

Problem 1.

Set 3.1

- 10.
- \forall
- positive integers
- m
- and
- n
- ,
- $m \times n \geq m + n$

$$m = 0, n = 1 \quad 0 \times 1 \stackrel{?}{\geq} 0 + 1$$

$$0 \not\geq 1$$

- 12.
- \forall
- real numbers
- x
- and
- y
- ,
- $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

$$x = 1, y = 1$$

$$m = 0, n = 1$$

$$\sqrt{1+1} \stackrel{?}{=} \sqrt{1} + \sqrt{1}$$

$$\sqrt{2} \neq 2$$

Problem 2.

Set 3.1

29.

- (a)
- $\exists x$
- such that
- $\text{Rect}(x) \wedge \text{Square}(x)$

There are geometric figures that are both rectangles and squares.

True; squares that are both rectangles and squares.

- (b)
- $\exists x$
- such that
- $\text{Rect}(x) \wedge \neg \text{Square}(x)$

There are geometric figures that are rectangles but not squares.

True; rectangles of unequal side lengths are rectangles but not squares.

- (c)
- $\forall x, \text{Square}(x) \rightarrow \text{Rect}(x)$

If a geometric figure is a square, it is a rectangle.

True; squares have all the criteria of rectangles but have the added criteria of equal side lengths.

Problem 3.

Set 3.1

33.

- (c)
- $ab = 0 \Rightarrow a = 0 \text{ or } b = 0$

True

- (d)
- $a < b \text{ and } c < d \Rightarrow ac < bd$

$$a = -1, b = 0, c = -1, d = 0$$

$$-1 \times -1 \stackrel{?}{<} 0 \times 0$$

$$1 \not< 0$$

False

Problem 4.

Set 3.2

- 10.
- \forall
- computer programs
- P
- , if
- P
- compiles without error messages, then
- P
- is correct.

 $\exists P$ such that P compiles without error messages and isn't correct.

- 17.
- \forall
- integers
- d
- , if
- $6/d$
- is an integer then
- $d = 3$
- .

 $\exists d$ such that $6/d$ is an integer and $d \neq 3$.

- 19.
- $\forall n \in \mathbb{Z}$
- , if
- n
- is prime then
- n
- is odd or
- $n = 2$
- .

 $\exists n \in \mathbb{Z}$ such that n is prime and n is even and $n = 2$.

- 21.
- \forall
- integers
- n
- , if
- n
- is divisible by 6, then
- n
- is divisible by 2 and
- n
- is divisible by 3.

 $\exists n$ such that n is divisible by 6 and not divisible by 2 and not divisible by 3.

23. If a function is differentiable then it is continuous.

There exists a function that is differentiable and not continuous.

Problem 5.

Set 3.2

40. Being divisible by 8 is a sufficient condition or being divisible by 4.
If n is divisible by 8, then n is divisible by 4.
42. Passing a comprehensive exam is a necessary condition for obtaining a master's degree.
If one does not pass a comprehensive exam, then one cannot obtain a master's degree.
44. Having a large income is not a necessary condition for a person to be happy.
 $\neg(\forall x(\text{Happy}(x) \rightarrow \text{HighIncome}(x)))$
 $\exists x(\text{Happy}(x) \wedge \neg \text{HighIncome}(x))$
 There exists a happy person that doesn't have a large income.
46. Being a polynomial is not a sufficient condition for a function to have a real root.
 $\neg(\forall x(\text{Polynomial}(x) \rightarrow \text{RealRoot}(x)))$
 $\exists x(\text{Polynomial}(x) \wedge \neg \text{RealRoot}(x))$
 There exists a polynomial function without a real root.
47. The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for reasonable [program] correctness.
 $\forall x(\text{Correct}(x) \rightarrow \neg \text{Error}(x) \wedge \neg \text{Error}(x) \not\rightarrow \text{Correct}(x))$
 There are no error messages whenever a program is correct, but there are incorrect programs without error messages.

Problem 6.

Set 3.3

- 41.
- (c) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}, x = y + 1$
True because \mathbb{R} is closed under addition.
- (d) $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+$ such that $xy = 1$

$$y = \frac{1}{x}$$
 True because \mathbb{R}^+ is closed under division.
- (f) $\forall x \in \mathbb{Z}^+$ and $\forall y \in \mathbb{Z}^+, \exists z \in \mathbb{Z}^+$ such that $z = x - y$
False because \mathbb{Z}^+ isn't closed under subtraction.
 $x = 1, y = 2$
 $z = 1 - 2 = -1 \therefore z \notin \mathbb{Z}^+.$
- (g) $\forall x \in \mathbb{Z}$ and $\forall y \in \mathbb{Z}, \exists z \in \mathbb{Z}$ such that $z = x - y$
True because \mathbb{Z} is closed under subtraction.
- (h) $\exists u \in \mathbb{R}^+$ such that $\forall v \in \mathbb{R}^+, uv < v$

$$u < 1$$
 True because $\{u \in \mathbb{R}^+ \mid u < 1\} \subset \mathbb{R}^+.$

Problem 7.

Set 3.4

13. For all students x , if x studies discrete mathematics, then x is good at logic.
Tarik studies discrete mathematics.
 \therefore Tarik is good at logic.
Valid by universal modus ponens.
14. If compilation of a computer program produces error messages, then the program is not correct.
Compilation of this program does not produce error messages.
 \therefore This program is correct.
Invalid by inverse error.

15. Any sum of two rational numbers is rational.

The sum $r + s$ is rational.

\therefore The numbers r and s are both rational.

Invalid by converse error.

