

OXFORD

MANUAL OF POLITICAL ECONOMY

A CRITICAL AND VARIORUM EDITION

VILFREDO PARETO

edited by Aldo Montesano, Alberto Zanni,
Luigino Bruni, John S. Chipman,
and Michael McLure

Manual of Political Economy

Manual of Political Economy

VILFREDO PARETO

*A Critical and
Variorum Edition*

EDITED BY

Aldo Montesano, Alberto Zanni, Luigino Bruni,
John S. Chipman, and Michael McLure

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

Great Clarendon Street, Oxford, ox2 6DP,
United Kingdom

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide. Oxford is a registered trade mark of
Oxford University Press in the UK and in certain other countries

© Aldo Montesano, Alberto Zanni, Luigino Bruni,
John S. Chipman, and Michael McLure 2014

The moral rights of the author have been asserted

First Edition published in 2014

Impression: 1

All rights reserved. No part of this publication may be reproduced, stored in
a retrieval system, or transmitted, in any form or by any means, without the
prior permission in writing of Oxford University Press, or as expressly permitted
by law, by licence or under terms agreed with the appropriate reprographics
rights organization. Enquiries concerning reproduction outside the scope of the
above should be sent to the Rights Department, Oxford University Press, at the
address above

You must not circulate this work in any other form
and you must impose this same condition on any acquirer

Published in the United States of America by Oxford University Press
198 Madison Avenue, New York, NY 10016, United States of America

British Library Cataloguing in Publication Data
Data available

Library of Congress Control Number: 2013956375

ISBN 978-0-19-960795-2

Printed and bound in Great Britain by
CPI Group (UK) Ltd, Croydon, CR0 4YY

CONTENTS

Introductory Editorial Materials

Foreword to the English Translation	VII
Foreword to the 2006 Italian Edition	IX
Editors' Introductory Note	XI
Photograph of Pareto and images of the 1906 and 1909 editions	XXIII

Manual of Political Economy

Index of Chapters	iii
Note	xii
Preface to the First (1906) Edition	xiii
Chapter I General Principles	1
Chapter II Introduction to Social Science	20
Chapter III The General Concept of Economic Equilibrium	72
Chapter IV Tastes	125
Chapter V Obstacles	145
Chapter VI Economic Equilibrium	173
Chapter VII Population	192
Chapter VIII Landed Capital and Capital Goods Proper	220
Chapter IX The Concrete Economic Phenomenon	232
Appendix to the Italian (1906) Edition	273
Appendix to the French (1909) Edition	309
Index of Subjects	398
Authors' Names Index	403

Detailed Editorial Materials

Annotations by John Chipman	406
Annotations from the 2006 Italian Edition	479
Editors' Notes	512
Notes to the French Appendix	621
Index of Authors Cited by the Editors	660

FOREWORD TO THE ENGLISH TRANSLATION

The Italian ‘edizione critica’ of Pareto’s *Manuale–Manuel* was launched at the *International Seminar: Vilfredo Pareto’s “Manuale di Economia Politica,” 1906–2006*, convened at Bocconi University on June 5–6, 2006.¹ By the time of that event the necessity for an English-language edition was already obvious. In fact, the economic work of Pareto is considered less frequently in the English-speaking world than the historical importance of his contribution to economics warrants. Above all, the fame that Pareto enjoys for his economic work is due mainly to Sir John Hicks’ meritorious mediations; and, conversely, what is least well known of Pareto’s economic contributions is that which has not been transmitted via Sir John’s mediation.

Shortly after the 2006 seminar, John Chipman and Michael McLure agreed to my proposal to integrate them within the editorial team to collaborate on the English edition. For this I requested, and obtained, contributions from the Italian Ministry of Universities and from Bocconi University, both of which I sincerely thank.

John Chipman informed me (16 October 2007) that an English translation was available to him that incorporated all the text from both the Italian *Manuale* and the French *Manuel* (and he sent me it in October 2008). This translation originates from an early 1970s *American Economic Association* project, supervised by John Chipman, to publish Pareto’s *Manual* and a selection of Pareto’s important articles.² For that project, John Chipman worked on the English translation of the 1909 *Manuel*, originally prepared by Roger Dehem (a Belgian economist and professor at the Laval University, Quebec, Canada, who wrote the foreword to the fourth edition of *Manuel* published in 1966 in Pareto’s *Oeuvres complètes*) and revised by John Cairncross (a British poet, living in Italy, and brother of the economist Alec Cairncross) in light of the original 1906 *Manuale*. The AEA had entered into a publishing contract with Academic Press, Inc. in 1974 and the manuscript of the *variorum* edition of the *Manual of Political Economy* was prepared by John Chipman, but it was never published. In 2007 the AEA reverted the rights to the translations prepared for that manuscript to John Chipman.

In view of this, the translation supplied by John Chipman has been adopted for the English ‘critical edition’, and the translation of Pareto’s *Manuale di Economia Politica / Manuel d’Économie Politique* published in this edition has been edited by John Chipman. Collaborating with John in that editorial task were Yuen Kit (Augustine) Mok, who formatted and inserted the diagrams, and Chin Tung (Grace) Chan, who provided assistance with annotations and other aspects of the publication. In regard to matters

¹ The proceedings of that Seminar have been published in the *RISEC-Rivista Internazionale di Scienze Economiche e Commerciali* (2006, LIII, n.4) and republished in the book *New Essays on Pareto’s Economic Theory*, (L. Bruni and A. Montesano eds., Routledge, 2009).

² Some of these translations have been recently edited by J. Chipman and published in the *Giornale degli Economisti* (67, n.3, 2008) and the *History of Economic Ideas* (XVII/2009/1).

of translation, John Chipman availed himself of comments from the other editors, particularly Michael McLure, and from Vincenzo Savini, who also provided helpful and intelligent comment on the translation of editorial text originally prepared for the 2006 Italian edition. Of course, in this English edition those editorial contributions have been partially revised to update them, in the first instance, and then adapt them, as far as possible, for the requirements of English-speaking readers.

As for the Italian edition, the Editors' Introductory Note is by Alberto Zanni. The other notes prepared by the editors have been divided into three sections: 1) the 'Annotations' are brief notes pertaining directly to Pareto's text. In this English edition these annotations have been divided in two subsections, the subsection concerning the annotations which have been prepared by John Chipman, and the subsection whose annotations derive from the 2006 Italian edition, which were substantially written by Alberto Zanni (although some of them were abandoned when adapting the annotations for the English edition, which was overseen by Luigino Bruni). 2) The 'Editors' Notes', which are due to Alberto Zanni (except for E.N. 58, the last one, which is due to Michael McLure), provide more extended commentary, with particular features highlighted to frame aspects of Pareto's work in relation to subsequent literature. 3) The notes on the mathematical appendix to the *Manuel*, due to Aldo Montesano, serve, above all, to clarify the contribution of Pareto. Michael McLure has followed the preparation of the entire volume, contributing useful suggestions in every section.

Pareto is certainly not an easy economist. Neither has the preparation of this 'critical and variorum edition' been an easy task. I must thank all those who have participated in the task, with devotion and patience; from my co-editors, Alberto Zanni, Luigino Bruni, John Chipman, and Michael McLure for their effective collaboration, to the translator, Vincenzo Savini, and to Ms Grace Chan (for collaborating with John Chipman). I also feel a great debt of gratitude to the team at Oxford University Press, which I wish to thank for following our progress with interest and being ready to smooth away any difficulties.

ALDO MONTESANO

FOREWORD TO THE 2006 ITALIAN EDITION

A few years ago, Alberto Zanni and I found ourselves in agreement on the opportuneness of publishing a critical edition of Pareto's *Manuale* (1906) that would also incorporate the variations that were later included in the *Manuel* (1909), which worked as both the French translation and the second edition of the *Manuale* itself. Two reasons lay behind our intention. On the one hand, the *Manual* is a work of crucial importance, in that it introduced the kind of analytical approach that would characterize most economic theory of the 20th century—in particular, the ordinalist representation of individual choice, the theme of general economic equilibrium, and the evaluation of goods allocations according to the Pareto optimum criterion. Further interesting analytic points can also be found in the *Manual*—on, for example, production, non-linear prices, non-competitive markets, and the process of competition—which have not ended up within mainstream economics following Pareto's approach, as neither did, in its most idiosyncratic aspects, the general outline proposed by Pareto for social sciences. On the other hand, as it is often the case with innovative works, the *Manual* is obscure in many places. Many arguments, including some mathematical ones, are elliptical. There is the need for an interpretation that may help readers—even help them reject Pareto's idea, as it may happen. Furthermore, in the *Manual* there were misprints and oversights that made reading cumbersome.

With the approach of the centennial of the publication of the *Manuale*, Alberto Zanni and I decided, together with Luigino Bruni, to bring our old plan to fruition. With this object in mind, we applied for, and obtained, some funding from the Italian Ministry of Universities and from Bocconi University, to both of which we are grateful. We also decided not to propose a bio-bibliographical update on Pareto, both because it went beyond the aims of the current edition, and because the works of Giovanni Busino and that of Fiorenzo Mornati already fulfil this need in an excellent way.

Zanni dealt mainly with the notes on the text of the Manual, wrote the longer editorial notes to the text, and translated the *Manuel* mathematical appendix. I took care of the notes to the latter appendix (there are no notes to the Italian appendix because its most noteworthy parts are also included in the French appendix). We have exchanged our opinions on a great number of questions. Bruni focused on the editorial layout and edited the indices.

Dr Antonio Bianco collaborated with extraordinary commitment in the preparation of the text.

ALDO MONTESANO

EDITORS' INTRODUCTORY NOTE

1. Editions and reprints of the *Manuale* and *Manuel*. | 1.1. Editions and reprints of the *Manuale*. | 1.2. Editions and reprints of the *Manuel*. | 1.3. The American translation of the *Manuel* (with some words on the present English edition). | 2. The present *Manuale-Manuel*. | 2.1. The *Manuel*: the substantive second edition of the *Manuale*. | 2.2. Contents of the present edition. | 3. Indices. | 3.1. The three Indices of the *Manuale* and of the *Manuel*. | 3.2. Index of Chapters: from the Italian *Indice dei capitoli* to the French *Table des matières*. | 3.3. Index of Chapters of the present edition. | 3.4. Index of Subjects: from the Italian *Indice alfabetico* to the French *Index alphabétique*. | 3.5. Authors' names index: from the Italian *Indice dei nomi di autori* to the French *Table des auteurs cités*. | 4. Paragraph numbering. | 5. The translation of some words. | 6. Acknowledgements. | 7. Some abbreviations.

1. EDITIONS AND REPRINTS OF THE “MANUALE” AND OF THE “MANUEL”

1.1 The first edition of the *Manuale di economia politica con una introduzione alla scienza sociale* appeared in 1906 in Milan, published by the Società Editrice Libraria (*via Kramer, 4A—Gall. De Cristoforis, 54–55*) as issue No. 13 in the series *Piccola Biblioteca Scientifica* [‘Little Scientific Library’]. The books from this series are so small (about 13x8 cm) that in the antique book market Pareto’s *Manuale* was often called “il Paretino”—literally, “the little Pareto.” An indication at the bottom of p. IV—*Milano, 1905, Tip. Indipendenza di A. Berni & C.i.*—reveals that the *Manuale* came out of the printing house in 1905. This date explains why some economists (A. Graziani and A. Loria, for instance) have indicated that they had already read the *Manuale* by the end of 1905. In the text that follows, when it cannot give rise to any doubt, this first Italian edition will at times be simply indicated by the date, 1906 or (1906).

The first edition of the *Manuale* included a mathematical *Appendix* followed by three indices: an *Index of Chapters*, an *Index of Subject*, and an *Authors’ Names Index*. The volume ended with an *Errata corrigé* regarding also the mathematical formulae of the *Appendix*, and missing letters.

A reprint of the *Manuale* appeared in 1909 in Milan, still published by the Società Editrice Libraria, whose address this time had changed to: *Milano, Via Ausonio, 22 Gall. De Crist., 54–55*. There is no indication about the printing house. The only difference between this reprint and the first edition lay in the fact that the printer restored the missing letters and corrected the mistakes indicated in the *Errata corrigé*, which was removed. This explains why the 1909 reprint contains 575 pages as opposed to the 579 pages of the first edition.

A subsequent 1919 reprint (at the bottom of page IV we read: *1919—Tip. Mariani—Via Stelvio, 21, Milano*) is perfectly identical with the 1909 reprint.

After Pareto’s death (1923), there appeared two further reprints of the *Manuale*. In 1965, on the initiative of the Roman publisher Bizzarri (printing house *La Nuova Grafica*,

Rome), a reprint appeared that was subsequently withdrawn from the bookstores because it had not been authorized by the Editrice Libraria of Milan. This unauthorized reprint included a fine *Introduzione* by Luigi Amoroso, which was divided into two parts. The first part reproduced an article by Amoroso on Pareto's law. The second part was Amoroso's passionate peroration of how he believed he had made Pareto's static equilibrium, dynamic. Apart from Amoroso's introduction, Bizzarri's edition shows discordant characteristics. Even though it contains the translation of the mathematical *Appendice* of the French *Manuel*, the references from the text (1906) to the *Appendice* (1909), and vice versa, are missing, because the many passages added by Pareto in the *Manuel* were not identified and translated into Italian. Furthermore, Bizzarri's edition is characterized by the complete lack of intervention in the text itself by the author of the introduction or by the translators. In a word, Bizzarri's edition is missing an *editor*, or, if one wishes, was conceived under the banner "leave everything as it is," including misprints and Pareto's oversights.

In 1974, in Padua, Cedam publishers (who availed themselves of the *Tipografia Ed. Gualandi di Vicenza, Corso San Felice 259*) published a reprint with a *Presentazione* by Emanuele Morselli. Unfortunately, this reprint does not include any of the *Manuale*'s three indices, on which we shall dwell further below. For this reason and because of the fact that it presents well with its beautiful exterior appearance, one holds on to the 1974 Cedam *Manuale* with contrasting feelings, as one would to a beautiful stringless violin. Furthermore, on several occasions Morselli did not resist the temptation to alter Pareto's text, often arbitrarily, without warning the reader. We may provide evidence of this on a subsequent occasion.

In 1992, a facsimile reprint of the *Manuale* (1906) appeared in Düsseldorf, published by *Verlag Wirtschaft und Finanzen*, with an Introduction by Bertram Schefold and two articles by Gottfried Eisermann and Edmond Malinvaud.

In 1994, in Pordenone, there appeared another facsimile reprint of the *Manuale* (1906), published by *Ediz. Studio Tesi*, with an *Introduzione* by Siro Lombardini.

1.2 In substantive terms, the only true second edition of Pareto's book is the *Manuel d'économie politique—traduit sur l'édition italienne par Alfred Bonnet (revue par l'auteur)*, which appeared in 1909 in Paris, published by V. Giard & E. Brière in the *Bibliothèque internationale d'économie politique publiée sous la direction de Alfred Bonnet*. In the *Manuel*, after the mathematical *Appendix*, between the *Table des matières* and the *Index alphabétique*, precisely on p. 684, Pareto inserted some *Additions* to draw attention to two works, one by V. Furlan and another by E. Barone, which had appeared when the *Manuel* was already being printed. On p. 697 there followed an *Errata*, mostly regarding mathematical symbols.

A reprint of the *Manuel* appeared in Paris in 1927, published by *Marcel Giard—Librairie-Éditeur; 16, rue Soufflot et 12, rue Toullier*. The title page reiterates: *traduit sur l'édition italienne par Alfred Bonnet (revue par l'auteur)*. Anyone who is familiar with Pareto's correspondence knows how much he reproached himself for having trusted Bonnet as his translator. However, anyone who knows Pareto—who was a "top of the class" type of person from his childhood onwards, although he was always desperate because of an inability to notice his own oversights: a weakness that he confessed to Mrs Emilia Peruzzi many years before meeting Pantaleoni and also confessing that same

weakness to him—also knows the weight that should be given to the words: *revue par l'auteur*. This reprint—it was indeed a reprint, even though it reads *deuxième édition*—was a reproduction of the first French edition of 1909. The second-last page, p. 699, reveals a technical aspect of this reprint: *Reproduit par les procédés Dorel, 45 rue de Tocqueville—Paris XVII^e*.

The intermediate steps leading from the *Cours* (1896–7) to the *Manuale* (1906); the troubled journey from the *Manuale* (1906) to the *Manuel* (1909); Pareto's desire for a new edition of the *Manuel* that took into account at least his own 1911 encyclopedic entry on *Économie mathématique*; the sceptical hope for an English translation (in Kolkata, of all places!) of the *Manuale*: all these issues can—we were saying—be relived through the beautiful *Corrispondenza Pareto–Pantaleoni*.

A new edition of the *Manuel*, followed by various reprints, appeared after the second World War—in 1963, in two volumes, and, then, in 1966, in a single volume, as tome VII of Pareto's *Œuvres complètes*, which were published by *Librairie Droz*, Geneva, under the courageous direction of Giovanni Busino. Compared with the 1927 reprint, the Droz edition is characterized by a *Foreword* by Roger Dehem and by the fact that it makes use of the *Errata corrigé* which was therefore removed.

1.3 In 1971, the *Manual of political economy* by Pareto was published as an English translation from the 1927 reprint of the French *Manuel*, published by *Augustus M. Kelley Publishers*, New York. The translator was Ann S. Schwier; the editors, Alfred N. Page and Ann Schwier herself. The *Translator's notes* do not exceed two pages (pp. 493–4).

The text of the present English edition is based on translations prepared in the 1970s under the direction of John Chipman for the *American Economic Association*. The writer [A.Z.], like the other editors of the critical Italian edition, only became aware of the Dehem–Cairncross–Chipman translation of the *Manual* after publication of the Italian edition in 2006. Consequently, the *Editors' Introductory Note* to this Italian edition contains no mention of the unpublished manuscript for the English-language variorum edition of the *Manual*.³

³ On reviewing the final draft of the present Introductory Note, Luigino Bruni kindly provided me with the following information.

On the occasion of the 50th anniversary of the first edition of Pareto's *Manuale di Economia Politica* (1906), the *American Economic Association*, encouraged “by the general enthusiasm with which the translation of Walras by Jaffé has been received by the profession,” considered whether “to arrange the translation of Pareto's *Manuale* from the Italian” (Letter from John Perry Miller, chairman Research and Publications committee AEA, to Austin Robinson, Secretary of the Royal Economic Society, 27 January 1956). In particular, Parry Miller sought to involve the *Royal Economic Society* in the process as “a cooperative venture with the A.E.A on this matter similar to the arrangement provided for Walras.” Several letters were exchanged between Robinson, Miller, the *Royal Economic Society*'s publisher, George Allen & Unwin, the University of Chicago Press, J. R. Hicks and Nicholas Georgescu-Roegen, some of which included references to advice given by Piero Sraffa, William Jaffé, George Stigler and others. Georgescu-Roegen had personally offered to support the translation project, but on 24 October 1956 he was informed by Perry Miller, in the last letter to make reference to the 50th-anniversary translation project, that “unfortunately this project was for awhile tangled up in various complications.” One of those complications appears to have been the difficulty in finding a suitable translator, as Alexander Morin, the managing editor of University of Chicago Press, reported to Miller on 19 July 1956 that “we have been unable to locate a suitable translator,” and the project was withdrawn.

Six years after the mooted 50th-anniversary translation project, the Asia Publisher House (London), wrote to Hicks in 1962 expressing its interest in publishing an English translation of *Manuale*. Hicks referred the letter to Robinson, with this interesting comment: “I feel, however, strongly that what is needed is a good

This is as far as the editorial details regarding past editions and reprints of Pareto's book go.

2. THE PRESENT *MANUALE—MANUEL*

2.1 The present edition—substantially a translation of the 2006 Italian critical edition—integrates the *Manuale* with the *Manuel*; this compels us to clarify what leads one to consider the 1909 French *Manuel* the only truly substantive second edition, in essence, of the 1906 Italian *Manuale*.

First of all, the mathematical Appendix of the *Manuel* (about 100 pages) is so different from that of the *Manuale* (about 36 pages) that it was decided to include them both in the present edition. Apart from these Appendices, the main difference between the content of the chapters in the *Manuel* compared to those of the *Manuale* is not confined to the *bis* paragraphs that Pareto explicitly added to the *Manuel*—*bis* paragraphs are included in: §107 of Ch. II; §36 of Ch. III; §12 of Ch. VIII; and §§32 and 35 of Ch. IX—it also extends to footnotes as well as the passages from the existing paragraphs of the *Manuale* that were extended, which, in total, add about 30 printed pages to the book.

2.2 In the 2006 Italian critical edition, the mathematical *Appendix* of the French *Manuel* was translated into Italian by its editors. Furthermore, all the passages that Pareto added in the *Manuel*, in the text or in the footnotes, were identified and translated into Italian; it was the first time that such a search and translation were carried out. Analogously, the passages that appear in the *Manuale* but not in the *Manuel* were also pointed out.

In the 2006 Italian edition we substituted parentheses for the few brackets that Pareto used in the literary part of the book (as opposed to the mathematical section); and used square brackets to enclose new and revised text that Pareto added to the *Manuel*, or to refer readers to some of the *Editors' Notes* (E.N.s). In the present English edition passages that Pareto added to the *Manuel* are demarcated by the symbols [and]. The editors' annotations are organized in two separate sections: annotations prepared by J.S. Chipman, which are indicated in the text by superscript boldface numerals; and annotations that derive from the 2006 Italian edition, which are indicated in the text by superscript letters in square brackets. At times, the latter annotations include reference to numbered E.N.s, each of which deal more fully with a range of complex historical, mathematical, and theoretical issues. In total, 58 *Editors' Notes* are included, which relate directly to the chapters of the *Manual*. However, editorial notes concerning the French mathematical appendix are not referred to as E.N.s; rather, those notes appear in a separate section of the book and are referred to as the *Notes to the French Appendix* (N.Fr.App.).

English translation, better printed than the French (where the diagrams are perfectly horrid)" (6th March 1962). Finally, Robinson replied to Asia House with some notes: "The French edition contained considerable additions by Pareto himself, and Mr. Sraffa thinks that it would be right to use the French edition for purposes of translation, though it may contain mistakes that were not discovered by Pareto ... Mr Sraffa is not quite certain whether the second Italian edition, which is, I understand, of same date as the French edition, incorporated the improvements made by Pareto for the French edition" (12th March 1962).

All letters are at the Austin Robinson Archive, LSE Library Archives (PA357, RES minute books), London.

As for the white top margin on each page, we first of all point to the following difference between the *Manuale* and the *Manuel*. In the *Manuale*, at the top of the page one finds the page number, the title of the current chapter, and the relevant paragraphs; *the number* of the relevant chapter is missing (for instance, 188 CONC. GEN. DELL'EQUIL. ECON. [§117–120]). As a consequence, when one opens the *Manuale* looking for a specific chapter number, it is not immediately evident which chapter corresponds to the opened page. In the French edition, on the contrary, the page number and the title and chapter *number* are immediately evident; but the paragraph numbers are missing (for instance, 192 CHAP. III—NOTION GÉNÉRALE DE L'ÉQUILIBRE ÉCONOMIQUE).

In order to make the transition from the Italian Index of Chapters to the text easier for the reader, the 2006 Italian edition follows the example of the French edition, that is, it shows the current page number and the number and title of the current chapter at the top. As for ease of consultation, it seems to us that the Kelley edition of the *Manual*—which shows MANUAL OF POLITICAL ECONOMY at the top of every even page, while only showing the title of the current chapter (for instance, GENERAL NOTION OF ECONOMIC EQUILIBRIUM) at the top of every odd page—is a step backwards compared not only to the French *Manuel*, but also to the Italian *Manuale*. In the present English edition the example of the French edition is followed at the top of every odd page.

3. THE INDICES

3.1 The following table shows the terminology used: 1) in the *Manuale* (1906) and in the Italian critical edition (2006); 2) in the *Manuel* (1909); 3) in the American edition (1971); and, finally, 4) in the present English edition.

<i>Manuale</i> (1906 and 2006) Italian editions	<i>Manuel</i> (1909) French edition	<i>Manual</i> (1971) American edition	Present English edition
Indice dei capitoli	Table des matières	Table of contents	Index of Chapters
Indice alfabetico	Index alphabétique	Alphabetical index	Index of Subjects
Indice dei nomi di autore	Index des auteurs cité	Table of authors cited	Authors' Names Index

The *Manuale* includes *a*) an *Indice dei capitoli* (Index of Chapters) that becomes the *Table des matières* in the *Manuel*; *b*) an *Indice alfabetico* (Index of Subjects) that becomes the *Index alphabétique*; *c*) an *Indice dei nomi di autori* (Authors' Names Index) that becomes the *Table des auteurs cités*. Let us consider them separately.

Index of Chapters: from the “Indice dei capitoli” to the “Table des matières”

3.2 The *Table des matières* of the *Manuel* is the translation of the *Indice dei capitoli* of the *Manuale*. It therefore does not take into account the changes that took place in the chapters in the transition from the *Manuale* to the *Manuel*. Evidence of this lies in the fact that the French *Table* does not indicate the paragraphs that were added in the *Manuel*, which Pareto had marked with a number followed by “bis”.

Furthermore, there are some discrepancies between the French *Table* and the Italian *Indice* even in cases where the paragraphs referred to have not undergone any change. The case of §§41–46 of Chapter I is a blatant example of this. First of all, the French *Table* omits to translate what the Italian *Indice* reads for §41:

There cannot be a faith more scientific than another.

Furthermore, for the paragraphs from 43 to 46, the Italian *Indice* reads:

- 43. Scientific truth and social usefulness are different things. – 44. Error in considering a supposed end of evolution. – 45. Intuition; its modes and forms. – 46. The universal consensus of mankind is not at all a criterion of scientific truth.

whereas in the French *Table* we read:

- 43. Intuition; its modes and forms. – 44, 45, 46. Universal consensus is not a criterion for scientific truth.

In the transition from the Italian *Indice* to the French *Table*, a variety of lesser omissions and minor instances of “unfaithfulness” also takes place.

As an example of lesser omission, let us quote §6 of Ch. II. Whereas the Italian *Indice* reads:

- 6. Objective relations and subjective relations

in the French *Table* we read:

- 6. Objective relations.

Another example can also be found in paragraphs from 196 to 204 of Ch. III. Whereas the Italian *Indice* reads:

- 196 to 204. Exchange equilibrium, in the general case

the French *Table* reads:

- 196 to 204. Equilibrium in general.

As for the minor instances of “unfaithfulness”, it suffices to mention a single example, with the following preliminary remark. In political economy, barter—in Italian, *baratto*, and in French, *troc*—is a particular form of exchange. But Pareto generally uses the term “barter” to mean “exchange.” Well, the French edition systematically translates “barattare” (that is, “to barter”) and “baratto” with “échanger” and “échange,” rather than with “troquer” and “troc.” In view of this systematic approach, it is not clear why the “baratto” we find in the Italian *Indice*, Ch. III, §83, in the French *Table* becomes “troc,” a word that is never used in the text of that very same §83.

After these clarifications, let us consider the criteria we followed with regard to the Italian *Indice* and the corresponding French *Table* [the Index of Chapters of the present edition].

3.3 We shall not use the French *Table des matières*, since it is no more than a translation—and, as we have seen, not always a complete and faithful translation—of the Italian *Indice dei capitoli*. It is therefore the latter that appears also in the present English edition

as Index of Chapters. In it, however, we shall indicate, in square brackets, the “*bis*” paragraphs added by Pareto in the *Manuel*.

Since the present edition includes both the mathematical Appendix of the *Manuale* and the mathematical Appendix of the *Manuel*, the latter Appendix will be accompanied by its relevant Index.

We shall insert the Index of Chapters and the indices of the two mathematical appendices just mentioned, at the beginning, rather than at the end as in Pareto’s book—the reader’s comfort takes precedence here over other considerations. Only the remaining two indices, the Italian *Indice alfabetico* (*Index alphabetique* in the French edition, the Index of Subjects of the present English edition) and the *Indice dei nomi di autori* (the French *Index des auteurs cités*, the Authors’ Names Index of the present edition), can therefore be read, as in Pareto’s work, at the end of the present volume.

Index of Subjects: from the “*Indice alfabetico*” to the “*Index alphabétique*”

3.4 Apart from the changes relating to the rewriting of the mathematical Appendix, the French *Index alphabétique* is the translation of the corresponding Italian *Indice alfabetico*. This conclusion has been reached after a long and tiresome check because of the numerous French words that start with a different letter compared with the corresponding Italian words. Using a recent expression for an old concept, it could be said that the alphabetical diversity just mentioned completely modifies the content of the lexicographical order of the *Index* compared with that of the *Indice*. But, apart from this diverse alphabetical sequencing of words, the subject index does not differ in any significant manner between the two editions.

The only differences between the two are a direct consequence of Pareto rewriting the mathematical *Appendix*. We decided only to refer to the Italian *Indice alfabetico* [the Subject Index of the present English edition], with the addition of double cross-references to the Italian mathematical Appendix (App.) and to the French mathematical Appendix (Fr. App.). Readers have the advantage, therefore, of only having one alphabetical index in front of them, with cross-references to the two mathematical appendices of the present edition. We have also complied with Pareto’s practice of referring to paragraph numbers rather than page numbers.

Authors’ Names Index: from the “*Indice dei nomi di autori*” to the “*Table des auteurs cités*”

3.5 Even just for the part regarding the authors’ names, the *Errata corrige* that appeared in the first edition of the *Manuale* was far from exhaustive. We had started by carrying out further corrections and integrations. More in particular, we had started by pointing out to the reader, by means of notes, when the corrections and the additions were attributable to the *Errata corrige*, and when they were attributable to intervention by the present editors. A similar problem presented itself regarding the *Errata* of the *Manuel*. Indeed, in the *Table* some omitted names are not indicated in the *Errata* (Ariosto, for instance). A. Cournot, by way of another example, appears in the *Errata* (1909) for p. 157, but not for p. 660.

We renounced the idea of making this initial work visible and fell back on the following solution: in the *Indice dei nomi* [the Authors’ Names Index of the present edition], an

asterisk placed next to the page number relating to an author's name indicates that the name is only included in the French *Manuel*, or, in other words, that it is not included in the earlier Italian *Manuale*.

The names of the authors quoted by the present editors are indicated separately. Rather than adopting an alternative solution, it was deemed preferable to keep Pareto's indices separate from those of his editors.

4. PARAGRAPH NUMBERING

In those paragraphs that wrongly show a number repeated twice (e.g. §92 of the French Appendix) we shall use square brackets to mark the editors' intervention. Thus, one will find §92 and §92[bis]. We shall insert a number in square brackets to complete the paragraphs that, due to an oversight or error, were without a paragraph number in the original publication.

5. TRANSLATION OF SOME WORDS

We have always translated, in the Italian edition 2006, the expression *prix de revient* with *costo unitario*, rather than with *prezzo di costo*, and we have suggested "unitary cost" (which corresponds to the average cost) for the present English edition.

In the Italian critical edition (2006) we have always used the Italian expression *punto di arresto* to translate the two French expressions *point terminal* and *point d'arrêt* ("terminal point"), which express a concept that we believe was introduced by Cournot (see E.N. 12) and was subsequently found in Edgeworth, which Pareto generalizes to such an extent that we find it in his most general definition of economic equilibrium (1906, p.186, 1909, p.190, p. 95 of the present edition).

The writer (A.Z.) has dedicated the E.N. 41 to the translation of the word "moneta" (money).

Pareto's expression *lignes d'indifférence*—Edgeworth's original *indifference lines*—in the Italian edition (2006) has been translated with *linee* or *curve di indifferenza* and in the present English edition generally with *indifference lines* or *indifference curves*. The editors will generally use the expression *preference lines* (or *preference curves*)—Edgeworth's original *preference lines*—in their comments, and leave it to the reader to infer from the context when they are obtained: *i*) from indifference lines according to Edgeworth's "cardinalist" logic, which is also found in Pareto's earlier works, that is, up to the *Cours* included; *ii*) from indifference lines according to the logic of Pareto the "ordinalist," that is, the Pareto of the theory of choice.

The translator (A.Z.) from the French to the Italian language came across *Les morticoles* in the *Corrispondenza Pareto-Pantaleoni*, Vol. III, p. 101. From p. 168 of that very volume he realized that Pareto italicized *les morticoles* with *i morticoli*. Professor F. Aubert confirmed to us that *Les morticoles* is the title of a novel published by Léon Daudet in 1894—a satire on quack-like academic doctors. In the Italian edition (2006) it was eventually decided to use *mediconzolo* to translate the word *morticole*, which is found again in Ch. IX, §32bis, p. 493 of the *Manuel*. In the present English edition it becomes "sawbone".

Apart from some instances of *ou* instead of *on*, and vice versa, commas that were patently out of place, or missing, or redundant, and other minor exceptions, all oversights and misprints found in the French passages translated in the Italian 2006 edition and in the present edition have been pointed out. Contrary to Morselli (see above §1.1), we were never tempted to modify Pareto's language in the Italian 2006 edition. Why, for instance, should one write *province* or *formule*, when Pareto in his time was writing *provincie* and *formole*? Every linguist who after Darwin maintained that "languages, too, are living organisms that evolve" would be surprised at seeing something like this. As for Pareto's stinginess with words—sorely commented upon by the valiant K. Wicksell more than anyone else—why and how should one modify it, when it was presumably due to a classic conciseness of expression still found in works by Dante and Machiavelli that Pareto knew by heart, rather than to Pareto's oft-quoted engineering degree? If Bernardino Davanzati (1529–1606) came back to life, he would remark with renewed defiance that not even the British could render with greater conciseness the Latin motto *auro suadente nil potest oratio* that was dear to Pareto.

6. ACKNOWLEDGMENTS

We thank Professor Françoise Aubert (1943–2011) for helping us in the translation of various French words—as chance would have it, she lived in Fiesole in an annex of Villa Krauss, where Pareto lived before he moved to Lausanne.

We have not forgotten that Professor Claudio Napoleoni (1924–1988), Professor Sergio Steve (1915–2006) and Professor Paolo Sylos Labini (1920–2005) encouraged us to study Pareto.

A special thank you goes to Giacomo Becattini. It was for him that we translated 99.9 percent of *The Economics of Industry* by Alfred and Mary Marshall, in the short time we worked with him to bring to life the quintessentially Tuscan IRPET. Our friend Becattini was perhaps thinking that we would concern ourselves with Marshall and Keynes (a passion of ours!) rather than with Pareto. However, we were convinced, and still are, that John Stuart Mill's sociological message inspired not one but two great responses, and that Pareto's response deserved to be studied as much as Marshall's. Without us asking for it, and almost without him letting us realize it, Becattini tried to assist us in this desire of ours. To our friend Giacomo, who has the virtue of infecting others with a typical disease of his (his passion for knowledge!) a heartfelt thank you.

We wish to thank Aldo Montesano separately. When, many years ago, the first glimmer came to us of the idea of a critical edition of the *Manuale*—a book we had read as a new *De rerum natura*, written *per intervalla insaniae*—it seemed to us that the person who could advise us and fill the gaps in our knowledge was Professor Aldo Montesano, from the Bocconi University. Even though it came at some personal cost, we persevered in our conviction and set to work starting from the least enjoyable part: tracing back the passages that only appear in the *Manuel* (1909), and those that from the *Manuale* (1906) did not make the transition into the French edition. To Aldo, to his serene and graceful way of making available his talent in the field of mathematical economics, to his words of advice, even the few we did not heed, a heartfelt thank you.

7. SOME ABBREVIATIONS

- A. Z. = Alberto Zanni.
e.g. = *exempli gratia* = for example.
E.N. = Editors' Note.
N. Fr. App. = Notes to the French mathematical Appendix.
V. P. = Vilfredo Pareto.
G.d.E. = *Giornale degli Economisti*, successively *Giornale degli Economisti e Rivista di Statistica*, finally *Giornale degli Economisti e Annali di Economia*.
C.W.J.M.K. = *The collected writings of John Maynard Keynes*, Macmillan St Martin's Press for the Royal Economic Society.
N.C.E. = *Nuova collana di economisti stranieri e italiani*, 12 vol. edited by G. Bottai e C. Arena.
New Palgrave = *The new Palgrave—a dictionary of economics*, 4 vol. edited by Eatwell, Milgate and Newman, London, Macmillan, 1987.
**G. Sensini,
Corrispondenza, 1948** = *Corrispondenza di Vilfredo Pareto*, Padova, Cedam, 1948.
Correspondence Walras = *Correspondence of Léon Walras and related papers*, 3 vol. Edited by William Jaffé, Amsterdam, North-Holland Publishing Company, 1965.
**A. de Pietri-Tonelli,
Corrispondenza, 1961** = *Scritti paretiani, con 47 lettere inedite di Vilfredo Pareto ad Alfonso de Pietri-Tonelli*, edited by Pietro de Pietri-Tonelli, Padova, Cedam, 1961.
**Accademia dei Lincei
1973** = International Meeting on *Vilfredo Pareto* (Roma, 25–27 ottobre 1973), Roma, Accademia Nazionale dei Lincei, 1975.
**Corrispondenza
Pareto-Pantaleoni** = *Lettere a Maffeo Pantaleoni 1890–1923*, 3 vol. edited by Gabriele De Rosa, Roma, Edizioni di Storia e Letteratura, 1962.
V. P., Œuvres = *Œuvres complètes*, many volumes of Pareto's writings edited by Giovanni Busino, Librairie Droz, Gèneve or Gèneve-Paris.
V. P., Considerazioni = "Considerazioni sui principi fondamentali dell'economia politica pura", G.d.E., I (May 1892), II (June 1892), III (August 1892), IV (January 1893), V (October 1893). After the publication of the Italian critical edition of the *Manuale-Manuel*, 2006, appeared an English translation: *Considerations on the fundamental principles of pure political economy*, R. Marchionatti and F. Mornati editors, V. Savini, J. Kinder, M. McLure translators, London and New York, Routledge, 2007.

- V. P., *Epistolario*, 1973 = *Epistolario 1890–1923*, 2 vol. edited by G. Busino, Roma, Accademia Nazionale dei Lincei, 1973.
- V. P., *Cours* = *Cours d'économie politique professé à l'université de Lausanne*, 2 vol., Lausanne, Rouge Editeur, 1896 and 1897.
- V. P., *Anwendungen der Mathematik*, 1902 = "Anwendungen der Mathematik auf Nationalökonomie", *Encyklopädie der Matematischen Wissenschaften*, pp. 1094–120 (translated in Italian by G. Sensini, "Applicazioni della matematica all'economia politica", *G.d.E.*, November 1906, and in English by R. Dehem and J. Cairncross, J. Chipman ed., "The Application of Mathematics to Political Economy", *History of Economic Ideas*, XVII/2009/1).
- V. P., *Systèmes, 1902–1903* = *Les systèmes socialistes*, Paris, Giarde et Brière, 2 vol., first edition 1902–3, second edition, with a Preface by H. Bousquet, 1926.
- V.P., *Trattato di sociologia or Sociologia* = *Trattato di sociologia generale*, Firenze, Barbera, 1916; second edition, 3 vol., Firenze, Barbera, 1923.
- V. P., *Scritti politici*, vol. I, 1974, vol. II, 1974 = *Scritti politici*, vol. I, *Lo sviluppo del capitalismo (1872–1895)*, vol. II, *Reazione, libertà, fascismo (1896–1923)*, edited by G. Busino, Torino, Utet, 1974.

For comments on the most complete bibliography of Pareto's writings, see the references in E.N. 3.

We will also resort to abbreviations for the following works:

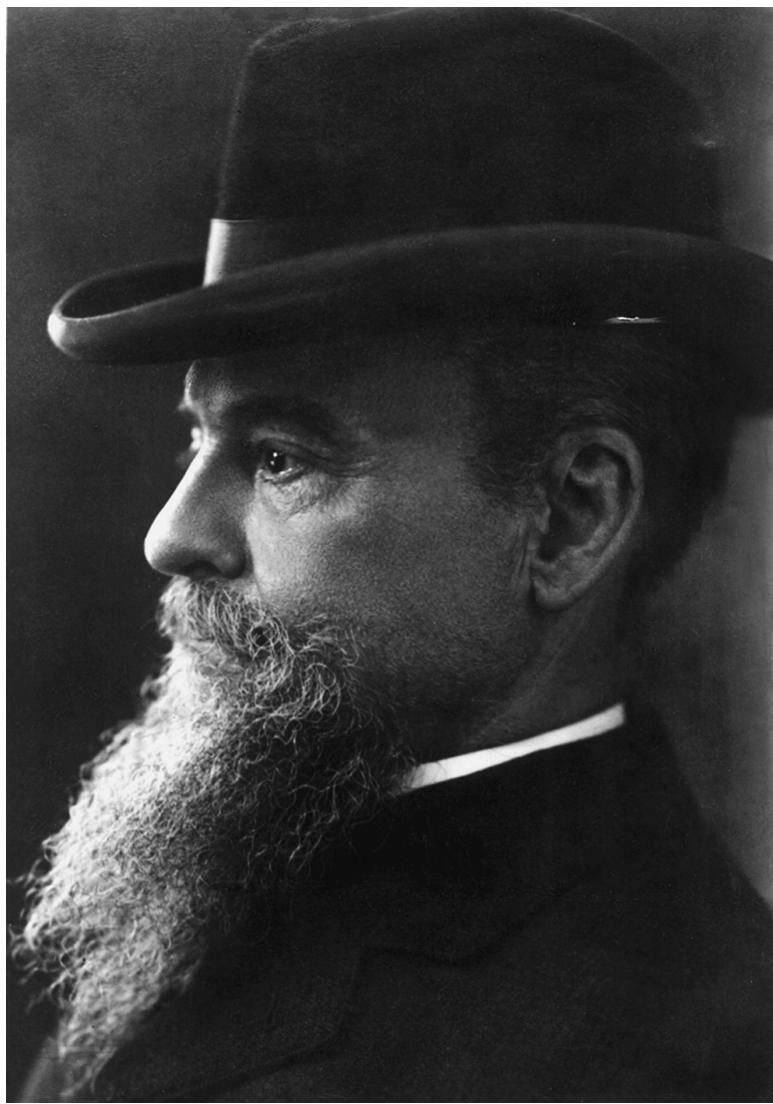
- Cournot, *Recherches* = A. Cournot, *Recherches sur le principes mathématiques de la théorie des richesses*, Paris, Hachette, 1838.
- M. Keynes, *Treatise on money*, 1930 = *A treatise on money*, vol. I, *The pure theory of money*, vol. II, *The applied theory of money*, London, Macmillan, 1930, successively in C.W.J.M.K., vol. V and VI, 1971.
- M. Keynes, *General theory*, 1936 = *A general theory of employment interest and money*, London, Macmillan, 1936, successively in C.W.J.M.K., vol. VII, 1973.
- A. Marshall, *Principles* = *Principles of economics*, 9a edit. (variorum), C. W. Guillebaud editor, London, Macmillan, 1961 (first edit. 1890).
- M. Pantaleoni, *Principii* = *Principii di economia pura*, Firenze, Barbera, 1889. An English translation is also available, but it is not identical to the Italian original. *Pure Economics*, MacMillan, 1898.
- P. Sraffa, 1960 = *Production of commodities by means of commodities, Prelude to a critique of economic theory*, Cambridge, Cambridge University Press, 1960. Italian edition, *Produzione di merci a mezzo di merci, premesse a una critica della teoria economica*, Torino, Einaudi, 1960.

In the *Editors' Notes* (E.N.s) and in the *Annotations from the 2006 Italian Edition* we shall recall these listed works in abbreviated form, followed by "cit."

In mulling, yet again, over the rich bibliography of the writings *by* Pareto (see E.N. 3) and, on the other hand, the persisting absence of a reasoned and critical bibliography of the writings *on* Pareto, the many occasions come to mind when we have put our trust in authors that were deservedly famous, only to realize that we had been sidetracked. Suffice it to mention the example of Edgeworth's box, discussed in E.N. 19. Who would not put their trust in a trio made up of Allais, Jaffé, and Georgescu-Roegen, who believe that the expression "Edgeworth's box" instead of "Pareto's box" constitutes theft from Pareto's legacy? Yet these three scholars ignored the *Cours*, Vol. I, §144, n.1, where Pareto draws two diagrams suggesting that the reader superimpose them, and limits himself to writing: "It is possible to represent geometrically the theories of exchange in quite elegant fashion by following the way opened by Mr Edgeworth . . ."

When confronted with the choice to enrich the E.N.s with a great number of bibliographical references that had not undergone any critical check, we ended up going the opposite way, renouncing the generous suggestions received also from Prof. Michael McLure. In short, we have ended up concluding many of our E.N.s with the formula SOME REFERENCES: it is meant as a wish, as an expression of trust in the future.

ALBERTO ZANNI



Vilfredo Pareto

Acknowledgement: This photograph was kindly provided by, and is printed with the permission of, the Centre de Recherches Interdisciplinaires Walras-Pareto, the University of Lausanne, Switzerland.

