Neural Networks

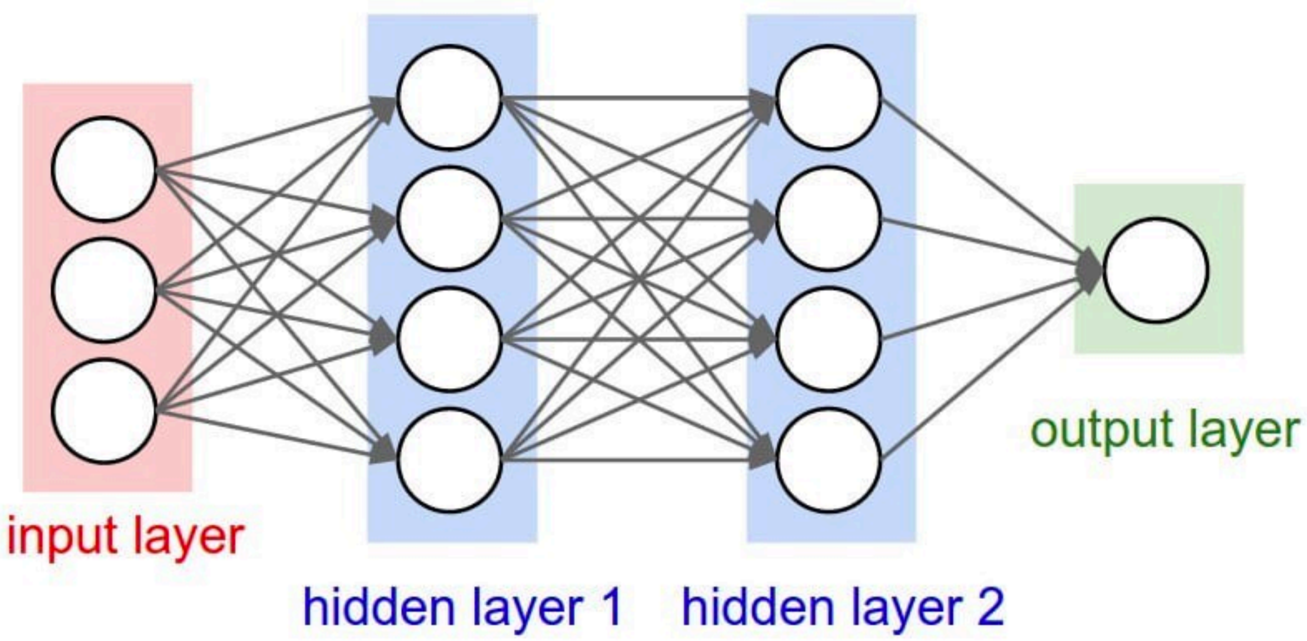
Lê Anh Cường

2020

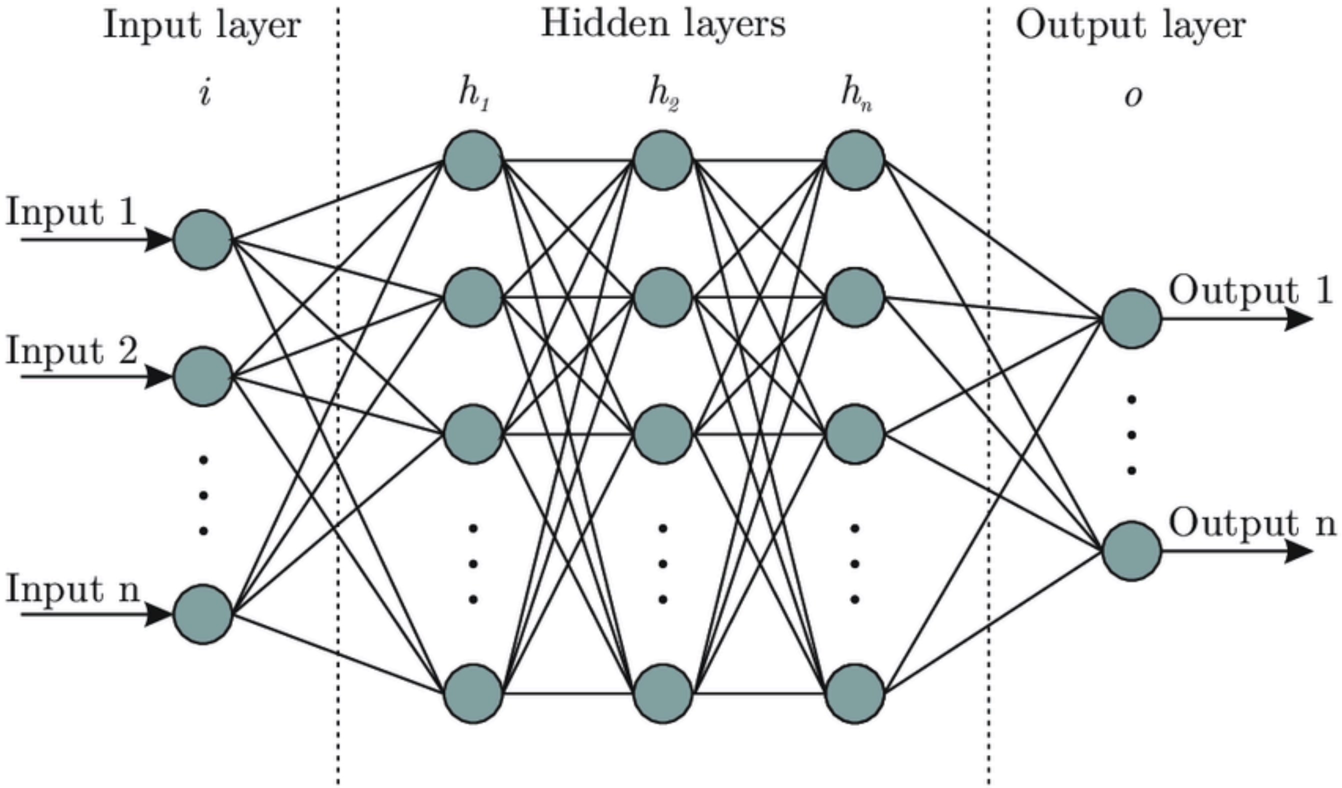
Outline

- General architecture of Neural Networks (NN)
- Forward computation for Feed-Forword Neural Networks (FFNN)
- Backpropagation for learning NN's parameters
- Implementation

