Risk Parity

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I. Introduction

Asset allocation strategies are crucial in investment management since they help investors reduce risk through diversifications and keep their long-term perspectives; there are multiple approaches to implement asset allocations. For many years, portfolios composed of 60% equities and 40% bonds have been regarded in the dominant position, since this kind of portfolios is dominated by equity risk, and stock volatility is much higher than bond volatility. Sometimes, this allocation may result in roughly 90% of the portfolio risk dedicated to equities, which is much higher than its capital allocation weight, like around 60%. This kind of allocation may lead to high tail risk and low risk-adjusted returns. For example, investors will suffer a huge impact under the dynamics of stock market because of equity's high proportion in the asset allocation. The influence of bonds, however, may be neglectable to some extents, no matter the bond market in what kind of position, because of its low weight in allocation, which makes this portfolio under-diversified.

Risk parity is a sort of portfolio strategies, that relies on risk-based diversification under multiple asset classes, to generate high and consistent returns. Comparing to traditional portfolios, a typical risk parity portfolio has a much lower exposure to equities, but more investments in other asset classes. Therefore, the risk budget is spread more evenly across different kinds of asset classes rather than mainly focusing on equities, and no asset class can be the dominant over the portfolio volatility, which means this kind of portfolio is capable to dealing with different economic circumstances under a good performance.

Risk parity strategy has received more and more attention over the past few years, since it has a more stable long-term performance relative to traditional portfolios with their typical large portion of allocations in equities. In the aftermath of the financial crisis in 2008, many investors are concerned about the tail risk in their portfolio. Risk parity strategies passed an acid test in 2008 by having a better performance comparing to traditional portfolios.

According to the theory of risk-based diversification, risk parity portfolio should have low volatilities and high Sharpe ratio, but does it really outperform any other allocation methods? In this paper, we will compare risk parity portfolio with 40/60 traditional portfolio, mean-variance portfolio and implied volatility portfolio.

i. We know risk parity approach attempts to minimize investment risks of portfolio diversification, so we will compare the historical performance of this strategy with other 3 to see whether it will yield a higher return.
ii. Theoretically speaking, a risk parity portfolio would have a relatively consistent performance, and could mitigate drawdown because of its better portfolio diversification strategy. Therefore, we will test its maximum drawdown comparing to other portfolios.
iii. Bonds and equities are two main composites of a portfolio, and there exists some assumptions that the bond-equity correlation has large impact on risk parity portfolios, and investors who has capability in spreading risk across multiple asset classes with negative correlations or low correlations could get a big reward, and here we want to test the effect of risk parity strategy in balancing risk budgets across multiple asset classes.

iv. Sharpe ratio is used to measure the adjusted return, we will examine the performance of risk parity portfolio under different market volatility situations.
v. We test the capital allocation of a single asset class (equities) against market volatility to see how efficiently the risk parity portfolio could react to adjust the weights.
vi. We using different financial indicators such as Sharpe ratio, maximum drawdown and VaR to test whether risk parity outperforms other investment strategies.
vii. [Various performance under different scenarios / economic environments]

II. Procedures

Our project mainly focuses on risk parity approach, and examine its performance comparing to S&P 500, mean-variance portfolio, inversed volatility portfolio, and 60/40 traditional portfolio. To analyze the benefits or limitations of risk parity, we use the same asset classes to test the performances of these portfolios under various economic scenarios.

2.1 Data Selection

The data we use for in our project are collected from Yahoo Finance for each asset classes. Because of the availability of data, the test covers the period between 08 December, 2011 and 31 December 2018 using daily returns of different ETFs. We think this period covers various market conditions, such as high volatility in 2012, and recession in 2014, so this period is appropriate to some extent.

The basic intuition is that traditional 60/40 portfolios are not actually diversified, but risk-based allocation theoretically delivers relatively more efficient diversification because it seeks to equalize the risk budget across different asset classes and it avoids managers to distribute a large amount of risks in a specific asset class. Therefore, we choose to include different asset classes which we represent through index: DGSIX – Global Allocation 60/40 Portfolio Institution Class.

