The curse of rationality in sequential scheduling games

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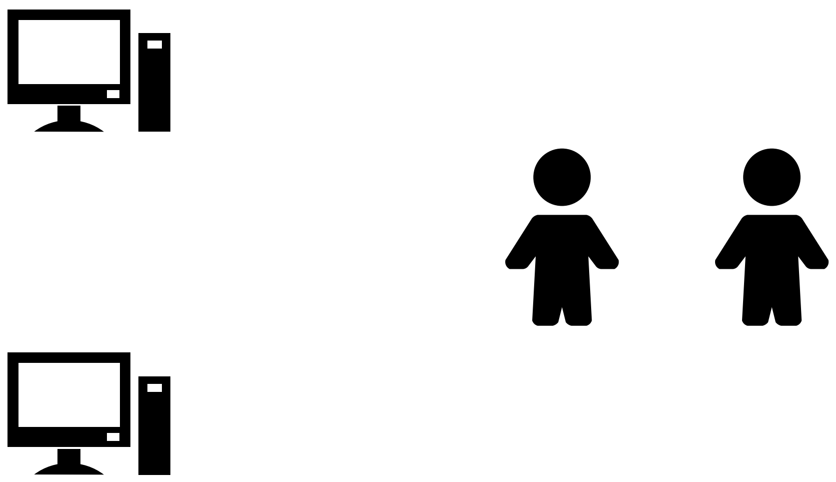
西安交通大學
XI'AN JIAOTONG UNIVERSITY

WINE 2020, December

Sequential Scheduling Games

A game of 2 machines and 2 users/jobs



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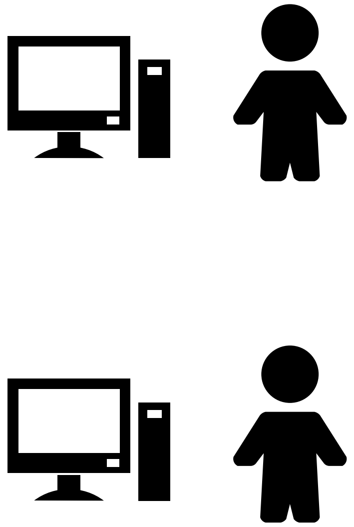

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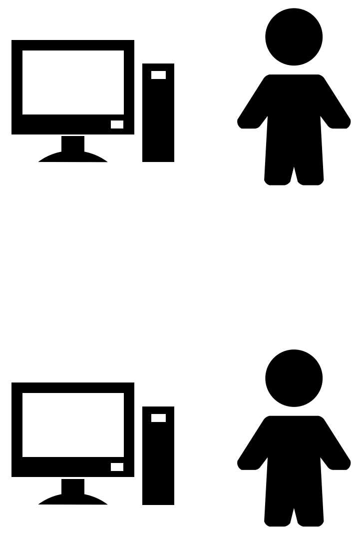

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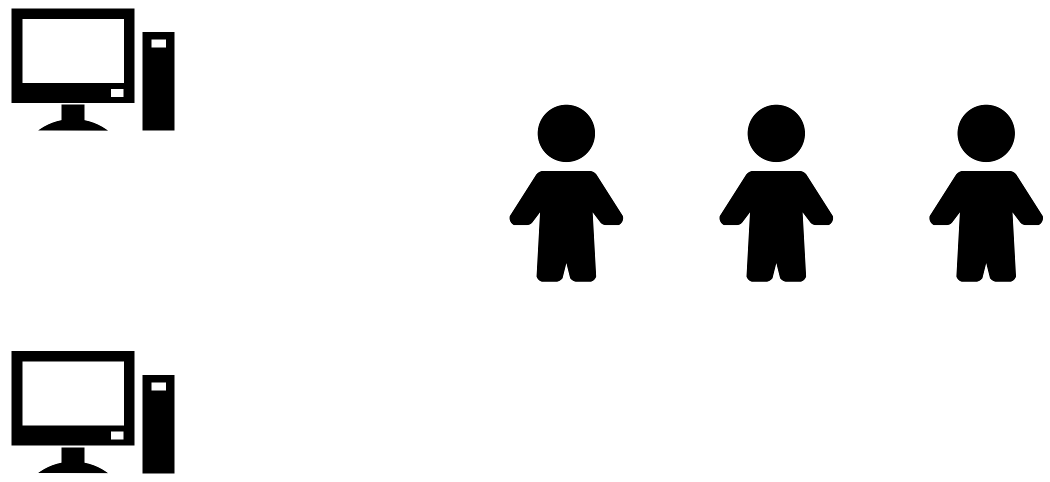

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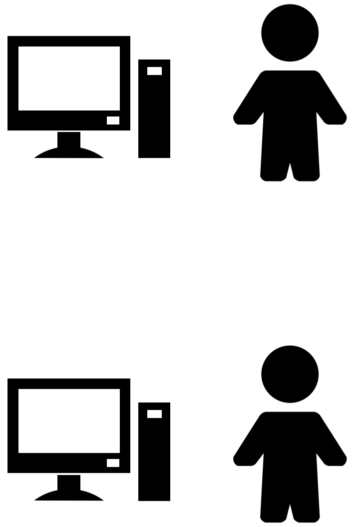

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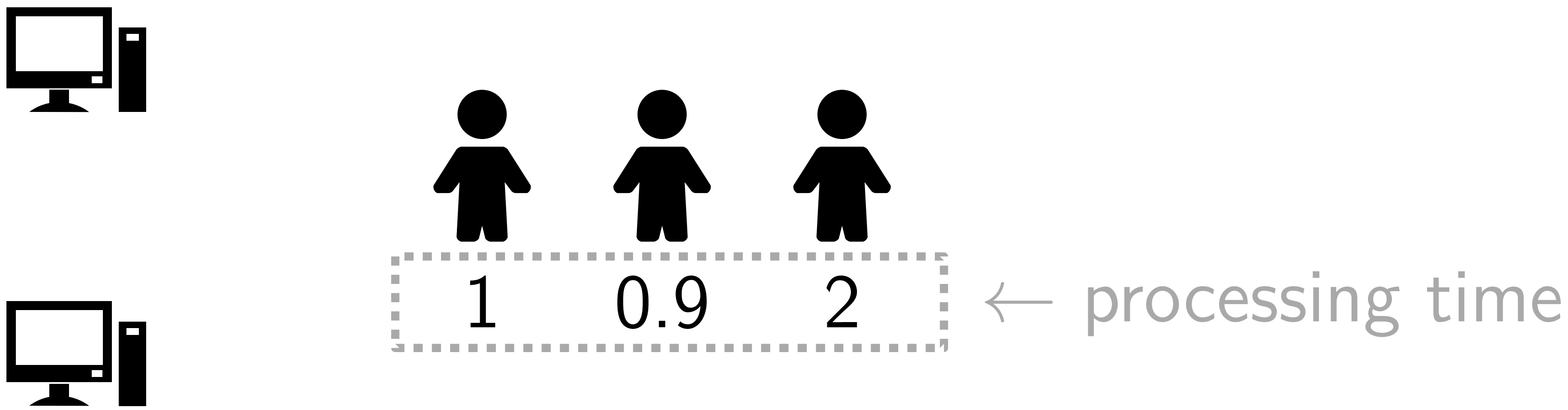

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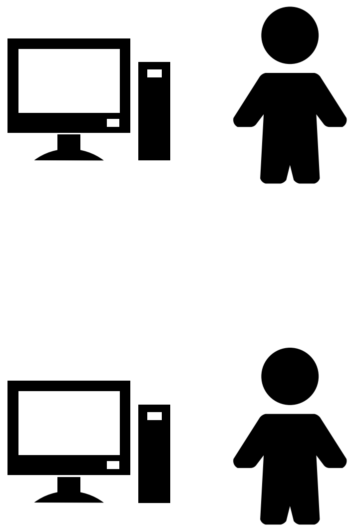

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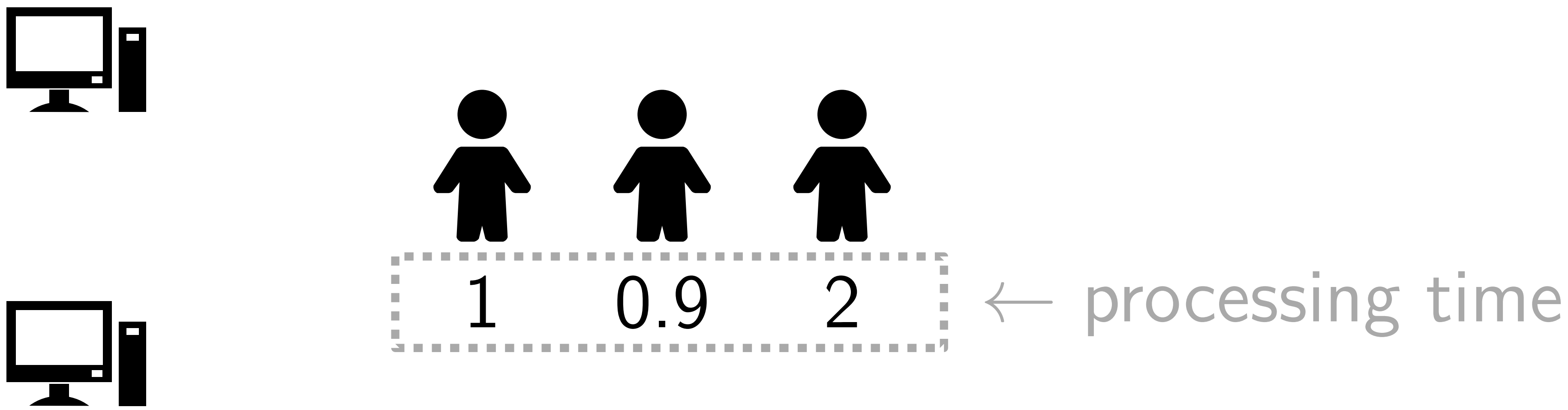

Individual cost: the load of the chosen machine

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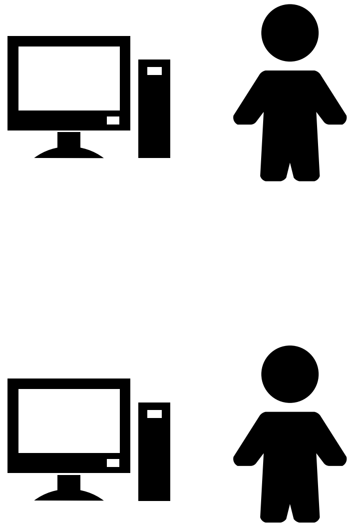

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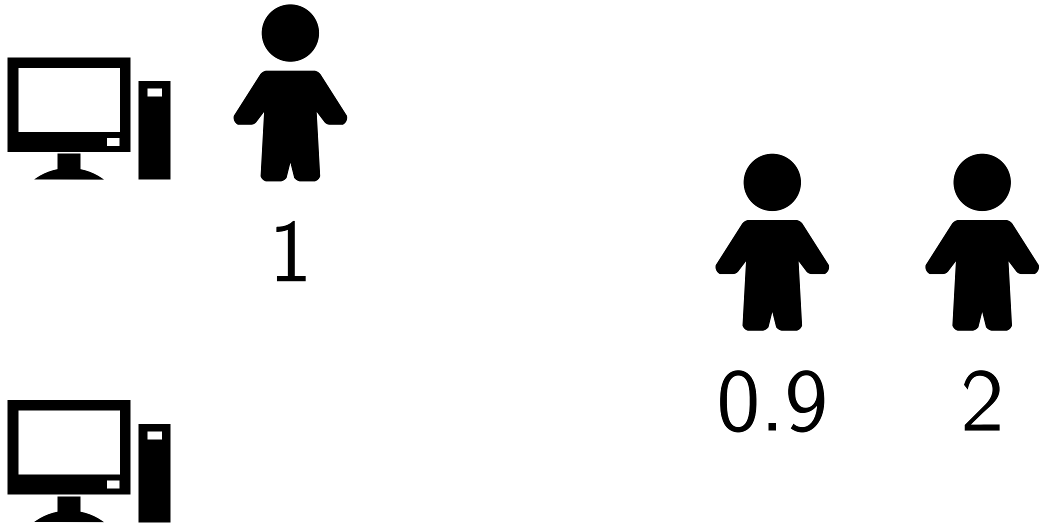

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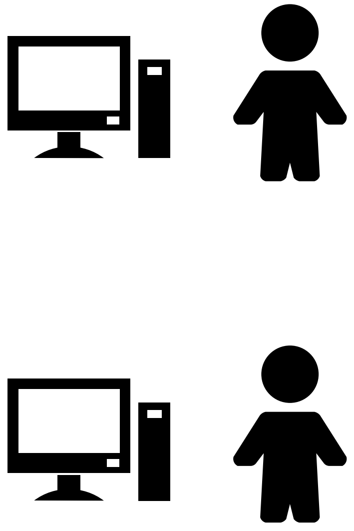

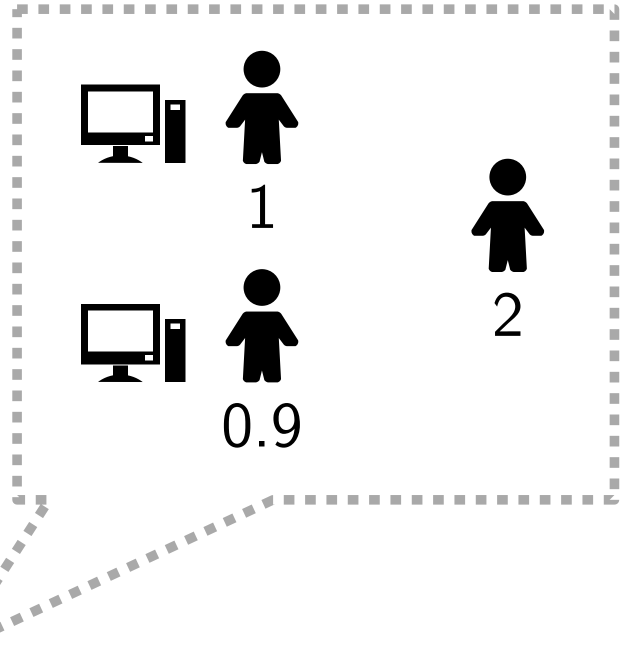

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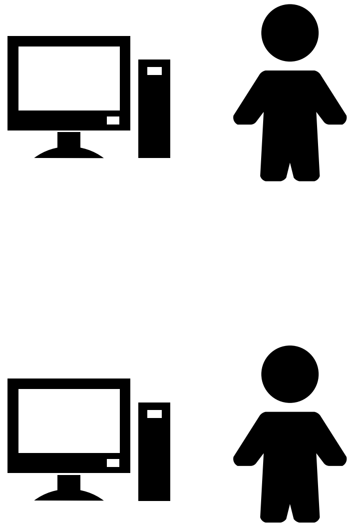

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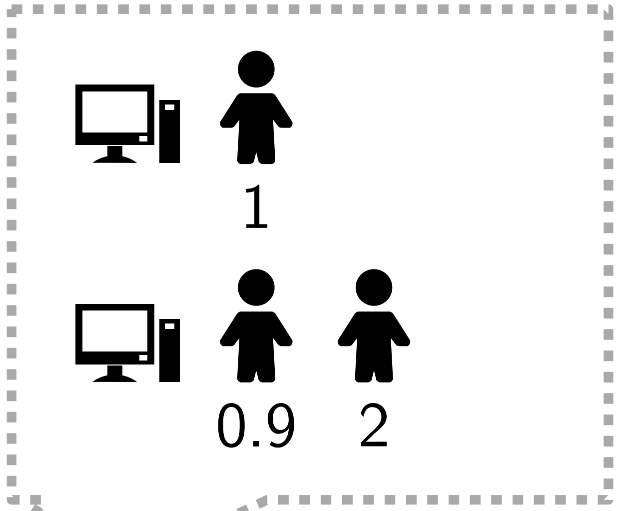

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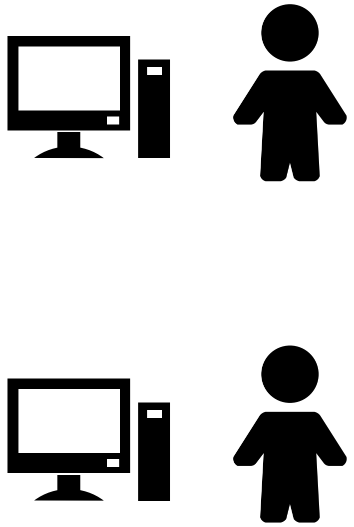

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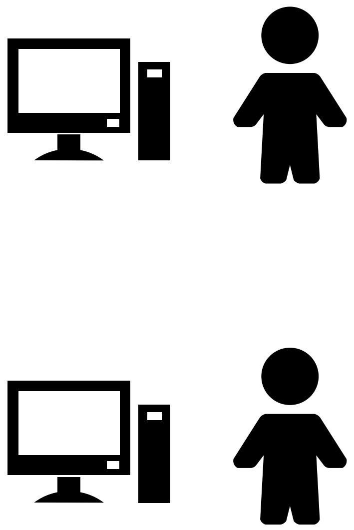

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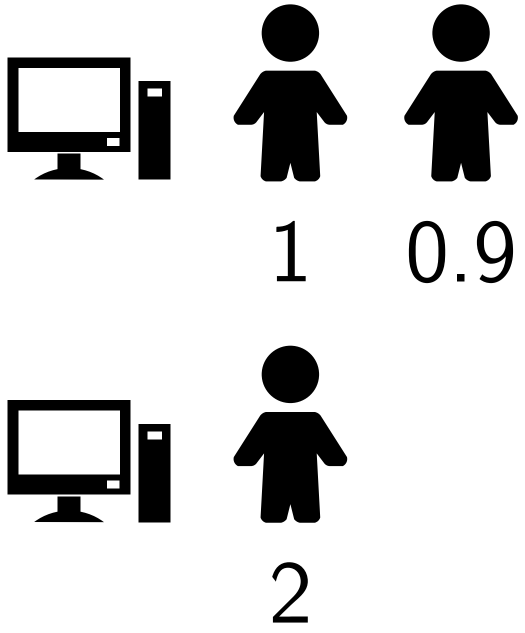

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A game of 2 machines and 3 jobs: ***unrelated machines setting***

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Game tree

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Backward induction

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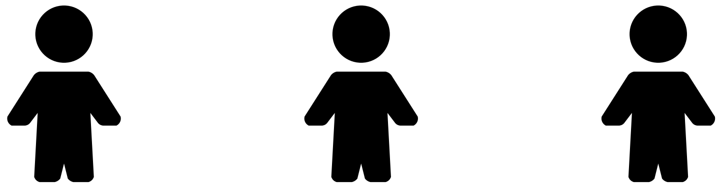

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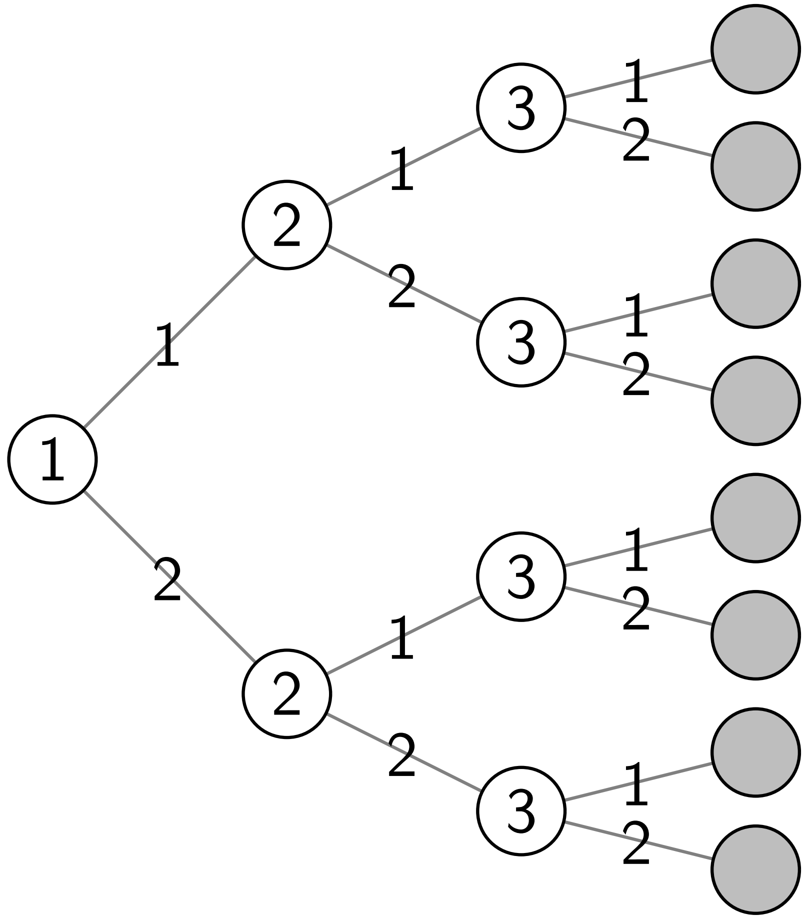