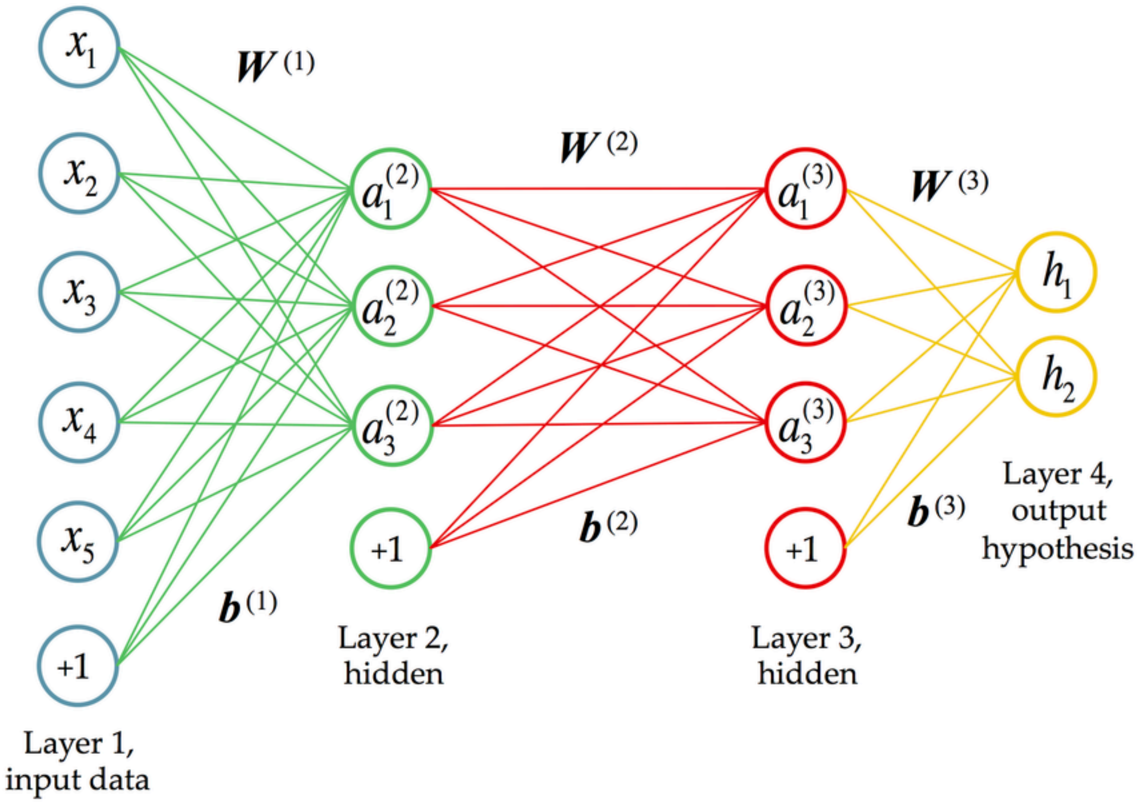
Neural Network



Neural Network

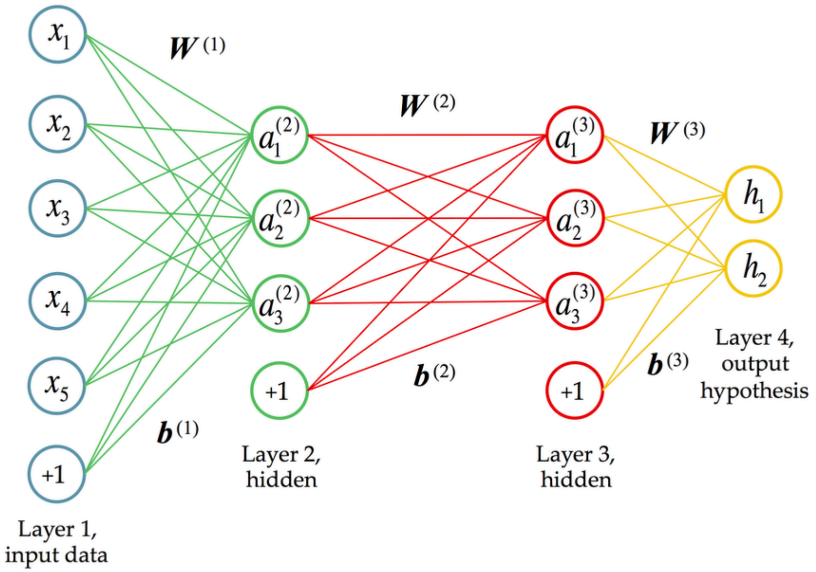


Neural Network

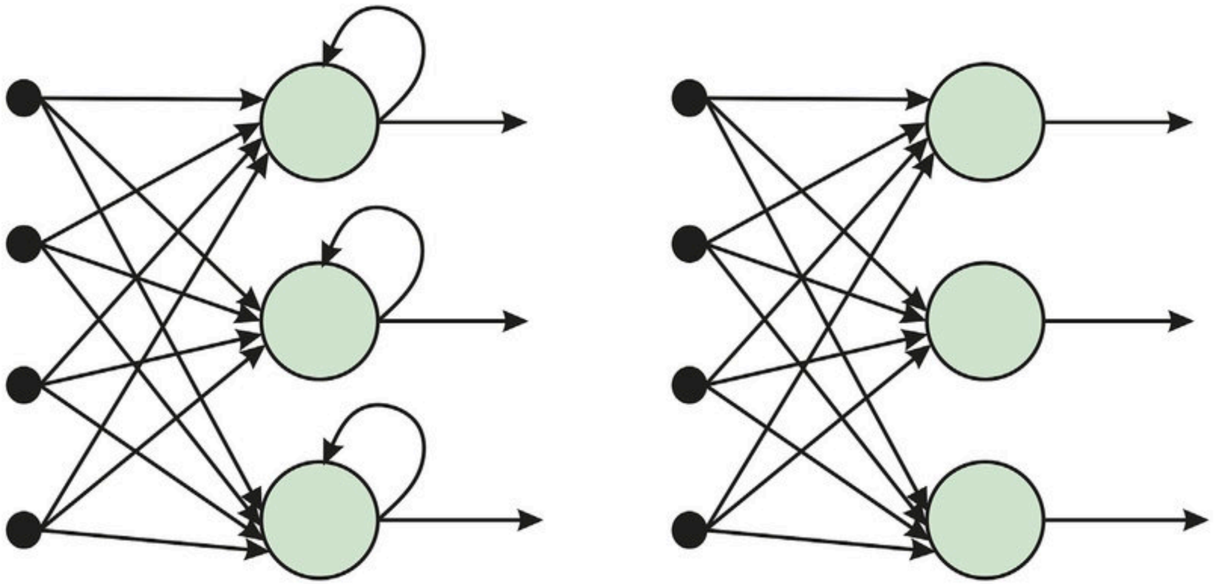


Neural Network: Definition

- **Artificial neural networks** (**ANNs**), usually simply called **neural networks** (**NNs**).
- An ANN is based on a collection of connected units or nodes called artificial neurons.
- Each connection can transmit a signal to other neurons.
- The connections are called edges which typically have a weight that adjusts as learning proceeds. The weight increases or decreases the strength of the signal at a connection.
- Different layers may perform different transformations on their inputs.

