Problem 1.

Set 3.1

10. $\forall$ positive integers m and n, $m \times n \geq m + n$

    $m = 0$, $n = 1$ $0 \times 1 \overset{?}{\geq} 0 + 1$

    $0 \ngeq 1$

12. $\forall$ real numbers x and y, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$
    $x = 1$, $y = 1$
    $m = 0$, $n = 1$
    $\sqrt{1+1} \overset{?}{=} \sqrt{1} + \sqrt{1}$

    $\sqrt{2} \neq 2$

Problem 2.

Set 3.1

29.
   (a) $\exists x$ such that $\text{Rect}(x) \wedge \text{Square}(x)$
   There are geometric figures that are both rectangles and squares.
   True; squares are both rectangles and squares.
   (b) $\exists x$ such that $\text{Rect}(x) \wedge \neg\text{Square}(x)$
   There are geometric figures that are rectangles but not squares.
   True; rectangles of unequal side lengths are rectangles but not squares.
   (c) $\forall x$, $\text{Square}(x) \rightarrow \text{Rect}(x)$
   If a geometric figure is a square, it is a rectangle.
   True; squares have all the criteria of rectangles but have the added criteria of equal side lengths.

Problem 3.

Set 3.1

33.
   (c) $ab = 0 \Rightarrow a = 0$ or $b = 0$
   True
   (d) $a < b$ and $c < d \Rightarrow ac < bd$
   $a = -1$, $b = 0$, $c = -1$, $d = 0$
   $-1 \times -1 \overset{?}{<} 0 \times 0$

   $1 \nless 0$

   False

Problem 4.

Set 3.2

10. $\forall$ computer programs P, if P compiles without error messages, then P is correct.
    $\exists P$ such that P compiles without error messages and isn't correct.
17. $\forall$ integers d, if $6/d$ is an integer then $d = 3$.
    $\exists d$ such that $6/d$ is an integer and $d \neq 3$.
19. $\forall n \in \mathbb{Z}$, if n is prime then n is odd or $n = 2$.
    $\exists n \in \mathbb{Z}$ such that n is prime and n is even and $n = 2$.
21. $\forall$ integers n, if n is divisible by 6, then n is divisible by 2 and n is divisible by 3.
    $\exists n$ such that n is divisible by 6 and not divisible by 2 and not divisible by 3.
23. If a function is differentiable then it is continuous.
    There exists a function that is differentiable and not continuous.

Problem 5.

Set 3.2

40. Being divisible by 8 is a sufficient condition or being divisible by 4.
 If n is divisible by 7, then n is divisible by 4.
42. Passing a comprehensive exam is a necessary condition for obtaining a master's degree.
 If one does not pass a comprehensive exam, then one cannot obtain a master's degree.
44. Having a large income is not a necessary condition for a person to be happy.
 $\neg(\forall x(\text{Happy}(x) \to \text{HighIncome}(x)))$
 $\exists x(\text{Happy}(x) \wedge \neg\text{HighIncome}(x)))$
 There exists a happy person that doesn't have a large income.
46. Being a polynomial is not a sufficient condition for a function to have a real root.
 $\neg(\forall x(\text{Polynomial}(x) \to \text{RealRoot}(x)))$
 $\exists x(\text{Polynomial}(x) \wedge \neg\text{RealRoot}(x)))$
 There exists a polynomial function without a real root.
47. The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for reasonable [program] correctness.
 $\forall x(\text{Correct}(x) \to \neg\text{Error}(x) \wedge \neg\text{Error}(x) \not\to \text{Correct}(x))$
 There are no error messages whenever a program is correct, but there are incorrect programs without error messages.

Problem 6.

Set 3.3

41.

(c) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $x = y + 1$
 True because $\mathbb{R}$ is closed under addition.

(d) $\forall x \in \mathbb{R}^+$, $\exists y \in \mathbb{R}^+$ such that $xy = 1$

$$y = \frac{1}{x}$$

 True because $\mathbb{R}^+$ is closed under division.