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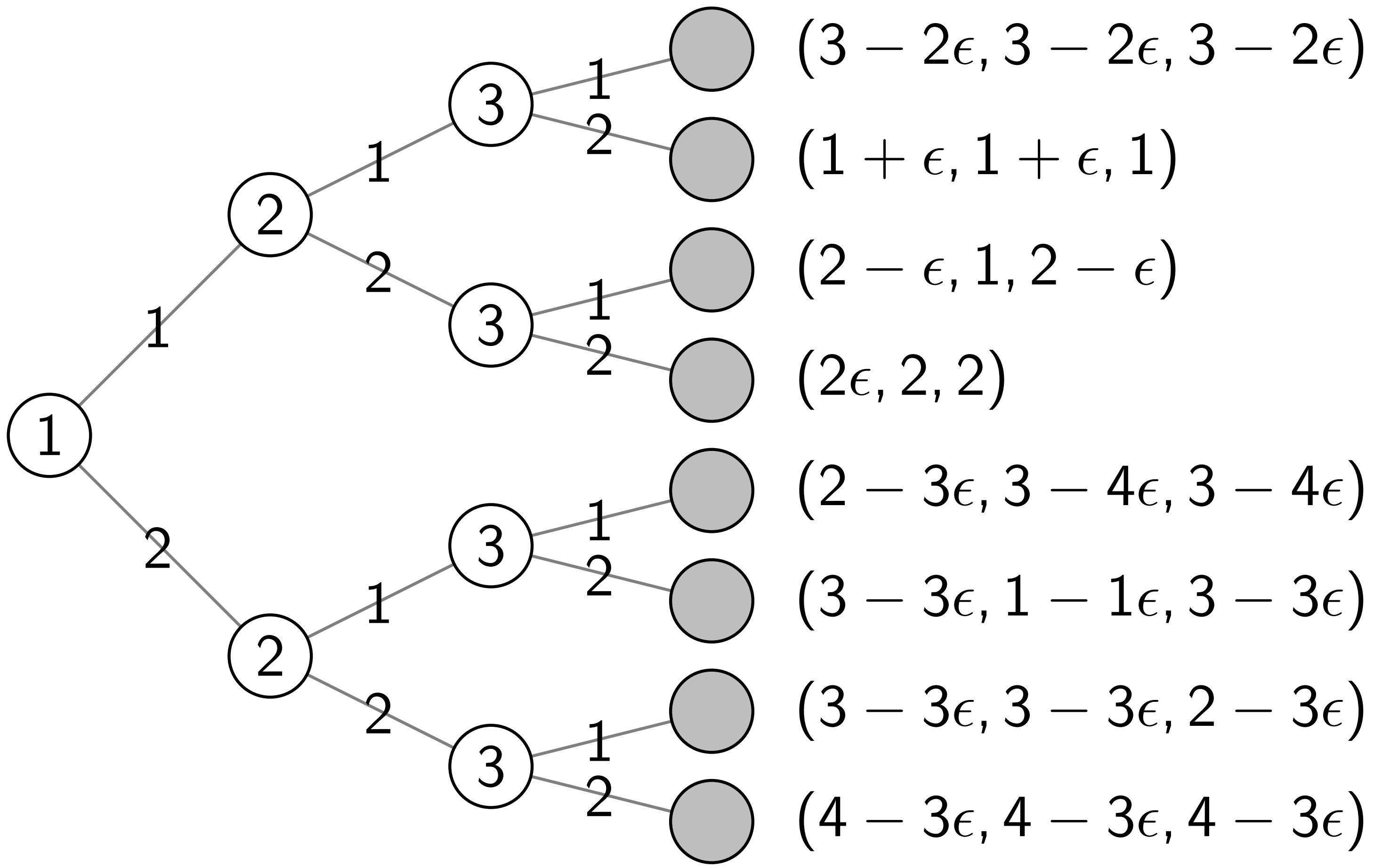

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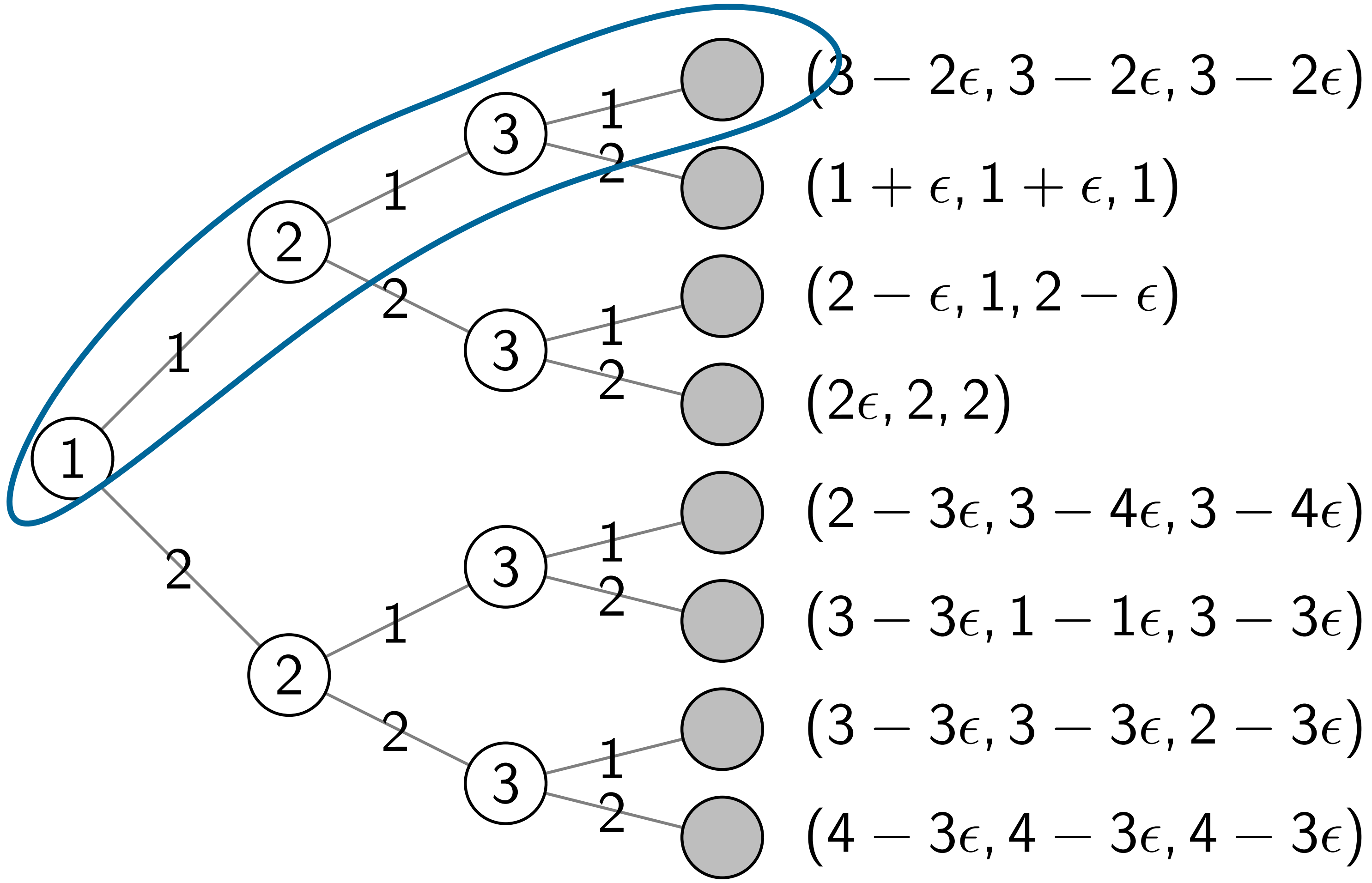

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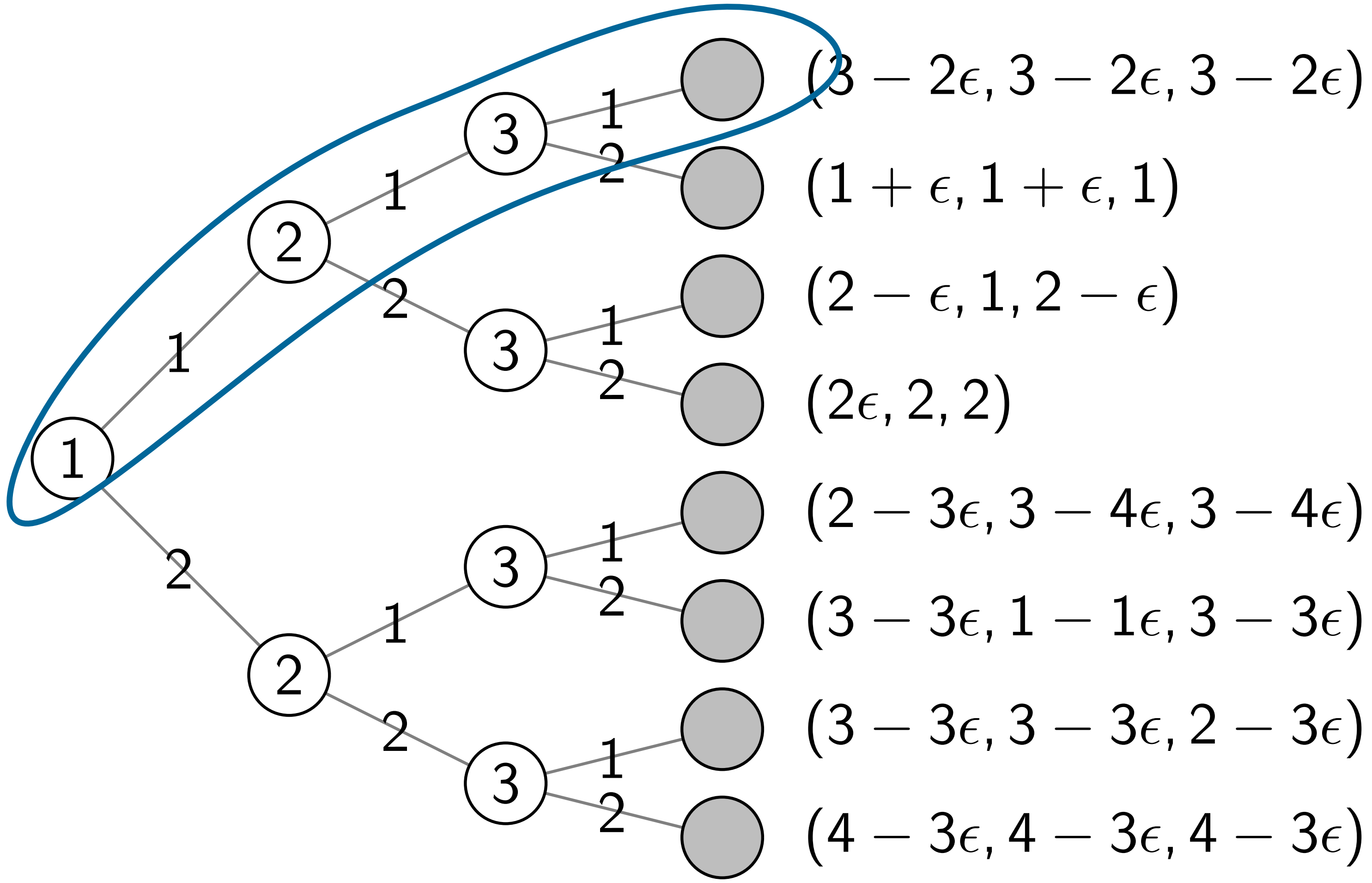

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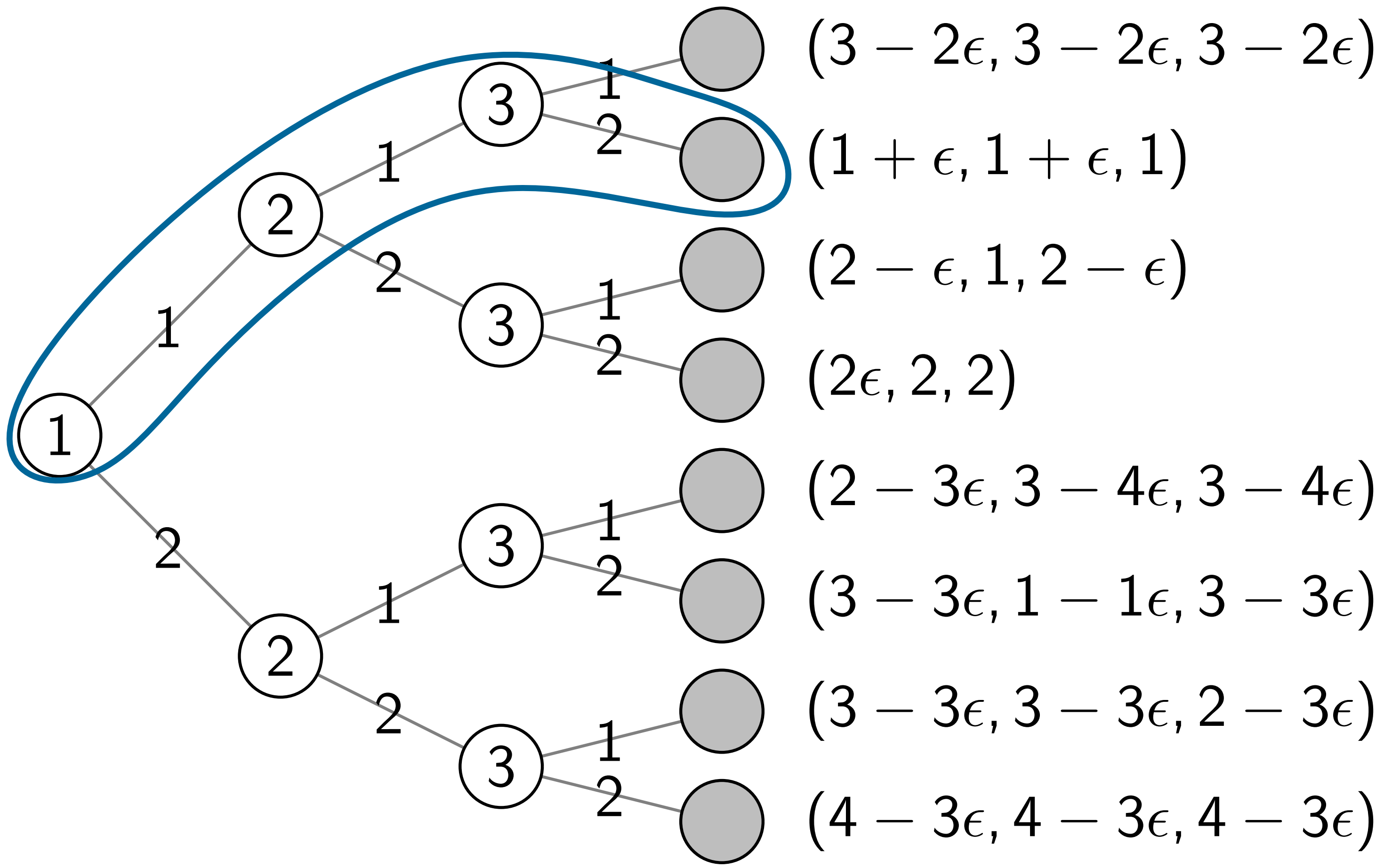

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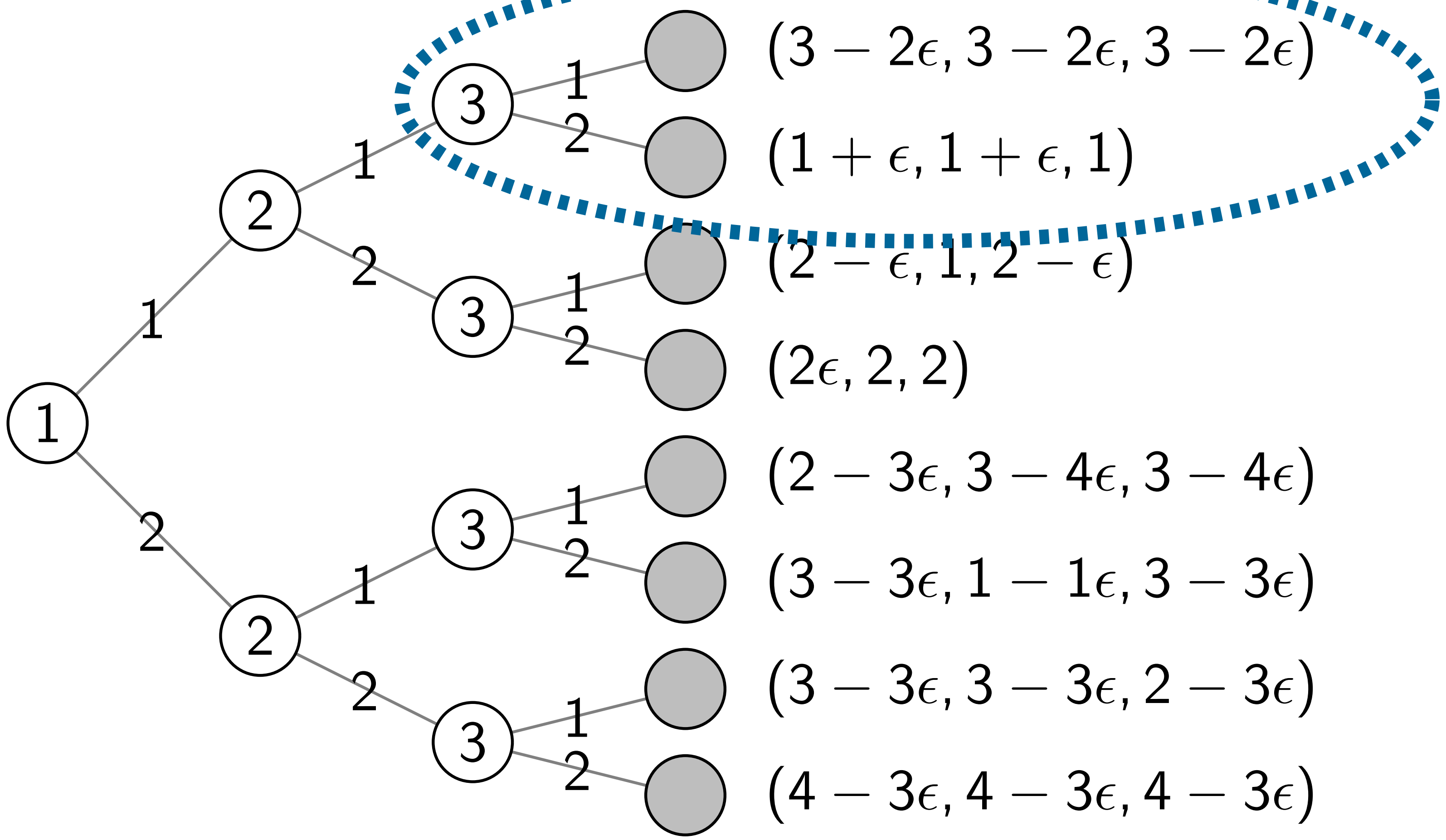

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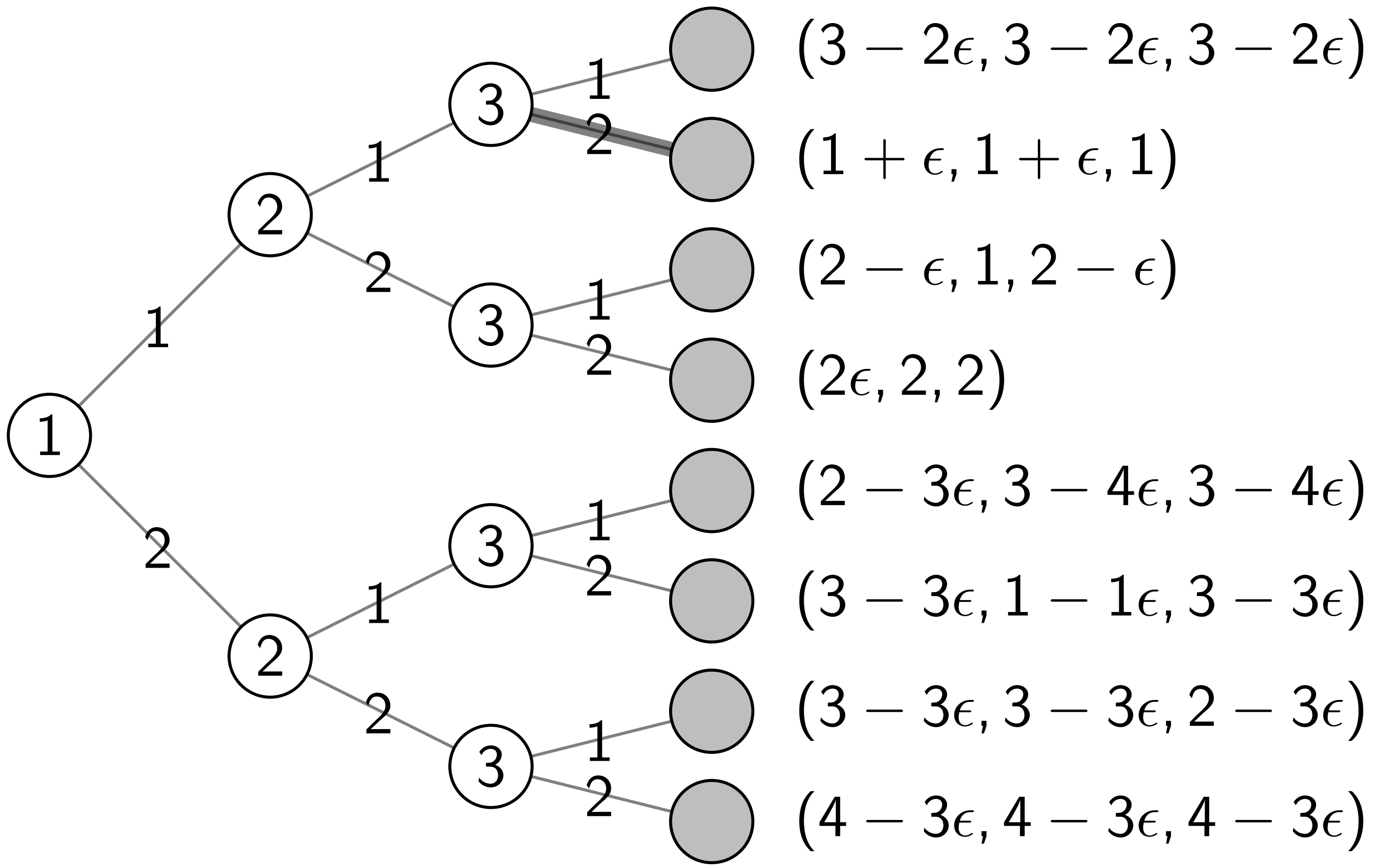