Name	Symbol	Asset Class
DFA US Core Equity 2 I	DFQTX	Equity
DFA Selectively Hedged Global F/I I	DFIEX	Equity
DFA Selectively Hedged Global F/I I	DFEOX	Equity
DFA US Core Equity 1 I	DFCEX	Equity
DFA Short-Term Extended Quality I	DFSHX	Bond
DFA Five-Year Global Fixed-Income I	DFEQX	Bond
DFA Intermediate-Term Extnd Qlty I	DFGBX	Bond
DFA Emerging Markets Core Equity I	DFTEX	Bond
DFA Inflation-Protected Securities I	DIPSX	Bond
DFA World ex US Government Fxd Inc I	DWFIX	Bond
S&P 500 Index	SPY	Equity

The indices we choose are top 10 holdings in DGSIX (98.13% of Total Assets.), and S&P 500 is our benchmark.

Besides, in order to improve the accuracy of model testing, we separate the available data into two portions – training and testing data. We use 3-month rolling data for training, and use 1-month rolling data for testing, where the training data is used to estimate or prepare the model, which is back tested in the next period – 1-month.

2.2 Leverage

Leverage plays an important role in risk parity, since it could help a risk parity investor to build a relatively more diversified and higher-return-for the-risk-taken portfolio – risk parity portfolio has low volatility and any portfolio with low volatility but well-diversified could use leverage to yield higher expected returns.

In order to make the comparison in performances of different strategies more convincible or increase the comparability, we leverage not only the risk parity portfolios but also the rest to have the same overall level of risk as S&P 500. Then, we compare their net returns to identify their performances.

Therefore, the amounts of leverage we obtained are based on the ratios of strategic portfolios' volatilities and the volatility of S&P 500 under the whole time period from 08 December 2011 to 31 December 2018.

2.3 Methodology

The main concept of the risk parity is to balance the risk contribution of each asset component, where the risk contribution can be interpreted in many ways. Here we implement the risk parity strategy using the CR (contribution to risk) notion of Maillard, Roncalli and Teïletche (2010). In this section, we first present a few unambiguous definitions of risk concepts:

Using ω and Σ to denote, respectively, the $NX1$ weight vector and the NXN covariance matrix, we can write down the analytical formula for the portfolio standard deviation (risk):

$$\sigma(\omega) = \sqrt{\omega^T \Sigma \omega}$$

Notice that this term can be written as the dot product of the weight vector and the partial derivatives vector of $\sigma(\omega)$ with respect to ω, and the latter term is defined as the total risk contribution (TRC) of i^{th} individual asset, of which the mathematic definition is given by:

$$TRC_i = \partial_{\omega_i}\sigma(\omega) = \frac{\omega_i\,(\Sigma\omega)_i}{\sqrt{\omega^T \Sigma \omega}}$$

As is shown to us, it allows us to break down the total portfolio risk into N separate parts. That's exactly what risk parity does—assign the same risk budget to each individual component. Therefore, we can define the contribution to risk of the i^{th} asset as follows:

$$CR_i = \omega_i \, TRC_i = \omega_i \frac{\omega_i \, (\Sigma\omega)_i}{\sqrt{\omega^T \Sigma \omega}}$$

By setting all risk contributions equal, we get the analytical solution for ω_i:

$$\omega_i = \frac{\sigma^2(\omega)}{(\Sigma\omega)_i N} \;\; \forall i$$

Therefore, we can get our weight vector by solving the following optimization problem. Notice that the risk parity is a fully investment strategy, so we set the constraint that the weights sum to 1. In addition, we add the short-sale restriction to guarantee all asset components will take long positions only:

$$\underset{\omega}{\operatorname{argmin}} \left[\omega_i - \frac{\sigma(\omega)^2}{(\Sigma\omega)_i \, N} \right]^2 \; s.t.\, \omega^T i = 1, \omega_i \geq 0 \; \forall \; i$$

This strategy is what we mainly focus on, and is called equal-weighted risk contribution strategy. For comparison, we briefly introduce two strategies here: inverse volatility weighting and mean-variance asset allocation. The first one is actually a naïve risk parity approach with regard only to the individual volatility. The weight of the i^{th} asset is calculated as:

$$\omega_i = \frac{\frac{1}{\sigma_i}}{\sum_{j=1}^{N} \frac{1}{\sigma_j}}$$

It’s intuitive but doesn't take correlation between assets into consideration. As the volatility of a portfolio is composed of idiosyncratic risk and systemic one, it is natural to use covariance rather than variance to allocate the weights. Therefore, we expect the true risk parity method to have a better performance than inverse volatility weighting.

