

Problem 1.

Exercise Set 6.1

25. Let $R_i = \left\{ x \in \mathbb{R} \mid 1 \leq x \leq 1 + \frac{1}{i} \right\} = \left[1, 1 + \frac{1}{i} \right]$ for all positive integers i .

(a) $\bigcup_{i=1}^4 R_i = [1, 2]$

(b) $\bigcap_{i=1}^4 R_i = \left[1, \frac{5}{4} \right]$

(c) Are R_1, R_2, R_3, \dots mutually disjoint? Explain.
No because they all contain 1.

(d) $\bigcup_{i=1}^n R_i = [1, 2]$

(e) $\bigcap_{i=1}^n R_i = \left[1, 1 + \frac{1}{n} \right]$

(f) $\bigcup_{i=1}^{\infty} R_i = [1, 2]$

(g) $\bigcap_{i=1}^{\infty} R_i = \{1\}$

Problem 2.

Exercise Set 6.1

26. Let $R_i = \left\{ x \in \mathbb{R} \mid 1 < x < 1 + \frac{1}{i} \right\} = \left(1, 1 + \frac{1}{i} \right)$ for all positive integers i .

(a) $\bigcup_{i=1}^4 R_i = (1, 2)$

(b) $\bigcap_{i=1}^4 R_i = \left(1, \frac{5}{4} \right)$

(c) Are R_1, R_2, R_3, \dots mutually disjoint? Explain.
No because R_1 and R_2 contain 1.25.

(d) $\bigcup_{i=1}^n R_i = (1, 2)$

(e) $\bigcap_{i=1}^n R_i = \left(1, 1 + \frac{1}{n} \right)$

(f) $\bigcup_{i=1}^{\infty} R_i = (1, 2)$

(g) $\bigcap_{i=1}^{\infty} R_i = \emptyset$

Problem 3.

Exercise Set 6.1

33.

- (a) $\mathcal{P}(\emptyset) = \{\emptyset\}$
- (b) $\mathcal{P}(\mathcal{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\}$
- (c) $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

35. Let $A = \{a, b\}$, $B = \{1, 2\}$, and $C = \{2, 3\}$. Find each of the following sets.

- (a) $A \times (B \cup C) = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), \}$
- (b) $(A \times B) \cup (A \times C) = \{(a, 1), (b, 1), (a, 2), (b, 2), (a, 3), (b, 3)\}$
- (c) $A \times (B \cap C) = \{(a, 2), (b, 2)\}$
- (d) $(A \times B) \cap (A \times C) = \{(a, 2), (b, 2)\}$

Problem 4.

Exercise Set 6.2

10. For all sets A , B , and C , $(A - B) \cap (C - B) = (A \cap C) - B$ **Proof:****Part 1:** $(A - B) \cap (C - B) \subseteq (A \cap C) - B$ Suppose $x \in (A - B) \cap (C - B)$.Prove $x \in (A \cap C) - B$. $x \in (A \cap B^c) \cap (C \cap B^c)$ by set difference law. $x \in A \cap C \cap B^c$ by idempotent law. $x \in (A \cap C) - B$ by set difference law.

Proof done.

Part 2: $(A \cap C) - B \subseteq (A - B) \cap (C - B)$ Suppose $x \in (A \cap C) - B$.Prove $x \in (A - B) \cap (C - B)$. $x \in (A \cap C) \cap B^c$ by set difference law. $x \in A \cap B^c \cap C \cap B^c$ by idempotent law. $x \in (A - B) \cap (C - B)$ by set difference law.

Proof done.

Both parts proven therefore $(A - B) \cap (C - B) = (A \cap C) - B$.

Problem 5.

Exercise Set 6.2

19. For all sets
- A
- ,
- B
- , and
- C
- ,
- $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Proof:**Part 1:** $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ Suppose $x \in A \times (B \cap C)$.Prove $x \in (A \times B) \cap (A \times C)$. $x \in (A \times B) \cap (A \times C)$ by distributive law.

Proof done.

Part 2: $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ Suppose $x \in (A \times B) \cap (A \times C)$.Prove $x \in A \times (B \cap C)$. $x \in A \times (B \cap C)$ by distributive law.

Proof done.

Both parts proven therefore $A \times (B \cap C) = (A \times B) \cap (A \times C)$.**Problem 6.**

Exercise Set 6.2

34. For all sets
- A
- ,
- B
- , and
- C
- , if
- $B \cap C \subseteq A$
- , then
- $(C - A) \cap (B - A) = \emptyset$

Proof:Let $B \cap C \subseteq A$.Assume $(C - A) \cap (B - A) \neq \emptyset$.Suppose $x \in (C - A) \cap (B - A)$. $x \in (C \cap B) \cap A^c$ as proven previously. $x \in B \cap C$ and $x \in A^c$ by definition of intersection.Since $B \cap C \subseteq A$, $x \in A$ by definition of subset.Contradiction because $x \in A$ and $x \notin A \Leftrightarrow x \in A^c$.Assumption is false therefore if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$.