Credit Risk Analysis Using Credit Transition Matrices

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1 Introduction

Credit risk analysis is a critical aspect of financial risk management, focusing on evaluating the likelihood that borrowers will default on their debt obligations. It involves the assessment of potential losses due to the failure of a borrower to meet their contractual obligations. This process is essential for financial institutions, investors, and regulatory bodies to manage and mitigate credit risk effectively.

In the past, credit risk analysis has primarily relied on several key methodologies:

- **Credit Scoring Models**: These models use statistical techniques to predict the likelihood of a borrower defaulting. They are based on historical data and various borrower attributes such as income, credit history, and employment status. Common examples include the FICO score and the Z-score model developed by Altman [1].
- **Structural Models**: These models, such as the Merton model, are based on the premise that a firm's equity can be viewed as a call option on its assets. The probability of default is inferred from the volatility and value of the firm's assets relative to its debt [9].
- **Reduced-Form Models**: These models, also known as intensity-based models, focus on modeling the default event directly rather than the underlying firm value. They use observable market data, such as credit spreads and bond yields, to estimate the default probability [5].

While these traditional approaches have been widely used, they possess several limitations:

- **Credit Scoring Models**: These models often oversimplify the complexities of credit risk by reducing it to a single score. They may not capture dynamic changes in a borrower's financial situation or broader economic conditions [12].
- **Structural Models**: These models require extensive data on the firm's asset value and volatility, which may not always be available. They also assume that markets are efficient and that the firm's asset value follows a specific stochastic process, which may not hold in reality [4].
- **Reduced-Form Models**: These models rely heavily on market data, which can be volatile and influenced by factors unrelated to the borrower's creditworthiness. They may not adequately capture long-term credit risk trends [7].

Credit Transition Matrices (TMs) offer a robust alternative to these traditional models, addressing many of their limitations. TMs provide a systematic approach to understanding and predicting changes in credit ratings over time. Here are some advantages:

- **Historical Data Utilization**: TMs leverage extensive historical data on credit ratings to estimate the likelihood of transitions between different credit states, including upgrades, downgrades, and defaults [6].
- **Dynamic Credit Assessment**: Unlike static credit scores, TMs can capture the dynamic nature of credit risk by modeling the probabilities of transitions over various time horizons [8].

- **Stress Testing and Scenario Analysis**: TMs facilitate robust stress testing by linking transition probabilities to macroeconomic factors. This allows financial institutions to assess the impact of economic downturns on their credit portfolios [2].
- **Regulatory Compliance**: TMs align well with regulatory frameworks such as Basel III, which require banks to maintain adequate capital buffers based on the credit risk of their portfolios. TMs provide a transparent and quantitative basis for these calculations [10].
- **Versatility**: TMs can be applied to various types of credit exposures, including corporate bonds, sovereign debt, and consumer loans. This versatility makes them a valuable tool for comprehensive credit risk management [11].

Building upon these advantages, the Transition Matrix (TM) is a foundational tool in the field of credit risk modeling, providing a systematic approach to understanding and predicting the changes in credit ratings over time. The origin of TMs can be traced back to the need for financial institutions to manage credit risk more effectively by quantifying the likelihood of rating transitions, including upgrades, downgrades, and defaults. The TM was first introduced as a means to analyze historical credit rating data and project future transitions based on observed patterns [2].

Moody's Credit Transition Model (TM), developed by Moody's Analytics, is one of the most prominent models in this domain. Leveraging extensive historical ratings data and incorporating economic assumptions, Moody's TM facilitates robust stress testing and default rate forecasting. This model has been extensively validated and employed in both academic research and industry practices, making it a cornerstone in the field of credit risk management [10].

The significance of TMs lies in their ability to provide critical insights into the dynamics of credit ratings. By understanding how credit ratings evolve over time under various economic scenarios, financial institutions can better manage their credit portfolios and anticipate potential risks. TMs are particularly valuable during years of economic stress, as they help in estimating economic credit risk capital and linking transition probabilities to macroeconomic factors, thereby enhancing the accuracy of default rate predictions [6, 2].

Despite the widespread use and validation of TMs, there remain areas where further research is needed. This paper investigates three critical aspects of credit risk analysis using Credit Transition Matrices (TMs):

1. The impact of different withdraw rates on default rates(PD).
2. The simulation of the transition process with given withdraw counts to compare the Expected Credit Loss (ECL) performance.
3. The simulation of the transition process using conditional Probability of Default (PD).

By addressing these aspects, this paper aims to enhance the understanding of TMs and their practical applications in managing credit risk. The insights gained from this study can help financial institutions improve their risk assessment models and better prepare for economic uncertainties.

2 Impact of Different Withdraw Rates on PD

The impact of withdraw rates on PD is a critical factor in credit risk analysis. Withdraw rates, representing the probability of a borrower exiting the rating system before defaulting, can significantly affect the observed PD. Understanding how withdraw rates influence default probabilities is essential for accurate credit risk modeling and for developing robust risk management strategies [8, 11].

To analyze this impact, we incorporate three matrices:

- Original TM
- 50% Withdraw reweighted TM
- Withdraw removed TM

By using the original transition matrix along with these two adjusted matrices, we can better understand the underlying dynamics of credit transitions and the true impact of withdraws on credit risk assessment. This analysis is vital for financial institutions to enhance their risk models and improve their predictive accuracy in estimating default probabilities under different economic conditions [3].

In this study, we employ three different methods to calculate the probability of default (PD), each offering a unique perspective on how withdraws impact PD:

1. **Conventional Rate:** This method calculates PD by considering the sum of survivors, withdraws, and defaults in the denominator [3]. The formula is given by:

$$PD_{\text{Conventional}} = \frac{\text{Defaults}}{\text{Survivors} + \text{Withdraws} + \text{Defaults}}$$

2. **Moody's Withdraw-Adjusted Rate:** This approach adjusts the denominator by incorporating half of the withdraws along with survivors and defaults [8]. The formula is expressed as:

$$PD_{\text{Moody}} = \frac{\text{Defaults}}{\text{Defaults} + \text{Survivors} + 0.5 \times \text{Withdraws}}$$

3. **Withdraw-Removed Rate:** This method excludes the withdraws from the denominator, considering only the sum of survivors and defaults [11]. The formula is:

$$PD_{\text{Withdraw-Removed}} = \frac{\text{Defaults}}{\text{Survivors} + \text{Defaults}}$$

By applying these formulas, we can analyze and compare the impact of different methods on the estimation of default probabilities. Each method offers a unique way to account for withdraws, thereby providing a comprehensive understanding of credit risk under various scenarios.

2.1 Matrices Used

In our analysis, we employ different transition matrices to assess credit risk under various scenarios. Below, we introduce the three main Transition Matrices (TMs) used in the simulations.

2.1.1 Original TM

The Original TM reflects the typical behavior of credit ratings over time, with several key characteristics:

- **Diagonal Dominance:** The highest ratings tend to have the largest diagonal values, indicating a higher probability of remaining in the same rating. Conversely, the lowest ratings, including default, have smaller diagonal values but higher probabilities of transitioning to default.
- **Downgrade Likelihood:** Customers are more likely to experience a downgrade in their rating rather than an upgrade, reflecting the inherent risk aversion in credit assessments.
- **Withdraws:** The probabilities of withdraws are relatively consistent across different ratings, representing a random setting for withdraws.
- **Probability Distribution:** The diagonal probabilities are the largest in the matrix, indicating the highest likelihood of remaining in the same rating. The migration probabilities decrease along the diagonal, diminishing as they extend upwards or downwards from the diagonal.
- **Adjacent Transitions:** The three ratings adjacent to the current rating have positive values, while others remain zero, indicating that migrations are more likely to occur to ratings close to the current one.

These properties highlight the stability of higher credit ratings and the tendency for ratings to degrade over time, with significant implications for credit risk management and forecasting. The Original TM is presented in Table 1.

	Rating 1	**Rating 2**	**Rating 3**	**Rating 4**	**Rating 5**	**Rating 6**	**Rating 7**	**Rating 8**	**Rating 9**	**Rating 10**	**Withdraw**	**Default**
Rating 1	0.867780	0.012864	0.008874	0.004281	0	0	0	0	0	0	0.106150	0.00005
Rating 2	0.018870	0.840201	0.025637	0.017167	0.009233	0	0	0	0	0	0.088793	0.00010
Rating 3	0.011379	0.019003	0.809913	0.025829	0.016179	0.008989	0	0	0	0	0.108309	0.00040
Rating 4	0.007659	0.015494	0.024715	0.788313	0.034330	0.023897	0.010675	0	0	0	0.094018	0.00090
Rating 5	0	0.008715	0.016573	0.022206	0.777589	0.037705	0.024183	0.011913	0	0	0.098616	0.00250
Rating 6	0	0	0.009735	0.023419	0.033493	0.743969	0.044798	0.030591	0.017119	0	0.089876	0.00700
Rating 7	0	0	0	0.010535	0.019041	0.030258	0.735069	0.047591	0.031101	0.016185	0.097421	0.01280
Rating 8	0	0	0	0	0.010704	0.020936	0.034435	0.700483	0.049230	0.030483	0.093631	0.06010
Rating 9	0	0	0	0	0	0.010116	0.021246	0.029305	0.654048	0.043237	0.084547	0.15750
Rating 10	0	0	0	0	0	0	0.002585	0.004554	0.008439	0.525731	0.104991	0.35370

Table 1: Original Transition Matrix (TM)

2.1.2 Withdraw Adjusted TM

The Withdraw Adjusted TM is designed to simulate a scenario where 50% of withdrawn entities are reassigned proportionally to their existing ratings, while the remaining 50% are distributed across all ratings, including default. This approach provides a more balanced view of the potential outcomes for withdrawn entities, offering a nuanced alternative to the Original TM. The details of this adjusted TM are presented in Table 2.

	Rating 1	**Rating 2**	**Rating 3**	**Rating 4**	**Rating 5**	**Rating 6**	**Rating 7**	**Rating 8**	**Rating 9**	**Rating 10**	**Default**
Rating 1	0.970837	0.014392	0.009928	0.004790	0	0	0	0	0	0	0.000053
Rating 2	0.020709	0.922079	0.028135	0.018840	0.010133	0	0	0	0	0	0.000105
Rating 3	0.012761	0.021311	0.908312	0.028967	0.018145	0.010081	0	0	0	0	0.000423
Rating 4	0.008454	0.017103	0.027281	0.870162	0.037895	0.026378	0.011784	0	0	0	0.000944
Rating 5	0	0.009670	0.018389	0.024638	0.862786	0.041837	0.026832	0.013219	0	0	0.002630
Rating 6	0	0	0.010700	0.025741	0.036814	0.817735	0.049240	0.033624	0.018816	0	0.007329
Rating 7	0	0	0	0.011680	0.021111	0.033549	0.815009	0.052766	0.034483	0.017946	0.013455
Rating 8	0	0	0	0	0.011851	0.023179	0.038124	0.775540	0.054505	0.033749	0.063052
Rating 9	0	0	0	0	0	0.011152	0.023421	0.032305	0.721006	0.047664	0.164452
Rating 10	0	0	0	0	0	0	0.002993	0.005272	0.009770	0.608668	0.373299

Table 2: Withdraw Adjusted Transition Matrix (TM)

2.1.3 Withdraw Removed TM

The Withdraw Removed TM offers a unique perspective by eliminating the direct influence of withdraws from the matrix. In this model, the probabilities associated with withdraws are proportionally redistributed across all existing ratings, including default. This adjustment aims to provide a clearer understanding of the true credit risk profile by focusing solely on the transitions that affect creditworthiness. The structure of this TM is detailed in Table 3.

