Section 2.4

1.

(a) $\mathbf{I}^2 = \mathbf{I}$

(b) $\mathbf{1} \cdot \mathbf{1} = n$

(c) $\mathbf{I1} = \mathbf{1}$

(d) $\mathbf{Ie}_i = \mathbf{e}_i$

(e) $\mathbf{1} \cdot \mathbf{e}_i = 1$

(f) $\mathbf{e}_i \cdot \mathbf{e}_i = 1$

(g) $\mathbf{e}_i \cdot \mathbf{e}_j = 0$

(h) $\mathbf{1I1} = \mathbf{1} \cdot \mathbf{1} = n$

(i) $\mathbf{e}_i\mathbf{Ie}_j = \mathbf{e}_i \cdot \mathbf{e}_j = 0$

2.

(a) $\begin{bmatrix} 3 & 5 & 7 \\ 2 & -1 & 1 \\ 1 & 6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 5 & 7 & -8 \\ 2 & -1 & 1 & -4 \\ 1 & 6 & -2 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

6.

(a) $\mathbf{p'} = \underbrace{\begin{bmatrix} 1.5 & -0.2 \\ 0.3 & 0.9 \end{bmatrix}}_{\mathbf{A}} \mathbf{p} + \underbrace{\begin{bmatrix} 100 \\ 50 \end{bmatrix}}_{\mathbf{B}}$

(c) $$\begin{aligned} \mathbf{p''} &= \mathbf{A}(\mathbf{Ap} + \mathbf{B}) + \mathbf{B} \\ &= \mathbf{A}^2\mathbf{p} + (\mathbf{A} + \mathbf{I})\mathbf{B} \\ \mathbf{p}^{(3)} &= \mathbf{A}(\mathbf{A}^2\mathbf{p} + (\mathbf{A} + \mathbf{I})\mathbf{B}) + \mathbf{B} \\ &= \mathbf{A}^3\mathbf{p} + (\mathbf{A}(\mathbf{A} + \mathbf{I}) + \mathbf{I})\mathbf{B} \\ &= \mathbf{A}^3\mathbf{p} + (\mathbf{A}^2 + \mathbf{A} + \mathbf{I})\mathbf{B} \end{aligned}$$

10. $\mathbf{Ax} = \mathbf{By} = [1, 1]$, $\mathbf{Ay} = [1, 0]$, $\mathbf{Bx} = [0, 1]$

(c) $$\begin{aligned} \mathbf{z} &= (\mathbf{A} + \mathbf{B})\mathbf{x} - 2(\mathbf{A} + \mathbf{B})\mathbf{y} \\ &= \mathbf{Ax} + \mathbf{Bx} - 2\mathbf{Ay} - 2\mathbf{By} \\ &= [1, 1] + [0, 1] - 2[1, 0] - 2[1, 1] \\ &= [-3, 0] \end{aligned}$$

(d) $$\begin{aligned} \mathbf{z} &= (3\mathbf{A} + \mathbf{B})(\mathbf{x} + \mathbf{y}) \\ &= 3\mathbf{Ax} + 3\mathbf{Ay} + \mathbf{Bx} + \mathbf{By} \\ &= 3[1, 1] + 3[1, 0] + [0, 1] + [1, 1] \\ &= [7, 5] \end{aligned}$$

Section 2.5

2.

(a) $|[1,1,1]|_e = \sqrt{3}$,
$|[1,1,1]|_s = 3$,
$|[1,1,1]|_{mx} = 1$

10.

(a) $\mathbf{A} = \begin{bmatrix} 1.3 & -0.1 & -0.2 \\ 0.4 & 0.8 & -0.3 \\ 0.1 & 0.1 & 1.1 \end{bmatrix}$
$\|\mathbf{A}\|_s = 1.8$, $\|\mathbf{A}\|_{mx} = 1.6$

(b) $\mathbf{p'} = \mathbf{Ap}$, $\mathbf{p} = [10, 10, 10]$
$|\mathbf{p'}|_s \leq (1.8|\mathbf{p}|_s = 54)$, $|\mathbf{p'}|_{mx} \leq (1.6|\mathbf{p}|_{mx} = 16)$
$\mathbf{p'} = [10, 9, 13]$
$|\mathbf{p'}|_s = 32$, $|\mathbf{p'}|_{mx} = 13$

26.