(f) $\forall x \in \mathbb{Z}^+$ and $\forall y \in \mathbb{Z}^+$, $\exists z \in \mathbb{Z}^+$ such that $z = x - y$
 False because $\mathbb{Z}^+$ isn't closed under division.
 $x = 1$, $y = 2$
 $z = -1 \therefore z \notin \mathbb{Z}^+$.

(g) $\forall x \in \mathbb{Z}$ and $\forall y \in \mathbb{Z}$, $\exists z \in \mathbb{Z}$ such that $z = x - y$
 True because $\mathbb{Z}$ is closed under subtraction.

(h) $\exists u \in \mathbb{R}^+$ such that $\forall v \in \mathbb{R}^+$, $uv < v$

$$u < 1$$

 True because $\{u \in \mathbb{R}^+ \mid u < 1\} \subset \mathbb{R}^+$.

Problem 7.

Set 3.4

13. For all students x, if x studies discrete mathematics, then x is good at logic.
 Tarik studies discrete mathematics.
 $\therefore$ Tarik is good at logic.
 Valid by universal modus ponens.
14. If compilation of a computer program produces error messages, then the program is not correct.
 Compilation of this program does not produce error messages.
 $\therefore$ This program is correct.
 Invalid by inverse error.

15. Any sum of two rational numbers is rational.
The sum $r + s$ is rational.
$\therefore$ The numbers r and s are both rational.

Invalid by converse error.

17. If an infinite series converges, then the terms go to 0.
The terms of the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ go to 0.
$\therefore$ The infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

Invalid by converse error.

18. If an infinite series converges, then the terms go to 0.
The terms of the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+1}$ do not go to 0.
$\therefore$ The infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+1}$ does not converge.

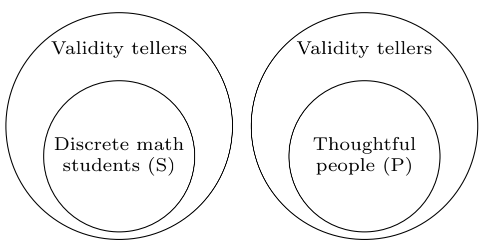
Valid by universal modus tollens.

Problem 8.

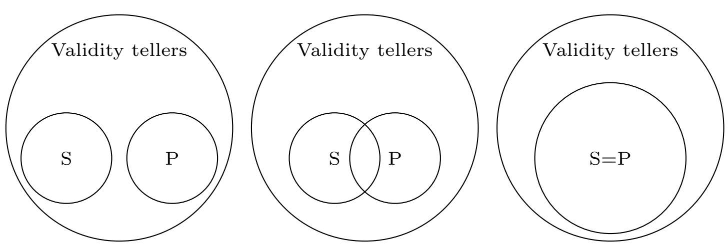
Set 3.4

22. All discrete mathematics students can tell a valid argument from an invalid one.
All thoughtful people can tell a valid argument from an invalid one.
$\therefore$ All discrete mathematics students are thoughtful.

Invalid.



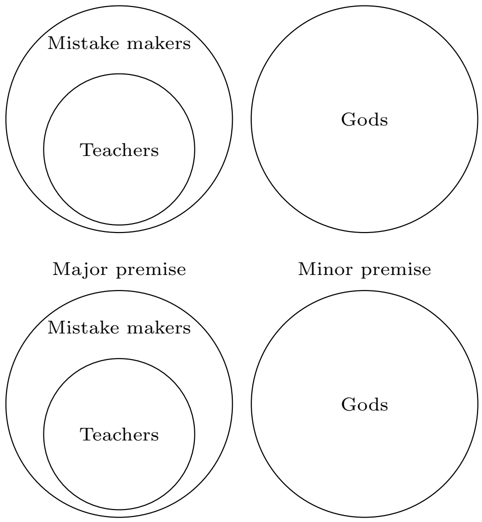
Major premise Minor premise



(a) (b) (c)