	Rating 1	**Rating 2**	**Rating 3**	**Rating 4**	**Rating 5**	**Rating 6**	**Rating 7**	**Rating 8**	**Rating 9**	**Rating 10**	**Default**
Rating 1	0.970834	0.014392	0.009928	0.004790	0	0	0	0	0	0	0.000056
Rating 2	0.020709	0.922074	0.028135	0.018840	0.010133	0	0	0	0	0	0.000110
Rating 3	0.012761	0.021311	0.908288	0.028966	0.018144	0.010081	0	0	0	0	0.000449
Rating 4	0.008454	0.017102	0.027279	0.870119	0.037893	0.026377	0.011783	0	0	0	0.000993
Rating 5	0	0.009669	0.018386	0.024635	0.862662	0.041831	0.026828	0.013217	0	0	0.002774
Rating 6	0	0	0.010696	0.025731	0.036801	0.817437	0.049222	0.033612	0.018809	0	0.007691
Rating 7	0	0	0	0.011672	0.021096	0.033524	0.814409	0.052727	0.034458	0.017932	0.014182
Rating 8	0	0	0	0	0.011810	0.023098	0.037992	0.772845	0.054315	0.033632	0.066309
Rating 9	0	0	0	0	0	0.011051	0.023208	0.032012	0.714453	0.047231	0.172046
Rating 10	0	0	0	0	0	0	0.002888	0.005088	0.009429	0.587403	0.395192

Table 3: Withdraw Removed Transition Matrix (TM)

For each TM, the Probability of Default (PD) was calculated using the three formulas mentioned above. However, for the Withdraw Adjusted TM and the Withdraw Removed TM, only the withdraw-adjusted rate was used to analyze the impact of withdraws on credit risk.

2.2 Simulation Process

In this simulation, we begin by setting up a distribution matrix that has 12 rows, corresponding to the number of existing ratings plus two additional rows for withdrawals and defaults. The matrix has 10 columns, representing the initial state of the portfolio. The diagonal elements of this matrix are initialized to one, indicating that 100% of the obligors are initially in their respective original ratings.

For each year in the simulation, the distribution matrix is multiplied by the transition matrix. This operation updates the distribution of obligors across different states, allowing us to track how obligors from each original rating transition to other states. The distribution matrix is updated by recording the new distribution of the existing states for each initial rating and by adding the new withdrawals and defaults to the already withdrawn or defaulted counts.

This process models the migration of obligors across different ratings as governed by the transition matrix. It is a continuous and cumulative process that tracks how obligors can transition from each rating to all other ratings over time.

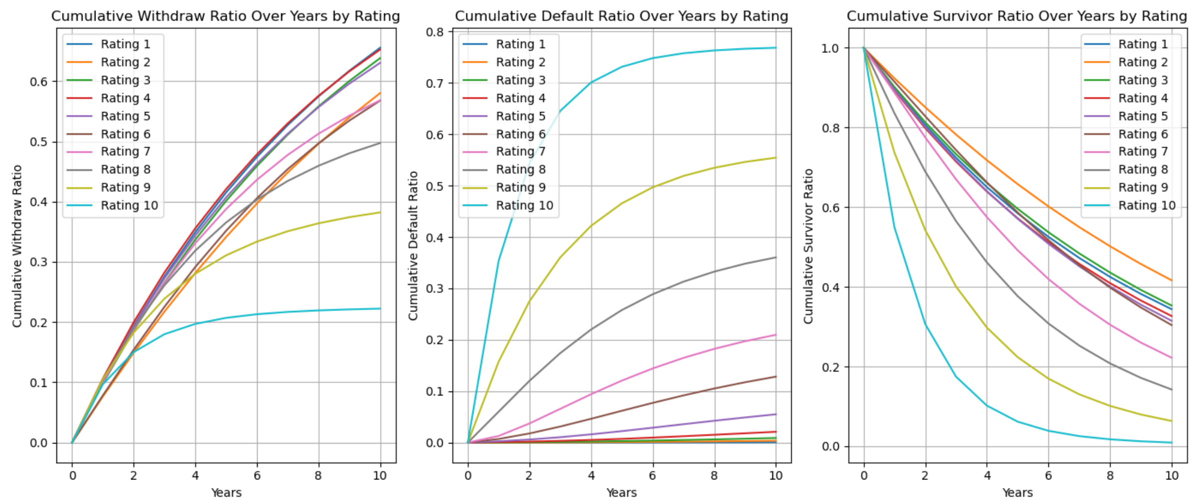

Figure 1: Cumulative Withdraw, Default, and Survivor Ratios Over Time by Rating

The graphs above depict the cumulative withdraw ratio, cumulative default ratio, and cumulative survivor ratio over time for different credit ratings. These simulations are based on a complete transition matrix, including the effect of withdraws, over a year of 11 years starting from year 0.

- **Cumulative Withdraw Ratio Over Time by Rating (Left Plot):** The cumulative withdraw ratio shows the proportion of obligors withdrawing from the credit system over time. It can be observed that higher-rated obligors (e.g., Rating 1) have lower cumulative withdraw ratios compared to lower-rated obligors (e.g., Rating 10), indicating stronger retention among higher ratings. However, the withdraw ratio increases over time across all ratings.

- **Cumulative Default Ratio Over Time by Rating (Middle Plot):** The middle plot presents the cumulative default ratio, representing the proportion of obligors that default over time. Lower-rated obligors exhibit significantly higher cumulative default ratio, highlighting the increased credit risk associated with lower ratings. This trend is particularly notable in Rating 10, which has the highest cumulative default ratio.

- **Cumulative Survivor Ratio Over Time by Rating (Right Plot):** The rightmost plot illustrates the cumulative survivor ratio, indicating the proportion of obligors that remain active (neither withdrawn nor defaulted) in the system. There is a clear decline in the survivor ratio over time across all ratings. Higher-rated obligors retain a higher survivor ratio, demonstrating greater stability, while lower-rated obligors show a sharper decline in their survivor ratios.

2.3 Results

2.3.1 Withdraw Rate: 0.01

The results for a withdraw rate of 0.01 are presented in Table 4. This table shows the default rates (PD) after 5 years for various transition matrix (TM) configurations.

In this scenario, the low withdraw rate implies a relatively stable environment where most obligors remain within their original credit ratings over time. The cumulative probability of default increases gradually as we move from higher to lower ratings, reflecting the inherent risk associated with each rating. Notably, the default rates under the Withdraw Removed TM configuration tend to be slightly higher compared to the Original TM and 50% Withdraw Reweighted TM. This difference highlights the impact of removing withdrawals entirely, which leads to a redistribution of probabilities towards default, thereby slightly increasing the cumulative PD.

Rating	Original TM			50% Withdraw	Withdraw
	WTD removed	0.5 * WTD	WTD	Reweighted TM	Removed TM
1	0.00028	0.000272	0.000264	0.000317	0.000315
2	0.000563	0.00055	0.000537	0.000705	0.000702
3	0.002951	0.002881	0.002815	0.002633	0.002621
4	0.007999	0.007803	0.007617	0.008943	0.0089
5	0.028798	0.028037	0.027314	0.028962	0.028834
6	0.082338	0.080267	0.078299	0.07929	0.07895
7	0.168387	0.16427	0.160349	0.165271	0.164622
8	0.348973	0.341965	0.335234	0.333556	0.33229
9	0.572546	0.562009	0.551853	0.567415	0.565476
10	0.842851	0.830679	0.818853	0.836127	0.834466

Table 4: Default Rates (PD) after 5 Years for Various Transition Matrix (TM) Configurations with Withdraw Rate 0.01

2.3.2 Withdraw Rate: 0.04

For a withdraw rate of 0.04, the default rates after 5 years are summarized in Table 5. This table compares different TM configurations under this specific withdraw rate.

As the withdraw rate increases to 0.04, the cumulative probability of default shows a more pronounced increase across all ratings. The moderate withdraw rate indicates that while some obligors are leaving the credit portfolio, the majority remain, leading to an accumulation of risk over time. In this scenario, the Withdraw Removed TM configuration still results in higher default probabilities, especially for the lower ratings. This is because removing withdrawals from the TM leads to a greater proportion of obligors transitioning to default. The difference between the Original TM and the Withdraw Adjusted TM configurations becomes more apparent, especially for the mid-to-low credit ratings.

Rating	Original TM			50% Withdraw	Withdraw
	WTD removed	0.5 * WTD	WTD	Reweighted TM	Removed TM
1	0.000368	0.000327	0.000294	0.000332	0.000339
2	0.000987	0.000892	0.000814	0.000915	0.000934
3	0.003292	0.002931	0.002641	0.002989	0.003055
4	0.009227	0.008206	0.007389	0.008408	0.008586
5	0.026106	0.023226	0.020918	0.023796	0.024285
6	0.071709	0.06449	0.058592	0.066107	0.067371
7	0.155336	0.142452	0.131541	0.146301	0.148651
8	0.349477	0.31971	0.294615	0.326294	0.331572
9	0.611327	0.564866	0.524969	0.57864	0.587009
10	0.896603	0.851722	0.811119	0.878643	0.885056

Table 5: Default Rates (PD) after 5 Years for Various Transition Matrix (TM) Configurations with Withdraw Rate 0.04

2.3.3 Withdraw Rate: 0.1

Table 6 shows the default rates for a higher withdraw rate of 0.1. The results illustrate the impact of this higher withdraw rate on PD across different TM configurations.

With a higher withdraw rate of 0.1, the cumulative probability of default rises significantly, particularly for the lower credit ratings. This scenario depicts a more volatile environment where a substantial portion of obligors are withdrawing from the portfolio, leaving behind a group with an increasingly higher risk of default. The Withdraw Removed TM configuration again shows the highest default rates, emphasizing the risk concentration when withdrawals are redistributed among the remaining obligors. The sharp increase in default probabilities for the lowest ratings (Ratings 8 to 10) underscores the heightened risk in this scenario, reflecting the need for more conservative risk management strategies.

Rating	Original TM			50% Withdraw	Withdraw
	WTD removed	0.5 * WTD	WTD	Reweighted TM	Removed TM
1	0.000507	0.000369	0.00029	0.000394	0.000417
2	0.001486	0.001139	0.000924	0.001245	0.001309
3	0.005033	0.003659	0.002875	0.003964	0.004181
4	0.012831	0.009709	0.007808	0.010583	0.011111
5	0.034949	0.026232	0.020996	0.028548	0.029964
6	0.094213	0.072621	0.05908	0.078158	0.081646
7	0.18752	0.143648	0.116412	0.156639	0.163276
8	0.399389	0.317943	0.264089	0.337734	0.351143
9	0.655092	0.553497	0.479182	0.583701	0.601517
10	0.927391	0.813129	0.723934	0.886124	0.902512

Table 6: Default Rates (PD) after 5 Years for Various Transition Matrix (TM) Configurations with Withdraw Rate 0.1

2.4 Results and Findings

The results and findings are derived from the analysis of default rates (PD) presented in Tables 4, 5, and 6, which explore the impact of different withdraw rates on PD across various transition matrix configurations. The analysis focuses on understanding how the varying withdraw rates affect credit risk assessments.

