

1.1

9.

$$C + R = 9$$

$$C - R = 5$$

$$2R = 4$$

$$R = 2$$

$$C - 2 = 5$$

$$C = 5$$

14.

$$J + S = 20 + 4(J - S)$$

$$\Rightarrow J + S = 20 + 4J - 4S$$

$$\Rightarrow -3J + 5S = 20$$

$$2S = 40 + J$$

$$\Rightarrow -J + 2S = 40$$

$$-3J + 5S = 20$$

$$-2(-J + 2S = 40)$$

$$-J + S = -60$$

$$\Rightarrow S = J - 60$$

$$2S = 40 + J$$

$$\Rightarrow 2(J - 60) = 40 + J$$

$$\Rightarrow 2J - 120 = 40 + J$$

$$\Rightarrow J = 160$$

$$S = J - 60$$

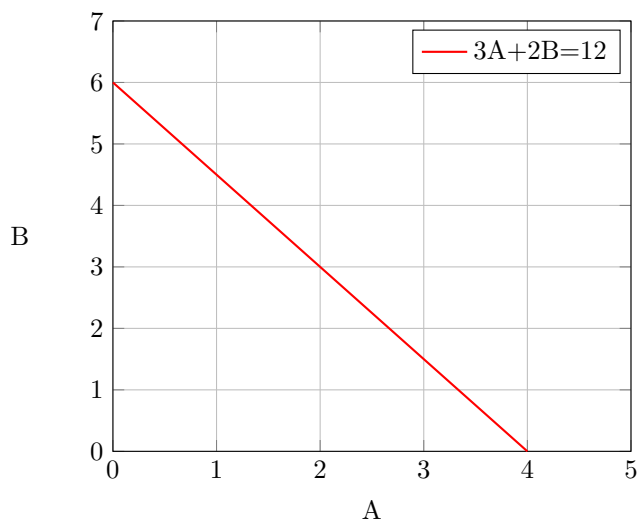
$$\Rightarrow S = 100$$

22.

$$A(12T + 6C) + B(8T + 4C) = 48T + 24C$$

$$\Rightarrow 6A(2T + C) + 4B(2T + C) = 24(2T + C)$$

$$\Rightarrow 3A + 2B = 12$$



There are 3 integer pair solutions.

1.2

4.

(a)

$$R_1 = 6H + 4D + 3G$$

$$R_2 = 3H + 6D + 2G$$

$$R_3 = 2H + 3D + 6G$$

$$x_1 R_1 + x_2 R_2 + x_3 R_3 = 280H + 350D + 350G$$

(b)

$$\begin{aligned} x_1 R_1 + x_2 R_2 + x_3 R_3 &= 280H + 350D + 350G \\ \Rightarrow x_1 (6H + 4D + 3G) \\ &+ x_2 (3H + 6D + 2G) \\ &+ x_3 (2H + 3D + 6G) = 280H + 350D + 350G \\ \Rightarrow H (6x_1 + 3x_2 + 2x_3) \\ &+ D (4x_1 + 6x_2 + 3x_3) \\ &+ G (3x_1 + 2x_2 + 6x_3) = 280H + 350D + 350G \end{aligned}$$

$$6x_1 + 3x_2 + 2x_3 = 280$$

$$4x_1 + 6x_2 + 3x_3 = 350$$

$$3x_1 + 2x_2 + 6x_3 = 350$$

x_1	x_2	x_3	H	D	G
30	20	20	280	300	250
25	25	25	275	325	275
20	25	40	275	350	350

6.

(a)

$$\begin{aligned}
 J &= 10C + 1P + 30V \\
 F &= 50C + 3P + 10V \\
 M &= 200C + 0.2P + 0V \\
 x_1J + x_2F + x_3M &= 600C + 20P + 200V
 \end{aligned}$$

1.3

3.