PICCOLA BIBLIOTECA SCIENTIFICA - 13

VILFREDO PARETO

MANUALE

DI

ECONOMIA POLITICA

CON UNA

INTRODUZIONE ALLA SCIENZA SOCIALE



MILANO

SOCIETÀ EDITRICE LIBRARIA

Via Kramer, 4 A - Gall. De Cristoforis, 54-55

1906

BIBLIOTHÈQUE INTERNATIONALE D'ÉCONOMIE POLITIQUE
Publiée sous la direction de Alfred Bonnet

MANUEL D'ÉCONOMIE POLITIQUE

PAR

VILFREDO PARETO

TRADUIT SUR L'ÉDITION ITALIENNE

Alfred BONNET

(REVUE PAR L'AUTEUR)



PARIS (3^e)
V. GIARD & E. BRIÈRE

LIBRAIRES-ÉDITEURS

16, RUE SOUFFLOT ET 12, RUE TOULLIER

—
1909

Manual of Political Economy

VILFREDO PARETO

Index of Chapters

(Numbers indicate paragraphs)

CHAPTER I, PP. 1–19

General Principles

§ 1. Possible aims for the study of political economy and sociology. – 2, 3. How the approaches to that study have been used. – 4, 5, 6. Uniformities, that is, laws. – 7. Apparent exceptions. – 8. The truth of uniformities and laws is only conditional. – 9. Conditions are partly implicit, partly explicit. – 10. A concrete phenomenon can never be known to us in all of its details. – 11. We can only have an approximate knowledge of it. – 12. It is therefore pointless to object that, in some details, the theory diverges from the concrete phenomenon. – 13. Example. – 14. Successive approximations. – 15, 16, 17. The facts cannot be known by reasoning on the concepts we have of them; it is necessary to resort to direct observation. – 18. Method of reasoning *by elimination* only; why it is erroneous. – 19. The results of theory will always differ, at least a little, from reality. – 20. Sciences that can have recourse to experience and sciences that must confine themselves to observation. A theory cannot have any criterion of truth other than the extent to which it agrees with reality. 21. Abstraction; how it is used in the sciences. – 22, 23, 24. It may have two forms; they are equivalent. – 25, 26. Science is essentially analytic; practice, synthetic. – 27, 28, 29, 30. Theory of a concrete phenomenon is only a theory of part of that phenomenon. Science disassembles the parts of a phenomenon and studies them separately; practice must reassemble the results obtained in this way. – 31. Uselessness of an exclusively negative criticism of a theory. – 32. Sometimes, for the sake of increased simplicity, we deliberately deviate, with theory, from the concrete phenomenon. – 33. Critical study of economic phenomena; where it is worthwhile, and where it is useless. – 34. Evolution. – 35. Uselessness of the discussions about the *method* of political economy. – 36. Statements that can be experimentally verified, and statements that cannot be experimentally verified. – 37, 38. Science only concerns itself with the former. – 39, 40. Anything that is a precept is not scientific, except when it is only a formal precept. – 41. There cannot be a faith more scientific than another. – 42. Confusion between science and faith. – 43. Scientific truth and social usefulness are different things. – 44. Error in considering a supposed end of evolution. – 45. Intuition; its modes and forms. – 46. The universal consensus of mankind is not at all a criterion of scientific truth. – 47. Error of the metaphysicians in wanting to transport absolute statements into scientific statements, which, by their own nature, are essentially subordinate, and which must always be intended as subject to the condition that they are only true within the limits of time and experience known to us. – 48. Absurdity of the idea of wanting to substitute science for faith. – 49, 50. Conclusions deduced from non-experimental premises. – 51. Invention.

CHAPTER II, PP. 20–71

Introduction to Social Science

§ 1. The study of sociology must still start from some empirical principles. – 2, 3. Non-logical actions and logical actions. – 4, 5. Tendency to see non-logical actions as logical and to reduce all the relations between phenomena to the relation of cause and effect. – 6. Objective relations and subjective relations. – 7, 8, 9. How and to what extent subjective relations diverge from objective relations. – 10, 11, 12. Various relations between real facts. – 13 to 15. Relations between imaginary facts and real facts. – 16, 17. How repeating an experience is useful in bringing subjective relations closer to objective relations. – 18, 19. Brief account of the theory of non-logical actions. Morals are a subjective phenomenon. – 20. Experimental research that can be usefully undertaken on moral sentiments and on religious sentiments. – 21. Relation between ethics and religion. – 22. In general, relations between non-logical sentiments. – 23. Logical relations and non-logical relations between ethics and religion. – 24 to 40. Analysis of the logical systems of ethics. They are pointless thoughts without any real content. – 41. Research that can be usefully undertaken on moral sentiments or other similar sentiments. – 42. These sentiments are essentially subjective. – 43. The dependence between these various sentiments is not a dependence based on logical reasoning but arises from these sentiments having remote and common causes. – 44. This dependence varies in time, in space, and, within the same society, according to the individuals. – 45, 46, 47. There is not one single set of morals, but there are as many as there are men. – 48. Contrast between the various non-logical sentiments, for instance, between moral sentiments and religious sentiments. How keen faith prevents one from seeing these contrasts. – 49. How and why this is not generally understood. – 50. Man tries to re-establish, between these non-logical sentiments, the logical relations that he imagines must exist. – 51, 52, 53. Certain circumstances are favorable for the development of certain sentiments, other circumstances are adverse to it; and they work in various ways according to the individuals. – 54, 55. How the sets of morals and the religions of the various social classes mutually change each other. – 56, 57. Examples from history. – 58, 59. This mutual action gives rise to rhythmic movements. – 60, 61. Evils that follow from the extension of the sentiments of the upper classes to the lower classes. – 62 to 74. Examples from history. – 75. General problem of sociology. – 76 to 79. Darwinian solution; how it is partly true and partly erroneous. – 80 to 82. Solution according to which a society is shaped in such a way as to procure the advantage of a social class. – 83. The research that aims to ascertain whether moral sentiments have individual or social origins is pointless. – 84. Knowing how sentiments arise, change, and disappear is more pressing than knowing their origin. – 85, 86, 87. Examples from history. – 88. Imitation and opposition. – 89 to 93. How the objective relations that have just been studied turn into subjective relations. – 94 to 96. The one and the same objective relation can be expressed through different subjective forms. Persistence of certain social phenomena under entirely different forms. – 97. Real movements and virtual movements. Problem lying in investigating how certain hypothetical changes in certain social facts act on other facts. – 98, 99. Analysis of this problem. – 100, 101. Objective difficulties and subjective difficulties that are encountered in this study. – 102. Society is not homogeneous. – 103. Circulation of the aristocracies. – 104, 105, 106. How the struggle between the various

social classes is interpreted in subjective terms. Objectively, the concept of the equality of man is absurd; subjectively, it plays no small part in social phenomena. – 107 [and 107bis]. How certain men, while acting to move in a direction, go instead in the opposite direction. – 108. Social and economic theories act on society not so much because of their objective value, but because of their subjective value. – 109. Prejudice of equality before the law. – 110 to 114. Social usefulness, according to men, of the variety of sets of morals and beliefs. – 115 to 123. How appearance differs from reality in the political system. Examples from history.

CHAPTER III, PP. 72–124

The General Concept of Economic Equilibrium

§ 1, 2. The object of political economy. – 3. Difficulties of the economic problem, and how it is worthwhile to use mathematics in order to remove some of these difficulties. – 4, 5, 6. How the economic problem is simplified; *pure economy*. – 7. Three parts of *pure economy* – 8, 9, 10. Economic statics. One studies a continuous phenomenon. – 11, 12. Two classes of theories; the first aims at comparing the sensations of a man, the second aims at comparing the sensations of different men. Political economy deals exclusively with the former. – 13. The way we shall follow in this study. – 14, 15. We shall study the tastes, the obstacles, and how, from their contrast, economic equilibrium is born. – 16, 17, 18. Economic goods and the sensations they provide. – 19. The elements that must be combined are tastes and obstacles. – 20, 21. Qualitative and quantitative combinations of economic goods. – 22. Definition of economic equilibrium; real movements and virtual movements. – 23, 24, 25, 26. Data pertaining to the equilibrium problem. – 27. How, in general, equilibrium is determined. – 28, 29. Men's tastes; the imperfect concept one used to have of it through the economists' notion of *value in use*. – 30, 31. How, by rectifying erroneous concepts, the theory of pure economy was born. – 32 to 36 [and 36bis]. Ophelimity. – 37. The constraints that exist between the conditions of the economic phenomenon. – 38. We shall endeavor to explain the theories of pure economy without using algebraic symbols. – 39. Direct and indirect effects of tastes. – 40 to 48. Types of phenomena derived from tastes; free competition; monopoly. – 49. Type of the socialist system. – 50, 51. How the types mix and how they must be considered. – 52 to 54. The indifference lines of tastes. – 55, 56. Indices of ophelimity. – 57, 58, 59. How the tastes of an individual are represented; the hill of pleasure. – 60, 61. How the condition of a man who successively has various quantities of an economic good may be illustrated with a path. – 62, 63, 64. Considerations on the paths; terminal points and points of tangency with the indifference lines. – 65, 66, 67. Continuous and discontinuous variations. – 68. Obstacles. – 69. First kind of obstacles. – 70, 71, 72. Transformations of economic goods. – 73, 74. Second kind of obstacles. – 75. Indifference lines of obstacles in objective transformations. – 76. The indifference lines of the producer. – 77 to 80. Analogies between the indifference lines of tastes and indifference lines of obstacles. – 81. The hill of profit. – 82. Competition. – 83. Competition in the case of exchange. – 84. Competition in the case of production. – 85. One starts by studying a community separated from the others. – 86 to 88. Modes of competition. – 89. Types of phenomena concerning

producers. – 90, 91, 92. Equilibrium in general. – 93 to 99. Equilibrium with regard to tastes; how the equilibrium on a path takes place at a terminal point, or at a point of tangency between the path and an indifference curve. – 100 to 104. Equilibrium of the producer. – 105. The line of maximum profit. – 106 to 111. Equilibrium of tastes and obstacles. – 112 to 115. General theory that determines the point of equilibrium. – 116 to 133. Modes and forms of exchange equilibrium. Various points of equilibrium. Stable equilibrium and unstable equilibrium. – 134. Maxima of ophelimity. – 135 to 151. Modes and forms of equilibrium in production. The line of maximum profit. Competition among producers. – 152 to 155. Prices. – 156, 157. Value in exchange. – 158. The price of a commodity in terms of another. – 159 to 166. Economic phenomena described using the concept of price. – 167 to 174. Prices and the second kind of obstacles. Constant prices and variable prices. – 175. The budget of the individual. – 176. The budget of the producer – 177, 178, 179. The cost of production. – 180 to 183. Supply and demand. – 184. Supply curve and demand curve. – 185, 186, 187. Supply and demand depend on all the circumstances of economic equilibrium. – 188 to 192. Equality between supply and demand, at the point of equilibrium. – 193. Ways in which supply and demand vary. – 194. Equality between the cost of production and the selling prices. – 195. Stable equilibrium and unstable equilibrium, with the concepts of supply and demand. – 196 to 204. Exchange equilibrium, in the general case. – 205 to 214. Exchange and production equilibrium, in the general case. – 215, 216. Stable and unstable equilibrium in the general case. – 217, 218. How and why the use of mathematics is helpful. – 219 to 226. Errors that had their origin from neglecting the use of mathematics where this use was indispensable. – 227. The search for one sole cause of value is useless and inconclusive. – 228. Only pure economy has so far succeeded in giving us a synthetic conception of the economic phenomenon.

CHAPTER IV, PP. 125–44

Tastes

§ 1. Aim of the present chapter. – 2 to 7. Tastes and ophelimity. Consumption is considered as voluntary only. – 8. Independent consumption and dependent consumption. Two kinds of dependence. – 9 to 13. Study of the first kind of dependence. It divides into two cases. – 14 to 18. Study of the second kind of dependence. – 19. Hierarchy of commodities. – 20 to 23. Ways of considering the second kind of dependence. Equivalences in consumption – 24. How extensive the phenomenon of the dependence of consumption is. – 25, 26. We can study the economic phenomenon only within a small region surrounding the equilibrium point. – 26 to 28. Indifference curves change as time and circumstances vary. – 29 to 31. Deviations between the theoretical phenomenon and the concrete phenomenon. – 32. Ophelimity and its indices. – 33, 34. Characteristics of ophelimity for independent consumption. – 35, 36. Dependent consumption. – 37 to 42. Characteristics of ophelimity in general. – 43 to 47. Characteristics of indifference curves. – 48 to 53. Relation between ophelimity or indifference curves and supply and demand. Relations with the income of the consumer. – 54 to 65. Various shapes of indifference

lines and of exchange lines. Consideration of the various kinds of dependence. – 66 to 68. The phenomenon of ophelimity, in general. – 69, 70. The hill of ophelimity.

CHAPTER V, PP. 145–72

Obstacles

§ 1. The study of production is more complex than the study of tastes. – 2 to 7. Division of labor and the firm. – 8 to 10. The objective towards which the firm strives. – 11. How, while aiming at a goal, it can sometimes reach another one. – 12. The type of the socialist system. – 13 to 16. The various courses open to the firm. – 17 to 24. Capital goods. How this concept is not rigorous; and how it can be made so. – 25 to 29. The theory of economic equilibrium with and without the concept of capital. – 30, 31, 32. Depreciation and insurance. – 33. The services of capital. – 34. Material and nonmaterial goods. – 35, 36, 37. Production coefficients. – 38. Transformations in space. – 39 to 42. Transformations in time. – 43 to 47. The budget of the firm and transformations in time. Various ways of considering these transformations. – 48 to 51. Interest on capital. – 52 to 57. Net interest and its causes. – 58. Net interest on different capital goods. – 59, 60. The balance sheet of the firm and yields on capital goods. – 61. The budget of the firm and the entrepreneur's labor and capital – 62 to 65. The firm and the owner of economic goods. – 66 to 69. Actual firms and their profits and losses. – 70 to 75. Variability in production coefficients. – 76, 77. Possible offset between variations in different coefficients. – 78 to 80. Distribution of output. – 81 to 87. General equilibrium of production. – 88. Production of capital goods. – 89. Successive equilibrium positions. – 90. *Rent*. – 91, 92. Acquired rent. – 93, 94, 95. Ricardo's *rent*; its relationship with the cost of production. – 96, 97. How this special case is part of the general theory of production.

CHAPTER VI, PP. 173–91

Economic Equilibrium

§ 1 to 18. Examples of equilibrium. Various forms of the law of the cost of production. How competition works. – 19 to 25. Usual forms of the indifference curves for exchange and production; Commodities with an increasing cost of production and commodities with a decreasing cost of production. – 26. Equilibrium of tastes and production. – 27 to 31. Equilibrium in general. Greater approximation to the concrete phenomenon. – 32, 33. Properties of equilibrium. Maximum of ophelimity. – 34 to 38. Properties of the equilibrium of exchange. When and how the maximum of ophelimity is reached – 39, 47. Properties of the equilibrium of production. When and how the maximum of ophelimity is reached. – 48. Argument in favour of collectivist production. – 49 to 51. How free competition works to determine production coefficients and to have the net interest for the various capital goods tend to be equal. – 52 to 61. Economic equilibrium in the collectivist society. – 62, 63, 64. Maxima of ophelimity for sectorial communities. – 65 to 69. Pure theory of international trade. – 70. Equilibrium and prices. – 71 to 79. Quantity

theory of money. How prices vary. – 80 to 89. Relations between equilibrium, the prices of factors of production, and the prices of products. – 90, 91. Subjective interpretation of the phenomena that have just been studied. – 92. Economic circulation. – 93 to 96. False interpretations of the competition among entrepreneurs. – 97 to 101. Erroneous conceptions of production.

CHAPTER VII, PP. 192–219

Population

§ 1. The economic phenomenon begins from man and ends up back to man. – 2. Social heterogeneity. – 3 to 10. The mean and the distribution of the deviations. The curve of errors. – 11 to 17. The curve of the distribution of incomes. – 18 to 22. Social circulation. – 23 to 25. Within certain limits of time and space, changes in the shape of the curve of incomes were very slight. – 26. The lower part has changed more than the upper part. – 27 to 31. Theoretical consequences of the facts that have just been noted. – 32 to 45. Relations between economic conditions and population. – 46, 47. How neglecting to consider economic crises may lead to very serious mistakes. – 48. Theory of correlation. – 49, 50. Effects of an increase in economic prosperity. – 51 to 56. The effect of a variation in the amount of wealth may be entirely different from the effect of the amount of this wealth. Study of this latter effect. – 57 to 60. Production of personal capital goods. – 61. Cost of production of the adult man. – 62 to 67. Obstacles to reproductive power. Exceptional increase in population in the 19th century. – 68 to 70. Means of subsistence and population. – 71 to 80. Nature of obstacles. Their direct and indirect effects. – 81 to 88. Subjective view of phenomena related to an increase in population. – 89 to 96. Malthus and his theories. – 97 to 101. Human society in general. The main facts that determine its nature are: hierarchy – the alternation of aristocracies – selection – the average proportion of wealth, or of capital goods, per individual. – 102, 103. Quantitative conditions for the utility of society and individuals. – 104 to 115. Stability and selection. Principle of stability and principle of mutability. – 116, 117. Subjective interpretation of the preceding facts.

CHAPTER VIII, PP. 220–31

Landed Capital and Capital Goods Proper

§ 1. Landed capital. – 2. The competition that exists between lands. – 3, 4. Main characteristics: *rent*. – 5 to 7. Forms of ownership and relationships between those who own the land and those who work it. – 8. Capital goods proper. – 9, 10. Savings. – 11. The production of savings is not determined by the yield that is obtained from savings. – 12 and 12bis to 16. Various kinds of savings, according to the use that is made of them. – 17 to 21. Yield on savings and the social system. – 22 to 25. Subjective interpretation of the phenomena. – 26, 27, 28. The alleged law of declining yield on capital. – 29. Money; various kinds of money. – 30. Functions that money performs. – 31 to 34. Foreign

exchange. – 35 to 39. The exchange rate and international trade. The circulation of paper currency. Equivalent positions of equilibrium. – 40. Gresham's law. – 41. Bimetallism. – 42. Money substitutes. – 43. Metallic money is a very small part of the wealth of a country. – 44, 45. Quantity of metallic money in circulation; and consumption of gold and silver for industrial purposes. – 46 to 50. Banks. The *guaranty* of banknotes of the banks of issue. The discount rate.

CHAPTER IX, PP. 232–72

The Concrete Economic Phenomenon

§ 1. How we move from the study of the abstract phenomenon to the study of the concrete phenomenon. – 2. Deviations between the abstract phenomenon and the concrete phenomenon, in consumption. – 3 to 6. Retail and the damage to society caused by its imperfect organization. – 7. Variations in retail and wholesale prices. – 8 to 15. Trusts and Syndicates. – 16. Collective agreements on production. 17 to 19. Some men spend their efforts to appropriate the goods produced by the others. – 20 to 22. This uniformity, which has been observed so far, shows no sign whatsoever of disappearing; its form may change, but its substance stays the same. – 23, 24. How and why in other times high prices were considered damaging, whereas now they are considered good for society. – 25 to 35 [and 32bis and 35bis]. Evolution that leads to the creation of a new caste. – 36 to 38. How the present economic state – and probably that of the future – is a mixture of competition and of *restrictionism*. – 39. International trade. – 40 to 54. Economic theory of international trade. – 41. Say's *law of markets*. – 42 to 52. Ricardo's theory of *comparative costs*. – 53. Duty on exports. – 54. Duty on imports. – 55. Indirect economic effects. – 56. Distributive effects. – 57. Social effects. – 58, 59. Fiscal effects. – 60. How complex the problem is of knowing whether free trade is preferable than protection. – 61. The fact that protection necessarily leads to a destruction of wealth is not enough reason to condemn it. – 62 to 66. Causes of protection. – 62. Ignorance of the findings of the economic science cannot be included among these causes. – 63 to 65. The members of leagues that procure protection. – 66. How it is that the attack to establish protection is easier than the defense to repel it – 67. Protection cannot be condemned for the only reason that it is advocated by those who benefit from it by appropriating other people's goods. – 68. The effects of protection cannot be known empirically. – 69. Exception that provides confirmation of what the theory tells us. – 70. Empirical errors with regard to protection. – 71, 72. How the destruction of wealth caused by protection could be offset, in some cases, by the increase in wealth depending on other causes. – 73, 74. Economic crises. – 75. Crisis is nothing but a particular case of the law of rhythm of economic and social phenomena. – 76, 77. How production adapts to consumption. – 78, 79. Two main kinds of causes of economic crises. – 80. Upward phase and downward phase. – 81. Crises cause less damage than is usually believed. – 82. The facts that are concomitant with the crises are erroneously considered as causes of the crises. – 83, 84. The supposed excesses of consumption, production, intensity of circulation, and use of credit. – 85 to 87. Symptoms of a crisis. JUGLAR's and DES ESSARS' theories. – 88. JEVONS' theory.

APPENDIX (1906), PP. 273–308

§ 1. Aim of the Appendix. – 2 to 4. Lines of indifference. – 5, 6, 7. Surface of the indices of pleasure, or of ophelimity, and its level curves. – 8. When it is possible to determine ophelimity through experience. – 9. Equivalence of elementary ophelimities. – 10. Properties of indifference lines. – 11. Characteristics of the indices inferred from those of the indifference lines. – 12. Another characteristic of ophelimity. – 13. Characteristics of indifference curves deduced from those of ophelimity. – 14. Several economic goods, the consumption of which is independent. – 15. Various kinds of dependence. – 16. Difficulty in the use of mathematics. – 17. Other lines that can be substituted for the indifference lines. – 18, 19. Obstacles. – 20 to 22. Equilibrium of exchange for two individuals. – 23. Nature of the equations that determine this equilibrium. – 24. Constant prices. – 25. Monopoly. – 26 to 29. Equilibrium of exchange in the general case. – 30. Equilibrium of exchange with constant prices. – 31 to 33. Production. – 34, 35. Equilibrium of production. – 36. Equilibrium of production in the case of free competition with variable prices. – 37, 38. Equilibrium of production in the case of free competition with constant prices. – 39. How production coefficients and allocation are determined. – 40. Individual production with constant prices. – 41, 42. Individual production with maximum ophelimity. – 43, 44. Collective production. – 45. Collective production with maximum ophelimity. – 46. Exchange. – 47. Properties of equilibrium. – 48, 49. Finite variations in the case of exchange.

APPENDIX (1909), PP. 309–97

§ 1. Aim of the Appendix. – 2 to 4. Indifference lines. – 5 to 7. Differential equation provided by observation. – 8, 9. Integral of the equation and its correspondence with pleasure. – 10 to 19. How it is possible to infer ophelimity from observation. – 22, 23. Equilibrium in the case of one individual and two goods. – 24, 25. The case of several economic goods. – 26, 27. Equation of the obstacles. – 28 to 33. Monopoly of one individual. – 34. Free competition. – 35. Comparison between the cases that have been considered. – 36. Three goods. – 37. Prices. – 38. Budget. – 39. Variable prices. – 40. Example. – 41. Equilibrium in the case of one individual, any number of goods, and constant prices. – 42. Ophelimity and supply and demand curves. – 43. Economic equilibrium inferred from supply and demand curves. – 44, 45. Properties of indifference lines. – 46 to 50. Characteristics of the indices. – 51. Complexity of indifference lines. – 52 to 55. The general laws of supply and demand. – 56 to 62. Error of the economists who considered the ophelimity of money as constant. – 63 to 67. General case of exchange with constant prices. – 68. Monopoly of one individual and one commodity. – 69, 70. Monopoly of two individuals and one commodity. – 71. Monopoly of two individuals and two commodities. – 72 to 76. Considerations on these different cases of monopoly. – 76bis. Example. – 77. Production. – 78. Production coefficients. – 79. Costs of production. – 80. Consumers' equilibrium. – 81. Equilibrium of the firms. – 82. Equilibrium of production. – 83. Free competition. – 84 to 88. Monopoly in production. – 89. Definition of maximum of ophelimity for a community. – 90 to 92. Determining the maximum

of ophelimity of production. – 93 to 100. Example. – 101 to 106. Variability of the production coefficients. – 107. Distribution of the quantities among firms. – 108. Usual errors with regard to the production coefficients. – 109 to 115. Properties of economic equilibrium and its relations with the maximum of ophelimity. – 116. The case of one individual. – 117 to 120. The conditions for the maximum of ophelimity. – 121 to 126. The maximum of ophelimity in relation to finite displacements. – 127 to 129. Economic meaning of the maximum of ophelimity. – 130 to 132. An overview: constraints. – 133. Decomposition of the economic system. 134 to 136. Index-functions. – 137. The indices of ophelimity. – 138. Index-functions in general. – 139 to 141. Economic types. – 142. Second derivatives of the index-functions. – 143. The system of equations that determines economic equilibrium. – 144. Properties of economic equilibrium; economic antagonisms. – 145 to 150. Maximum of ophelimity. – 151. Finite variations. – 152. The different points of equilibrium.

NOTE^[a]

When a section of a chapter is quoted in that chapter, the section is indicated simply by the sign §. If the section belongs to another chapter, the chapter number is given before the section mark.

Examples: In Chapter I, p. 3, (§4) indicates section 4 of that chapter. Again, in Chapter I, p. 24, (II, 6) indicates section 6 of Chapter II.

In the quotations, *Cours* indicates my *Cours d'économie politique*, Lausanne, 1896–7; and *Systèmes socialistes* indicates my book, *Les Systèmes socialistes*, Paris, 1903.

PREFACE TO THE FIRST (1906) EDITION^[a]

The aim of the present Manual is to set forth the principles of political economy in a volume of small dimensions.¹ I have begun by providing a very brief outline of these principles, solely for the purpose of showing, by means of examples rather than by a bare statement, how economic phenomena stand in relation to other social phenomena.

The subject matter is set out somewhat differently here than it was in the *Cours* which I published some years ago; and it behoves me to provide the reader an explanation of that difference; which I will do unconstrainedly, without allowing myself to be held back by regard for anything or anyone, and as if the *Cours* had been written by someone else.

First of all, the part dealing with pure theory diverges more markedly from the so-called classical methods in the present Manual than in the *Cours*. In the *Cours*, the division of the economic phenomena into exchange and production is still fundamental; true, such a division is perhaps the best that can be obtained empirically, but it is perhaps not equally satisfactory in a scientific investigation of the intrinsic relationships in the phenomena. In order to reach such a goal, it seems to me better to avail oneself of the way followed in the present Manual, which is to consider the economic phenomena as arising out of the contrast between human tastes and the obstacles encountered in satisfying them—a route which also leads immediately to shedding light on the concept of economic equilibrium.

In the *Cours*, the concept of capital also remains fundamental, as it was in so-called classical economics and still is in the new mathematical economics. And here too I shall reiterate that it would be difficult to consider the phenomena empirically in a very different way. But scientifically, the loose concept of capital must give way to the much more precise one of the transformations of economic goods. This approach does not mean that we are foregoing the benefit that can be obtained from empirical doctrines; but we come back to them after having conferred precision and rigor on the concepts which they use. It is convenient to speak about capital, but that must be done only after the real objects to which this term corresponds have been clearly defined; and besides, it is extremely useful to show how the whole theory of economic phenomena can be established without any need to have recourse to the term and the concept of *capital*.

Similarly, the concept of *price* is not essential, and it is possible, although with greater difficulty, to do without it. That is much better brought out here than in the *Cours*.

Concretely, economic phenomena almost always occur at constant prices for the successive portions exchanged; variable prices are much less frequently observed. For that reason, in the *Cours* I considered the former almost exclusively; and while this may be sufficient for an empirical study, in a scientific study we must on the contrary consider variable prices, not only because in this way the nature of the phenomena emerges better but also because the empirical material itself, thanks to such considerations, becomes clearer and can be better understood. Here I have therefore given ample scope, within the limits of the restricted space available, to the study of variable prices.

I say nothing of other minor divergences which the discerning reader will immediately see should he wish to compare the present Manual with the *Cours*.

Next, the study of concrete phenomena is also different in the two works; and not only do we find, as in theory, different ways of considering the same thing, but in some passages of the *Cours* there are erroneous ways.

Such errors spring from two main sources. The first is an incomplete synthesis in working back from the scientific analysis to concrete doctrine. The author has noted the need for such a complete synthesis, but then, without realizing it, has partly ignored it on some occasions, if not explicitly, at least implicitly. The example of free trade and protection will serve for all these cases. Scientifically, it can be proved that protection² usually causes a destruction of wealth. The study of past and present facts shows that protection is brought about in large part through the efforts of those who derive advantage from it in appropriating other people's property. But is that sufficient to condemn protection concretely? Certainly not; it is essential to note the other social consequences of that regime and to take a stand only after having completed this study.

I believe that such an answer would also have been given by the author of the *Cours*; hence, the mistake is not really explicit, but the author often expresses himself as if, in the concrete case, free trade were good in all cases and protection bad in all cases, and such assertions presuppose that the starting point is some proposition which is vitiated by the error in question.

The defective synthesis also comes out in other cases, for example in the footnote to §221 [of the *Cours*]³ in which the increase in the English public debt is blamed on wars which are damned out of hand as pointless and capricious. Nor can the author be excused by pointing out that that footnote reproduces someone else's writings; because the author, by failing to disassociate himself from the opinions expressed in that work, shows that he endorses them, at least in part.

Moreover, this is not only a sin of omission since, throughout the *Cours*, it can be seen that here and there the author regards peace and economic and political freedom as the best means of obtaining people's welfare. But he does not and cannot provide any scientific proof of that proposition, that is, a proof which is based solely on facts; hence that belief transcends, at least for the time being, objective reality, and seems to originate largely in sentiment. Precisely for that reason, it should have been absolutely excluded from a work whose sole aim was to study the facts scientifically; and the author should not have allowed it to intrude, even if only in passing and casually.

The other source of errors is less obvious, and is widely—indeed almost universally—regarded as a source not of error but of truth. It consists in having considered the propositions enunciated from an almost exclusively objective point of view without taking due account of the subjective phenomena.

Let there be given any proposition—and to leave it indeterminate we shall express it by saying "A is B." It is customary for the theorist to hold the view—whereas empiricists instinctively avoid such errors—that the effect of the proposition on a society depends solely on whether it is objectively true or false and on whether or not it is known to the bulk of the people making up that society. Hence, when we have found what we believe is the truth, our only remaining preoccupation is to propagate this doctrine. This opinion, which is extremely general, stands out fairly clearly in the writings of the French *philosophes* of the end of the 18th century, and in many of the writings of the

so-called classical economists. It blends perfectly with the sentiments of humanity and philanthropy, and would that it were really true! But unfortunately there is perhaps no opinion in all of social science which is more at odds with the facts and clashes with them. In case upon case, men's faith, or belief, by its very nature and independently of its objective content, impels them to act in a certain way. To the two investigations to which reference has been made, I must therefore add a third, and inquire into the effect of certain beliefs on men. Then it will be of advantage to study the ways in which these beliefs arise and spread—which two processes exist by themselves and without necessarily being linked to the objective reality of the proposition “A is B” to which these men subscribe.

All this was understood by the author of the *Systèmes socialistes*, but not always borne in mind by that same author while he was writing the *Cours*; which therefore turns out to be defective in that respect.

In the present volume, I have tried as best I could to avoid similar errors.

None of the facts cited in the *Cours* has, as far as I know, been shown by the criticism of others to be incorrect.¹ Nor has this been revealed by a searching revision to which I have subjected the work. If I had discovered that any of these facts did not hold, I would not have the slightest hesitation in adding a confession of that error to my present confession of other errors.

The facts from which a theory is deduced can be selected from the past or from the present. Each of these approaches has its merits and defects. Present-day facts are often better known, especially in detail, and hence, objectively, they are sometimes to be preferred to those of the past. On the other hand, they have a stronger impact on the passions, and hence, subjectively, they are less helpful; and past facts can usefully replace them.

Anyone who aims to convince others that A is B must take the greatest care to avoid any unnecessary conflict. He has of necessity to battle with those who affirm that A is not B; but he does not feel the same necessity to contradict someone who believes that X is Y, or that T is U. Hence, if for the purpose of proving that A is B he can call up on either the former or the latter proposition, it is to his advantage to choose the proposition which is repugnant to the least number of persons he seeks to convince. Suppose that the proposition “A is B” is objectively proved much more effectively by recourse to the proposition “X is Y” than to the proposition “T is U”; and suppose moreover that the former proposition is absolutely repugnant to the sentiments of the people one wants to convince, whereas the latter is a matter of indifference to them. Anyone whose aim is solely to discover the relations between objects, in order to prove that A is B, prefers to start from the proposition “X is Y.” But anyone whose aim instead is to persuade others—to propagate the science—will prefer to start from the proposition “T is U.”

¹ Without in any way contesting the correctness of the facts some worthy persons, and also some hypocrites—some wolves in sheep's clothing—criticized me for having reported them. Because I spoke the truth about the second-rate politicians who are tearing Italy apart, I was accused of having slandered Italy.⁴ This pettiness leaves me unmoved, and I laugh it to scorn with a quotation from Boileau:

“Qui méprise Cotin n'estime point son roi

* Et n'a, selon Cotin, ni Dieu, ni foi, ni loi!”⁵

In the present work, my sole intention is to seek out the relations among the facts, and the uniformities or laws which are observed in those relations; for this reason I have always had a preference for the first kind of proof; and only when two proofs are equally good objectively have I chosen the one which is less likely to offend the sentiments of the greatest number.

In order to prove certain propositions of economic science, it is necessary to have recourse to mathematics. The proofs of that nature have been collected in the *Appendix*, and all the rest of the volume may be read even by those who are not familiar with mathematical disciplines.

CHAPTER I

General Principles

1. Among the aims that the study of political economy and sociology^[a] may pursue, we may note the following three: (1) Such a study may consist in a collection of recipes¹ that are useful to individuals and public authorities in their economic and social activity. The author has then only this usefulness in view, in the same way that the author of a handbook on the breeding of rabbits simply tries to be useful to those who breed these creatures. (2) The author may possess a doctrine that he considers excellent, and such as to procure all kinds of benefits to a nation, or even to mankind, and he may set out to propagate it, with apostolic zeal, in order to make people happy, or simply, as a well-worn expression has it, “to do a bit of good.” The aim here is still utility, but a more general, less down-to-earth kind. Between these two types of study, there is, generally, the difference that may exist between a collection of recipes and a treatise on morals. It is exactly the same, but rather less obvious, when the author takes for granted the doctrine that he considers to be the best one and simply declares that he is studying the phenomena for the good of humanity.¹ By following such a path, botany would study plants in order to identify the ones that are useful to man, geometry would study lines and surfaces for purposes of land surveying, etc. It is true that sciences have begun in this way; they were initially arts, but gradually set about studying the phenomena independently of any other aim. (3) The author may have as his sole aim to study and ascertain the uniformities present in the phenomena, i.e., their laws (§4), without having any direct practical usefulness in view, without being concerned with providing recipes or precepts, without even seeking the happiness, the utility, or the welfare of mankind, or of one part of mankind. The aim, in this case, is exclusively scientific: it is to seek knowledge and nothing else.

¹ In 1904, Mr. G. de Greef still gave the following definition (*Sociologie économique*, p. 101): “Economics is that fundamental part of social science which has as its object the study and the knowledge of the working and the structure of the nutritive system of societies, with a view to their preservation and also of their perfection through the progressive reduction of human effort and of dead weight, and through increase in the net results, in the interest and for the common happiness of the individual and the species organized around society.”

[On this we may observe:]² (1) First, it is strange to use a metaphor (*nutritive system*) as a definition. (2) Is economics not concerned with the production of poisons, the construction of railways, railway tunnels [such as the Saint Gothard,]² battleships, etc.? If not, which science should these concern? If so, are all these fine wares eaten by society (*nutritive system*)? What an appetite! [May Santa Lucia preserve Madame Society's eyesight!]³ (3) This study is made with a practical humanitarian aim (*in view*); it is thus the definition of an art and not of a science. (4) As we know, definitions are not to be disputed; therefore, they cannot contain any theorems. Our author has, nevertheless, included a number of them. There is the improvement that will result from the reduction of *dead weight* (which of course includes capitalists, who are thus condemned by definition), and there is a common happiness of the individual and the species, and he thus, by definition, disposes of the difficult problem of finding out when this common happiness exists, and when, on the contrary, the happiness of the individual is in opposition to that of the species, or conversely. We could still make a number of observations about this definition, but I shall restrict myself to the above.

I should warn the reader that in this Manual I shall be concerned only with procuring this third objective. This does not imply in the least that I mean to run down or disparage the other two; I only wish to make a distinction among alternative ways of analyzing the material, and to [indicate the one that will be adopted in this book].⁴

I also warn the reader that I shall endeavor, as far as possible—and knowing how difficult it is, [I fear I shall not be able to attain my goal]⁵—to use only words that clearly fit real, well-defined objects, and never to use words that may influence the reader. I repeat that I do not in the least wish to run down or disparage the latter procedure, which I hold, on the contrary, to be the only one that is capable of carrying conviction with a large number of individuals, and which must be resorted to if one is aiming at this result. But in this book, I am not trying to convince anybody; I am looking for the uniformities in the phenomena. Those with another objective will have no difficulty in finding plenty of books that will give them satisfaction; they may [draw nourishment from these and]⁶ dispense with reading this one[—which, as Boccaccio said of his tales, will not run after anybody to get read. As the proverb says, the world is beautiful because it is varied.]⁶

2. In nearly every branch of human knowledge, the phenomena have been studied from the three points of view we have just indicated; and, usually, the chronological order of these points of view corresponds to that of my enumeration; however, the first one is often mixed with the second, and, in certain very practical matters, the second one is of little use.

Cato's work, *De re rustica*, belongs to the first kind; in the Preface, however, he sometimes adopts the second point of view. The works published in England around the end of the 18th century advocating new methods of cultivation belong, in part, to the second kind and in part to the first. The treatises on agricultural chemistry and on other similar sciences belong to a large extent to the third kind.

Pliny's natural history contains chemical and physical recipes; other recipes are to be found in books on alchemy. Modern works on chemistry, on the other hand, belong to the third kind.

3. In most works devoted to political economy, the three methods are still in use, and science has not yet been separated from art. Not only is this third point of view not clearly and frankly adopted in the treatises on political economy, but most authors disapprove of the exclusive use of this method. Adam Smith clearly states that “political economy, considered as a branch of the science of a statesman or legislator, proposes two distinct objects: first, to provide a plentiful revenue or subsistence for the people, or, more properly, to enable them to provide such a revenue or subsistence for themselves; and, secondly, to supply the state or commonwealth with a revenue sufficient for the public services. It proposes to enrich both the people and the sovereign.”⁷ This would amount to adopting our first point of view exclusively; fortunately, Smith does not keep to his definition, and most of the time he adopts our third point of view.

John Stuart Mill declares that “Writers on Political Economy profess to teach, or to investigate, the nature of wealth, and the laws of its production and distribution.”⁸ This definition belongs to the third kind; but Mill often adopts the second point of view and preaches the cause of the poor.

M. Paul Leroy-Beaulieu says he has reverted to Adam Smith's method. Perhaps he has even gone further. In his *Traité*,⁹ he mostly keeps to the first method; he sometimes uses the second one, but rarely the third.

4. Human actions reveal certain uniformities, and it is only because of this property that they can be the object of scientific inquiry. These uniformities bear still another name; they are called *laws*.

5. Anyone studying a social science, anyone who makes an affirmation about the effects of such an economic, political, or social measure, implicitly acknowledges the existence of these uniformities; otherwise, his inquiry would have no object, his affirmations would be baseless. If there were no uniformities, one could not, with any degree of approximation, draw up the budget of a state or of a municipality any more than one could, say, that of an industrial company.

Some authors, although not acknowledging the existence of economic uniformities (laws), propose nevertheless to write the economic history of the people of such and such a country; but this is an obvious contradiction. In order to make a choice among the facts that occurred at a moment in time, and to separate those that are to be retained from those that are to be discarded, the existence of certain uniformities must be acknowledged. If the facts A, B, C, . . . are to be separated from the facts M, N, P, . . . , this is because it has been observed that the former occur in a uniform sequence, whereas this is not the case with the latter; to assert this is to assert a law. If the man who describes the sowing of wheat does not acknowledge the existence of uniformities, he will have to tell us, for instance, whether the sower's hair is red or black, as well as to note that sowing takes place after plowing. Why is the first fact omitted, and the second taken into account? Because, one would say, the first has nothing to do with germination or the growth of wheat. But what does this mean? Simply that wheat germinates and grows in the same way, whether the sower's hair is black or red, i.e., that the combination of these two facts presents no uniformity. On the contrary, such a uniformity exists between the fact of the soil having or not having been tilled and the fact that the wheat grows well or badly.

6. When we assert that A has been observed together with B, we do not usually say whether or not we consider this coincidence fortuitous. It is on the strength of such an ambiguity that those writers learn who want to construct a political economy while denying that it is a science. If you point out the existence of a uniformity or a law, they reply: "We are simply relating what happened." But, after having secured acceptance for their proposition in this sense, they use it in another and state that the economic or social phenomena A and B have in some cases been linked in the past, one draws the conclusions that they will also be linked in the future, one is obviously in the process of asserting a uniformity, a law; after that it is ridiculous to seek to deny the existence of economic and social laws.

If one does not admit that there are uniformities, knowledge of the past and the present is mere curiosity, and nothing may be inferred from it regarding the future; the reading of [a knightly romance or of the *Three Musketeers*]¹⁰ would have the same value as that of Thucydides' history. If, on the contrary, from knowledge of the past, one claims one can draw the slightest inference regarding the future, it is because one admits, at least implicitly, the existence of uniformities.

7. Strictly speaking, there can no more be exceptions to economic and social¹¹ laws than to other scientific laws. A uniformity that is not uniform is meaningless.

But scientific laws have no objective existence. The imperfection of our minds does not permit us to apprehend the phenomena in their entirety,² and we are obliged to study them separately. Consequently, instead of general uniformities, which are and will always remain unknown, we are compelled to consider an infinite number of partial uniformities, which crisscross each other, are superimposed on and clash with each other in a thousand ways. When one of these uniformities is considered, and when its effects are modified or hidden by the effects of other uniformities, which we do not intend to consider, we usually say, although the expression is improper, that the uniformity or the law that is considered admits of exceptions.^[a] If this way of putting the point is accepted, physical [and chemical]^[13] laws, and even mathematical laws,³ admit of exceptions in the same way as economic laws.

According to the law of gravity, a feather thrown up into the air should fall toward the center of the earth. On the contrary, under the influence of the wind, it is often blown upward. One could thus say that the law of gravity admits of exceptions; but this is an improper expression, which physicists do not use. We are simply in the presence

² [A highly talented author, Benedetto Croce, made certain criticisms on the occasion of the publication of the Italian edition of this work. These criticisms should be noted here, not in a polemical spirit, which is usually quite pointless, but because they may serve as examples to clarify general theories.

[This author writes: "What does the *imperfection* of the human mind mean? Does one, by any chance know of a perfect mind, in comparison with which one could show that the human mind is *imperfect*?"

[One might reply that, if the use of the term "imperfect" is legitimate only when one can, by antithesis, point to something "perfect," the term "imperfect" should be banned from the dictionary; one will never have the opportunity of using it: perfection is not of this world, people say.

[But this reply could be only a formal one. One should go deeper, and see what lies behind the words.

[Croce, being a Hegelian, has obviously been offended by my coupling the offensive epithet *imperfect* with the human mind. The human mind cannot possibly be imperfect, so he argues, as it is all that exists in the world.

[But if one takes the trouble to look into the meaning of the terms in our text, one will immediately notice that the meaning remains absolutely the same if, instead of saying: "The imperfection of our minds does not allow us, etc.," one were to say: "The nature of our minds does not allow us, etc." In a discussion that is objective and not purely verbal, it is thus useless to adhere to the term: *imperfection*.

[It could then be objected that, if you admit that the term *imperfection* is not essential to express your thinking, why do you not cut it out of the French translation? You could thus without much effort please the admirers of the human mind.

[This calls for some general observations that should be made once and for all.

[The use of ordinary language, instead of the technical language of certain sciences, has great disadvantages, its lack of precision not being the least of them. But it has some advantages too; and, if we must suffer from the first, we may as well benefit from the second. Among these is the ability to suggest, by the use of a word, accessory connotations which, if they were developed at excessive length, might distract attention from the main subject.

[The use made here of the word *imperfection* suggests that it refers to something that might possibly be more or less imperfect, that varies by degrees. Indeed, men may consider a more or less extensive portion of the phenomena: some synthetic minds embrace a larger part of them than the other, more analytic minds; but, in any case, they are all able to embrace only an often very restricted part of the whole.

