# Privacy-Aware Similarity Search

## A Glance at Approximate $k$-Nearest Neighbors Search

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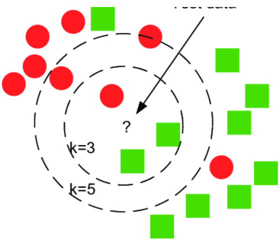
**Nearest neighbor (NN), $k$-NN, Approximate $k$-NN**

Shared by Jing last week...



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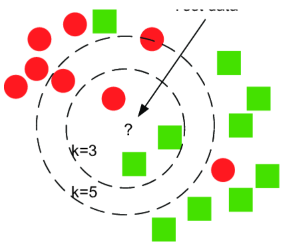
- Recommendation System, Large-Scale Machine Learning
- Face Recognition, Biometric identification
- General-Purpose Similarity Search



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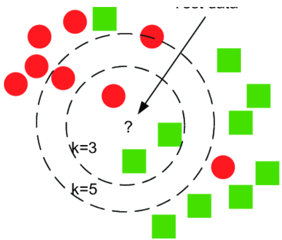


## Sensitive Data

Disclosure of data is not always acceptable.

- Biometrics: Face, Genetic Sequence, Clinical Data; User Profiles...
- Laws and regulations (GDPR in EU, Data Security Law in China...)

# Privacy-Aware (Approximate) $k$-NN: Scenarios

## Data Owner

- Client: Outsource encrypted data to remote DB server (Searchable Encryption)
- **Server Provider (SP)**: The provider itself holds the data (stored in plaintext)

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- **Single**: Most of works follow this trend
- **Dual**: Good for lightweight protocol
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## Threat Models

- Semi-Honest (SH): Parties follow the protocol specification but try to infer information from what they received
- Malicious Adversaries (Client/Server) (MA/MC/MS): Parties may deviate from protocol, collude with others, or otherwise behave maliciously to learn more about the database.
- Mixed: Some parties are SH and some parties are MA.

# Privacy-Aware (Approximate) $k$-NN: State of the Art

| | TM/ #Srv/ DO | NN approach | Comm. | Comp. | Rounds | Crypt. Tools | Efficiency |
|---|---|---|---|---|---|---|---|
| [SFR20] | SH/ Single/ SP | $k$-ish NN | $\log N$ | $N$ | 1 | FHE | ★☆☆☆ |
| SANNS [Che+20] | SH/ Single/ SP | $k$-Means | $N/k$ | $N$ | $N$ | AHE/ GC/ ORAM | ★★☆☆ |
| [ZS21] | SH/ Single/ Both | Linear Scan | $\log N$ | $N^2$ | 1 | FHE | ★★☆☆ |
| PP-AkNN [BT21] | SH/ Single/ Client | extended LSH | $\log^2 N$ | N/A | $N$ | ODS | N/A |
| [SLD21] | MC/ Dual/ SP | LSH | $\sqrt{N}$ | $N$ | 1 | DPF | ★★★★ |

Table: Comparison between 5 recently proposed approaches. TM stands for *Threat Model*. DO stands for *Data Owner*. $N$ is the database size.

# To be discussed in this talk

- SANNS: Scaling Up Secure Approximate $k$-Nearest Neighbors Search (USENIX Security 20')[Che+20]
- What else can we do to exceed them (in some ways)?

# SANNS: Scaling Up Secure Approximate k-Nearest Neighbors Search

Heavy cryptographic tools-based solution

# Previous works

## Linear scan: Secure $k$-NN in a 2PC setting

- Compute the distance between q and all the points in $X$
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## Distance Computation

Some of the previous works use

- Paillier[Pai99] (in [Bar+10; ESJ14; Erk+09; Hua+11]) (no SIMD acceleration)
- BFV[Bra12; FV12] (in [JVC18]) (used by SANNS)
- Oblivious Transfer (OT)-based multiplication (in [DSZ15]) (less computation but more communication)

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## Top-$k$ selection

SANNS achieves this task by garbling a new top-k circuit ($O(n + k^2)$ comparators), some of the previous works use:

- The naive circuit of size $O(nk)$ (in [Ash+18; SGB18; Son+15])
- Homomorphic encryption such as BGV[BGV14] (in [SFR18; SFR20])

# Contributions

## SANNS

**S**ecure **A**pproximate $k$-**N**earest **N**eighbor **S**earch

- The server (data owner): learns nothing
- Client (query maker): learns nothing but the final result
- Query (query point $q$ and the result) and database are kept confidential

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## Two protocols

Based on **approximate top-$k$ selection** (semi-honest model)

- Optimized linear scan
- Sublinear-time clustering-based algorithm

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## Scalability

SANNS scales up to databases with 10 million entries

- Tests on various datasets: SIFT, Deep1B, Amazon reviews text.

