Problem 1.

(a) Set 2.1

13. $\neg(p \wedge q) \vee (p \vee q)$

p	q	$\neg(p \wedge q)$	$(p \vee q)$	$\neg(p \wedge q) \vee (p \vee q)$
F	F	T	F	T
F	T	T	T	T
T	F	T	T	T
T	T	F	T	T

17. $\neg(p \wedge q) \stackrel{?}{\equiv} \neg p \wedge \neg q$

p	q	$\neg(p \wedge q)$	$\neg p \wedge \neg q$
F	F	T	T
F	T	T	F
T	F	T	F
T	T	F	F

$\neg(p \wedge q) \not\equiv \neg p \wedge \neg q$ because they do not have identical truth values for all possible substitutions.

(b) Set 2.1

22. $p \wedge (q \vee r) \stackrel{?}{\equiv} (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ because they have identical truth values for all possible substitutions.

24. $(p \vee q) \vee (p \wedge r) \stackrel{?}{\equiv} (p \vee q) \wedge r$

p	q	r	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
F	F	F	F	F
F	F	T	F	F
F	T	F	T	F
F	T	T	T	T
T	F	F	T	F
T	F	T	T	T
T	T	F	T	F
T	T	T	T	T

$(p \vee q) \vee (p \wedge r) \not\equiv (p \vee q) \wedge r$ because they do not have identical truth values for all possible substitutions.

(c) Set 2.1

42. $((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$

p	q	r	$\neg p \wedge q$	$q \wedge r$	$(\neg p \wedge q) \wedge (q \wedge r)$	$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q$
F	F	F	F	F	F	F
F	F	T	F	F	F	F
F	T	F	T	F	F	F
F	T	T	T	T	T	F
T	F	F	F	F	F	F
T	F	T	F	F	F	F
T	T	F	F	F	F	F
T	T	T	F	T	F	F

Contradiction.

43. $(\neg p \vee q) \vee (p \wedge \neg q)$

p	q	r	$\neg p \vee q$	$p \wedge \neg q$	$(\neg p \vee q) \vee (p \wedge \neg q)$
F	F	F	T	F	T
F	F	T	T	F	T
F	T	F	T	F	T
F	T	T	T	T	T
T	F	F	F	T	T
T	F	T	F	T	T
T	T	F	T	F	T
T	T	T	T	T	T

Tautology.

(d) Set 2.1

46.

(a)
$$\begin{aligned} p \oplus p &\equiv (p \vee p) \wedge \neg (p \wedge p) \\ &\equiv T \wedge \neg T \\ &\equiv F \end{aligned}$$
$$\begin{aligned} (p \oplus p) \oplus p &\equiv F \oplus p \\ &\equiv (F \vee p) \wedge \neg (F \wedge p) \\ &\equiv p \wedge T \\ &\equiv p \end{aligned}$$

(b) $(p \oplus q) \oplus r \stackrel{?}{\equiv} p \oplus (q \oplus r)$

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r$	$q \oplus r$	$p \oplus (q \oplus r)$
F	F	F	F	F	F	F
F	F	T	F	T	T	T
F	T	F	T	T	T	T
F	T	T	T	F	T	F
T	F	F	T	T	F	T
T	F	T	T	F	T	F
T	T	F	F	F	T	F
T	T	T	F	T	F	T

$(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$ because they have identical truth values for all possible substitutions.

(e) Set 2.2

6. $(p \vee q) \vee (\neg p \wedge q) \to q$

p	q	$p \vee q$	$\neg p \wedge q$	$(p \vee q) \vee (\neg p \wedge q)$	$(p \vee q) \vee (\neg p \wedge q) \to q$
F	F	F	F	F	T
F	T	T	F	T	T
T	F	T	T	T	F
T	T	T	F	T	T

8. $\neg p \vee q \to r$

p	q	r	$\neg p \vee q$	$\neg p \vee q \to r$
F	F	F	T	F
F	F	T	T	T
F	T	F	T	F
F	T	T	T	T
T	F	F	F	T
T	F	T	F	T
T	T	F	T	F
T	T	T	T	T

13.