23. All teachers occasionally make mistakes.
No gods ever make mistakes.
∴ No teachers are gods.

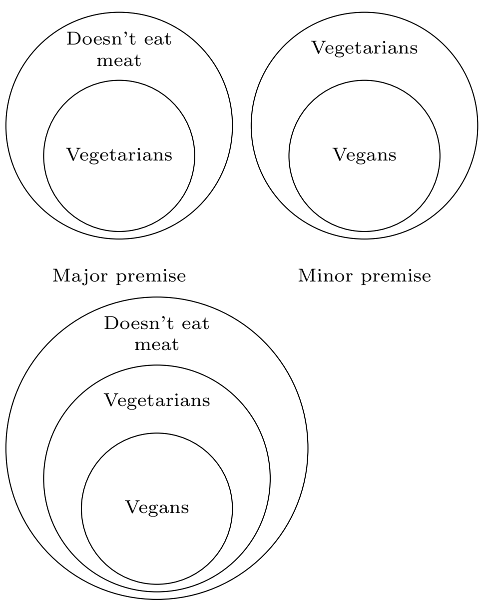
Valid.



(a)

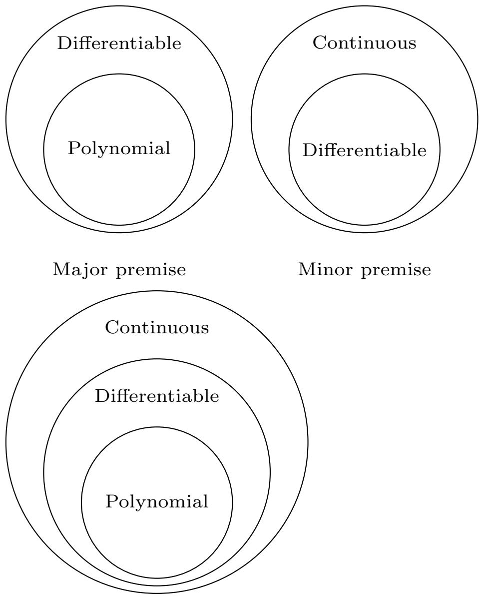
24. No vegetarians eat meat.
All vegans are vegetarian.
∴ No vegans eat meat.

Valid.



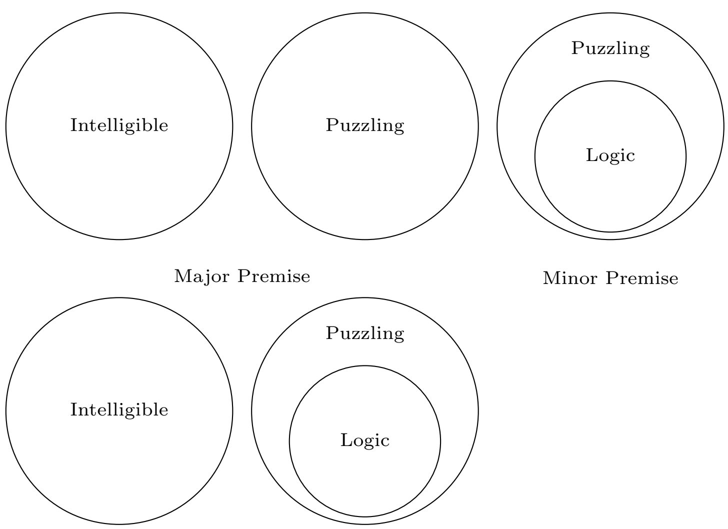
(a)

26. All polynomial functions are differentiable.
All differentiable functions are continuous.
∴ All polynomial functions are continuous.
Valid.



(a)

27. Nothing intelligible ever puzzles me.
Logic puzzles me.
∴ Logic is unintelligible.
Valid.



(a)