For the mean-variance portfolio construction, we choose the maximum portfolio Sharpe Ratio as our optimization target, which can be written as (short-sale is allowed):

$$\max_{\omega} \frac{\omega(\mu - R_f i)}{\sqrt{\omega^T \Sigma \, \omega}} \;\; s.t.\, \omega^T i = 1$$

Where μ and $\boldsymbol{i}$ denote, respectively, the $NX1$ return vector and $NX1$ all-ones vector. By solving the above optimization problem, we can get the analytical solution for this maximum Sharpe Ratio portfolio, as known as tangency portfolio:

$$\omega_T = \frac{V^{-1}(\mu - R_f i)}{i^T V^{-1}(\mu - R_f i)}$$

Ignoring the estimation error first, this method does give us the highest Sharpe Ratio, however, it says nothing about the risk. In addition, there are lots of noise in historical return, which gives rise to a difficulty in predicting returns accurately. Its performance in real world is hard to tell.

2.4 Performance Analysis

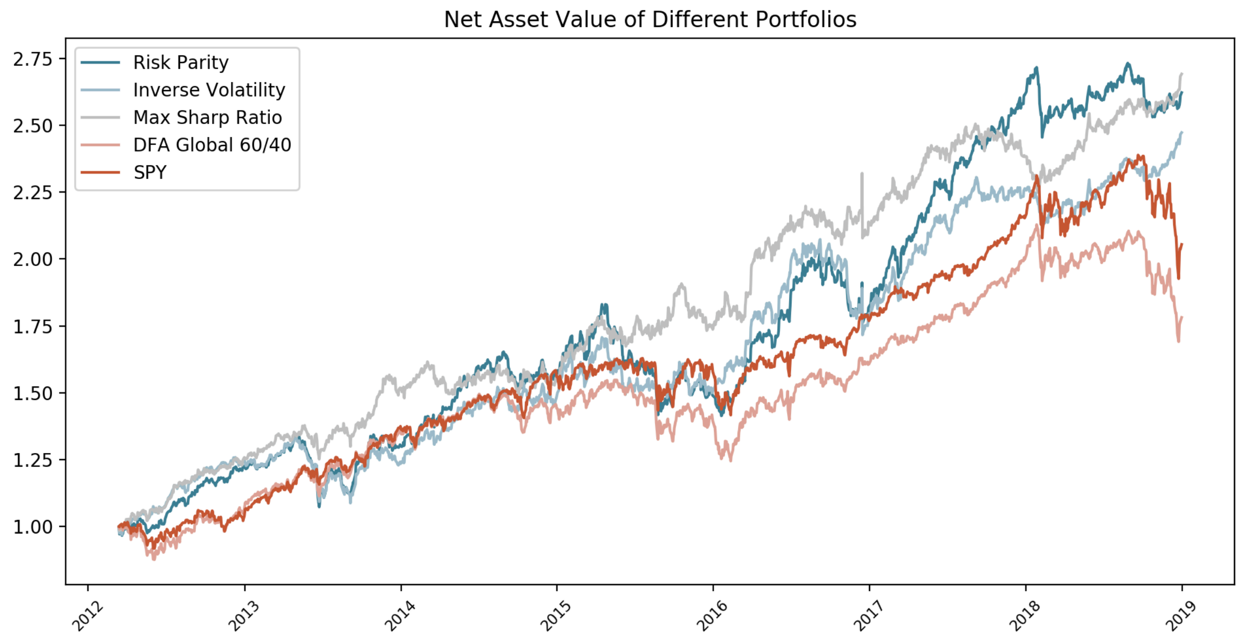
According to the theory of risk-based diversification, risk parity portfolios should have lower volatilities and higher Sharpe ratios. However, does risk parity really outperform a risk inverse volatility weighting, a passive investment or any other allocation methods? To test its efficiency, we backtest this strategy and compare it with the market and those mentioned strategies under rolling-window.

Here we set the 3-month rolling data as our training set, and 1-month rolling data as the test set. The backtesting statistics are: Sharpe Ratio, MDD (Maximum Drawdown) and VaR (Value at Risk). As is mentioned before, risk parity portfolio is constructed to have low volatility. For comparison convenience, we leverage these three optimal portfolios (risk parity, inverse volatility weighting, mean-variance) to match the market (S&P) volatility. By doing so, we are able to figure out whether it delivers a higher return given the same risk level.