(a) $p \to q \equiv \neg p \vee q$

p	q	$p \to q$	$\neg p \vee q$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

They have identical truth values for all possible substitutions.

(b) $\neg (p \to q) \equiv p \wedge \neg q$

p	q	$\neg (p \to q)$	$p \wedge \neg q$
F	F	F	F
F	T	F	F
T	F	T	T
T	T	F	F

They have identical truth values for all possible substitutions.

(f) Set 2.2

10. $(p \to r) \leftrightarrow (q \to r)$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \leftrightarrow (q \to r)$
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	F	F
F	T	T	T	T	T
T	F	F	F	T	F
T	F	T	T	T	T
T	T	F	F	F	T
T	T	T	T	T	T

11. $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$
F	F	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	T	T	T	T	F	T	T
T	F	F	T	T	F	T	T
T	F	T	T	T	F	T	T
T	T	F	F	F	T	F	T
T	T	T	T	T	T	T	T

(g) Set 2.2

30. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
F	F	F	F	F	T
F	F	T	F	F	T
F	T	F	F	F	T
F	T	T	F	F	T
T	F	F	F	F	T
T	F	T	T	T	T
T	T	F	T	T	T
T	T	T	T	T	T

31. $p \to (q \to r) \equiv (p \wedge q) \to r$
$p \to (q \to r) \leftrightarrow (p \wedge q) \to r$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$p \to (q \to r) \leftrightarrow (p \wedge q) \to r$
F	F	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	T	T	T	T	F	T	T
T	F	F	T	T	F	T	T
T	F	T	T	T	F	T	T
T	T	F	F	F	T	F	T
T	T	T	T	T	T	T	T

(h) Set 2.2

25. A conditional statement is not logically equivalent to its inverse.
$a \to b \not\equiv \neg a \to \neg b$

a	b	$a \to b$	$\neg a \to \neg b$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

27. The converse and inverse of a conditional statement are logically equivalent to each other.
$b \to a \equiv \neg a \to \neg b$

a	b	$b \to a$	$\neg a \to \neg b$
F	F	T	T
F	T	F	F
T	F	T	T
T	T	T	T

Problem 2.

(a) Set 2.1

26. Sam isn't an orange belt nor is Kate a red belt.
28. The units digit of 4^{67} is not 4 and not 6.
29. This computer program doesn't have a logical error in the first ten lines and it isn't being run with an incomplete data set.
30. The dollar isn't at an all-time high nor is the stock market at a record low.
31. The train isn't late and my watch isn't fast.

(b) Set 2.1

33. $-10 \geq x \geq 2$
35. $x < -1$ and $x \leq 1$
37. $0 \leq x < -7$
39. ($num\_orders \geq 50$ or $num\_instock \leq 300$) and
($50 > num\_orders \geq 75$ or $num\_instock \leq 500$)

(c) Set 2.2

20.

(a) P is a square and P isn't a rectangle.
(b) Today is New Year's Eve and tomorrow isn't January.
(c) The decimal expansion of r is terminating and r isn't rational.
(d) n is prime and n isn't positive nor 2.
(e) x is nonnegative and x isn't positive and not 0.
(f) Tom is Ann's father and Jim isn't her Uncle nor is Sue her aunt.
(g) n is divisible by 6 and n isn't divisible by 2 nor 3.

Problem 3.

(a) Set 2.2

33. If this integer is even, then it equals twice some integer; and if this integer equals twice some integer, then it is even.
35. If Sam is allowed on Signe's racing boat, then he is an expert sailor.
If Sam isn't an expert sailor, then he isn't allowed on Signe's racing boat.
38. If it doesn't rain, then Ann will go.
39. If the security code isn't entered, then the door won't open.

