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AMS210.03 SAMPLE MIDTERM 1

February 26, 2015

Show all work to receive full credit.

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1) (15 points) Let

$$\underset{\sim}{A}=\begin{bmatrix}1&2&3&4\\2&4&6&8\\3&5&7&9\end{bmatrix}\qquad \underset{\sim}{B}=\begin{bmatrix}1&0&-1\\2&-2&0\\0&1&-1\end{bmatrix}\qquad \underset{\sim}{C}=\begin{bmatrix}5&4&1\\1&0&2\\3&2&1\\0&1&3\end{bmatrix}$$

Perform the following multiplications if possible. If not possible, please indicate that it is not possible:

a) $\underset{\sim}{A}\,\underset{\sim}{B}$ not possible

b) $\underset{\sim}{B}\,\underset{\sim}{A}=\begin{bmatrix}-2&-3&-4&-5\\-2&-4&-6&-8\\-1&-1&-1&-1\end{bmatrix}$

c) $\underset{\sim}{C}\,\underset{\sim}{A}=\begin{bmatrix}16&31&46&61\\7&12&17&22\\10&19&28&37\\11&19&27&35\end{bmatrix}$

d) $\underset{\sim}{C}\,\underset{\sim}{B}=\begin{bmatrix}13&-7&-6\\1&2&-3\\7&-3&-4\\2&1&-3\end{bmatrix}$

e) $\underset{\sim}{A}\,\underset{\sim}{A}$ not possible

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2) (15 points) Suppose that we are given the following matrices involving the costs of fruits at different stores, the amounts of fruit different types of people want, and the numbers of people of different types in different towns.

$$\begin{bmatrix} & StoreA & StoreB\\ Apple & .10 & .15\\ Orange & .15 & .20\\ Pear & .10 & .10\end{bmatrix}\qquad \begin{bmatrix} & Apple & Orange & Pear\\ PersonA & 5 & 10 & 3\\ PersonB & 4 & 5 & 5\end{bmatrix}$$

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$$\begin{bmatrix} & PersonA & PersonB \\ Town1 & 1000 & 500 \\ Town2 & 2000 & 1000 \end{bmatrix}$$

a) (5 pts.) What matrix product tells how many each fruit will be purchased in each town.
b) (10 pts.) Compute the matrix and label it appropriately.

$$\underset{\sim}{A} = \begin{bmatrix} .10 & .15 \\ .15 & .20 \\ .10 & .10 \end{bmatrix} \qquad \underset{\sim}{B} = \begin{bmatrix} 5 & 10 & 3 \\ 4 & 5 & 5 \end{bmatrix} \qquad \underset{\sim}{C} = \begin{bmatrix} 1000 & 500 \\ 2000 & 1000 \end{bmatrix}$$

Answer:

a) $\underset{\sim}{C}\,\underset{\sim}{B}$ or $\underset{\sim}{B}^T\,\underset{\sim}{C}^T$

b) $\begin{bmatrix} & Apple & Orange & Pear \\ Town1 & 7000 & 12500 & 5500 \\ Town2 & 14000 & 25000 & 11000 \end{bmatrix}$ or $\begin{bmatrix} & Town1 & Town2 \\ Apple & 7000 & 14000 \\ Orange & 12500 & 25000 \\ Pear & 5500 & 11000 \end{bmatrix}$

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3) (15 points)

The matrix $\underset{\sim}{A} = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$

has eigenvectors

$$\underset{\sim}{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \underset{\sim}{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

a) (5 pts.) Determine the eigenvalue associated with $\underset{\sim}{v}$.

b) (5 pts.) Write $\underset{\sim}{x} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ as a linear combination of $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

c) (5 pts.) The eigenvalue for $\underset{\sim}{u}$ is 4. Use your result in (b) to determine $\underset{\sim}{A}^3\,\underset{\sim}{x}$.

Answer:

a) $\underset{\sim}{A}\,\underset{\sim}{v} = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 1\underset{\sim}{v}$, so $\lambda = 1$. Eigenvalue associated with $\underset{\sim}{v}$ is 1.

b) $\underset{\sim}{x} = \begin{bmatrix} 1 \\ 7 \end{bmatrix} = 3\underset{\sim}{u} - 2\underset{\sim}{v}$

c) $\lambda_1 = 4$, $\underset{\sim}{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\lambda_2 = 1$, $\underset{\sim}{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$$\underset{\sim}{x} = \begin{bmatrix} 1 \\ 7 \end{bmatrix} = a\underset{\sim}{u} + b\underset{\sim}{v}, \; a = 3, \; b = -2$$

$$\underset{\sim}{A}^3 \underset{\sim}{x} = a\lambda_1^3 \underset{\sim}{u} + b\lambda_2^3 \underset{\sim}{v} = 3 \cdot 4^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2 \cdot 1^3 \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \cdot 4^3 - 2 \\ 3 \cdot 4^3 + 4 \end{bmatrix}$$