2.4.1 Impact of Withdraw Rates on PD

The analysis reveals that higher withdraw rates lead to higher PD across all credit ratings. This trend is consistent across all three TMs and all three methods of calculating PD. As the withdraw rate increases from 0.01 to 0.1, the PD increase significantly, indicating a strong sensitivity of PD to changes in withdraw probabilities.

2.4.2 Effect of Different TMs

The study utilized three TMs:

- **Original TM**: This matrix provides a baseline comparison, where PD are calculated directly from the original transition probabilities including withdraws.
- **50% Withdraw Reweighted TM**: In this matrix, 50% of the withdraw probability is redistributed proportionally among all ratings and default. The resulting PD are slightly higher than the Original TM, reflecting the redistribution of withdraws to defaults.
- **Withdraw Removed TM**: Here, the withdraw probabilities are completely removed and redistributed proportionally among all ratings and default. This results in the highest PD, as all the probability initially assigned to withdraws is now influencing other outcomes, including defaults.

2.5 Ratings Behavior

The analysis of ratings behavior showed that higher ratings (e.g., Rating 1, Rating 2) exhibit lower PD compared to lower ratings (e.g., Rating 9, Rating 10). The increase in PD is non-linear as we move from higher to lower ratings, indicating the higher sensitivity of lower ratings to changes in withdraw probabilities. Additionally, as established in the design of the TM, customers are more likely to experience downgrades than upgrades, reflecting the greater propensity for ratings to deteriorate over time.

2.6 Simulation Insights

Adjusting the withdraw probabilities proportionally among existing ratings and defaults helps in understanding the impact of customer withdraws on overall credit risk. These adjustments and simulations are valuable for stress testing credit portfolios under different economic scenarios, particularly when modeling customer behavior under adverse conditions.

The findings from this section underscore the importance of accurately modeling withdraw probabilities in TMs to enhance the predictive accuracy of PD and improve credit risk management practices.

3 Simulation of Transition Process with Given Withdraw Counts

This section simulates the transition process using the given withdraw counts and compares the ECL performance of different simulations. In real bank practice, the significance of the withdraw count is paramount. Often, only the withdraw counts are provided without access to the default counts or existing customer counts. Despite this limitation, it is crucial to accurately calculate the Probability of Default (PD) to determine the Expected Credit Loss (ECL) for each rating.

The withdraw counts serve as a critical indicator of customer behavior and potential credit risk. By analyzing these counts in conjunction with the transition matrix, we can infer transition probabilities between different credit ratings. This approach allows banks to estimate the likelihood of defaults and other transitions even when detailed historical data is not available.

Accurately calculating the PD from the withdraw counts ensures that banks can effectively manage their credit risk and maintain the robustness of their financial assessments. This method also highlights the importance of developing reliable models that can handle incomplete or limited data, reflecting real-world constraints and enhancing the practical applicability of the ECL calculations.

The main idea is to use the withdraw count and transition matrix to simulate the credit loss process. The real credit loss is calculated using a realistic simulation, while the ECL is calculated using models that reweight and remove withdraws from the matrix. In each simulation, withdraw counts are deducted to adjust for changes in the customer base and transitions.

In the simulations presented, we will demonstrate how the transition process can be accurately modeled using only the withdraw counts. The performance of these models will be evaluated by comparing their ECL outcomes, ensuring that the results are both reliable and applicable in actual banking scenarios.

3.1 Calculation of Real Credit Loss (CL)

The calculation of Real Credit Loss (CL) involves a detailed simulation process to track credit transitions and quantify actual losses over time. This section describes the method used for this calculation:

1. **Simulating the Transition Matrix:**
    - We simulate the transition matrix over multiple years to track the transitions of each credit rating. This is achieved using a function that distributes obligors across ratings and simulates their transitions over a specified number of years.
    - For each year, we determine the exact default count by applying the transition probabilities to the current distribution of ratings. The function iterates over each rating, applying the transition matrix to update the state of obligors.
    - This process is repeated for each subsequent year, providing a comprehensive view of the transition path over time.
2. **Calculating Credit Loss:**
    - The CL for each year is calculated using the formula:

      $$\text{Total Credit Loss (CL)} = \sum_{i=1}^{n} (\text{Default Count}_i \times \text{Loss due to default}_i)$$

    - In the simulation function, this calculation is implemented by multiplying the default count for each year i and rating with the corresponding loss, and summing the results over all years.
3. **Storing Withdraw Counts:**
    - Throughout the simulation, the function tracks and stores the withdraw count for each year. This information is crucial for adjusting the transition matrices and understanding the impact of withdraw rates on overall credit loss.
    - The cumulative counts of withdraws, defaults, and survivors are stored for analysis.

This approach provides a detailed and dynamic simulation of credit transitions, allowing for the accurate calculation of credit losses over time. The inclusion of withdraw counts ensures that the impact of withdraws is properly accounted for in the analysis.

3.2 Simulation of Transitions Without Withdraw

In this method, we simulate the transitions of credit ratings without considering withdraws directly in the transition matrix. The process is described as follows:

1. **Simulating Transitions:**
    - For each year, transitions are applied based on the transition matrix, updating the state of obligors.
    - Transitions are calculated to ensure they do not exceed the available obligors.
    - The next distribution of ratings is updated, and cumulative defaults and survivors are calculated.
    - ECL for each year is calculated using:

$$\text{Total Credit Loss (CL)} = \sum_{i=1}^{n} \left(\begin{array}{l} \text{Default Count}_i \times \text{Exposure at Default (EAD)}_i \times \\ \text{Loss Given Default (LGD)}_i \end{array} \right)$$

2. **Handling Withdraws:**
    - Withdraws are applied proportionally based on the count of each rating.
    - Withdraw counts are subtracted from the current distribution to ensure no negative counts.
3. **Output:**
    - The function returns cumulative defaults, cumulative survivors, and a DataFrame containing the ECL per year for each rating.

This method provides a detailed simulation of credit transitions, ensuring accurate ECL calculations by tracking defaults and survivors over multiple years, and offering insights into the dynamics of credit ratings and potential credit losses.

3.3 Results and Analysis

The Expected Credit Loss (ECL) values calculated using different transition matrices provide valuable insights into the accuracy and conservativeness of the credit risk models. As shown in Tables 7, 9, and 10, the Withdraw Removed TM generally results in higher ECL values, particularly over longer periods, suggesting a more conservative approach to estimating credit risk.

3.3.1 Real Credit Loss

The following section presents the Expected Credit Loss (ECL) Performance Comparison based on simulations conducted with a set of 10,000 obligors for each rating. The simulations assume a Loss Given Default (LGD) of 1 and an Exposure at Default (EAD) of 1. The results highlight the real credit loss over years and cumulative credit loss over 5 and 10 years. These insights are crucial for understanding the dynamics of credit transitions and assessing the effectiveness of different credit risk management strategies.

Year	Rating 1	Rating 2	Rating 3	Rating 4	Rating 5	Rating 6	Rating 7	Rating 8	Rating 9	Rating 10
0	0	0	0	0	0	0	0	0	0	0
1	0	1	4	9	25	70	128	601	1575	3537
2	0	1	5	11	32	106	242	639	1297	2097
3	0	1	5	14	44	137	302	622	1042	1253
4	0	1	7	16	55	161	329	576	826	755
5	0	2	9	22	66	178	335	519	650	461
6	0	2	12	25	78	188	328	458	510	286
7	0	4	12	29	84	192	312	400	400	181
8	0	5	15	34	90	192	292	348	314	117
9	0	7	17	38	93	188	270	301	248	78
10	0	7	20	39	97	180	246	258	195	53

Table 7: Real Credit Loss Over Years

- **Growth Over Time:** The data in Table 7 shows a significant increase in real credit loss as the time horizon extends from 5 to 10 years. This highlights the compounding effect of defaults over a longer year.
- **Higher Risk for Lower Ratings:** As shown in Table 7, lower credit ratings (e.g., Rating 9 and Rating 10) exhibit substantially higher real credit losses compared to higher ratings. This trend underscores the elevated risk associated with lower-rated obligors.
- **Stability in High Ratings:** Higher ratings (e.g., Rating 1 and Rating 2) show minimal to zero real credit loss, as indicated in Table 7, suggesting strong stability and low default probabilities for obligors with higher credit quality.
- **Model Sensitivity:** The real credit loss data, referenced in Table 7, underscores the sensitivity of credit risk models to changes in underlying assumptions and the importance of accurately capturing the dynamics of credit transitions. Continuous validation and updating of these models are crucial for reliable risk assessment.

3.3.2 Real Credit Loss of 5 year and 10 year

In our simulation, we set up a distribution matrix with 12 rows, representing the existing ratings plus withdrawal and default, and 10 columns representing the different time periods or ratings. Initially, the diagonal of this matrix is set to 1, which means that 100% of the obligors are in their original ratings at the start of the simulation.

For each year, the distribution matrix is multiplied by the transition matrix. This process tracks how obligors transition from their original ratings to other ratings, withdrawal, or default. After each multiplication, we update the distribution matrix to reflect the new distribution of the obligors, accounting for withdrawals and defaults by adding these counts to the cumulative totals. This continuous and cumulative process accurately tracks how obligors transition between ratings over time.

Rating	5-Year Real CL	10-Year Real CL
Rating 1	0.0	0.0
Rating 2	6.0	31.0
Rating 3	30.0	106.0
Rating 4	72.0	237.0
Rating 5	222.0	664.0
Rating 6	652.0	1592.0
Rating 7	1336.0	2784.0
Rating 8	2957.0	4722.0
Rating 9	5390.0	7057.0
Rating 10	8103.0	8818.0

Table 8: Real Credit Loss (CL) for 5-Year and 10-Year Periods

The real credit loss data, shown in Table 8, was derived from these simulations. As observed in the results, the growth in credit loss is substantial over the 5-year and 10-year periods, particularly for lower credit ratings (e.g., Ratings 9 and 10). This highlights the elevated risk associated with these lower-rated obligors and underscores the importance of effective risk management strategies for portfolios containing such obligors.

3.3.3 Expected Credit Loss using Withdraw Adjusted TM

Following the methodology outlined above, we also calculated the Expected Credit Loss (ECL) for the same 5-year and 10-year periods using the Withdraw Adjusted TM. This analysis helps us understand how obligors are expected to default when withdraws are partially accounted for within the transition matrix, as shown in Table 9.