[These considerations are secondary; they have a place in a footnote; they could not be inserted in the text without detriment to the clarity of the argument.]¹²

³ Suppose that a mathematician can observe, at the same time, Euclidean and non-Euclidean space. He will see that the theorems of geometry which depend upon Euclid's postulate do not hold in the latter case; consequently, by accepting the form of expression discussed in the text, he will say that these theorems involve exceptions.

of other phenomena that are superimposed upon those to which the law of gravity refers.⁴

8. A law or a uniformity is true only under certain conditions, which serve precisely to indicate which phenomena are to be singled out. For instance, chemical laws that depend upon affinity are different according to whether the temperature is kept within certain limits, or whether it exceeds those limits. Up to a certain temperature, two bodies will not combine; above that temperature, they combine; but if the temperature rises beyond a certain limit, they dissociate.

9. Some of these conditions are implicit; others are explicit. Among the first, one should include only those that can be easily inferred by everybody without the least possibility of error; otherwise, we should have a conundrum and not a scientific theorem. There is no proposition that cannot be verified under certain specified conditions. The conditions of a theorem¹⁴ are an integral part of the theorem¹⁴ and cannot be separated from it.

10. We do not know, and may never know, a concrete phenomenon in every detail; a gap always remains.⁵ This is sometimes materially verified. For instance, it was thought that the entire composition of the atmosphere was known; but one fine day argon was discovered, and a little later, in the same way, a large number of the other gases were discovered in the atmosphere. What could be simpler than the fall of a solid body? However, we do not know and shall never know all its particulars.

11. A great many important conclusions flow from the preceding observation.

As we do not have complete knowledge of any concrete phenomenon, our theories of these phenomena are only approximative. We know only ideal phenomena, which more or less approximate the concrete phenomena. We are in the position of someone who knows of an object only through photographs; however perfect, they will always differ in some way from the object itself. We should thus never judge the value of a theory by trying to find out whether it deviates from the real facts, because no theory can stand or will ever stand such a test.

[It should be added that theories are only a means of knowing and investigating phenomena. A theory may be good for achieving a certain aim; another may be so for

⁴ *Les Systèmes socialistes*, II, pp. 75 et seq., Paris, 1903.

⁵ [Here, Croce asks: "And who can know it but man?"]

[All believers are sensitive on the subject of their faith; Croce must have seen here again (§7, footnote 2) another blasphemy on the human mind. But I had really no evil intention of this kind. One has only to read this paragraph, even very superficially, to see that it simply expresses the idea that new details of a given phenomenon are continuously brought to our knowledge. The example of the atmosphere seems to me to express this clearly.]

[Perhaps Croce thought that I wanted to solve, incidentally, the weighty question of the objective world. The partisans of the *existence* of the external world will express themselves by saying that argon existed before it was discovered; the partisans of the *existence* of human concepts alone will say that it has existed only since the day it was discovered.]

[I must warn the reader that I do not intend to indulge in discussions of this kind. One should therefore never try to find in this volume a solution to those problems, which I leave entirely to the metaphysicians.]

[I repeat that I am only opposed to the intrusion of metaphysicians into the field of the *θεορία φυσική*—this expression being intended to cover everything real; if they stay outside or beyond the *θεορία φυσική* I do not wish to molest them, and I even admit that, exclusively in this domain, they obtain results that are beyond the reach of those who are adept in the experimental method.]

[Finally, the question of the intrinsic value of certain doctrines has nothing to do with their social utility. There is no relation between the one issue and the other.]¹⁵

reaching another aim. But, in any case, they should fit the facts, for otherwise they would be useless.]¹⁶

For a qualitative inquiry, one should substitute a quantitative one, and see to what extent the theory deviates from the real world. Given two theories, we must choose the one that deviates the least. We must never forget that a theory must be accepted only temporarily; the one that we hold to be true today will have to be discarded tomorrow if another is discovered that fits the facts better. Science is in constant evolution.

12. It would be absurd to consider the existence of Mont Blanc as an objection to the theory that the earth is a sphere, because the height of that mountain is negligible in relation to the diameter of the terrestrial sphere.⁶

13. In representing the earth as a sphere, we come closer to reality than when we figure it as being flat or cylindrical, as some writers did in antiquity;⁷ consequently, the theory that the earth is a sphere must be regarded as preferable to the theory that it is flat or cylindrical.

When we represent the earth as an ellipsoid, we come closer to the facts than when we consider it as a sphere. It was thus a step forward when the theory of the ellipsoid replaced that of the sphere.⁸

But even this theory of the ellipsoid must be abandoned today, for modern geodesy teaches us that the shape of the terrestrial spheroid is much more complex. New studies bring us closer to the real world every day.

Nevertheless, in certain rough calculations, we still use the ellipsoid form. In so doing, we make a mistake, but a smaller one, we know, than others to which these calculations¹⁸ are subject, and, to simplify the computations, we may disregard the differences between the ellipsoid and the terrestrial spheroid.

14. This way of approximating the real world by theories that keep conforming more closely to it, and that subsequently become generally more and more complex, is what is called the method of successive approximations; it is used, implicitly or explicitly, in all sciences (§30, footnote 14).

15. Another consequence is this: it is a mistake to believe that one can discover exactly what the properties of concrete facts are by reasoning about a priori ideas regarding these facts, without modifying these concepts by comparing these conclusions with the facts *a posteriori*. This error is analogous to the one that a farmer would make if he were to imagine that he could make up his mind about buying an estate after seeing only a photograph of it.

⁶ Pliny was mistaken in his estimate of the height of the Alps; and so, regarding Dicaearchus' observation that the height of the mountains is negligible compared to the size of the earth, he says: "Mihi incerta haec videtur conjectatio, haud ignaro quosdam Alpium vertices, longo tractu, nec breviore quinquaginta millibus passuum assurgere."¹⁷ *Historiae Mundi*, II, 65. This would give a height of about 74,000 meters, whereas in fact Mont Blanc is only 4,810 meters high.

⁷ Anaximenes thought it was flat; Anaximander believed it was cylindrical.

⁸ Paul Tannery, *Recherches sur l'histoire de l'astronomie ancienne*, p. 106, in discussing the postulate of the spherical shape of the earth says: "Nevertheless, considering its objective aspect, it has the value of a first approximation, just as, for us, the hypothesis of the revolving ellipsoid constitutes a second approximation. The great difference is that, after the measurements and the observations are carried out at different points of the globe, we can assign limits to the discrepancies between this approximation and real facts, whereas the ancients were not able to do so seriously."

The idea we form of a concrete phenomenon partly tallies with the phenomenon and in other respects differs from it. Equivalence between the ideas of two phenomena does not imply the equivalence of the phenomena themselves.

Obviously, any phenomenon can be known only through the idea that it arouses in us; but precisely because the image of the real world thus obtained is imperfect, we always have to compare the subjective phenomenon, i.e., the theory, with the objective phenomenon, i.e., the experimental fact.

16. Moreover, the ideas we have of the phenomena, without further experimental verification, constitute the materials that are most accessible to us, since they exist in ourselves, and since something can occasionally be obtained from these materials. It follows that, especially in the early stages of a science, men have an irresistible tendency to reason about the ideas they already possess about the facts, without being concerned with rectifying these ideas by experimental research. Similarly, they seek to find in etymology the properties of the objects expressed by the words. They experiment on the names of the facts instead of experimenting on the facts [themselves]¹⁹. Some truths may well be discovered in this way, but only when a science is in its earliest stages; when it is somewhat more developed, this method becomes absolutely useless, and to acquire ideas that keep coming closer to the facts, one should investigate these facts directly, rather than by looking at them through some a priori ideas, or through the meaning of the words that denote them.

17. The natural sciences have now all reached the point at which the facts are studied directly. Political economy has also reached that stage, to a large extent at least. Only in the other social sciences does one still persist in [interrelating concepts and]²⁰ words;⁹ but we must get rid of this method if we want these sciences to progress.

⁹ [Croce remarks: "As if even Pareto's *Manuale* were not a tissue of *concepts* and *words*; man thinks by means of concepts and expresses them in words."

[This is another verbal criticism, like those I already noted (§7, footnote 2; §10, footnote 5). Of course, I never intended to deny that any work is a tissue of concepts and words; but I might distinguish between words expressing dreams and those expressing realities.

[Now, if some metaphysician is shocked by the term "realities," I can only advise him not to go on reading this book. I warn him—if he has not yet realized it—that we speak two different languages so that neither of us understands that of the other. For my part, I think I make myself sufficiently clear when I say that one should distinguish a real gold louis from an imaginary one; and if somebody were to say that there are no differences, I could propose a simple exchange: I will give him some imaginary gold louis, and he will give me real ones in return.

[Finally, leaving aside any discussion of how things are to be named, there are several types of "tissues of concepts and words." There is one type which is for the use of metaphysicians, and from which I try to keep away as much as possible; there is another type which is to be found in works on the physical sciences, and it is to this type I want to approximate, in dealing with the social sciences.

[Hegel says: "Diamonds are the typical crystal; at the sight of this product of the earth the eye rejoices because it sees in it the first-born of light and weight. Light is the abstract and completely free identity. Air is the identity of the elements. The subordinate identity is a passive identity with respect to light, and is the transparency of the crystal." (This translation is not mine; it is that of a famous Hegelian: A. Vera, *Philosophie de la Nature*, II, p. 21.)

[This explanation of transparency must be excellent, but I humbly admit that I do not understand it at all, and it is a model that I am very keen not to imitate.

[The proof given by Hegel of the laws of celestial mechanics (*Systèmes*, II, p. 72) seems to me the height of absurdity, whereas I perfectly understand books like *Les méthodes nouvelles de la mécanique céleste*, by H. Poincaré. When this author says: "The final aim is to be able to explain all astronomical phenomena; the only way to achieve this is to make observations that are as precise as possible and to compare them with the results of the computations" (i, p. 1), I find "a tissue of concepts and words" that is quite different from those

18. Another consequence. The method of reasoning which might be called the method of *elimination*, and which is still often used in the social sciences, is inaccurate. Here is what it amounts to. A concrete phenomenon X has a certain property Z. According to what we already know, this phenomenon is made up of parts A, B, C. It can be proved that Z belongs neither to B nor to C, and one concludes that it necessarily belongs to A.

The conclusion is inaccurate because the enumeration of the parts of X is never, and can never be, complete. In addition to the A, B, C, that we know—or which are all the author of the reasoning knows, which are all he is considering—there may be others, D, E, F, . . . , which we do not know, or which the author of the reasoning has overlooked.¹⁰

19. Another consequence. When the results of a theory are applied in practice, one may be sure that they will always be more or less modified by other findings, which depend upon phenomena that are not considered by the theory.

20. From this point of view, there are two large categories of sciences: those which, like physics, chemistry, and mechanics, can resort to experiments, and those which, like meteorology, astronomy, and political economy, cannot, or can hardly, have recourse to experiments, and which must be content with observation. The first are able to separate out materially the phenomena that correspond to the uniformity or law they want to investigate; the second category can only separate them mentally, theoretically; but in either case, it is always the concrete phenomenon that decides whether a theory should be accepted or rejected. There is not, and cannot be, another criterion for the truth of a theory than the more or less perfect accord of the latter with the concrete phenomena.

[When I speak of the experimental method, I am expressing myself elliptically; I mean the method that uses either experimentation, or observation, or both together if that is possible.]²²

The sciences that can use only observation separate, by means of abstraction, certain phenomena from certain others; the sciences that can also use experimentation carry this abstraction out materially; but abstraction constitutes for every science the preliminary and indispensable condition of any research.

21. Such abstraction arises out of subjective necessity; there is nothing objective about it. It is thus arbitrary, at least within certain limits, because the purpose which it is supposed to serve must be taken into account. [The same remark applies to classifications.]²³ Consequently, a certain abstraction or a certain classification does not necessarily exclude another abstraction or another classification. Both may be used, according to the object one has in view.

Rational mechanics, when it reduces bodies to simple points of matter, and pure economics, when it reduces real men to the *homo economicus*, are using completely similar abstractions¹¹ imposed by similar necessities.

encountered in Hegel, Plato, and other similar authors; and my aim is precisely, for the social sciences, to make “observations that are as precise as possible and to compare them with the results of the theories.”

[An author owes it to his readers to warn them of the path he intends to follow; and it is only to this end that I have written this first chapter.]^{21[a]}

¹⁰ *Systèmes*, II, p. 252.

¹¹ Vito Volterra, *Giornale degli Economisti*, November 1901.²⁴

Chemistry, when it deals with chemically pure bodies, also has recourse to abstraction, but it has the possibility of obtaining real bodies artificially which more or less give material form to the abstraction.

22. Abstraction may appear in two forms which are exactly equivalent. In the first, one considers an abstract being who possesses only the qualities that it is desired to investigate; in the second, these properties are considered directly and are separated from the others.

23. Real man performs economic, moral, religious, aesthetic, and other actions. Exactly the same idea is expressed, whether we say: "I study economic actions, and I set the others aside," or "I study *homo economicus* who performs only economic actions." The same idea is also expressed in the following two ways: "I study the [reactions]²⁵ between concrete sulfur and oxygen, abstracting from the foreign bodies they may contain," or by saying "I study the relationships between chemically pure sulfur and oxygen."

The same body that I consider as chemically pure for purposes of a chemical study, I can consider as a material particle for purposes of a mechanical study; I may consider only its shape for purposes of a geometrical study, etc. The same man, whom I consider as a *homo economicus* for an economic study, may be considered as a *homo ethicus* for a moral study, as a *homo religiosus* for a religious study, etc.

The concrete body comprises the chemical body, the mechanical body, the geometrical body, etc.; the real man comprises the *homo economicus*, the *homo ethicus*, the *homo religiosus*, etc. In short, considering these different bodies, these different men, amounts to considering the different properties of this real body, of this real man, and tends only to cut into slices the matter to be investigated.

24. It is thus a grave mistake to accuse the student of economic actions—or of the *homo economicus*—of disregarding, or even of disdaining, moral, religious, or other actions, i.e., the *homo ethicus*, the *homo religiosus*, etc.; this would amount to saying that geometry ignores or despairs the chemical properties of bodies, their physical properties, etc. It is the same mistake when political economy is accused of not taking morals into account; it is as if one were to accuse the theory of chess of not taking account of the culinary art.

25. By studying A separately from B, one simply²⁶ submits to an absolute necessity of the human mind; but it is not because one studies A that one affirms its preeminence over B. By separating the study of political economy from that of ethics, one does not at all assert that the former is superior to the latter. By writing a treatise on chess, one does not at all assert the preeminence of chess over the culinary art, or over any other science or art.

26. Coming back from the abstract to the concrete, one has to bring the parts together again that were first separated to make them easier to study. Science is essentially analytical; practice is essentially synthetical.¹²

Political economy need not take ethics into account; but anyone proposing a practical measure should take into account not only the economic consequences, but also the moral, religious, political, and other consequences. Rational mechanics need not take

¹² An example—in which the synthesis is, however, not yet complete—will be given in Chapter IX where free trade and protection are discussed.

into account the chemical properties of bodies; but anyone seeking to predict what will happen when a given body is placed in contact with another one will have to take account of not only the findings of mechanics, but also those of chemistry, physics, etc.

27. In the case of certain concrete phenomena, the economic aspect outweighs all the others; one will then be justified, without serious risk of error, in accepting the conclusions derived from economic science alone. There are other concrete phenomena whose economic aspect is insignificant; in their case, it would be absurd to accept the conclusions of economic science alone; on the contrary, these should be ignored. Between these two types, there are intermediate phenomena; economic science will show us a fairly important aspect of them. In any case, it is a question of degree, or more or less.

In other words, one may say: sometimes concrete man's actions are, with a slight margin of error, those of *homo economicus*; at times they coincide almost completely with those of *homo ethicus*; sometimes they coincide with those of *homo religiosus*, etc.; at still other times they partake of the actions of all of these.

28. When an author is unmindful of this observation, it is customary, in taking issue with him, to oppose theory and practice. This is an imperfect way of expressing oneself. Practice need not be opposed to theory; it brings together the different theories which apply to the case considered, and it uses them for a concrete purpose.

For instance, an economist who champions a piece of legislation while taking only its economic effects into account is not being too theoretical; rather, he is not being theoretical enough, since he is disregarding other theories which he should blend with his own, in order to form a judgment of that practical case. A person who recommends free trade by considering its economic effects alone is not formulating an inaccurate theory of international trade, but making an inaccurate application of a theory that is intrinsically correct; his error consists in ignoring other political and social effects, which are the subject matter of other theories.¹³

29. To distinguish the different aspects of a phenomenon in order to study them separately, then to bring them together again into a synthesis, is a procedure that is applied, and that can be applied, only when a science has reached a very advanced stage; at the beginning, all the aspects are studied together; analysis and synthesis are merged.

This is one of the reasons why sciences first appear in the form of art, and it is also one of the reasons why sciences separate and split up as they progress.

30. In his *Introduction à l'économie moderne*, Sorel proposes to go back to the stage of science where analysis is not distinguished from synthesis. His attempt is understandable when we consider the inadvanced state of the social sciences; but that would be moving upstream, and not downward with the current. It should, moreover, be observed that one is thus engaged in implicit theorizing. Indeed, Sorel not only proposes to describe

¹³ G. Sorel is [only]²⁷ partly right when he says: "The statesmen will usually be very insensitive to the demonstration by which one proves to him that protection always destroys wealth if he believes that protection is the least costly way of acclimatizing industry and the spirit of enterprise in his country . . ." (*Introduction à l'économie moderne*, p. 26).²⁸

For this qualitative comparison, a quantitative one needs to be substituted; one should say: "I shall lose so many millions per year, and I shall gain so many," and then come to a decision. If wealth worth 500 million per year were thus destroyed in order to gain a mere 100 million, it would be bad business. I notice, too, that Sorel states the problem solely from an economic point of view, whereas there is a very important political and social aspect which should be taken into account.

the past, he also wants to know the future; but, as we have already shown, the future can be linked to the past only if certain uniformities are explicitly or implicitly acknowledged, and these uniformities can only be revealed through the process of scientific analysis.¹⁴

31. An essentially negative critique of a theory is perfectly pointless and sterile; to be useful, a negative statement should be followed by a positive one; a more accurate theory should be substituted for the wrong one. If this is not always the case, it is simply because the more accurate theory is presented in one's mind.

When somebody denies that the earth is flat, he contributes nothing to the sum of our knowledge, as he would do if he were to declare that the earth is not flat, but round.

Let us, moreover, observe that, if we want to be perfectly rigorous, any theory is wrong, in the sense that it does not fit the real facts and it will never be able to do so (§11). It is thus a pleonasm to repeat in connection with a particular theory something that holds for all of them. The choice before us is not between a more or less approximate theory and one which fits the concrete facts to perfection, for such a theory does not exist. We have to choose between two theories, one of which is less close and the other closer to the concrete world.

32. It is not only because of our ignorance that theories diverge more or less sharply from the concrete world. We often depart from concrete reality in order to achieve greater simplicity, to make up for this divergence.

The difficulties encountered in the study of a phenomenon are of two kinds: viz., objective and subjective; they depend upon the nature of the phenomenon and upon the difficulty we have in apprehending a somewhat extensive range of objects or particular theories.

The economic phenomenon is exceedingly complex, and there are great objective difficulties in discovering the theories of its different parts. Let us suppose for a moment that these difficulties have been overcome, and that, for instance, the laws of the prices of all

¹⁴ Sorel says, *op cit.*, p. 25: "One cannot . . . imagine a method of successive approximations to settle the question of whether it is better to marry an intelligent but poor girl rather than a rich but not very bright heiress."

We would point out that this problem is one of private interest and that it is usually solved, not by reason, but by sentiment. However, if one wishes to proceed by reasoning, one could perfectly well imagine the method that might be followed:

First approximation—The material and moral conditions of the future spouses will be examined. A man, for instance, attaches more importance to material goods than to intellectual ability. He will be right to marry the wealthy heiress.

Second approximation—Let us take a closer look at the nature of this wealth. Formerly, if a man and a woman possessed neighboring pieces of land, a marriage that led to the union of these estates was considered very advantageous. Let us see whether the woman of means is not perhaps used to living above her income. What kind of intelligence do we find in the woman who is poor? If she has a good head for business and if the prospective husband is in charge of a trading firm or an industry which he is unable to manage, and which could be properly managed by this woman, it might be advantageous to marry the poor but intelligent woman.

Third approximation—We have spoken of wealth and intelligence; but one ought to take into account health, beauty, sweetness of character, etc. For many men, these qualities will rank first. And there still remains an infinite number of circumstances to be considered.

If the problem were social instead of individual, i.e., if we were to ask ourselves whether it was useful for a nation that young men should choose their life-mate according to her wealth or intelligence, one would arrive at similar considerations, consisting essentially in analyses (separation into parts), successive approximation, and finally synthesis, i.e., the reunion of elements that were initially separate.

commodities are contained in some thick folio volumes. We shall be far from having an idea of the price phenomenon. The very abundance of information to be found in all these volumes would not enable us to form the slightest idea of the price phenomenon. The day when someone, after having leafed through these documents, will tell us that *demand decreases when the price rises*, he will have given us a very valuable piece of information, although it would, in fact, take us much, much farther away from the concrete world than the documents studied by him.

Consequently, the economist—like all those, I might add, who study very complex phenomena—must constantly solve the problem of knowing up to what point he should carry the study of particulars. The point at which it is desirable to stop cannot be determined absolutely; this point depends upon the object one is aiming at. The brick producer who wants to know at what price he can sell them will have to take account of other elements than those considered by the scientist looking for the general law of the prices of building materials; still other elements have to be considered by someone looking, not for the laws of specific price, but for the law of prices in general.

33. The origin of economic phenomena has been carefully studied by many modern scientists, and such an inquiry is certainly useful from a historical point of view, but it would be a mistake to think that one could in this way arrive at an understanding of the relations between the phenomena in our society.

Ancient philosophers made the same error by always wanting to go back to the origin of things. Instead of astronomy, they studied cosmogonies; instead of trying to acquire an experimental knowledge of the animals, vegetables, and minerals that they could see, they inquired into how these entities had been generated. Geology has become a science and has progressed only since the day when it started to study present phenomena, and then work back to past phenomena, instead of following the opposite path. To understand a tree completely we may start with the roots and work up to the leaves, or we may start with the leaves and work down to the roots. Ancient [metaphysical]²⁹ science largely followed the first method; modern experimental science has made use of the second one exclusively, and the facts have shown that this is the only one conducive to knowledge of the truth.

It is of no use to know how private property grew up in prehistoric times, in order to study the economic role of property in our modern societies. We would not deny that one of these facts is closely linked to the other, but the chain connecting them is so long and is lost in such obscurity that we cannot reasonably hope to follow it, at least for the time being.

We do not know from what plant wheat is derived; but, even if we did, it would be of no avail if we wanted to know the best way to grow and produce wheat. However much one studies the seed of the oak, the beech, and the limetree, this study will never, for anyone wanting building timber, replace the direct study of the properties of the wood of these trees. Nevertheless, in this case, we have a perfect grasp of the relation between the facts at each extremity of the phenomenon, between the origin and the end. No doubt, the acorn will grow into an oak. Nobody has ever seen an acorn giving birth to a limetree, or the seed of a limetree developing into an oak. The relation between the oak and its origin is known to us with a degree of certainty which we shall never attain as regards the links between the origin of private property and ownership in our times, or, in general, between the origin of an economic phenomenon and that phenomenon in our day.

But it is not enough in order to be able to infer the properties of the latter from those of the former.

34. The study of the evolution of economic phenomena in times close to our own and in societies that do not differ vastly from ours is much more useful than the study of their origins; and this is so from two points of view. It enables us first to replace direct experimentation, which is impossible in the social sciences. When we are able to carry out experiments, we try to bring about the phenomenon under study, in various circumstances, in order to see how the latter act upon it, whether they modify it or not. But when we cannot proceed in this way, the only thing we can do is to see whether we can find in a natural state in space and time those experiments which cannot be performed artificially.

Secondly, the study of the evolution of the phenomena may be useful to us in facilitating the discovery of the uniformities that are present in this evolution, and in enabling us to predict the future on the basis of the past. It is obvious that the longer the chain of deductions between the past and the future facts, the more these deductions become uncertain and doubtful; it is thus only on the basis of the very recent past that one can predict the very near future; and, unfortunately, even within these narrow limits, predictions are very difficult to make.¹⁵

35. Discussions about the “method” of political economy are a waste of time. The aim of science is to discover the uniformities in phenomena; one should, consequently, have recourse to whatever procedure, whatever method, that attains this objective. Good methods can be distinguished from bad ones only by testing them. The one that achieves our aim is good, at least so long as a better one has not been discovered. History is useful in that it extends the experience of the present into the past, and makes up for the experiments we are unable to make; the historical method is thus a good one. But the deductive method, or the inductive method, which applies to present facts, is no less good. Where, in deduction, ordinary logic suffices, one makes do with it; where it does not, one makes no bones about replacing it by the mathematical method. Finally, if an author prefers such and such method, we shall not pick a quarrel with him on that score; we shall simply ask him to reveal to us scientific laws, without worrying too much about the path followed in order to discover them.

36. Some authors commonly assert that political economy cannot use the same tools as the natural sciences, “because it is a moral science.” This very imperfect expression conceals concepts that call for analysis. First of all, as to the *truth* of a theory, there can be no other criterion than its agreement with the facts (Chapter II, 6), and there is only one way to ascertain this agreement: from this point of view, there cannot be any differences between political economy and other sciences.

But some maintain that beyond this experimental truth, there lies another one, which is not subject to experimentation, and which they hold to be superior to the former. Those with time on their hands may argue about words; those who have more substantial things to do will refrain from arguing. We shall not dispute the use that it is desired to make of the word “truth” [and will let others use it as they please]³⁰; we shall simply say that one can classify all propositions into two categories. In the first, which we shall call X, we shall

¹⁵ *Cours*, 578.

put statements that can be tested experimentally; in the second, which we shall call Y, we shall put those that cannot be experimentally verified. We shall, moreover, split this last category into two: we shall call Y_α the statements that cannot be experimentally tested at present, but which could be in the future. This category will, for instance, comprise the statement that the sun, with its train of planets, will lead us, some day, into a four-dimensional space. We shall call Y_β the statements which will not, as far as we can foresee with the help of our scanty knowledge, be capable of experimental verification either in the present or in the future. This category would include the assertion of the immortality of the soul, and other affirmations of that kind.

37. Science concerns itself only with propositions X, which are by themselves capable of proof; everything that is not included in this category X remains beyond the scope of science. Moreover, I do not propose to extol one category in order to disparage the other; I wish only to distinguish them. However much scientific propositions may be disparaged, and the others extolled as warmly as the most fervent believers desire, it will always be true that the two categories are essentially different from one another. They cover different domains, which have nothing in common.

38. Anyone who affirms that Pallas Athene, *invisible and intangible*, inhabits the Acropolis of the city of Athens, asserts something that cannot be verified experimentally and is thus beyond the scope of science; science cannot be concerned with this assertion, either to accept it, or to reject it; and the believer is perfectly right to disdain the [negations]³¹ that a pseudoscience seeks to challenge him with. The same is true of the proposition that Apollo inspires the priestess of Delphi; but not of the other proposition that the priestess' oracles agree with certain future facts. That proposition can be verified by experience; consequently, it comes within the scope of science, and faith has nothing to do with it.

39. Anything that looks like a precept is not scientific, unless it is like a precept in form only, and it is in substance a statement of facts. These two propositions: To obtain the area of a rectangle, *one should* multiply the base by the height,¹⁶ and: *one should* love one's neighbor as oneself,¹⁷ are essentially different. In the first one, the words: *one should* could be left out, and it might simply be said that the area of a rectangle is equal to the base multiplied by height; in the second one, the idea of duty cannot be removed. The second proposition is not scientific.

Political economy tells us that bad money drives out good. This proposition is a scientific one, and it is for science only to verify whether it is true or not. But, if one

¹⁶ [From the point of view we have adopted, geometrical truths are experimental ones, logic itself being experimental.]

[In this case, it may, moreover, be remarked that the area of a concrete rectangle will be closer to the product of the base and the height, the closer the concrete rectangle is to the abstract rectangle considered by geometry.]³²

¹⁷ [It has been objected that "every right-thinking person believes so." First of all, this proposition differs from the one in the text. The two propositions: "A is equal to B," and: "Every man—or some men—think that A is equal to B, or should be equal to B," express things that are absolutely distinct.]

[Next, it is a known fact that some men—like, for instance, followers of Nietzsche—are far from acknowledging the truth of this proposition. If one replies that these are not "right-thinking people" one is bound—which appears to be very difficult if not impossible—to provide a proof of that statement, which will not, in the last resort, amount to asserting that they are not right-thinking people because they do not love their neighbor; for the demonstration would simply be circular!]³²

were to say that the state *ought not* to issue bad money, the proposition would not be scientific. It is because political economy has so far contained propositions of this kind that people can be excused for claiming that political economy, being a moral science, is not subject to the rules of the natural sciences.

40. It should, moreover, be observed that the above proposition might be elliptical; and, in this case, it might become scientific if the ellipsis were removed. If one were to say, for instance, that the state *ought not* to issue bad money, if it wishes to obtain the maximum [benefit]³³ for society; and if one were to provide a factual definition of what is meant by *maximum benefit*, the proposition would be capable of experimental verification, and would consequently become scientific (§49, footnote).

41. It is absurd³⁴ to assert, as some do, that one's faith is *more scientific* than that of others. Science and faith have nothing in common, and the latter cannot contain more or less of the former. In our days, a new faith has been born according to which every human being *ought to* sacrifice himself for the good of "the lowly and humble;" and its believers speak with contempt of the other creeds, which they consider unscientific. These worthy people do not see that their precept has no more scientific basis than any other religious precept.

42. From the earliest times up to the present, men have always tended to merge and confuse propositions X with propositions Y, and this is one of the most serious obstacles to the progress of the social sciences.

Those who believe in propositions Y constantly encroach upon the domain of propositions X. For most cases, this is due to the fact that they do not keep the two things separate; for many others, it is a weakness of their faith, which summons experimentation to its aid. The materialists are in the wrong to ridicule the *credo quia absurdum*,³⁵ which, in a certain sense, recognizes the distinction between the two types of propositions; this is what [our]³⁶ Dante expressed so well:¹⁸

State contenti, umana gente, al *quia*;
Che se potuto aveste veder tutto,
Mestier non era partorir Maria.³⁷

43. One should be on one's guard against a certain way of confusing propositions X and Y, which is based on an ambiguity similar to the one in §40. Suppose that the proposition: "A is B" does not belong to the realm of experience and consequently of science; some imagine they provide a scientific proof of it by showing how useful it is for men to believe that A is B. But these propositions are not at all identical; so even if the second proposition is shown to be experimentally true, this does not allow one to conclude anything about the first one. It is all very well for some people to assert that only the *true* is useful; but if the word *true* is given the meaning of *experimentally true*, this proposition does not conform at all to the facts, which on the contrary contradict it at every turn.

¹⁸ *Purgatorio*, III, 37–39. And *Paradiso*, II, 43–44:

Le si vedra cio che tenem per feder,
Non dimostrato, ma fia per se noto,
A guisa del ver primo che l'uom crede.³⁸

44. Another ambiguous procedure is the following one. It is proved—or more exactly, it is believed possible to prove—that “evolution” brings A closer to B, and it is believed that this proves that everyone *ought* to strive to make A equal to B, or even that A is equal to B. These are three different propositions, and the proof of the first does not in the least carry with it the proof of the other two. Let us add that the proof of the first is usually very imperfect.¹⁹

45. The confusion between propositions X and Y can also arise from attempts to show that, since they may have a common origin, they have a common nature and common characteristics; this is a form of argument that has long been used, and which returns to the surface every now and then. People have sometimes seen such a common origin in the universal consensus, or in some other similar fact; in our day, it is most often found in *intuition*.

Logic is of use in proof, but rarely—almost never—in invention (§51). A man receives certain impressions; under their influence he enunciates, without being able to say either how or why—and if he tried, he would be mistaken—a proposition that can be verified experimentally, and which is therefore of the kind I have just called propositions of the type X. Once this verification has subsequently taken place, and the event has occurred as predicted, the operation just described is called INTUITION. When a peasant, looking at the sky in the evening, says: “It will rain tomorrow,” he is said, in the event it really does rain tomorrow, to have had an intuition that it was bound to rain; but one would not have said so had the weather turned out to be fair. If an individual who has had experience with sick people says about one of them: “Tomorrow, he will die,” and if the man in fact dies, this individual will be said to have had an intuition of that death; but this could not have been said had the patient recovered his health.

As I have already so often remarked, and must continue to repeat, it is quite useless to argue about the names of things. Consequently, if somebody wants to also designate as intuition the operation by which rain is predicted when on the contrary fair weather ensues, or the death of someone who instead recovers, he is free to do so; but in that case, one ought to distinguish the correct intuitions from the incorrect ones, and this can be done by experimental verification. The former will be useful, and the latter [inconclusive]³⁹.

By the same operation, which yields propositions that are capable of experimental proof and that later may be recognized as true or false, one may also obtain propositions that are not capable of experimental proof; and, if one wishes, this operation may be called *intuition*.

We shall thus have three kinds of intuition: (1) intuition that leads to propositions X, which are then verified by experience; (2) intuition that leads to propositions X, which are subsequently contradicted by experience; and (3) intuition that leads to propositions of type Y, which experience can neither verify nor contradict.

By thus giving the same name to three very different things, it becomes easy to confuse them; one is careful to make the confusion between the third and the first, deliberately forgetting the second. Some say: “by intuition, man arrives at the knowledge of *truth*, be

¹⁹ *Systèmes*, I, p. 344; *Cours*, II p. 578.

it experimental or not," and in this way, they attain the desired aim, which is to confuse propositions X with propositions Y.

If the following two questions had been put to Pericles: "What do you think the Athenians will do in such circumstances?" and "Do you believe that Pallas Athene protects your city?" he would intuitively have given two absolutely different answers, because the former might have been verified experimentally, whereas the latter could not have been.

The origin of these answers is the same; they both, without Pericles having been aware of it, reflect certain of his impressions. But this reflection has quite a different value in each case. Pericles' opinion was of great importance in the first question, whereas the opinion of any Scythian (who did not know Athenians) would have been worthless; but, in the case of the second question, Pericles' opinion and that of a Scythian had the same value, for, if the truth must be told, neither had any relation with Pallas Athene.

Knowing the Athenians as well as he did, Pericles had many opportunities to verify, correct, and adjust his forecasts concerning their behavior, and the outcome of his past experience translated itself into a new intuition and gave it value. But the same could obviously not have been the case with respect to his intuitions concerning Pallas Athene.

If someone who knows nothing about arboriculture tells us, on looking at a tree, that the tree will die, we need not attach more importance to these words than if they had been pronounced at random. If, on the contrary, the judgment is expressed by an experienced arboriculturist, we can regard his intuition as sound, because it is based on experience. And, even if both these men have *a priori* the same knowledge, but we know by experience that the one is rarely wrong in his forecasts or intuitions, whereas the other is more often wrong, we may grant the former a measure of confidence which we shall deny the latter. But, where experience cannot be brought in, the forecasts or intuitions of both will have the same value, and this value is experimentally equal to zero.

Intuitions of [experimental facts]⁴⁰ may be contradicted by the facts themselves; intuitions must therefore agree with the facts. Non-experimental intuitions are contradicted only by other intuitions [of the same kind]⁴¹; so, for both types to coincide, it is enough for some men all to hold the same opinion. Agreement in the former case is objective; in the second one, subjective. If the one is confused with the other, this is due to the common mistake that causes man to consider himself the center of the universe and the measure of all things.

46. The universal consensus of man does not have the virtue of making a proposition experimental when it is not, even if this consensus extends over time, and if it includes all men who have ever existed. Thus the principle which says that what cannot be conceived of cannot be real is absolutely worthless, and it is absurd to imagine that the possibilities of the universe are limited by the capacity of the human mind.

47. Metaphysicians, who use propositions Y, usually assert that these are necessary in order to draw any conclusion from propositions X, because, without a superior principle, the conclusions would not *necessarily* flow from the premises. They are thus reasoning in a vicious circle, because they make the precise assumption that one wants to put propositions X in the category of propositions that partake of the nature of necessity and absolute truth.²⁰ And, in fact, it is true that if one wants to confer on category X the

²⁰ I am employing these expressions because others use them, but I do not quite know what these terms are supposed to mean.

characteristics of propositions Y, these have to be injected either into the assumptions or into the way conclusions are drawn. But, if it is held that propositions X are strictly subordinate to experience, and that they are never accepted once and for all, but only so long as experience does not contradict them, there is no need to resort to propositions Y. From this point of view, logic itself is regarded as an experimental science.

48. On the other hand, those who are concerned with propositions X also often encroach on the territory of propositions Y, whether by laying down precepts in the name of "science," which would thus seem to be delivering oracles like a God, or by denying propositions Y, over which science has no power. It is this intrusion that partly justifies Brunetière's statement that "science is bankrupt." Science has never been bankrupt as long as it has been kept within its territory, which is that of propositions X; it has always gone bankrupt—and always will—when it encroaches on the territory of propositions Y.

[If one wanted to answer the question: 'Why does hydrogen, combined with oxygen, produce water?' one would have no option but to say: 'Because hydrogen has the property of being capable of generating water.' Thus, only the question of *why* is absurd, since it gives rise to an answer that looks naive or ridiculous. It is better to admit that we do not know, and that this indicates the limits of our knowledge. We may know how, and in what conditions, opium sends one to sleep, but we shall never know why." (Claude Bernard, *La science expérimentale*, pp. 57, 58)]⁴⁴

49. Quite a different situation from the one we have just dealt with is faced when conclusions are deduced [logically]⁴⁵ from a premise that cannot be verified by experience. The conclusions cannot be experimentally verified either, but they are linked to the premise in such a way that, if the premise is a proposition that one can test experimentally later on, i.e., one of the propositions indicated by Y_α in §36, the conclusions will also become experimental. If the premise is a proposition of Y_β , the conclusions will always

[Croce invites me to find out, and for that purpose he urges me to read Plato, Aristotle, Descartes, Leibniz, Kant, and other metaphysicians. Alas! I must give up any hope of my ignorance being dispelled, because it is precisely after a careful study of these authors that the term *absolute* appeared incomprehensible to me . . . and, I am afraid, to them too.

[Moreover, I must confess that, as it seems to me, many of Plato's reasonings might be grouped into two classes. Those that are understandable are puerile; those that are not are incomprehensible. If one wishes to see the length to which this author is led by the mania for purely verbal explanations, one needs only read the *Cratylus* again. It is hard to imagine anything more absurd than this dialogue. The gloomiest individual will laugh when he learns that the gods were named $\theta\epsilon\omega\acute{\iota}$ because they are always running!]

[It is related that when Diogenes was arguing with Plato about ideas, and the latter spoke of the *τραπεζότης* (essence of the table, quality of being a table, the table *in se*), and the *κυαθότης* (essence of the cup, quality of being a cup, the cup *in se*), he said: "I—Oh Plato!—see the table (*τράπεζα*) and the cup (*κύαθος*), but I do not at all see the *τραπεζότης* and the *κυαθότης*." To which Plato replied: "That is right, because you have eyes to see the table and the cup, and not eyes to see the *τραπεζότης* and the *κυαθότης*."

[*Πλάτωνος περὶ ἴδεῶν διαλεγομένου, καὶ ὄνομάζοντος τραπεζότητα καὶ, Ἐγὼ, εἶπεν, ὦ Πλάτων, τράπεζαν μὲν καὶ κύαθον ὄρῳ. τραπεζότητα δὲ καὶ κυαθότητα, οὐδαμῶς* (*Diog. Laer.*, VI, 53).]⁴²

[I must confess to the reader that I am nearly as blind as Diogenes was, and that the *essence* of things escapes me entirely.]

[Claude BERNARD, *La science expérimentale*, p. 53, writes: "Newton said that anyone looking for the prime causes proves thereby that he is not a scientist. Indeed, such research remains sterile, because it raises problems that cannot be tackled by means of the experimental method."]

[In the study of political economy and sociology, I intend to use only the experimental method; I shall thus restrict myself exclusively to the only problems that it can solve.]⁴³

remain beyond the scope of experience, while being linked to the premise in such a way that anyone accepting the latter must be linked to the premise in such a way that anyone accepting the latter must also accept the former.²¹

50. For this mode of reasoning to be possible, the premises must be clear and precise. For instance, the space in which we live is Euclidean, or differs only slightly from it, as innumerable facts of experience prove. Non-Euclidean spaces can, however, be imagined, and, so, starting from precise premises, it has been possible to build non-Euclidean geometries, which are beyond the scope of experience.

When the premises are not precise, as is the case for all those which the moralists would like to introduce into social science and into political economy, it is impossible to draw any rigorously logical conclusions from them. These imprecise premises might not be useless if their conclusions could be tested at every step, and their imprecision thus gradually corrected; but where such verification is lacking, the attempted pseudo-reasoning ends by having no other value than that of a disconnected dream.

51. We have so far dealt only with proof; invention is quite different. The latter may sometimes originate in ideas which have nothing to do with the real world, and which may even be absurd. Chance, bad reasoning, imaginary analogies, may lead to correct propositions. But in seeking to prove them, there is no other means but to see whether, directly or indirectly, they agree with experience.²²

²¹ This proposition is elliptical, and is of the nature of those we discussed in §40. We should add: "if one wants to reason logically." Obviously, nothing could be proved to a person who did not accept this condition.

²² *Systèmes*, II, p. 80, footnote: Paul Tannery (*Recherches sur l'histoire de l'astronomie ancienne*, p. 260) who, besides having a tendency to go somewhat beyond the facts in order to defend certain metaphysical ideas, speaking of the theories of the solar system, declares: "Here is a noteworthy instance which cannot be too strongly emphasized of the great importance of *a priori* (metaphysical) ideas in the development of science. When science is firmly constituted, it is easy to brush aside the considerations of the simplicity of the laws of nature, etc., which guided the founders . . . But one forgets that this is not the way that great discoveries were made, or the main advances achieved."

CHAPTER II

Introduction to Social Science

1. The foundation of political economy and, in general, of every social science, is evidently psychology. A day may come when we shall be able to deduce the laws of social science from the principles of psychology, in the same way that some day, perhaps, the principles of the constitution of matter will give us, by deduction, all the laws of physics and chemistry. But we are still very far from this state of affairs, and we must adopt another approach. We have to start from certain empirical principles to explain the phenomena of sociology, as well as those of physics and chemistry. At a later date, psychology, by extending the chain of its deductions, and sociology, by going back to principles that are more and more general, may combine and form a deductive science, but these hopes are still far from being realized.

2. To put some order into the infinite variety of human actions that we must study, it will be useful to classify them according to certain types.

Two of these at once suggest themselves. A well-brought-up man enters a drawing room; he takes off his hat, says a few words, and makes certain gestures. If we ask him why, all he will be able to tell us is: it's the custom. He behaves in the same way in certain matters of far greater importance. If he is Catholic and goes to Mass, he will perform certain actions "because this is what is done." He will justify many other actions of his by saying that this is what moral principles require.

But let us imagine this same individual in his office, engaged in buying a large quantity of wheat. He will no longer say that he is acting in a certain way because such is the custom, but his purchase of wheat will be the final outcome of a process of logical reasoning that is based on certain experimental data. If the data should happen to change, so would the conclusion, and he might refrain from buying, or he might even sell wheat instead of buying it.

3. We can thus, by abstraction, distinguish between (1) nonlogical actions and (2) logical actions.

I say "by abstraction" because in real actions the categories are nearly always mixed, and an action may to a large extent be nonlogical and to a small extent logical, or conversely.

For example, a speculator's actions on the Stock Exchange are certainly logical; but they depend, even if only in small part, on the individual's character, and because of this they are also in part nonlogical. It is a known fact that some individuals are generally bullish; others are more bearish.

It should be noted, moreover, that nonlogical does not mean illogical; a nonlogical action might have been the best that could be found, given observed facts and logic, for adjusting the means to the end; but this adjustment was obtained by a procedure other than that of logical reasoning.

It is known, for instance, that the cells in a honeycomb end in the form of a pyramid, and that with a minimum surface, hence with the minimum outlay of wax, they achieve the maximum volume, that is, they can hold the largest quantity of honey. Nobody, however, supposes that this is so because the bees have solved a maximum problem by syllogisms and mathematics; it is obviously a nonlogical action, although the means are perfectly adjusted to the end, and consequently the action is far from being illogical. The same observation could be made about a great many other actions, which are usually called instinctive, either by men or by animals.

4. It should be added that man has a very marked tendency to consider as logical actions that are nonlogical. It is because of a similar tendency that man animates and personifies certain material objects and phenomena. Both these tendencies are found in everyday language, which, in retaining traces of the sentiments that prevailed when it was being formed, personifies things and facts, and presents them as the outcome of a logical will.