$k = 10$ over $10M$ entries with accuracy 0.9 in a bunch of seconds

## Secret Sharing

- Arithmetic: $x \in \mathbb{Z}_t \longmapsto (x_A, x_B)$ such that $x_A + x_B \equiv x \pmod{t}$
- Boolean: $x \in \{0, 1\}^\tau \longmapsto (x_A, x_B)$ such that $x_A \oplus x_B \equiv x$

# Preliminaries

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## AHE

A (private-key) additive homomorphic encryption

- Operations: add two ciphertexts, add/multiply a ciphertext by a constant
- SANNS uses the **BFV**[Bra12; FV12] implementation in SEAL[a]

[a] https://github.com/Microsoft/SEAL

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## GC

Garbled circuit[Yao86]: achieves generic secure two-party computation for arbitrary Boolean circuits.
(A method that enables two parties with private inputs $x$ and $y$ to jointly compute a function $f(x, y)$, useful especially in comparison)

## DORAM

Distributed version of oblivious RAM

- 2 parties hold secret shares of an array
- oblivious read and write operations are supported
- SANNS uses Floram[DS17], which leverages Functional Secret Sharing to mask the database

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## $k$-means clustering

Iterative algorithm[Llo82]

- Find a clustering $X = C_1 \cup C_2 \cup ... \cup C_k$
- Find centers $\mathbf{c_1}, \mathbf{c_2}, ..., \mathbf{c_k} \in \mathbb{R}^d$ which approximately minimize

$$\sum_{i=1}^{k} \sum_{\mathbf{x} \in C_i} ||\mathbf{c_i} - \mathbf{x}||^2$$

# SANNS: optimized linear scan

## First algorithm: optimized linear scan

- Compute the distance between $q$ and all the points in $X \rightarrow$ AHE
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## Approximate top-$k$ selection

Being used in both optimized linear scan and clustering based algorithm

- Given a list of $n$ numbers ($b$-bits), output the $k \leq n$ smallest elements in the sorted order
- Augmented functionality: output the ID together with the $k$ smallest values

# Approximate top-$k$

## Approach: the output has to be **approximately correct** (query point $q$)

1. Shuffle inputs in uniformly random order

$$x_1, .., x_n \longmapsto x_{\pi(1)}, ..., x_{\pi(n)}$$

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## Approximate distances

Discard the $r$ low-order bits of the $b$-bit inputs

- Circuit size: $O(b \cdot (n + kl)) \rightarrow O((b - r)) \cdot (n + kl))$

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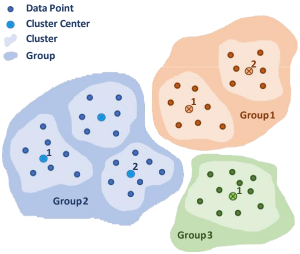

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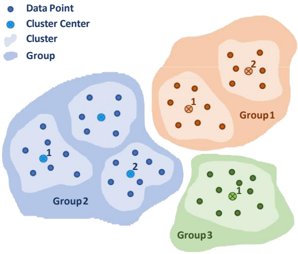


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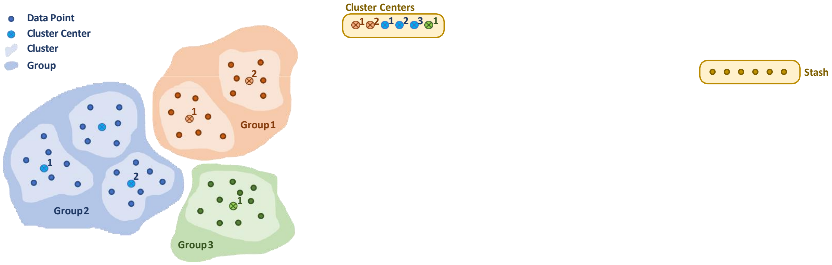
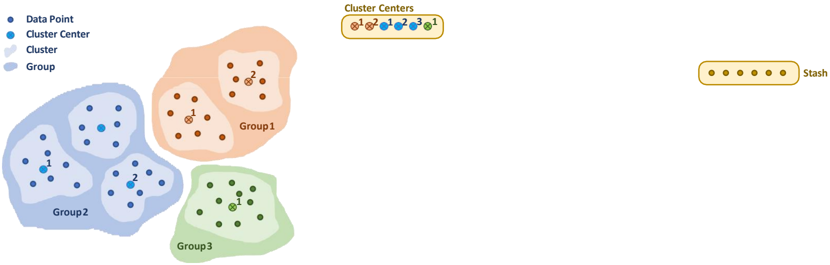