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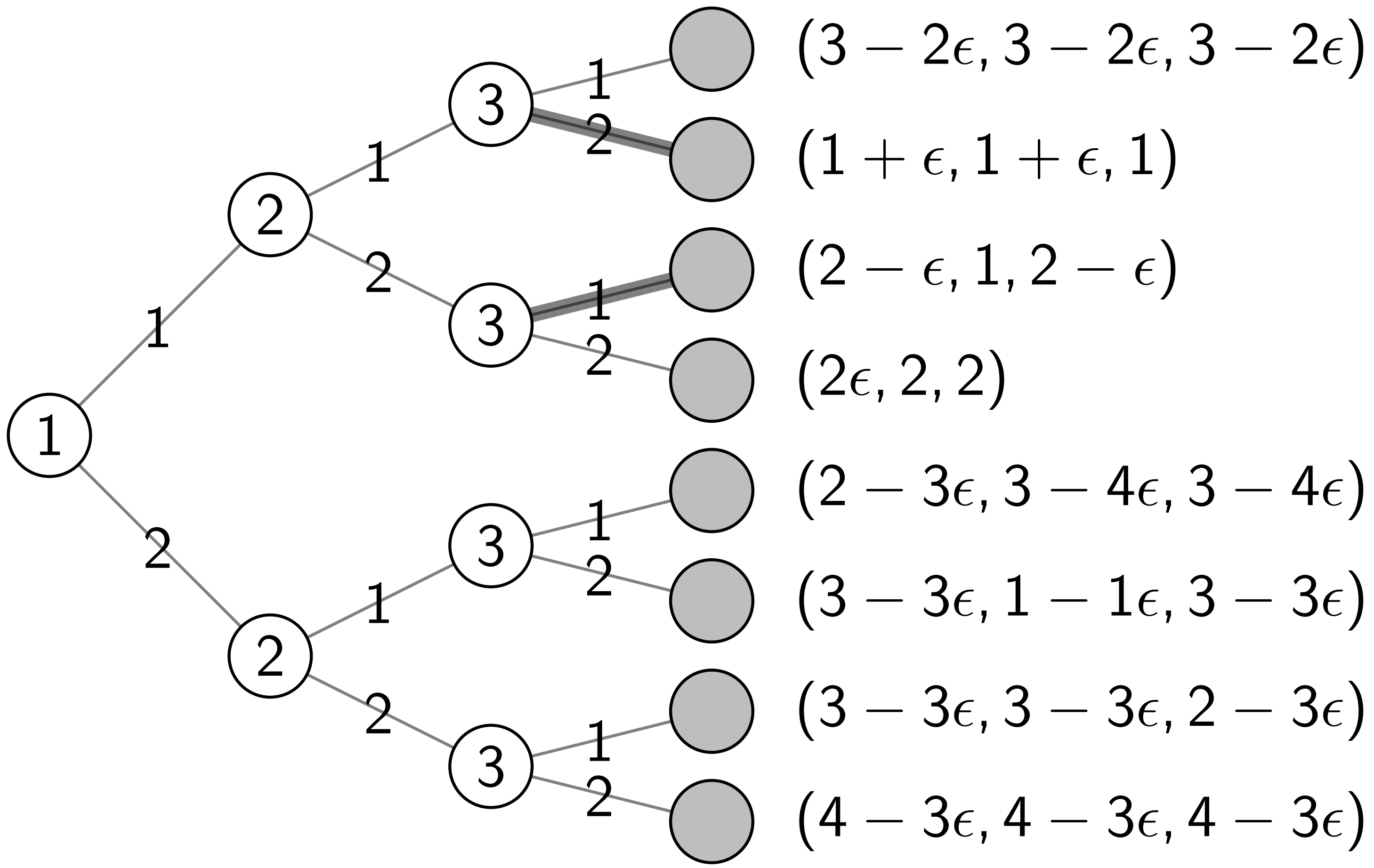

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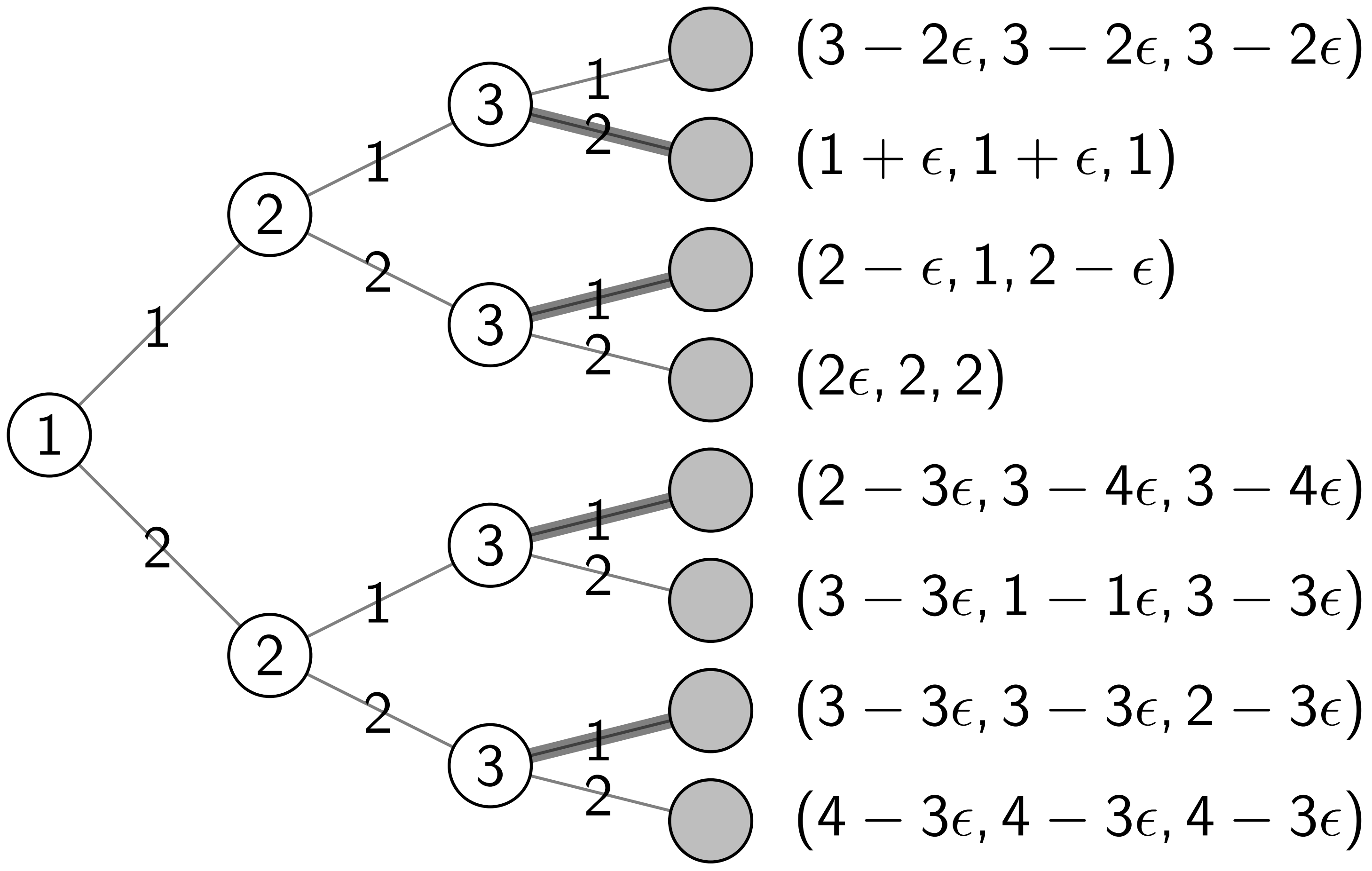

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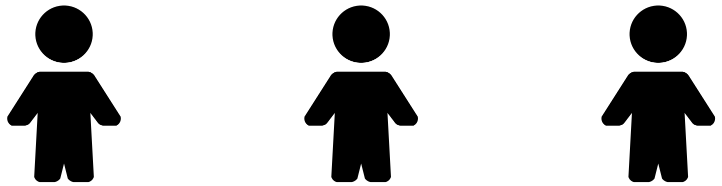

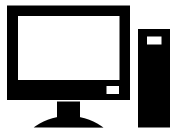

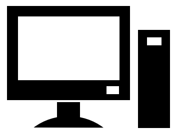

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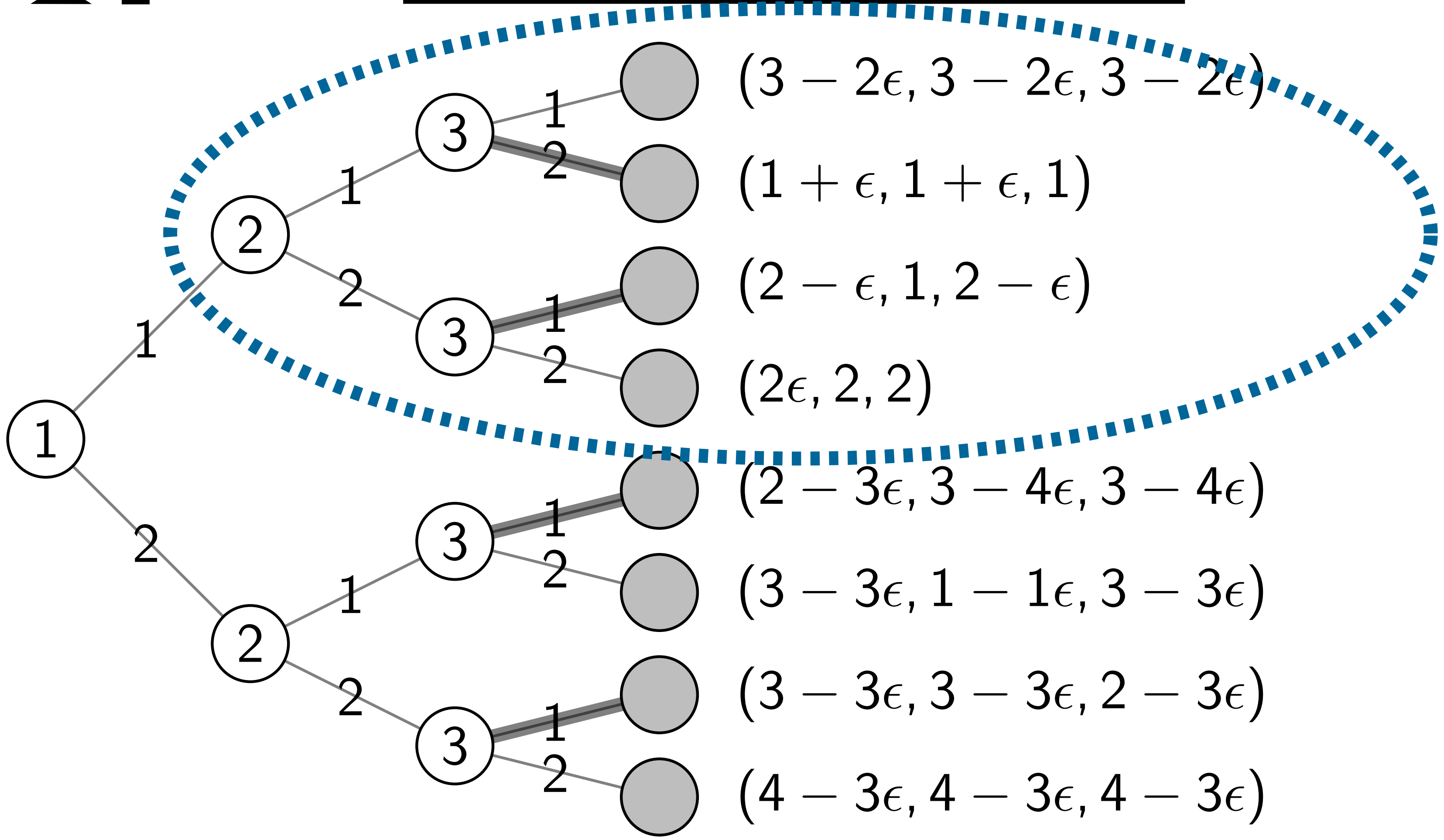

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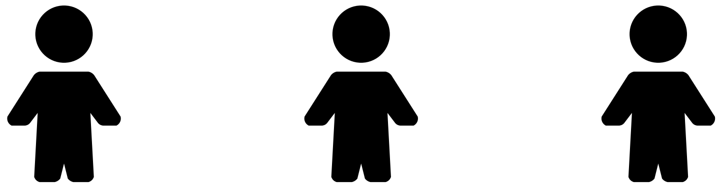

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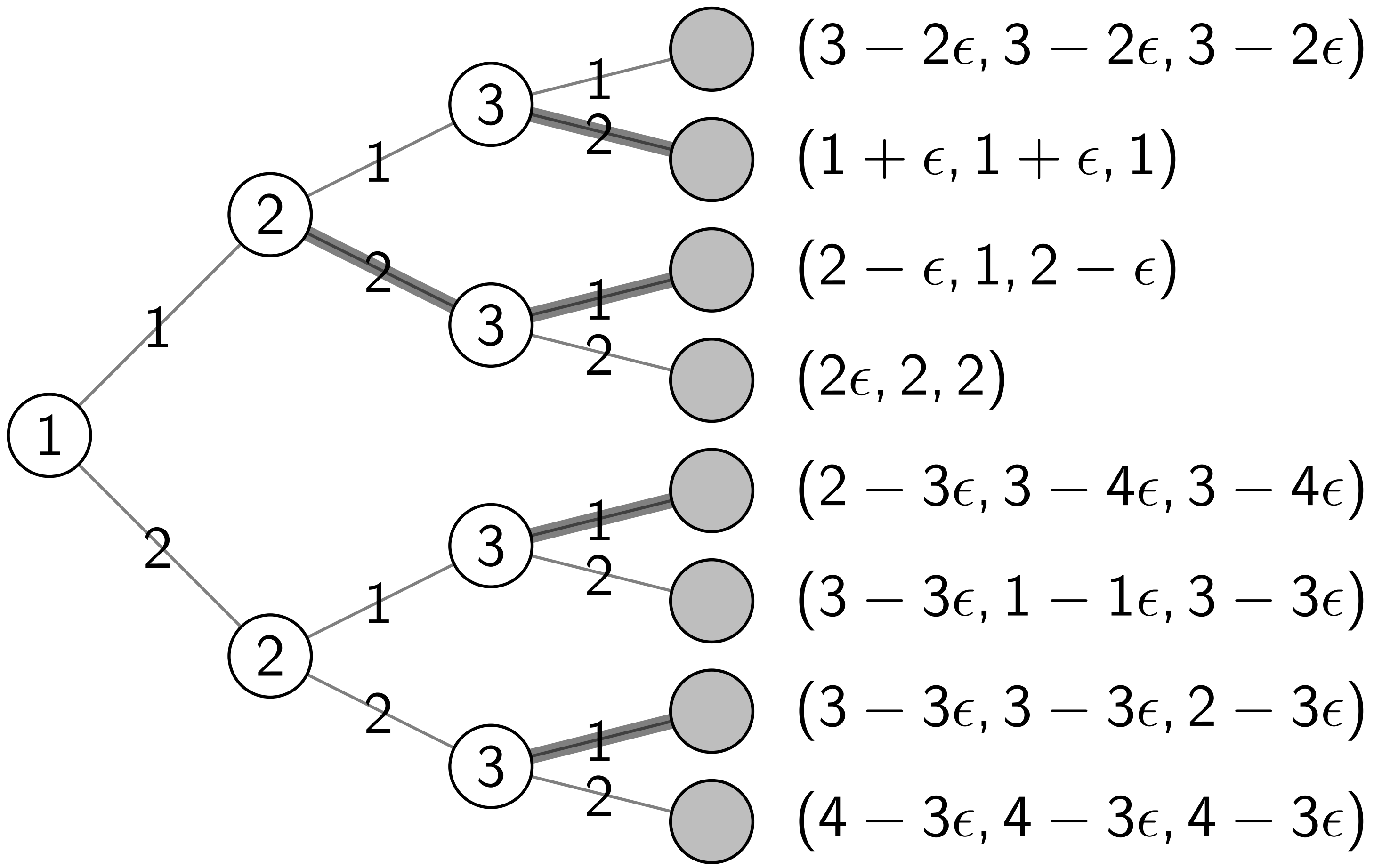

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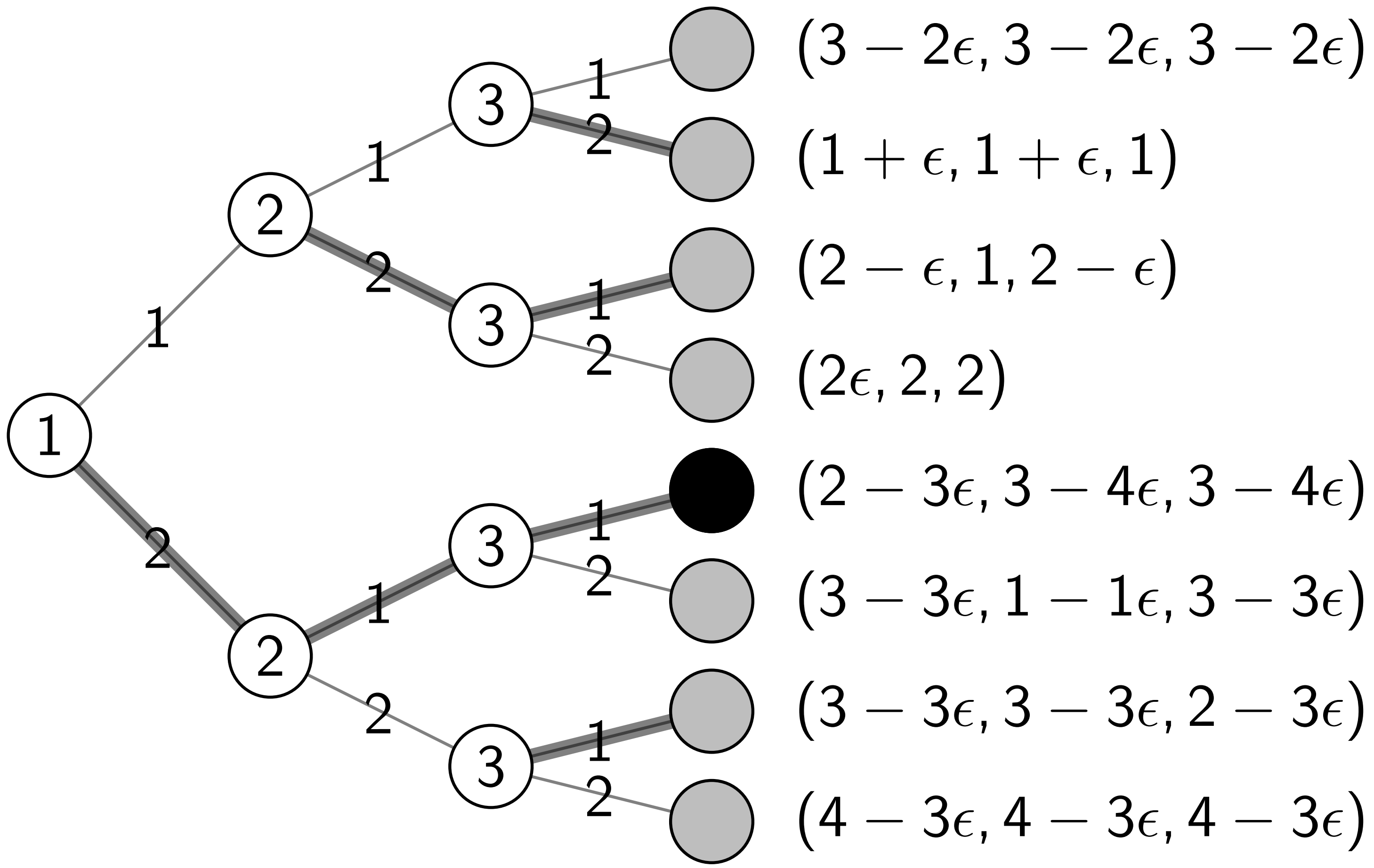

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Sequential scheduling game on unrelated machines

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- Farsighted players: when a job makes decision, he knows the choices made by his predecessors as well as the processing times of his successors.
- The game always possesses **subgame-perfect equilibria (SPE)**, which can be calculated by backward induction.