5. This tendency to consider nonlogical actions as logical lessens and turns into a tendency—which is equally mistaken—to consider the relations among phenomena as having solely the form of relations of cause and effect, whereas the relations among social phenomena are much more often relations of mutual dependence.¹ Let us note in passing that causal relations are much easier to study than relations of mutual dependence. Ordinary logic is usually adequate for the former; while, for the latter one often has to resort to a special form of reasoning: i.e., mathematical reasoning.²

6. Let *A* be a real fact and *B* another real fact, which have a causal relation between them, or one of mutual dependence. This we shall call an *objective* relation.

To this relation there corresponds, in man's mind, another relation, *A'B'*, which is strictly speaking a relation between two concepts in the human mind, whereas *AB* was a relation between two things. This relation *A'B'* we shall call *subjective*.

If we find in the minds of men in a given society a certain relation, *A'B'*, we may investigate the following things: (*a*) What the nature is of this subjective relation—whether the terms *A'*, *B'* have a precise meaning, whether there is or there is not a logical link between them. (*b*) What objective relation, *AB*, corresponds to the subjective one, *A'B'*. (*c*) How this subjective relation, *A'B'*, arose and how it was determined. (*d*) In what way the relation *AB* was transformed into the relation *A'B'*. (*e*) What effect the existence of these subjective relations *A'B'* has upon society—whether they correspond to something objective, *AB*, or are entirely imaginary.

When *A'B'* corresponds to *AB*, the two phenomena develop in a parallel way; when the former becomes somewhat complex, it is called a *theory*. It is considered as *true* (I, 36) when *A'B'* corresponds to *AB* throughout its course, that is, when theory agrees with experience. There is not and cannot be any other criterion of [the truth of a theory].¹

¹ *Cours d'économie politique*, I, §225, Lausanne, 1896–1897.

² This is not understood by many economists who speak of the “mathematical method,” without having the least idea about it. They have conjured up all kinds of reasons for explaining, in their own way, the use of this unknown monster to which they have given the name of “mathematical method,” but they have never thought of this one, even though it was explicitly indicated in Vol. I of the *Cours d'économie politique*, published at Lausanne in 1896.

[The same facts may, moreover, be explained by an infinite number of theories, all of them equally true, because all of them reproduce the facts to be explained. This is what Poincaré meant by saying that when a phenomenon admits of one mechanical explanation it admits of an infinite number of them.

[More generally, it may be observed that establishing a theory amounts in a sense to fitting a curve through a certain number of determinate points. An infinite number of curves can satisfy this condition.^{3]}²

7. We have already remarked (I, 10) that we cannot know any natural phenomenon in all its particulars; consequently, the relation $A'B'$ will always be incomplete in comparison with the relation AB —if for no other reason than this, that it will never be possible for these relations to completely coincide; the subjective phenomenon will never be a strictly faithful copy of the objective phenomenon.

8. But these phenomena can diverge from one another for quite different reasons. Whereas, for the scientist who studies natural facts experimentally in his laboratory, the subjective phenomenon approaches the objective phenomenon as closely as possible; for the man who is affected by feeling and passion, the subjective phenomenon may diverge from the objective one to the point of having nothing in common with it.

9. It should be noted that the objective phenomenon appears to our minds only in the form of a subjective phenomenon, so that it is the latter rather than the former which is the cause of human actions. To be able to influence human actions, the objective phenomenon first has to be transformed into a subjective one;⁴ hence the great importance to sociology of the study of subjective phenomena and their relation to objective phenomena.

Relations among subjective phenomena are rarely a faithful copy of the relations among the corresponding objective phenomena. The following difference is very often observed. Under the influence of circumstances, some people perform certain actions, P, \dots, Q ; afterwards, upon reasoning about them they discover, or believe they have discovered, a principle common to P, \dots, Q ; they then imagine that they have derived P, \dots, Q as a logical consequence of this principle. In reality, P, \dots, Q are not the consequence of the principle, but it is the principle which is the consequence of P, \dots, Q . Admittedly, once the principle is established, actions R, \dots, S ,³ which are deduced from it, follow; thus, the disputed proposition is only partly false.

The laws of language provide us with a good example. Grammar did not precede, but followed, the formation of words; nevertheless, once established, grammatical rules give rise to certain forms, which have been added to the existing forms.

To sum up, we may group the actions P, \dots, Q and R, \dots, S ³ into two sets: the first, P, \dots, Q which is the most numerous and the most important, is preexistent to the principle which seems to govern these actions; the second, R, \dots, S , which is secondary and often of very little importance, is the consequence of the principle; or, in other words, it is an indirect consequence of the same causes which directly produced P, \dots, Q .

10. The phenomena A' and B' of §6 do not always correspond to the real phenomena A, B ; it very often happens that A' or B' , or even both, do not correspond to anything

³ [*Rivista di Scienza*, Bologna, No. 2, 1907, *Les doctrines sociales et économiques considérées comme science.*]

⁴ *Systèmes socialistes*, I, p. 15.

real, and are purely imaginary entities. Moreover, the relation between A' and B' may be logical only in appearance and not in reality.⁵ Thus, there are different cases which ought to be distinguished.

11. Let A be a real phenomenon, of which another phenomenon B , also a real one, is the consequence. There is an objective relation of cause and effect between A and B . If an individual has some rough approximations of A and B , and puts them in a relation of cause and effect, he obtains a relation $A'B'$, which is a more or less faithful image of the objective phenomenon. The relations a scientist discovers in his laboratory are of this kind.

12. One may be unaware that B is the consequence of A , and believe that it is, on the contrary, a consequence of another real fact, C ; or one may, knowing that B is a consequence of A , deliberately choose to consider it to be a consequence of C .

Scientific errors come within the first case; there will always be such instances of them because man is liable to error. Examples of the second case are to be found in *legal fictions*, in the arguments used by political parties to keep one another down, or in other similar circumstances; this is how the wolf in the fable argued when he was about to eat the lamb. Most of the arguments justifying the imposition of taxes belong to the same category; it is that one wishes to set taxes, B , in accordance with certain principles of justice or of general interest [C];⁴ but in reality, B is linked, by a relation of cause and effect, to the interests, A , of the dominant class. Finally, the origin of casuistry may be found at least in part in this same kind of reasoning.⁶

13. We have dealt so far with three real facts: A , B , and C ; but entirely imaginary facts often form part of human speculations.

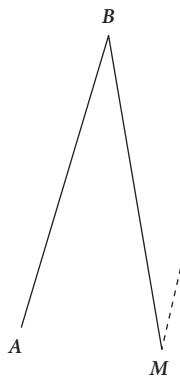


FIG 2

One of these imaginary possibilities, M , may be placed in a logical relation with a real fact, B ; this error, which is still frequent in the social sciences, was once common in the physical sciences. For example, one removes the air contained in a tube connected to a vessel full of water; the pressure of the air on the surface of the water is fact A ; the rising of the water in the tube is fact B . Now, this fact was formerly explained by another, completely imaginary one, M , i.e., by the fact that "nature abhors a vacuum," of which, moreover, B is a logical consequence. At the beginning of the 19th century, the "life force" explained an infinite number of biological facts. Contemporary sociologists explain and prove [an infinite number of things by bringing in the idea of]⁵ "progress." "Natural laws" have had and continue to have great importance in the explanation of social facts. For many people who have picked up socialist theories [a little]⁶ like parrots, "capitalism" explains everything and is the cause

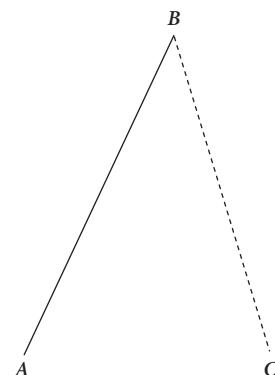


FIG 1

⁵ *Systèmes*, I, p. 22.

⁶ *Systèmes*, I, p. 178, 27.

of all the evil observed in human society. Others speak of “free land,” which nobody has ever seen; and we are told that all the evils of society sprang up on the day when “man was separated from the means of production.” When was that? We do not know; perhaps it was the day Pandora opened her box, or else in the times when [the wolf and the lamb]⁷ could talk.

14. When one brings in imaginary facts, *M*, which can be chosen at will, it would seem that one should at least take care that the link *MB* be logical; yet this is not always done, either because some men are repelled by logic, or because they set out to influence sentiments. It follows that the imaginary fact *M* is often related to another imaginary fact *N* by means of a logical link, or even by an illogical one. We find numerous instances of the latter kind in metaphysics and theology, and in certain philosophical works such as Hegel’s *Philosophy of Nature*.⁷

Cicero (*De natura deorum*, II, 3) cites a process of reasoning by which, from the existence of divination, *M*, one infers the existence, *N*, of the gods. In another work, he cites an inverse process of reasoning by which divination is inferred from the existence of the gods.⁸

Tertullian knows how it happens that devils are able to predict rainfall: it is because they live in the air and feel the effects of the rain before it reaches the earth.⁹

In the Middle Ages, when men wanted to construct a theory, they were almost irresistibly disposed to reason—or better, to indulge in unreason—in that fashion; and if by any chance—which was rarely the case—someone dared express a doubt, he was persecuted as an enemy of God and men, by those who were undoubtedly in complete conflict with common sense and logic. The unbelievable arguments about predestination, effectual grace, etc., and in our time the flow of nonsense on solidarity, prove that men do not easily rid themselves of these fantasies, which only the physical sciences have succeeded in ridding themselves of, whereas they still weigh down the social sciences.

[In our day there has come to be a tendency to justify these modes of reasoning. The element of truth in this new point of view is the concept that all theories are relative, combined with a reaction against the feeling that modern scientific theories have an absolute value.

[The theory of universal gravitation does not have any absolute real content to set against the “error” of the theory that each celestial body has an angel guiding its movements. Moreover, this second theory can be rendered as true as the first one by adding that those angels, for reasons unknown to us, cause the celestial bodies to move *as if* they were attracted by one another in direct proportion to their masses and inversely as the squares of their distances. But then the intervention of angels is superfluous, and must be eliminated, because, in science, every useless assumption is harmful. Some day, perhaps, the concept of universal gravitation will be dispensed with on the same grounds; but—and this is what matters—the equations of celestial mechanics will continue to hold.¹⁰]¹⁰

⁷ *Systèmes*, I, p.71 et seq.

⁸ *De divinatione*, I, 5: “Ego enim sic existimo: si sint ea genera divinandi vera, de quibus accepimus, quaeque colimus, esse deos; vicissimque, si dii sint, esse, qui divinent.”⁸

⁹ *Apologeticus adversus gentes pro Christianis*, 22: “Habent de incol tu aëris, et de vicinia siderum, et de commercio nubium coelestes sapere paraturas ut et pluvias quas jam sentiunt, repromittant.”⁹

¹⁰ J.H. Poincaré, *La science et l’hypothèse*, pp. 189–190: “No theory seemed to be sounder than that of Fresnel, which explained light by the movements of ether. However, Maxwell’s theory is now preferred. Does that mean

15. If an objective relation, AB , coincides approximately with a subjective one, $A'B'$, in a person's mind, he will be able by means of logical reasoning to draw other conclusions C', D', \dots , from A' which are not too far removed from the real facts C, D, \dots . Assuming on the contrary that M is an imaginary cause, or even a real fact different from A , and that the objective relation AB corresponds to the subjective relation MB' in a person's mind, then, again by means of logical reasoning, he will draw certain conclusions N, P, Q, \dots , which will have nothing in common with reality. If he then compares his deductions with reality, with the intention of seeking only for truth and without being blinded by some strong emotion, he will perceive that M is not the cause of B ; and thus, little by little, by constantly experimenting and comparing his theoretical deductions with reality, he will modify the subjective relation MB' and replace it by another one $A'B'$, which will come closer to reality.

16. The scientist's experimental studies are of this kind, as are many of man's practical actions, including those studied by political economy. Such actions are repeated a great many times, under varied conditions, so that a large number of consequences of A , or of M , are available for examination, and it is therefore possible [when other subjective conditions agree,¹¹] to arrive at a faithful concept of the objective relations.

17. On the other hand, a person who seldom operates according to the relation AB , or does so always under identical conditions, or lets himself be strongly swayed by his sentiments, may have a partly imaginary notion MB' ,^[a] of the relation AB , and sometimes an entirely imaginary notion MN .

18. A theory of the first kind of action is essentially different from one of the second kind. I shall provide only a short account of the latter, as the main object of this Manual is the study of the former.

It should [first]¹¹ be noted that, in social life, the second kind of action is very widespread and is of the utmost importance. What are called MORALS and CUSTOMS depend entirely upon such actions. It is a fact that up to now no nation has had a scientific or experimental ethics. The attempts of modern philosophers to reduce ethics to such a form have been in vain; but even if these attempts had been successful, it would still be true that they have not penetrated beyond a very restricted circle, and that most, in fact almost all, people are completely unaware of them. Similarly, every now and then someone comes along who remarks upon the anti-scientific, anti-experimental character of certain habits and customs; and while this may give rise to [some moderately

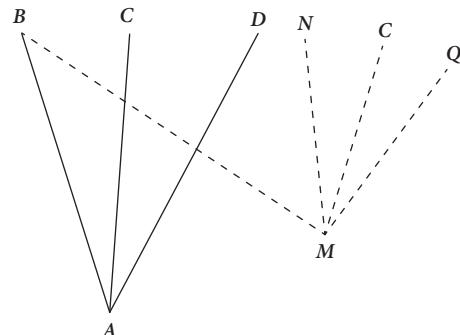


FIG 3

that Fresnel's work has been in vain? No, because Fresnel's aim was not to find out whether ether really exists, whether or not it is made of atoms, whether these atoms really move in this or that direction; it was to predict optical phenomena. Now, Fresnel's theory can still do this, just as well today as before Maxwell's time. The differential equations are always true; we can still integrate them by the same procedures and the results of this integration still remain valid.

impressive works of literature, it does not usually]¹² have the slightest influence on these habits and customs, which change for altogether different reasons.

There are some phenomena, which [in our society]¹³ are called ETHICAL OR MORAL, which everybody believes he knows all about, and which nobody has ever known how to define in a rigorous way.

They have hardly ever been studied from an entirely objective point of view. Everyone concerned with them has some principle which he would like to impose on others, and which he deems superior to any other. Such people look not for what men at a given time and place have called moral, but for what, according to them, ought to be called by that name; and when they so much as deign to study some other systems of morals, their view is colored by their prejudices, and they are content to compare it with their own, which becomes the measure and the standard of all the others. From this comparison, there follow a number of implicit or explicit theories. The moral code has been considered as something absolute, revealed or imposed by God, according to the general view, or derived from man's nature, according to some philosophers. If some nations do not follow [and practice]¹⁴ it, it is because they are unaware of it, and missionaries have the duty to teach it to them and unseal the eyes of these unfortunates to the light of truth; or else philosophers will bestir themselves to remove the thick veils which prevent weak mortals from knowing absolute *Truth*, *Beauty*, and the *Good*—words that are used, although nobody has ever known what they mean, or to what real objects they correspond. Those who draw fine distinctions on these matters regard the different types of morality—some say also the different types of religion—as an effort by *Humanity* (another abstraction of the same kind as the preceding ones, although slightly less unintelligible) to attain the knowledge of the supreme *Good* and *Truth*.

These ideas have changed in our time, perhaps much more in form than in substance, but in any case, they have come a little closer to reality, and we have had a theory of evolutionary ethics. However, the idea of a moral code has not been abandoned on that account; it has simply been placed at the final stage of evolution, whether in an absolute or a temporal sense. This moral code, chosen and adopted by the author who puts it forward, is obviously better than all those that preceded it. This can be proved, if one likes, with the aid of another very fine and, in our day, very powerful metaphysical entity, called *Progress*, which vouches for the fact that each stage of evolution marks an improvement upon the preceding one, and which, thanks to certain of its occult yet very effective virtues, prevents the next stage from becoming worse.

In fact—and leaving aside all these empty or pointless considerations—this moral code is nothing else but the product of the sentiments of the man who makes it his own: sentiments which for the most part are drawn from the society in which he lives, and which to a trifling extent are specifically his own. It [is a nonlogical outcome that]¹⁵ gives form to the sentiments, and reasoning modifies it only slightly; and it has no other value than as an expression of these sentiments and this reasoning.

But this is not how its author sees it. He has accepted this moral code while swayed by his sentiments, and he sets himself the problem: how can he prove it by experience and logic? He thus necessarily falls into purely verbal wrangles, because this problem is by its very nature insoluble.

19. Men, and probably also animals that live in a community, have certain sentiments which, in determinate circumstances, provide standards for their actions. Man's

sentiments have been divided into various categories, among which those called religion, morality, law, and custom should be noted. Even today precise boundaries cannot be drawn between these different categories, and there was a time when all of them were blurred and formed a nearly homogeneous whole. They have no precise objective counterpart, and are only a product of our mind; it is therefore pointless to try, for instance, to find an objective basis for morality, or justice. And yet, men have in all ages reasoned as though morality and justice existed in their own right, being impelled by a very strong tendency to confer an objective character on subjective facts, and by an overpowering need to cover the relations between their sentiments with a veneer of logic. This is how most theological disputes originate, as well as with the truly monstrous idea of a scientific religion.

Morality and justice were first regarded as subordinate to the divinity; subsequently they acquired an independent existence, and by inverting the terms of the problem even the Almighty himself was required to submit to these laws.¹¹ Here, we have a manifestation of the vacillating role faith plays in men's minds. When faith is all-powerful, the idea of the Divinity is dominant; when faith weakens, the idea of the Divinity gives way to metaphysical concepts such as those we have indicated (§48), and later on to experimental concepts. This movement does not always take place in the same direction, but is subject to wide swings. Even in his day, Plato took action against the Olympian gods in the name of metaphysical abstractions; this was then followed by a revival of faith and by other swings of the pendulum. Finally, in modern times there are theologians for whom belief in God has become nothing more than a belief in "solidarity," and religion dissolves in a nebulous humanitarianism. These theologians imagine that their reasoning is scientific because they have virtually stripped their language of any suggestion of positive religion; however, they do not realize that it remains just as devoid of any experimental concepts, and is composed of nothing but meaningless words suitable only to arouse in some people, by their mere sound, indeterminate, vague feelings of the kind that come to us when we drowse. If one compares the life story of any saint written in the Middle Ages with this empty talk, it will be seen that both are equally devoid of any experimental concept, but the former at least is intelligible, whereas the latter is incomprehensible.

20. Useful research which could be undertaken on these sentiments would cover their nature, their origin, their history, their relation to each other and with other social facts, and the relation they may have to the utility of the individual and of the species (§6).

Even when one engages in this kind of research, it is very difficult to proceed in an entirely dispassionate and scientific fashion; for this is opposed by the profound emotions which the subject arouses. As a rule, those who reason about these sentiments distinguish

¹¹ In our day this is a general opinion. Montesquieu had already written in the *Lettres persanes*, LXXXIII: "If there is a God, my dear Rhédi, He must necessarily be just; because, if He were not, He would be the worst and the most imperfect of all beings. Justice is a relation of propriety that is really to be found between two things: this relation is always the same, for whatever being that is considered, be it God, an angel, or lastly a man."¹⁶

Note first a contradiction. The Almighty has created, in addition to things, this "relation of propriety" between them; He then finds himself obliged to submit to this "relation of propriety."

Observe next the common error of conferring an objective value on what is only subjective. This relation of propriety exists nowhere but in man's mind. This error explains, and in part removes, the contradiction I have pointed out.

two categories, putting those they approve of, which are called true and good, in the first, and those they disapprove of, which are called false and wicked, in the second. These opinions affect all their judgments and leave their stamp on all their research. In Europe, from the Middle Ages until about the 18th century, it was forbidden to speak about religions other than Christianity, except as grievous errors; in our day a humanitarian-democratic religion has emerged, which is the only true and good one; all others, Christianity included, are false and pernicious. Those who adhere to these concepts naively imagine that they are scientifically far superior to those who in the past acted with the same intolerance in a different way.¹²

Among modern writers who have studied the evolution of these sentiments, very few are free from such blemishes, because they have a faith to which they more or less subordinate the facts, and seek to prove that evolution develops in the direction they hanker after and support. Nevertheless, their works have greatly benefited science thanks principally to the facts they have gathered, arranged, and illustrated, and also because these types of studies have helped to create the habit of considering these sentiments objectively, at least to a slight extent. In any case, the evolution, or history, of these sentiments is the best known—or [perhaps I should say]¹⁸ the least unknown—in sociology. Thus, given the limited space at my disposal, this part of the subject is the one on which I intend to dwell the least, preferring instead to turn to other lesser-known topics; I shall not even go fully into these, but only examine certain special cases which will furnish examples of the general theories.

21. There have long been disputes concerning the relations between religious and moral sentiments. The two extreme opinions are: (1) Ethics is an appendage of religion. (2) On the contrary, ethics is autonomous; this gives rise to the doctrine of “independent ethics.”

We must hasten to observe with respect to these disputes what goes on behind the scenes. Those who hold the first of these opinions propose to prove the usefulness of religion as the source of ethics; those who advocate the second seek to prove the uselessness of religion or, more precisely, of a certain religion which is not to their liking. Let us leave such considerations aside for the present; if we examine the problem on its merits, we shall see that it is wrongly posed, because it combines two different problems in one and, as we shall see, they may have different solutions. Here, as in all similar cases, we must distinguish between the logical relations that it may suit us to establish between sentiments, and the factual relations that exist between them; that is, we should, as usual, distinguish between subjective and objective relations.

22. Let us imagine that an individual has certain sentiments A, B, C, \dots ;¹⁹ if they could subsist together only if there were a logical link among them, the two problems we have just distinguished would be reduced to one. This is why such a reduction is usually carried out. It is a common opinion, whether implicit or explicit, that men are guided only by reason, and that all their sentiments are therefore logically linked. But such an opinion

¹² Alfred de Musset, *L'Espoir en Dieu*:

“Absolute kings worshipped a despot God;
Today, we are told God is republican.”

Nowadays the talk is of a socialist God; and there are Christians who admire Christ only as a precursor of Jaurès.¹⁷

is erroneous and is refuted by innumerable facts. This leads to the adoption of another extreme opinion—which is, however, just as erroneous—that man is guided exclusively by his sentiments and not by reason. These sentiments have their origin in the nature of man, combined with the circumstances in which he has lived; and it is not legitimate to assert a priori that there is a logical link among them. There is a logical relation between the shape of a pheasant's bill and the nature of its food; but there is none, or at least none known to us, between the shape of its bill and the color of the male's feathers.

23. The problem posed in §21 may thus be divided into the following two: (1) if we assume—an important premise—that one wishes to prove logically that man ought to follow certain moral standards, what argument appears to be formally the most rigorous? (2) Are religious sentiments or—to restrict somewhat the scope of a problem which may be too general—sentiments caused by a positive religion admitting of a personal God (sentiments which we shall call *A*), always, or usually, accompanied by moral sentiments *B*? That is, is *A* found always, or usually, with *B*, or does *B* occur alone, or usually without *A*?

The first problem belongs to those we have indicated by (*a*) in §6; the second is one of those we have indicated by (*b*).

24. Consider the first problem. Usually, reasoning tends to induce a man to do a certain thing, *A*, which is not agreeable in itself, or which is not sufficiently agreeable to impel him to do it directly. Moreover, in general, *A* denotes not only a positive action but also abstention from something else.

25. Among the innumerable arguments presented concerning the first problem, it will be worthwhile to consider those that are divided into the following categories: (I) It is proved that *A* is ultimately to man's advantage: (*Ia*) because a supernatural being, or even simply a natural or supernatural law (Buddhism), rewards those who do *A*, punishes those who fail to do *A*, whether (*Ia*1) in this life, or (*Ia*2) in the next; or (*Ib*1) to the individual, or (*Ib*2) to the species. (II) It is proved that *A* is the consequence of a certain principle, usually metaphysical, of some precept acknowledged a priori, of some other moral sentiment. For instance: (*IIa*) *A* coincides with what *nature* requires; or, for [several]²⁰ modern authors, with evolution, with the doctrine of "solidarity," etc.; (*IIb*) *A* is the consequence of the precept that we *ought to* endeavor to come closer to perfection; that we should "procure the happiness of mankind, or rather, of all sentient beings";¹³ or that we ought to do everything that might improve and glorify humanity; or that "we ought to act in such a way that the maxim of our will can take the form of a principle of universal legislation" (Kant), etc.

26. Arguments of type (*Ia*) are the most logical, and the best of them are the types (*Ia*2). When Ulysses, in order to prove that guests ought to be well treated, says that they come from Zeus,¹⁴ he uses an argument which, if the premise is accepted, is perfectly logical. It can only be refuted by those who, like Cyclops, believe themselves to be as

¹³ John Stuart Mill, *Logic*, VI, 12, §7.²¹

¹⁴ *Odyssey*, VI, 207, 208.

$\pi\rho\circ\varsigma\gamma\grave{\alpha}\rho\Delta i\circ\varsigma\epsilon'\sigma\iota\tilde{\alpha}\pi\alpha\tau\epsilon s$
 $\xi\epsilon\hat{\nu}\nu\acute{\iota}\tau\epsilon\pi\tau\omega\chi\acute{\iota}\tau\epsilon$

"Because it is from Zeus that all strangers and all beggars come." To Cyclops (IX, 275), he says:

$Z\epsilon\hat{\nu}\varsigma\delta'\dot{\epsilon}\pi\tau\iota\mu\acute{\jmath}\tau\omega\pi\acute{\iota}\kappa\epsilon\tau\alpha\omega\tau\epsilon\,\xi\epsilon\acute{\iota}\nu\omega\tau\epsilon,$

mighty as Zeus, but those who know they are weaker have no escape; it should be noted that they are defeated by their own weapons; it is because of their selfishness that they are afraid of Zeus's omnipotence.

27. The logical link is extremely strong; let us examine the premise implied in the statement that Zeus avenges strangers. In case (I α_1) this proposition can be tested experimentally (I, 36); and consequently it can be easily demolished by the observations of a certain Diagoras,¹⁵ or by those that Cicero puts in Cotta's mouth (*De natura deorum*, III, 34 and *passim*)²³; but in (I α_2) the proposition, not being experimental, is not subject to any experimental verification, and the reasoning becomes so strong that it can only be opposed by a *non liquet*; it is impossible to refute it by proving the contrary.

28. Arguments of type (I β) and especially type (I β_1) lead to obvious sophistries. At bottom, if we strip away the metaphysical veils, to assert that the individual pursues his own advantage by behaving according to moral rules comes down to asserting that virtue is always rewarded and vice punished, which is manifestly false. The proof that is usually employed since Plato's day¹⁶ consists in replacing the pleasant or painful sensations felt by man by abstractions defined in such a way as to make them depend on his having acted morally. This is reasoning in a circle: if happiness is the consequence of moral behavior, it is not hard to conclude that moral behavior is conducive to happiness.

29. These errors originate in the unwillingness to acknowledge pleasurable or unpleasant sensation as a basic fact, which cannot be deduced by reasoning. When a man experiences a sensation, it is absurd to try to prove to him that he experiences a different one. If a man feels unhappy, it is utterly ridiculous to try to prove to him that he is happy, and conversely.

It seems [impossible]²⁴ that a man of Spencer's intelligence should have fallen into such a gross error; but his whole treatise on ethics is unworthy of his intellect [and thus best shows up the defect in the method followed]²⁴. In §79 of his *Evolutionary Ethics*,²⁵ he tries to prove that "other-regarding actions conduce to self-regarding gratifications by generating a genial environment." This is a *petitio principii*. Either a man experiences pleasure in seeing others happy, and, in such a case, it is quite unnecessary to prove to him that he will procure pleasure by making others happy—it is as if one said: "Wine gives you pleasure; consequently, to obtain pleasure, drink wine." Or else this man feels

"Zeus avenges beggars and strangers."

Cyclops replies (IX, 275):

Oὐ γὰρ Κύκλωπες Διὸς αἰγιόχου ἀλέγουσιν.

"The Cyclops do not pay heed to [the aegis-bearing]²² Zeus."

¹⁵ It is said that Diagoras became an atheist because some individual who, by perjury, had done him harm went unpunished. *Sextus Empiricus, Adversus physicos*, p. 562; *Scholia in Aristophanes, Nub. (The Clouds)*, p. 830.

¹⁶ *Republic*, I, pp. 353–4: "Socrates: Is not justice the virtue of the soul, and injustice the vice of the soul? Thrasymachus: Granted. Socrates: Thus, the just man and the just soul will lead a good life, and the unjust man a bad one. Thrasymachus: So it appears from your argument. Socrates: But he who lives a good life is content and happy and the reverse is true for him who does not? Thrasymachus: Of course. Socrates: Then the just man is happy and the unjust unhappy.—'Ο μὲν δίκαιος ἄρα εὐδαιμων, ὁ δὲ ἀδίκος ἀθλιος.' This is what he again paraphrases in III, pp. 444–5.

We do not know what Socrates's real views were; but Xenophon's Socrates nearly always regards the good and the useful as identical, and similarly the evil and the harmful. Anyone who speaks like this flies in the face of the facts; and to prove his affirmation he can only resort to sophistries.

no pleasure in seeing others happy, and in such a case it is not at all true that by benefiting others he will make himself happy. It is as if one were to say: "You do not like wine; but, if you did like it and were to drink some, you would feel happy; so drink wine, and you will be happy."

In §80, Spencer tries to prove that "The sensitiveness to purely personal enjoyments is maintained at a higher pitch by those who minister to the enjoyment of others, than it is by those who devote themselves wholly to personal enjoyments."²⁶ This, again, is reasoning in a circle; one takes as a premise exactly what is to be proved. It is a strange pretension on Spencer's part that he should try to demonstrate logically that we feel what we don't! Here is a man who eats chicken; one tries to prove to him that he would get more pleasure by eating only half of it and giving the other half to his neighbor. He replies: "Certainly not; I have already tried that, and I can assure you that I experience more pleasure by eating the whole chicken myself than by giving half of it to my neighbor." You may call him wicked, you may insult him [as much as you wish]²⁷, but you cannot prove to him logically that he does not have this sensation. The individual is the sole judge of what he likes or dislikes; and if, for instance, he is a man who dislikes spinach, it is utterly ridiculous and absurd to try to prove to him, as one proves Pythagoras's theorem, that he likes it. One may well succeed in convincing him that, by undergoing a certain disagreeable sensation, he will obtain another agreeable one; that, for instance, by eating spinach every day, he will be cured of a certain disease; but he always remains the sole judge of whether or not such a compensation exists between this pleasure and that pain, and nobody can prove to him logically that the compensation exists, if he feels that it does not.

I do not discuss [here the so-called]²⁸ phenomena of the power of suggestion, which have nothing to do with logical proofs.

30. In arguments of type (Iβ2), a premise is generally implied; the complete argument would be: "The individual ought to do whatever benefits the species; A benefits the species. Hence, the individual ought to do A." This premise is usually suppressed, because few would unreservedly subscribe to the statement that the individual ought to do whatever benefits the species; and the introduction of reservations would force us to solve a difficult problem, since the utility of the individual and the utility of the species are heterogeneous quantities which do not easily lend themselves to comparison. Selection operates by sacrificing the individual to the species (VII, 99). It very often happens that what is good and useful for the individual runs absolutely counter to certain circumstances that are favorable to the species. To be sure, the individual cannot exist without the species, and conversely; hence if the species is destroyed, so are the individuals, and conversely. But this is not sufficient to identify the good of the individual with that of the species. An individual may live and be happy by doing harm to all the other individuals who make up the species. Arguments of the kind outlined above are generally lacking on the side of logic.

31. Arguments of type (II)—as well as, for that matter, those of type (I)—may be considered from two points of view. It might be held that the principle to which one wants the moral sentiments to be linked is simply the standard prevailing sentiment. Likewise, there exists an infinity of crystals all of which can be deduced from the cubic system. But the proponents of arguments of type (II) do not usually have this in mind; and, if they did, it would be impossible for them to demonstrate that all the moral sentiments

that now prevail or that have existed in the past can be deduced from the principle they uphold. It is difficult to see how one and the same principle could give rise to the precept “avenge thine enemy,” which prevails in many societies, or even just the Greek precept: “Hate the man who hates you, love deeply the man who loves you,”¹⁷ and the other one, “Forgive thine enemies; love thy neighbor as thyself.” In general, authors seek to furnish the standard of sentiments that *ought* to exist, rather than of those that have existed. This gives rise to the second point of view from which these arguments can be envisioned; they have as their object to describe not what *is*, but what *ought* to be, and this is why they are devoid of logical value.

Herbert Spencer gets around the difficulty by describing as *pro-ethical*²⁹ the habits and customs that observation proves exist or have existed; and he reserves the term *ethical* for something absolute which *ought* to exist. [But he does not, and cannot, establish the proposition in which this term “ought” enters.]³⁰ He criticizes a priori ethics, such as Christian ethics; but his own ethics are fundamentally just as a priori as those he criticizes, and he himself is forced to recognize that observation gives us only the “pro-ethical.”³¹

He is convinced, for instance, that war is immoral. This proposition may satisfy his sentiments and those of other men, but it cannot be proved scientifically, and nobody can say whether war will ever disappear from the earth. Spencer’s abhorrence of war and warlike sentiments is purely subjective; but, following the usual procedure, he elevates it to the level of an objective principle, which he uses to judge the ethics of the different societies. He does not see that he is thus only imitating the religious man, for whom any religion other than his own is false. Spencer simply professes the religion of peace; and this religion is neither better nor worse than Islam, Buddhism, or any other religion.

For part of the way, Spencer applies the rules of scientific reasoning; he then abandons this approach, impelled by a powerful force that induces men to give an objective value to subjective facts, and he moves into the realm of faith, in which he gradually becomes immersed.

32. In cases such as this, the principle adopted by the authors is no more obvious than the conclusions they wish to reach; and they end up proving something dubious by deducing it from something more dubious still. Let us not inquire whether such and such a thing is in conformity with man’s *nature*,¹⁸ or *end*, or with some other similar imaginary entity, or if it is in conformity with evolution, or with some other similar abstraction; because, even if we could be assured that it was, which is not the case, we should not be entitled to draw the conclusion that a given individual ought to do such a thing. Let us go on to arguments of type (IIβ), in which the gaps seem to be less serious.

33. The latter have a common defect, from the point of view of logic, in that their premises lack precision and that no real meaning corresponds to them. We do not realize this at first, because the premises agree with some of our sentiments; but, when we take a closer look at them, the more we try to understand what they mean, the less intelligible they become.

¹⁷ Μισοῦντα μίσει, τὸν φιλοῦνθ' ὑπερφίλει.

¹⁸ *Systèmes*, II, p. 21.

34. Let us, for instance, look at one of the least objectionable of the phrases cited above, namely that of Mill. Let us leave aside the last part, concerning sentient beings—which would prevent us from eating meat and fish, and even from walking, for fear of treading on some insect—and let us consider the theory in its most reasonable form, that of the pursuit of happiness of mankind. The terms are beguiling; they seem clear, but they are not. “Mankind” is not an individual with simple sensations of happiness or unhappiness, but is a totality of individuals who have such sensations. The definition in question implicitly assumes: (1) that one knows exactly what is meant by mankind, whether it comprises only the individuals who live at a given moment, or those who have lived in the past and those who will live in the future; (2) that the conditions for the happiness of each individual in a given community are not contradictory—otherwise, the problem of assuring the happiness of that community would be like that of describing a square triangle; (3) that the amounts of happiness enjoyed by all the individuals are homogeneous, so that they may be added—otherwise, it is not clear exactly how one could determine the sum of happiness enjoyed by a community; and, if this sum is not known, we have no criterion for knowing whether the community is happier in some circumstances than in others.

35. (1) In reality, those who speak of mankind usually mean instead their own country, or, as an extreme case, their own race; and highly moral civilized nations have destroyed and continue to destroy savage or barbarous peoples without the slightest scruple. [Leaving this aside,]³² let us assume that by mankind is meant all men. Extremely serious questions remain to be solved: When the happiness of men presently living is in contradiction with that of men to come, which should prevail? When, as is often the case, the happiness of present individuals clashes with that of the species, must the latter give way to the former, or conversely? It should be noted that European civilization is the fruit of innumerable wars and of very extensive destruction of the weak by the strong; it is at the cost of these sufferings that our present prosperity has been obtained: is this a blessing or an evil? The principle postulated does not by itself suffice to solve these questions.

36. (2) Let us imagine a community composed of a wolf and a lamb; the happiness of the wolf consists in eating the lamb, that of the lamb in not being eaten. How can this community be made happy? Mankind is composed of warlike and peaceful peoples; the happiness of the former consists in conquering the latter; and the happiness of the latter consists in not being conquered. We have to resort to some other principles, and eliminate, for instance, the happiness of the warlike peoples, i.e., deem it less worthy than that of peaceful peoples, who alone are to be taken into account. In that case, this fine principle which was set up expressly to solve moral problems, when put to the test, is cast aside and serves no useful purpose.

The Romans' happiness lay in the destruction of Carthage; that of the Carthaginians perhaps lay in the destruction of Rome, or in any case, in the preservation of their city. How could the happiness of the Romans and Carthaginians have been achieved?

37. (3) It might be answered: the total happiness, if the Romans did not destroy Carthage or the Carthaginians Rome, would be greater than if one of those cities were destroyed. Such an assertion is [without foundation]³³ and cannot be supported by the slightest evidence. How is one to compare those pleasant, or painful, sensations, and add them up? But, to make the utmost concession, let us assume that this is possible, and consider, for example, the problem of determining whether or not slavery is moral.

If the masters' pleasant sensations constitute a sum (?) that is greater than the painful sensations of the slaves; and conversely, if there are few masters and many slaves. But such a solution would, in the former case, certainly not be accepted by those who wish to employ the principle of the greatest happiness of mankind. To know whether theft is, or is not, moral, should we compare the painful sensations of the robbed with the pleasant ones of the robbers, and investigate which have the greater intensity?

38. In order to be able to use Mill's principle, one is led to combine it implicitly [or explicitly]³⁴ with some other principle; for instance, with principles of the class typified by Kant's^[a]. But even then, the difficulties which at first seem removed reappear as soon as one tries to reason with any degree of rigor. There cannot be a truly universal principle of legislation in a society, such as human society, composed of individuals who differ in sex, age, physical and intellectual activities, etc. And if this principle is subject to reservations which take account of these and other similar circumstances, the main problem then consists in knowing which of the reservations should be adopted and which should be dismissed; then the posited premises become entirely useless.

Are the dispositions to be found in Gaius, *De conditione hominum*, I, §§9, 10, 11¹⁹ of the nature of a "principle of universal legislation" or are they not? If they are, slavery is justified; if they are not, it is not even legitimate to arrange for some men—elected, say, and appointed to certain offices—to command while others obey. From a formal point of view, both dispositions are identical, and they differ only in the nature and in the form of the restrictions.

39. It is remarkable how sentiment has such a great sway on men in such matters as to make most of them lose the use of plain reason. For instance, in France at present a large number of men, who seem in other respects to be reasonable, admire the meaningless words of the famous *Declaration of the Rights of Man*. The first paragraph bears some resemblance to a principle of universal legislation. It declares that: "Men are born and remain free and with equal rights; social distinctions can be based only on the common good."^[a] Let us pass over the fact that this liberty and this equality simply mean that men are born and remain free, except with respect to matters in which they are subject to certain constraints; and equal in every respect, except in matters in which they are unequal—i.e., in hardly anything. Let us consider only the proposition according to which social distinctions can be based only on the common good. This is of very little help in solving [the problem of social order, and only disposes of]³⁶ the difficulty, which now consists in determining what is the common good. We need merely read Aristotle to see how slavery can be defended by maintaining that it is in the common good;²⁰ one could equally justify the feudal system, so much hated by the revolutionaries who composed that [fine]³⁷ *Declaration*. In our day the French Jacobins justify the distinction they make between citizens who belong to Christian religious orders and those who belong to Masonic lodges in terms of the common good; but the Athenians also held that the distinction they made between barbarians and the citizens of Athens was based on the common good.

¹⁹ §9. Et quidem summa divisio de iure personarum haec est, quod omnes homines aut liberi sunt aut servi.

§10. Rursus liberorum hominum alii ingenui sunt; alii libertini.

§11. Ingenui sunt, qui liberi nati sunt; libertini, qui ex justa servitate manumissi sunt.³⁵

²⁰ *Systèmes*, II, p. 110.

In short, all these pseudo-scientific arguments are less clear and have less value than the Christian maxim: "Love thy neighbor as thyself." This maxim is, moreover, to be found at very different times and among absolutely different peoples; it is even to be found in the Chinese Lun-Yu.²¹

40. The metaphysical arguments which we have been considering have no objective value, because they are concerned with things that do not exist. They are of the same kind as the argument used in order to find out whether Eros preceded Chaos, Earth, and Tartarus, or if he was Aphrodite's son. It would be futile to inquire what the true situation really was; we can only try to ascertain how the Greeks conceived it; their views are for us facts whose history we can recount.

Numerous ethical systems have held sway in the past and hold sway today, and [for all the time their adherents have contended with one another]³⁸ none of them has acquired a decisive supremacy over the others. [Thus]³⁸ it has remained and still is an open question which system is the best, as in the case of the three rings Boccaccio discusses in one of his short stories; nor could it be otherwise, for there is no experimental or scientific criterion to settle such a question.

The only experimental or scientific content of all these systems consists in the fact that some men have experienced certain sentiments and have expressed them in certain ways.

41. It is from an analogous point of view that in the preceding paragraphs I have considered men's thoughts concerning certain abstractions; but other and more useful³⁹ inquiries remain to be made. We can investigate the nature of these sentiments and the relations that really exist among them, disregarding the imaginary relations that men think exist. Then we can investigate how and in what manner the actual relations have been transformed into imaginary ones. This brings us to problems (β), (γ), (δ) of §6.

42. Let us first see whether these sentiments have any objective existence independent of the diversity of human minds, or whether they are subordinate to this diversity. It is easy to see that only the second hypothesis can be accepted. Even though sentiments regarding religion, morality, patriotism, etc., are given a common expression—both literally and formally—by a great many people, they are interpreted by them in different ways. Plato's Socrates (§65) and Theophrastus's superstitious man had the same religion, but they certainly understood it in very different ways.²² Moreover, without

²¹ *Lun-Yu or Philosophical Colloquies*. Pauthier's [French] translation, I, 4, 15 reads: "Our master's doctrine consists only in having upright intentions and in loving one's neighbor as oneself." The [French] translator adds: "Readers will find it difficult to believe that my translation is accurate; I doubt, however, that there could be a more faithful one."

We also find in the *Mahabharata* the statement that we ought to treat others as we ourselves wish to be treated. More or less similar maxims can be found among a number of peoples. They stem from feelings of kindness toward others, and from the need of the weak to appeal to feelings of equality in order to defend themselves.

²² G. Boissier, *La religion romaine*, I, p. 179, speaking of the apotheosis of the emperors said: "In general, the common herd thought that the Caesars were gods like the others; they attributed the same power to them, and believed that this power revealed itself in the same way; by apparitions and in dreams. Enlightened people, on the contrary, drew a certain distinction between them and the other deities; for them, the Caesars were more like the heroes or demi-gods of the ancient Greeks. In short, they did not accord them more privileges than the Stoics attributed to their sages after death."

having recourse to history, anyone who wants examples has only to look about him to find as many as he wishes. Thus, when we speak, for instance, of love of one's country, we have in mind an abstract category of sentiments, which is made up of particular sentiments prevailing among different individuals; and no more does this category have any objective existence than does that of the mammals, which consists of particular animals which have only an individual existence. For the people who constitute a nation, these [individual]⁴⁰ sentiments, even though they may differ in part, nevertheless have something in common.

43. [Even]⁴⁰ sentiments that belong to different categories appear to us not to be entirely independent of one another. This dependence is not generally logical, as most men wrongly imagine, but arises from the fact that these sentiments have remote and common causes; and that is why they seem to us so many branches from the same trunk.

The dependence appears [mainly]⁴⁰ between actions of the same kind: [thus,]⁴⁰ non-logical actions are favored or thwarted together, as are logical actions. The man who [frequently]⁴⁰ gives way to one kind of sentiment will [subsequently]⁴⁰ give way more easily to other types; a person who is accustomed to [frequent]⁴⁰ use of reasoning in some cases will more readily employ it in other cases.

44. Thus, if, as we shall do in the case of wealth (VII, 11), we arrange people in strata according to their qualities of intelligence and character, putting in the upper strata those who possess both these qualities in the highest degree, and in the lower strata those who possess one or both of these qualities in only a small degree, we shall see that the various sentiments become less dependent the higher one ascends into the upper strata, and more dependent the lower one descends into the lower strata. To pursue the analogy, we may say that the branches are quite distinct and separate in the upper strata, whereas in the lower strata they are all intertwined.