(c) $p_2 = 0.25, p_3 = 0.25, p_4 = 0.5$, all other $p_i = 0$

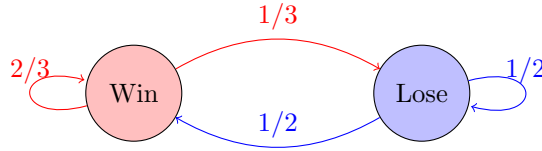
$$\begin{aligned}
 p'_1 &= 0.50p_1 + 0.25p_2 &= 0.0625 \\
 p'_2 &= 0.50p_1 + 0.50p_2 + 0.25p_3 &= 0.1875 \\
 p'_3 &= 0.25p_2 + 0.50p_3 + 0.25p_4 &= 0.3125 \\
 p'_4 &= 0.25p_3 + 0.50p_4 + 0.25p_5 &= 0.3125 \\
 p'_5 &= 0.25p_4 + 0.50p_5 + 0.50p_6 &= 0.125 \\
 p'_6 &= 0.25p_5 + 0.50p_6 &= 0
 \end{aligned}$$

(d) $p_1 = 0.1, p_2 = 0.2, p_3 = 0.2, p_4 = 0.2, p_5 = 0.2, p_6 = 0.1$

$$\begin{aligned}
 p'_1 &= 0.50p_1 + 0.25p_2 &= 0.1 \\
 p'_2 &= 0.50p_1 + 0.50p_2 + 0.25p_3 &= 0.2 \\
 p'_3 &= 0.25p_2 + 0.50p_3 + 0.25p_4 &= 0.2 \\
 p'_4 &= 0.25p_3 + 0.50p_4 + 0.25p_5 &= 0.2 \\
 p'_5 &= 0.25p_4 + 0.50p_5 + 0.50p_6 &= 0.2 \\
 p'_6 &= 0.25p_5 + 0.50p_6 &= 0.1
 \end{aligned}$$

5.

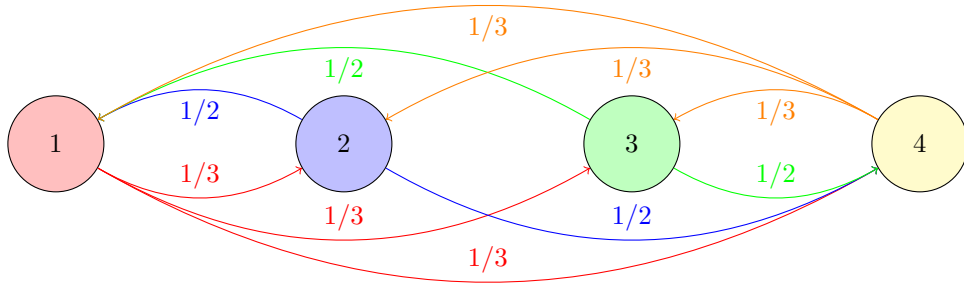
(a)



(c) $P(\text{winning the game after the next}) = \frac{2}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2} = \frac{11}{18}$

8.

(a)



(b) Starting at room 1:

$$p_1 = 0, p_2 = \frac{1}{3}, p_3 = \frac{1}{3}, p_4 = \frac{1}{3}$$

$$p'_1 = \frac{1}{2}p_2 + \frac{1}{2}p_3 + \frac{1}{3}p_4 = \frac{4}{9}$$

$$p'_2 = \frac{1}{3}p_1 + \frac{1}{3}p_4 = \frac{1}{9}$$

$$p'_3 = \frac{1}{3}p_1 + \frac{1}{3}p_4 = \frac{1}{9}$$

$$p'_4 = \frac{1}{3}p_1 + \frac{1}{2}p_2 + \frac{1}{2}p_3 = \frac{3}{9}$$

$$p''_1 = \frac{1}{2}p'_2 + \frac{1}{2}p'_3 + \frac{1}{3}p'_4 = \frac{6}{27}$$

$$p''_2 = \frac{1}{3}p'_1 + \frac{1}{3}p'_4 = \frac{7}{27}$$

$$p''_3 = \frac{1}{3}p'_1 + \frac{1}{3}p'_4 = \frac{7}{27}$$

$$p''_4 = \frac{1}{3}p'_1 + \frac{1}{2}p'_2 + \frac{1}{2}p'_3 = \frac{7}{27}$$

$$P(\text{in room 4 after two periods}) = \frac{7}{27}$$