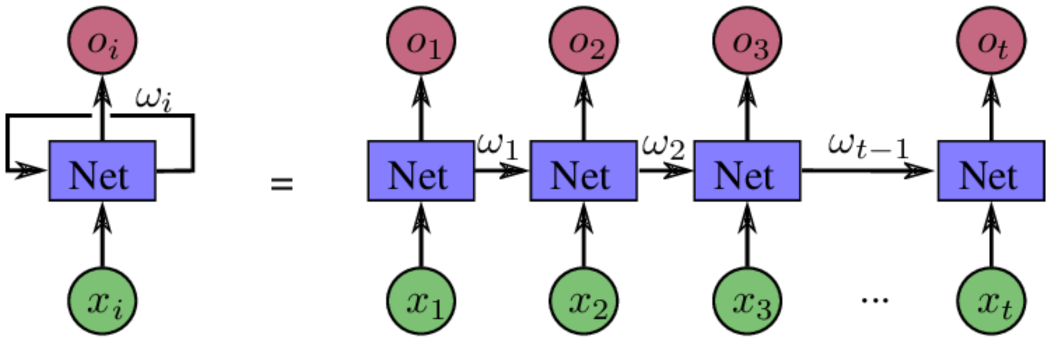


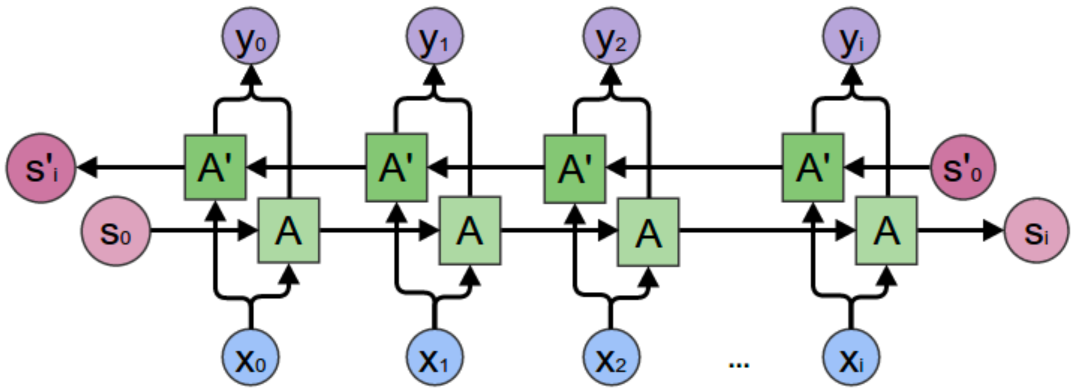
NN Types



(a) Recurrent Neural Network (b) Feed-Forward Neural Network

Recurrent Neural Network





Multiple Perceptron vs Neural Networks

A **multilayer perceptron** (MLP) is a class of feedforward artificial neural network (ANN). The term MLP is used ambiguously, sometimes loosely to *any* feedforward ANN, sometimes strictly to refer to networks composed of multiple layers of perceptrons (with threshold activation); see § Terminology. Multilayer perceptrons are sometimes colloquially referred to as "vanilla" neural networks, especially when they have a single hidden layer.[1]

An MLP consists of at least three layers of nodes: an input layer, a hidden layer and an output layer. Except for the input nodes, each node is a neuron that uses a nonlinear activation function. MLP utilizes a supervised learning technique called backpropagation for training.[2][3] Its multiple layers and non-linear activation distinguish MLP from a linear perceptron. It can distinguish data that is not linearly separable.[4]

https://en.wikipedia.org/wiki/Multilayer_perceptron

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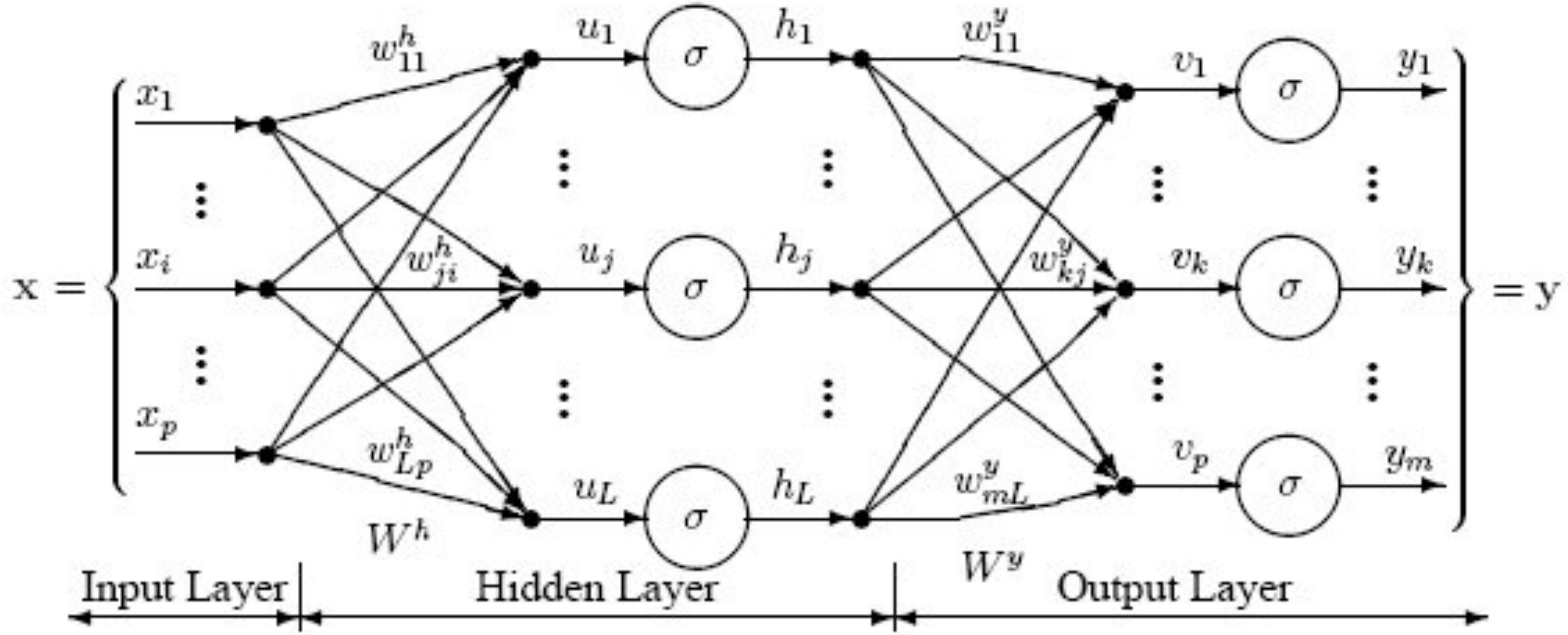
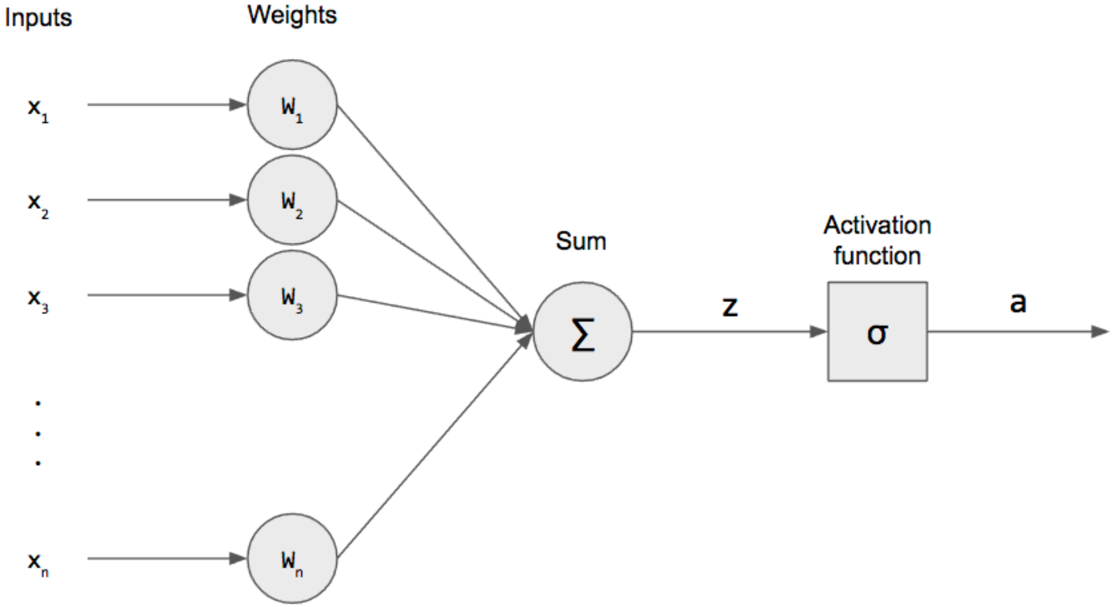
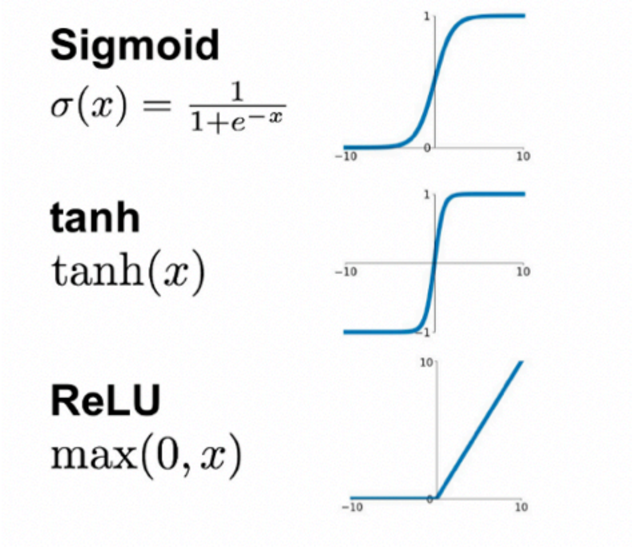
FFNN and forward Computation