Motivations

Computational ablility of players

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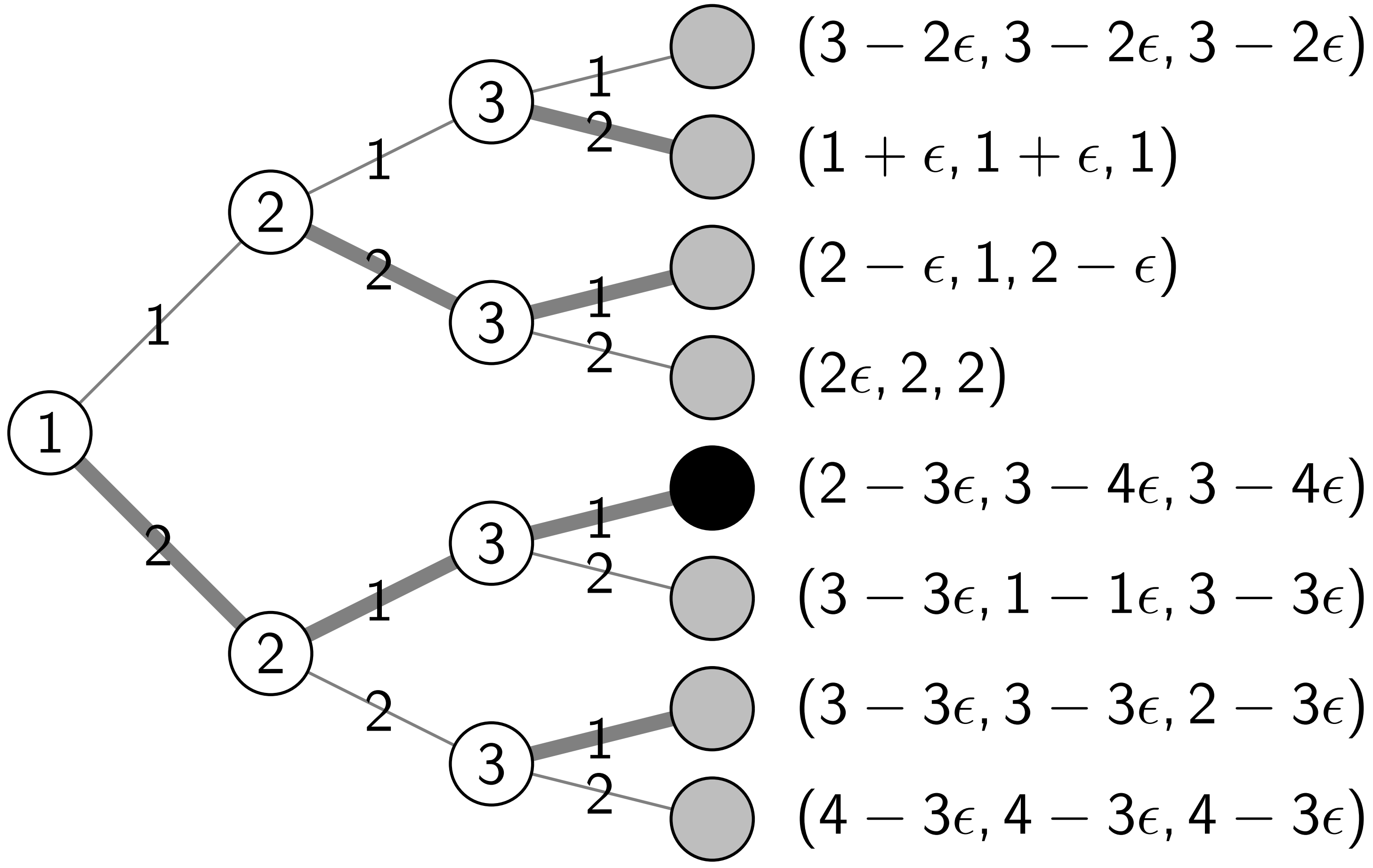

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- $2^3 = 8$ leaves for 2 machines and 3 jobs
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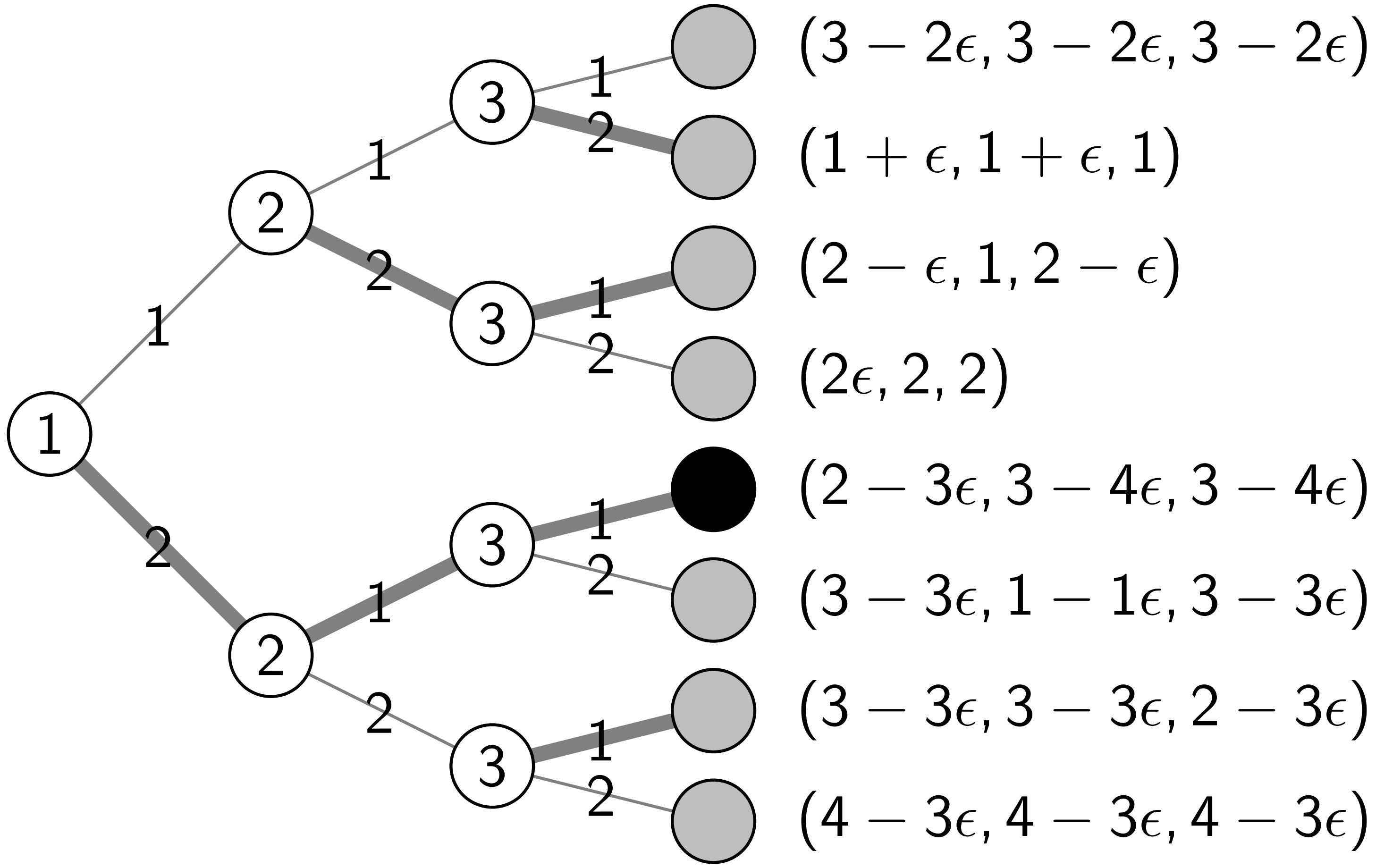

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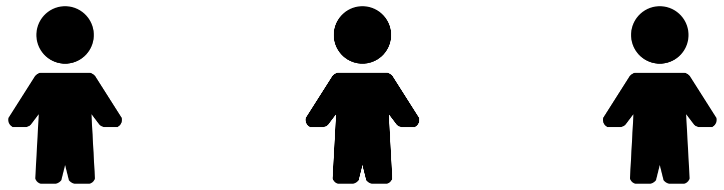

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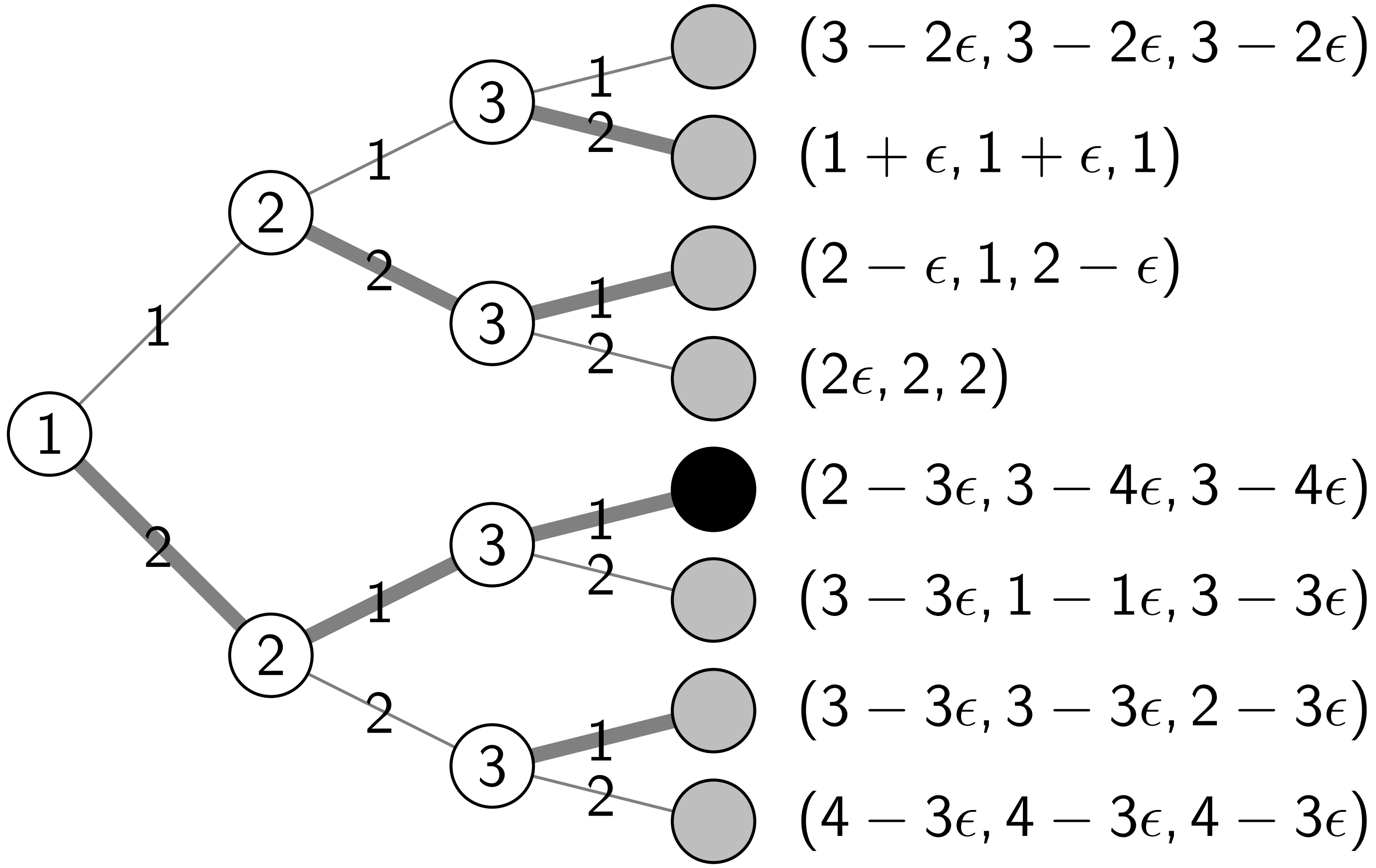

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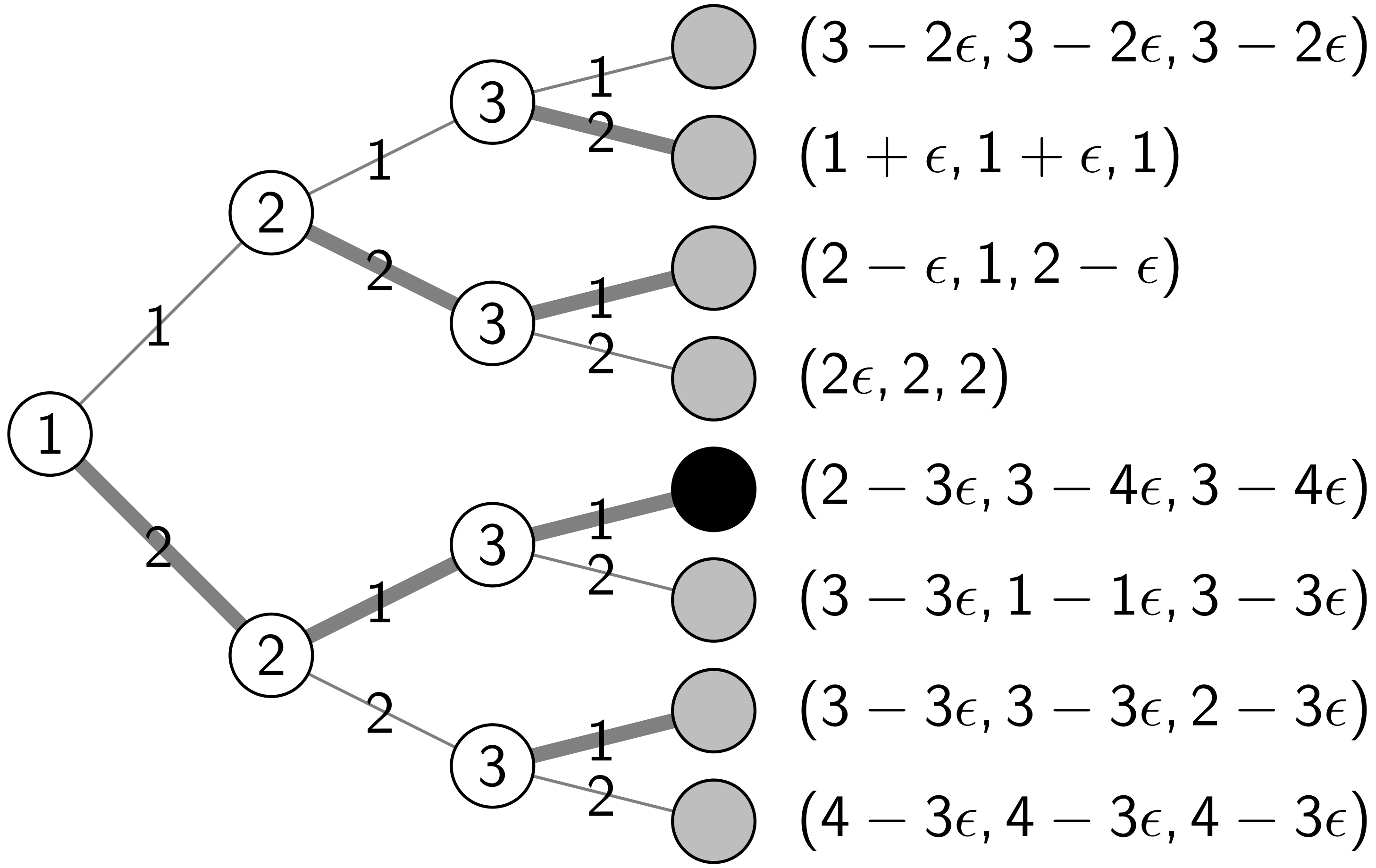

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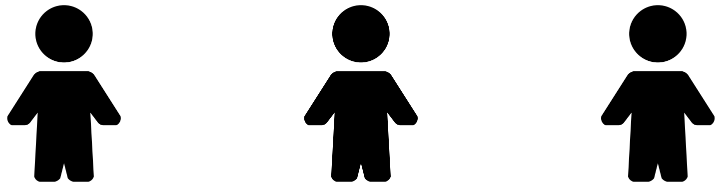

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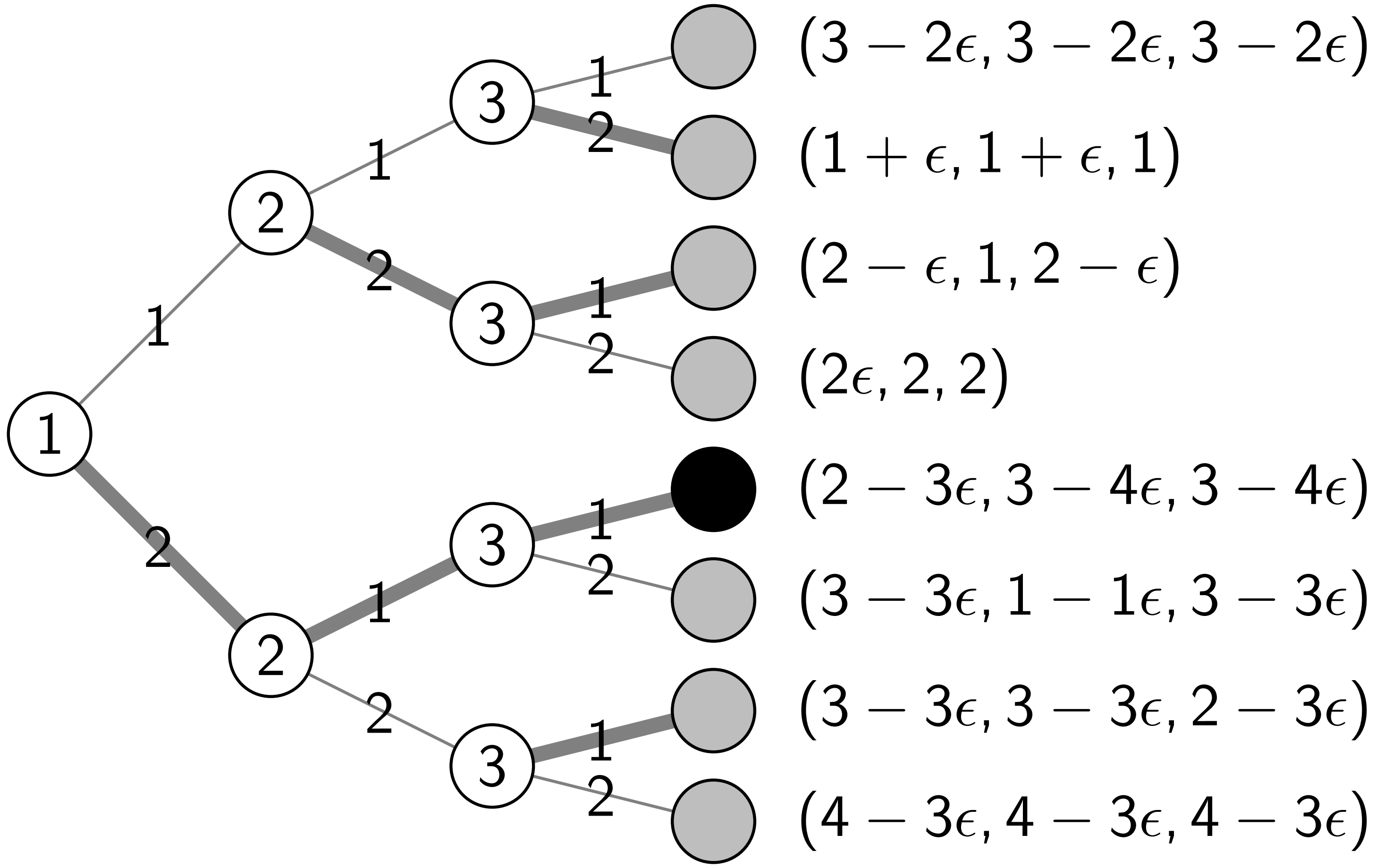

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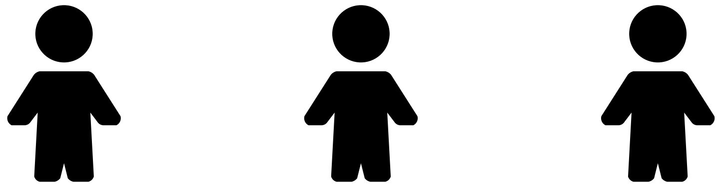

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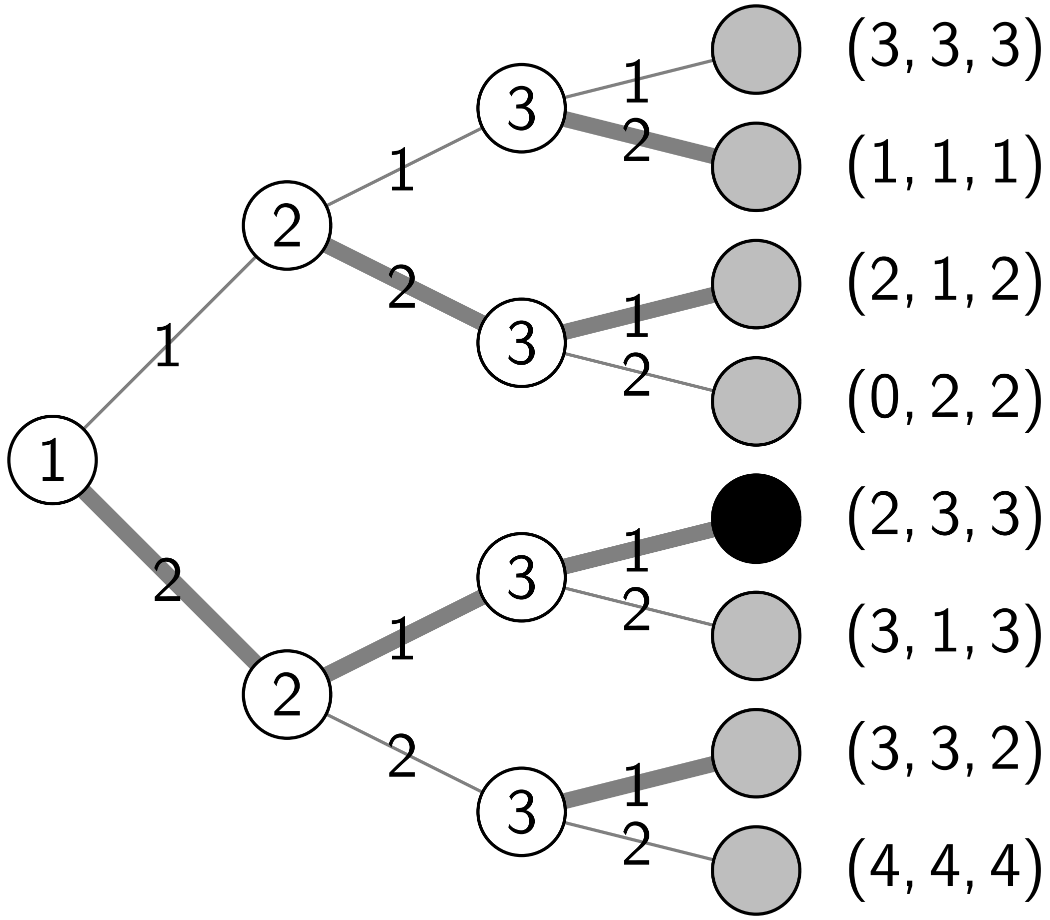