(b) Set 2.2

41. If this triangle has two 45° angles, then it is a right triangle.
43. If Jim didn't do homework regularly, then he won't pass the course.
If Jim passes the course, then he did his homework regularly.
45. If this computer program isn't correct, then it produces error messages during translation.
If this computer program doesn't produce error messages during translation, then it is correct.
46. X is boiling $\rightarrow$ temperature $\geq 150°$C
(a) temperature $\geq 150°$C $\not\rightarrow$ X is boiling
Not necessarily true.
(b) temperature $< 150°$C $\rightarrow$ X isn't boiling
Always true.
(c) X is boiling $\not\leftrightarrow$ temperature $\geq 150°$C
Not necessarily true.
(d) X isn't boiling $\rightarrow$ temperature $< 150°$C
Not necessarily true.
(e) X is boiling $\not\leftrightarrow$ temperature $\geq 150°$C
Not necessarily true.
(f) X is boiling $\rightarrow$ temperature $\geq 150°$C
Always true.

Problem 4.

(a) Set 2.3

11. $p \to q \vee r$

$\neg q \vee \neg r$

$\therefore \neg p \vee \neg r$

p	q	r	Premise 1 $p \to q \vee r$	Premise 2 $\neg q \vee \neg r$	Conclusion $\neg p \vee \neg r$
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	T	T
F	T	T	T	F	
T	F	F	F	T	
T	F	T	T	T	T
T	T	F	T	T	F
T	T	T	T	F	

Invalid argument because true premises can lead to a false conclusion.

Problem 5.

(a) Set 2.3

38.

(b) Another two natives C and D approach you but only C speaks.

C says: Both of us are knaves.
What are C and D?

Suppose C is a knight.

$\therefore$ What C says is true — definition of knight

$\therefore$ C and D are knaves — what C said

$\therefore$ C is a knave — specialization

$\therefore$ C is a knight and a knave — contradiction

$\therefore$ The supposition is false

Suppose C is a knave.

$\therefore$ What C says is false — definition of knave

$\therefore$ C is a knight or D is a knight — opposite of what C said

$\therefore$ D is a knight — by elimination

$\therefore$ C is a knave and D is a knight

(c) You then encounter natives E and F.

E says: F is a knave.
F says: E is a knave.
How many knaves are there?

Suppose E and F are knaves.	
$\therefore F$ is a knight	opposite of what E said
$\therefore F$ is a knight and a knave	contradiction
$\therefore$ The supposition is false	
Suppose E is a knave and F is a knight.	
$\therefore F$ is a knight	opposite of what E said
$\therefore E$ is a knave	what F said
$\therefore$ 1 knight and 1 knave	
Suppose E is a knight and F is a knave.	
$\therefore F$ is a knave	what E said
$\therefore E$ is a knight	opposite of what F said
$\therefore$ 1 knight and 1 knave	
Suppose E and F are knights.	
$\therefore F$ is a knave	what E said
$\therefore F$ is a knight and a knave	contradiction
$\therefore$ The supposition is false	

There is 1 knave.

Problem 6.

(a) Set 2.3

44.

$p \to q$		(1)
$r \vee s$		(2)
$\neg s \to \neg t$		(3)
$\neg q \vee s$		(4)
$\neg s$		(5)
$\neg p \wedge r \to u$		(6)
$w \vee t$		(7)
$\therefore r$	elimination by (2) and (5)	(8)
$\therefore \neg t$	modus ponens by (3) and (5)	(9)
$\therefore \neg q$	elimination by (4) and (5)	(10)
$\therefore \neg p$	modus tollens by (1) and (10)	(11)
$\therefore u$	modus ponens by (11), (8) and (6)	(12)
$\therefore w$	elimination by (7) and (9)	(13)
$\therefore u \wedge w$	conjunction by (12) and (14)	(14)

Problem 7.

(a) For the circuit corresponding to the following Boolean expression, there is an equivalent circuit with at most two logic gates. Find such a circuit.
$(\neg P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$

Using DeMorgan's Laws to rewrite ands using only nots and ors:
$\neg (P \vee Q) \vee \neg (P \vee \neg Q) \vee \neg (\neg P \vee Q)$