1.1

9.

$$\begin{aligned} C+R&=9\\ C-R&=5\\ 2R&=4\\ R&=2\\ C-2&=5\\ C&=5 \end{aligned}$$

14.

$$\begin{aligned} J+\ S&=20+4\,(J-S)\\ \Rightarrow\quad J+\ S&=20+4J-4S\\ \Rightarrow -3J+5S&=20 \end{aligned}$$

$$\begin{aligned} 2S&=40+J\\ \Rightarrow -J+2S&=40 \end{aligned}$$

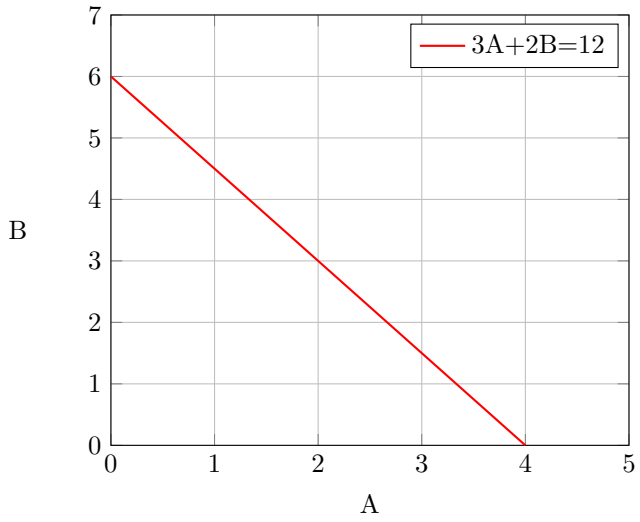
$$\begin{array}{rl} -3J+5S= & 20\\ -2(\ -J+2S= & 40)\\ \hline -J+\ S= & -60\\ \Rightarrow\quad S= & J-60 \end{array}$$

$$\begin{aligned} 2S&=40+J\\ \Rightarrow 2(J-60)&=40+J\\ \Rightarrow\ 2J-120&=40+J\\ \Rightarrow\quad J&=160 \end{aligned}$$

$$\begin{aligned} S&=J-60\\ \Rightarrow S&=100 \end{aligned}$$

22.

$$\begin{aligned} A\,(12T+6C)+B\,(8T+4C)&=48T+24C\\ \Rightarrow\ 6A\,(2T+C)+4B\,(2T+C)&=24\,(2T+C)\\ \Rightarrow\quad 3A+2B&=12 \end{aligned}$$



There are 3 integer pair solutions.

1.2

4.

(a)

$$\begin{aligned} R_1 &= 6H + 4D + 3G \\ R_2 &= 3H + 6D + 2G \\ R_2 &= 2H + 3D + 6G \\ x_1R_1 + x_2R_2 + x_3R_3 &= 280H + 350D + 350G \end{aligned}$$

(b)

$$\begin{aligned} & x_1R_1 + x_2R_2 + x_3R_3 = 280H + 350D + 350G \\ \Rightarrow \quad & x_1\left(6H + 4D + 3G\right) \\ & +x_2\left(3H + 6D + 2G\right) \\ & +x_3\left(2H + 3D + 6G\right) = 280H + 350D + 350G \\ \Rightarrow \quad & H\left(6x_1 + 3x_2 + 2x_3\right) \\ & +D\left(4x_1 + 6x_2 + 3x_3\right) \\ & +G\left(3x_1 + 2x_2 + 6x_3\right) = 280H + 350D + 350G \end{aligned}$$

$$\begin{aligned} 6x_1 + 3x_2 + 2x_3 &= 280 \\ 4x_1 + 6x_2 + 3x_3 &= 350 \\ 3x_1 + 2x_2 + 6x_3 &= 350 \end{aligned}$$

x_1	x_2	x_3	H	D	G
30	20	20	280	300	250
25	25	25	275	325	275
20	25	40	275	350	350

6.

(a)

$$\begin{aligned} J &= 10C + 1P + 30V \\ F &= 50C + 3P + 10V \\ M &= 200C + 0.2P + 0V \\ x_1 J + x_2 F + x_3 M &= 600C + 20P + 200V \end{aligned}$$

1.3

3.