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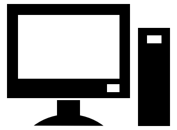

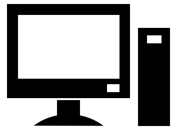

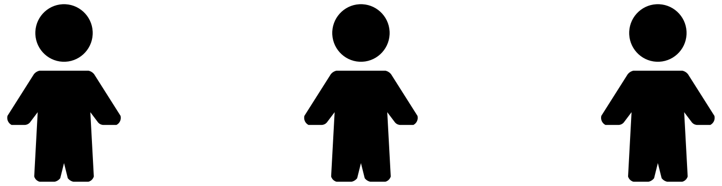

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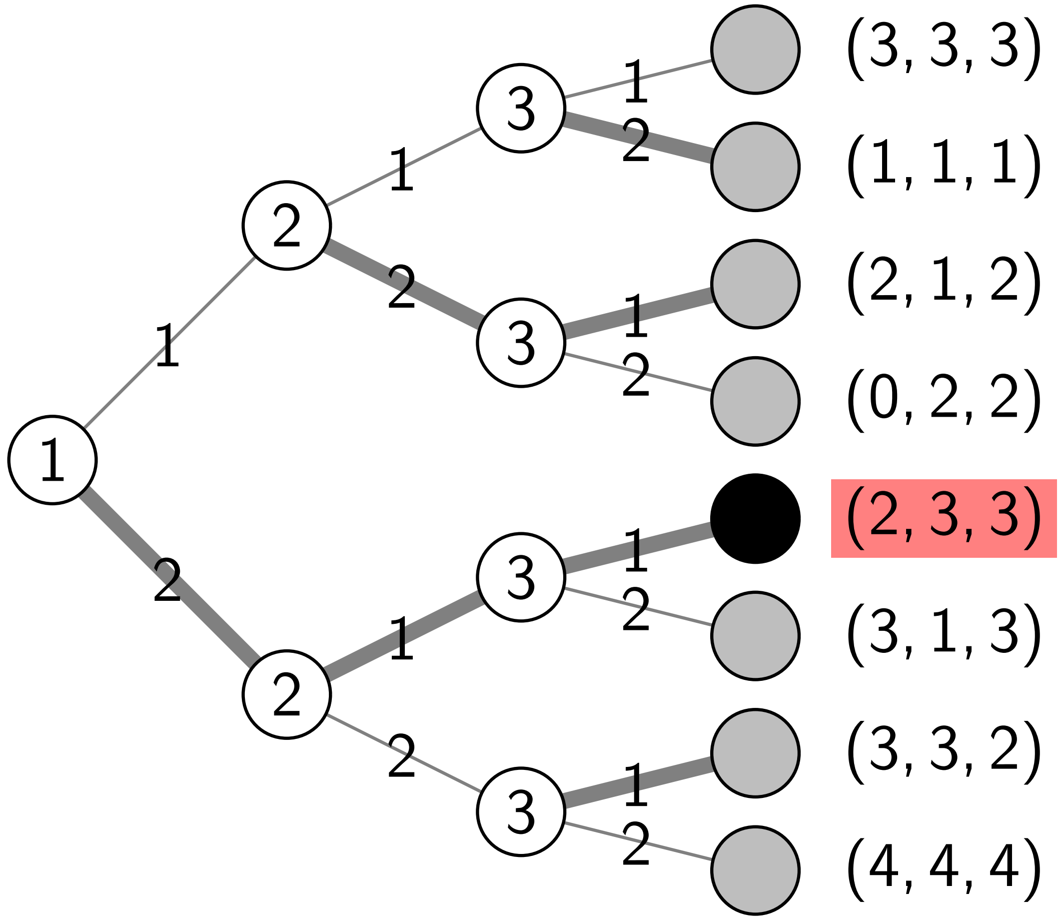

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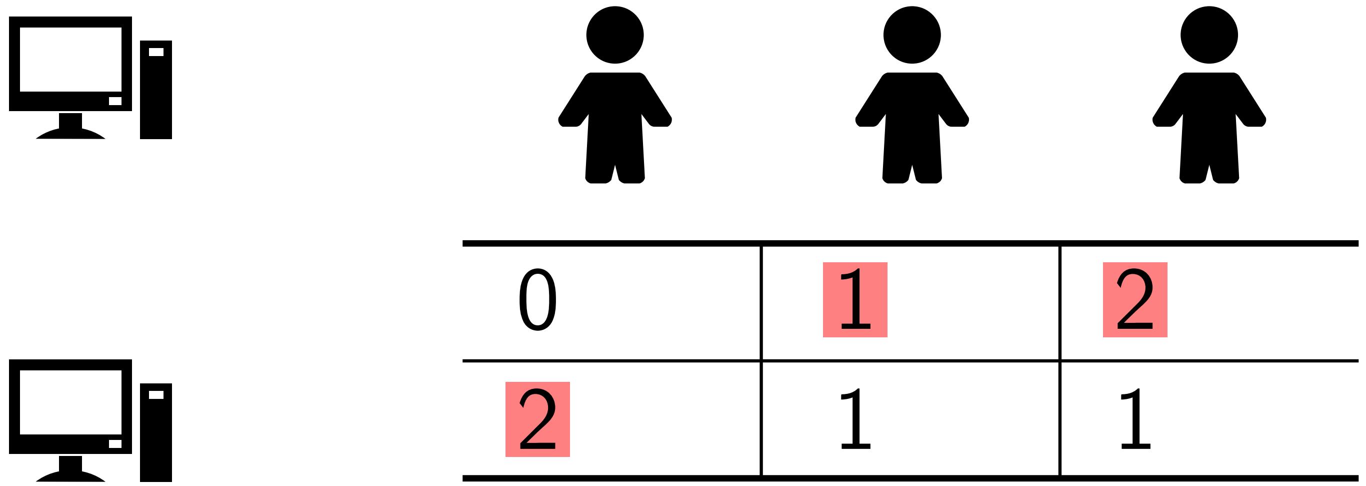
It is reasonable to reconsider the common assumption that all players have unlimited computational capacity (full rationality)



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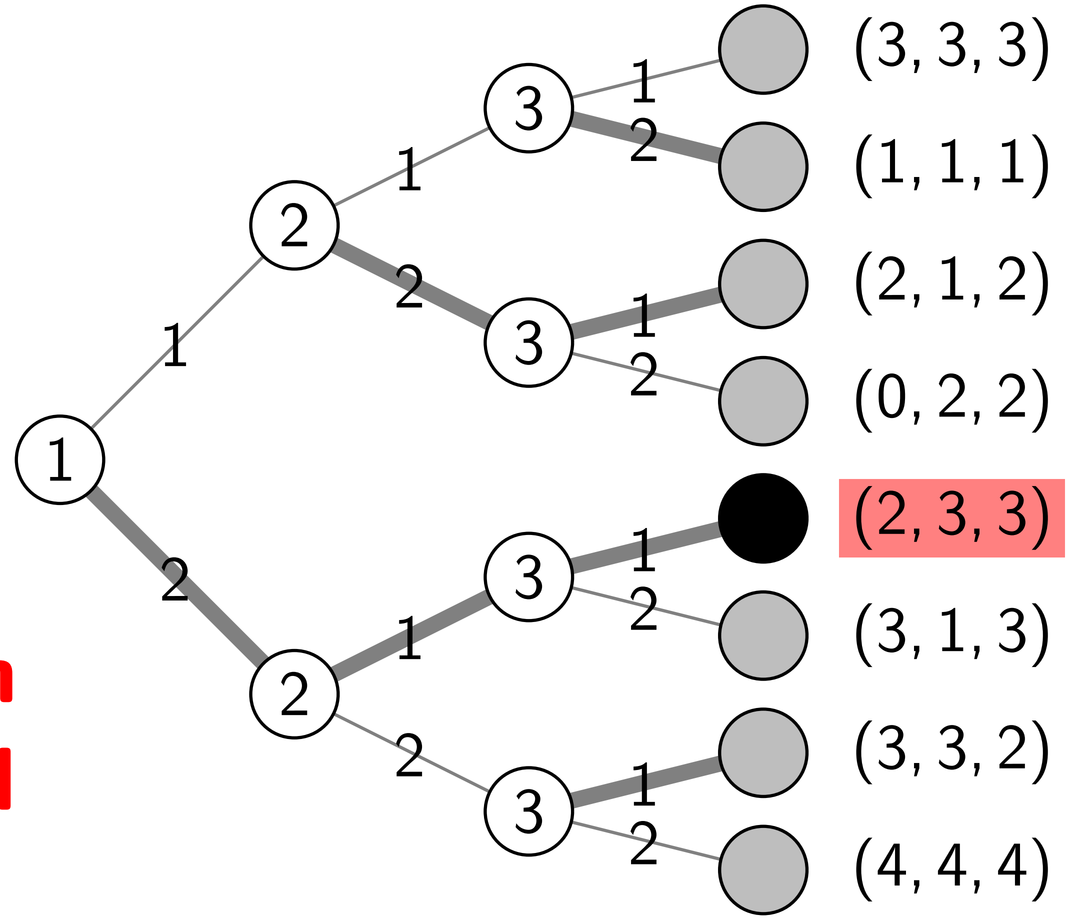

Motivations

Efficiency of equilibrium

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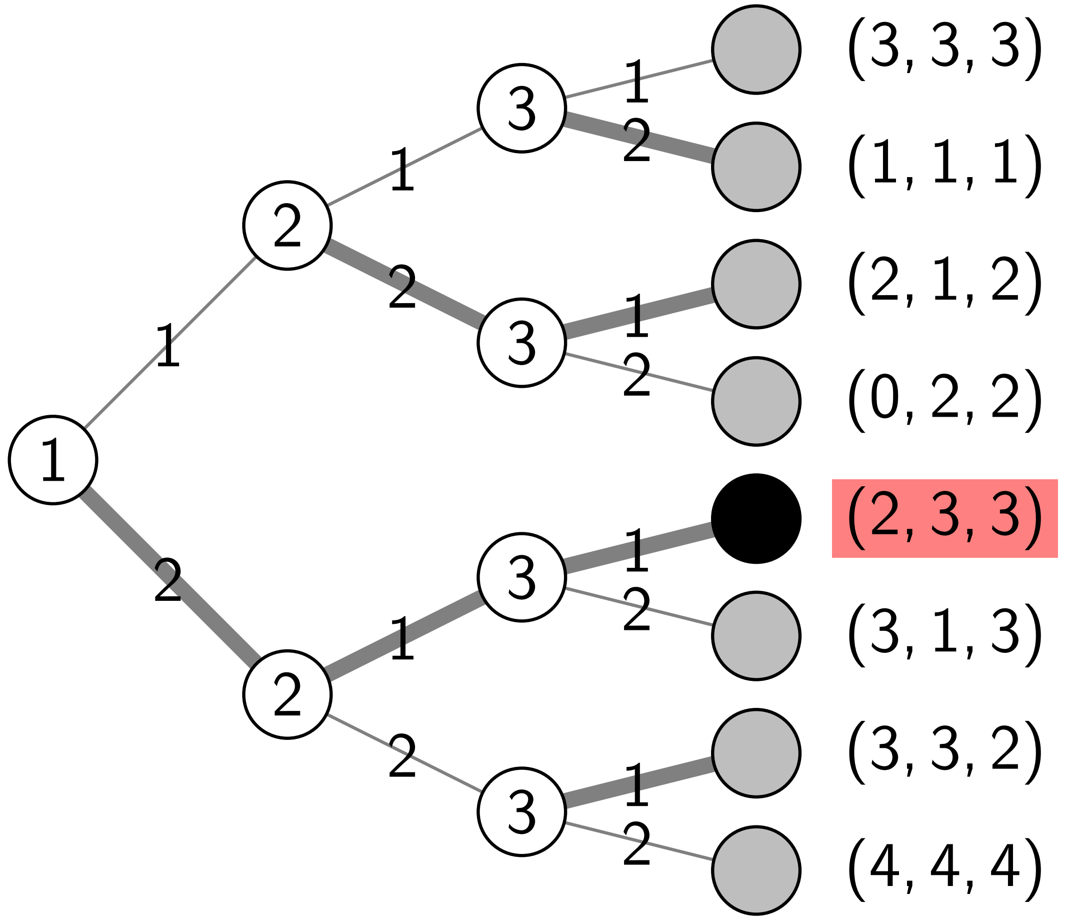

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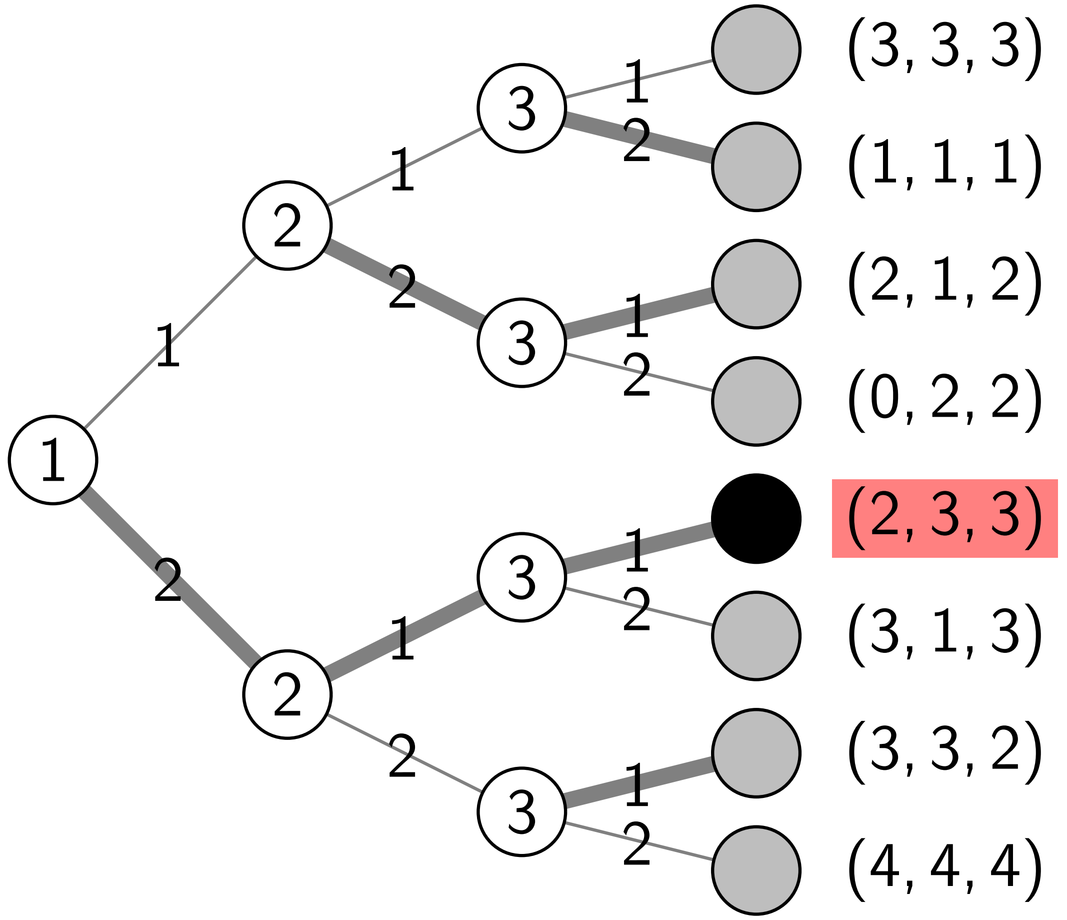

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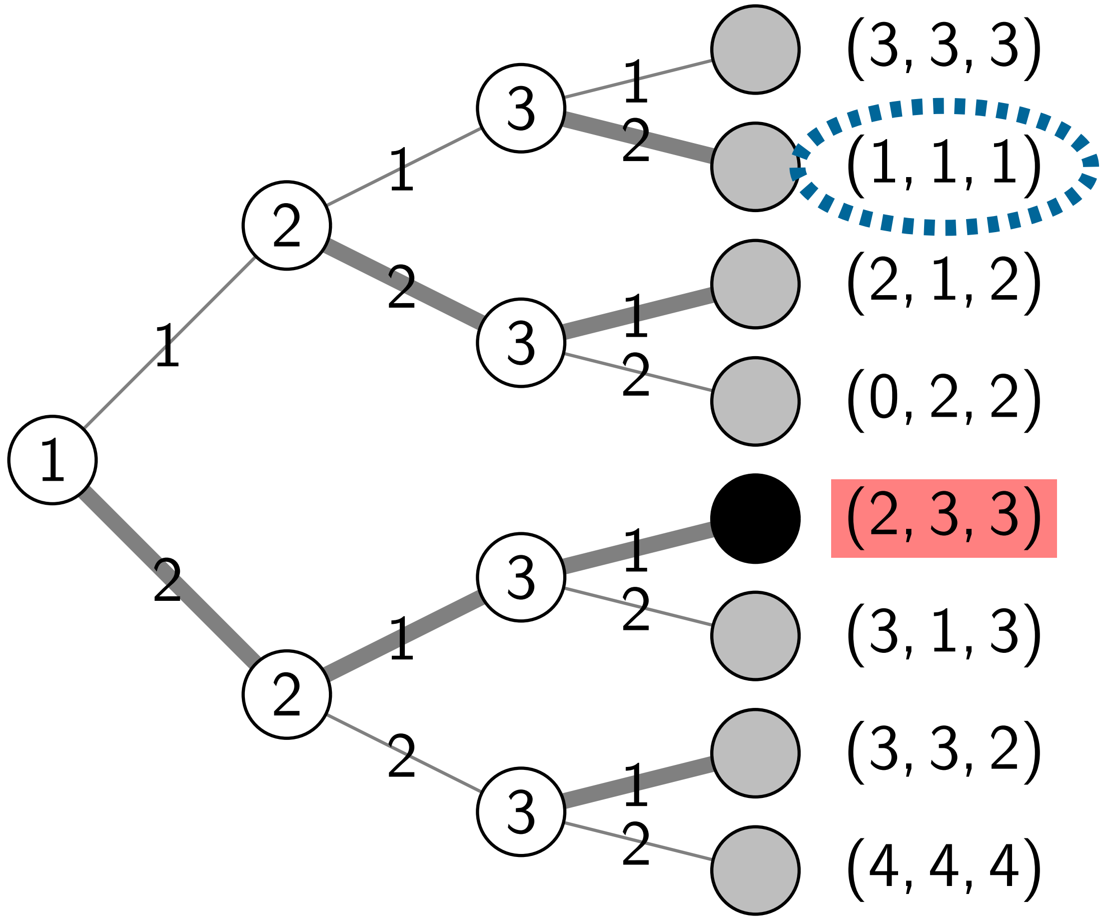

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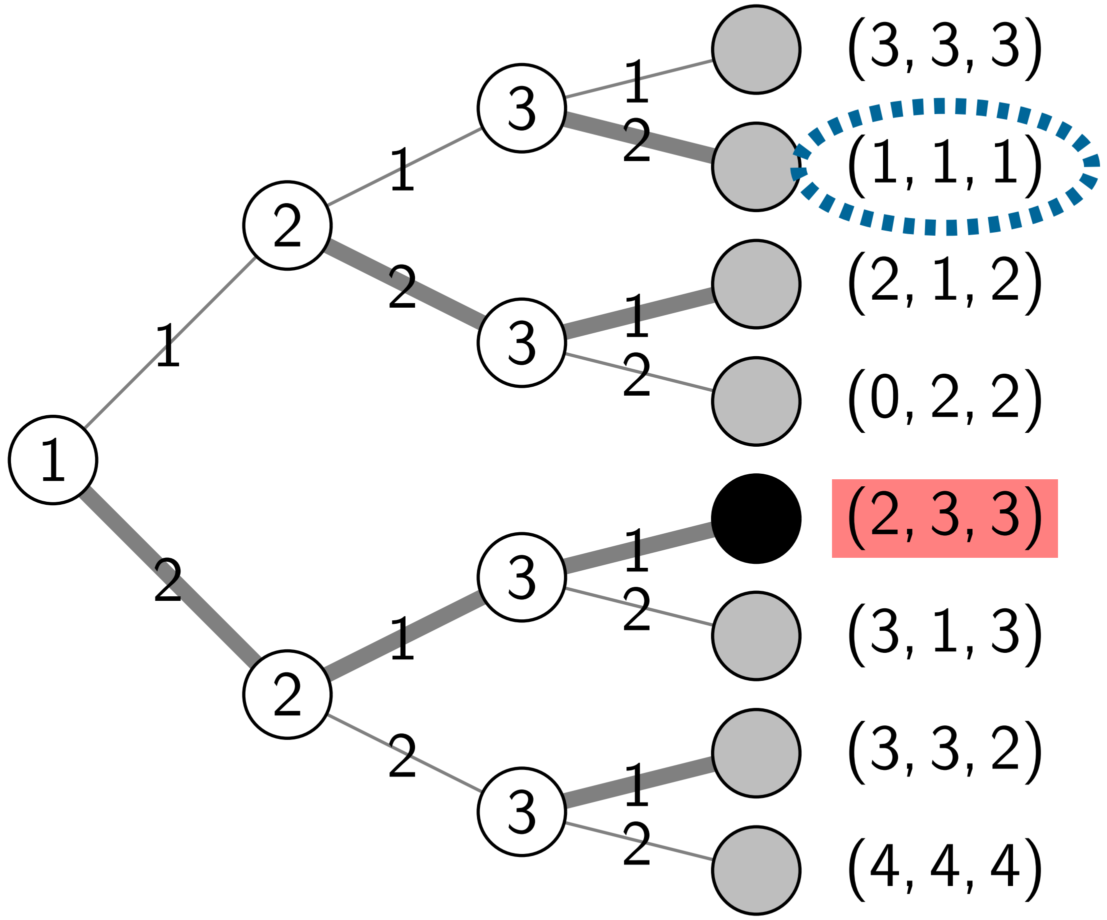