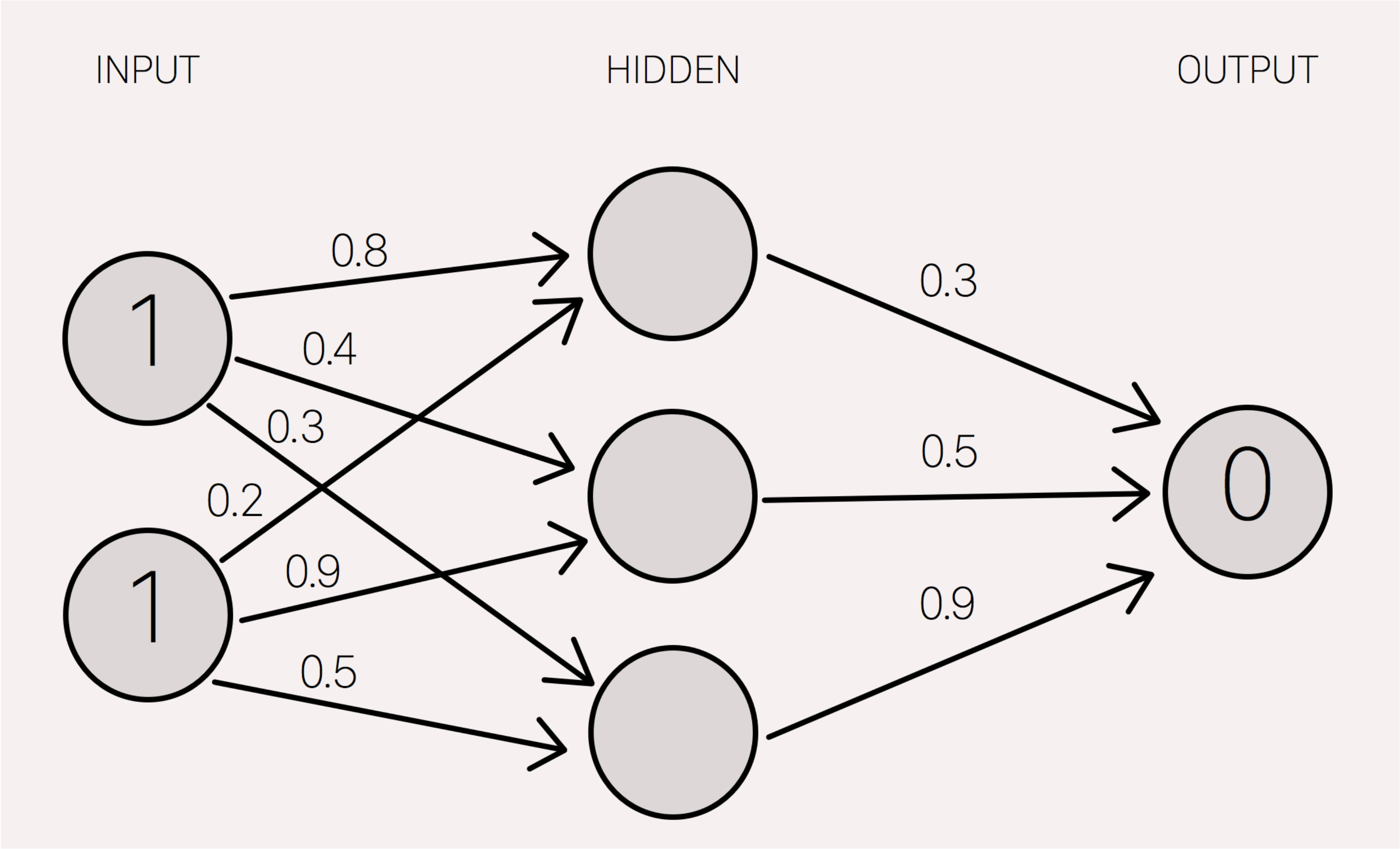


A Neural


Inputs
Weights
x1
x2
x3
xn
w1
w2
w3
wn
Sum
Σ
z
Activation function
σ
a

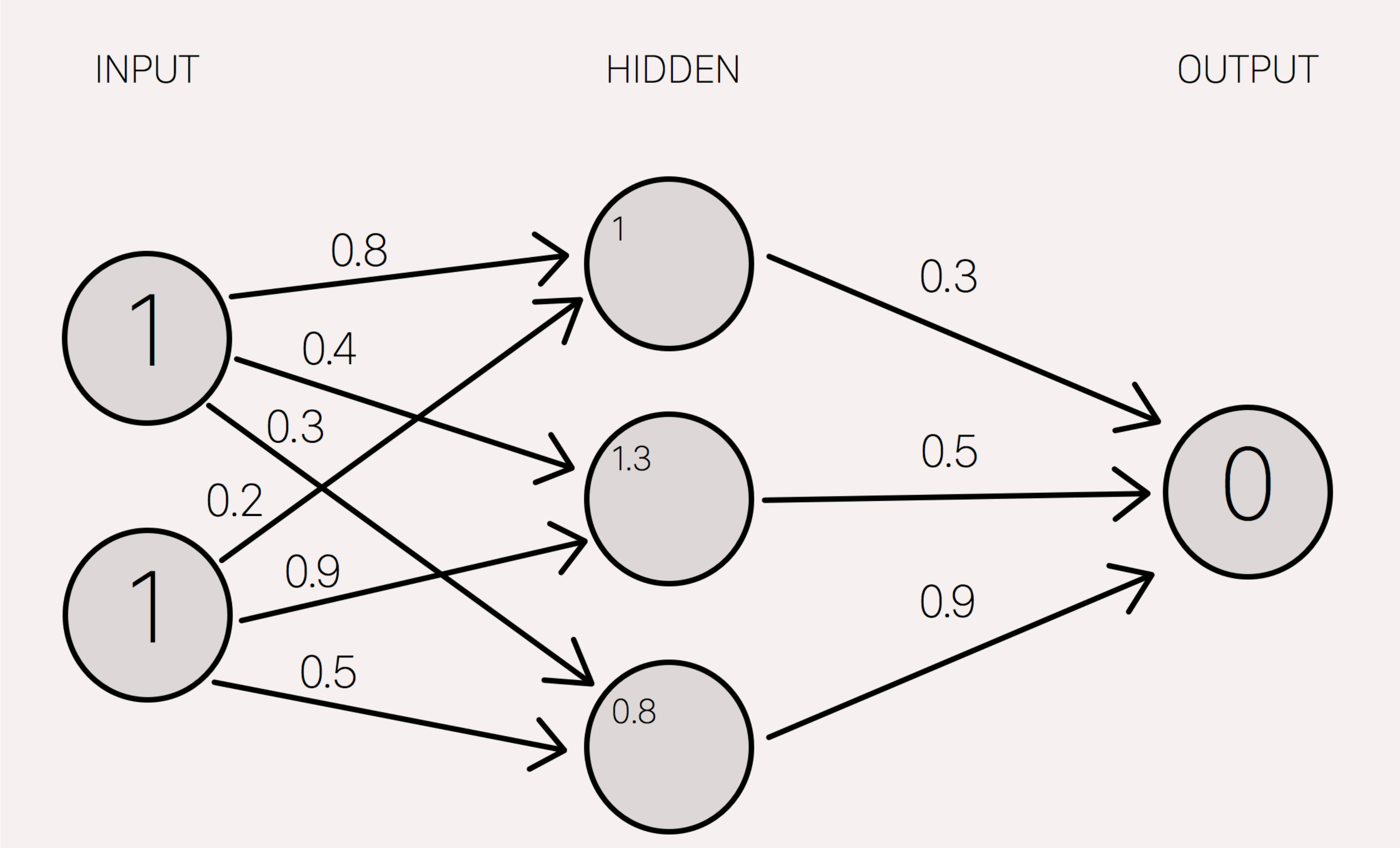

Sigmoid
$\sigma(x) = \frac{1}{1+e^{-x}}$
tanh
$\tanh(x)$
ReLU
$\max(0, x)$

Example

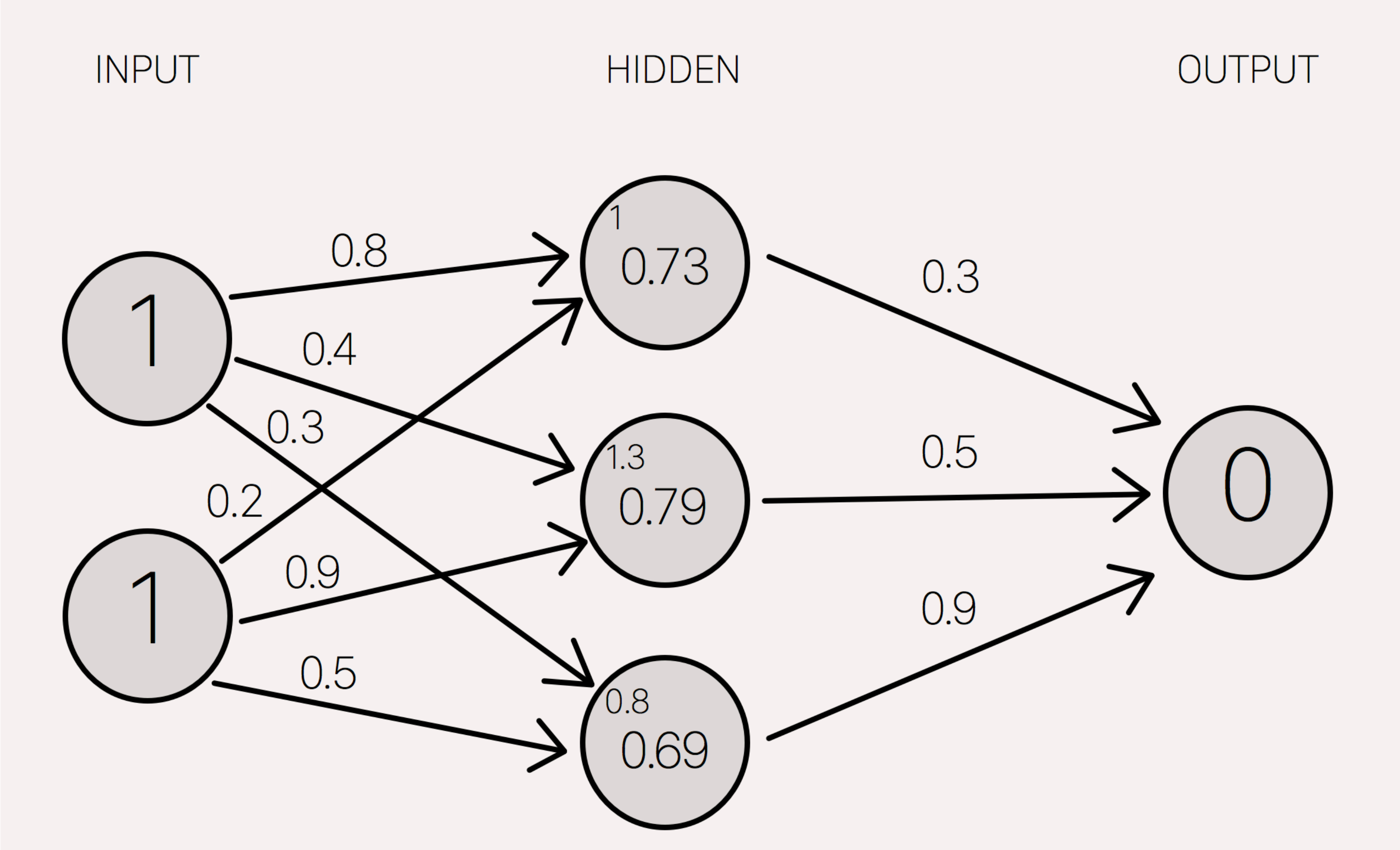


https://stevenmiller888.github.io/mind-how-to-build-a-neural-network/

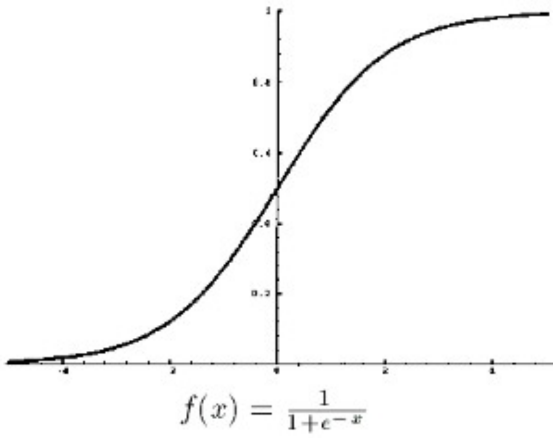
Example



Example

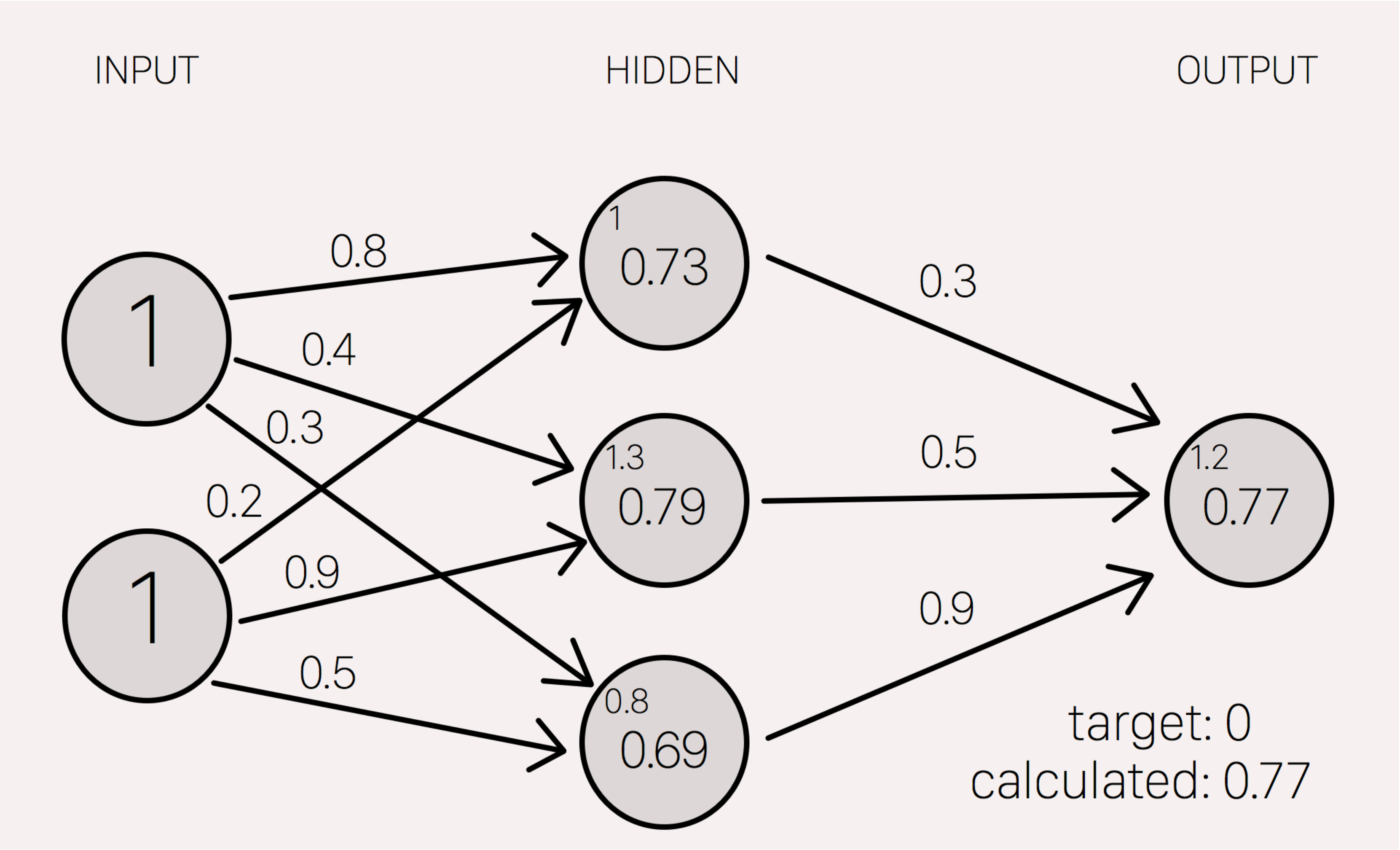