(c) $p_2 = 0.25$, $p_3 = 0.25$, $p_4 = 0.5$, all other $p_i = 0$

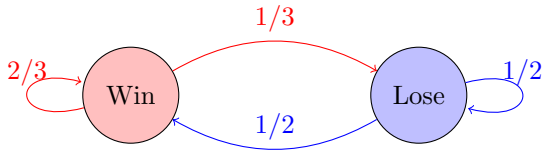
$$\begin{aligned} p_1' &= 0.50p_1 + 0.25p_2 = 0.0625 \\ p_2' &= 0.50p_1 + 0.50p_2 + 0.25p_3 = 0.1875 \\ p_3' &= 0.25p_2 + 0.50p_3 + 0.25p_4 = 0.3125 \\ p_4' &= 0.25p_3 + 0.50p_4 + 0.25p_5 = 0.3125 \\ p_5' &= 0.25p_4 + 0.50p_5 + 0.50p_6 = 0.125 \\ p_6' &= 0.25p_5 + 0.50p_6 = 0 \end{aligned}$$

(d) $p_1 = 0.1$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.2$, $p_5 = 0.2$, $p_6 = 0.1$

$$\begin{aligned} p_1' &= 0.50p_1 + 0.25p_2 = 0.1 \\ p_2' &= 0.50p_1 + 0.50p_2 + 0.25p_3 = 0.2 \\ p_3' &= 0.25p_2 + 0.50p_3 + 0.25p_4 = 0.2 \\ p_4' &= 0.25p_3 + 0.50p_4 + 0.25p_5 = 0.2 \\ p_5' &= 0.25p_4 + 0.50p_5 + 0.50p_6 = 0.2 \\ p_6' &= 0.25p_5 + 0.50p_6 = 0.1 \end{aligned}$$

5.

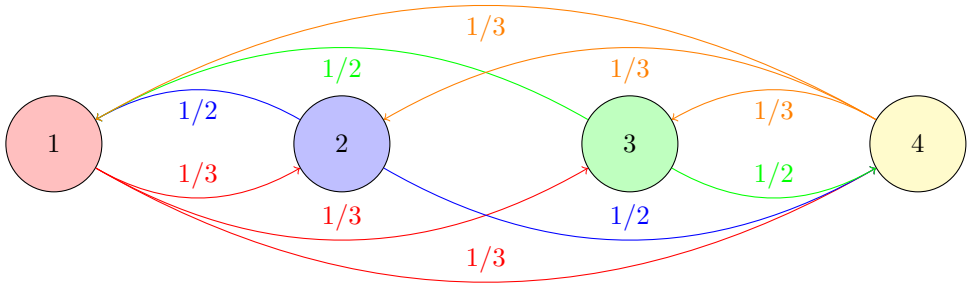
(a)



(c) $P(\text{winning the game after the next}) = \frac{2}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2} = \frac{11}{18}$

8.

(a)



(b) Starting at room 1:

$p_1 = 0$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{3}$, $p_4 = \frac{1}{3}$

$$\begin{aligned}
p_1' &= \quad \frac{1}{2}p_2 + \frac{1}{2}p_3 + \frac{1}{3}p_4 = \frac{4}{9} \\
p_2' &= \frac{1}{3}p_1 \quad + \quad \frac{1}{3}p_4 = \frac{1}{9} \\
p_3' &= \frac{1}{3}p_1 \quad + \quad \frac{1}{3}p_4 = \frac{1}{9} \\
p_4' &= \frac{1}{3}p_1 + \frac{1}{2}p_2 + \frac{1}{2}p_3 \quad = \frac{3}{9}
\end{aligned}$$

$$\begin{aligned}
p_1'' &= \quad \frac{1}{2}p_2' + \frac{1}{2}p_3' + \frac{1}{3}p_4' = \frac{6}{27} \\
p_2'' &= \frac{1}{3}p_1' \quad + \quad \frac{1}{3}p_4' = \frac{7}{27} \\
p_3'' &= \frac{1}{3}p_1' \quad + \quad \frac{1}{3}p_4' = \frac{7}{27} \\
p_4'' &= \frac{1}{3}p_1' + \frac{1}{2}p_2' + \frac{1}{2}p_3' \quad = \frac{7}{27}
\end{aligned}$$

$P(\text{in room 4 after two periods}) = \frac{7}{27}$