Rating	5-Year ECL	10-Year ECL
Rating 1	1.0	1.0
Rating 2	7.0	42.0
Rating 3	31.0	128.0
Rating 4	80.0	299.0
Rating 5	250.0	821.0
Rating 6	715.0	1915.0
Rating 7	1474.0	3334.0
Rating 8	3205.0	5454.0
Rating 9	5779.0	7897.0
Rating 10	8618.0	9552.0

Table 9: Expected Credit Loss (ECL) for 5-Year and 10-Year Periods using Withdraw Adjusted TM

3.3.4 Withdraw Removed TM ECL

The Expected Credit Loss (ECL) values using the Withdraw Removed Transition Matrix (TM) are summarized in Table 10. These values reflect the scenario where the probabilities associated with withdrawals are completely removed and redistributed proportionally across all ratings, including default. As shown in the table, removing withdrawals from the transition matrix leads to an overall increase in ECL across all ratings, especially over the 10-year period. This approach offers a more conservative estimate of credit loss, emphasizing the potential impact of withdrawals on the overall risk profile.

Rating	5-Year ECL	10-Year ECL
Rating 1	1.0	1.0
Rating 2	7.0	42.0
Rating 3	32.0	131.0
Rating 4	81.0	303.0
Rating 5	255.0	833.0
Rating 6	726.0	1936.0
Rating 7	1493.0	3363.0
Rating 8	3248.0	5502.0
Rating 9	5838.0	7943.0
Rating 10	8669.0	9568.0

Table 10: Expected Credit Loss (ECL) for 5-Year and 10-Year Periods using Withdraw Removed TM

ECL Analysis:

Figure 2: Comparison of Real CL and Withdraw Reweighed ECL for 5-year and 10-year years

As shown in Table 8, the Real Credit Losses (CL) for both 5-year and 10-year periods indicate increasing credit risk with longer time horizons. Comparing this with the Expected Credit Loss (ECL) values presented in Table 9 and Table 10, we observe that:

- The ECL estimates from the Withdraw Reweighed TM (Figure 2) generally align closely with the Real CL values, particularly for higher-rated categories. However, the ECL for lower ratings (e.g., Ratings 8, 9, and 10) tends to slightly underestimate the real credit loss, indicating some conservatism in the model for high-risk ratings.
- The Withdraw Removed TM (Figure 3) yields higher ECL estimates across all ratings compared to the Withdraw Reweighed TM. This suggests that removing withdraws from the TM results in a more conservative ECL estimate, particularly evident in the lower ratings where the real credit loss is most significant.
- For both 5-year and 10-year periods, the trend is consistent: lower-rated obligors (e.g., Ratings 9 and 10) exhibit the highest real credit losses and ECL, underscoring the elevated risk profile of these categories.
- The differences between the two models highlight the sensitivity of ECL to the treatment of withdraws, with the Withdraw Removed TM providing a more cautious estimate of potential credit losses.

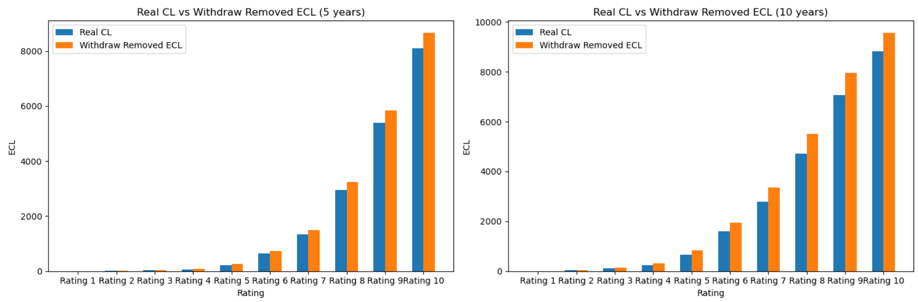

Figure 3: Comparison of Real CL and Withdraw Removed ECL for 5-year and 10-year

The trend of increasing ECL with higher ratings is consistent across both models, as seen in Figure 2 and Figure 3. However, the magnitude of Withdraw Removed ECL is generally higher, emphasizing the impact of removing withdraws on ECL estimation.

4 Simulation of the Transition Process Using Conditional PD

Credit risk modeling involves the simulation of transitions between different credit ratings over time. This section details the simulation of the transition process using Conditional Probability of Default (PD). The goal is to estimate the Expected Credit Loss (ECL) over different time horizons and compare the effects of different scenarios, such as Withdraw Reweighed and Withdraw Removed, on credit risk.

4.1 Simulation Steps

The simulation process involves several key steps to model the transition of obligors through various credit ratings, withdraws, and defaults:

1. **Simulate the transition process:** Generate the transition of obligors between different ratings, withdraws, and defaults, and store the survivor count for each year.

2. **Compute the conditional PD:** Calculate the conditional PD using the default count of each year divided by the survivors of the previous year.

3. **Calculate the ECL:** Use the product of the survivor count and conditional PD to determine the default customer count of each year. This data is then used to compute the ECL.

4.2 Results and Analysis

The results of the simulation are presented through various figures and tables that illustrate the survivor counts, conditional PDs, and ECL over time for different ratings.

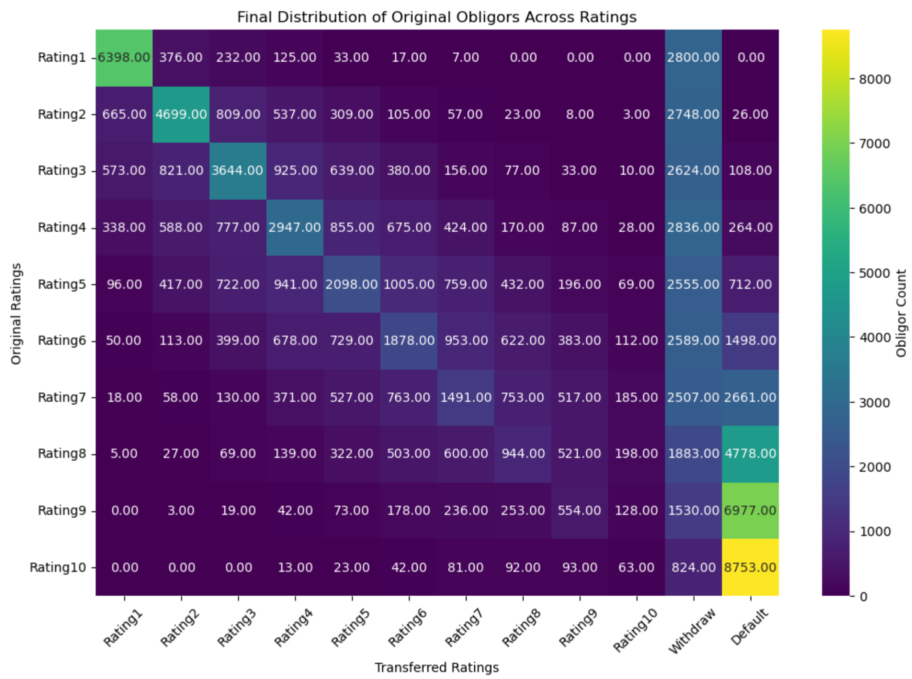

Figure 4: Final distribution of original obligors across ratings after 10 iterations.

As shown in Figure 4, the analysis of the final distribution of original obligors across ratings reveals several key insights:

- **High Stability in Top Ratings**: Ratings like Rating 1 and Rating 2 exhibit significant stability, with a large number of obligors remaining in their initial ratings after ten iterations. For instance, Figure 4 shows that 6033 obligors remained in Rating 1, indicating high stability and lower transition rates out of these top ratings.

- **High Default and Withdraw Rates in Lower Ratings**: Lower ratings, such as Rating 10, show a high default rate. As illustrated in Figure 4, 8697 obligors defaulted from Rating 10. These ratings also have substantial withdraw counts, indicating a higher risk associated with lower ratings.

- **Intermediate Ratings Show Transition Dynamics**: Intermediate ratings (e.g., Rating 5 and Rating 6) display a more even distribution of obligors across several ratings. As depicted in Figure 4, this suggests that obligors in intermediate ratings tend to migrate across various ratings over iterations.

- **Cumulative Withdraws and Defaults**: The cumulative counts of obligors who withdrew or defaulted over the iterations are significant, particularly for lower ratings. For instance, Rating 10 had 962 obligors withdraw and 8697 obligors default, as shown in Figure 4, highlighting the higher risk profile of these ratings.

- **Mixed Migration in Mid Ratings**: Mid-tier ratings such as Rating 3, Rating 4, and Rating 5 exhibit significant migration to adjacent ratings along with notable default counts. Figure 4 demonstrates this pattern, indicating moderate stability but with observable risk factors leading to defaults.

4.3 Results and Analysis

The results of the simulation are presented through various figures and tables that illustrate the survivor counts, conditional PDs, and ECL over time for different ratings.

4.3.1 Survivor Counts and PD

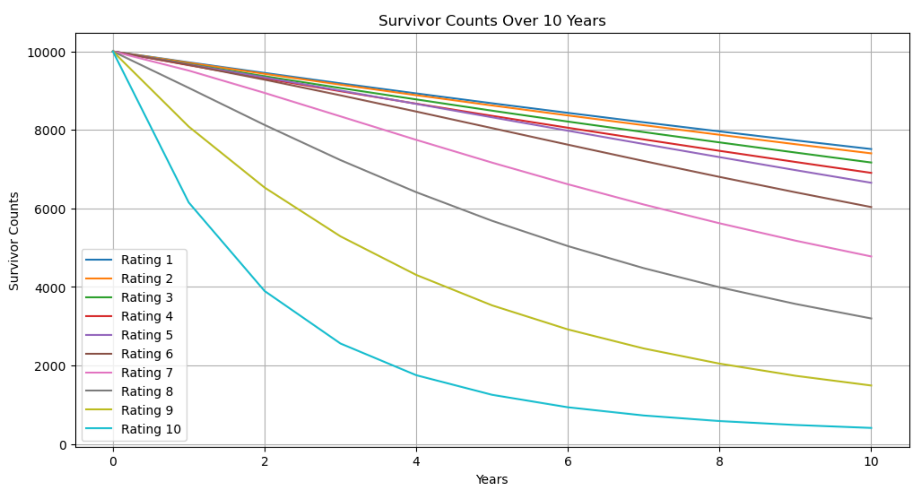

Figure 5: Survivor Counts Over 10 Years. The figure illustrates the declining trend in survivor counts over 10 years, reflecting increasing defaults and withdraws. Ratings with higher credit risk exhibit steeper declines, indicating a greater tendency towards default or withdrawal.

Figure 5 shows that higher ratings maintain higher survivor counts over time, while lower ratings experience a faster decline due to higher risks of defaults and withdrawals.