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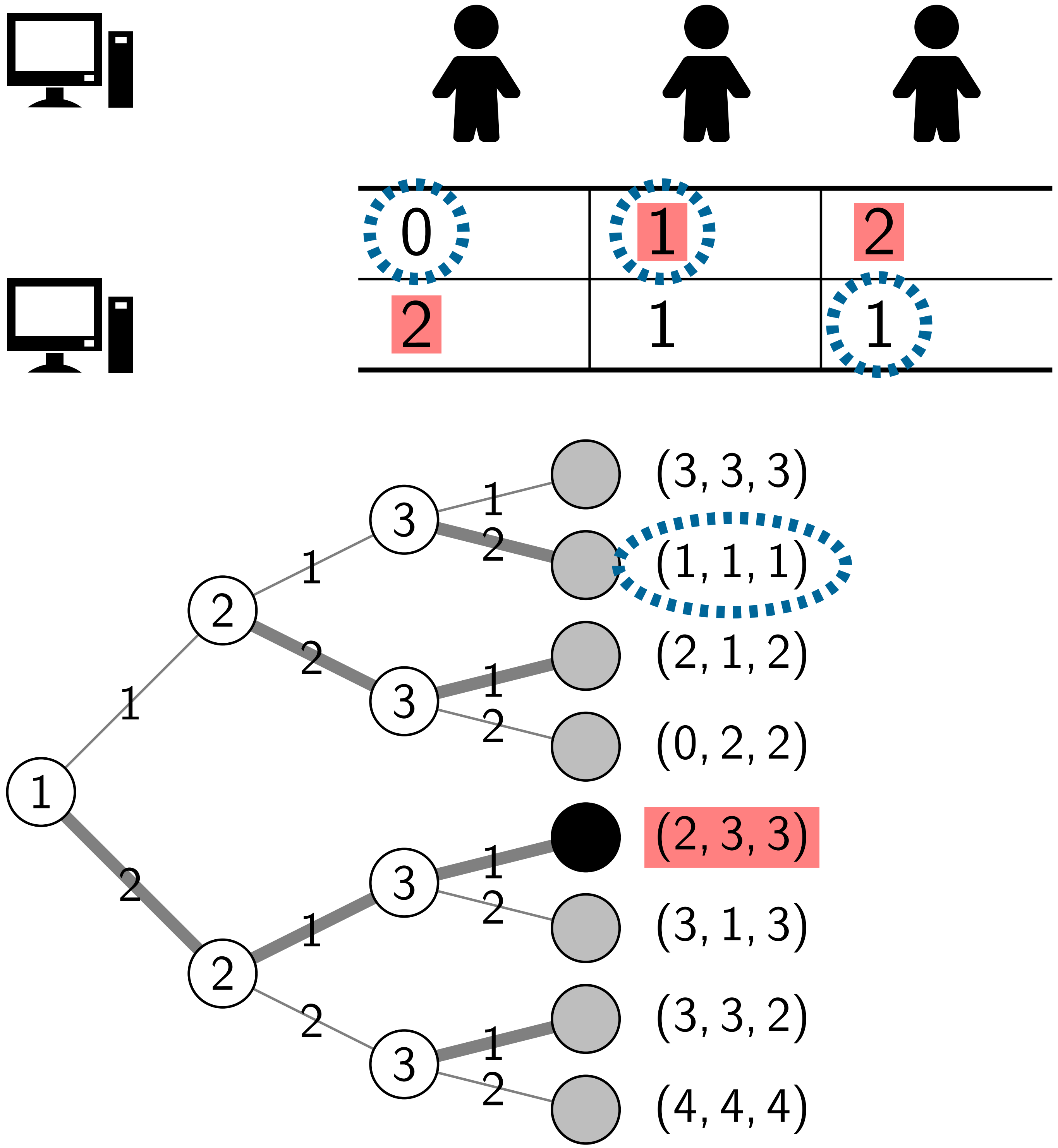
$$\text{PoA} = \frac{\text{worst Nash Equilibrium}}{\text{OPT}}$$



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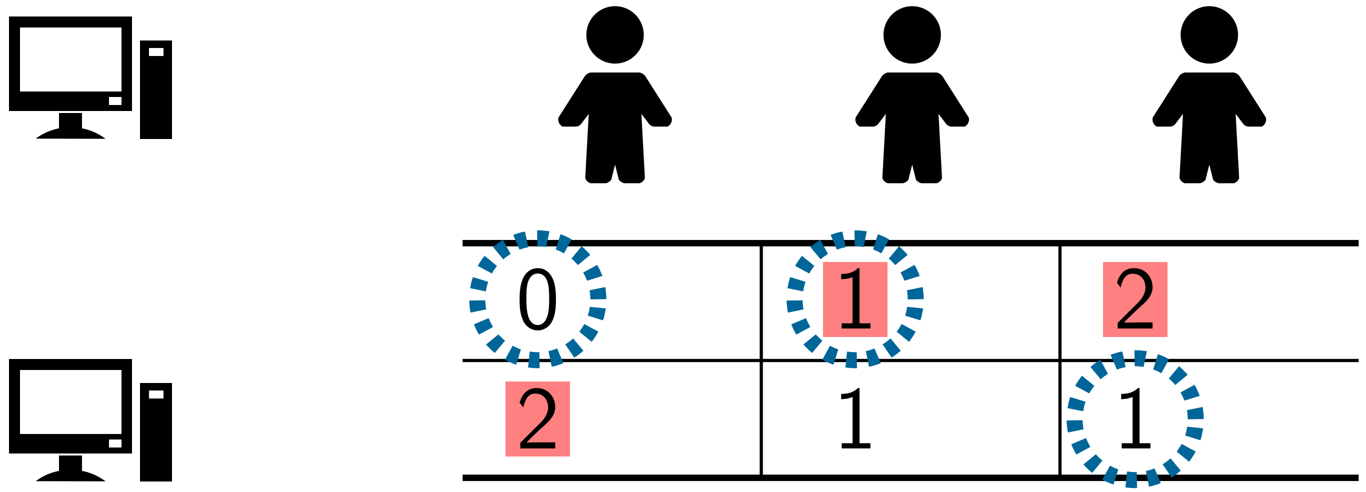

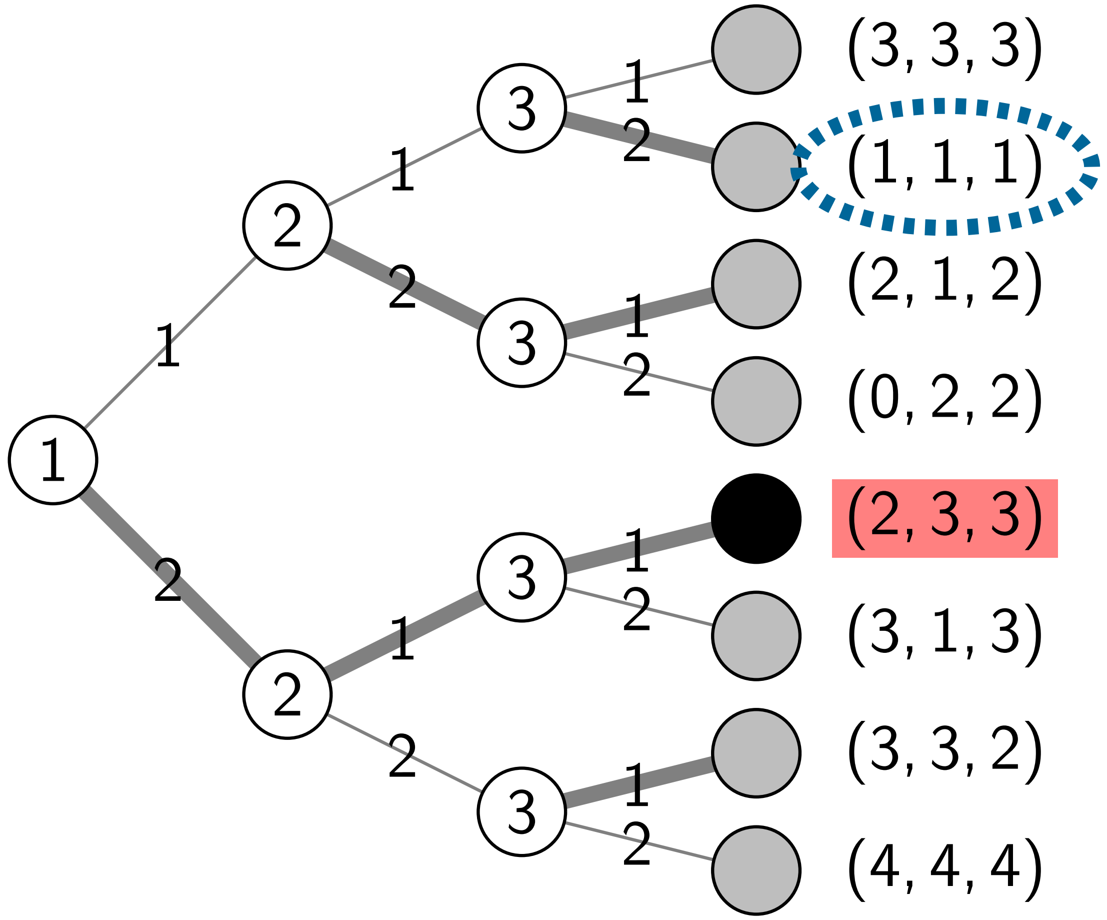

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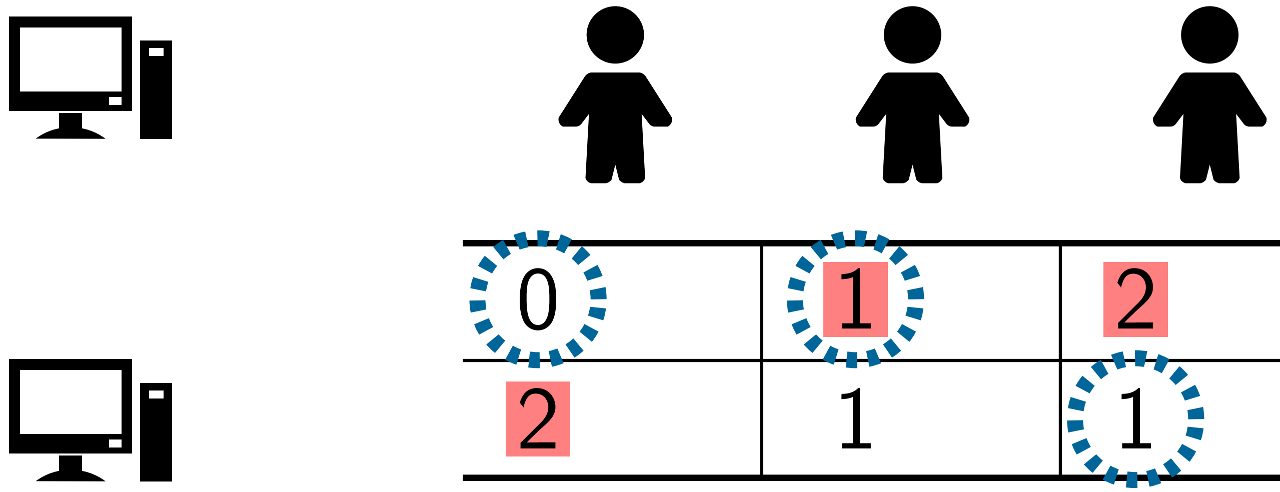
e.g., $\text{SPoA} = \frac{3}{1} = 3$



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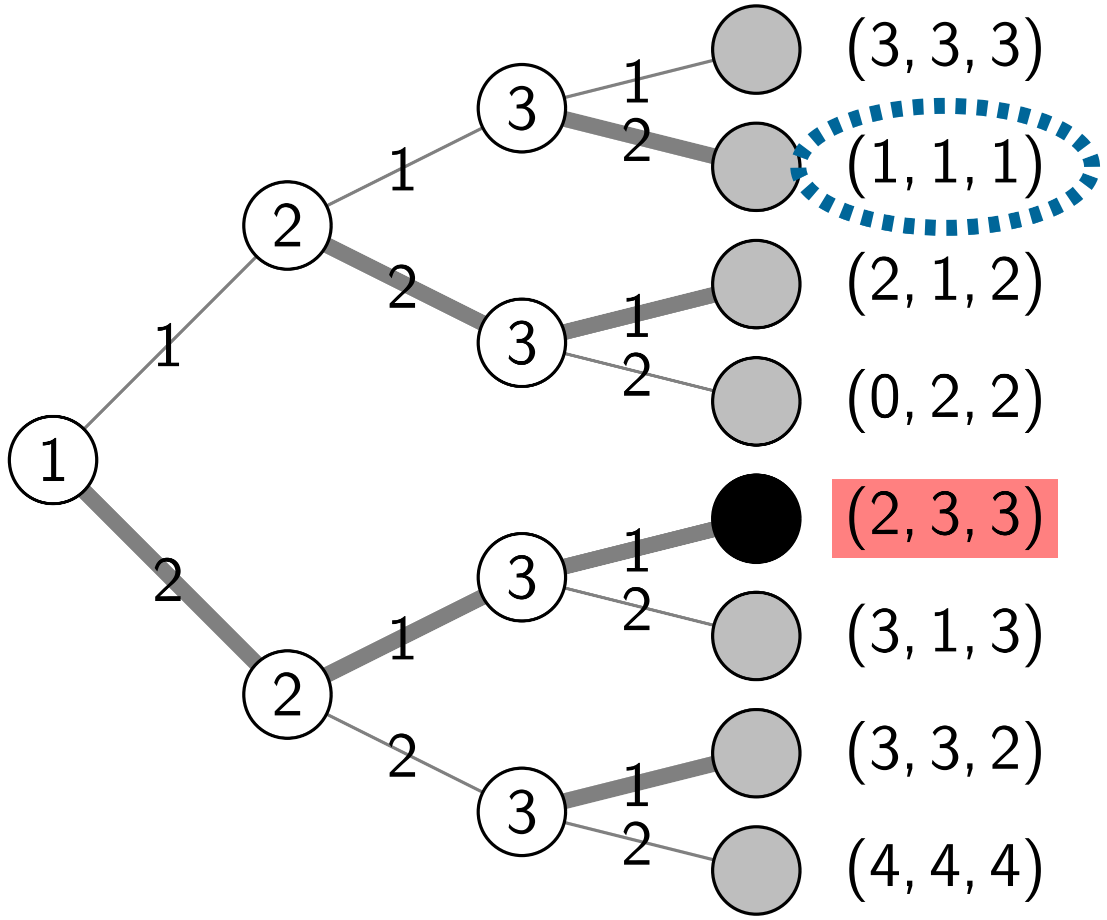

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General version: m machines, n jobs, and arbitrary p_{ij}

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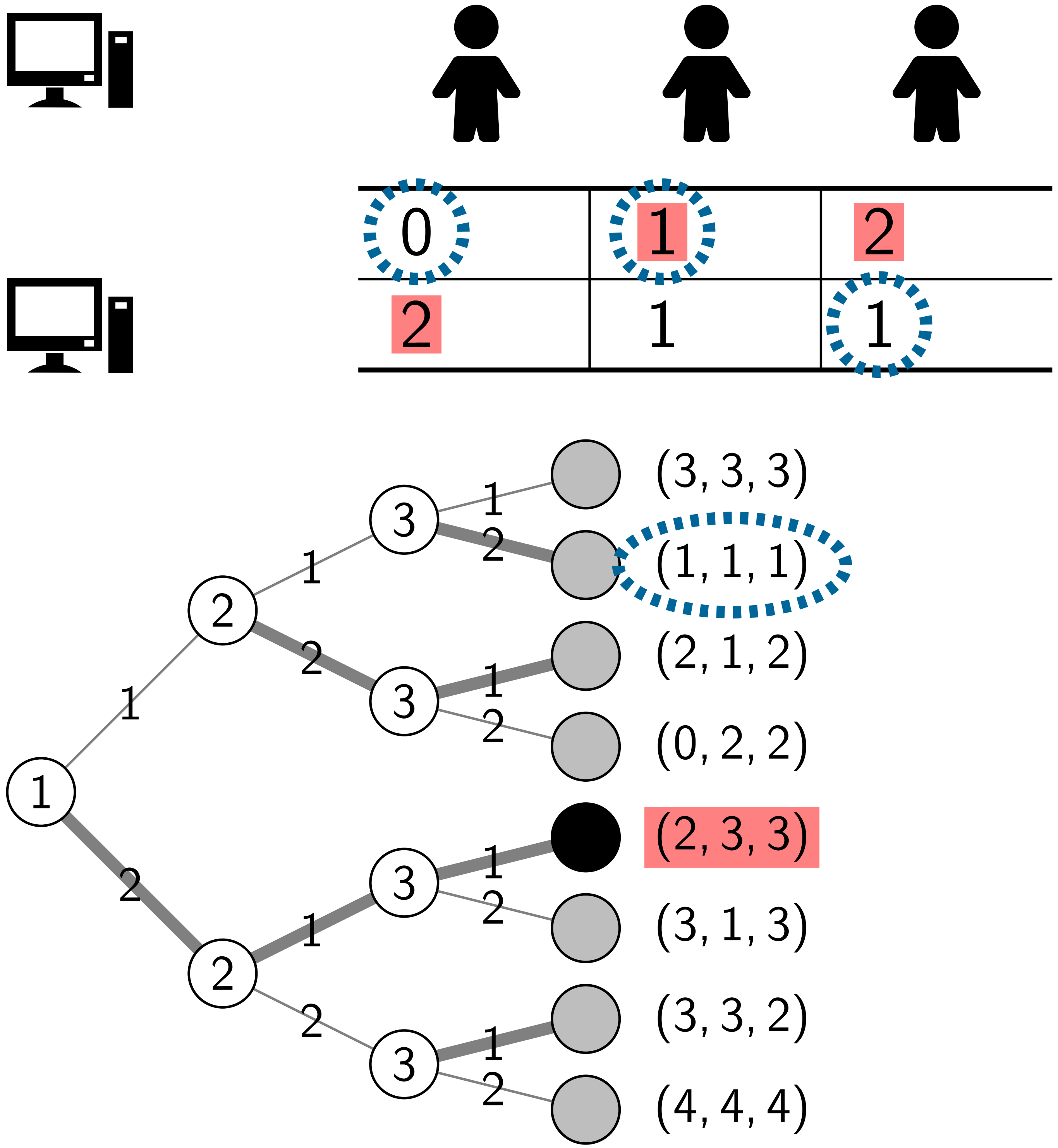

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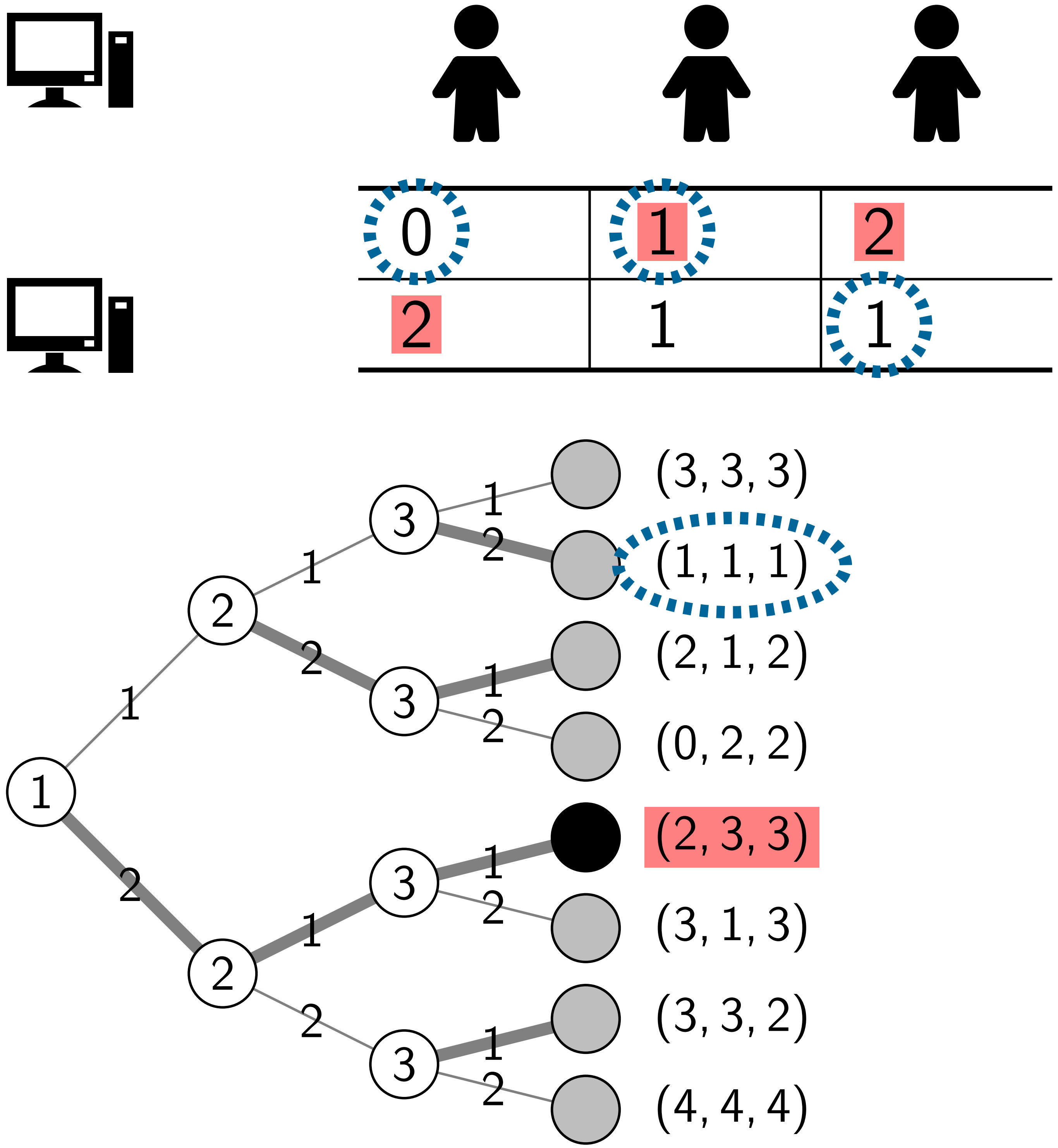

