) temperature $< 150^\circ\text{C} \rightarrow X$ isn't boiling
Always true.
 - (c) X is boiling $\not\rightarrow$ temperature $\geq 150^\circ\text{C}$
Not necessarily true.
 - (d) X isn't boiling \rightarrow temperature $< 150^\circ\text{C}$
Not necessarily true.
 - (e) X is boiling $\not\rightarrow$ temperature $\geq 150^\circ\text{C}$
Not necessarily true.
 - (f) X is boiling \rightarrow temperature $\geq 150^\circ\text{C}$
Always true.

Problem 4.

(a) Set 2.3

11. $p \rightarrow q \vee r$

$\neg q \vee \neg r$

$\therefore \neg p \vee \neg r$

p	q	r	Premise 1 $p \rightarrow q \vee r$	Premise 2 $\neg q \vee \neg r$	Conclusion $\neg p \vee \neg r$
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	T	T
F	T	T	T	F	
T	F	F	F	T	
T	F	T	T	T	T
T	T	F	T	T	F
T	T	T	T	F	

Invalid argument because true premises can lead to a false conclusion.

Problem 5.

(a) Set 2.3

38.

(b) Another two natives C and D approach you but only C speaks.

C says: Both of us are knaves.

What are C and D ?

Suppose C is a knight.

\therefore What C says is true

$\therefore C$ and D are knaves

$\therefore C$ is a knave

$\therefore C$ is a knight and a knave

\therefore The supposition is false

Suppose C is a knave.

\therefore What C says is false

$\therefore C$ is a knight or D is a knight

$\therefore D$ is a knight

$\therefore C$ is a knave and D is a knight

definition of knight

what C said

specialization

contradiction

definition of knave

opposite of what C said

by elimination

(c) You then encounter natives E and F .

E says: F is a knave.

F says: E is a knave.

How many knaves are there?

Suppose E and F are knaves.

$\therefore F$ is a knight

opposite of what E said

$\therefore F$ is a knight and a knave

contradiction

\therefore The supposition is false

Suppose E is a knave and F is a knight.

$\therefore F$ is a knight

opposite of what E said

$\therefore E$ is a knave

what F said

\therefore 1 knight and 1 knave

Suppose E is a knight and F is a knave.

$\therefore F$ is a knave

what E said

$\therefore E$ is a knight

opposite of what F said

\therefore 1 knight and 1 knave

Suppose E and F are knights.

$\therefore F$ is a knave

what E said

$\therefore F$ is a knight and a knave

contradiction

\therefore The supposition is false

There is 1 knave.

Problem 6.

(a) Set 2.3

44.

$$p \rightarrow q \quad (1)$$

$$r \vee s \quad (2)$$

$$\neg s \rightarrow \neg t \quad (3)$$

$$\neg q \vee s \quad (4)$$

$$\neg s \quad (5)$$

$$\neg p \wedge r \rightarrow u \quad (6)$$

$$w \vee t \quad (7)$$

$$\therefore r \quad \text{elimination by (2) and (5)} \quad (8)$$

$$\therefore \neg t \quad \text{modus ponens by (3) and (5)} \quad (9)$$

$$\therefore \neg q \quad \text{elimination by (4) and (5)} \quad (10)$$

$$\therefore \neg p \quad \text{modus tollens by (1) and (10)} \quad (11)$$

$$\therefore u \quad \text{modus ponens by (11), (8) and (6)} \quad (12)$$

$$\therefore w \quad \text{elimination by (7) and (9)} \quad (13)$$

$$\therefore u \wedge w \quad \text{conjunction by (12) and (14)} \quad (14)$$

Problem 7.

- (a) For the circuit corresponding to the following Boolean expression, there is an equivalent circuit with at most two logic gates. Find such a circuit.
- $$(\neg P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$$

Using DeMorgan's Laws to rewrite and using only nots and ors:

$$\neg(P \vee Q) \vee \neg(P \vee \neg Q) \vee \neg(\neg P \vee Q)$$