Human society thus presents in space an appearance similar to (but not identical with) the appearance it presents in time; for it is well known that in primitive times the various sentiments—which are now completely distinct—formed a homogeneous mass (§81, footnote 53).

45. It is not just qualities of intelligence and character that behave in [the manner just set out];⁴¹ many other circumstances produce the same effect. [Among the principal ones are the kinds of occupations people engage in.]⁴² Those who govern, from the lowest to the highest steps of the ladder, from a private industrial company to the state, have sentiments that are generally more distinct and more independent than those who are governed; this comes from the fact that the former must necessarily have a larger view than the latter; and precisely because they see things from a higher vantage point, they acquire through practice concepts which are lacking in those who are engaged in occupations of a more restricted scope.²³

46. This new classification coincides in part with the preceding one, and in part also with the classification of men according to their wealth;²⁴ but these categories also differ

²³ It should be noted that by those who govern I do not mean politicians; on the contrary, habits acquired by the man who has for a long time governed any part, large or small, of human activity, and habits acquired by the glib orator, the intriguer, and flatterer of Demos, are essentially different.

²⁴ Those who own and administer great wealth govern a substantial part of human activity, and consequently they usually acquire the habits of the function they perform. A man who merely enjoys his wealth and entrusts

in part. First, it may be observed that there are elements on their way down from the upper strata, while others are on their way up from the lower strata. Moreover, there are members of the intellectual aristocracy who do not use their talents to earn themselves material goods, but who concern themselves with art, literature, and science; there are the idle, the unfit, and those who expend their minds and vigor on sports, etc. In short, innumerable circumstances may lead to men with the same qualities of intelligence and character being allocated differently in the social hierarchy.

47. It should be noted—and this is a new analogy with what happens over time (§81, footnote 53)—that the faculty of abstraction increases as one rises from the lower to the upper strata. It is only in the upper strata that we find the general principles which epitomize the different kinds of actions. With the appearance of these principles, the contradictions that may exist between the actions come to light—contradictions which are more easily hidden as between the concrete cases from which the principles are abstracted.

48. The human mind is so constituted that in periods of ardent faith it does not notice any contradiction between its ideas concerning religion and its other thoughts concerning morality or facts of experience. These various thoughts, although sometimes absolutely at variance with one another, can coexist in the same mind. But when faith wanes, or else when, for example, in passing from the lower to the upper strata in the same society, the different kinds of sentiments become more independent (§19), this coexistence becomes unpleasant and painful, and a man will attempt to suppress it by removing the contradictions he only then notices.

In the minds of the ancient Hellenes,⁴³ the scandalous adventures of their gods coexisted—without the slightest conflict—with fairly high moral principles. The same mind could encompass [without butting against each other,]⁴⁴ the belief that Kronos cut off his father Uranus's genitals with a jagged scythe,²⁵ and the belief that a man who insulted his aged father was hated by the gods.²⁶ In Plato's day, on the contrary, the discord had become acute, and one of these beliefs was on the point of driving out the other. Plato did not want Zeus to be represented as marrying his sister Hera without his parents' knowledge, nor did he want it admitted that "we believe or allow it to be said that Theseus, son of Poseidon, and Pirithoüs, son of Zeus, had tried to abduct Persephone, or that some other scion of the gods, or some other hero, was guilty of impiety or of the crimes recounted by the poets." As time passed, there developed a mania for providing artificial interpretations of ancient beliefs and for changing their meaning. Meanwhile, as Grote well observed: "The doctrine, supposed to have been originally symbolized and subsequently overclouded, in the Greek myths, was in reality first intruded into them by the unconscious fancies of later interpreters. It was one of the various roads which instructed men took to escape from the literal admission of the ancient myths, and to arrive at some new form of belief, more consonant with their ideas of what the attributes and character of the gods ought to be."⁴⁵

its administration to a manager does not belong to this class, just as the politician does not belong to the governing class.

²⁵ Hesiod, *Theogony*, 180.

²⁶ Hesiod, *Opera et dies*, 329.

Similarly, Christians in the Middle Ages did not see—and could not see—those discrepancies between the stories of the Bible and moral principles which were so maliciously brought to light by the *philosophes* of the 18th century.²⁷

49. The discrepancy just pointed out is only one instance of a much more general phenomenon. Barbarous peoples, and the common folk in civilized nations, have other things to do than to study their own sentiments. If some philosopher follows the maxim “know thyself,” the great majority of men pay no heed. Moreover, a man who has certain concepts, and experiences certain sentiments, is generally not very interested in relating them to one another. Even when, with the progress of time, a few men who have the habit of reasoning give the matter some thought, they easily rest content with any relation suggested to them by their imagination. Thus, in certain societies, everything man has to do is commanded by God; and this commandment is the link that determines the relation between completely different facts. Those who speculate further bring out some metaphysical link. In the end, it is only after great strides have been made by civilization that an extremely small number of men try to investigate the experimental links among these facts.

If this is not generally realized, it is because one falls into the error pointed out in §9. That is, it is assumed that facts are the logical consequence of a principle; it then seems very strange that they can be mutually contradictory. It is assumed that man acts under the influence of these logical deductions; and it cannot then be conceived how it is that his various actions can, in part, fail to be mutually related.

50. Under the influence of these preconceptions, man always tries to re-establish among the facts those logical relations which, he fancies, must necessarily exist and could have been obscured only as a result of gross error or profound ignorance.

The attempts made to reconcile faith with reason, religion with science, experience with history, provide us with remarkable instances of such a procedure.

It should be noted that so far none of these attempts has had a great success; indeed, one can lay down as a general rule that the more any faith tries to reconcile itself with science, the more rapidly will it decay.²⁸ This is natural, because one need only open one's eyes a little to see that no one ever became a believer as a result of a proof of the kind used in a theorem in geometry.

Likewise, the metaphysical religions have little or no practical value, because they lack the necessary qualities to act on the reason and senses of the common people.

The *Salvation Army*, by employing means appropriate to those it ministers to, has much greater social effectiveness than that of the most subtle and learned metaphysical discussions.

²⁷ It is well known that, although deeply Christian, Dante believed that vengeance [of relatives]⁴⁶ was just (*Inferno*, XXIX, 31–36).⁴⁷

O Duca mio, la violenta morte
Che non gliè vendicata ancor, diss'io,
Per alcun che dell'onta sia consorte,
Fece lui disdegnos; onde sen gio
Senza parlarmi, si com'io stimo:
Ed in ciò m'ha e' fatto a sè più pio.

²⁸ This is what happened to a certain “liberal protestantism,” which is no longer even a theism. A professor of theology defined religion as “the totality of all solidarities.”

Those who want to introduce historical biblical criticism into the Christian religion fail to understand what absolute divergence there is between science and religion, between reason and faith, and how they correspond to different human needs. The sacred books of all religions derive their value not from their historical precision, but from the sentiments they can arouse in those who read them; and the man who, overwhelmed with grief, calls religion to his aid, does not need a learned historical dissertation about which he would understand nothing, but words of comfort and hope. What religion has become for certain humanitarian theologians turns into nothing but a toy for the use of men of letters and metaphysicians.

If we consider present-day societies, we shall see that this need to reconcile religious and other sentiments is to be found in the upper social strata only; and the latter, in order to make their lucubrations acceptable to the lower strata, have to present them in an entirely different light, i.e., as means of reconciling faith with material interests—which are naturally the main concern of the lower strata. In this way we see, for example, the origin and emergence of the doctrine of the *Christian Democrats*.

Trade unionists wish to be considered as at least equal to the middle class, by virtue of the principle that all men are equal; but later on they jettison this fine principle and regard themselves as far superior to the nonunion workers and scabs.⁴⁸ When the seamen of the port of Marseilles went on strike, they held that the government would have infringed their right to strike if it had replaced them with sailors from the navy. When, subsequently, the captains and officers of the merchant marine struck in their turn, the seamen asked the government to send naval officers to take command of their ships; they had forgotten all about the principle of the right to strike. In a similar fashion a Bushman once said to a traveller: "When someone carries off my wife, he commits a bad deed; when I carry off someone else's wife, I do a good deed."

In the lower strata among the socialists, no contradiction is perceived between the arguments of trade unionists and those of the Marseilles seamen; and if it does not cross their minds, no one pauses to think about it. Only the leaders see the contradiction, and they resolve it forthwith by the use of subtle casuistry; some of them may even be acting in good faith.

[A most glaring and rather comic contradiction is that of people who, on the one hand, ask for the abolition of military courts in the name of equality of citizens before the law, and, on the other hand, ask for a privileged tribunal: conciliation boards for manual and blue-collar workers.

[The same persons who approved of the bizarre decrees of President Magnaud, which explicitly and deliberately contravened the law, were indignant at the timid reservations made in other decrees regarding the law of separation. In the first case, they said: "The judge should let himself be guided by his sense of equity, without bothering about the law;" in the second case, they asserted, no less resolutely, that "the judge has only to apply the law strictly, and if his sentiments are at variance with it, he can simply take leave." Feelings take precedence over reason and prevent us from seeing such an obvious contradiction, or at least from taking account of it.

[In Italy, the decisions of the courts in matters of private libel are null and void when the guilty are socialist members of Parliament; and this state of affairs is approved by believers in a rigorously absolute equality of citizens before the law.]⁴⁹

The “intellectuals” who [in France]⁵⁰ ferociously denounced the military courts for their conduct in a famous trial, and who made the world echo with their complaints, listen without protest to Attorney General Bulot when he declares that there is a *raison d'État*⁵¹ to which a judge must submit or else be dismissed.²⁹ And in spite of the clear words of Mr. Bulot who went so far as to speak explicitly of the “*fait du prince*,”⁵² there are people who believe that the republic is exempt from such faults, which were peculiar to the monarchy.

Other “intellectuals” imagine, in good faith, that only the Catholics threaten “freedom of thought;” so, to win this freedom, they unreservedly approve the persecution of the Catholics, and they admire Mr. Combes. And, even when the latter unambiguously declares that his aim is to establish a new and uniform faith, which is as intolerant as the others,³⁰ they are not aware of the contradictions they are falling into.

In a number of [foreign]⁵³ countries, teetotalism has become a religion, with a fierce band of sectarians; some of these sectarians also subscribe to the religion of materialism or some other similar religion, so that they are fiercely hostile to Catholicism and they deride the obligation to abstain from meat! If they are told that, at bottom, to compel someone to abstain from meat on certain days is a prescription of the same kind—though less irksome—as that of forbidding the consumption of a small quantity of alcoholic beverages, they think they can resolve the contradiction by saying that their prescriptions carry the approval of worthy, democratic, progressive, and sacrosanct “science;” which simply means that certain physicians include this among their many more or less reasonable pieces of advice; and these sectarians forget, or seem to forget—or fail to see—that their “science” nowadays confirms the Catholic prescriptions, by showing that certain illnesses can be avoided by abstaining from meat.³¹ An abundance of similar

²⁹ Official press release of the June 24th session of the Parliamentary commission of inquiry into the case of the Carthusian monks.

“SEMBAT: You too, Mr. Attorney General, have spoken of a higher interest. Is there then a *raison d'État* to which a magistrate is bound to submit?”

“BULOT: Or he may be dismissed, of course.” (Laughter)

“BERTHOULAT: How could the investigation proceed, although you did not have the name which you declared was indispensable to the Prime Minister?”

“BULOT: It did not continue for long, and it ended in a nonsuit, because one could go no further; I submitted to the *raison d'État*, to the *fait du prince* if you like.”

If one admits the “*fait du prince*,” one understands why the magistrates were so lenient to the Humberts, and so hard on the victims of those famous crooks.

Funck-Brentano, in *L'affaire du collier*, Paris, 1901, p. 325, writes: “And such was the absolute power of the monarchy under the *ancien régime* . . . The queen's honor is at stake, the crown may be threatened. The king entrusts the judgment to a court, none of the judges being appointed by him; to magistrates over whom he has no power and will never have any at any point in their career, in any way; to magistrates who, in spirit and by tradition, are hostile to him. As Bugnot has shown, the King's Attorney himself is not, in Parliament, freely chosen by the King. Indeed, we even have the Comptroller General, assisted by the King's librarian . . . who in such serious circumstances directly challenges the King's interests and authority. Nobody is astonished at this. Is there, today, a government that has the courage to see such a freedom flourish under its eyes?”

The government that granted such a freedom was the government of a decadent class, and it fell; the government that is suppressing freedom today is that of a rising and prosperous aristocracy. And the bourgeoisie, ignorant and cowardly, is helping it along with its money.

³⁰ See footnote 64 in §94.

³¹ In 1904, in a long study presented to the Paris Academy of Medicine, Dr. Lucas-Championnière concluded that eating meat causes intestinal disorders as well as appendicitis after a bout of influenza (*grippe*); he recommended a vegetarian diet from time to time, i.e., occasional abstinence from meat.

examples could be cited concerning all species of fanatical sectarians, in all ages and in all countries.

Herbert Spencer remarks "how absolute throughout Europe is the contradiction between the codes of conduct adjusted respectively to the needs of internal amity and external amity."³² But to reconcile these opposite precepts, he takes an expeditious course: without further ado, he throws out the latter, in the name of *his* ethics, and it does not enter his mind that these precepts may be as useful, and even as indispensable, as the former ones.

51. Some circumstances promote the development of [various kinds of sentiments];⁵⁷ other circumstances operate against them. In this way, one of the main attributes of the interdependence of these sentiments is revealed, namely, their common origin. The interdependence between religious and moral sentiments is exactly of this kind, as was already noted in §43; thus, they often tend to be promoted or thwarted together; and the same must be said, in even more precise terms, about all similar sentiments.³³ Analogously, rain in a meadow causes various species of graminaceae to grow; a prolonged drought is harmful to them. It is in this manner that the sentiments we have dealt with are linked, rather than by one type of sentiment depending directly upon the other (§70).

[It is to these general principles that Mr. S[amuel] Reinach's observations should be related; he sees the origin of ethics in *taboos*.

[The primitive religion of Rome was no more than a religious observance almost devoid of theological conceptions; and this circumstance is not without some bearing on the Romans' spirit of discipline, and consequently also on their sway over the entire Mediterranean basin.]⁵⁸

52. This is not to say that we can expect to find the same sentiments in all societies, or that they all increase or decrease in the same degree. It simply means that those sentiments which, for innumerable reasons, can be found in a society are subject to certain circumstances which act upon them all. For example, one society may have certain sentiments A, B, C, . . . and another society may have sentiments B, C, . . . without the sentiment A. If certain circumstances change, the sentiments of the former will become A', B', C' . . . , their intensity having changed, but not in the same degree; and the same will be true of the sentiments in the other society.

At the time when my *Systèmes socialistes* appeared, Lord Salisbury had secured the rejection of one of the numerous absurd bills put forward by the teetotalers, but [once this statesman had retired]⁵⁴ his successors put through a law along the same lines. (*Systèmes socialistes*, I, p. 274)

[When Mr. Yves Guyot asked for proof that absinthe is a poison, a good humanitarian replied by proposing the following experiment, in order to settle the question. "Each of us will drink, every 24 hours, he, two liters of absinthe; I, two liters of water."]

[If the humanitarians deigned to listen to reason, it could be pointed out that according to this proposition, the way to decide whether a substance is toxic or not is to compare the effects produced by the absorption of equal quantities of this substance and of water. Yves Guyot could then make a counterproposition to his opponent, and ask him to consume, every 24 hours, two liters of salt (sodium chloride), whereas Guyot would be content with drinking two liters of water. Table salt would thus be classified as a toxic substance, whose use should be banned.]⁵⁵

³² *The Ethics of Diverse Societies*, §115.⁵⁶

³³ This fact stands in a remote but not unimportant connection with another well-known fact that someone who has often been hypnotized loses his power of resistance and can be lulled to sleep by a mere signal.

53. Not only do these sentiments differ from one society to another, but in the same society they vary according to individuals; and the circumstances that act on these sentiments have effects that differ from one individual to another. For persons whose sentiments have greater independence, it is easier for some types of sentiments to be favored [separately];⁵⁹ for those whose sentiments are less independent, the various sentiments are favored and thwarted together. This is why in the upper strata of the population, it is easy to find people who are lacking in one kind of sentiment, but who possess the other kinds in a high degree.³⁴

54. If men lived completely apart from one another, they might have quite distinct religious, moral, patriotic, and other sentiments; but men live in society and, consequently, more or less in a state of communism as regards those sentiments. The material heritages may be entirely separate one from another; the heritage of sentiments and the intellectual heritage are common, at least in part.

55. Changes in the sentiments of one social class act so as to bring about further changes in the sentiments of other classes. The process may be more or less rapid, sometimes even very slow. As a rule, the sentiments are undermined and weakened by reasoning, in the upper classes; and it is only indirectly that this tendency later extends to the lower classes. Its character and its form then often change; the skeptical reasoning of the upper classes may, in the lower classes, be transformed into a new faith. Conversely, the sentiments of the lower classes act upon the minds of the upper classes, who transform them into pseudo-scientific reasoning.³⁵

56. The ancient Spartans had the [virtues of patriotism]⁶¹ to an eminent degree; they were apparently also rather religious, but they were not moral in the same degree.³⁶ This, as a matter of fact, might be said of most Hellenes; it is the more remarkable to observe—which confirms our general proposition even more strongly—that, when circumstances changed, all these sentiments were weakened together, the strong as well as the weak.

57. In the cases of Athens, thanks to their literary works, we can follow the decadence of the religious sentiments in the intellectually superior classes, from Aeschylus' time on through Euripides, up to the time of the Cynics, the Epicureans, and the Skeptics. The lower class resisted irreligion and only gradually followed the example set from above. A great many facts testify to this resistance, among which we need only mention the condemnation of Diagoras, Socrates, and others of the same kind. A similar phenomenon took place in Rome at the time of Cicero when, incidentally, the resistance of the common people was merely passive; but it became active, and it extended to the upper class with the spread of the oriental cults and with the final triumph of Christianity, which persecuted the philosophers. Reactions of the same kind were observed at the time when the mendicant orders were founded; they were observed again when the irreligion of the cultured classes—in the Latin world particularly—was repudiated by the great religious

³⁴ Bayle, *Pensées diverses... à l'occasion de la comète*, 4th ed., p. 353: "... I wish to observe that the few persons who, among the Ancients, openly professed atheism, such as Diagoras, Theodorus, Euhemerus, and some others, did not live in a way that would have led people to talk about the looseness of their morals. I do not note any accusation of their having distinguished themselves by the dissoluteness of their lives..."

This argument, often cited and given a general value (it is also found in Spencer, *Facts and Comments*)⁶⁰ has only the very restricted value indicated in the text.

³⁵ [a] We can find as many instances as we like in antiquity, the Middle Ages, and in modern times.

³⁶ Fustel de Coulanges, *Nouvelles recherches sur quelques problèmes d'histoire*, p. 92: "There is no Greek city where history mentions so many cases of corruption." And he goes on to cite a large number of facts.

reaction of Protestantism; and once again, in France, when the irreligion of the upper classes led to the revolution of 1789, which was, as has been rightly remarked by de Tocqueville, a *religious* revolution[, the religions being the humanitarian one and that of the Jacobins].⁶²

58. It should be noted that, in all these cases, and in other similar ones that could be cited, the religious reaction is accompanied by a moral reaction.³⁷ The description of these phenomena is always the same: the use of reason weakens the religious sentiments in the upper classes and simultaneously the moral sentiments—sometimes those of patriotism too—and then cosmopolitanism emerges; and in general, it may be said that many non-rational sentiments also lose force. The movement gradually spreads to the lower classes; it then provokes in these classes a reaction that leads to a revival of religious and moral sentiments, and often of patriotic sentiments too. This [movement],⁶³ thus originating in the lower classes, extends little by little to the upper classes, where religious sentiments acquire renewed vigor. And then, again, these new sentiments weaken, as did the old ones. A cycle similar to the one just described starts anew. This is how the rhythmic variations arise which have long been observed in the intensity of religious sentiments.³⁸

59. We should not forget that we are dealing with sentiments, and that we should not confuse them with the form these sentiments may assume. It often happens that the popular reaction, while reviving and stimulating religious sentiments, gives them a new form; in that case, it is not the old religious fervor that reappears, but a new faith. Nor should one confuse religious sentiments with worship; the latter⁶⁴ may decay while the former⁶⁴ flourish. Neither should it be believed that religious sentiments necessarily have a personal god as their object; the case of Buddhism should be sufficient to prevent us from falling into such a gross error; in our own day, there is the example of socialism, which may properly be regarded as having turned into a religion (§85, footnote 57).

60. If the upper classes were able and willing to keep the fruit of their reasoning to themselves, this series of actions and reactions would perhaps be less frequent and less intense. But, as a result of the very conditions of social life, it is difficult for the upper classes to do this; they do not even do the little that they could, because, apart from those who betray their class for illicit gain, other individuals in the upper classes who are honest in material matters are impelled by an urge to make the lower classes take part in their reasoning. And what is more, they are blinded by their envy and hate for the old doctrines relating to sentiment, which they mistakenly insist on judging solely in the light

³⁷ G. Boissier, *La Religion romaine*, II, p. 377, treats as an exceptional fact what is, on the contrary, the rule. Speaking of Roman society in the 3rd century of our era, he says: "What makes the changes in religious opinions at that time so remarkable is that they coincide with those observed in public morality."

Lea, *Histoire de l'Inquisition*, translated by S. Reinach, Vol. I, p. 126 of the translation (p. 111 of the original), gives an example of the rebirth of morals along with that of religious sentiments: "One afternoon as he (Gervais de Tilbury) was escorting his archbishop Willem, on horseback, his attention was attracted by a pretty girl who was working alone in a vineyard. He lost no time in making advances to her, but she rebuffed him, saying that if she were to listen to him, she would be irrevocably damned. So austere a virtue was a clear sign of heresy; the archbishop immediately ordered the girl to be imprisoned as suspect of Catharism."

Machiavelli, *Discorso sulla prima decade di Tito Livio*, Vol. I, p. 12, when discussing his own times, blames the Roman Church for Italy's evils, because "through the bad example of that court, this province has lost all devotion and all religion, which entails innumerable disorders . . . We Italians have as our main debt to the Church and the priests that we have become irreligious and wicked . . ."

³⁸ *Systèmes socialistes*, I, p. 30.

of intrinsic logic. Since they completely fail to understand the great social value of these doctrines, they consider them to be pointless and empty superstitions, thus displaying a stupidity which seems to them to be wisdom.

61. By acting in this manner, and to the extent that they achieve their goal—which generally consists in weakening certain forms of religious sentiment in the lower classes—they achieve another result which was certainly not intended: which is to weaken their moral sentiments too. When, as time passes, they see the emergence of a reaction on the part of religious sentiments, whether in the old form or in a new one, their reason is permeated,⁶⁵ offended, and overpowered, and in short they arrive where they least intended to go.

62. In Athens the resistance of the lower classes did not turn into a reaction that reached the upper classes; this was probably because the phenomenon was disturbed by the Roman conquest. This coexistence for a certain time of an upper class in which reason prevailed and a lower class where sentiment ruled is not the least of the reasons for the extraordinary development of civilization in Athens at that time.³⁹

63. Already in Pericles' circle people were meeting and freely discussing popular beliefs, and their conversations in Aspasia's house call to mind the French salons on the eve of the Revolution; in both cases, philosophy mixed gracefully with loose morals.⁴⁰ The charges brought against Aspasia and Anaxagoras might have originated in political hatred of Pericles; but the form of the accusation, which was a charge of impiety, must have had some support in the facts; this is obvious in Anaxagoras' case. It was by associating with this philosopher, according to Plutarch's *Pericles*, 6, that Pericles profited by learning about the vanity of the popular superstitions surrounding miracles. And, as early as Anaxagoras, patriotism was declining along with religion;⁴¹ finally, Diogenes, the forerunner of our *internationalists*, openly declared himself a cosmopolitan.⁴²

64. From the philosopher's talk and from stage productions, irreligion spread among the people, but not without meeting some resistance. Euripides started his drama of *Melanippe* thus: "Zeus, whoever he may be, . . . for I know him only by report;" but the public was so shocked that he had to change this verse.⁴³ Numerous passages of his

³⁹ See, in another sense, which is however analogous, the case of Scipio and his companions. *Systèmes socialistes*, I, p. 303.

⁴⁰ Plutarch, *Pericles*, 24, recounts that Aspasia [reared prostitutes]⁶⁶, *Athenaeus*, XIII, p. 570: καὶ Ασπασίᾳ δὲ ἡ Σωκρατικὴ ἐνεπορεύετο πλήθη καλῶν γυναικῶν, καὶ ἐπλήθυνεν ἀπὸ τῶν ταύτης ἔταιρόδων ἡ Ἑλλὰς . . .⁶⁷

"Aspasia, the Socratic, carried on a trade in scores of attractive women, and, thanks to her, Greece was full of prostitutes." The story has been embellished by the comic writers, but on the whole there seems to be no doubt about the fact; or, at least it is no more or no less probable than nearly all the facts of Greek history.

Plutarch, in *Pericles*, 32, recounts how Aspasia was accused of impiety (*ἀσεβεία*) by Hermipphe, and also of pandering, having procured free women for Pericles. Phidias was also accused of similar pandering on behalf of Pericles (*ibid.*, 13).

⁴¹ *Diogenes Laertes*, II, 6: "To someone who asked him: 'Are you not concerned with the fatherland?' he answered: 'I am greatly concerned with my fatherland,' and pointed to the sky."⁶⁸

⁴² *Diogenes Laertes*, VI, 63: "When he was asked where he came from, he replied: 'Cosmopolita.' ἐρωτηθεὶς πόθεν εἴη; Κοσμοπολίτης, ἔφη."⁶⁹ See also: Lucian, *Vita auctione* (The sale of lives). Similarly, Epictetus, *Arian*, *The discourses of Epictetus*, III, 24, and Antigenes, *Philo the Jew*. The same story has been told of Socrates, but this seems very unlikely.

⁴³ He replaced it by the following verse: "Zeus, as you are called in truth, [as the voice of truth declares]?" Plutarch, *Amatorius*, XIII, 4.⁷⁰ See also Lucian, *Jupiter tragicus*, 41; *Iustin Martyr* p. 41.

dramas are directed against religion, at least as the common people understand it; he even questions the foundations of morality.⁴⁴

65. The case of Socrates is [quite remarkable]⁷¹. He was deeply respectful of popular religious beliefs, very moral, submitting to the laws of his country to the point of accepting death rather than escape those laws. And yet, his work was involuntarily directed towards the destruction of religion, morals, and patriotism, because, through his dialectics, by urging men to use their reason to inquire into the motives and the nature of these sentiments, he destroyed them at their root. Here we have a typical example of the general theory set forth in §43.

66. One thus arrives at an apparently paradoxical conclusion. Whereas the charges brought against Socrates are false from a formal point of view and in their particulars, they are in substance and in general true. Of all the accusations made by Aristophanes in his comedy *The Clouds*, none is literally even remotely true; and yet, the general impression that *The Clouds* was to give rise to in the minds of those who heard it, i.e., that Socrates' work was at in the last analysis contrary to religious and moral sentiments—is fully justified. Similarly, it is untrue that Socrates "did not consider as gods those that the city honored as such," as was alleged in the accusation that led him to his death. It is even more untrue that he "had corrupted the youth,"⁴⁵ if the word *corrupt* is interpreted in the sense that his accusers had in mind. It is nevertheless absolutely true that, disputing as he did everything with everybody, he unwittingly undermined belief in the gods of the city, and he corrupted the youth in the sense that he weakened in them the faith that was essential if they were to act for the good of the city. Furthermore, the very circumstance which does Socrates most honor, and which, in an abstract perspective, seems to increase his credit considerably, i.e., that he did not charge fees for his lessons, is precisely what made his teaching most dangerous for the city. Indeed the Sophists, who charged high fees for their services, could only have had a small number of listeners, who belonged mostly to the intellectual aristocracy; they could consequently undermine the patriotic beliefs of only a small number of persons; and [to some of these]⁷³ the Sophists could even do more good than harm, because their disciples were accustomed to making use of their reason. Socrates, on the contrary, addressed the artisans, men whose day-to-day preoccupations made it impossible for them to follow with advantage long, subtle, and abstruse arguments, and he destroyed their faith without being able in the least to replace it by useful applications of reason.

67. This insidious and baneful activity was keenly felt by contemporaries, who instinctively understood all the evil that it might do; that is why Socrates had enemies, as much among the partisans of oligarchy as among those of democracy. The Thirty forbade him explicitly to talk with the youth;⁴⁶ the democrats sentenced him to death.

⁴⁴ *The Phenician Women*, 504, 525; *Io*, 1051, etc. Moreover, the words he puts into Hyppolitus' mouth, i.e., "It was my tongue but not my mind that spoke," were often held against him by his contemporaries as being extremely impious, since they really meant that one should not be held to a promise obtained by fraud and trickery. This may, within certain limits, be granted. But it is an example of casuistry: *Systèmes socialistes*, I, p. 29, Aristotle *Rhetic*, I, 15, 19.

⁴⁵ *Diogenes Laertes*, II, 40: "... ἀδικεῖ δὲ καὶ τοὺς νέους διαφθείρων."⁷²

⁴⁶ Xenophon, *Memorabilia*, I, 2, 36. The Thirty summoned Socrates before them. Pretending not to understand, Socrates asked whether in buying something from a man under thirty, he did not need to ask the price. Charicles replied that he was free to do so, "but you have a habit, Socrates, of asking questions when you know

68. As is noted by Zeller (*Philosophie der Griechen*, Vol. III,⁷⁴ 2nd ed., p. 193), the evil was general and was not confined to Socrates' teaching: "Had not all the cultured men of that time received instruction in independent criticism, which was destructive of the traditional faith and morals?" Aristophanes himself, who wanted to lead his contemporaries back to the old ideas, "is soaked in the concepts of his time."⁷⁵

69. One should not overlook a circumstance which is of no great importance for the history of this period, but which acquires significance through the analogy it reveals with other subsequent phenomena: while the old beliefs were weakening, the practices of the *Mysteries* gained considerable ground. Thus we see a slight hint of a different kind of resistance which was the major factor in other phenomena: that is, we see the resistance of the religious sentiments showing itself in a new form (§59).

70. It remains to be seen how the moral and patriotic sentiments declined in intensity together with religious sentiments. Let us emphasize that I am speaking only of sentiments that are linked to positive religions and not of those linked to metaphysical religions, which, by their very nature, are professed by only a very small number of persons (§50).

If we compare the age of Marathon with that of Socrates, opinions diverge. Some, like Grote, deny that morals had become decadent; others, like Zeller, on the contrary, consider that they had become worse. But, if we come down to the times of Demetrius Poliorcetes, for instance, the decay of morals is manifest, and denied by none.⁴⁷ This is enough to support the general proposition that religious, ethical, and patriotic sentiments often decline and rise together; whereas the question of knowing whether the decay had set in in Socrates' day matters only in establishing the rapidity with which the movement originating in the upper classes had spread to the lower ones.

71. If we could rely on the comparisons made by contemporaries between morals of Antiquity and those of their own day, we would be forced to conclude that from Socrates' time on, or even earlier, morals were already in a full decline. But such comparisons must be discounted—even when made by such men as Thucydides (III, 82, 83)—because there was a general prejudice among the ancient writers that the present was worse than the past.⁴⁸

the answer perfectly well; give those questions up." Continuing, Critias, another member of the Thirty, said: "It would be better, Socrates, if you stopped busying yourself with shoemakers, carpenters, and blacksmiths, because they are tired of your talk."

⁴⁷ There is a vast difference between the Athenians who had refused the "earth and water" requested by Darius, and had subsequently withheld the shock of the attack by the powerful Persian fleet at Salamis, and the Athenians who prostrated themselves in a cowardly way before Demetrius Poliorcetes. They enrolled Demetrius and Antigonus among their *savior gods*, and replaced the Archon—whose name was used to designate the [years]⁷⁶—by the priest of the *savior gods*. The spot where Demetrius first alighted from his chariot was consecrated, and an altar to *Demetrius the Savior* was erected there. It was decreed that the persons sent on missions to Demetrius should be called not ambassadors, but *theori*, like those sent to Pythones and Olympia. They even changed the name of one of their months, calling it Demetrian. The rest can be read in Plutarch, *Demetrius*, 10, 11, 12.

⁴⁸ Horace, *Odes.*, III, VI, summarizes a century-old opinion in the lines

Aetas parentum, pejor avis, tulit
Nos nequiores, mox datus
Progeniem vitiosiorem.

"Our fathers were worse than our forefathers, we are worse than our fathers, and we shall leave sons who are worse than we are."

In our day, the contrary opinion has become an article of faith.

We must therefore completely reject this facile but fallacious corroboration of our general proposition, and look for some other way to find out whether it agrees with the facts.

72. We need only fall back on history. The contrast between the heroes of Salamis and Demetrius Poliorcetes' foolish courtiers is too great—and there are too many additional facts of a similar kind—for us to have the slightest doubt on the matter.

73. Let us add that any doubts one might have about comparisons between past and present no longer apply to comparisons made between contemporary facts; and concerning these we have Polybius' [most valuable]⁷⁷ testimony. He observes⁴⁹ that "the excess of religion, which other peoples regard as a vice, is what sustains the Roman Republic. Religion is glorified and possesses extraordinary power in all private transactions. This will astonish many people, but I myself feel that this is brought about on account of the masses.⁵⁰ If it were possible to have a republic composed only of wise men, such a system would perhaps not be necessary . . . Consequently, it seems to me that it was neither accidentally nor rashly that the ancients introduced such beliefs concerning the gods and the punishments of hell into the minds of the common people, whereas it is with much recklessness and senselessness that they have been cast away by the moderns.⁵¹ Thus, to mention only one point, those among the Greeks who control the public purse do not keep their oath when they are entrusted with a single talent, even when they have ten sureties, ten seals, and twice as many witnesses. Among the Romans, on the other hand, those who administer considerable sums as magistrates and as legates keep their word, out of respect for their oath." Before long, however, at the time of Sallust and Cicero, the Romans became as the Greeks were at the time of Polybius.

74. Two things should be observed concerning Polybius' statement. (1) the facts—and there is no valid reason to question them; (2) the interpretation—which falls into the usual error of establishing a relation of cause and effect between religious and moral sentiments, whereas there is only a relation of dependence on common origins and causes (II, 43).

75. (§6, γ) Let us inquire how these sentiments arise and are sustained; and, with this object in view, let us consider a more general problem: i.e., how and why certain facts A, B, C, . . . , exist in society, be they sentiments, institutions, customs, or other similar things.

76. A solution to this problem has recently been presented which, if it could be accepted, would be perfect and would at a single stroke turn sociology into one of the most advanced sciences. This solution is obtained by extending to social facts the theory proposed by Darwin to explain the form of living beings; and there are undoubtedly similarities between the two cases. Following this path, we may then say that the sentiments, institutions, and customs of a given society are those best suited to the society's given circumstances; that there is, in short, a perfect adjustment of the one to the other.

⁴⁹ VI, 56, 57 ff.⁷⁸

⁵⁰ Ἐμοί γέ μήν δοκοῦσι το πλήθους χάριν τούτο πεποιηκέναι.⁷⁹

⁵¹ Scipio, the African, had a [select]⁸⁰ group of friends around him, which included Polybius, and it is very probable that he reproduced the ideas of that group.

Later, Cicero, *De haruspicum responso*, 9, espoused an idea that was current in Rome, by observing that it was because of their religion that the Romans had vanquished other peoples: *omnes gentes nationesque supervimus*.⁸¹

77. The facts seem to confirm this solution, because it indeed contains an element of truth, which is precisely the one that can be found in the analogous theory of the forms of living beings, and which was brought to light by the neo-Darwinists. Accordingly we must assume that selection operates only by destroying the worst forms—those that depart too far from the forms that have adapted to the circumstances of living beings, or societies. Thus, selection does not determine the forms precisely, but only sets certain bounds on these forms.

It is thus certain that a warlike people cannot have absolutely cowardly sentiments, excessively peaceful institutions, or unwarlike customs; but within⁸² these limits, its sentiments, institutions, and customs may vary considerably, and consequently are determined by other circumstances extraneous to selection.

78. The institutions of somewhat civilized peoples are the more lenient to debtors the more capital goods^{83[a]} these peoples possess. This fact, if considered superficially, seems to lend complete confirmation to the theory of §76, and we can say: The fewer capital goods a society possesses, the more valuable they are; and the greater the need to preserve and increase them; consequently, the more rigid must the regulations be that have this objective.

This reasoning is partly true, but also partly false. It is true to the extent that if the less wealthy societies did not have social institutions to prevent the destruction of wealth, they would quickly decline into barbarism. It is false to the extent that such social institutions do not follow precisely the course of the increase in wealth. Thus, they do not keep becoming less rigid as wealth increases; rather, it could well happen that for a short while they remain equally rigid, or even become more rigid, while wealth is increasing. The correspondence between the two phenomena is not perfect, but only roughly approximate.

It should also be noted that this correspondence between the two phenomena does not come about solely through the selection process. In a society in which capital goods are scarce, any destruction of them that takes place will cause grave hardships, and will directly give rise to sentiments calling for measures to prevent similar destruction. This will not be the result of logical reasoning, but of a process similar to the one that induces not only men, but also animals, to ward off anything that might cause them grief.

79. A society in which each individual hated his fellow men obviously could not subsist, and would dissolve. A certain minimum of goodwill towards one's fellows is thus [necessary for the society to maintain itself. There is also another minimum, higher than the preceding one, that is]⁸⁴ necessary for the members of this society, by a process of mutual assistance, to resist the onslaughts of other societies. Above⁸⁵ this minimum, sentiments of goodwill may vary to a greater or lesser extent.

80. Another very simple solution, of the same kind as the preceding one, can be obtained by supposing moral, religious, and other sentiments to be the ones most beneficial to the dominant social class.

This solution has an element of truth, but relatively less than the preceding one, and a larger measure of error. Moral precepts often have as their object the consolidation of the power of the ruling class, but also very often its mitigation.⁵²

⁵² *Systèmes socialistes*, II, p. 115.

81. Among the facts that determine general moral maxims, the instinct of sociability is certainly the most important. We do not know why this instinct exists in some animals, and not in others; it must thus be considered as a primitive fact, which we do not trace further back.

It seems probable that in ethics as well as law⁵³ this instinct was first manifested in disjointed facts, which were then joined together and summarized in moral maxims; in this way the maxims appear to be the result of experience. In a certain sense, the divine sanction given to these maxims can also be considered from the same point of view, because anyone who did not observe them showed himself to be lacking in sentiments necessary in the circumstances of social life in which he found himself; sooner or later, he could pay the penalty. Thus it was not entirely a pretence to say that, for instance, Zeus avenged supplicants.

One usually reasons as if moral maxims originated exclusively from [the sentiments of]⁸⁷ people upon whom these maxims imposed certain rules of action or abstention, but in reality they also originate from [the interests of]⁸⁷ people who derive some advantage from them. Someone who wishes others to do something for him rarely expresses his desire outright; he feels it more advisable to give it the form of a general concept or of a moral maxim. This can be seen very clearly in our own days when we consider the new ethics of solidarity.

82. Social problems are essentially quantitative, whereas we express solutions to them qualitatively. As a result of this fact, some moral maxims are literally opposed to one another, their object being essentially to check excessive deviations in either direction, so as to bring one to the point that is considered to be quantitatively the best. Thus, the maxim "Love thy neighbor as thyself" is set off against the other one, that "charity begins at home."⁵⁴ Some maxims in a society are favorable to the dominant class, but others are unfavorable to it;⁵⁵ in societies where usury is at its most cruel, one finds ethical maxims that are all the more opposed to it. In all these cases, what men regard as a social evil is corrected by certain facts, which are then epitomized in the form of maxims or precepts. Maxims or precepts that apply to certain social classes, to certain castes, to certain communities, etc., can be traced to a similar origin.

What is, rightly or wrongly, considered harmful to a more or less closely knit community is forbidden by a precept of the ethics peculiar to that community; likewise, what is considered useful to it is imposed upon it. There then arise phenomena of interposition among these different ethics, and between these ethics and general ethics.

⁵³ See Post, *Grundriss der ethnologischen Jurisprudenz*, and especially Sir Henry Sumner Maine, *Ancient Law*. He points out that in remote Greek antiquity, the *θέμιστες* were sentences dictated to the judge by the divinity. "Parities of circumstances were probably commoner in the simple mechanism of ancient society than they are now, and in the succession of similar cases awards are likely to follow and resemble each other. Here we have the germ or rudiment of a Custom, a conception posterior to that of Themistes or judgments. However strongly we, with our modern associations, may be inclined to lay down *a priori* that the notion of a Custom must precede that of a judicial sentence, [and that a judgment must affirm a Custom or punish its breach],"⁸⁶ it seems quite certain that the historical order of the ideas is that in which I have placed them."^[a]

⁵⁴ Theognis of Megara says (181–2) that "it is better for man to die than to be poor and to live in dire poverty," and a little farther (315–18) he notes that many of the wicked are rich and many of the poor are good, and adds: "I would not exchange my virtue for their wealth."

⁵⁵ *Systèmes socialistes*, II, p. 315.

83. It is pointless to inquire whether the origin of moral sentiments is *individual* or *social*. The man who does not live in society is an extraordinary being and is almost, or rather, entirely unknown; and a society that is detached from individuals is an abstraction which corresponds to nothing in the real world.⁵⁶ Consequently, all the sentiments that can be observed among men living in society are individual from one point of view and social from another. Social metaphysics, which forms a substratum to this kind of research, is simply socialist metaphysics, and is directed towards favoring certain a priori doctrines.

84. It would be much more important to know how sentiments arise, change, and disappear at present than to know their origin. After all, knowing how certain sentiments arose in primitive societies merely satisfies our curiosity (I, 33), and has hardly any other utility. Likewise, it is of no use to the seaman to know what the boundaries of the seas were in remote geological epochs, whereas it is very important for him to know the seas as they are now. Unfortunately, we know very little about the natural history of the sentiments of our time.

85. (§6, γ) In France, where democracy is [progressing faster],⁸⁹ remarkable changes have taken place before our very eyes in the second half of the 19th century. Religious sentiments seem to have increased in intensity; but they have partly changed their form, and a new Jacobin-socialist religion is flourishing.⁵⁷

The following changes may be observed in moral sentiments: (1) a general increase in morbid pity, which has been termed *humanitarianism*; (2) more particularly, a feeling of pity and even of benevolence toward evildoers, while there is increasing indifference to the misfortunes of the upright man who is their victim; (3) a considerable increase in indulgence for and approval of loose morals in women.

The facts which are related to these changes are the following: (1) the increase in the country's wealth, which makes it possible to waste part of it on *humanitarianism* and indulgence to evildoers; (2) a greater degree of participation of the poorer classes in government; (3) the decadence of the middle class; (4) an unbroken period of peace that has existed for thirty-four years.

The relations that depend on the first fact belong to the kind dealt with in §§76–79. Those that depend on the second belong to the kind cited in §80.

In short, the movement started in the intellectually superior classes; it manifested itself in literature, it then extended to the lower class, and it has taken on a practical form.

86. The sentiments of blame for evildoers, especially thieves, are certainly much weaker; and today, one regards as good judges those who, with little knowledge and no

⁵⁶ *L'individuel et le social*. Rapport au Congrès international de Philosophie, Geneva, 1904.⁸⁸

⁵⁷ The following is one example, out of the many that could be cited, of the way in which most people understand the new faith. M. Pidoux, *La jeunesse socialiste*, Lausanne, 15 January 1903: "Socialism is itself a religion. It is the religion *par excellence*, the human religion which no longer hypocritically believes in a better world; it wants men, in mutual solidarity, to unite their efforts to make this earth a paradise where mankind can enjoy the greatest possible happiness . . . This religion is well worth the one which, for twenty centuries, has planted its cross on the earth . . . Our religion wishes to establish equality among men . . . It is the religion of man, of science, of reason . . . Our religion kindles in men's hearts love of one's neighbor and hatred of evil. It also kindles the revolt that liberates and consoles . . . It kindles revolt against the society in which we live, and prepares the transformation of that society on the basis of collectivism. Two religions are facing each other. The one is the religion of selfishness and [envy],⁹⁰ the other is that of solidarity and science. The latter will be the religion of the future."

scruples, eager only for an unhealthy popularity, protect evildoers and are severe and harsh only towards upright people. This attitude would hardly have been understood by most Frenchmen living, for instance, in 1830, although at that time it had already found its way into literature, but it looked then as if it were merely a series of literary exercises.]⁹¹

The same is true of loose morals. It may be that morals are not, in fact, worse than they were [some]⁹² fifty years ago, but the theory [has certainly changed].]⁹³

This change also took place in the intellectual part of society, and appeared first in an exclusively literary form; it was accepted [by many]⁹⁴ as being only an intellectual diversion, it not being believed that it might some day become part of social morality.

Later on, all these changes turned into so many weapons for the parties bent on subverting the present social order; they found support in socialist doctrines, which they strengthened. Meanwhile they were welcomed by a decadent middle class, eager for perverse pleasures, as often happens with degenerates.