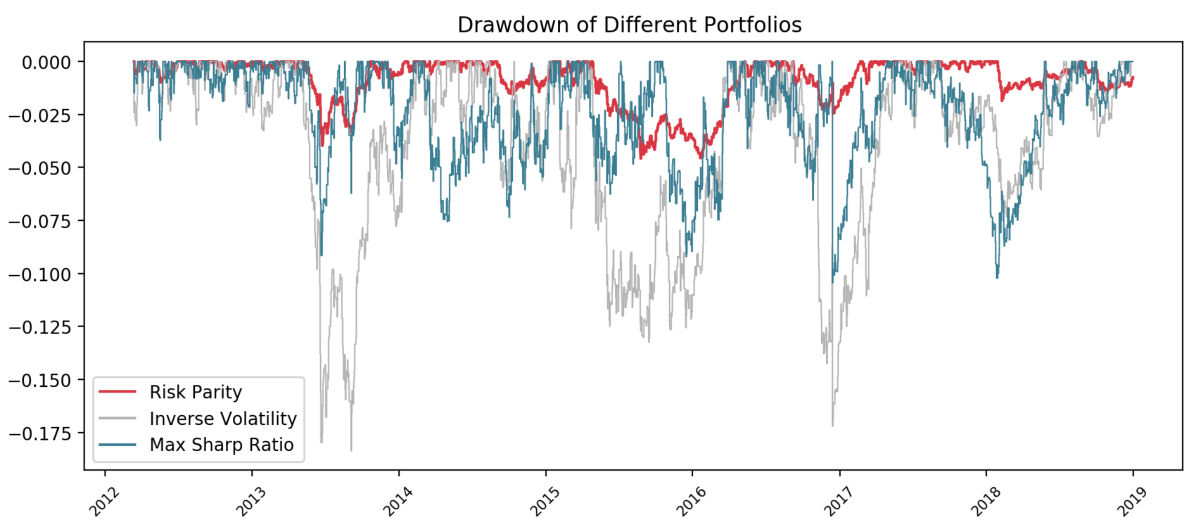
Besides, we can test the robustness of this strategy. Compared to the market, risk parity leverages up low risk assets and deleverage high risk assets, which reduces its sensitivity to market turbulence. After adding a leverage ratio, we can better analyze the influence of the shocks and to see how it acts in different market circumstances. Notice that all statistics given below are based on the levered strategies.

To better analyze the benefits of risk parity, we set four benchmarks for it, including the market index (S&P 500), a traditional 60% equity / 40% fixed income portfolio (a typical asset allocation), tangency portfolio and inverse volatility weighting portfolio. Notice that the last two active investment portfolios are constructed with the same asset classes as risk parity. Then we get their net value and maximum drawdown plot of 2011-2019.

As is shown in the net value plot, risk parity is able to produce more superior risk-adjusted returns than other strategies, especially when there is a large market turbulence, i.e. the end of 2016 and the end of 2018. Risk parity outperformances the others given the same risk level. It can be seen that the downside protection of risk parity helps with the long-term performance.