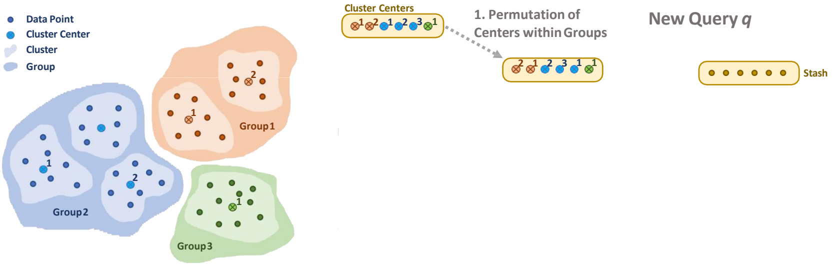



**Dataset:** Floram(init)

**Clusters**

**Stash**

# SANNS: clustering based algorithm

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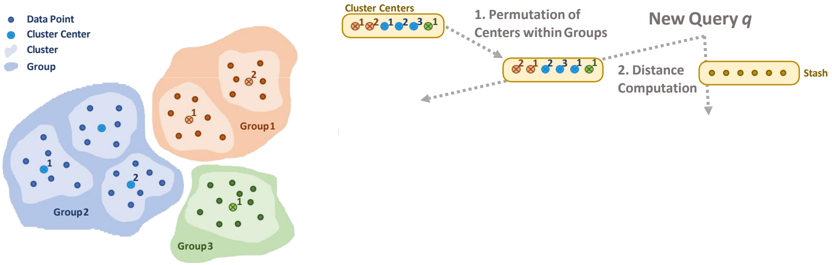


**Dataset:** Floram(init)

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- **Step 2:** AHE

**Stash**

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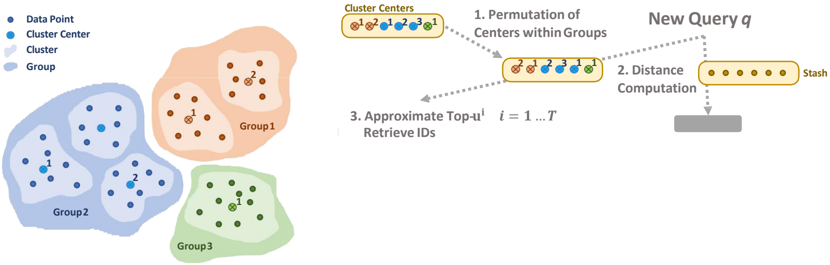
# SANNS: clustering based algorithm




**Dataset:** Floram(init)

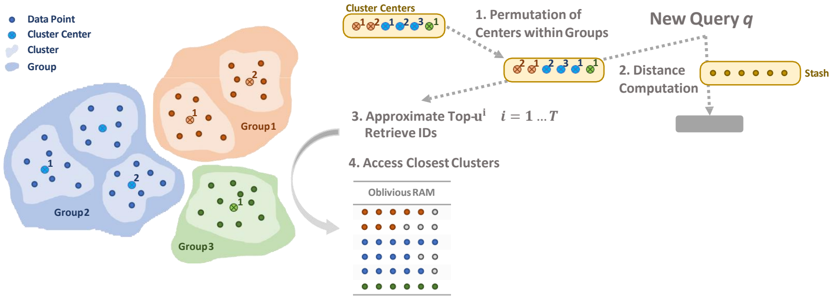
**Clusters**

- **Step 2:** AHE
- **Step 3:** GC

**Stash**

- **Step 2:** AHE

# SANNS: clustering based algorithm




**Dataset:** Floram(init)

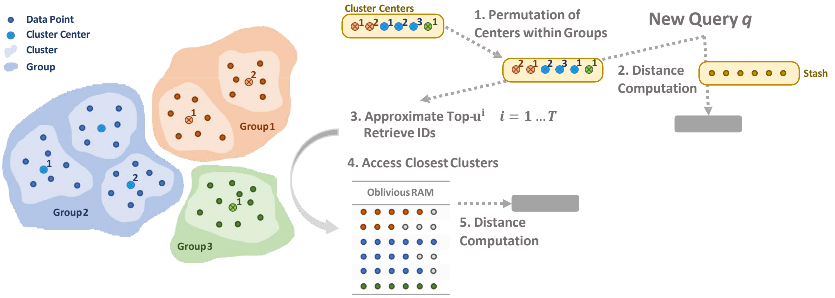
**Clusters**

- **Step 2:** AHE
- **Step 3:** GC
- **Step 4:** Floram(read)