Positive law has followed only sluggishly behind this evolution of morality; and so, some judges, thirsting for praise from the gallery, and eager to get into the new rulers' good graces, openly despise the code and the laws, and for the grounds of their judgments they look to George Sand's^[a] novels and Victor Hugo's *Les Misérables*.

87. The lessened blame felt toward thieves may have some relation to the progress of theories attacking private property, but this relation is by no means certain; instead, the relation to democracy and universal suffrage seems to be much more obvious.⁵⁸ It should be noted here that, even if criminals were present in the same proportion in the upper and the lower classes, the effects would still be different, depending on whether power lies more in the hands of the one class or the other.

In the upper classes, the aim is to maintain laws and moral rules, while transgressing them; in the lower classes, the aim is to change those same laws and rules. This comes about because the strong place themselves above the law and morality, whereas the weak submit to them.

In France, cases [have become]⁹⁶ so numerous in which members of Parliament have to intervene to protect petty criminals among their electorate from the consequences of their misdeeds, that they have finally given rise to general maxims, which form an unwritten body of laws, parallel to but different from the written law; and judges who wish to avoid being mistreated by the government, or who wish to curry favor with it, follow the former laws and not the latter. [The history of swindlers who consistently go unpunished provided they have some political protection is particularly edifying].⁹⁷ In fact, a host of offenses are no longer prosecuted, although they would still be punishable by law (IX, 32 etc.). Magistrates banter gaily about adultery. "Why go on with your speech?" one of these judges asked the lawyer who was handling a case. "Surely you know the Court's fee. It is twenty-five francs, and that is that." This is also the fee charged by the other French judges; and even someone who has won the reputation of being a *good judge*, because of his benevolence to the evildoers, imposes the light fine of one franc for

⁵⁸ In Australia, thefts of gold in the mines go unpunished, because thieves are legion, and owing to their voting power they have an appreciable influence in the government.

[The softening of the penal laws in many European countries has considerably increased the number of criminals who keep their voting rights].⁹⁵

adultery; and is delighted at this new insult to the law, to the institution of the family, and to morals.

Many⁹⁸ of those prostitutes, so dear to the hearts of [certain judges,]⁹⁹ are charging more. Much harsher treatment is meted out to the poor women who once belonged to a religious congregation, and are accused of [violating the law by]¹⁰⁰ feigning not to belong to it any longer. In particular, their faithfulness to their vows of chastity is held as evidence against them.

The growth of democracy has strengthened the sentiment of equality between the two sexes; but it is probable that the cessation of war has had still more to do with it, because it is war that above all brings out man's superiority. This sentiment of equality has given rise to the theory of a single sexual standard for both man and woman. Some visionaries interpret this in the sense that men ought to be more chaste; but the great majority, who stick to reality, interpret it to mean that chastity has become out of date for women.

There is even a writer who has claimed the "right to immorality" for women. Girls have become freer and freer, and their way of life is such that it certainly does not put obstacles in the way of irregular union. This is, however, denied by many who see only what they wish and what their faith in "progress" bids them see, and what happens in reality, as gynecologists well know, since modern, free girls are good customers of theirs.

The ease of abortion in some great modern cities recalls Rome as described by Juvenal; and the public goes and listens, without disapproval or disgust, to a play that indirectly justifies abortion, for which it holds "society" responsible.

All these phenomena are related to the decadence of the middle class. This decadence is nothing but one particular case of a much more general fact: the constant circulation of the [élites].¹⁰¹

88. The example set by France has influence on the sentiments of peoples—such as the Italians for instance—who have numerous and frequent personal and intellectual relations with that country; whence there appears a new cause of change in sentiments, namely imitation.

Imitation takes place not only between one people and another, but also among the various social classes and among the various individuals of which the people are composed; it is in this way that a movement, starting at any point in society, spreads by imitation; and it continues to spread where circumstances are favorable to it; or it dies out if they are unfavorable.

Imitation has its counterpart in opposition.⁵⁹ When a doctrine is generally accepted, somebody comes along to attack it. Many people, after hearing the same thing constantly repeated, feel an urge to assert the opposite. A theory that goes too far in one direction necessarily gives rise to another that goes too far in the opposite direction. The theories of *humanitarianism* and of the equality of men have their necessary counterpoise in the egoistic theories of Nietzsche's *superman*. In the Middle Ages, the witches were an outgrowth of religious exaltation.

⁵⁹ On imitation and opposition, readers may consult Tarde's works *Les lois de l'imitation* and *L'opposition universelle*, which, however, show an extraordinary lack of scientific precision.

I would remind the reader that, for reasons of space, I must deal in a few words with theories to which it would be possible to devote volumes.

89. (§6, δ) Let us see how the objective relations we have just studied are transformed into subjective relations. In general, the following uniformities are observed:

(1) A double transformation takes place. A real objective relation, A, is transformed, without man being aware of it, into a subjective relation, B. Then, owing to the tendency in man to transform subjective relations into objective ones, the relation B is transformed into another objective relation, C, which is different from A and generally imaginary.¹⁰²

(2) Man is always inclined to confer an absolute value on what is only contingent. This tendency is in part satisfied by the transformation of the contingent fact, B, into the imaginary one, C, which is much less contingent, and which may even be absolute. (3) Man always tends to establish a logical relation between different facts which he feels to be dependent on one another, without knowing how or why. Moreover, this logical relation is usually one of cause and effect. With the exception of mechanics and related sciences, relations of mutual dependence are very rarely used. (4) Man is guided by particular interests, and mainly by sentiments, while he imagines and makes others believe that he is guided by general interests [and by pure reason].¹⁰³

It very often happens that A (see Figure 4) is a particular interest which, without man's being aware of it, is transformed into B; and then B is turned into a general interest, C, which is imaginary. It often happens, too, that the transformation AB starts out by taking place consciously, i.e., that man is aware that he is guided by a particular interest; and then, little by little, this slips his mind, and for the relation AB he substitutes the relation CB, i.e., he believes he is moved by a general interest.

This can more easily be explained by an example. A stands for sentiments of sociability and certain relationships that are beneficial to the individual and to the species; B represents sentiments of kindness toward guests; C represents the explanation given for these sentiments, by saying that the guest comes from Zeus. The following is another example: A stands for a poor man's sentiments of cupidity; B is the sentiment that the rich ought to give to the poor; C is the principle of "solidarity" among men.

90. It must be added that belief in the imaginary cause C is, in its turn, a psychic fact; and it thus finds its place among the real facts of type A, which give rise to B. One is thus in the presence of a series of actions and reactions. The case of language brings this out admirably.

The phenomena of phonetics and syntax certainly did not originate in some preexisting grammatical rules; on the contrary, the rules were derived from phonetics and syntax. However, after this operation was completed, the existence of these rules acted, in turn, on phonetics and syntax. The same is true of the facts of law. Although there are still some people who attribute them to imaginary causes and, for example, see their origin in a certain "juridical sense," it is now beginning to be realized that, on the contrary, it is the facts of law that have given rise to the abstract principles (§80), and—it can be argued—to this "juridical sense" too; but, once these principles and this sense had come into existence,¹⁰⁴ they became facts like any others, and acted as such in determining human actions. Indeed, in this particular case, this action rapidly became

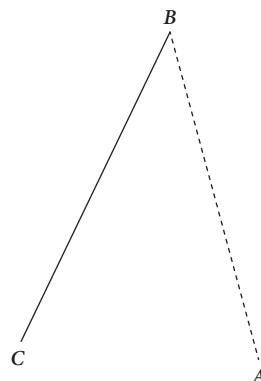


FIG 4

by far the most important and predominant one, because these principles were imposed by force.

91. When for C we take the principle that what is moral is everything that can be taken as a general rule of human actions (or some other similar principle), we can verify that all the uniformities referred to in (§89) are present. (1) The moral sentiments to be explained in this way arose from certain other objective facts, A, as we have already seen. (2) The principle laid down is absolute; it is restricted neither by time nor by place; it applies to the lowest Negro as well as to the most civilized European, to prehistoric and to modern man. The relation CB is of the same nature as a geometrical theorem that holds for all times and places. The metaphysicians do not see the absurdity of this conclusion. (3) The relation between this fine principle of the general rule of human actions and the conclusion B that one seeks to derive from it is logical, at least in appearance, and to the extent that the nature of the principle, which has no real content (§38), allows. Moreover, it is a relation between a cause C and an effect B. (4) This reasoning is used mainly to request others to consent to some sacrifice, or to induce the public authority to impose it on them. If one were to say: "Give me such and such a thing, because it will be useful for me," one would not often get it; one must instead say, "Give me such and such a thing because it will be beneficial to all," and one will then find allies. It should be noted that the word *all* does not generally include the person from whom the thing is taken; but it often means the majority; and this is enough for the incongruity of the expression to pass unnoticed in this pseudo-scientific reasoning.

Striking workmen combat the factory owners and beat up their comrades who wish to work, all in the name of "solidarity." It is pretty clear that this solidarity, while it may well exist among the strikers, does not exist between them and the employers and the scabs. And yet the theorists speak of solidarity among all men; and then they extend the propositions they have arrived at to [that other solidarity which]¹⁰⁵ might more accurately be called a small clique. Solidarity is always [invoked in order to receive, never to give].¹⁰⁶ The workman who earns ten [lire]¹⁰⁷ a day reckons that, in the name of solidarity, the rich man ought to share his wealth with him; but he would find it ridiculous if he himself were to be asked, in the name of this same solidarity, to share his earnings with [someone who earns a wage of a lire or slightly more]¹⁰⁸ a day.

"Democracy" in the United States of America has, as a principle, that all men are equal; that is why in that [civilized]¹⁰⁹ country Negroes and Italians are *lynched*, and Chinese immigration is forbidden, whereas war would be declared on China if Americans were excluded from that country. [In New York, immigrant women are examined by midwives; those who are not properly married are rejected, in order to prevent them from corrupting American innocence.]¹¹⁰ Australian socialists want to come to the aid of "the lowly and humble," and middle class cowards aids and abets them. But in 1894,¹¹¹ when a missionary was murdered by natives, the Australians organized an expedition which mercilessly destroyed a large number of those wretched, perfectly innocent people. French socialists have a mania for peace; they regard war as a crime, [but]¹¹² they openly preach the extermination of the middle class. [In the meantime, they injure the gendarmes, while killing officers and soldiers whom the government has moved in to maintain order. The looting of factories goes unpunished. In Russia, attempts on the lives of factory managers can no longer be counted. At the beginning of 1907, some workers put their manager into an iron tube and roasted him to death.

The European and American humanitarians did not breathe a word about this; but they shriek at the top of their lungs if the police happen to beat up the assassins they are arresting. The humanitarians' sympathy stops at evildoers and does not extend to upright people. The decadent middle class willfully close their eyes and ears in order not to see or hear; and while their adversaries ready themselves to destroy them, they are in raptures over the idea of the coming of a "new and better humanity."^{113[a]}

92. It should be noted that, with the pseudo-logic that often serves to establish the relations CB, the equality of M to N does not imply the equality of N to M, as would be the case in ordinary logic. For instance, in modern democracies, the poor must enjoy the same rights as the rich, because all men are equal; but they are no longer equal if the same rights are claimed for the rich as for the poor. The workmen now have special, privileged courts, the conciliation boards, which, in some countries, always rule against the employers or the middle class and always for the workmen.⁶⁰ If an employer or a bourgeois were to set fire to a workman's house, he would no doubt be sentenced to the penalty laid down by the law; but French strikers and their friends may set fire to and loot the employers' and bourgeois' homes without the government daring to turn the police against them. In Italy, socialist lawyers and their friends indulge in the kind of violence and abuse towards magistrates which would not be tolerated on the part of others. In July 1904, at Cluses, there was a strike of workers in a watch factory. As a condition of taking the workmen back, one of the employers asked them to pay for the panes they had broken at the beginning of the strike. The workmen were extremely indignant at this strange claim; this is understandable because every man looks after his own interest; but the humanitarian middle class was also thoroughly indignant about it—and this would be harder to understand if one did not stop to consider what a contemptible and decadent lot of people they are. [The French proverb: "He who breaks, pays,"]¹¹⁴ applies, of course, to the bourgeois [middle class] only, and not to the workmen, and still less to the [sacrosanct]¹¹⁵ strikers. The factory was besieged, the little child of one of the owners was hit by a stone, in his mother's arms; to defend themselves, the owners fired at the aggressors. The plant was then ransacked and set ablaze, and the armed forces which surrounded it did nothing to halt the attack. Proceedings were instituted against a few of the looters [and incendiaries]¹¹⁶—chosen, however, from among the minor offenders. But since a general strike would have been declared if they had been arrested, they were allowed out on bail. The owners who had defended themselves were, on the contrary, placed under custody awaiting trial. They were then convicted,⁶¹ whereas the looters [and incendiaries]¹¹⁶ were acquitted.

At the end of 1903, the French Parliament voted an amnesty for all acts committed during or connected with the strikes. While this amnesty was being discussed, some individuals, confident of impunity, went up to Paris and looted shops. Two of them were summoned before the court, which declared that the amnesty was applicable to them; the others were therefore left undisturbed. Had a shopkeeper looted the house of one of these evildoers, he would certainly have been convicted by the courts. And yet there are people who in [perfectly]¹¹⁷ good faith believe that this is the regime of equality among

⁶⁰ *Systèmes socialistes*, I, p. 136.

⁶¹ Even Combes' government was finally ashamed; and four months later, it pardoned these poor wretches.

citizens, and who go into ecstasies at the thought of its superiority over the *ancien régime*, under which some citizens were privileged.

93. People who want to convince others that they are guided by the general interest and not by a particular one, are not always sincere. Among the sophistries used extremely frequently when one wants particularly to strike at a certain object, E, while pretending instead to lay down a general measure, the following one is noteworthy. The object E has certain characteristics, M, N, P, . . . ; one of them, e.g., M, is chosen which, in appearance, serves to distinguish this object from the others; and it is asserted that the measure is of a general nature and is directed against M. Republics in ancient times often passed laws which seemed to be general but which were essentially aimed at a small number of individuals, or even one individual.

Sparta, at the start of the Peloponnesian war, sent ambassadors to Athens to ask “the Athenians to avenge the sacrilege to the goddess.”⁶² This was a roundabout way of requesting them to drive out Pericles, who, through his mother, was descended from the Alcmeonides who were considered guilty of this sacrilege.

The sophistry becomes even more obvious when M is also found in some other object, F, to which the measure taken against E, allegedly because of M, does not apply. For instance, [in 1906],¹¹⁸ in France, in order to exclude the religious orders from teaching, some people said that the only object of the prohibition was to bar unmarried people from teaching. But it is obvious that if [the males who belong to religious orders do not have wives, and the females do not have husbands],¹¹⁹ it is also true that not all celibate people are members of religious orders. If it was intended to strike at them, this would have to have been done directly and not through the intermediary of religious orders.

94. The very same idea may be expressed in different languages and, in the same language, in several ways. The same discussion which some centuries ago would have taken on a theological form would today take on a socialist form. When people say in modern jargon that a law displays “broad humanity,” this must be translated by saying that it favors the lazy and the good-for-nothings at the expense of hard-working and upright people. If someone wishes to express the idea that a man seems to him to deserve blame, one would say, in the language of the Middle Ages, that he is an excommunicated heretic; in the language of the Jacobins at the end of the 18th century, that he is an aristocrat; in the language of the modern Jacobins, that he is a reactionary.⁶³ These are simply different ways of expressing the same thing.

More generally, it can be observed that in society a phenomenon can remain fundamentally the same and assume various and often very different forms in the course of time; in other words, the same phenomenon persists in various forms.⁶⁴

⁶² Thucydides, I, 126: “. . . ἐκέλευον τοὺς Ἀθηναίους τὸ ἄγος ἐλαύνειν τῆς θεοῦ.”

⁶³ As the Paris correspondent of the *Journal de Genève* (29 January 1905) [so well expressed it].¹²⁰ “For the word ‘clerical’ has lost its real meaning today just as much as that of ‘aristocrat’ under the Committee of Public Safety.”

⁶⁴ Numerous facts supporting this theory can be found in my *Systèmes socialistes* and in the table of contents: *Persistance des mêmes phénomènes sociaux*. We shall only add one fact which occurred after the first edition of that book had appeared in print.

In the sitting of the French Senate of June 24th 1904, the President of the Council, M. Combes, defending the law which bars the religious orders from teaching, said: “We believe that we are not being visionaries in regarding it as desirable and practicable to achieve in contemporary France what the *ancien régime* had so firmly established in France of yore. One king, one faith: such was their motto. This maxim was the basis of the power

95. The foregoing remarks show that there is some truth in G. Sorel's observation that anything concerning the fatherland and tradition is of a mythical character,⁶⁵ and that "myths are necessary for a precise exposition of the conclusions of a social philosophy that does not wish to deceive itself . . ."⁶⁶¹²² Indeed, whenever we try to understand what certain men were or are thinking, we have to know the language and the forms in which they expressed their thoughts. Grote, for instance, showed clearly that we cannot understand the history of the ancient Greeks if we do not try, as much as possible, to absorb the myths which formed the intellectual background in which they lived.

Similarly, anyone who wants to act effectively on men has to speak their language and employ forms that are acceptable to them, and hence has to use the language of myths.

96. But Sorel's theory is incomplete, for besides these subjective phenomena, there are objective ones, and one cannot prevent others from being concerned with them. Sorel's error arises out of the precept he lays down: "What sociology needs is to adopt, from the start, a frankly subjective approach; it should know what it wants to do, and it should subordinate all its inquiries to the kind of solution it wishes to advocate."⁶⁷ This may well be the object of propaganda, but not of science. Let us not argue about words; let this thing bear any name one likes! How can somebody be forbidden to inquire into the objective phenomena which underlie these subjective facts, or even simply from looking for uniformities in these ways of considering subjective facts?

Sorel himself provides an example of the two kinds of considerations involved in a subjective fact. He says that "it is probable that Marx had put forward the concept of the catastrophe [the destruction of the middle class resulting from the concentration of wealth] only as a myth, which was to provide a very clear illustration of the class struggle and social revolution."⁶⁸

Marx may have had any concept he pleased; but it is still legitimate for us to inquire whether that catastrophe happened, or did not happen, within the limits of time assigned to it. I cannot conceive how or why men should be forbidden to deal with this objective fact.

Moreover, if Marx had wanted to express himself in myths, he would have done well to make this known before events contradicted his predictions; otherwise the prophet's job becomes too easy. Someone makes a prophecy; if it is confirmed by the facts, one admires its author's perspicacity; if it is contradicted, it is said to be a myth.

97. (§6e) Our inquiries have so far borne on facts that actually occurred and upon movements that may be called REAL, as distinct from other hypothetical ones, and which we shall call VIRTUAL (III, 22).

The subject is not exhausted by a study of how certain facts take place. There remains a problem of great importance: if, by hypothesis, one among the facts which are related to

of our monarchical governments. We should try to find a similar one that corresponds to the requirements of the present time."

A great many people, in France, think in the same way; the persistence of this state of mind is remarkable from the time of the repeal of the Edict of Nantes (not to go further back) up to the present day. The form changes but the substance remains the same.

⁶⁵ *La ruine du monde antique*, p. 213.¹²¹

⁶⁷ *Introduction à l'économie moderne*, p. 368.

⁶⁶ *Introduction à l'économie moderne*, p. 377.¹²³

⁶⁸ *Introduction à l'économie moderne*, p. 377.

each other is modified, what changes will the others undergo? This problem is a necessary prelude to the solution of a second problem: under what circumstances is maximum utility procured for a society, for a part of society, [for a social class],¹²⁴ or for some particular individual—given, of course, that one defines at the outset what is meant by this utility?

98. These problems arise in all man's actions, and thus also in those that are the object of POLITICS. These are in practice more important than all the others; or rather—still from the practical point of view—they are the only ones that matter, and any other study is useful only as a preparation for this one. But they are the most difficult ones; we shall come across them in political economy, and in that subject we shall be able at least to arrive at approximate solutions. On the other hand, to the extent that they have to do with actions that depend upon sentiments and upon politics, no solutions to these problems exist—not even roughly approximate ones. This difference is the main reason why political economy is more advanced than other branches of the social sciences.

99. The basis of all reasoning on the present subject is to be found in the following problem: what effects will certain given measures have upon sentiments? Not only are we not in any position to solve this problem theoretically, in general; but we are even lacking in practical solutions which, in the history of human knowledge, usually precede theoretical solutions and often constitute the material from which the latter are derived. Even the most brilliant statesmen almost always go astray when they look for such solutions. We need merely recall the example of Bismarck. He tried to solve the following problem: what measures would bring about a weakening of the sentiments that give the Catholic and Socialist parties their strength? He thought he had found the solution in the measures of the *Kulturkampf* and in the emergency laws against the Socialists. The facts proved that he had committed an enormous blunder. The subsequent developments were precisely the opposite of what he had intended: the Catholic party has become the most powerful one in the Reichstag; the Socialist party has gone from strength to strength, and it has been collecting an increasing number of votes in each election. Not only did Bismarck's measures fail to prevent these consequences, but instead they greatly contributed to bringing them about.⁶⁹

100. The difficulties in the way of setting up a theory in this field are in part objective, in part subjective.

Among the objective difficulties, the following may be noted:

(1) Phenomena come about very slowly, and consequently do not have the necessary frequency to give rise, by confirmation and reconfirmation, to a theory. All the sciences have made extraordinary progress; nevertheless, in the subject under discussion, the best analysis available is still to be found in the works of Aristotle and Machiavelli. Among the many reasons for this fact, not the least important is that each of these authors lived in an age when political changes were rapid, numerous in space, and frequent in time. Aristotle found in the numerous republics abundant material for his studies, as did Machiavelli in the numerous Italian states. If experiences similar to those just mentioned with reference

⁶⁹ After all, what has been established with the greatest degree of certainty on this subject is already in Machiavelli: "... men have to be coaxed or exterminated, because they take vengeance for slight offenses; for grave offenses they cannot; so that the offense done to a man should be such that one need not fear his vengeance." *Il principe*, ch. III.

to Bismarck had been numerous and [repeated]¹²⁵ over a small number of years, we might have been able, perhaps, by comparing them and inquiring into what they had in common and in what respects they differed, to discover in the end some uniformity which could be the beginning of a theory. But we have had to wait until now for another experience of this kind; namely, the struggle of the French Jacobins with the Catholics. If this should result in something similar to what followed the German *Kulturkampf*, we would have an indication of a uniformity. But what a poor indication that rests on only two facts!

(2) Phenomena related to sentiment cannot be measured with precision; we therefore lack the statistics, which are so useful in political economy. The statement that certain sentiments become weaker or stronger is always a little arbitrary; it always depends to some extent on the author who is judging the events.

(3) [The phenomena are]¹²⁶ much rarer and more complex than those studied in political economy; and they are the result of many more causes, or, more exactly, they are in mutual relation with a larger number of other phenomena.

(4) As they are very often nonlogical (§3), we cannot put them into a mutual relation by means of logical deductions, as can be done in political economy. The difficulty is all the greater because men have a habit of attributing nonreal logical motives to their actions.

(5) It is extremely difficult to ascertain anyone else's sentiments, and even one's own, with any precision; hence the material that should serve as the basis of the theory is always somewhat uncertain. For example, in §99, we cited as proof of the strength of the socialist sentiments in Germany the fact that the number of votes picked up by the socialist party was increasing. But this is only one indication, which needs to be supported by others, because many of these voters are not socialists, but radicals, liberals, and malcontents.

101. Let us now pass on to the subjective difficulties:

(1) The authors almost never search for the truth, but look for arguments to defend what they consider in advance to be the truth, which is for them an article of faith. Investigations of this kind are always sterile, at least in part. Not only do authors act in this way because they involuntarily succumb to their passions, but they often do it deliberately; nor do they refrain from sharply censuring those who refuse to act in this way. What foolish [and fatuous]¹²⁷ accusations have been levelled at Machiavelli! This difficulty also exists in political economy; likewise, the difficulties we are about to discuss are common to political economy and sociology. Most economists study and set forth their material with the intention of arriving at a foregone conclusion.

(2) There are countless prejudices and *a priori* concepts derived from religion, morals, patriotism, etc., to prevent us from reasoning scientifically on social matters. The Jacobins, for instance, seriously believe that "kings and priests" are the cause of all the evils of mankind,⁷⁰ and they look at the whole of history through these tinted glasses. Many of them imagine that Socrates was the victim of "priests," even though priests had absolutely no part in Socrates' death. For many socialists, any misfortune, small or great, descending on man is an undeniable consequence of "capitalism." [Theodore] Roosevelt is convinced that the American people are far superior to other peoples; and

⁷⁰ *Systèmes socialistes*, II, p. 491.

he does not see how ridiculous it is to cite Washington to tell the world that “to be prepared for war is the most effective means to promote peace” (*American Ideals*, ch. VIII^[a], entitled: “Washington’s Forgotten Maxim” [!!] ¹²⁸).⁷¹ Truly we poor Europeans had imagined that, some time before Washington ever existed, certain inhabitants of a wretched little country called Latium had, in their idiom, already said: *si vis pacem*,¹³⁰ and so on, but it seems that we were mistaken, and that the Latins must have copied Washington and repeated what he was the first to say.

Other difficulties of the same kind will be found in Herbert Spencer’s *Introduction to Social Science*.¹³¹

The same difficulties are encountered in the study of political economy. The “ethical” economists, with great affectation, casually discuss matters they do not understand.¹³² Another writer, puffed up with conceit, struts about like a peacock to hide his ignorance, as he announces to the public that he is applying the “historical method.”¹³³ Still another, speaking of the “mathematical method,” passes judgment on it and tears it to pieces, but he is as familiar with what he is talking about as an Athenian at the time of Pericles could have been about the Chinese language.¹³⁴

(3) The subjective difficulty pointed out in No. 5 of §100 is related to an analogous subjective difficulty: we find it very difficult to avoid judging the actions of others through our own sentiments. It is only in recent history that it has come to be understood that, in order to have a clear idea of the facts about a given people at a given time, one has had to try, as far as possible, to see them with the sentiments and the ideas of a man belonging to that people at that time. It was thus discovered that there are things which, although bearing the same name, are essentially different, depending on the place and time when they were observed. The French Jacobins of the first revolution believed, and some of their present successors still believe, that the French republic is similar [to and almost identical with]¹³⁵ the Roman republic or the Athenian republic.

(4) Faith alone spurs men on strongly to act; it is therefore by no means desirable, for the good of society, that the majority of men, or even a large number of them, should deal scientifically with social matters. Hence there is a conflict between the conditions of *action* and those of *knowledge*.⁷² And from this we derive a new argument (§60) which shows us how unwisely people act who seek to extend knowledge to everybody, without distinction or discrimination. It is true that the harm that might ensue is corrected in part by the fact that what they call *knowledge* is simply a particular kind of sectarian faith; and we ought to dwell less on the evils of skepticism than on those that flow from that faith.

⁷¹ “A century has passed since Washington wrote: ‘To be prepared for war is the most effectual means to promote peace.’ We pay this maxim the lip service we so often pay to Washington’s words; but it has never been graven on our hearts.”^{129[b]}

⁷² For instance, Roosevelt’s book, *American Ideals*, may perhaps be useful in inducing the citizens of the United States to take action, but it certainly adds nothing to our knowledge, and its scientific value is very close to zero.

The author believes that his country is the foremost in the world; “To bear the name of American is to bear the most honorable of titles;”¹³⁶ others think the same about their own country; an Englishman may think the same about England; a German about Germany, etc. Logically the two following propositions: A is superior to B, and B is superior to A, are contradictory, and cannot both hold;¹³⁷ but they both may well be upheld if their sole object is to induce men to take action.

(5) The conflict between the conditions of action and those of knowledge also appears in the fact that, in action, we conform to certain rules of custom and morality; it would, indeed, not be possible to do otherwise, if only because we should have neither the time nor the means to go back to the origins, in each particular case, and work out a complete theory of it. On the other hand, to know or to *understand* the relations between things, one must discuss these very principles.

For instance, in a warlike people, customs promote warlike sentiments. Assuming that this people is to remain warlike, it is in the people's interest that individuals' actions conform to these sentiments, at least within certain limits. One is then justified, at least within these limits, in judging a given activity to be harmful simply because it conflicts with those sentiments. But this conclusion is no longer warranted if one inquires whether it is advantageous for this people to be warlike or peace-loving.

Similarly, under a system of private property, certain sentiments are offended when property rights are violated; and, as long as it is deemed useful to maintain the system, it is logical to condemn acts which conflict with those sentiments. These thus become a valid criterion of what is good or bad in that society. But they cannot play this role when instead there is some question as to whether private property should be preserved or abolished. To raise the objection, as some authors did in the first half of the 19th century, that socialists are evildoers because they want to abolish private property, is indeed reasoning in a vicious circle, and turning the accused into a judge. One would commit a similar error if one were to judge free love by invoking sentiments of chastity, decency, and modesty.

In a society organized in a certain manner, where certain sentiments, A, exist, it may reasonably be considered that something, B, that is contrary to these sentiments, may be harmful; but since experience tells us that some societies are organized in a different way, in one of these there may exist certain sentiments, C, that are favorable to B, and B may be useful to that society. Consequently, when it is proposed to establish B, in order to pass from the first system to the second, it can no longer be objected that B is contrary to sentiments, A, prevailing in the first system.

It should be further observed that the universal consensus of man—assuming for the sake of the argument that it could be known—would not alter this conclusion in the least, even if one disregards the consideration that [today's universal consensus need not be the same as yesterday's, nor remain the universal consensus tomorrow].¹³⁸

(6)¹³⁹ To persuade someone in scientific matters, it is necessary to present facts that are, as far as possible, certain, and relate them logically to the consequences one wishes to draw from them. To persuade someone in matters of sentiments—and nearly all reasoning about society and human institutions belongs to this category—it is necessary to present facts that can arouse the sentiments so that they will suggest the conclusion one wishes to draw from them. It is obvious that these two modes of reasoning are completely different.

The following is an example. Brunetière, replying to René Bazin, in the sitting of the French Academy of April 29th, 1904, starts out by proving that art must be *human*: "It is possible for us, indeed, everything leads us to believe that, if we did not exist, the planets would nonetheless describe their orbits in space; and it does not seem likely that if we were some day to disappear from the surface of the globe, nature and life would vanish and disappear with us. But what is art without man? What would be its

purpose? Indeed what would be its material? Art has properly speaking no existence or reality but for man and through man . . . This is why the basic condition of art is to be human, even before being art.” Note that, in this context, *human* simply means: what belongs to man; and in this sense the proposition stated is not disputable. But scarcely had Brunetière demonstrated his proposition in one sense, than he uses it in another; and, by a sleight of hand, *human* is changed into *humanitarian*, which is not the same thing at all. “The naturalists finally understood it [the proposition in the meaning indicated above] . . . they have realized that the naturalist novel, freed from its former constraints, could not but develop sooner or later into the social novel.” Here the new meaning comes to the fore. “*By plunging in among the populace*, to use La Bruyère’s expression, it was therefore inevitable that naturalism would make discoveries . . .” And here we see *social* assuming a special sense, meaning: that which belongs to certain social classes; further on, this meaning becomes more and more specific, and *human* art becomes not only *humanitarian art*, but even *humanitarian* in the sense that suits Brunetière: “You have realized that curiosity about pleasure or the suffering of others would be nothing but indiscreet or even perverse if we were not thereby looking for reasons and means of establishing or tightening the bonds of solidarity which bind us to them.” It would seem that the wretched middle class do not rank as men, and thus that what appertains to them is not *human*. Brunetière asks whether it has been noticed that in Bazin’s novels: “One can only catch a glimpse in the background of some lightly sketched-in figures of bourgeois heroes. But the real characters, the ones you love, the ones your heart and your talent prefer . . . are all drawn from the people, from the real people, those who work with their hands, farmers, factory workers . . . It is in the narrow circle of their profession that you have set the drama of their existence. One does not even see, in *La Terre qui meurt*, the owner of the farm tilled by the Lumineaus . . .” If he had come on the scene, the novel would not have been *human*; the owner is not considered as a man. Finally, in a lyrical outburst, our author, addressing Bazin, declares: “I hardly know, in contemporary literature, a work that is less aristocrat and less bourgeois, more popular, than yours. Not one of the masters of the contemporary theater or novel has shown a greater readiness to study the humble, with a more concerned or more passionate curiosity about their ills.”⁷³

In short, our author’s reasoning boils down to this: art should concern itself with things related to man, it should be *human*; thus, it should be concerned only with the people, with workmen, and have as its purpose solidarity, i.e., be *humanitarian*.

Logically, this reasoning is absurd; it was nonetheless favorably received and applauded by the worthy bourgeois who listened to it. This follows simply because they did not pay attention to the reasoning; they paid attention only to the words, which gradually

⁷³ To grasp the intention of this speech, it should not be forgotten that there is strong competition between Brunetière’s Catholic socialism and the other brands of socialism. The supporters of any of these doctrines are always racking their brains to prove that they are more concerned with the good of the people than the followers of the other doctrines. Everyone tries to bring grist to his mill by flattering and cheating Demos.

Brunetière reserves the title of “social novel” to the works of his preference and denies it to his opponents’ novels; “because I am giving the name of ‘social novel’ neither to the *Mystères de Paris*, nor to the *Compagnon du Tour de France*, nor to the *Misérables*.” For their part, the socialists do not grant Brunetière the right to call himself a socialist.

Anyone who can call himself a “genuine and undisputed socialist” without having this title challenged by anyone [has solved the most insoluble of problems.]¹⁴⁰

stirred up their sentiments. These good souls believe that, by prostrating themselves before the people, by humble adulation, they will come back to power. Moreover, they are completely lacking in any moral vigor; hence, in order to experience pleasant sensations, all they need is to hear some passable literary work studded with words like: people, workers, the lowly and the humble, human solidarity, etc.

In many peoples, reasoning on social matters stops at the point where it appears that certain facts are, or are not, acceptable to the religious sentiments. At present, among civilized peoples, this point is to be found where it appears that the facts agree, or do not agree, with humanitarian sentiments; and it does not occur to anyone to follow the scientific procedure of examining these sentiments themselves.

For instance, Herbert Spencer's sentiments are absolutely opposed to war; hence, after carrying his reasoning up to the point where he shows that certain facts clash with these sentiments, there is nothing for him to add; and these facts are condemned.⁷⁴ Other authors stop at the point where they can prove that something is contrary to "the equality of man," and it does not occur to them that this equality may perfectly well be disputed.

102. Human society is not homogeneous; it is formed of elements which differ in greater or lesser degree, not only in their obvious characteristics, such as sex, age physical strength, health, etc., but also in their less observable, but not less important, characteristics, such as intellectual and moral qualities, activity, courage, etc.

The statement that men are objectively equal is so absurd that it does not even call for a refutation. On the other hand, the subjective concept of the equality of man is a fact of great importance, and one which acts powerfully in determining the changes that society undergoes.

103. Just as one distinguishes rich and poor in a society—even though incomes increase by imperceptible degrees from the lowest to the highest level—one can draw a distinction in a society between the élite or *aristocratic* part, in the etymological sense of the word (*ἀριστος* = better), and the part consisting of the common people; but it must always be remembered that the transition from one to the other is imperceptible.

The concept one forms of an élite is dictated by the qualities one looks for in it. There may be an aristocracy of saints, just as there may be an aristocracy of brigands; an aristocracy of the learned, or an aristocracy of knaves, etc. When, then, we consider the totality of qualities that are favorable to prospering and being dominant in a society, we have what we shall call simply an *aristocracy*, or *élite*^[a].

This élite actually exists in all societies, and governs them, even when in outward appearances the regime is very broadly democratic.

⁷⁴ In *The Ethics of Diverse Societies*,¹⁴¹ §127, our author writes: "The epithet 'great' as applied to Alexander, Karl [i.e., Charlemagne], [the czar] Peter [of Russia], Frederick [of Prussia], Napoleon, is applied notwithstanding all the atrocities they committed." It does not occur to him that [at least some]¹⁴² of these actions may have contributed greatly to human civilization. Moreover, he blames Lord Wolseley, a general in the British Army, for having told his soldiers that "they must believe that their duties are the noblest that fall to man's lot."¹⁴³ But how could a general express himself otherwise? Should he tell his soldiers: "You are evildoers, because you fight; you should take to your heels"?

Spencer himself acknowledges, in his *Principles of Sociology*,¹⁴⁴ that in other times war has been useful to civilization. Apparently we are now in a time when that is no longer the case; war has become harmful. This proposition may be true—it may be also be false—but it is certainly not so obvious as to become an axiom by which one may judge all men's actions in our own time.

In accordance with a very important law—and one that provides the principal explanation of many social and historical facts—these aristocracies do not last, but are continually renewed; there thus comes about a phenomenon which may be called the [*circulation of the élites.*]^{145[b]}

We shall have to come back to all this when we deal with population; it is sufficient for us here to call to mind in a very condensed form those facts which we shall need in the considerations that follow.

104. Let us imagine a society composed of a part A, which dominates, and a part B which is subordinate, each decidedly hostile to the other.

Both may appear to be as they really are. But it will more often happen that the dominant part, A, tries to appear to be acting for the common good, because it hopes thus to diminish the opposition of B; whereas the subordinate part, B, will openly claim the advantages it seeks to obtain.

Similar facts may be observed when the two communities are of different nationalities: for instance, the English and the Irish, the Russians and the Poles.

The phenomenon becomes much more complex in a society with a homogeneous nationality or—which amounts to the same thing—one which is considered to be such by those who constitute it.

First, in such a society, between the two opposing parts, A and B, there is a part C, which partakes of both, and which is sometimes on the one side and sometimes on the other. Then part A breaks into two: one, which we shall call A α , has still enough power and energy to defend its share of authority; the other, which we shall call A β , is composed of degenerate¹⁴⁶ individuals, of weak intelligence and will; or *humanitarians*, as they are now called. Similarly, part B may be separated into two groups: the one, which we shall call B α , constitutes the new rising aristocracy; it is also joined by elements from A which, from cupidity or ambition, betray their own class and put themselves at the hand of the enemy. The other part, which we shall call B β , is composed of the common herd which forms the bulk of mankind.⁷⁵

105. Objectively, the struggle consists solely in the B α 's trying to take the place of the A α 's; everything else is subordinate and secondary.

In such a war the leaders—i.e., the B β 's and the B α 's—need soldiers, and each one tries to enlist them as best as he can.

The A α 's try to give the impression that they are working for the common good, but in the present case this weapon becomes a two-edged sword. For if, on the one hand, it still helps to diminish the resistance of the B β 's, on the other hand, it also decreases the energy of the A β 's, who accept as true what is mere fiction, and which can be useful only as such. In the long run, it may happen that the B β 's believe less and less in the slogans of the A α 's whereas the A β 's adopt it increasingly as the rule guiding their real conduct, and, in that case, the stratagem used by the A α 's has rebounded on them and in the end does them more harm than good. This can be seen at the present time in the relations between the middle class and the common people in certain countries.⁷⁶

⁷⁵ In fact, one passes by imperceptible degrees from one of these classes to the other. The observation made in §103 is pertinent.

⁷⁶ *Systèmes socialistes*, p. 396.

106. As for the Ba's, they appear as the defenders of the B β 's and, better still, as the advocates of measures useful to all citizens. So that the dispute, which, objectively, is a struggle for power between the Aa's and the Ba's, subjectively assumes the form of a struggle for liberty, justice, law, equality, and such like; and it is in this form that history records the events.

From the point of view of the Ba's, the particular advantages of this kind of action are that they attract not only the B β 's, but part of the C's and also most of the A β 's.

Suppose that the new élite clearly and simply announces its intention, which is to supplant the old élite; nobody would come to its aid, and it would be beaten before it gave battle. As against this, the new élite seems not to ask anything for itself, knowing very well that, without asking for it in advance, it will obtain what it wants as a result of its victory; it affirms that it is waging war only in order to obtain equality between the B's and the A's, in general. Thanks to this fiction, it acquires the favor, or at least the benevolent neutrality of the intermediate part C, which would not have consented to back the particular aims of the new aristocracy. Then, at the next stage, not only has it the bulk of the people on its side, but it [wins the support of the degenerate part of the former élite as well.]⁷⁷ It should be observed that this part, although degenerate, is always superior to the common people: the A β 's are superior to the B β 's, and they have, besides, the money necessary to finance war. It is generally agreed that nearly all revolutions have been the work, not of the common herd, but of the aristocracy—and particularly the debased part of the aristocracy. This can be seen in history, from Pericles' time on until the first French Revolution; and even today, we see that a part of the middle class strongly supports socialism, all of whose leaders, moreover, are bourgeois. The élites usually end by committing suicide.

The foregoing is only a summary of numerous facts; it has no more value than these facts themselves. But, for lack of space, I would refer the reader to my book, the *Systèmes*, where these facts are partly set forth.⁷⁷

We can now see the great subjective importance of the concept of the equality of man; this importance does not exist objectively. This concept is the commonly used means, particularly in our days, of getting rid of one aristocracy and replacing it by another.

107. It should be observed that the degenerate part of the élite, i.e., the A β 's, is the one which is really deceived and ends up where it did not want to go. The common herd, i.e., the B β 's, often ends by gaining something, either during the battle, or when it happens to change masters. The élite of the former aristocracy, i.e., the Aa's, is not tricked; it succumbs to force; the new aristocracy carries the day.

The work of the humanitarians of the 18th century, in France, paved the way for the murders of the Terror; the liberals of the first half of the 19th century paved the way for the demagogic oppression that is dawning.

Those who demanded equality of citizens in the eyes of the law certainly did not foresee the privileges which are now enjoyed by the popular class; the special jurisdictions of the old régime have been suppressed, and a new one has just been established: that of conciliation boards which favor the workers.⁷⁸ Those who demanded the freedom to

⁷⁷ A very large number of other facts will be found in my *Sociologie*.¹⁴⁸

⁷⁸ *Systèmes socialistes*, I, p. 136.

strike did not imagine that, for the strikers, freedom would consist in the right to knock those who want to go on working on the head, and to burn down factories with impunity. Those who demanded equality of taxation in favor of the poor did not imagine that this would bring about progressive taxation at the expense of the wealthy, and lead to a system in which taxes are voted by those who do not pay them, so that the following shameless reasoning is sometimes heard: "Tax A hits only the rich, and will finance expenditures that will benefit only the less well-to-do; it is thus certain to be approved by a majority of the voters."

[The simpletons who, in some countries, disrupted the army, by letting themselves be carried away by tirades about justice and equality, are later astonished and indignant at the rise of antimilitarism, for which they are responsible. Their mind is unable to understand that one reaps what one has sowed.]¹⁴⁹

107 bis. The great mistake of the present age is to believe that men can be ruled by pure reason, without resorting to force, which is, on the contrary, the foundation of any social system. It is even strange to note that the antipathy felt by the contemporary middle class toward force results in leaving the field for violence. Evildoers and rioters, being assured of impunity, do just about anything they want. The most peaceful people are driven to combine in an association and to have recourse to intimidation and violence by governments which leave them no other way to defend their interests.

[The humanitarian religion will very probably disappear when it has accomplished its work of social disintegration, and when a new élite has risen on the ruins of the old one. The naive lack of conscience of a decadent bourgeoisie is the only reason why this religion is strong, and it will no longer be of any use the day the opponents of the bourgeoisie have become strong enough to stop playing an underhand game.

[That is, it should be added, just what the best of them are already doing; and trade unionism already gives a foretaste of what the strength and the dignity of the new élite may one day be.

[One of the most remarkable works of our time is that which G. Sorel has published under the title *Réflexions sur la violence*.⁷⁹ It anticipates the future by completely abandoning meaningless humanitarian tirades, and by adopting a scientific approach to the world as it really is.]¹⁴⁹

108. The economic and social theories used by those taking part in social struggles should be judged not by their objective value, but mainly by their effectiveness in arousing emotions. A scientific refutation of the theories, however objectively correct it may be, is pointless.

Not only so, when it serves their purpose, men may believe in a theory which they know of scarcely more than by name; this phenomenon by the way is common to all religions. Most of the Marxist socialists have not read Marx's works. In particular cases, this can be definitely proved. For instance, before these works were translated into French and Italian, the French and Italian socialists, who did not know German, could clearly not have read them. The last parts of Marx's *Capital* were translated into French at a time when Marxism was on the decline in France.

⁷⁹ *Le Mouvement socialiste* from January 1906 on, and especially May–June 1906.^{150[a]}

All the scientific discussions—for or against—free trade have had no influence or only a very weak one on the practice of free trade or protection.

Men are moved by sentiment and interest, but they like to imagine that they are moved by reason; hence, they seek—and always find—a theory which, *a posteriori*, gives a veneer of logic to their actions. If this theory could be scientifically refuted, the result would simply be that a new theory would take the place of the old one, and it would achieve the same purpose; a new veneer would take the place of the old one, but the actions would remain the same.