(c) $\mathbf{A} = \begin{bmatrix} 1 & 6 \\ -2 & -6 \end{bmatrix}$, $\mathbf{u}_1 = [-2, 1]$, $\mathbf{u}_2 = [-3, 2]$
$\mathbf{Au}_1 = \lambda_1 \mathbf{u}_1$
$\mathbf{Au}_1 = [4, -2] = -2\mathbf{u}_1$, $\lambda_1 = -2$
$\mathbf{Au}_2 = \lambda_2 \mathbf{u}_2$
$\mathbf{Au}_2 = [9, -6] = -3\mathbf{u}_2$, $\lambda_2 = -3$

28.

(c) $\mathbf{u}_1 = [1, 1]$, $\mathbf{u}_2 = [-5, 6]$
$\lambda_1 = 7$, $\lambda_2 = -4$
$\mathbf{v} = 7\mathbf{u}_1 + \mathbf{u}_2$

$$\begin{aligned} \mathbf{A}^3\mathbf{v} &= \mathbf{A}^3(7\mathbf{u}_1 + \mathbf{u}_2) \\ &= 7\mathbf{A}^3\mathbf{u}_1 + \mathbf{A}^3\mathbf{u}_2 \\ &= 7(7^3)\mathbf{u}_1 + (-4)^3\mathbf{u}_2 \\ &= [7^4 + 4^3 \cdot 5, 7^4 - 4^3 \cdot 6] \end{aligned}$$

Section 2.6

6.

(b) $G_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$G_2 = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix}$

7. $$M = \text{To}\quad \begin{array}{cc} & \text{From} \\ & \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} & \begin{bmatrix} 0 & 1/2 & 1/2 & 1/3 & 0 & 0 & 0 & 1/4 \\ 1/4 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 \\ 1/4 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 \\ 1/4 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 1/4 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/4 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/4 \\ 1/4 & 0 & 0 & 0 & 1/3 & 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1/4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} \begin{bmatrix} 0 & 1/2 & 1/2 & 1/3 \\ 1/4 & 0 & 0 & 1/3 \\ 1/4 & 0 & 0 & 1/3 \\ 1/4 & 1/2 & 1/2 & 0 \end{bmatrix} & A \\ A^T & \begin{bmatrix} 0 & 1/2 & 1/2 & 1/4 \\ 1/3 & 0 & 0 & 1/4 \\ 1/3 & 0 & 0 & 1/4 \\ 1/3 & 1/2 & 1/2 & 0 \end{bmatrix} \end{bmatrix}$$

Section 3.1

2.

(b) $\underbrace{\begin{bmatrix} 2 & -3 \\ -5 & 6 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

$\det(A) = -3$

$x = \dfrac{\begin{vmatrix} 5 & -3 \\ 10 & 6 \end{vmatrix}}{\det(A)}$, $y = \dfrac{\begin{vmatrix} 2 & 5 \\ -5 & 10 \end{vmatrix}}{\det(A)}$

$x = \dfrac{60}{-3}$, $y = \dfrac{45}{-3}$

$x = -20$, $y = -15$

23.

(a)

(i) $\mathbf{A} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$$\begin{aligned} \det(\mathbf{A} - \lambda \mathbf{I}) &= \begin{vmatrix} 4-\lambda & 0 \\ 2 & 2-\lambda \end{vmatrix} \\ &= (4-\lambda)(2-\lambda) - 0 \cdot 2 \\ &= (4-\lambda)(2-\lambda) \end{aligned}$$

Eigenvalues are 4 and 2.

(iii) $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

$$\begin{aligned} \det(\mathbf{A} - \lambda \mathbf{I}) &= \begin{vmatrix} 2-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix} \\ &= (2-\lambda)(3-\lambda) - 1 \cdot 2 \\ &= 6 - 5\lambda + \lambda^2 - 2 \\ &= \lambda^2 - 5\lambda + 4 \\ &= (\lambda - 1)(\lambda - 4) \end{aligned}$$

Eigenvalues are 1 and 4.