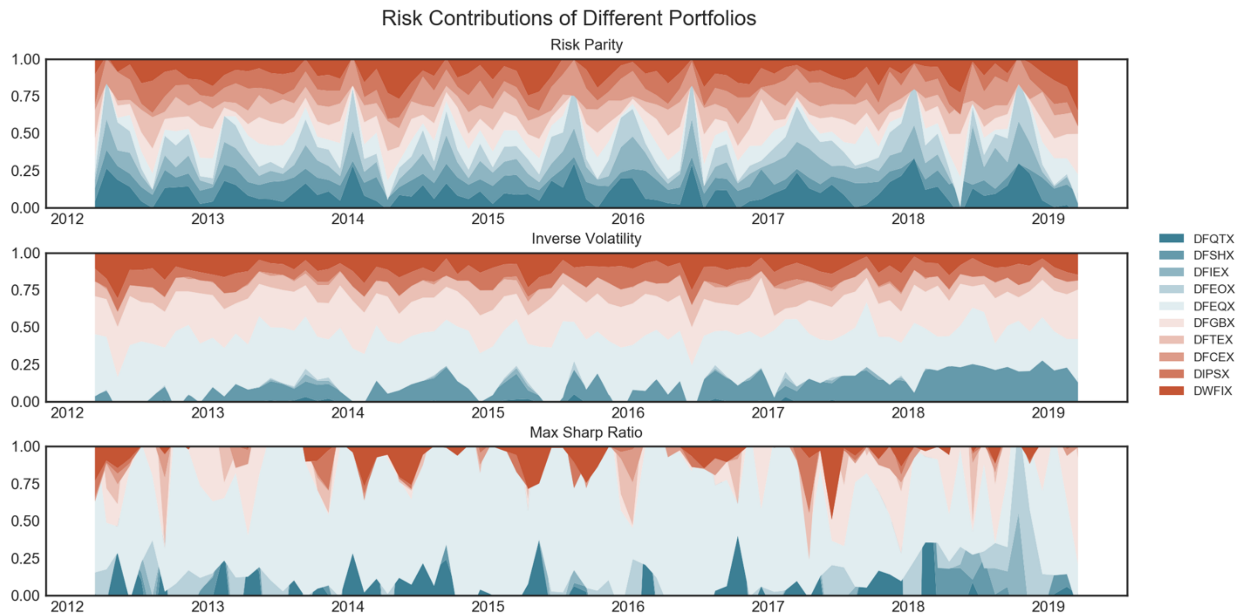


Then we take a look at its risk exposure. Given the test dataset, risk parity presents a consistent performance, and is more resilient when compared to other strategies. It may be explained by its nature of risk-diversification. When allocating capital, it's actually allocating the risk. It's worth noting that inverse volatility weighting performs badly and has the highest maximum drawdown in these portfolios, which proves it to be flawed to overlook the correlation between asset classes.



To better analyze the dynamics driving the consistent performance of risk parity (as shown below), we plot the risk contribution of it and other two approaches (inverse volatility weighting & max Sharpe Ratio optimization) in the testing period. Compared with other two risk-based allocation methods, max Sharpe Ratio strategy assigns heavy weight to a single asset class, showing no risk-diversification at all.

In addition, risk parity reduces the problem of over-concentration of assets to the extent. By adding the concerns of correlation between assets classes into framework, risk parity shows a perfect risk diversification over time. It presents a more robust out-of-sample performance when compared to the naïve risk allocation strategy.



Recall that risk parity is designed to help with risk diversification, we have reason to believe that it has a better performance when the market is more volatile as it isolates the effects of economic environment shifts. Based on this intuition, we analyse the performance of risk parity under different scenarios, and find that the framework of risk parity benefits the most from risk-diversification in periods of recession. Here we extract the period of 2012-2014 for illustration, a typical period including years of high-volatility (2012), low-volatility (2013) and recession (2014). The annual returns of risk parity and four comparable portfolios are given below.

Financial Indicators of different portfolios (2012-High Volatility)					
	SPY	Risk Parity	Inverse Volatility	Mean Variance	60/40
Sharpe Ratio	2.59	0.52	0.00	1.29	2.05
Max Drawdown (%)	5.60	4.00	15.27	11.20	5.70
VaR (%)	-1.00	-0.28	-0.20	-0.10	-0.70

Financial Indicators of different portfolios (2014-Recession)					
	SPY	Risk Parity	Inverse Volatility	Mean Variance	60/40
Sharpe Ratio	1.18	1.84	1.62	0.31	0.59
Max Drawdown (%)	7.30	1.50	5.40	6.90	5.90
VaR (%)	-1.10	-0.10	-0.20	-0.10	-0.70

Our results show that risk parity approach achieves the highest Sharpe Ratio in both 2012 and 2014.

Closely look at the how strategies behave during the years 2012 and 2014 in those two tables. It was clear that, when the volatility reached the highest in 2012: Risk parity and Inverse Volatility performed better, since they both concerned about risk diversification. Especially during the recession year 2014, we could obviously notice that risk parity strategies achieves the highest Sharpe ratio with the lowest VaR among all these portfolios.

Even the overall performance of risk parity is favorable, it is not safe to blindly assume that the risk parity approach outperforms the other allocation algorithms one all recession periods. In a recession time, risk parity tends to underperform over well-structured institutional portfolios, which may incur great loss when the long-term interest rate rises. This potential risk term is even significant when considering the tail-risk resulted from leverage.

III. Conclusion

Since the economics depression, the risk-based allocation has become the new incarnation in asset allocation process, and the stable framework of risk parity helps it stand out. Comparing with traditional dollar allocation portfolios, risk parity method is able to present a consistent performance even in high-volatility and recession periods, as it reduces equity concentration and tail risk. In addition, risk parity also shows competitive advantages when comparing with other risk-based strategies (60/40 and inverse volatility). It achieves a balanced framework by assigning weights to asset classes in terms of their risk contribution, which helps with its robustness in different economic environment.

There are also some considerable problems with risk parity in reality. The necessity of leverage in order to have risk parity across different asset classes may exclude some investors, particularly retail investors using discount brokers who do not offer margin accounts, which will reduce the practicality of this method. However, the leverage can be adjusted so that investors can meet their target expected returns. Also, the rolling correlation of bonds and equities may change over time, which might be a potential risk when we build our risk parity strategies.

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