It is, then, mainly to sentiment and interest that one must address oneself in order to move men to act and follow the path desired. Little is as yet known about the theory of similar phenomena, and we cannot dwell further on this point.

109. The equality of citizens in the eyes of law is a dogma for many people, and as such it is not subject to experimental criticism. But if we wish to discuss this scientifically, we shall immediately see that it is not at all *a priori* obvious that such equality is advantageous to society; indeed, given the heterogeneity of society itself, the contrary appears to be more likely.

If, in modern societies, this equality has taken the place of personal status in ancient societies, it is perhaps because the evils caused by equality are less than those caused by the contradiction between personal status and the sentiment of equality that prevails in modern times.

Moreover, this equality is often a fiction. Workers are granted new privileges every day; they thus obtain a personal status that is of no little utility to them. As we have already noted, from the proposition that the worker is equal to the bourgeois it does not follow, thanks to the logic of sentiment, that the bourgeois is equal to the worker.⁸⁰

110. A consequence of the heterogeneity of society is that rules of conduct, beliefs, and morals, should—at least in part—be different for the different parts of society, in order to provide the maximum advantage¹⁵² to society. In reality, such is more or less the case in our societies, and it is only by a fiction that one speaks of a unique system of ethics. Governments, for instance, have ideas about honesty that are quite different from those of individuals. One need merely cite the espionage to which they resort to intercept the secrets of national defense,⁸¹ the manufacture of forged money, which is now replaced by issues of paper money, etc.

Among individuals, we can observe various “professional ethics,” which differ from each other in greater or lesser degree.

⁸⁰ [To be informed as to what *equality* is in the most advanced of modern democracies, one has only to read M. Deschanel's speech in the French Assembly, on May 8th 1907.]

[In that connection, Mr. G. de Lamarzelle writes: “So, under so-called democratic regimes, it is never the bulk of the people, but always a minority, that directs everything, that is master of everything.”]

[“This minority . . . has succeeded in dominating everything in France, and it uses its dominant position above all—as is abundantly proved by Mr. Steeg's speech—to satisfy the personal interests and appetites of its members.”]

[The conclusions drawn by these statesmen from contemporary events had already been inferred by me, in a general way, from the facts of all of history, in *Systèmes socialistes*, published in 1902; and much earlier, Sir Henry Sumner Maine had called attention to this uniformity in history.]¹⁵¹

⁸¹ In 1904, many French newspapers spoke in terms of the highest praise, as if she were a heroine, of a certain woman who, while in the German ambassador's service in Paris, betrayed him and handed papers over to agents of the French government which she had been stealing from the embassy.

Such differences do not prevent these various ethics from having something in common. The problem, like all problems of sociology, is essentially quantitative.

111. If the different classes of human societies were materially separated, like those of certain insects (termites), these various ethics might subsist without too serious a clash. But the classes of human societies are intermixed. Moreover, there is nowadays a very strong sentiment of equality among people, which could not be contravened without very serious damage. These essentially different ethics must therefore have the appearance of not being so.

It should be added that it is difficult for a class of people to appear indefinitely to have sentiments other than their true ones; these different ethics must thus be considered the same by the very people who follow them. Casuistry, which has existed at all times and among all peoples, makes some provision for this. A general principle is set forth, which is accepted by everybody; all the necessary exceptions are then made, thanks to which the principle is general in appearance only. All the Christians in the Middle Ages [reverently and obsequiously]¹⁵³ admitted the divine principle of the pardon of offenses, but the feudal lords were violent in avenging the insults received. Nowadays, everybody declares himself [a faithful upholder]¹⁵⁴ of the equality of man, but this does not prevent workmen from obtaining new privileges every day.

112. The means which serve to keep systems of ethics separate are very imperfect; such systems mingle in reality, and we thus deviate from the conditions which would enable society to thrive.

113. The lower classes need a humanitarian ethic, which also helps to attenuate their hardships. If the upper classes honor this only formally, there is no great harm; but if, on the contrary, they actually conform to this ethic, the result may be very harmful for society. In the past, it has often been pointed out that people need to be governed with an iron hand in a velvet glove. Justice must be rigid and appear to be lenient. The surgeon comforts his patient with nice words while he cuts him up with a steady and pitiless hand.

114. In a more restricted society—that of the socialists of our time—we see the leaders and in general the most cultured socialists having beliefs hardly different from those of the lower orders. Whereas the latter dream of a future golden age which “collectivism” will bring about, the former, enlightened by the way their society is governed, or by the public administration, have less faith in the panacea of collectivism, and prefer to occupy themselves with more immediate reforms.⁸² This diversity of faith is most useful to the socialists, because in this way everyone has the faith best suited to his nature and the work he has to do.

115. The diversity in men’s nature, combined with the need to give satisfaction in some way to the sentiment of equality, has resulted in democracies trying to give the

⁸² [About the end of 1906, Jaurès was summoned, in the Assembly, to state precisely what legislation was needed to establish the collectivism that he had been demanding for so long. He asked for a period of three months to prepare his statement, which is already most astonishing, even from the logical point of view alone, because it might be expected that a party leader would know exactly what he wanted. But this is not all; the three months have long ago elapsed, and 1907 has started without Jaurès having yet disclosed his plan which still remains shrouded in thick clouds.]

[Such conduct may seem absurd from an objectively logical point of view; it is on the contrary perfectly sensible and reasonable from a subjective point of view when one’s purpose is to act upon sentiments, for reasons that have been given in the text.]¹⁵⁵

appearance of power to the people and the reality of power to an élite. Up to now, the democracies where this has been possible are the only ones to have prospered; but this equilibrium is unstable, and after many changes it usually leads to some radical upheaval.

116. The legend told by Dionysius of Halycarnassus¹⁵⁶ is typical of many subsequent historical phenomena. Servius Tullius deceived the common people with the centuriate assemblies, and took the government of the commonweal away from them. "They imagined that they all had an equal share in the government of the city, because each man, in his century, was called upon to give his opinion; but they were mistaken, because each century had only one vote whether it was composed of a large number of citizens, or only a few,"⁸³ and, moreover, because the poor were the last to be called, and even then only when the vote of the first centuries was not decisive.

According to Cicero, freedom consists in giving the people power to bestow its confidence [honorably]¹⁵⁷ on good citizens;⁸⁴ and this is properly speaking the principle that modern representative regimes proposed to put into effect. But neither in Rome nor in the modern states was this achieved; and the common people wanted something more and better than simply the power to elect the [patricians who were]¹⁵⁹ to govern them.

117. History shows that the ruling class has always tried to speak to the people in the language that it believed not closest to the truth, but best suited to the aim it had in view.⁸⁵ This happens even in the most advanced democracies, such as the French one. Here we have a new and remarkable instance of the persistence, in various forms, of the same social phenomena.

118. For reasons it is pointless to examine, the ruling class in France is composed of two parts, which we shall call A and B. The A's, to overthrow the B's, called in the socialists, but with the fixed intention of yielding very little or no ground to the people, by throwing smoke in its eyes and by lavishly bribing only those leaders they wanted to have in their service. In order that such behavior should not be too apparent and to divert attention from their artifices, they thought up the anticlerical campaign; and, using this bait, they hooked a few simpletons, to whom it was then easy to join the [many]¹⁶⁰ humanitarians of scanty energy and intelligence. In short, in France, there are presently "capitalists" who are becoming rich and powerful by using the socialists.⁸⁶

⁸³ *Ant. Rom.*, IV, 21: "Υπελάμβανον μὲν γὰρ ἄπαντες ἵσον ἔχειν τῆς πολιτείας μέρος, κατ' ἄνδρα διερωτώμενοι τᾶς γνώμας ἐν τοῖς ιδίοις ἔκαστοι λόχοις· ἔξηπατώντο δὲ τῷ μίᾳν εἶναι ψῆφον ὅλου τοῦ λόχου, τοῦ τε ὀλίγους ἔχοντος ἐν αὐτῷ πολίτας καὶ τοῦ πάνυ πολλούς.

⁸⁴ This is why he wanted the people to show their ballots and offer them to the best citizen. *De legibus*, III, 17: "habeat sane populus tabellam, quasi vindicem libertatis, dommodo haec optimo cuique et gravissimo civi ostendatur, ultroque offeratur; uti in eo sit ipso libertas, in quo populo potestas honeste bonis gratificandi datur."¹⁵⁸

⁸⁵ Aristotle describes the artifices used by the oligarchies, *Politics*, IV, 10, 6: Ἐστι δ' ὅσα προφάσεως χάριν ἐν ταῖς πολιτείαις σοφίζονται πρὸς τὸν δῆμον πέντε τὸν ἀριθμόν. "In the republics the common people are misled in five ways, by pretexts." And he adds that in the democracies it had recourse to similar expedients.

⁸⁶ See an excellent article by G. SOREL in Colajanni's *Rivista popolare*: "The experience of the anticlerical policy followed with so much obstinacy by the French Government in the past two years is one of the most important social phenomena that the philosopher can study." The author stresses the cowardice of Combes' opponents, which, it should be added, is merely a particular case of the general law of decadence of aristocracies. "When the expulsion of the monks began, it was announced that there would be an energetic resistance . . . but after a few attempts in Brittany everything calmed down . . . The opponents' courage did not stretch to legal resistance . . . The *Libre Parole* has noted several times that the Catholic world has not curtailed its feast days

119. The further down one goes in the social strata, the more is misoneism in evidence, and the more men are loath to act for other considerations than for their direct and immediate interest. It was on this interest that the upper class established its rule, in Rome as well as in modern nations. But this cannot last forever, for the lower class ends up having a better understanding of its own interest, and it turns against those who have exploited its ignorance.

120. This phenomenon can be conveniently studied in modern England. Digging deeper and deeper into the social strata whose support was needed to win the elections, the Tory party has helped to extend the suffrage ever further, and it has rewarded its allies by measures which have rightly been called: "Tory socialism." The Whigs, who formerly defended liberal principles, now compete with the Tories in order to get into the good graces of the masses. They court the socialists, and go much further than the honeyed and humanitarian socialism of the Tories. Both parties vie with each other in humbling themselves before the lower orders, and each contrives to outbid the other in its adulation. This came out even in the tiniest details. During the election campaigns, candidates are not ashamed to send their wives and their daughters to beg for votes and to hold out their hands and offer their lips to dirty and unmannerly people.¹⁶⁴ The common man is charmed when this conduct is new and unexpected; and surprised at so much love and goodwill; but, when it is repeated, such behavior ends by nauseating anyone who sees through the self-seeking flattery.

121. When a social stratum understands that the intention of the upper class is simply to exploit it, this class sinks even lower in order to recruit other supporters; but obviously the day will come when it will not be possible to go on like this because there will be no voting fodder left. When the vote is given to all men, including lunatics and criminals, when it is extended to women and, if you like, to children, one will have to stop there;

or modified its social relations . . . In vigorous articles, Urban Gohier has denounced all kinds of horsetrading on the part of the *Petite République*; and, if many young people have turned socialist, it is no doubt because they were sure of making a good bargain. They would be really curious to know the names of the capitalists who recently granted rather large sums to enable the *Petite République* to reform itself and the *Humanité* to come into existence; nobody imagines, I suppose, that the capitalists hand money over to socialist newspapers out of love of collectivism! One does not give a million francs to businesses of this kind, if the donors are not sure of some profit. Parliamentary socialism has become excellent business, and its shares stand high in stock exchange circles."

The author has a clear understanding of the way political evolution works: "Thus, material questions are hidden under a double layer of sentiments, which prevent people from seeing that there is much more selfishness and a larger dose of evil passions in political conduct than they think . . . In general, politics is dominated by the interests of those who run it and who intend to profit from it. Interests combine easily; and it is for this reason that, nearly everywhere, liberal governments draw their support from people who are out to obtain something for themselves, for their electoral councils, or for the social groups whose votes they are bidding for."

Mr. Germain, who was a director of the *Crédit Lyonnais*, spoke very accurately, as early as 1883, of politicians, "of those men who value only one thing: having a majority and controlling the French budget for the benefit of their clientèle."

We can add certain facts which were brought to light in an inquiry on the Carthusian monks (Chartreux).¹⁶¹ First, someone declared that, with some friends, he had contributed one hundred thousand francs¹⁶² for the [election of government candidates],¹⁶³ and he added that "he is not concerned with politics." It is this other fact discussed by Mr. Aynard in the Assembly, on July 12th 1904: ". . . we would also like to know what are the funds of the Mascaraud committee which is a handmaiden of the government. We would like to know who is the eccentric who keeps an admirable account of his banquets, indeed especially of his banquets, and of his comings and goings but who keeps no account of the funds entrusted to him."

But this is nothing compared with what goes on in the United States at election time.

one will not be able to go still lower, unless the vote is granted to animals, [which would be easier to decide than to get them to vote.]¹⁶⁵

122. In Germany the universal franchise was introduced partly to fight the liberal bourgeoisie; the phenomenon is thus similar to what happened in England. A number of social laws were also promulgated in the hope of taking supporters away from the socialist party; but this attempt failed, for the common people realized what the game was. The upper class has now started complaining about the universal franchise, and is looking for a way to go back on it.⁸⁷

123. At the beginning of the democratic evolution which developed in the course of the 19th century and which seems bound to finish in the 20th century, some thinkers realized very clearly how it would end; but their predictions have been forgotten, now that they are being fulfilled, and now that, at this final stage, the dregs of the people will understand and seek to put into effect this logical observation: "if the arbitrary expression of my will is the principle of legal order, my happiness is perhaps also the principle of the distribution of wealth."⁸⁸

But history will not stop at the end of the present evolution; and, if the future is not to be completely different from the past, the present evolution will be followed by another, which will be in the opposite direction.^[a]

⁸⁷ Professor von Jagemann, who for ten years was a member of the Imperial Federal Council representing the Baden government, and who is now professor of public law at the University of Heidelberg, has written an interesting work in which he examines the legal means which could be used in Germany to replace universal suffrage by restricted suffrage.

⁸⁸ Stahl, *Rechtsphilosophie*, II, 2, p. 72.¹⁶⁶

CHAPTER III

The General Concept of Economic Equilibrium

1. The preceding considerations had as their aim [to provide not the theory, but only a few examples,]¹ of a very broad class of phenomena, which one can rarely abstract from in practical matters. We are now going to study quite a different class of phenomena, of which we propose to construct the theory.

We shall study logical actions repeated a great number of times, by which men procure things that satisfy their tastes.

Let us examine a relation of the kind indicated by AB in §89, Chapter II. We shall not deal, at least in pure economics, with relations of the type BC, nor with the reactions of these on B. Rather, we shall deal only with certain relations between objective facts and the corresponding subjective phenomena, which are mainly people's tastes. Moreover, we shall further simplify the problem by assuming that the subjective fact is perfectly adjusted to the objective one. We can do this because we take into account only actions that are repeated; this allows us to presume that the link between these actions is a logical one. A man who purchases a certain type of food for the first time may buy more of it than is necessary to satisfy his tastes, taking account of the price. But, when making a second purchase, he will correct his mistake, at least in part, and so on, until little by little, he obtains exactly the quantity he wants. We shall consider him when he has reached this situation. Similarly, if at first he makes a mistake in his reasoning to obtain what he wants, he will correct his reasoning when repeating it, and will end up making it completely logical.

2. In this way we have vastly simplified the problem by considering only a part of men's actions and by assigning certain characteristics to them. This study will be the object of political economy.

3. But on the other hand the problem is very complex because objective facts are very numerous and partly dependent on one another. By reason of this mutual² dependence, ordinary logic becomes powerless once one moves beyond the elementary stage. We must therefore have recourse to a special kind of logic, appropriate to this kind of study, namely mathematical logic. There is therefore no basis for speaking of a "mathematical method" that is in OPPOSITION to other methods; it is a question of a method of research and proof that is ADJOINED to the others.

4. Furthermore, because of the difficulties inherent in the problem itself, we must split the subject up and begin by eliminating everything that is not strictly essential, and consider the problem reduced to its main and essential elements. We are thus led to divide the subject into pure and applied economics. The first [can be represented by]³ a graph containing only the main lines to which the second adds the details. These two parts of economics are analogous to the two parts of mechanics: rational and applied mechanics.^[a]

5. One proceeds in a substantially similar fashion in almost every branch of human knowledge. Even in grammar, one begins by stating the main rules of phonetics, and then goes on to provide the particulars. When, in Greek grammar, it is said that the argument is the sign of the past indicative of the historical tenses, one has a rule that could be called one of "pure grammar." But this rule is not enough by itself to enable us to know what these past tenses really are; to find out, we have to add a great many particulars.

6. The first problem we now propose to study is thus a very special one, and we have to solve it before we can pursue our research any further.

7. The study of pure economics is composed of three parts: a static part; a dynamic part that considers successive equilibria; and a dynamic part that studies the movement of the economic phenomenon.

This division corresponds to concrete reality. What will the average price of 3 percent French bonds be on the Paris Stock Exchange today? This is a problem in statics. Here are a few others of the same kind: What will these average prices be tomorrow, the day after, etc.? According to what laws do these average prices fluctuate? Is there an upward or a downward trend? These are problems of successive equilibria. What laws govern the price movements of the 3 percent French bonds, i.e., how does an upward movement go beyond the equilibrium point and itself become the cause of a movement in the opposite direction? How do these prices vary, quickly, or slowly, with acceleration or deceleration? This is a problem in economic dynamics.

8. Static theory is the most advanced; there are very few and scanty accounts about the theory of successive equilibria. Except for the special case of business cycles, nothing is known about dynamic theory.^[a]

9. We shall first deal exclusively with static theory. Here, we may consider an isolated economic phenomenon, e.g., the production and consumption of a certain quantity of a commodity; or we may study a continuous economic phenomenon, such as the production and consumption of a certain quantity of a commodity per unit of time. As we have already seen, political economy is concerned with phenomena that repeat themselves (§1), not with accidental or exceptional ones, but with average ones. Consequently, the real world can be further approximated by the study of the continuous economic phenomenon. Will Mrs. So and So buy a certain diamond today, or will she not? This may well be a psychological problem, but it is certainly not an economic one. How many diamonds are sold, on the average, in England, in a month or in a year? Here we have an economic problem.

10. When it is clearly understood that the phenomenon we are studying is a continuous one, there is no need to burden the text by continually repeating "per unit of time." When therefore we speak, for instance, of the exchange of ten kilograms of iron against one kilogram of silver, it should be understood that it takes place "per unit of time," and that we are not discussing an isolated exchange, but a repeated one.

11. There are two large classes of theories. The first one aims at comparing the sensations of a man placed in a variety of conditions, and at determining which of these conditions that man will choose. Political economy is mainly concerned with this class of theories. Since it is usually assumed that in making choices men are guided exclusively by their own interest, it is said that this class is formed of the theories of egoism. But it could just as well be formed of the theories of *altruism* (if it were possible to define rigorously what this term means), and in general of the theories based upon any rule which man

follows to compare his sensations. It is not essential to this class of theories that a man having to choose between two sensations should choose the more agreeable one; he could choose another one, according to an arbitrarily determined rule. The essential property of this class of theories, then, is that it compares the different sensations of one man, and not the sensations of different men.

12. The second class of theories compares one man's sensations with another's, and determines the conditions under which men should be ranked with respect to one another if certain aims are to be achieved. This study is still one of the most imperfect in the social sciences.¹

13. Two roads are open to us for the study we wish to undertake; each of them has its advantages and its disadvantages. We can either study each subject successively in depth, or else start by forming a general, and inevitably superficial, idea of the phenomenon, and then come back to the topics already looked at in general in order to study them in detail; we may complete our inquiry by achieving a still closer approximation to the phenomenon under consideration. The first method would make for a more orderly and non-repetitious exposé; but it would make it difficult to obtain immediately a clear view of the whole phenomenon in all its complexity. By following the second method, we obtain such a view, but we have to resign ourselves to making fleeting reference to certain details and to postponing the study of them until later. Despite its drawbacks, this last method should not be ignored; it is particularly fruitful in cases when, as is the case in economic science, the details have so far been more thoroughly investigated than the general phenomenon, which has been completely or almost completely ignored. The day may come, perhaps in a few years, or much later, when such will no longer be the case; it will then be better to opt for the first method.

14. The main object of our study is economic equilibrium. We shall soon see that this equilibrium is the result of the opposition between men's tastes and the obstacles to their satisfaction. Our study can thus be subdivided into three entirely distinct parts: (1) the study of tastes; (2) the study of obstacles; (3) the study of the way these two factors combine to attain an equilibrium.

15. The best order to follow would be to begin with the study of tastes and exhaust this subject; then to proceed to the study of obstacles, and exhaust that as well; and finally, to study the equilibrium process, without reverting to the study of tastes or obstacles.

But it would be difficult to proceed in this way—for the writer as well as for the reader. It is not possible to exhaust any one of these subjects without making extensive use of concepts that properly belong to the other two subjects. If these concepts are not fully explained, the reader will be unable to follow the discussion; if they are explained, arguments will be confounded which it was intended to keep separate. Moreover, the reader wearis of a long study whose aim is not clear. The writer is aware of this, and will deal with tastes and obstacles, not at random, but only to the extent that it may be useful for determining equilibrium. The reader naturally likes to be made aware of this plan, and to know where the long road he is made to follow is leading him.

Precisely in order to show where we are going, and to develop certain concepts that will be of use later on, we shall in this chapter provide a general outline of the three parts

¹ *Cours d'économie politique*, II, § 654.

of the phenomenon. Tastes and obstacles will be dealt with only insofar as necessary to obtain certain insights into economic equilibrium. Then, after we have acquired a rough knowledge of the whole, we shall go back and take up each part of it separately. Chapter IV will be devoted to the study of tastes, and Chapter V to that of obstacles; finally, in Chapter VI we shall see how these elements are combined in the equilibrium process.

16. Let us imagine that individuals find themselves [in relation to]⁴ certain objects capable of satisfying their tastes, and which we shall call economic goods. If we pose the problem of how to distribute one of these goods among these individuals, we have a question pertaining to the second class of theories (§12). In fact, each man experiences only one sensation: the one that corresponds to the quantity of the economic good assigned to him. We are thus not dealing with different sensations of a given individual, which we could compare among themselves; we can only compare the sensation experienced by one individual with that experienced by another.

17. If two or more things are available, each individual will experience two or more different sensations, according to the quantities at his disposal. The question then arises of comparing these sensations and determining from among the different possible combinations the one that will be chosen by the individual. This question pertains to the first class of theories (§11).

18. If all the quantities of goods available to the individual increase (or decrease), we shall at once see that, except in one case that will be dealt with later (IV, 34), the new position will be more (or less) advantageous than the former from the point of view of the individual in question, so that in this case there is no problem to solve. But if, on the contrary, certain quantities decrease while others increase, it is necessary to find out whether the new combination is, or is not, advantageous to the individual. It is to this class that economic problems belong. We see them arise in the real world, in the case of an exchange, when one thing is given and another received, and in the case of production, when certain things are transformed into certain others. We shall deal with precisely these problems first.

19. The elements we must combine are, on the one hand, man's tastes, and, on the other, the obstacles to their satisfaction. If, instead of men, we had to study ethereal beings without tastes or wants, without even the need for food and drink, there would be no economic problem. The same would be true if, going to the opposite extreme, we supposed that no obstacle stood in the way of the satisfaction by men of any of their tastes and desires whatever. The man who has everything up to satiety does not give rise to an economic problem.

The problem arises because tastes encounter certain obstacles, and it is all the more difficult the more varied and complex these tastes and these obstacles are. The question is further complicated by the fact that there are many ways to satisfy these tastes and to overcome these obstacles. It is therefore necessary to find out how and why one [particular]⁵ way is preferred by individuals to others.

Let us then proceed to examine the phenomenon more closely.

20. If one were to choose only between two things, or among a small number of things, the problem to solve would be a qualitative one, and its solution would be easy. Do you prefer a barrel of wine or a watch? The answer is immediate. But, in fact, there are a very large number of things to choose among and even for only two things, the combinations of quantities to choose from are countless. In one year, a man may drink

100, 101, 102 . . . liters of wine; he may, if his watch does not run too well, buy another one at once, or wait one month, two months, . . . one year, two years, . . . before making the purchase, and in the meantime keep the watch he has. In other words, there are infinitely many cases in which the things one has to choose among vary in quantity by minute, almost imperceptible degrees. We must therefore construct a theory which is of use in solving this kind of problem.

21. Let us consider a series of these combinations of different quantities of goods. A man may pass from one of these combinations to another, and finally settle for one of them. It is very important to know which one this is, and we can find out by means of the theory of economic equilibrium.

22. Economic equilibrium. Economic equilibrium can be defined in different ways, which amount essentially to the same thing. Economic equilibrium can be said to be a state which would be maintained indefinitely if there were no change in the conditions under which it is observed. If, for the moment, we consider a position of stable equilibrium only, we may say that it is determined in such a way that if it is only slightly modified it immediately tends to be restored and revert to its initial state. The two definitions are equivalent.

For instance, an individual under certain circumstances or conditions purchases one kilogram of bread every day. If one day, he were to be compelled to buy only 900 grams, and the following day he were left free again, he would again buy one kilogram. If there is no change in his conditions, he will continue indefinitely to buy one kilogram of bread. This is what we call a state of equilibrium.

We shall have to express mathematically the fact that, once the equilibrium position is reached, these variations or movements do not occur, which amounts to saying that the system is maintained indefinitely in the given state.

Movements that are necessary to actually reach equilibrium may be called *real*. Movements away from equilibrium that might hypothetically take place but do not in fact occur, because equilibrium persists, may be called *virtual*.

Political economy studies actual movements in order to know how facts come to pass; and it studies virtual movements in order to understand the properties of certain economic states.

23. If it were possible to make arbitrary movements away from a given economic state, it would be possible to continue indefinitely the movements that increase the quantities of all the goods a man can desire, and we would thus reach a state in which the man had everything to satiety. This would, of course, be a position of equilibrium; but obviously this does not describe what happens in real life, and we shall have to look for other equilibrium positions on which we shall fix attention, because not all movements, but only certain ones, are possible. In other words, there are obstacles or constraints that restrict the movements, that divert people from following certain paths, and that prevent certain variations from taking place. Equilibrium results precisely from this opposition between tastes and obstacles. The two extreme cases, which have already been considered but which are not met with in the real world, would be those in which either there are no tastes, or there are no obstacles.

24. If the obstacles or the constraints were such as to determine each movement precisely, we would not have to bother about tastes; the consideration of obstacles would be enough to determine equilibrium. This is, however, not the case in fact, at least not

in general. The obstacles do not determine every movement in an absolute way; they only fix certain limits, and impose certain restrictions, but they allow the individual to move according to his own tastes in a more or less restricted field; and among all the movements permitted in this way, we shall look for those that are likely to occur in reality.

25. Tastes and obstacles refer to each of the individuals under consideration. For an individual, the tastes of other people with whom he is in contact are part of the obstacles he faces.

26. To have all the data pertaining to the equilibrium problem, it is necessary to add to tastes and obstacles the factual conditions that determine the state of the individuals and of the transformation of goods; for instance, the quantities of commodities in the possession of the individuals, the means by which goods can be transformed, etc. This will become clearer as we continue our study.

27. To determine equilibrium, we shall lay down the condition that, at the point where it obtains, the movements allowed by the obstacles are prevented by the tastes; or conversely, what amounts to the same thing, when the variations allowed by the tastes are prevented by the obstacles. In fact, it is clear that these are just two ways of saying that no movement takes place; and this is, by definition, the property of equilibrium.

[At an equilibrium point, we must therefore determine which movements are prevented and which are allowed by the tastes; and similarly which movements are prevented and which are allowed by the obstacles.]⁶

28.⁷ **Men's tastes.** A way must be found to submit men's tastes to calculation. The idea has therefore been put forward that they could be deduced from the pleasure that certain things arouse in men. If something satisfied a man's wants or desires, it was said that it had a *value in use*, or *utility*.

29.⁸ This concept was imperfect or ambiguous in several respects. (1) It was not sufficiently clear. It brought out that this *value in use*, or *utility*, was exclusively a relation between a man and an object. That is why many spoke of it, perhaps even unawares, as if it were an objective attribute of things. Others, who came closer to the truth, but not close enough, used these expressions as if they referred to a relation between men in general and an object. (2) They did not realize that this *value in use* depended upon (was a function of, as mathematicians say) the quantity consumed. For instance, to speak just of the *value in use* of water is meaningless; it is not enough, as we have just seen, to add that this *value in use* refers to a given individual; it is very different according as the man is dying of thirst, or has already drunk as much as he wanted. To be precise, we should speak of the value in use of a certain quantity of water added to a known quantity that has already been consumed.

30. It was mainly in the process of correcting this error in early economics that pure economics was born. With Jevons, it appeared as a correction of the then prevailing theories of "value;" with Walras it became—and this was a great step forward—the theory of a special case of economic equilibrium, i.e., that of free competition. Another case, that of monopoly, had already been studied, but in quite a different fashion, by Cournot. Marshall, Edgeworth, and Irving Fisher^[a] have continued tackling the economic phenomenon in a more comprehensive and more general way, until in my *Cours*, it has become the general theory of economic equilibrium, and I shall go still further along

this path in the present book.² (3) The word utility had come to mean something quite different in political economy from what it means in ordinary language. Thus, morphine is not useful, in the ordinary sense of the word; on the contrary it is harmful to the addict; on the other hand morphine is economically *useful* to the addict, for it satisfies one of his wants, however unhealthy. Although the older economists had already noticed this double meaning, every now and again there would be some who lost sight of it. It is therefore as well to stop using the same word to mean such different things. I suggested in my *Cours* the term *ophelimity*, which has since been adopted by others.

31. It will be useful at this point to make a general observation that applies not only to the present case but also to many others that we shall consider later on. The criticism I have made applies to the old theories today, but not at the time when they were elaborated. It would be a serious error to think that it would have been better had these erroneous theories never existed. Those or similar theories were indispensable for arriving at better theories. Scientific conceptions keep being modified to approximate truth more closely. Theories are continuously being amended. Some imperfect propositions are first advanced, and one then goes ahead to study the science; next, one takes a step back to correct these propositions. It is only recently that Euclid's postulates have been reexamined. What would have happened to geometry if the ancients had stubbornly and obstinately refused to go beyond the examination of these postulates, and had absolutely set aside any urge to advance the study of the science? There is a world of difference between the astronomical theories of Newton, Laplace, and more modern theories; but the former were a necessary stage in the development of the second, and the second in the development of the third. The old theories of economics were necessary to the development of new ones; and these, though still very imperfect, will help us to develop better theories, and so on. But improving theory is something quite different from trying to destroy it by silly, pedantic quibbling. The first approach is sensible and useful; the second is foolish and futile, and anyone with no time to waste would do better not to bother about it.

32. The ophelimity, for an individual, of a certain quantity of an object, added to another given quantity (which could be zero), already in his possession, is the pleasure which that quantity procures him.

33. If this quantity is very small (infinitely small), and if we divide the pleasure it yields by that same quantity, we obtain the *elementary ophelimity*.

34. Finally, if we divide the elementary ophelimity by the price, we obtain the WEIGHTED ELEMENTARY OPHELIQUITY.

35. The theory of ophelimity has been further improved. There is a weak point in all the reasoning leading up to it that has been brought to light mainly by Professor Irving Fisher. We have assumed that what is called *pleasure*, *value in use*, *economic utility*, or *ophelimity* is a quantity; but this has not yet been proved. And supposing it were, how should we go about measuring this quantity?^[a] [It was a mistake common to both Professor Fisher and myself]^[10] to believe that we can in general infer the value of ophelimity from the law of supply and demand. This can only be done in one particular

² For further details on the history of the theories of pure economics I must refer the reader to my article: *Anwendungen der Mathematik auf Nationalökonomie*, in *Encyclopädie der mathematischen Wissenschaften*.^{9[b]}

case, in which only the unit of measurement of ophelimity remains arbitrary; this is the case with commodities each of whose ophelimity depends solely upon the quantity of that commodity, and is independent of the quantities of other commodities consumed (Appendix [, 8]^{11[b]}). But, in the general case, i.e., when the ophelimity of a commodity, A, consumed together with commodities B, C, . . . , depends not only on the consumption of A but also on the consumption of B, C, . . . the ophelimity remains indeterminate, even after the unit of measurement has been fixed (see the Appendix [, 5]).¹²

36. In what follows, when we speak of ophelimity, it must always be understood that we simply refer to one of the systems of ophelimity indices (§ 55).

36 bis.¹³ The concepts of *value in use*, *utility*, *ophelimity*, *ophelimity indices*, etc. make it much easier to set forth the theory of economic equilibrium, but they are not necessary for the construction of this theory.

Thanks to the use of mathematics, this whole theory, as it is developed in the Appendix, rests on only one fact of experience, namely, the determination of the quantities of goods that form combinations to which an individual is indifferent (§ 52).³ The theory of economic science thus acquires the rigor of rational mechanics; it deduces its conclusions from experience, without bringing in any metaphysical entity.

37. As we have already remarked, there may be certain constraints that prevent economic phenomena from adapting to tastes. For instance, there were formerly governments that compelled their subjects to purchase a certain quantity of salt every year. It is obvious that in this particular case, and for this commodity, there was no need to take tastes into account. They might be ignored for all commodities, if everyone had to purchase a fixed amount of each of them every year. If such were the case in practice, it would be useless to waste time on the development of a theory of tastes. But the most unsophisticated observation is enough to show that it is not so in the real world. Even when certain constraints are imposed, as, for instance, when a state monopoly fixes a price, or creates obstacles to production, sale, free transit, etc., they do not entirely prevent the individual from acting according to his tastes within certain limits. Consequently, everyone has always to solve certain problems to determine his acts of consumption according to his tastes; the poor man will ponder whether it is better for him to buy some sausage or a little wine; the rich man will consider whether he prefers to buy a car or a jewel; but just about everybody solves problems of this kind. Hence the need to consider the abstract theory that corresponds to these concrete facts.

38. I shall try to explain the results that have been reached by mathematical economics without using algebraic symbols. [Thus,¹⁴ symbols will be used in the Appendix only], and the reader will be able to read the text by itself and leave the Appendix aside].¹⁴ Here, we need merely recall certain principles, of which the main one is, for the moment, the following. The conditions of a problem are expressed algebraically in equations. They contain known and unknown quantities. To determine a certain number of unknowns, an equal number of distinct conditions (equations) are needed, i.e., conditions such that one of them is not the consequence of the others. Moreover, they must not be contradictory. For instance, if one looks for two unknown numbers, and if the conditions

³ This is what literary and metaphysical economists cannot grasp. They nevertheless take it upon themselves to give us their views; and the reader who has some knowledge of mathematics will be amused to hear the nonsense of which they will deliver themselves in connection with this paragraph and § 8 in the Appendix.

(equations) specify that the sum of these numbers must be equal to a given number, and the difference be equal to some other given number, the problem is fully determined, because there are two unknowns and two conditions (equations). But if, on the other hand, in addition to the sum of the two numbers, the sum of the double of each of these numbers were specified, the second condition would be a consequence of the first; because if, for example, 4 is the sum of two unknown numbers, 8 will be the sum of twice each of these numbers. We do not, in such a case, have two distinct conditions (equations), and the problem remains indeterminate. In economic problems, it is most important to know whether certain conditions determine the problem completely, or leave it indeterminate.^[a]

39. Direct and indirect effects of tastes. Many hypotheses could be formulated as to the way man is guided by his tastes; and each of them could provide the basis for an abstract theory. In order not to risk wasting time on the study of useless theories, we shall examine concrete facts and look for types of abstract theories that fit them.

Take an individual who buys 3 percent French bonds at 99.35; let us ask him why he does so. He may say that it is because he thinks that at that price the purchase looks attractive to him. Having put the expenditure of 99.35 francs in one of the scales, and 3 francs of annual income in the other, he considers that, for him, the purchase of the income is worth the expenditure. If the price were 98, he would buy 6 francs of income instead of 3 francs. He does not consider whether he would prefer to buy 3 francs of income at 99.35 or 6 francs at 98; it would be an idle question, since the determination of that actual price does not depend on him; he investigates what amount to buy at a given price, because that is all that depends on him. Let us turn our attention to the seller. He may be guided by entirely identical reasons. In that case, we still have the same type of transaction. But, toward the end of 1902, we might have run into someone who would have said to us: "I am selling in order to force bond prices down, and thus to irritate the French government." There is always someone to be found who would say: "I am selling (or buying) in order to force the price down (or up), and make a profit on it." Anyone who acts in this way is guided by quite different motives from those we considered previously; he is trying to change the price, and is mainly comparing the positions he has reached with different prices. This is another type of transaction.

40. Types of phenomena derived from tastes. The two types of phenomena just mentioned are of great importance for the study of political economy; let us look at their properties and designate the first type by (I), and the second by (II).^[a] Let us start by considering the case in which the man who transforms economic goods has only his own personal interest in view; later (§49), we shall deal with cases based on different assumptions.

We may say that the man who purchases or sells a commodity may be guided by two entirely different kinds of considerations.

41. He may seek only the satisfaction of his tastes, given any particular state of the market. Without directly intending to, he contributes to the modification of these conditions, because, depending on the different states of the market, he is disposed to transform a smaller or a larger quantity of one commodity into another. He compares successive transformations under given market conditions, and he looks for conditions such that these successive transformations will lead him to a point where his tastes are satisfied. We thus have type (I).

42. The individual considered may instead try to modify the conditions of the market to his own advantage, or for any other purpose. In a given state of the market, the exchange leads to an equilibrium point; in another state equilibrium takes place at another point. One compares these two positions and one looks for the one that is best suited to the proposed aim. After the choice has been made, one tries to modify the conditions of the market so as to bring them into line with this choice. We thus have type (II).

43. Whereas type (I) may obviously be that of any individual who happens to be operating on a market, type (II), on the contrary, fits only those who know how—and are able—to modify the conditions of the market, which is certainly not the case with everybody.

44. Let us pursue our investigation, and we shall see that type (I) comprises a very large number of transactions, which include most—and even all—transactions related to domestic consumption. When has a housewife about to purchase chicory or coffee ever been seen to concern herself with anything but the price of these articles, or been heard to say: "If I purchase chicory today, that may cause its price to rise in the future, and I have to consider the loss that my present purchase of chicory will cause me in the future"? Who has ever refrained from ordering some clothing to be made, not in order to avoid the expenditure, but only in that way to cause the price of clothing in general to drop? If someone went to the market and said: "I should be pleased if the price of strawberries were as low as 30 centimes a kilogram; I will therefore stick to this price," he would be laughed at. Instead, he would say; "At 30 centimes per kilogram, I shall buy 10 kilograms; at 60 centimes only 4 kilograms; at a price of 1 franc I shall buy none;" and he will thus make a deal with the seller. Type (I) then, corresponds to a very large number of concrete facts, and we will not be wasting our time by constructing a theory to take account of them.

45. We also find no lack of examples of type (II). On the Stock Exchange, powerful banking companies and syndicates conform to this type. Those who—thanks to substantial means—try to corner commodities, obviously want to modify the conditions of the market in order to profit from the move. When the Italian¹⁵ government fixes the price of the tobacco it sells to the public, it operates according to type (II). All those who enjoy a monopoly, and who know how to profit from it, act according to that type.

46. If we look at actual cases it will be seen that type (I) is encountered where there is competition among those who conform to this type. The persons with whom they bargain may not be in competition and consequently may not follow type (I). Type (I) is observed in a much purer form where competition is more extensive and more perfect. It is precisely because each day on the Paris Stock Exchange there are many people who buy and sell French bonds, that it would be foolish to speak about modifying conditions in that market by buying or selling a few bonds. Of course, if all those who sell (or buy) should come to an agreement, they could indeed modify these conditions to their advantage; but they do not know each other, and everyone acts on his own account. Amidst such confusion and competition, each individual has no alternative but to mind his own business and to seek to satisfy his own tastes, in accordance with the various conditions prevailing on the market. All those who sell (or buy) French bonds do modify prices, but they modify them without any prearranged plan; not as a goal, but as the effect of their own actions.

47. Phenomena of type (II) can be observed where there is no competition, and where there is cornering, monopoly, etc. When someone is operating to alter the conditions of the market to his advantage, he must be sure, if his attempts are not to be in vain, that others will not upset his plans, and, therefore, he must somehow or other get rid of his competitors. This he can do either with the aid of the law, or because he alone possesses certain commodities, or because—by influence, intrigue, deception, or intelligence—he edges out his competitors. His competitors may also be negligible because they have slight importance, or for other reasons.

Finally, it should be noted that it often happens that a certain number of people form an association precisely in order to dominate the market; this too is a case of type (II), since the association itself may from certain points of view be regarded as a single individual.

48. Another analogous, though not identical, case, is that in which a number of persons or associations agree among themselves to alter certain market conditions, while leaving their associates complete freedom of action with regard to other conditions. Often the selling price is fixed, while everyone remains free to sell as much as he can. Sometimes the quantity each one is allowed to sell is predetermined, either absolutely or in such a way that when the limit is exceeded a certain sum has to be paid to the association; it may also be stipulated that a premium will be paid to those whose sales stay under the limit. As to the price, it is freely determined by each seller; it is only exceptionally that selling conditions are fixed beforehand.^[a]

For instance, the trade unions sometimes impose a uniform level of wages: an employer who hires ten workers at a certain wage rate cannot hire an eleventh one at a lower rate. Moreover, as a rule the trade unions also fix the wage rate, so that not only the conditions but also the terms are fixed, and this brings us back to one of the previous cases.

The law sometimes requires every portion of a commodity to be sold at the same price. This is the case in almost every country for the railroads, which cannot charge the tenth traveler any more or less in identical conditions than is being asked of the first one. A philanthropist may sell below the price in order to help consumers or a certain class of consumers. We shall come across other cases in dealing with production; and naturally these cases may be very numerous, since they relate to the very varied conditions that can be altered in the economic phenomenon.

49. We have, then, to examine different kinds of type (II) phenomena. We must straight away set aside one of these kinds, which we shall call type (III); this is the one that would exist if the whole economic system were organized in such a way as to provide maximum welfare for all those who are part of it. We shall, moreover, have to define precisely in what this welfare consists (VI, 33, 32). Type (III) corresponds to the collectivist organization of society.

50. It should be noted that types (I) and (II) refer to individuals; it thus may happen—and it usually does—that when two persons bargain, one behaves according to type (I) and the other according to type (II); or if a large number of persons are bargaining together, some behave according to type (I) and others according to type (II). This can also be the case for type (III), if the collectivist state leaves its subjects some freedom of choice.

51. A person who follows type (II) stops, by the very definition given of this type, at a point where his tastes are not *directly* satisfied. Consequently, by comparing the situation which an individual behaving according to type (I) would reach with the one to which

behavior of type (II) would lead, one will see that the second differs from the first by a certain greater or smaller quantity of commodities. One might thus also define type (I) as follows: it is the case in which the quantities of commodities satisfy tastes directly; type (II) covers the case in which the quantities of commodities are such that a positive or negative residual remains after tastes have been directly satisfied.

52. The indifference lines of tastes. Let us consider a man who is guided by his tastes alone, and who possesses one kilogram of bread and one kilogram of wine. Depending on his tastes, he is disposed to have a little less bread and a little more wine, or conversely. He agrees, for example, to have only 0.9 kilograms of bread, provided he can have 1.2 kilograms of wine. In other words, this means that for him both these combinations—namely, one kilogram of bread and one kilogram of wine, 0.9 kilograms of bread and 1.2 kilograms of wine—are equivalent. He does not prefer the second to the first, or the first to the second; he would not know which to choose; he is *indifferent* as to whether he enjoys the one or the other of these combinations.^[a]

Starting from the combination of one kilogram of bread and one kilogram of wine, we can determine a large number of others between which choice is indifferent; we could for example, have:

Bread	1.6	1.4	1.2	1.0	0.8	0.6
Wine	0.7	0.8	0.9	1.0	1.4	1.8

This series, which could be called an indifference series, could be extended indefinitely.

53. The use of graphs will make it very much easier to follow the argument. Let us draw two perpendicular axes, OA and OB ; let us measure the quantities of bread along OA and the quantities of wine along OB . For example,

Oa represents one unit of bread, and Ob one unit of wine; the point m , defined by these coordinates, indicates the combination of one kilogram of bread and one kilogram of wine.