INPUT
HIDDEN
OUTPUT
1
1
0.8
0.4
0.3
0.2
0.9
0.5
1
0.73
1.3
0.79
0.8
0.69
0.3
0.5
0.9
0

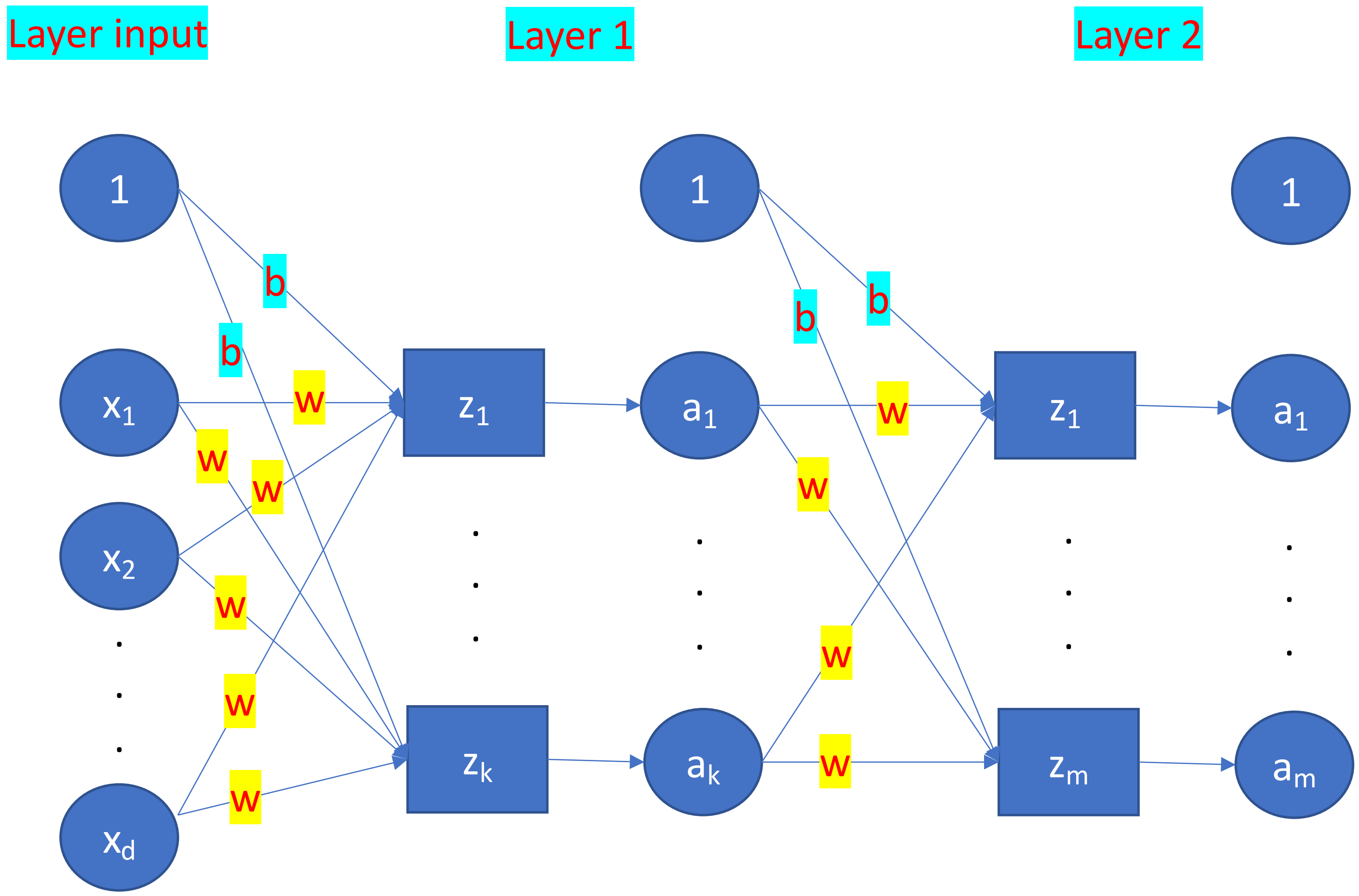


$f(x) = \frac{1}{1+e^{-x}}$

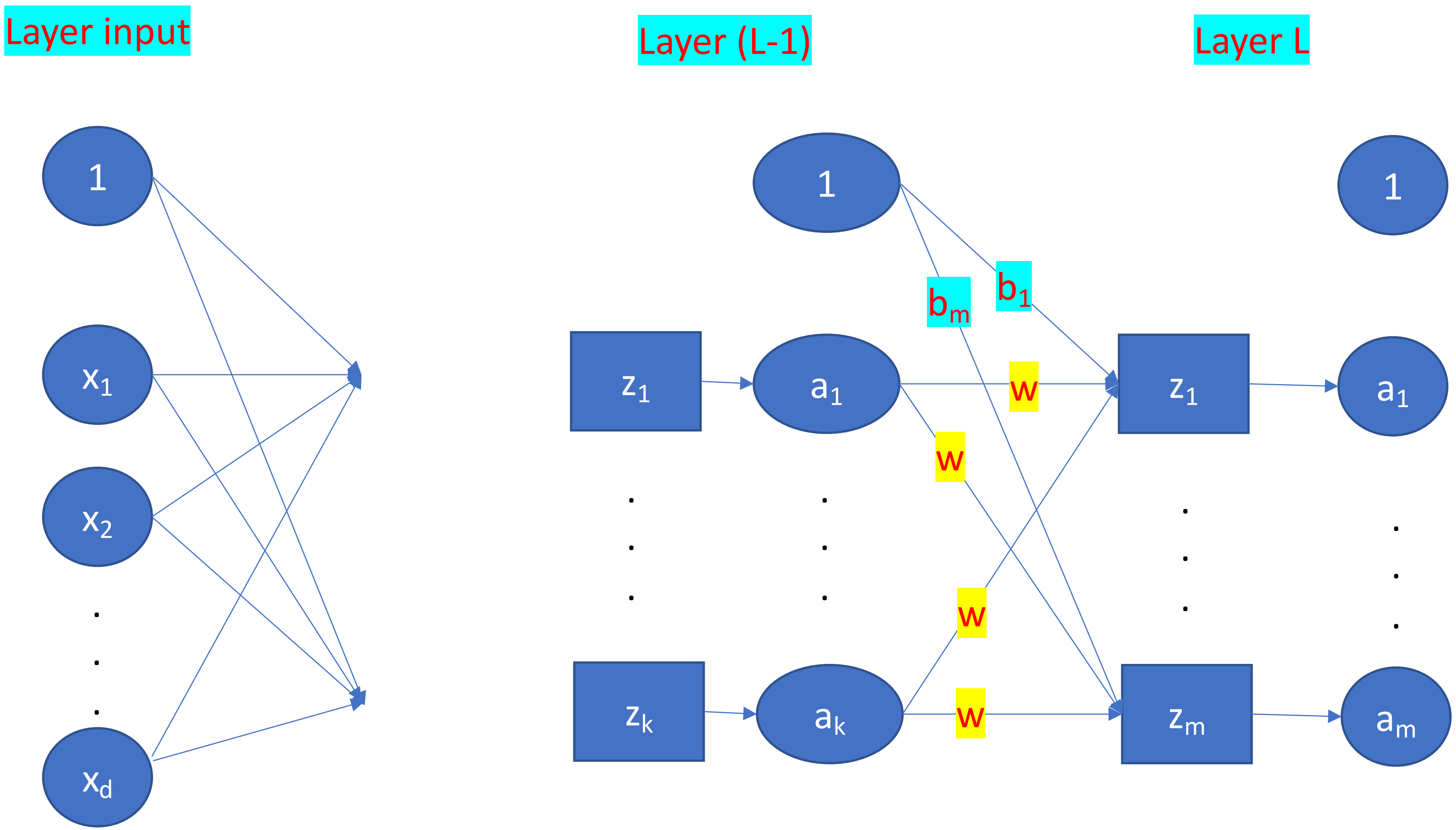
Example



INPUT
HIDDEN
OUTPUT
1
1
0.8
0.4
0.3
0.2
0.9
0.5
1
0.73
1.3
0.79
0.8
0.69
0.3
0.5
0.9
1.2
0.77
target: 0
calculated: 0.77



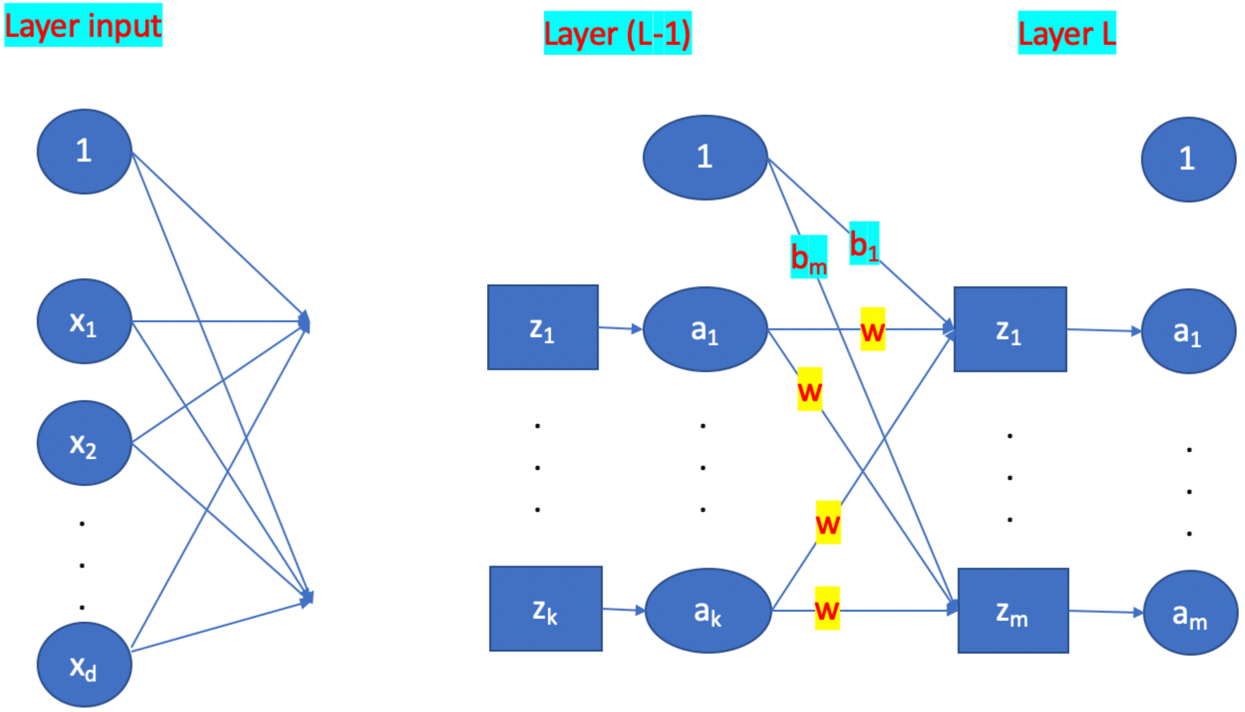
w^l_{jk} is the weight from the k^{th} neuron in the $(l-1)^{\text{th}}$ layer to the j^{th} neuron in the l^{th} layer



$$z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l$$

$$a_j^l = \sigma\left(\sum_k w_{jk}^l a_k^{l-1} + b_j^l\right),$$

Forward Computation



$$z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l$$

$$a_j^l = \sigma\left(\sum_k w_{jk}^l a_k^{l-1} + b_j^l\right),$$

$$z^l \equiv w^l a^{l-1} + b^l$$

$$a^l = \sigma(z^l)$$

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Backpropagation for NN

- The backpropagation algorithm was originally introduced in the 1970s, but its importance wasn't fully appreciated until a famous 1986 paper by David Rumelhart, Geoffrey Hinton, and Ronald Williams.
- The paper describes several neural networks where backpropagation works far faster than earlier approaches to learning, making it possible to use neural nets to solve problems which had previously been insoluble.
- Today, the backpropagation algorithm is the workhorse of learning in neural networks.