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myopic player

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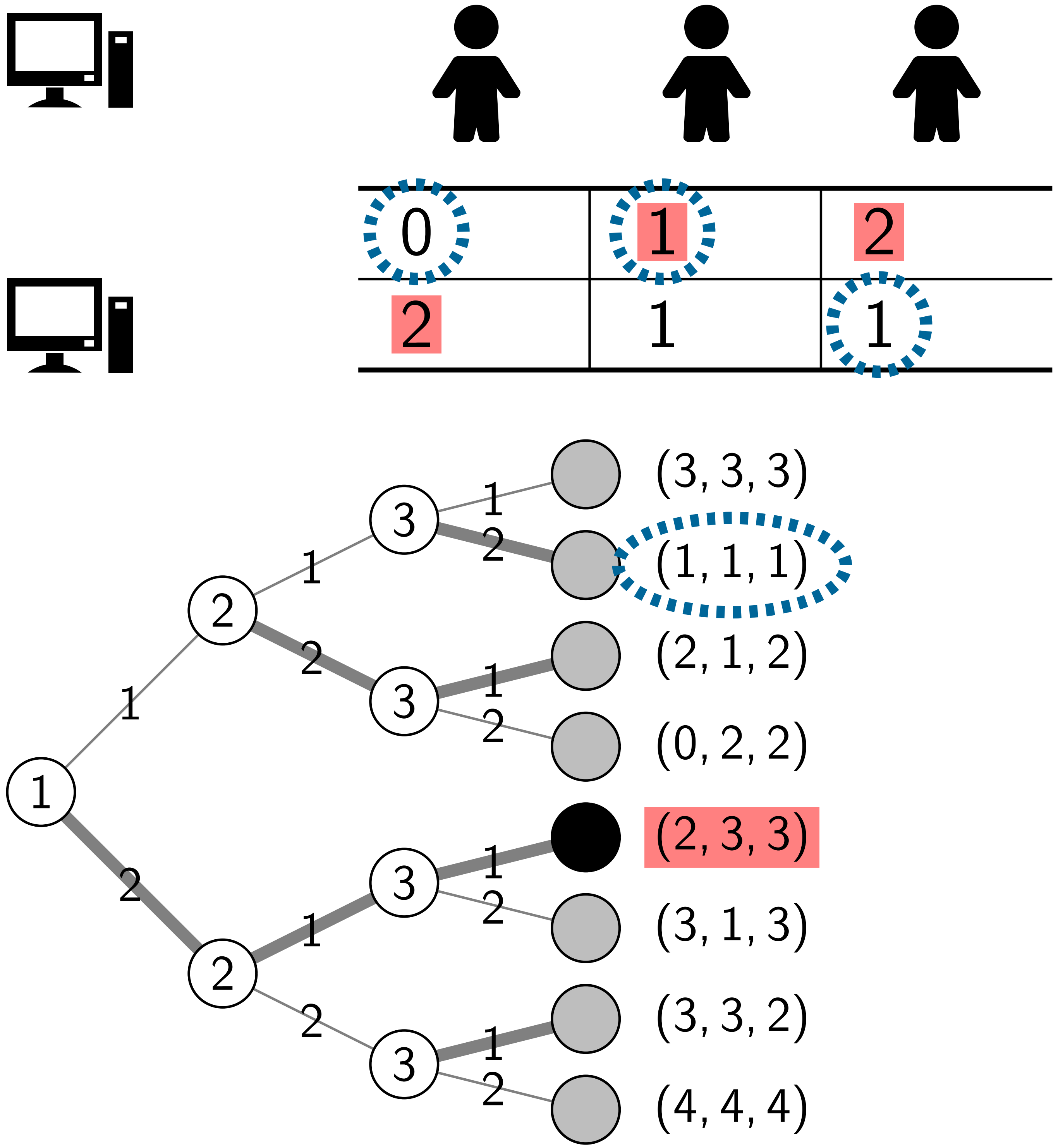

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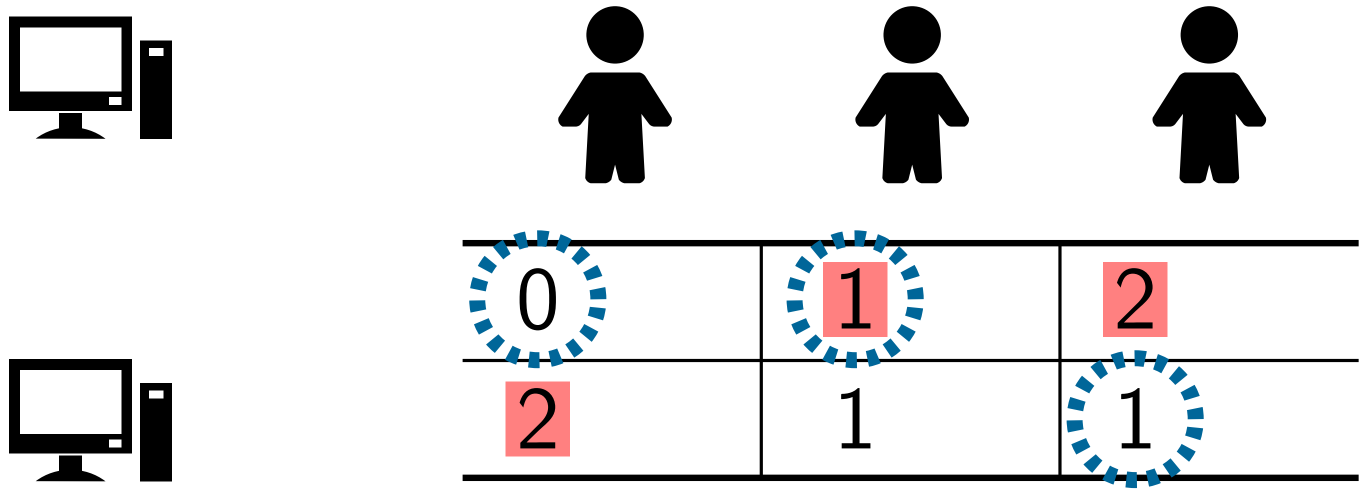

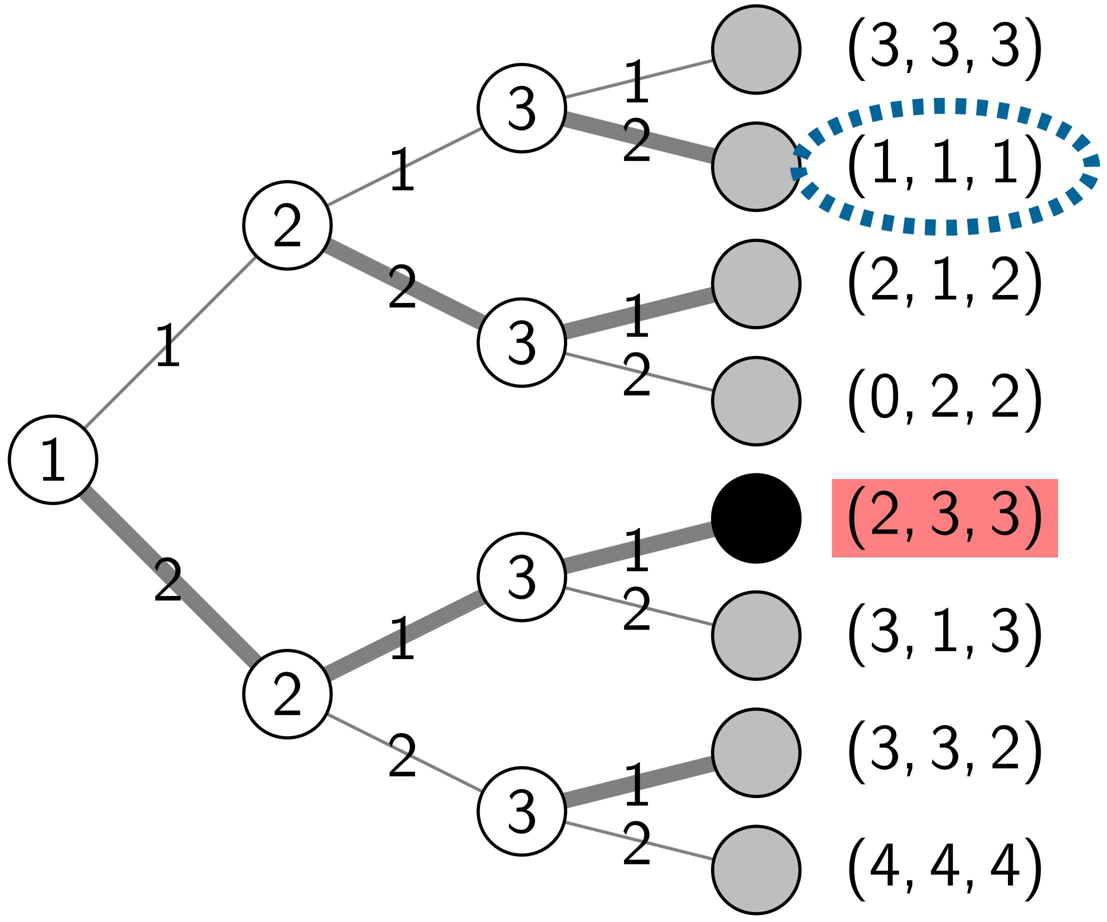

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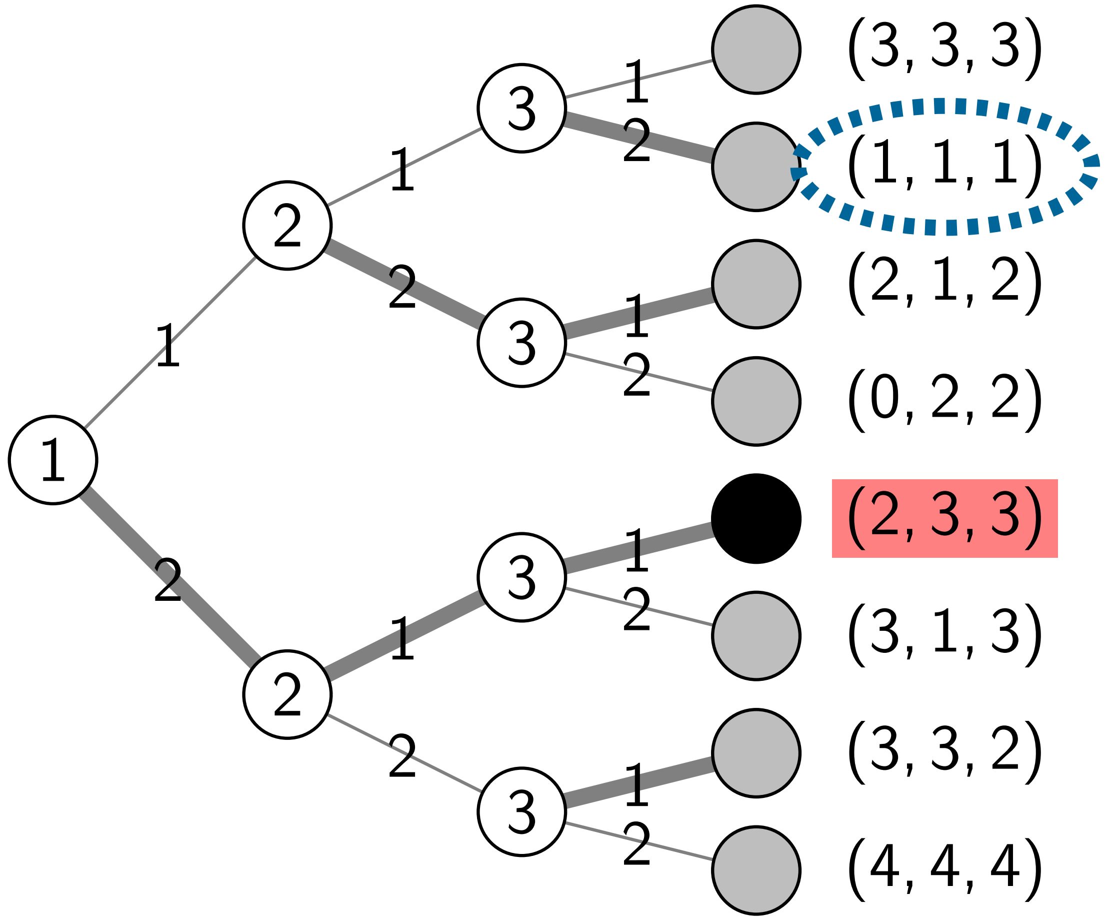
myopic player — full rationality

$\text{SPoA} = 2 \longleftrightarrow \text{SPoA} = \Theta(n)$

curse of rationality

0	1	2
2	1	1



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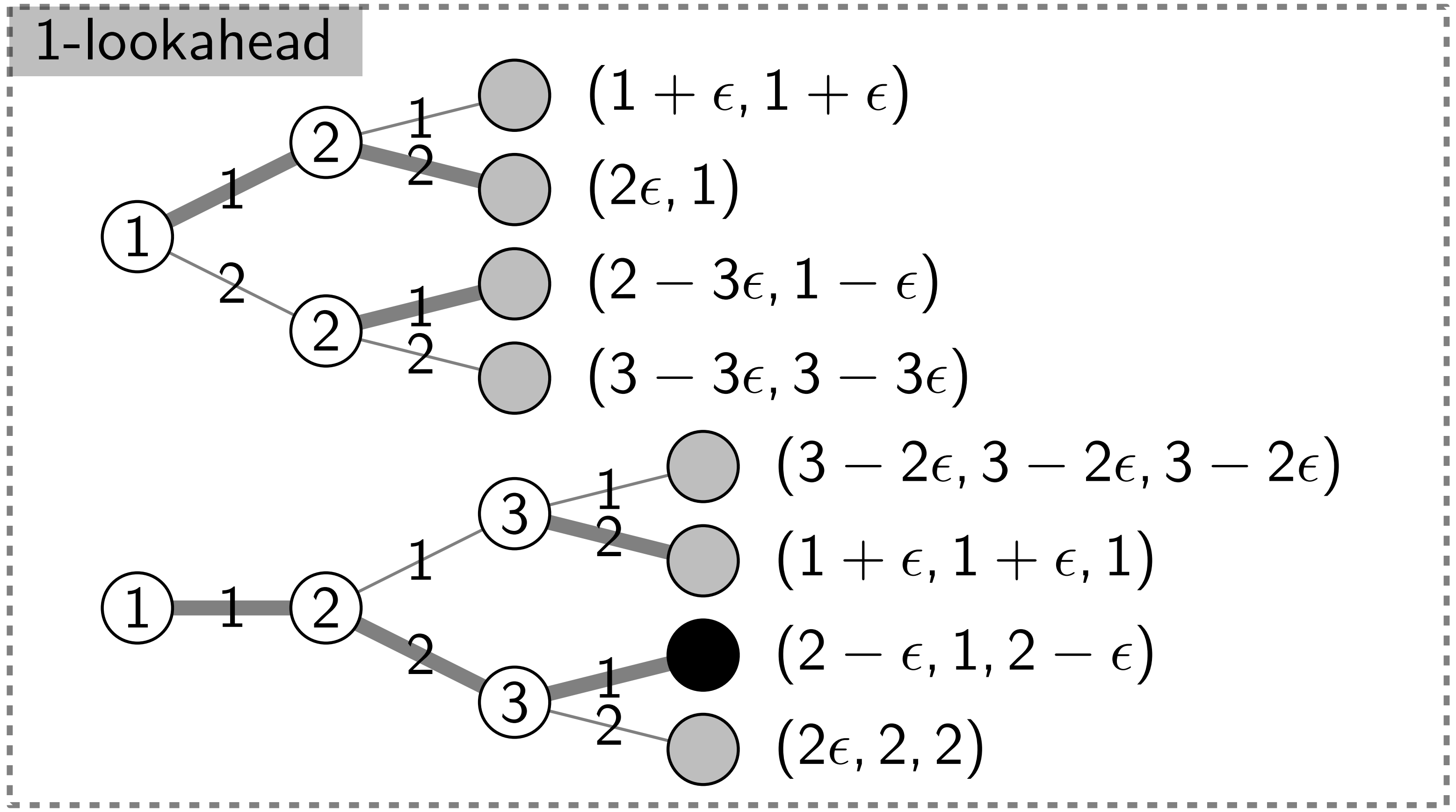

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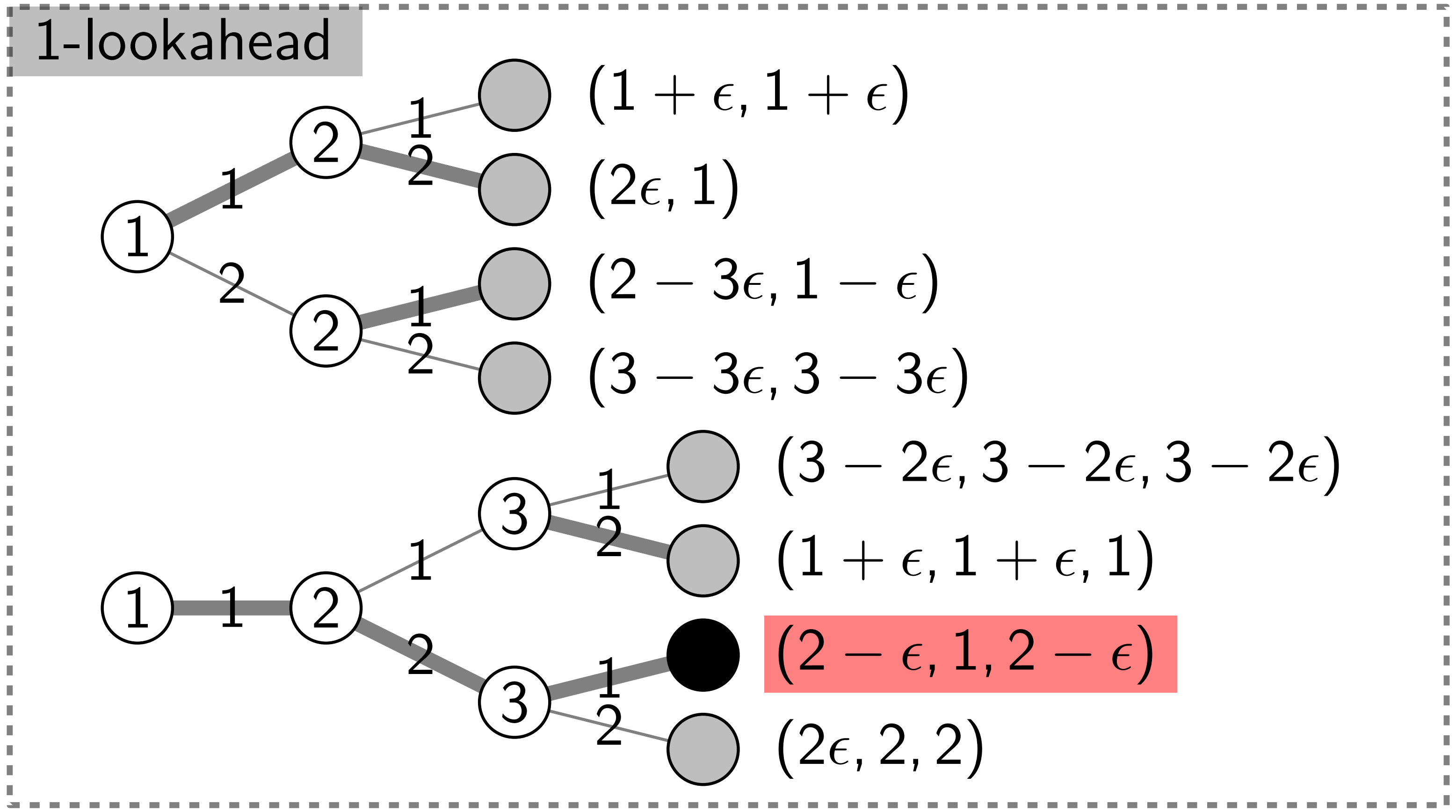

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  Similar settings can also be found in:
- [Mirrokni, Thain, Vetta, 2012]
- [Bilò, Fanelli, Moscardelli, 2017]
- [Groenland, Schäfer, 2018]
- [Kroer, Sandholm, 2020]