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4) (20 points)

Consider the following system of linear equations (rabbit-fox equations).

$$R' = R + 0.1R - 0.15F$$
$$F' = F + 0.2R - 0.3F$$

a) (5 pts.) Write the rabbit-fox equations in matrix form using $\underset{\sim}{p} = \begin{bmatrix} R \\ F \end{bmatrix}$, $\underset{\sim}{p}' = \begin{bmatrix} R' \\ F' \end{bmatrix}$ and $\underset{\sim}{A} = \begin{bmatrix} 0.1 & -0.15 \\ 0.2 & -0.3 \end{bmatrix}$

b) (5 pts.) Rewrite the equation from part (a) in the form $\underset{\sim}{p}' = \underset{\sim}{Q}\,\underset{\sim}{p}$ where $\underset{\sim}{Q}$ is some matrix expression.

c) (10 pts.) Find the sum and maximum norm of the matrix $\underset{\sim}{Q}$.

Answer:

a) $\underset{\sim}{p}' = \underset{\sim}{p} + \underset{\sim}{A}\,\underset{\sim}{p}$

b) $\underset{\sim}{p}' = \underset{\sim}{p} + \underset{\sim}{A}\,\underset{\sim}{p} = \underset{\sim}{I}\,\underset{\sim}{p} + \underset{\sim}{A}\,\underset{\sim}{p} = (\underset{\sim}{I} + \underset{\sim}{A})\underset{\sim}{p} = \underset{\sim}{Q}\,\underset{\sim}{p}$ so $\underset{\sim}{Q} = \underset{\sim}{I} + \underset{\sim}{A}$

c) $\underset{\sim}{Q} = \underset{\sim}{I} + \underset{\sim}{A} = \begin{bmatrix} 1.1 & -0.15 \\ 0.2 & 0.7 \end{bmatrix}$

$$\left\|\underset{\sim}{Q}\right\|_S = 1.3 \quad , \quad \left\|\underset{\sim}{Q}\right\|_{\max} = 1.25$$

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5) (35 points)

$$\underset{\sim}{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

a) (10 pts.) Write the matrix $\underset{\sim}{B}$ in partitioned form. Name the submatrices appropriately and give descriptions for each.

b) (10 pts.) Compute $\underset{\sim}{B}^2$ in partitioned form, then write out of all the entries of $\underset{\sim}{B}^2$.

c) (5 pts.) Draw the graph whose adjacency matrix is $\underset{\sim}{B}$, use a, b, c, d, e, f as nodes.

d) (5 pts.) How many paths of length 2, in the graph from (c), are there between the “a” node and the “e” node?

e) (5 pts.) Which element of $\underset{\sim}{B}^2$ corresponds to the value found in part (d)?

Answer:

a) $\underset{\sim}{B} = \begin{bmatrix} \underset{\sim}{C} & \underset{\sim}{D} \\ \underset{\sim}{D} & \underset{\sim}{C} \end{bmatrix}$, $\underset{\sim}{C} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $\underset{\sim}{D} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $\underset{\sim}{C}$ and $\underset{\sim}{D}$ are all 3x3 matrices.

b) $$\underset{\sim}{B}^2 = \begin{bmatrix} \underset{\sim}{C} & \underset{\sim}{D} \\ \underset{\sim}{D} & \underset{\sim}{C} \end{bmatrix} \begin{bmatrix} \underset{\sim}{C} & \underset{\sim}{D} \\ \underset{\sim}{D} & \underset{\sim}{C} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{C}^2 + \underset{\sim}{D}^2 & \underset{\sim}{C}\underset{\sim}{D} + \underset{\sim}{D}\underset{\sim}{C} \\ \underset{\sim}{D}\underset{\sim}{C} + \underset{\sim}{C}\underset{\sim}{D} & \underset{\sim}{D}^2 + \underset{\sim}{C}^2 \end{bmatrix}$$

$$\underset{\sim}{C}^2 + \underset{\sim}{D}^2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\underset{\sim}{C}\underset{\sim}{D} + \underset{\sim}{D}\underset{\sim}{C} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

$$\underset{\sim}{B}^2 = \begin{bmatrix} 2 & 0 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 2 & 0 & 2 \\ 1 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 1 \\ 2 & 0 & 2 & 0 & 3 & 0 \\ 0 & 2 & 0 & 1 & 0 & 2 \end{bmatrix}$$

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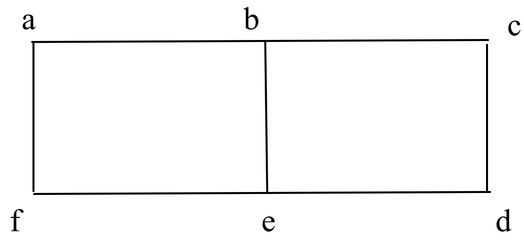
c)



d) 2 paths

e) b_{15} or b_{51} of $\underset{\sim}{B}^2$ corresponding to the value in d).

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