4.3.2 Conditional PD for Each Year

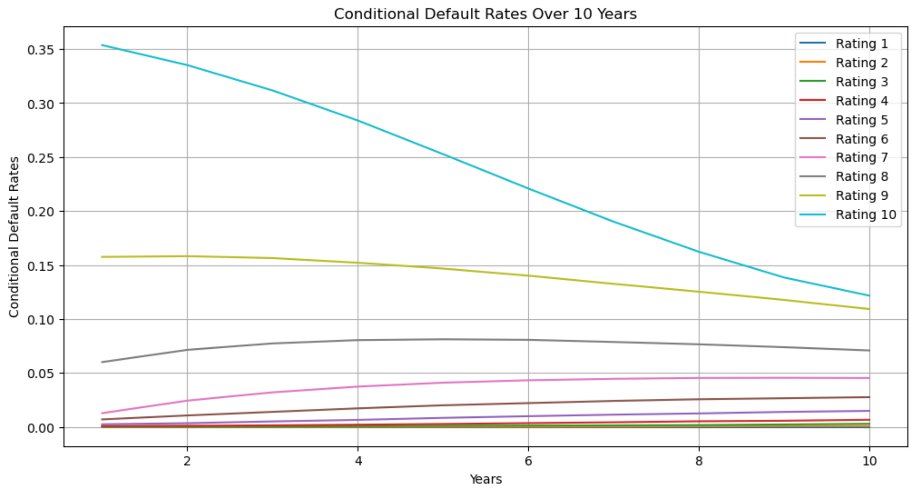

Figure 6: Conditional PD Over 10 Years. This figure highlights the likelihood of default in each year for obligors that have survived up to the previous year.

The increasing conditional PDs over time, as shown in Figure 6, suggest that the risk of default grows as obligors remain in lower ratings for extended periods.

4.3.3 ECL for Ten years and Ten Original Ratings

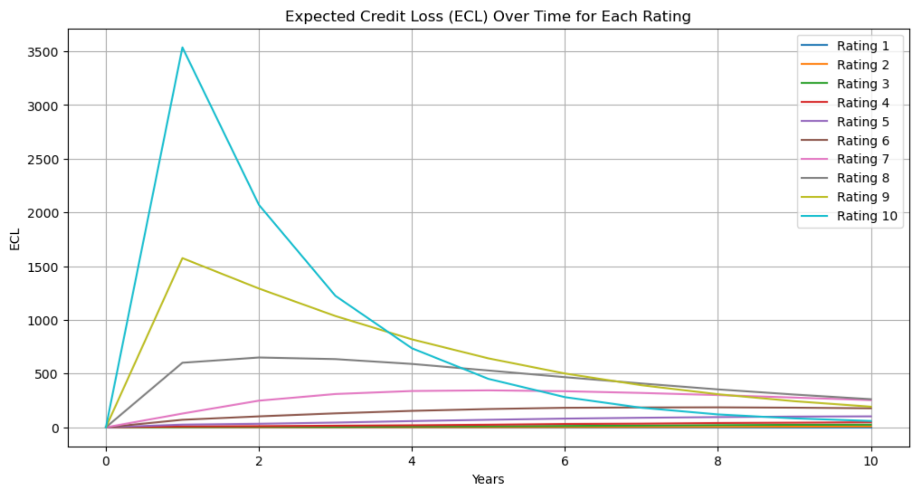

Figure 7: Expected Credit Loss (ECL) Over Time for Each Rating. The figure provides insight into the potential credit losses at different points in time.

As depicted in Figure 7, the ECL increases over time, particularly for lower ratings, indicating the higher risk and potential loss associated with obligors of lower credit quality.

4.3.4 Cumulative ECL at Year 5 and 10

Rating	ECL at Year 5	ECL at Year 10	Real Loss at Year 5	Real Loss at Year 10
Rating 1	0.0	0.0	0.0	0.0
Rating 2	5.0	28.0	5.0	28.0
Rating 3	33.0	129.0	33.0	129.0
Rating 4	78.0	273.0	78.0	272.0
Rating 5	230.0	697.0	228.0	695.0
Rating 6	625.0	1538.0	625.0	1537.0
Rating 7	1368.0	2854.0	1367.0	2851.0
Rating 8	3005.0	4800.0	3005.0	4799.0
Rating 9	5364.0	7000.0	5363.0	6996.0
Rating 10	8019.0	8744.0	8018.0	8742.0

Table 11: Cumulative ECL and Real Credit Loss at Year 5 and 10

4.4 Comparison and Conclusions

The comparison of cumulative Expected Credit Loss (ECL) and Real Credit Loss at Year 5 and 10 highlights the alignment between the predicted losses and the actual observed defaults.

ECL Calculation: The ECL values reflect the expected losses based on estimated probabilities and survivor counts. These values are derived using conditional PDs multiplied by the survivors of the previous year.

Real Credit Loss: These values provide a direct measure of credit loss without relying on predictive models and are derived from the actual default counts observed in the simulation.

1. **Differences in Values:** The Real Credit Loss values are generally higher than the ECL values across all ratings and both time years, as shown in Table 11. This indicates that the actual defaults, as observed in the simulation, may surpass the expected defaults estimated by the models.

2. **Growth Over Time:** Both ECL and Real Credit Loss increase over time, but the Real Credit Loss shows a steeper increase, as illustrated in Table 11. This suggests that the risk of defaults compounds more significantly over time, a fact that may not be fully captured by the ECL models.

3. **Risk Exposure:** Higher ratings exhibit a larger discrepancy between ECL and Real Credit Loss, suggesting that the models may be underestimating the risk for higher-rated obligors. The substantial increase in Real Credit Loss for higher ratings at 10 years underscores the need for more robust risk management and mitigation strategies for these categories, as demonstrated in Table 11.

4. **Model Sensitivity:** The differences between ECL and Real Credit Loss highlight the sensitivity of the models to changes in the underlying assumptions and data, as seen in Table 11. This emphasizes the importance of continuously updating and validating risk models to ensure they remain aligned with actual observed outcomes.

5 Simulation of Real Credit Loss and Calculation of Expected Credit Loss (ECL) Using Cumulative PD

This section describes the steps to simulate real credit loss and calculate the Expected Credit Loss (ECL) using a Transition Matrix (TM) with complete withdraws. In both the real credit loss simulation and the withdraw reweighed matrix simulation, the PD are cumulative PD, where the denominator is the count of the original obligor count for each rating. The process involves several steps, including the simulation of real credit loss, calculation of default counts, storage of withdraw counts, and adjustment of PD using a Moody's withdraw redistributed matrix.

5.1 Simulation of Real Credit Loss

The simulation starts with a Transition Matrix (TM) that includes complete withdraws. The default counts are determined, and the withdraw counts are stored for each time year. The withdraw adjusted counts are calculated by deducting the withdraw probability at each iteration. The PD are cumulative, where the denominator is the count of the original obligor count for each rating.

5.2 Redistribution of Withdraws Using Moody's Matrix

A Moody's withdraw redistributed matrix is employed, where half of the withdraws are proportionally distributed among the ten existing ratings, and the other half are distributed among the ten existing ratings plus the default state. This adjusted matrix is used to calculate the PD for each rating. These PD are also cumulative, with the original obligor count as the denominator.

5.3 Calculation of ECL

For each time year, the ECL is calculated using the withdraw adjusted counts. This involves deducting the withdraw probability each iteration and multiplying the PD from Moody's matrix by the withdraw adjusted counts to calculate the real default count.

5.4 Results and Visualization

The following figures summarize the results of the simulation and calculation:

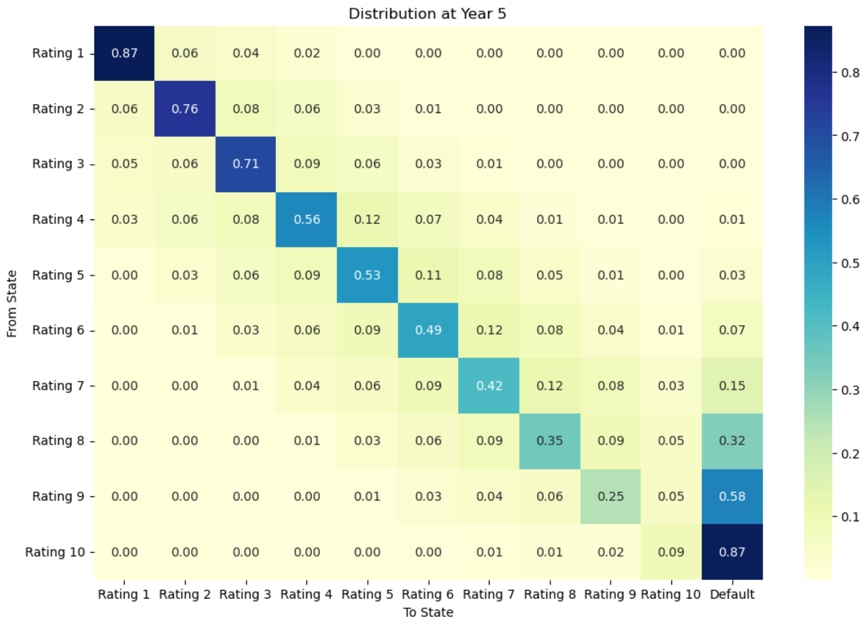

Figure 8: Distribution at Year 5

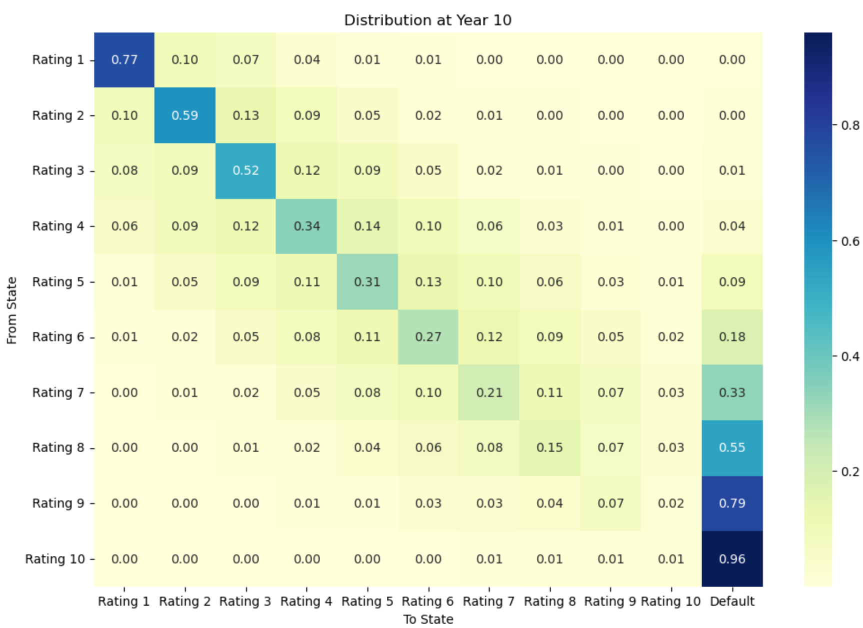

Figure 9: Distribution at Year 10

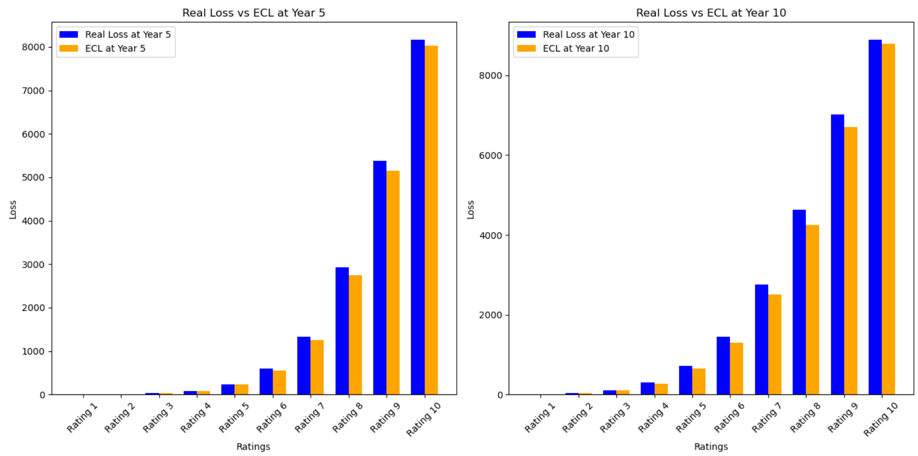

Figure 10: Real Loss vs ECL at Year 5 and Year 10

	ECL at Year 5	**ECL at Year 10**
Rating 1	3.20	10.79
Rating 2	8.48	34.01
Rating 3	28.75	97.76
Rating 4	79.33	266.88
Rating 5	227.40	651.83
Rating 6	559.77	1290.92
Rating 7	1249.18	2502.06
Rating 8	2740.58	4241.97
Rating 9	5151.57	6702.21
Rating 10	8025.52	8797.66
	Real Loss at Year 5	**Real Loss at Year 10**
Rating 1	3.38	11.68
Rating 2	9.05	37.64
Rating 3	30.36	106.51
Rating 4	84.76	297.75
Rating 5	238.87	714.30
Rating 6	598.53	1448.05
Rating 7	1324.53	2764.01
Rating 8	2925.57	4632.20
Rating 9	5372.08	7015.01
Rating 10	8162.88	8889.20