17. If an infinite series converges, then the terms go to 0.

The terms of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ go to 0.

\therefore The infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

Invalid by converse error.

18. If an infinite series converges, then the terms go to 0.

The terms of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n+1}$ do not go to 0.

\therefore The infinite series $\sum_{n=1}^{\infty} \frac{1}{n+1}$ does not converge.

Valid by universal modus tollens.

Problem 8.

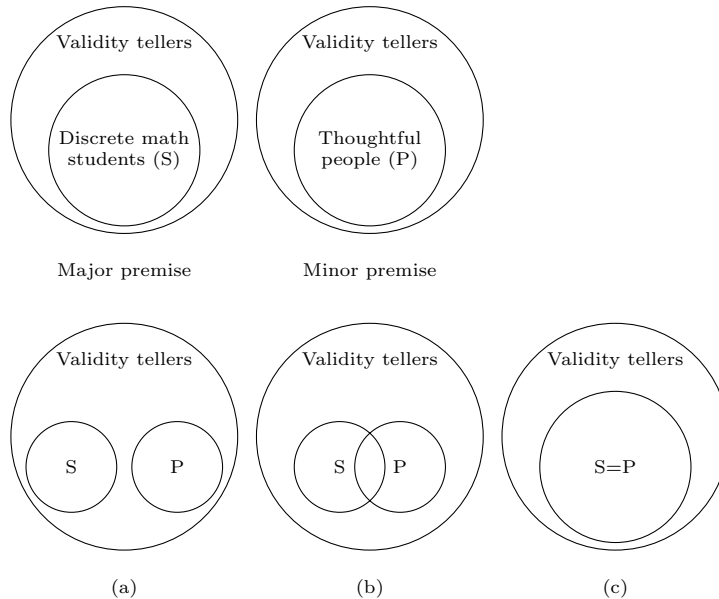
Set 3.4

22. All discrete mathematics students can tell a valid argument from an invalid one.

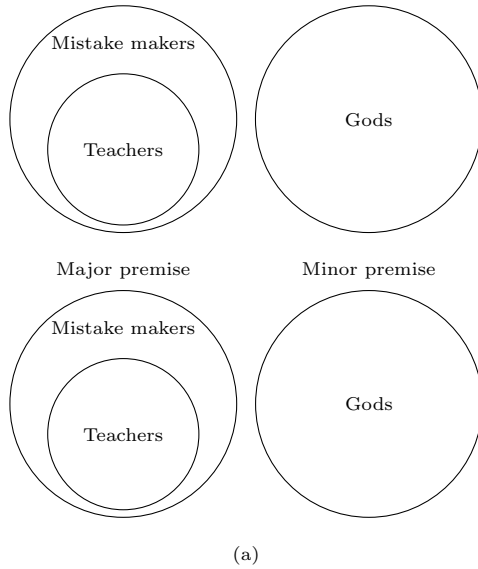
All thoughtful people can tell a valid argument from an invalid one.

\therefore All discrete mathematics students are thoughtful.

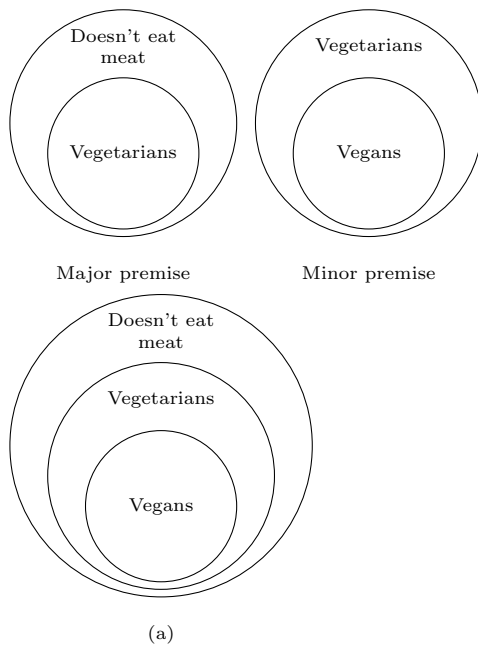
Invalid.



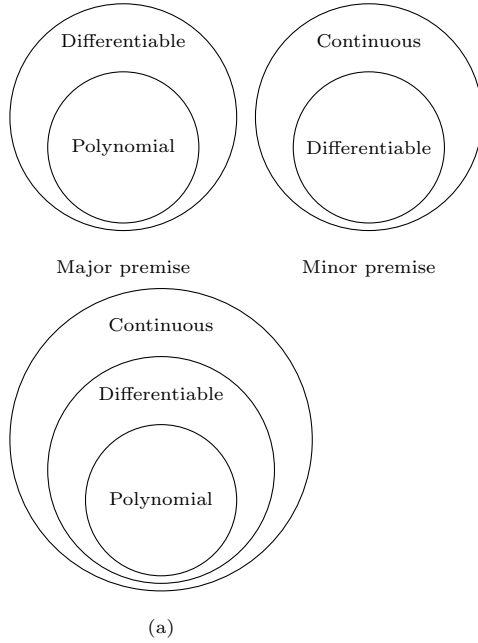
23. All teachers occasionally make mistakes.
 No gods ever make mistakes.
 \therefore No teachers are gods.
 Valid.



24. No vegetarians eat meat.
 All vegans are vegetarian.
 \therefore No vegans eat meat.
 Valid.



26. All polynomial functions are differentiable.
 All differentiable functions are continuous.
 \therefore All polynomial functions are continuous.
 Valid.



27. Nothing intelligible ever puzzles me.
 Logic puzzles me.
 \therefore Logic is unintelligible.
 Valid.