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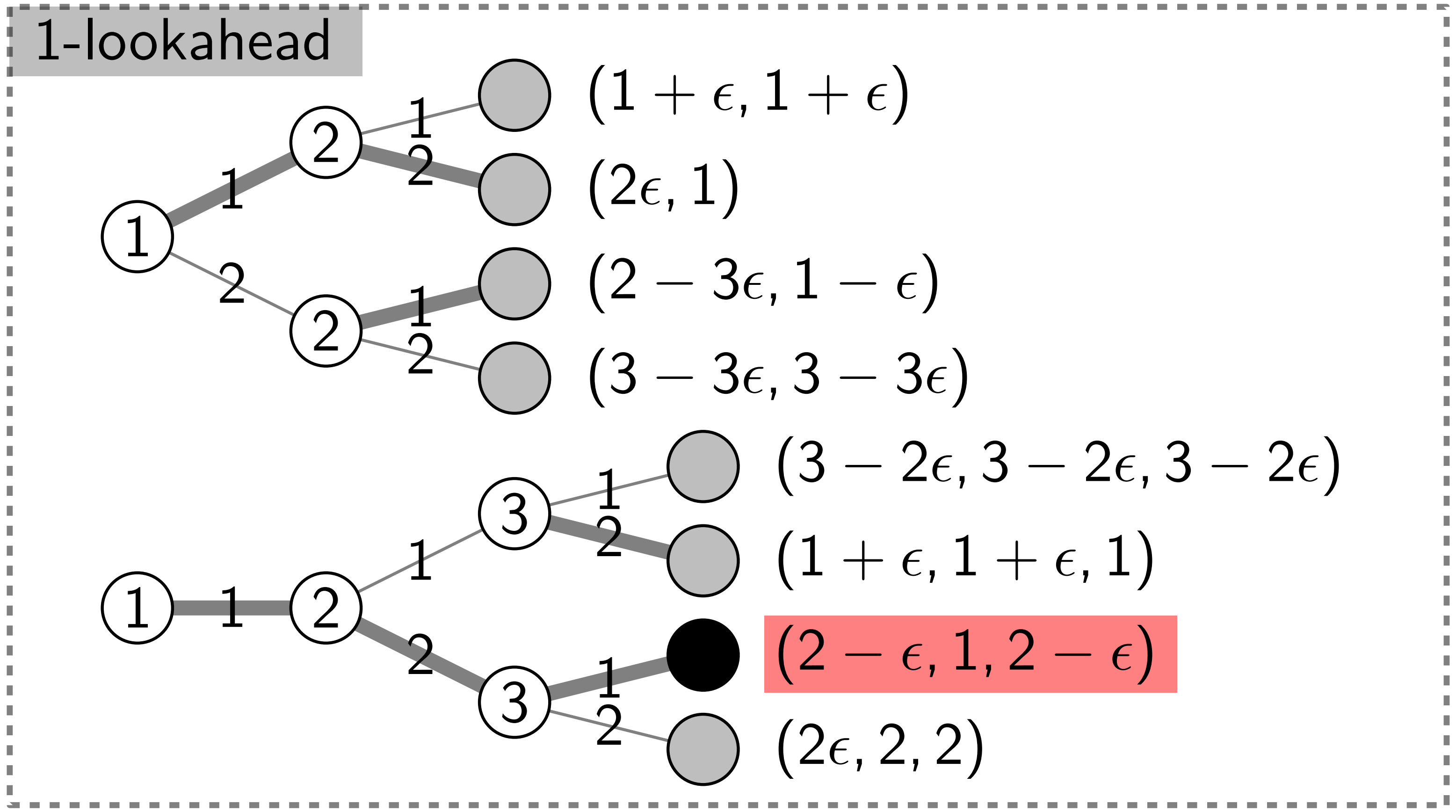

Modeling the bounded rationality

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- A simple-minded player simply assumes the successors will choose machines with minimum processing times, so he/she can easily find a best choice.

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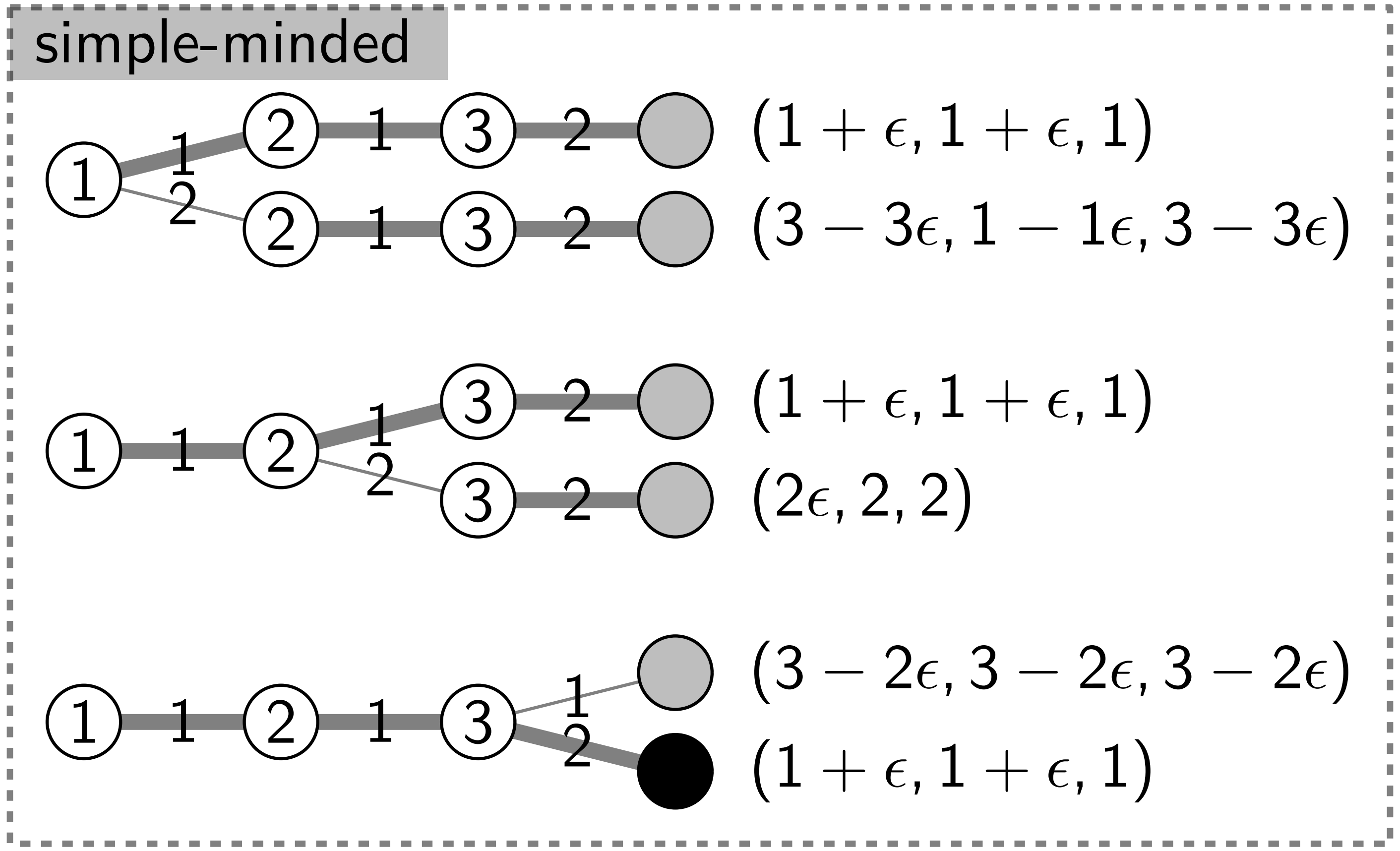

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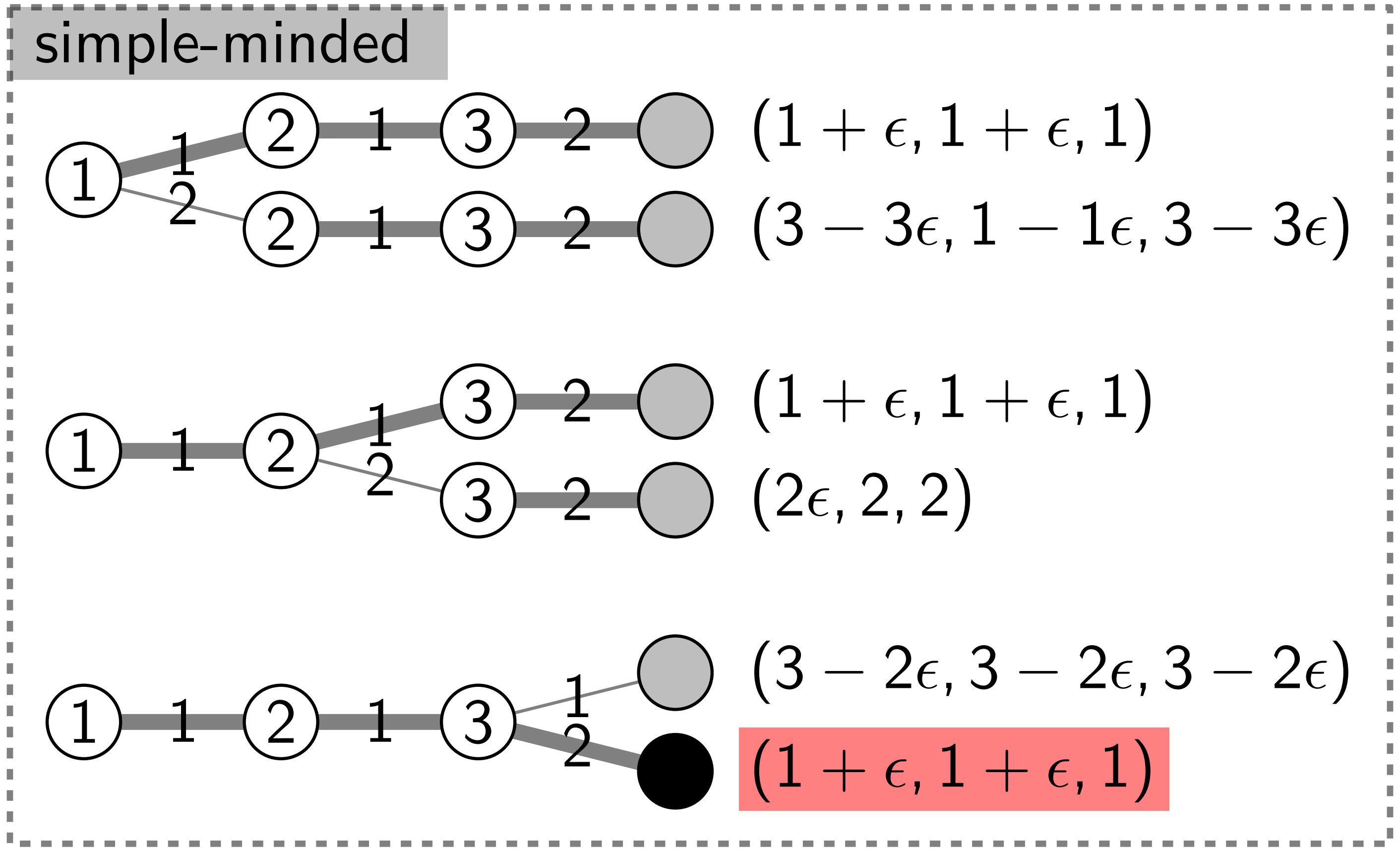

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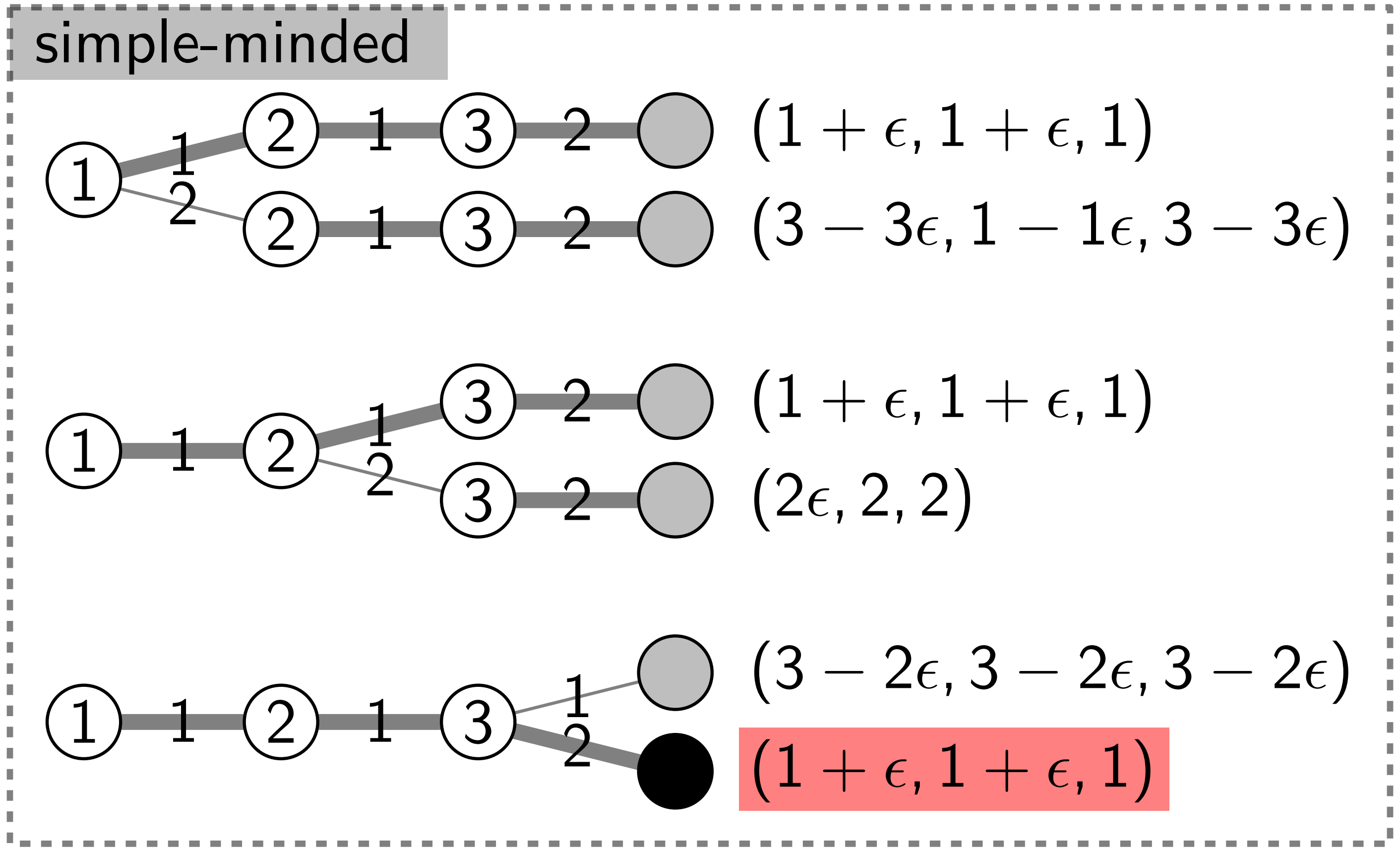

- full rationality: SPoA ≈ 3
- 1-lookahead: SPoA ≈ 2
- simple-minded: SPoA $= 1$

Warm-up: 1-lookahead on 2 unrelated machines

- **Lemma**:

$L_{\max} \leq \sum_{j \in N} p_j$, where $p_j = \min_{i \in M} p_{i,j}$, i.e., the minimum processing time of job j

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For the sequential scheduling game on two unrelated machines where players have 1-lookahead, SPoA $= 2$

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Proof:

(1) SPoA $= \frac{L_{\max}}{OPT} \leq \frac{\sum_{j \in N} p_j}{\sum_{j \in N} p_j / 2} = 2$

(2)

	job 1	job 2
machine 1	$1 + \epsilon$	2
machine 2	1	$1 + \epsilon$

$\Rightarrow$ SPoA ≥ 2

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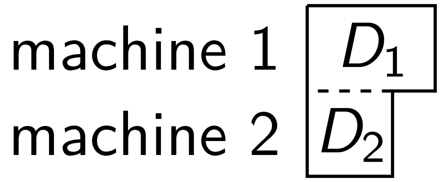

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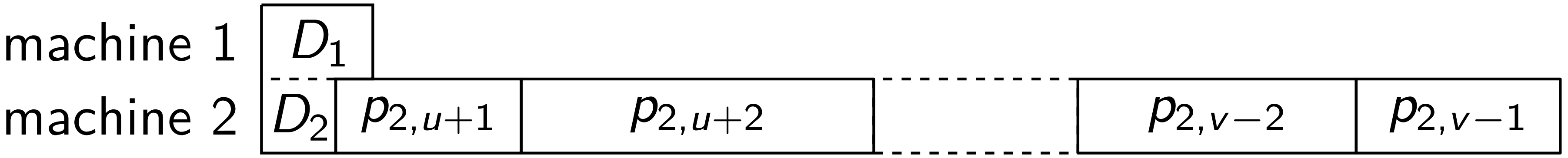

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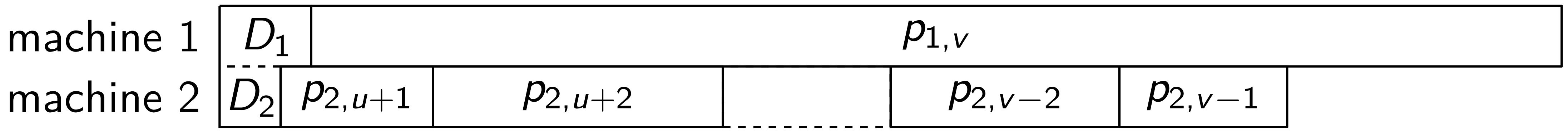

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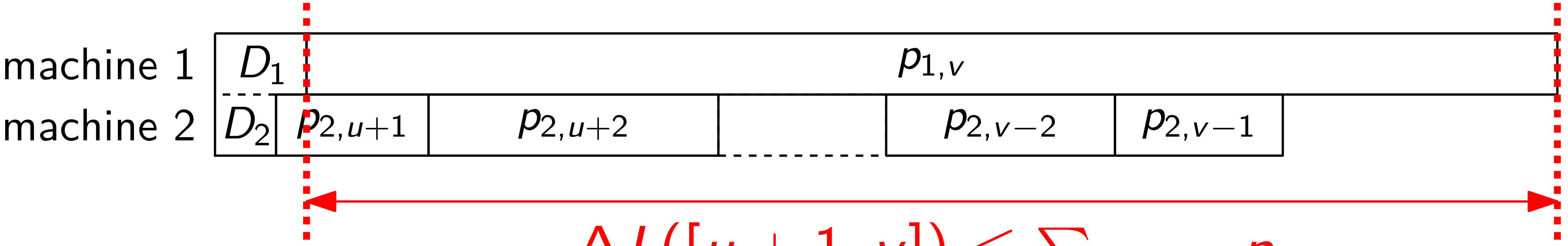

$\Delta L([u+1, v]) \leq \sum_{u<j\leq v} p_j$

k-lookahead on m machines

- **Lemma**:

$\Delta L([\ell : n]) - \Delta L([\ell + 1 : n]) \leq p_\ell + \Delta L(K_\ell)$ for $\ell = 1, 2, \dots, n$

k-lookahead on m machines

- **Lemma**:

$\Delta L([\ell : n]) - \Delta L([\ell + 1 : n]) \leq p_\ell + \Delta L(K_\ell)$ for $\ell = 1, 2, \ldots, n$

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- $\Delta L(K_j)$: k players with full rationality

k-lookahead on m machines

- [Chen, Giessler, Mamageishvili, Mihalák, Penna, 2020]:

  For 2 unrelated machines: $\Delta L(K_j) \leq (k-1) \sum_{j \in K_j} p_j$

- [Leme, Syrgkanis,Tardos, 2012]:

  For m unrelated machines: $\Delta L(K_j) \leq 2^k \sum_{j \in K_j} p_j$

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- **Theorem**:

For the sequential scheduling game where players have k-lookahead, the SPoA is at most $O(k^2)$ for the two unrelated machines case, and at most $O(2^k \cdot \min\{mk, n\})$ for the m unrelated machines case.

Simple-minded Players

- **Theorem**:

  For the sequential scheduling game on m unrelated machines where players are *simple-minded*, SPoA $= m$.

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Proof (of SPoA $\leq$ m):

- Define: $A_i(j) = D_i(j) + P_i([j+1:n])$

  $D_i(j)$: the load of machine i due to the first j jobs

  $P_i([j+1:n])$: the total processing time of the jobs who are assumed by job j to choose machine i (i.e. the jobs have minimum processing time on machine i)

- Claim: $A_{\max}(\ell) \leq A_{\max}(\ell - 1)$

- SPoA $= \frac{L_{\max}}{OPT} = \frac{A_{\max}(n)}{OPT} \leq \frac{A_{\max}(0)}{OPT} \leq \frac{\sum_{j=1}^{n} A_j(0)}{OPT} \leq \frac{\sum_{j=1}^{n} p_j}{\sum_{j=1}^{n} p_j/m} = m$

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For the sequential scheduling game on m unrelated machines where players are *simple-minded*, SPoA $= m$.

Proof (of SPoA $\geq$ m):

	job 1	job 2	job 3	job 4
machine 1	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
machine 2	$4-5\epsilon$	1	∞	∞
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$OPT = 1$

- SPoA $\geq$ m

Summary

A summary of the results

	2 machines	m machines
0-lookaahead (online greedy)	2	m
1-lookaahead		
k-lookaahead		
n-lookaahead (full rationality)	$\Theta(n)$	$O(2^n)$
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Concluding remark

- reconsidering the "perfect rationality" assumption for the players

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Concluding remark

- reconsidering the "perfect rationality" assumption for the players
- Future work:

  - to improve the bounds for the SPoA of k-lookahead model

  - to further understand the role that bounded rationality plays in other games

THANK YOU!