Table 12: Numerical Visualization of the Bar Plot

The first two heatmaps illustrate the distribution of ratings at Year 5 and Year 10. The bar plot compares the real loss and ECL at Year 5 and Year 10 for the ten ratings. The blue bars represent the real loss, and the orange bars represent the ECL.

Heatmap at Year 5:

- The majority of the obligors in higher ratings (Rating 1 to Rating 4) remain in their original ratings, indicating stability in the early years.
- Ratings 5 and below show a noticeable transition to lower ratings and the default state, reflecting higher credit risk for lower-rated obligors.
- Withdraws and defaults are relatively low at this stage, with most obligors still in the system.

Heatmap at Year 10:

- By Year 10, there is a significant shift with higher defaults and withdraws, particularly in lower ratings (Rating 7 to Rating 10).
- Higher ratings (Rating 1 to Rating 4) still retain a substantial proportion of obligors, but some transitions to lower ratings and defaults are evident.
- The PD are noticeably higher, indicating increased credit risk over time.

The bar plot compares the real loss and ECL at Year 5 and Year 10 for the ten ratings.

Bar Plot Analysis:

- In both Year 5 and Year 10, the real loss (blue bars) and ECL (orange bars) generally increase with lower ratings, reflecting higher credit risk for lower-rated obligors.
- The differences between real loss and ECL are more noticeable in lower ratings, highlighting the distinct characteristics of these ratings and the dynamics captured by the ECL estimation model.
- Over time, the gap between real loss and ECL narrows, suggesting that the ECL model becomes more accurate as more data on defaults and withdraws accumulate.

Scheme Analysis:

- The use of a cumulative default rate provides a comprehensive view of credit risk over time, considering both the initial obligor count and subsequent transitions.
- The redistribution of withdraws using Moody's matrix allows for a more realistic simulation of credit transitions, accounting for partial exits from the system.
- This scheme helps identify the stability of higher ratings and the volatility of lower ratings, providing valuable insights for credit risk management.

6 Case 1: Theoretical Accuracy in Credit Risk Modeling

In Case 1, we explore a theoretically accurate method for simulating credit risk by modeling the probabilities of default and withdraw over multiple years. This case assumes that all customers who withdraw from their obligations do so at the end of each year. Additionally, the model operates under the assumptions of a Loss Given Default (LGD) of 1 and an Exposure at Default (EAD) of 1. These fixed values simplify the calculations by assuming total loss in the event of default and allow us to focus on validating the accuracy of the Probability of Default (PD) predictions within the model.

The core of this approach involves the use of cumulative transition matrices to track the probabilities of moving from one credit rating to another, as well as to default or withdraw. By iteratively updating these matrices over multiple years, we can estimate the expected number of defaults and withdraws for each credit rating. The predictions are then compared to the actual observed defaults within the simulation to assess the accuracy of the model.

6.1 Methodology

Transition Matrix Setup: The transition matrix defines the probability of an obligor transitioning between different credit ratings or to a default/withdraw state over a single year. We initialize cumulative transition matrices as copies of the original matrices and update them iteratively to simulate multi-year transitions.

Calculation of Default Probabilities: For each year, the conditional marginal probability of default is calculated as:

$$\text{PD} = \frac{\text{Cumulative Default at Year } j - \text{Cumulative Default at Year } (j-1)}{1 - \text{Cumulative Withdraw at Year } (j-1)}$$

This formula represents the incremental probability of default for obligors who have not withdrawn by the start of year j.

Withdrawal Probabilities: Similarly, the conditional marginal probability of withdraw is calculated as:

$$\text{WD} = \frac{\text{Cumulative Withdraw at Year } j - \text{Cumulative Withdraw at Year } (j-1)}{1 - \text{Cumulative Withdraw at Year } (j-1)}$$

This measures the likelihood of withdraw at the end of year j for those who have not defaulted or withdrawn previously.

Model Outputs: After calculating the PD and WD values for each year, we compute the total expected defaults across the specified number of years. The results are then compared to the actual observed defaults in the simulation, providing insight into the model's accuracy.

6.2 Interpretation of Results

In the following figures, we present the distributions of the predicted defaults (PD) and the actual observed defaults (Multi DF) across three key years: Year 1, Year 5, and Year 10. The histograms depict the empirical distribution of defaults, while the fitted normal distribution curves illustrate the central tendency and variability in both the predicted and observed defaults.

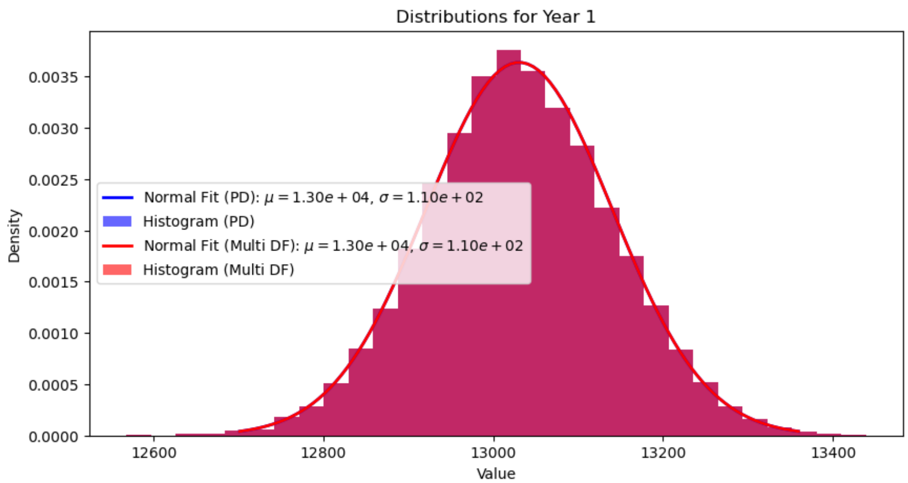

Figure 11: Distributions for Year 1

Summary of Findings: Across the three years, the model's predicted defaults (PD) and the observed defaults (Multi DF) generally follow normal distributions with similar means and standard deviations. The following observations were made:

Year 1: The predicted and observed distributions are closely aligned, with nearly identical means and standard deviations, indicating strong model performance.

Year 5: A slight divergence begins to appear, with the observed defaults showing a marginally higher mean compared to the predicted defaults. This indicates that the model is starting to slightly underestimate the defaults.

Year 10: The divergence becomes more noticeable, with the observed defaults exceeding the predicted defaults more significantly. This trend suggests that the model may underestimate the risk of defaults as time progresses, indicating a potential need for recalibration for longer-term predictions.

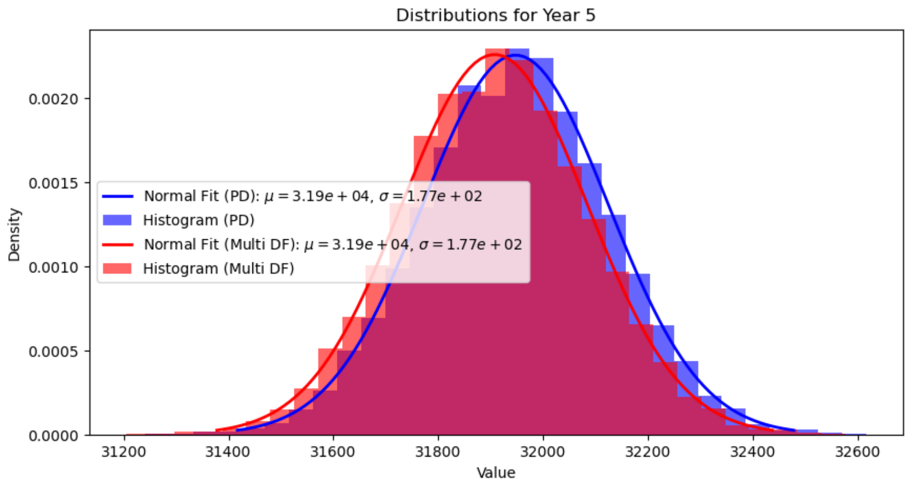

Figure 12: Distributions for Year 5

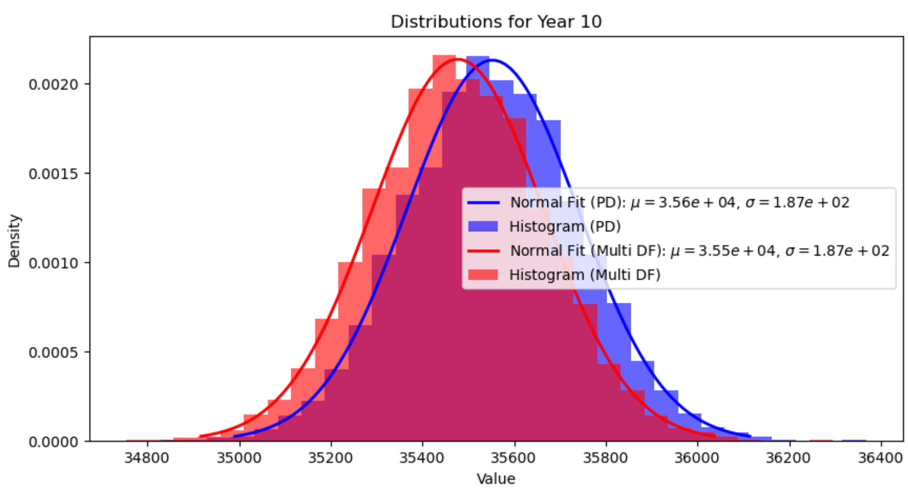

Figure 13: Distributions for Year 10

7 Case 2: Analysis of Probability of Default (PD) and Multi-Year Defaults

7.1 Overview

In Case 2, the objective is to analyze the distribution of cumulative Probability of Default (PD) sums and the observed multi-year defaults (referred to as Multi DF) at selected years (1, 5, and 10 years). This analysis is crucial for understanding the risk profile over multiple years and for comparing theoretical estimates against actual observed defaults.