**Stash**

- **Step 2:** AHE

# SANNS: clustering based algorithm




**Dataset:** Floram(init)

**Clusters**

- **Step 2:** AHE
- **Step 3:** GC
- **Step 4:** Floram(read)
- **Step 5:** AHE

**Stash**

- **Step 2:** AHE

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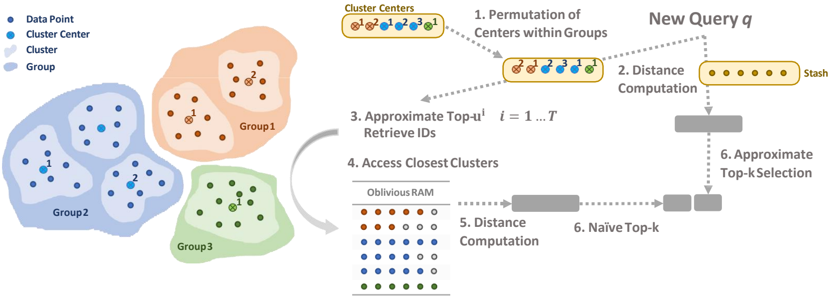


**Dataset:** Floram(init)

**Clusters**

- **Step 2:** AHE
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- **Step 4:** Floram(read)
- **Step 5:** AHE
- **Step 6:** GC

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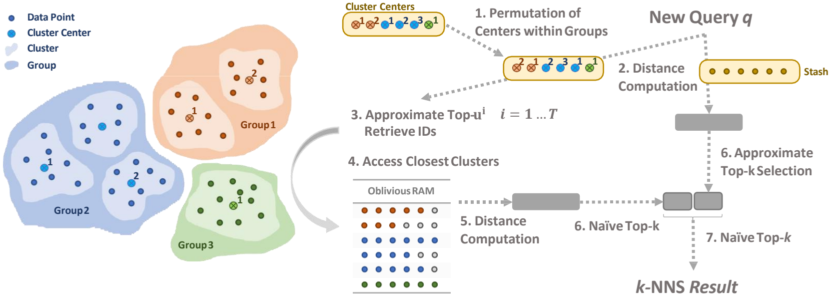


**Dataset:** Floram(init)

**Clusters**

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- **Step 3:** GC
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- **Step 5:** AHE
- **Step 6:** GC

**Step 7 - final top-$k$:** GC

**Stash**

- **Step 2:** AHE
- **Step 6:** GC

# Experiments

## Datasets

SIFT[Low99]

- Standard dataset of image descriptors (similarity between images)
- $1M$ entries of size $d = 128$
- 8-bits

Deep1B[BL16]

- Dataset of image descriptors (feature vectors extracted from a DNN)
- $1B$ entries of size $d = 96$
- Quantized to 8-bits
- Deep1B-1M: first $1M$ images, Deep1B-10M: first $10M$ imagess

Amazon[McA+15]

- Dataset of reviews on Amazon
- $1M$ entries of size $d = 50$
- Quantized to 9-bits

# Possible improvements?

- Computing distance (especially euclidean distance) using HE is really expensive.
  - Differential private Euclidean Distance Approximation[Sta21]
  - LSH-based solution avoiding heavy cryptographic tools(An e-print[SLD21])
- How to leverage state-of-the-art plaintext approach?
- How to excatly and accurately quantize the information leakage?
- Other distance metrics? (Hamming, Cosine similarity...)
- Distributed System for extremely large database?

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# Why is clustering algorithm appealing?

Compared with graph-based ANN algorithms, in clustering algorithms

- Most memory accesses are adaptive
    - SOTA graph-based algorithms[MY20; Fu+19] query by following edges in certain carefully constructed graphs
    - Certainty (distinguished by queries) $\rightarrow$ non-adaptive memory access $\rightarrow$ more rounds of interaction protected by ORAM
    - hence inefficient :(
- Many distances are computed at once (large batches of points)
    - On the contrary, those graph-based approaches adaptively compute *individual* distance
    - Hard to accelerate via vectorized SIMD optimization