Loss/Cost function

$$C = \frac{1}{n} \sum_x C_x$$

the cost for a single training example is $C_x = \frac{1}{2} \|y - a^L\|^2$.

$$y = y(x)$$

$$a^L = a^L(x)$$

Loss/Cost function

$$C = \frac{1}{2}\|y - a^L\|^2 = \frac{1}{2}\sum_j (y_j - a_j^L)^2,$$

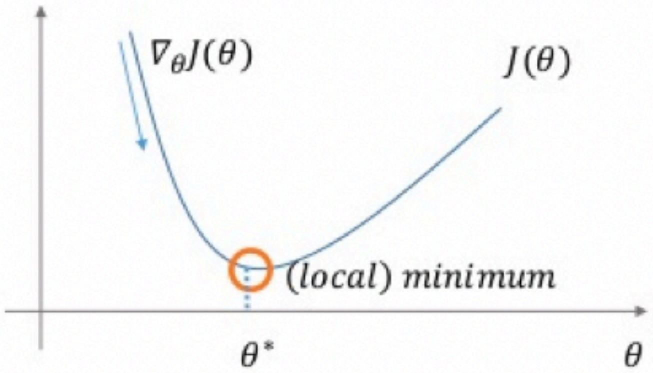
$$C = \frac{1}{2n}\sum_x \|y(x) - a^L(x)\|^2,$$

Learning Weights by Gradient Descent

- Gradient descent is a way to minimize an objective function $J(\theta)$
  - $J(\theta)$: Objective function
  - $\theta \in R^d$: Model's parameters
  - η: Learning rate. This determines the size of the steps we take to reach a (local) minimum.

Update equation

$$\theta = \theta - \eta * \nabla_\theta J(\theta)$$



Learning Weights by Gradient Descent

1. *Intialize random weights* W
2. *Repeat until convergence*

   3. *Calculate gradient* $\frac{\partial J(W)}{\partial W}$

   4. *Update weights* $W \leftarrow W - \alpha \frac{\partial J(W)}{\partial W}$

5. *Return weights* W

$$w_i = w_i - \mu \, \frac{\partial \mathrm{C}}{\partial w_i}$$

Gradient Calculation

$$z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l$$

$$C = \frac{1}{2}\|y - a^L\|^2 = \frac{1}{2}\sum_j (y_j - a_j^L)^2,$$

$$a_j^l = \sigma\left(\sum_k w_{jk}^l a_k^{l-1} + b_j^l\right),$$

$$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1}\delta_j^l.$$

$$\delta_j^l \equiv \frac{\partial C}{\partial z_j^l}.$$

Gradient Calculation

$$z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l$$

$$a_j^l = \sigma\left(\sum_k w_{jk}^l a_k^{l-1} + b_j^l\right),$$

$$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l.$$

$$C = \frac{1}{2}\|y - a^L\|^2 = \frac{1}{2}\sum_j (y_j - a_j^L)^2,$$

$$\delta_j^l \equiv \frac{\partial C}{\partial z_j^l}.$$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L}\frac{\partial a_j^L}{\partial z_j^L}.$$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L}\sigma'(z_j^L).$$

Gradient Calculation

$$z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l$$

$$a_j^l = \sigma\left(\sum_k w_{jk}^l a_k^{l-1} + b_j^l\right),$$

$$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l.$$

$$\delta_j^l \equiv \frac{\partial C}{\partial z_j^l}.$$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} \sigma'(z_j^L).$$

$$C = \frac{1}{2}\|y - a^L\|^2 = \frac{1}{2}\sum_j (y_j - a_j^L)^2,$$

$$\frac{\partial C}{\partial a_j^L} = (a_j^L - y_j).$$

$$\Rightarrow \quad \delta^L = (a^L - y) \odot \sigma'(z^L).$$

Gradient Calculation

$$z_j^l = \sum_k w_{jk}^l a_k^{l-1} + b_j^l$$

$$a_j^l = \sigma\left(\sum_k w_{jk}^l a_k^{l-1} + b_j^l\right),$$

$$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l.$$

$$\delta_j^l \equiv \frac{\partial C}{\partial z_j^l}.$$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} \sigma'(z_j^L).$$

$$C = \frac{1}{2}\|y - a^L\|^2 = \frac{1}{2}\sum_j (y_j - a_j^L)^2,$$

$$\frac{\partial C}{\partial a_j^L} = (a_j^L - y_j)$$

$$\delta^L = (a^L - y) \odot \sigma'(z^L).$$

$$\delta_j^l = \frac{\partial C}{\partial z_j^l}$$

$$= \sum_k \frac{\partial C}{\partial z_k^{l+1}} \frac{\partial z_k^{l+1}}{\partial z_j^l}$$

$$= \sum_k \frac{\partial z_k^{l+1}}{\partial z_j^l} \delta_k^{l+1},$$

Gradient Calculation

$$\delta^L = (a^L - y) \odot \sigma'(z^L). \qquad \text{(BP1)}$$

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l), \qquad \text{(BP2)}$$

By combining (BP2) with (BP1) we can compute the error δ^l for any layer in the network. We start by using (BP1) to compute δ^L, then apply Equation (BP2) to compute δ^{L-1}, then Equation (BP2) again to compute δ^{L-2}, and so on, all the way back through the network.

Gradient Calculation

$$\frac{\partial C}{\partial w^l_{jk}} = a^{l-1}_k \delta^l_j.$$

$$\delta^L = (a^L - y) \odot \sigma'(z^L). \tag{30}$$

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l), \tag{BP2}$$

Summary: the equations of backpropagation

$$\delta^L = \nabla_a C \odot \sigma'(z^L) \tag{BP1}$$

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l) \tag{BP2}$$

$$\frac{\partial C}{\partial b^l_j} = \delta^l_j \tag{BP3}$$

$$\frac{\partial C}{\partial w^l_{jk}} = a^{l-1}_k \delta^l_j \tag{BP4}$$

$$\delta^L = \nabla_a C \odot \sigma'(z^L).$$

$$\delta^L = (a^L - y) \odot \sigma'(z^L)$$

The backpropagation algorithm

1. **Input x:** Set the corresponding activation a^1 for the input layer.

2. **Feedforward:** For each $l = 2, 3, \ldots, L$ compute $z^l = w^l a^{l-1} + b^l$ and $a^l = \sigma(z^l)$.

3. **Output error δ^L:** Compute the vector $\delta^L = \nabla_a C \odot \sigma'(z^L)$.

4. **Backpropagate the error:** For each $l = L-1, L-2, \ldots, 2$ compute $\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l)$.

5. **Output:** The gradient of the cost function is given by $\frac{\partial C}{\partial w^l_{jk}} = a^{l-1}_k \delta^l_j$ and $\frac{\partial C}{\partial b^l_j} = \delta^l_j$.

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