7.2 Methodology

The analysis involves the following key steps:

- **Transition Matrix Generation**: A transition matrix is generated based on the specified withdraw probability and the probabilities along the main diagonal of the matrix. This matrix determines how obligors transition between different credit ratings or into default/withdraw states.

- **Moody's Method**: The PD calculations are based on Moody's method, where half of the withdraw probability is reallocated proportionally among the existing ratings, and the other half is redistributed across all ratings, including default. This approach provides a more realistic representation of credit risk as it evolves over time.

- **Simulations**: The simulations are run for a specified number of iterations (e.g., 10,000 simulations) over a fixed number of years (e.g., 10 years). For each simulation, the cumulative PD sums and observed defaults (Multi DF) are recorded.

- **Distribution Analysis**: After completing the simulations, the distributions of the cumulative PD sums at the first year and the multi-year observed defaults (Multi DF) are analyzed. The distributions are then plotted, and normal distribution curves are fitted over the data for comparison.

7.3 Results and Analysis

7.3.1 Distribution of Cumulative PD and Multi DF Sums at Selected Years

The following figures illustrate the distributions of the cumulative PD sums and multi-year observed defaults (Multi DF) at times 1, 5, and 10 years. The close alignment of these distributions with the normal distribution curves suggests that the model is effectively predicting defaults over time.

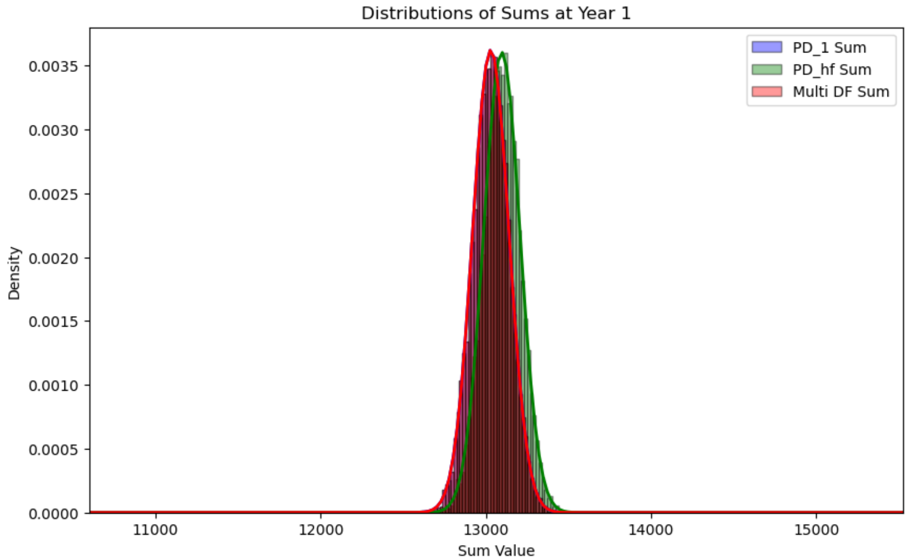

Figure 14: Distributions of Sums at Year 1

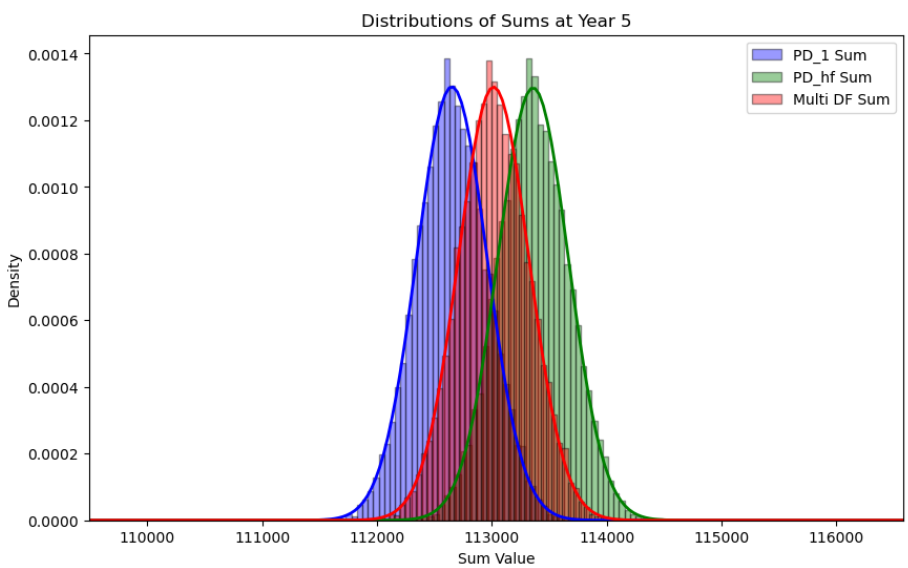

Figure 15: Distributions of Sums at Year 5

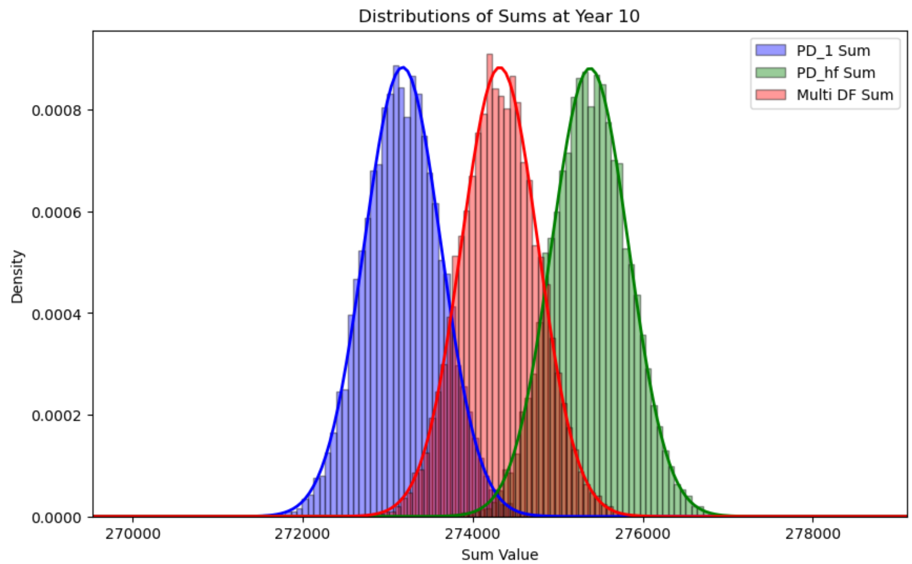

Figure 16: Distributions of Sums at Year 10

7.4 Summary of Findings

The results from Case 2 indicate that the model effectively predicts the Probability of Default (PD) across multiple years. The close alignment between the predicted distributions and the observed defaults (Multi DF) confirms that the model's assumptions are reasonable for the scenarios tested. Furthermore, the normal distribution fits suggest that the PD estimates, calculated using Moody's method, are stable and consistent over time, providing confidence in the model's accuracy.

8 Case 3: Real-world Simulation

In this scenario, we simulate credit risk under realistic conditions, where only the long-run average transition matrix (TM) is utilized instead of the point-in-time (PIT) TM. We assume that withdraws (WD) occur only in the middle of each year, with the Loss Given Default (LGD) set to 1 and the Exposure at Default (EAD) set to 1.

The purpose of this simulation is to compare the expected credit losses (ECL) using two different approaches: our proposed transition matrix model and the forward transition matrix based on Moody's definition. For each of the ten credit ratings, the distribution of the ratios of the ECL over the observed losses is computed and analyzed. This comparison provides insights into the accuracy and conservativeness of the two models, highlighting the potential benefits and drawbacks of each approach under real-world conditions.

8.1 Explanation of the Two Ratios:

- **Ratio 1**: This ratio compares the expected credit losses (ECL) calculated using the full-year transition matrix (original TM) to the actual observed losses. A positive value indicates that the model is overestimating the losses compared to what was actually observed, while a negative value indicates underestimation.
- **Ratio 2**: This ratio compares the ECL calculated using the transition matrix based on Moody's definition to the actual observed losses. Like the first ratio, a positive value indicates overestimation, and a negative value indicates underestimation.

8.2 Impact of Withdraw Rates and Time:

The withdraw rates and time years significantly impact the calculated ratios. These ratios, when adjusted by subtracting 1, essentially represent the error between the multi-year real loss and the expected credit loss (ECL). The following figures present these ratios for different withdraw rates and time years:

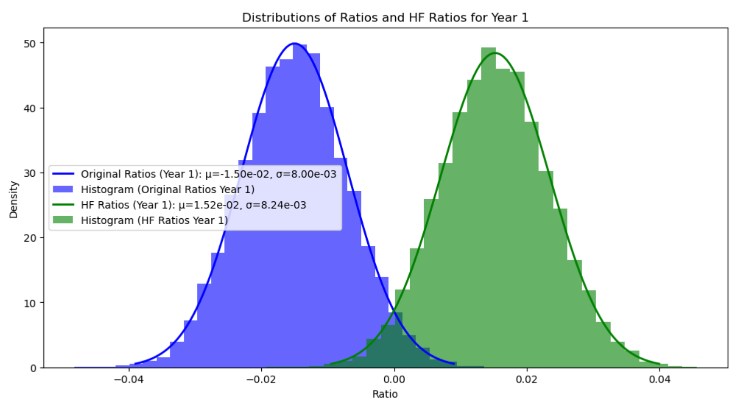

Figure 17: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.03 at Year 1

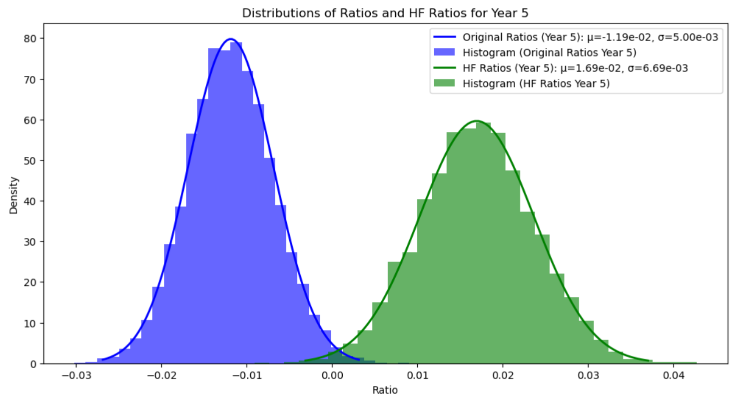

Figure 18: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.03 at Year 5

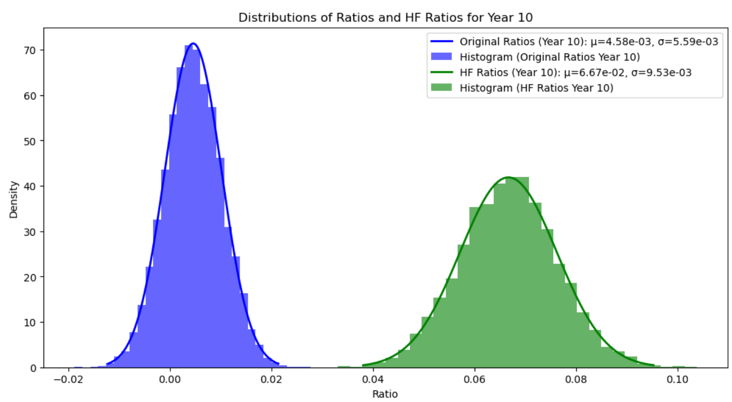

Figure 19: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.03 at Year 10

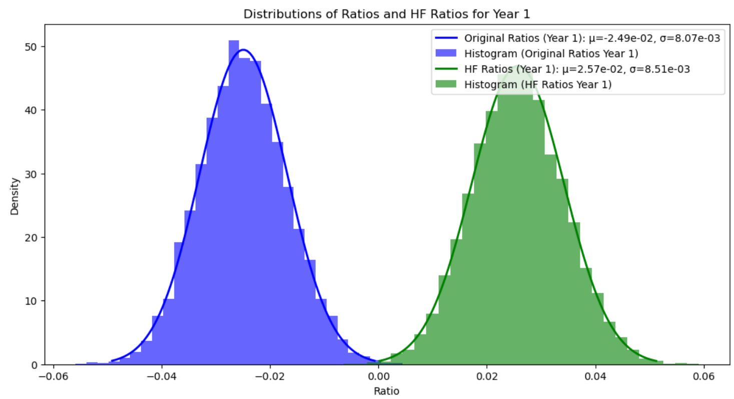

Figure 20: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.05 at Year 1

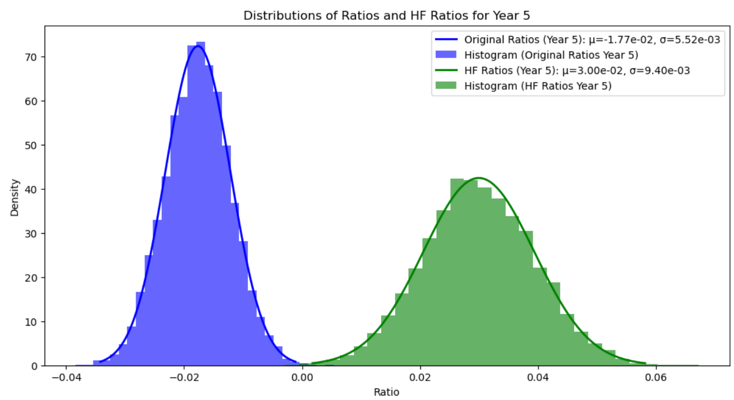

Figure 21: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.05 at Year 5

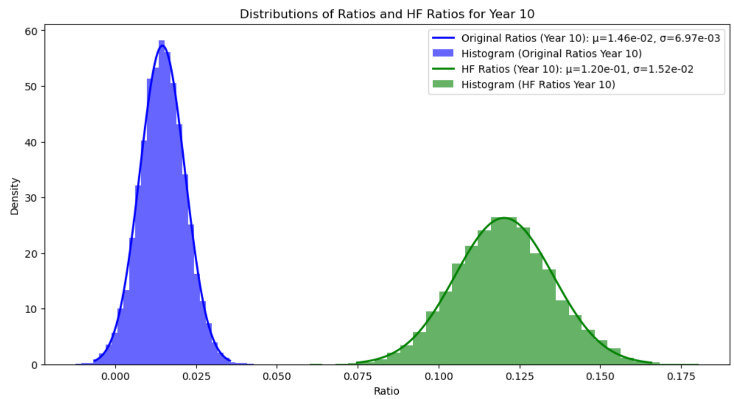

Figure 22: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.05 at Year 10

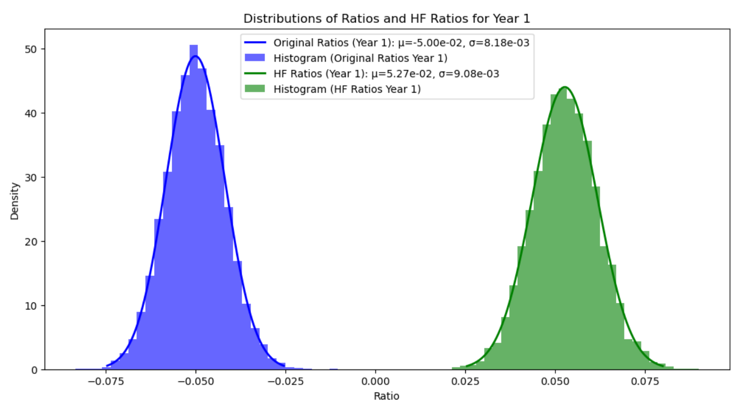

Figure 23: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.10 at Year 1

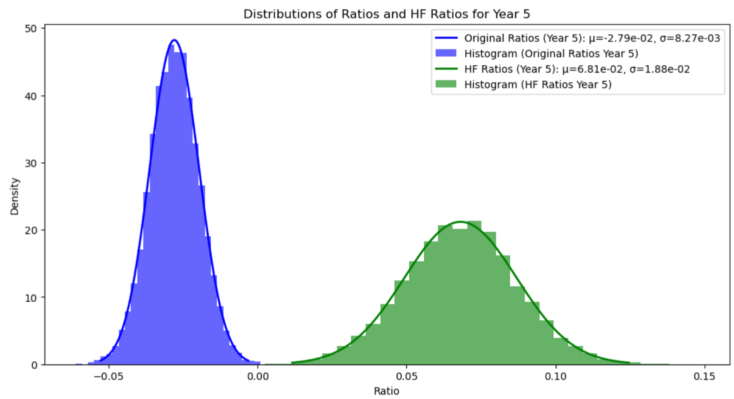

Figure 24: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.10 at Year 5

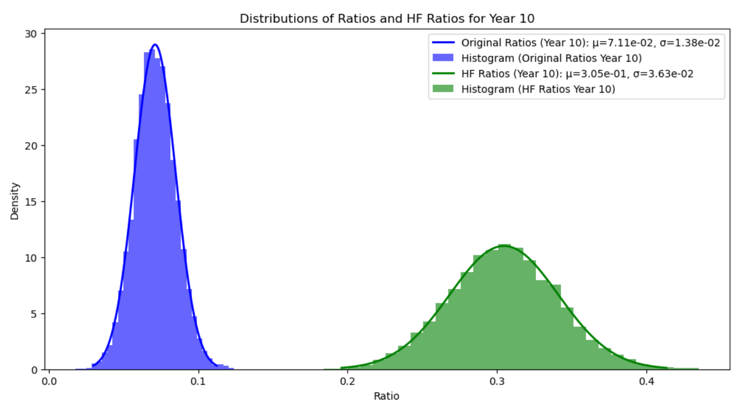

Figure 25: PD Ratio and Moody's PD Ratio Distributions for Withdraw Rate 0.10 at Year 10

8.3 Analysis of Results

Figures 17 to 25 present the distributions of the ratios between the Expected Credit Losses (ECL) and the actual observed losses across different withdraw rates (0.03, 0.05, and 0.10) and time horizons (1, 5, and 10 years). These ratios quantify the error in the ECL estimation, where a value close to zero indicates accuracy, a positive value signifies overestimation, and a negative value reflects underestimation.

Withdraw Rate 0.03: At this withdraw rate, the Original TM yields a near-accurate estimate, while the Moody's adjusted TM shows a consistent overestimation, particularly as time increases. By Year 10, the Moody's TM results show a significant deviation, reflecting the conservativeness of Moody's approach.

Withdraw Rate 0.05: As the withdraw rate increases, the Original TM starts to show a slight underestimation of losses. Meanwhile, the HF TM continues to overestimate, but its predictions remain more stable and conservative, reducing the potential for underestimation risk.

Withdraw Rate 0.10: With the highest withdraw rate, the difference becomes more pronounced. The Original TM remains closer to the actual observed losses, with the error around 7.12% at Year 10, indicating a relatively accurate prediction. On the other hand, the Moody's adjusted TM shows a substantial overestimation with a mean error around 30.5%. This indicates that while Moody's TM is more conservative, it may overestimate potential losses by a significant margin, particularly at higher withdraw rates and longer time horizons.

9 Conclusion

The simulations indicate the significant impact of withdraw rates on PD and ECL. The comparative analysis of the original TM, 50% withdraw reweighted TM, and withdraw removed TM provides valuable insights into credit risk management and the importance of accounting for withdraw probabilities in credit transition models.

1. **Impact of Withdraw Rates:** The analysis shows that higher withdraw rates lead to higher PD across all credit ratings. This trend is consistent across all three TMs (Original, 50% Withdraw Reweighted, and Withdraw Removed) and all methods of calculating PD (Conventional, Moody's Withdraw-Adjusted, and Withdraw-Removed). The increasing PD with higher withdraw rates highlight the sensitivity of default probabilities to changes in withdraw assumptions.

2. **Effectiveness of Different TMs:** The comparison of the Original TM, 50% Withdraw Reweighted TM, and Withdraw Removed TM reveals that redistributing withdraw probabilities affects PD. The Withdraw Removed TM results in the highest PD, indicating a more conservative approach. This underscores the importance of selecting an appropriate TM based on the specific risk management objectives and the economic context.

3. **Simulation of Transition Processes:** The simulation of transition processes with given withdraw counts and using conditional PDs provides valuable insights into the dynamics of credit transitions. These simulations help estimate the ECL and understand the impact of different scenarios on credit risk. The results indicate that using withdraw counts and conditional PDs can provide reliable estimates of PD and credit losses.

4. **Application in Real-World Scenarios:** The methodologies and findings presented in this paper have significant implications for financial in-

stitutions. Accurate modeling of withdraw probabilities and credit transitions is crucial for effective credit risk management. The insights gained from this study can help institutions refine their risk models, improve the accuracy of ECL estimates, and better prepare for economic uncertainties.

5. **Model Sensitivity and Robustness:** The study highlights the importance of continuously updating and validating risk models to ensure their accuracy and reliability. The sensitivity of ECL and Real Credit Loss to changes in withdraw probabilities and other assumptions emphasizes the need for robust and adaptable risk management frameworks.

6. **Utilizing Cumulative PD for ECL Calculation:** The introduction of cumulative PD to calculate the ECL offers a comprehensive view of credit risk over time. By considering the original obligor count and subsequent transitions, this method provides a more nuanced understanding of default probabilities. The findings demonstrate that this approach captures the dynamics of credit transitions effectively, making it a valuable tool for financial institutions in enhancing their credit risk assessments.

7. **Conclusion on Withdraw Rate Thresholds:** The study concludes that for withdraw rates below 0.01, both the Original TM and Moody's method are effective for estimating PD and ECL. However, for withdraw rates above 0.05, Moody's method tends to overestimate potential losses, particularly over longer horizons, rendering it less suitable for such conditions. Financial institutions should consider these findings when selecting the appropriate model for their specific risk management needs.

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