Problem 1.
Exercise Set 6.1

25. Let $R_i = \left\{x \in \mathbb{R} | 1 \le x \le 1 + \dfrac{1}{i}\right\} = \left[1,\, 1 + \dfrac{1}{i}\right]$ for all positive integers i.

(a) $\displaystyle\bigcup_{i=1}^{4} R_i = [1,\, 2]$

(b) $\displaystyle\bigcap_{i=1}^{4} R_i = \left[1,\, \frac{5}{4}\right]$

(c) Are $R_1, R_2, R_3, \ldots$ mutually disjoint? Explain.
No because they all contain 1.

(d) $\displaystyle\bigcup_{i=1}^{n} R_i = [1,\, 2]$

(e) $\displaystyle\bigcap_{i=1}^{n} R_i = \left[1,\, 1 + \frac{1}{n}\right]$

(f) $\displaystyle\bigcup_{i=1}^{\infty} R_i = [1,\, 2]$

(g) $\displaystyle\bigcap_{i=1}^{\infty} R_i = \{1\}$

Problem 2.
Exercise Set 6.1

26. Let $R_i = \left\{x \in \mathbb{R} | 1 < x < 1 + \dfrac{1}{i}\right\} = \left(1,\, 1 + \dfrac{1}{i}\right)$ for all positive integers i.

(a) $\displaystyle\bigcup_{i=1}^{4} R_i = (1,\, 2)$

(b) $\displaystyle\bigcap_{i=1}^{4} R_i = \left(1,\, \frac{5}{4}\right)$

(c) Are $R_1, R_2, R_3, \ldots$ mutually disjoint? Explain.
No because R_1 and R_2 contain 1.25.

(d) $\displaystyle\bigcup_{i=1}^{n} R_i = (1,\, 2)$

(e) $\displaystyle\bigcap_{i=1}^{n} R_i = \left(1,\, 1 + \frac{1}{n}\right)$

(f) $\displaystyle\bigcup_{i=1}^{\infty} R_i = (1,\, 2)$

(g) $\displaystyle\bigcap_{i=1}^{\infty} R_i = \emptyset$

Problem 3.
Exercise Set 6.1

33.

(a) $\mathscr{P}(\emptyset) = \{\emptyset\}$

(b) $\mathscr{P}(\mathscr{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\}$

(c) $\mathscr{P}(\mathscr{P}(\mathscr{P}(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

35. Let $A = \{a, b\}$, $B = \{1, 2\}$, and $C = \{2, 3\}$. Find each of the following sets.

(a) $A \times (B \cup C) = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), \}$

(b) $(A \times B) \cup (A \times C) = \{(a, 1), (b, 1), (a, 2), (b, 2), (a, 3), (b, 3)\}$

(c) $A \times (B \cap C) = \{(a, 2), (b, 2)\}$

(d) $(A \times B) \cap (A \times C) = \{(a, 2), (b, 2)\}$

Problem 4.
Exercise Set 6.2

10. For all sets A, B, and C, $(A - B) \cap (C - B) = (A \cap C) - B$
Proof:

Part 1: $(A - B) \cap (C - B) \subseteq (A \cap C) - B$
Suppose $x \in (A - B) \cap (C - B)$.
Prove $x \in (A \cap C) - B$.
$x \in (A \cap B^c) \cap (C \cap B^c)$ by set difference law.
$x \in A \cap C \cap B^c$ by idempotent law.
$x \in (A \cap C) - B$ by set difference law.
Proof done.

Part 2: $(A \cap C) - B \subseteq (A - B) \cap (C - B)$
Suppose $x \in (A \cap C) - B$.
Prove $x \in (A - B) \cap (C - B)$.
$x \in (A \cap C) \cap B^c$ by set difference law.
$x \in A \cap B^c \cap C \cap B^c$ by idempotent law.
$x \in (A - B) \cap (C - B)$ by set difference law.
Proof done.

Both parts proven therefore $(A - B) \cap (C - B) = (A \cap C) - B$.

Problem 5.
Exercise Set 6.2

19. For all sets A, B, and C, $A \times (B \cap C) = (A \times B) \cap (A \times C)$
    Proof:

    Part 1: $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$
    Suppose $x \in A \times (B \cap C)$.
    Prove $x \in (A \times B) \cap (A \times C)$.
    $x \in (A \times B) \cap (A \times C)$ by distributive law.
    Proof done.

    Part 2: $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$
    Suppose $x \in (A \times B) \cap (A \times C)$.
    Prove $x \in A \times (B \cap C)$.
    $x \in A \times (B \cap C)$ by distributive law.
    Proof done.

    Both parts proven therefore $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Problem 6.
Exercise Set 6.2

34. For all sets A, B, and C, if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$
    Proof:
    Let $B \cap C \subseteq A$.
    Assume $(C - A) \cap (B - A) \neq \emptyset$.
    Suppose $x \in (C - A) \cap (B - A)$.
    $x \in (C \cap B) \cap A^c$ as proven previously.
    $x \in B \cap C$ and $x \in A^c$ by definition of intersection.
    Since $B \cap C \subseteq A$, $x \in A$ by definition of subset.
    Contradiction because $x \in A$ and $x \notin A \Leftrightarrow x \in A^c$.
    Assumption is false therefore if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$.