54. We can in this way represent all the preceding series, and by joining together the points of this series, we obtain the line nms , which may be called the INDIFFERENCE LINE, or INDIFFERENCE CURVE.⁴

55. Let us attach to each of these combinations an index which must satisfy the following two conditions and is otherwise arbitrary: (1) two combinations with respect to which choice is indifferent must have the same index; (2) of two combinations, the preferred combination must have a higher index.⁵

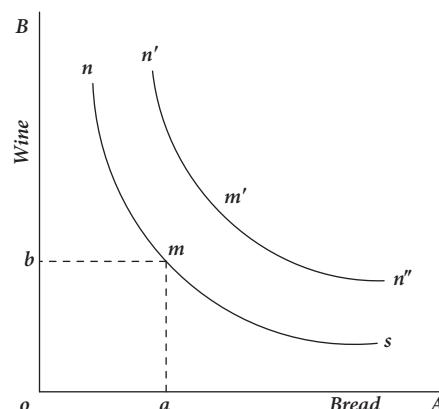


FIG 5

⁴ This expression is due to Professor F. Y. Edgeworth. He assumed the existence of *utility* (ophelimity) and deduced the indifference curves from it; I, on the contrary, consider the indifference curves as a factual datum and infer from them all that is necessary for the theory of equilibrium, without resorting to ophelimity.

⁵ See IV, 32 for another condition that can usefully be added, but which is not necessary to have recourse to here.

We thus obtain INDICES OF OPHELIMITY, or of the pleasure enjoyed by an individual from the combination corresponding to a given index.

56. From the foregoing, it may be concluded that all the combinations in an indifference series have the same index, i.e., that all the points on an indifference line have the same index. Let 1 be the index of the line nms in Figure 5; and m' (e.g., 1.1 units of bread and 1.1 of wine) another combination that the individual prefers to combination m , and let us attach to it the index 1.1 . Starting from the combination m' , we derive another indifference series, i.e., we draw another curve $nlm'n''$. We may continue in this way, considering, of course, not only the combinations that are better for this individual than combination m , but also those that are worse. We shall thus obtain a number of indifference series, each one having its index. In other words, we shall cover the part of the plane OAB that we are considering with an infinite number of indifference curves, each one having its index.^[a]

57. In this way we obtain a complete description of our individual's tastes with respect to bread and wine; and this is all that is needed in order to determine economic equilibrium. The individual may disappear, provided he leaves this photograph of his tastes.

What has been said of bread and wine can of course be repeated for all the commodities.^[a]

58. The reader who is familiar with topographical maps knows that it is customary to use certain curves indicating the common elevation above sea level, or any other level, of all the points on the same curve.

The curves in Figure 5 are contour lines, provided the ophelimity indices are understood as representing the height above the assumed horizontal OAB plane of the points on a hill. This we may call the hill of the indices of pleasure. There are an infinite number of other similar ones, depending on the arbitrary system of indices chosen.

If pleasure can be measured—if ophelimity exists—one of these sets of indices will be precisely that of the values of ophelimity (1906 Appendix 4; 1909 Appendix 3)^{16[a]}), and the corresponding hill will be the hill of pleasure or of ophelimity.

59. An individual who enjoys a certain combination of bread and wine may be represented as a point on this hill. The pleasure felt by this individual will be represented by the height of this point above the OAB plane. The individual will enjoy a greater pleasure the higher he climbs^[a] [the "blissful mountain"]¹⁷; of two combinations he will always prefer the one represented by a higher point of the mountain.¹⁸

60. **The paths.** Let us imagine an individual who possesses a quantity of bread represented by oa and a quantity of wine represented by ab . We shall say that the individual is at a point on the hill which is projected onto b on the horizontal plane xy ; or elliptically, for the sake of brevity,

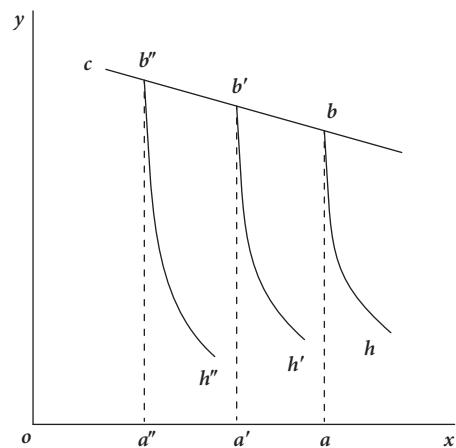


FIG 6

that he is at b . Let us suppose that at another moment the individual has oa' of bread and $a'b'$ of wine; he will leave b and move to b' . If, thereafter, he has oa'' of bread and $a''b''$ of wine, he will have moved from b' to b'' . . . and so on, as far as c . Let us suppose that points b, b', b'', \dots , are very close to each other, and let us join them by a line. We shall say that the individual who has successively had the quantities oa of bread and ab of wine, oa' of bread and $a'b'$ of wine, etc., has followed, on the hill, along a *path*, or route, or road, which is projected on the horizontal oxy plane as the line $bb'b''c$, or, in short, that he has followed the path bc .^[a]

61. Note that if an individual should follow an infinite number of paths, such as $hb, hlb, hl'bl', \dots$, and if he should stop at points b, b', b', \dots , he must be considered as in fact following the path b, b', b'', \dots, c .

62. Let us consider a path mn , which is tangential to an indifference curve $t'l'$ at c ; and let us suppose that the ophelimity indices keep increasing from t to $t'l'$, and that the path ascends from m up to c , and then descends from c to n . A point a which, starting from m , precedes the point c , and beyond which the obstacles do not allow the individual to go, will be called a TERMINAL POINT^[a]. It is encountered only on going up from m towards c , and not in coming down from c towards n . Thus, b would not be a terminal point for an individual following the path mn ; but it would be one for an individual following the path nm , i.e., for somebody who, starting from n , would proceed toward m .

63. The terminal point and the point of tangency have one property in common, namely that of being the highest point that can be reached by an individual following the path mn . The point c is the highest one on the whole path; the point a^{19} is the highest one on the portion of the segment ma^{19} of the path that the individual is allowed to follow.

64. We shall see later how convenient it is for the exposition of economic theories to represent economic phenomena by indifference curves and paths.

65. Continuous and discontinuous variations. Indifference curves and paths could be discontinuous; indeed, so they are in reality, since variations in quantities are brought about in a discontinuous manner. An individual passes from a state in which he has 10 handkerchiefs to one in which he has 11, and not through intermediary states in which he has for example, 10 and one hundredth of a handkerchief, 10 and two hundredths, etc.

In order to come closer to reality, one would therefore have to consider finite variations; but here there is a technical difficulty.

Problems concerning quantities that can be varied in infinitesimal degrees are much easier to handle than those in which the quantities undergo finite variations. One should therefore, whenever possible, substitute the first type of problem for the second, as is in

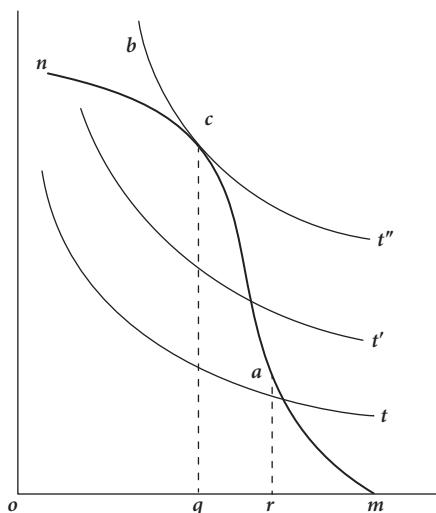


FIG 7

fact done in all physical and natural sciences. We know that we thereby commit an error; but we may disregard it, either when it is small in an absolute sense, or when it is smaller than other inevitable errors, which makes it pointless to look for a degree of precision in one direction which eludes us in another. This is exactly the case in political economy, where only average phenomena and those referring to large numbers are considered. When we speak of an individual, it is not with the purpose of discovering what this individual actually consumes or produces, but only in order to consider one element of an aggregate as a preliminary to summing the consumption and output of a large number of individuals.^[a]

66. When we say that an individual consumes one and one tenth of a watch, it would be ridiculous to interpret this expression literally. A tenth of a watch is an unknown object, which has no use. But we simply take these words to mean that one hundred individuals consume 110 watches.

When we say that equilibrium takes place when one individual consumes one and one tenth of a watch, we simply mean that equilibrium takes place when out of 100 individuals some consume one, some two watches, some more or none at all, so that, in the aggregate they consume approximately 110, or an average of 1.1 each.

This way of expressing oneself is not peculiar to political economy; it is common to a good many sciences.

In insurance, one speaks of fractions of human beings, for example, 27 and 37 hundredths human beings. Obviously, there is no such thing as 37 hundredths of a human being.

If one could not agree to substitute continuous for discontinuous variations, it would not be possible to develop the theory of the lever. A lever with equal arms, such as a pair of scales, for instance, is said to be in equilibrium when it sustains equal weights. Take a balance which is sensitive to a centigram, and place a milligram more in one of the scales than in the other; it will then be seen that, notwithstanding the theory, it remains in equilibrium.

The scale in which people's tastes are weighed is such that for certain commodities it is sensitive to a gram, for others only to a hectogram, and for others still to a kilogram, etc.

The only conclusion to be drawn from this is that one should not expect more precision from these scales than they are able to provide.

67. Besides, since this is only a technical difficulty, those with time to waste may amuse themselves with the consideration of finite variations; and after hard and protracted labors they will obtain results which, within the limits of possible error, do not differ from those that could be reached easily and quickly by assuming infinitesimal variations, at least in the usual cases. Our only purpose is to investigate objectively the relations between phenomena, and not to please pedants.

68. Obstacles. There are two kinds: some are immediately apparent, others are less obvious.

69. Among the first kind are the tastes of the persons with whom an individual carries out transactions. If a given quantity of a commodity is to be distributed among various individuals, the fact that this quantity is fixed constitutes an obstacle. If the commodity has first to be produced, the fact that it can be obtained only by using other commodities is also an obstacle; likewise, an obstacle is created by the fact that a commodity is not available at the time and place required. Finally, some obstacles are inherent in the social system of organization.

70. In general, when an individual gives up a certain quantity of a commodity to procure another one, we shall say that he TRANSFORMS the first commodity into the second. He may do this by barter, giving up the first commodity to someone and receiving the second; or through production, actually transforming the first commodity by himself into the second one. Or again, he may have recourse for this operation to a producer, i.e., someone who transforms commodities.

71. I shall reserve the terms PRODUCTION OR TRANSFORMATION for the latter operation and use the terms OBJECTIVE PRODUCTION OR TRANSFORMATION to describe a production process, when abstraction is made from the person who carries it out as when, for example, the individual who enjoys the commodity transformed produces it himself.

72. As to the objective transformation, we must distinguish, [at least in the abstract,]²⁰ three categories of transformations, namely:

(1) Material transformation: for example, transformation of wheat into bread, of the grass of the meadows (to which must be added the use of land and buildings) into sheep wool; etc.

(2) Transformation in space: for example, coffee in Brazil transformed into coffee in Europe.

(3) Transformation in time: for example the present crop of wheat stored and transformed into wheat available in a few months, and conversely, present consumption of wheat compensated subsequently by the yield of the future crop, by which means the future crop is transformed into a spot commodity (V, 48).

73. But this is not all; the subject is not exhausted, for there are other constraints or obstacles which form the SECOND KIND OF OBSTACLE. An individual, for example, possesses 20 kilograms of wheat; he exchanges 10 of them for 15 kilograms of wine, and a further 10 for 15 kilograms of wine. In all, he has exchanged 20 kilograms of wheat for 30 kilograms of wine. Or he may start by exchanging 10 kilograms of wheat for 10 kilograms of wine, and then exchange an additional 10 kilograms of wheat for 20 of wine. In all, he will have exchanged 20 of wheat for 30 of wine. The end result is the same; but it is reached in two different ways. The individual may be free to choose the way that is the most convenient to him or he may not. The latter is generally the case. What prevents an individual from choosing freely between them is an obstacle of the second kind.⁶

74. There are an infinite number of paths starting from m and leading to n such as msn , $ms'n$, $ms''n$, etc.

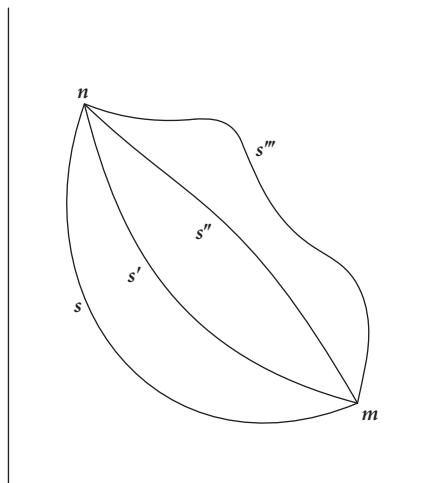


FIG 8

⁶ Most literary economists have only a vague idea of this kind of phenomenon.

One of these paths may be a straight line, or any type of curve. The second kind of obstacle has the effect, in some cases, of determining a unique path that can be followed starting from m , and in some cases only the type of path that may be followed. For instance, we shall consider a case (§172) in which an individual may leave m only by following a unique line. We shall come across another case (§172) in which this straight line may be arbitrary, i.e., the individual may choose among an infinite number of paths passing through m , provided they are all linear. We shall come across other cases in which the individual follows a broken line (VI, 7).

75. Indifference lines of obstacles, in objective transformations. In the case of obstacles of the first kind, there are certain lines analogous to indifference lines of tastes.

Let us suppose that a commodity A is transformed into another one, B, and that we know the quantities of B that can be obtained from $1, 2, 3, \dots$, units of A.

Let us draw two coordinate axes (Figure 9), and for each quantity oa of A let ab indicate the quantity of B produced. We thus obtain a curve $bb'b''$ which we shall call the INDIFFERENCE LINE OF OBSTACLES. We shall attach the index zero to this line, to indicate that along this curve transformations take place without leaving any residual.

Let the segments $bc, b'c', \dots$ of straight lines parallel to the oA axis have length equal to unity; we then obtain another indifference line, cc' , to which we attach the index 1. If the quantity oa'' of A is available, and a transformation is carried out resulting in $a''c'$ of B, there still remains $a'a''$ of A, that is, a residual of one unit of A; this is why we attach the index 1 to the line $cc' \dots$

Likewise, let $bd, b'd', \dots$, have length 1 and let us join the points $dd' \dots$; we obtain another indifference line to which we shall attach the index -1 , because on this line instead of gaining a unit, precisely one unit is lost, when in transforming oa of A into ab of B, only oa'' of A is obtained.

Proceeding in this way, we will cover the whole plane with indifference curves, some with positive indices and others with negative ones, separated by the line with index zero. This line deserves special consideration; we shall call it THE LINE OF COMPLETE TRANSFORMATIONS, because transformations along it take place without leaving any residual either positive or negative.^[a]

76. The indifference lines of the producer. If we consider a single producer, the lines we have just discussed are also the producer's indifference lines, because on each one of them he obtains the same profit if the index is positive, or the same loss if the index is negative; he neither gains nor loses if the index is zero, i.e., on the line of complete transformations. But when there are several producers, their very number may form part of the obstacles, and in that case the indifference lines vary (§147).²¹

77. Analogies between the indifference lines of tastes and indifference lines of obstacles. These lines are similar in certain respects, and different in others. They are analogous in that the individual attempts, insofar as he is permitted, to pass from one

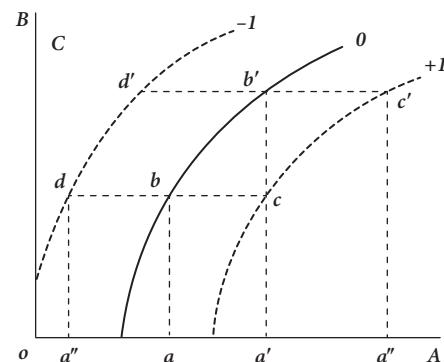


FIG 9

indifference line to another with a higher index, and in that the producer does the same.

78. It should be noted, moreover, that the individual who satisfies his own tastes is guided by considerations of ophelimity; the producer, on the other hand, is guided by considerations of quantities of commodities (§76).

79. Besides, the producer is often prevented by certain circumstances from rising above the line of complete transformations; and he cannot remain for long below it because he incurs losses; so in the end he is compelled to remain on this line. This is an essential difference from the phenomena relating to tastes.

80. Finally, the shapes of the indifference lines of tastes are generally different from those of the indifference lines of obstacles: a rough idea of this can be obtained by comparing Figure 5 with Figure 9.

81. If one considers the producer's indifference lines as the projections of the contours of a surface all of whose points have, on the plane, a height indicated by the index of the point, we obtain a HILL OF PROFIT, which is in part analogous to the hill of pleasure (§58), but differs from it in that it lies partly above and partly below the reference plane. It is like a hill partly under water; the surface of the hill emerges in part above sea level, but it also extends below it.

82. Competition. We have already referred to competition in §16; we must now define it more precisely.^[a]

One must distinguish competition among those who exchange from competition among those who produce; the latter can itself be subdivided into different types.

83. A person who exchanges tries to climb the hill of pleasure as high as he can. If he has an excess of A, he tries to obtain a larger quantity of B, and to achieve this he gives up a larger amount of A for the same quantity of B; i.e., if he is at the point *l*, he will decrease the slope of *ml* on the *oa*-axis. If he has an excess of B, i.e., if he finds himself at *r*, he gives up less of A for the same amount of B; he thus increases the slope of *mr* on the axis *oA*. In short, the individual moves in the directions shown by the arrows. This will be the case whether he is alone or in competition with others.

Competition has the effect of preventing him from comparing positions on two different paths, and of limiting his choice to positions on the same or closely neighboring path or to adjacent positions. Moreover, competing individuals keep moving up to the point where everybody is satisfied; and it is enough if only one individual is not satisfied to compel the others to move.

84. The producer attempts to ascend the hill of profit as high as he can (§81); i.e., he tries to obtain the greatest possible residual of A; he never has an excess of A. Consequently, he keeps moving in the same direction, and not sometimes in one direc-

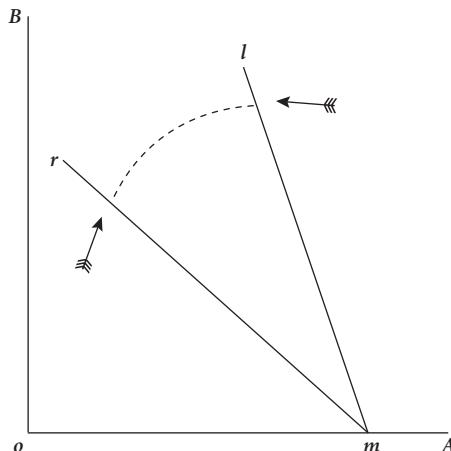


FIG 10

tion and sometimes in another, as in Figure 10. A change in the direction of his movement can only be the result of a change in the direction in which a larger quantity of A can be obtained.

85. One generally starts with the study of an isolated community that is not in contact with others. In such a community, the number of those who exchange is constant; on the other hand, the number of producers is essentially variable, for those who incur losses finally stop producing, whereas, if some make gains, new producers will soon emerge to share in the profits. Something similar happens in the case of consumers, and we shall have to take this into account when we deal with population, but the production of men does not obey the same laws as that of manufactures;²² in particular, it extends over a broader span of time; for this reason we shall devote a separate study to it.

86. With or without competition, the producer cannot remain on the side of the negative indices, where he is incurring losses. In the absence of competition he can, on the other hand, stay on the side of the positive indices, where he makes a profit; indeed, always with a tendency to move in the direction of maximum profit. Competition tends to reduce this profit, by pushing the producer toward the negative indices.

This competition may take place whether the technical conditions of production are assumed to be constant or variable. In the present chapter, we shall consider only the first type of competition.

87. Suppose there are two consumers. The first one has oa of A, the second one oa' ; between them they thus hold oA , which is the sum of these two quantities.²³ Let us assume that these two consumers can only move along the parallel lines $ad, a'd'$. They will transform ab of A into²⁴ $b'd$ of B, and the second $a'b'$ of A into²⁴ $b'd'$ of B. Summing the amounts thus transformed, we see that the consumers between them transform AB of A into²⁴ BD of B, by following a path parallel to ad and $a'd'$. Instead of these two consumers, we can thus consider a single one, who follows the path AD . The same reasoning applies to any number of consumers, who may thus be replaced by a single fictitious consumer, who represents them all.^[a]

88. We may proceed in the same way with the producers, provided we may disregard the modifications in the obstacles brought about by their numbers.^[a]

89. **Types of phenomena concerning producers.** As for consumers, one must consider types (I) and (II), to which type (III) may be added. The properties are the same. Type (I) is always that of competition; there are differences between competition among consumers and among producers.

90. **Equilibrium.** As we have already seen (§27), equilibrium comes about when the movements brought about by tastes are hindered by obstacles, and conversely. The general problem of equilibrium can thus be subdivided into three others: (1) deter-

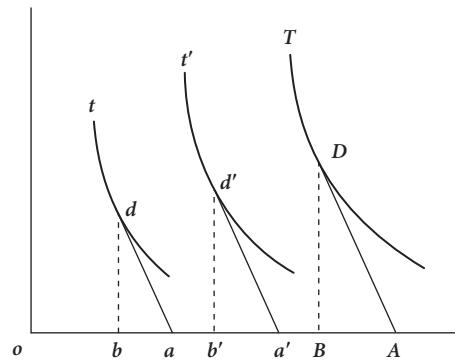


FIG 11

mining equilibrium with respect to tastes; (2) determining equilibrium with respect to the obstacles, or with respect to producers; (3) finding a point common to these two equilibria which will be a point of general equilibrium.

91. As to the paths, we have to consider (1) equilibrium on a given path; (2) equilibrium on a class of paths, and then see how the path to be followed is chosen.

92. As regards the different types of phenomena, we must first study type (I) for the trader and the producer. Afterwards, we shall investigate type (II) which, as a rule, is met with only in the case of individuals who trade with others acting according to type (I).

93. **Equilibrium with respect to tastes.** Let us start by considering an individual who follows a given path, and who tries to reach a point on it where his tastes are best satisfied.

94. If obstacles of the first kind fix a point on this path beyond which it is not possible to go, and if the positions preceding this point are less advantageous to the individual, he will naturally go up to that point, and there he will stop.^[a]

At that point, there will be equilibrium with respect to tastes. The point may be one of tangency between the path and an indifference curve, or a terminal point (§62); at any rate it is the highest point on the segment of the path the individual is permitted to follow.

95. The point of tangency could also be the lowest one on the path. At such a point, equilibrium would be unstable. We shall not deal with this case for the moment.

96. From now on we shall consider linear paths only, because they are the most frequent in the real world. But our reasoning is quite general and may, with slight alterations or restrictions, be applied to other kinds of paths.

97. Let us consider an individual whose tastes are represented by the indifference curves t, t', t'', \dots , the ophelimity indices increasing from t to t'' .²⁵ Each week this individual has a quantity om of A. Let us suppose that, in order to transform A into B, he follows the linear path mn . At the point a , where the path cuts the indifference curve t , there is no equilibrium, because the individual would be better off if he moved from a to b , on the curve t' , where he would have a higher index of ophelimity.

The same can be said of all the points where the path cuts indifference curves, but not of point c'' , where the path is tangential to an indifference curve. Indeed, from c'' the individual can move only towards b or towards b' , and in both cases the ophelimity index decreases. Tastes therefore preclude any further movement by the individual along the path mn once he has reached c'' ; consequently, c'' is an equilibrium point. The same is true of similar points, c, c', c'', c''' , located on other paths which the individual may be assumed to follow. If these points are

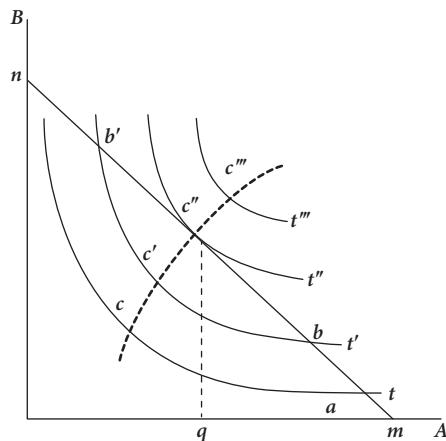


FIG 12

joined together by a line, we obtain a line of equilibrium with respect to tastes; it is also called the EXCHANGE LINE.^{7[a]}

Terminal points which, coming from m , precede the points on the exchange line may also be equilibrium points.

98. It can happen that a path leads up to a position with a zero quantity of A without being tangential to any indifference line. In such a case there would be a terminal point where the path cuts the ob -axis; this would mean that along this path, not only is the individual willing to dispose of the whole amount of A he possesses to have some of B, but even if he had a larger quantity of A, he would give it up for more of B.^[a]

99. By summing the quantities of the commodities transformed by each individual, one obtains the exchange line for this community of individuals. And if one likes, one can also draw the indifference curves for this community; they will be derived from the indifference curves of the individuals who make it up.

100. Equilibrium of the producer.

The producer tries to obtain maximum profit, and, if nothing gets in his way, he will ascend the hill of profit as high as he can. By following a path ol , the producer can reach a point c where this path is tangential to an indifference curve of obstacles; and this point may have a higher profit index than neighboring points on the path. In such a case, the producer's equilibrium is reached at the point c , on the path ol , analogously to the case of the consumer. We shall say that in this case competition is incomplete.^[a]

101. It may happen, on the other hand, either that the path ol' is not tangential to any indifference curve of obstacles, or that ol is tangential at c to one of these curves and the index of c is less than that of neighboring points on the path. In that case, competition is complete.

The producer will try to continue along the path ol up to any terminal point which is imposed on him by the other conditions of the problem.

102. Let us consider two categories of commodities: (1) There are certain commodities such that the amount of B obtained per unit of A increases with the total amount of A transformed; (2) There are other commodities for which the amount of B per unit of A instead decreases.⁸

103. In the first case, one has lines somewhat similar to the lines t, t', \dots in Figure 14, on each of which the corresponding index has been indicated. Obviously, no path

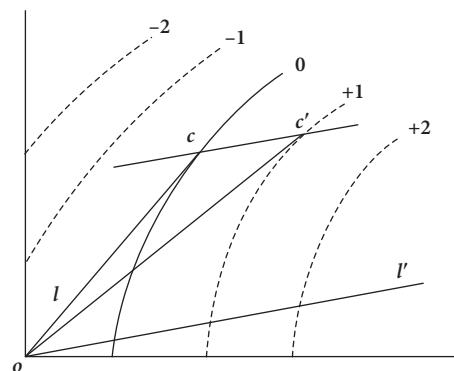


FIG 13

⁷ One could cover the plane with a great many exchange lines, thus obtaining a picture of the hill of opelimity indices which would be analogous to the one obtained by covering the plane with indifference lines (1906 Appendix 17, 1909 Appendix 42).

⁸ The first category is that of commodities B whose cost of production decreases with the increase in output; the second category is that of commodities whose cost of production increases.

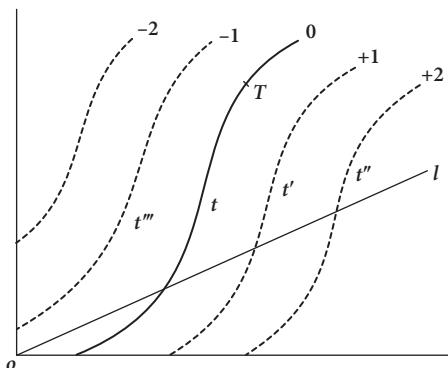


FIG 14

In fact, the producer may stop at any point where he is making a profit. Moreover, he tries to increase this profit as much as possible, i.e., he tries to go as far as possible along path ol ; in this case, equilibrium is reached at terminal points (§ 62) instead of tangency points. For these commodities competition is complete.

104. It is rare, moreover, for the indifference lines to have indefinitely the form indicated. Usually, beyond a certain point T , which is more or less distant, the phenomenon changes from the first category to the second. Point T and other similar ones may lie beyond the limits considered and in that case they might as well not exist.^[a]

105. The second category of commodities referred to in §102 has indifference lines shaped like those in Figure 13. Some paths, like oc , are tangential to an indifference curve; others, like ol' , cannot be tangential to any such curve. By joining the points of tangency, $cc' \dots$, we obtain a line that we shall call the LINE OF MAXIMUM PROFIT. It corresponds to the exchange line that is derived from the indifference curves of tastes. The region of indifference curves with positive index is, usually, the region of possible equilibrium; but obviously the producer will stay on the line of maximum profit if he can. For these commodities, competition is incomplete (V, 96).

Paths that do not meet the line of maximum profit and that end in some point with a negative index cannot be followed when there is competition (§137).

106. Equilibrium of tastes and obstacles. Let us consider a certain number of consumers and a single producer, or a certain number of producers, but with the restriction that their number does not affect the obstacles. Let us draw, for the consumers,

of the kind ol can be tangential to an indifference curve with positive index. The line t with index zero, i.e., the line of complete transformations,^[a] divides the plane into two parts or regions; in one region are the lines with negative indices; in the other, the lines with positive indices. The producer cannot stay in the first region, or at least not for long, because there he incurs losses; and obviously, he will not be willing or, for that matter, generally able to incur losses indefinitely. Equilibrium is thus not possible in this region. It is, however, possible in the second one, which we shall therefore call the REGION OF POSSIBLE EQUILIBRIUM.

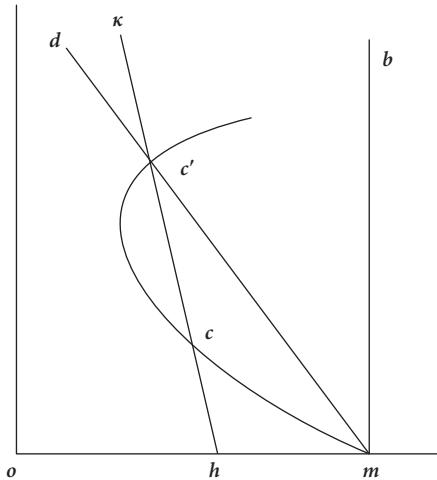


FIG 15

the exchange line mcc' for the total quantities of commodities, i.e., let us consider the community as if it were a single individual (§87).

For producers, let us draw the line nk , which will be the line of complete transformations for the commodities of the first category (§102), i.e., under complete competition; and which will be the line of maximum profit for commodities of the second category (§102), i.e., under incomplete competition. Let us consider the phenomena of type (I).^[a]

107. If there is a line of maximum profit, and if it intersects the consumers' exchange line, the producers will stay on this line of maximum profit, because that is to their advantage. Otherwise, as we shall see (§141), they will be driven onto the line of complete transformations. The line hk is thus the one on which the producers will stop; and the equilibrium points will be the points c, c' , where this line intersects the producers' exchange line.^[a]

108. All this naturally holds when the paths are straight lines starting from m , because it is precisely to these paths that the exchange line and the line of maximum profit refer. If the paths change, the lines change too. For instance, if the producer were forced to follow the line of complete transformations, equilibrium would take place at the point where this line is tangential to an indifference curve of tastes.

109. If two individuals exchange commodities between themselves, the equilibrium points are located at the intersections of the two individuals' exchange lines, the coordinate axes being positioned so that the path followed by the first individual coincides with that followed by the other (§116).

The same will be true if, instead of two individuals, we consider a community.

110. The abstract case which we have often considered of two individuals acting according to the phenomena of type (I) does not correspond to the real world. Two individuals who had to bargain between themselves would probably be guided by motives very different from those we have assumed. To correspond to the real world, we need to assume that the pair considered are not isolated, but form one element of a whole comprising many pairs. Let us begin by studying one of them, precisely so that we can see thereafter what happens when there are several of them. We thus suppose that the pair considered do not behave as if they were isolated but as if they formed part of a community.

The same restriction should apply in the consideration of a single producer and a single consumer.^[a]

111. When an individual operates according to phenomena of type (II), he imposes on others the path that is most advantageous to him, and the point of equilibrium occurs at the intersection of this path with the line of equilibrium of the other individuals.

112. From the foregoing, we can deduce the following general theorem:

For phenomena of type (I), if a point exists where a path followed by the contracting individuals is tangential to the indifference curves of these individuals, that is a point of equilibrium.

For, if two individuals bargain, the points at which their exchange lines intersect are equilibrium points; but, at these points, the paths are tangential to the indifference curves of tastes, since it is precisely this condition that determines those lines (§97). Naturally, the axes should be positioned in such a way that the individuals follow the same path (§116). The same reasoning applies to two communities.

113. If consumers bargain with producers who have a line of maximum profit (§105), the intersections of this line with the consumers' exchange line will give the equilibrium points; but at these points, the paths are tangential to the indifference curves of tastes and to the indifference curves of obstacles, since it is precisely this last condition that determines the line of maximum profit. The theorem is therefore proved.

114. If there is no point of tangency, the theorem does not apply, and is replaced by the following theorem, which is more general, and which implies the preceding one:

Equilibrium takes place at the points of intersection of the line of equilibrium of tastes and of the line of equilibrium of obstacles. These lines are the locus of the points of tangency of the paths with the indifference lines, or the locus of the terminal points of these paths.

115. For phenomena of type (II), we have the following theorem:

If an individual acts according to phenomena of type (II) with others who operate according to those of type (I), equilibrium takes place at the point that is most advantageous to the first individual, this point being one of those at which the paths intersect the curve that represents the locus of possible equilibrium points.

116. Modes and forms of exchange equilibrium. Let us now study in detail the phenomena that we have studied in broad outline.

Let us suppose that the obstacles consist only in the fact that the total amount of each commodity is constant, and that only its distribution between two individuals can vary. This is the case in the problem of exchange.

Let us assume that the first individual, whose situation is depicted in Figure 16, possesses om of commodity A, while the other individual has a certain amount^[a] of B, and none of A. The coordinate axes of the first individual are oA and oB ; those of the second are ωa and $\omega \beta$; the distance om being equal to the quantity of B in the hands of the second individual. The indifference curves of the first individual are t, t', t'', \dots and those of the second are s, s', s'', \dots . Given the way the figures are drawn, a single line indicates the path followed by both individuals. The ophelimity indices rise from t to t'' , and from s to s'' .^[b]

117. Let us investigate phenomena of type (I). If a path mc is tangential at c to a curve t and to a curve s , c is an equilibrium point. Thus, if the obstacles of the second kind impose, not a particular path, but only the kind of path, both individuals will try different paths of this kind, until they find one similar to mc .

To determine the point c , we may proceed as follows. For each individual we shall draw the exchange line (§97), and thus obtain, for each individual, the locus of the points at which the equilibrium can take place. The point at which the exchange line of the first individual cuts that of the second is, of course, the equilibrium point that we are looking for, since it is a point of equilibrium for both individuals.

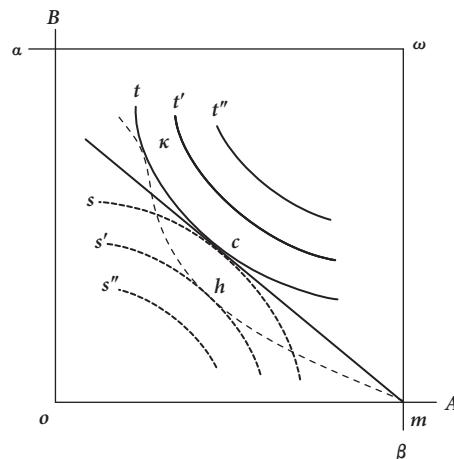


FIG 16

118. Should the obstacles impose a particular path, mhk , tangential at h to one of the curves s, s', \dots and at k to one of the curves t, t', \dots , the points of equilibrium would be different for the two individuals; consequently, if neither is able to impose his will on the other, i.e., if it is a type (I) phenomenon, the problem before us is insoluble.^[a] If the first individual can impose his terms on the second, he will force the latter to follow him up to the point k , where equilibrium will take place.

119. It should be noted that this case is not to be confused with the one in which an individual is able to compel another individual to follow a particular path (§128). In the first case, the path is predetermined, and an individual may force someone to move a certain distance along it. In the second case, the path is indeterminate, and an individual may determine it as he likes; but he cannot then force the other one to move a certain distance along it.^[a]

120. We have said that several paths may be tried before the one is found that leads to the point of equilibrium. Let us examine this matter more closely.

If we draw the exchange curves of two individuals, we shall see that in a fairly large number of cases these curves have shapes similar to those in Figure 17, and that they intersect more or less as indicated in those diagrams; the first one has three points of intersection, the other only one. They are of three different types, which we shall call $\alpha\beta\gamma$; they are indicated in greater detail in Figure 18.

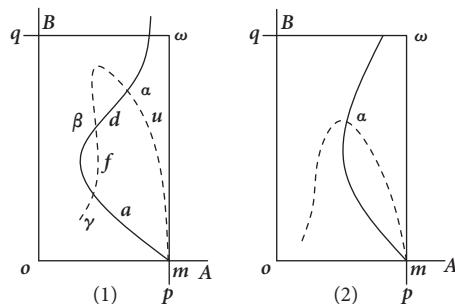


FIG 17

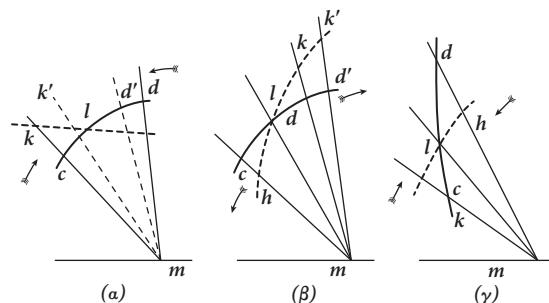


FIG 18

The exchange line for the first individual whose coordinate axes in Figure 17 are oA and oB , will always be indicated by cd in Figure 18. That line, for the second individual whose coordinate axes are indicated by $\omega\alpha$ and $\omega\beta$ in Figure 17, will always be indicated by hk in Figure 18.^[a] The point of intersection of these two contract lines,^[b] which is the equilibrium point, is denoted by l .

121. Let us consider the equilibrium for the first individual. In the case of the points (α) and (γ) , the points on the line lh precede those of the line cd , and are therefore terminal points (§62) for the first individual; the line on which he may find himself in equilibrium is thus clh .^[a] For a similar reason, the line on which the second individual may be in equilibrium, still in the case of the points (α) and (γ) , is also clh . In the case of the point (β) , this equilibrium line is for the first individual as well as the second, hld . We thus need to consider only what happens on these lines.

122. Let us consider the points (α) and (γ) .^[a] The first individual finds himself in an equilibrium position at h . Since we are considering type (I), the individual compares only the conditions in which he would find himself at the different points of the path mhd ; and sees that he would be better off at d than at h . But he cannot reach d , being prevented because of the second individual's tastes. If a large number of individuals are in competition with a large number of other individuals, and if our pair is not isolated, the first individual has a means of proceeding, if not as far as d , at least up to a point very close to it. He follows a path md' , a little less steep than md on the ox axis, i.e., he gives up a larger amount of A for the same quantity of B. In this way, he attracts customers away from the second individual; he receives some of B from other individuals, and he is able to reach d' , which is the highest point on the path, and where he settles in equilibrium.

Let us see what happens to the second individual. He was first at h , which is the highest point he can reach on the path. The loss of customers pushes him back; they bring him less of A,^[b] because the first individual receives more of it. The second individual is thus thrust back, for instance to h' . By comparing only the conditions in which he would find himself at different points along the path mhd , he sees that his position has worsened, and that he must try to watch out for his own interests and return to h , or at least to a point very close to it. To this end, he will follow the first individual's example, and will pay him back in his own coin. He will follow a path that is much closer to him but a little less steep than md' , and thus will settle, for instance, at a point h'' on the line kh .

Now it is up to the first individual to watch after his own interests, and he will follow a less sloping path. In this way, both individuals will come closer to the point l , by moving in the direction of the arrow.

A similar process could be started from the point c . The second individual who finds himself at c , which is a terminal point for him, tries to come closer to k which is the highest point on the path mck ; therefore he would be content with a little less of A for a given quantity of B. He thus follows a path such as mk' which is steeper than mk on the ox -axis. The first individual is compelled to do the same; and thus, little by little, both individuals move towards l .^[c]

123. The equilibrium point is therefore l , and we shall call this a point of STABLE EQUILIBRIUM, because, if both individuals move away from it they will tend to come back to it later.

124. Let us now consider the point (β) . As we have seen, hld is the line of equilibrium. Let us suppose that both individuals are at d ; the second individual would like to move

from this point, which is a terminal point for him, toward the point k ; for this purpose he cannot but agree to receive less of A for the same quantity of B, i.e., he follows a path $md'k'$, steeper than mk with respect to the ox -axis, and he will move away from l . The first individual is forced to follow his example; they will thus move in the direction of the arrow. The same is true on the other side of l . If both individuals are at h , the first one will try to come closer to c , and will therefore give up a larger amount of A for the same quantity of B; he will thus follow a path which is less steep than mc , and will move away from l . The second individual has to follow suit, and so on. Both individuals thus tend to move away from l . The point l is one of UNSTABLE EQUILIBRIUM.

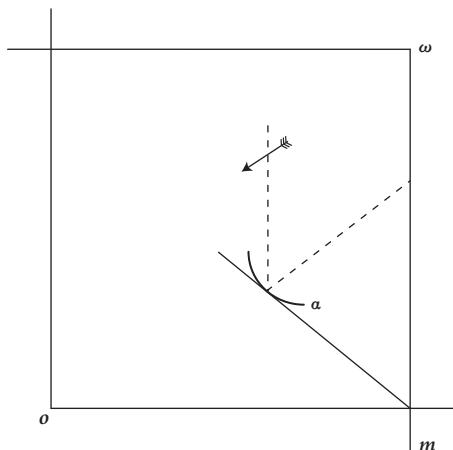


FIG 19

125. Let us come back to Figure 17. For the second individual, there is only one point of equilibrium and it is one of stable equilibrium. For the first individual, there are two points of stable equilibrium; namely, (α) and (γ), and one point to one or the other of the two points of stable equilibrium. The equilibrium line is the line $muadßlyam$.^{27[a]}

126. Let us consider as positive the direction of rotation indicated by the arrow in Figure 19; it causes the angle α to increase. If, in the direction of negative rotation, before the two equilibrium lines cross, the line of the individual who exchanges A for B precedes the line of the individual who exchanges B for A, the equilibrium is stable. In the contrary case, it is unstable.

127. From Figure 18, it will be seen that each individual always attempts to climb the hill of pleasure, to increase his ophelimity, by proceeding directly along the path followed; but competition deflects his course causing him to slide towards l in the case of stable equilibrium, and away from l in the case of unstable equilibrium.

Between these two equilibrium points, the problem is to know whether, by starting from the equilibrium point, and by moving in the positive direction of rotation, the first individual will be able to remain on his contract line, or whether he has to move onto that of the second individual, the points of which become terminal points for him. In the first case, we have the points (α) and (γ) in Figure 18; in the second, the point (β). We may express this also in the following way, which amounts to the same thing: in the case of negative rotation, if the first individual is not able to maintain himself on the exchange line, but has to pass on to that of the second individual (points (α) and (γ)), the equilibrium is stable; if, on the contrary, he can stay on his own exchange line (point (β)), the equilibrium is unstable.

128.^[a] Let us now consider phenomena of type (II). Let us assume that the second individual acts according to this type, while the first individual continues to behave according to type (I).

For the first individual, the equilibrium curve is still $matsb$, which joins the points of tangency of the different paths, starting from m with the indifference curves. The second individual is able to choose the path mde , but he cannot compel the first one to go beyond the point d , as far as e . Moreover, the second individual could stop before reaching d , and thus compel the first individual, too, to stop. In short, equilibrium is possible in the whole space lying between $m\omega$ and $mayts\beta db$. The way equilibrium is reached is different in these two cases. In phenomena of type (I), the individuals were led to this point by competition; whereas in phenomena of type (II) one of the individuals chooses the point that suits him best among those at which equilibrium is possible.

129. The second individual, who finds himself at d , no longer tries here, as previously, to move to e , or even to a point very close to it: he compares the situation in which he finds himself at d with one in which he would be at any other point where equilibrium is possible, and he chooses the point that suits him best, by imposing upon the other individual the path that necessarily leads to this point.

130. The point where the second individual enjoys the best situation is, of course, the one with the highest ophelimity index, the highest point among all those he can choose from, that is, the highest point on the second individual's hill of pleasure. Now, the points comprised between ωm and $\omega ayts$ are obviously not as high as those which lie beyond $mayts$. This curve may be considered as a path; its highest point on the second individual's hill of pleasure will be point t at which it is tangential to an indifference curve. It is thus the point where the second individual should stop.

131. Determining this point is very difficult in practice. That is why anyone who acts according to type (II) often sets himself another aim; namely, to obtain the greatest possible amount of A.^[a] The point which satisfies this condition is the point of tangency s of the common line of equilibrium and a line parallel to the oy -axis. This point is easily determined by trial and error, because the individual's own budget indicates the amount of A he obtains.

132. When commodity A is much more desirable [*ofelima, ophélimé*]²⁸ than commodity B for the second individual, the point s almost merges with the point t ; they would be identical if only A were desirable²⁸ for the second individual, because in that case the indifference lines would run parallel to the oy -axis (IV, 54).

Other conditions could be specified, and we would then obtain other points of equilibrium.

133. If, instead of following linear paths indicated by the prices, the individual follows the transformation line imposed by the obstacles or in general any given path, equilibrium may be stable or unstable. Let acb ²⁹ be a transformation line, and c the point

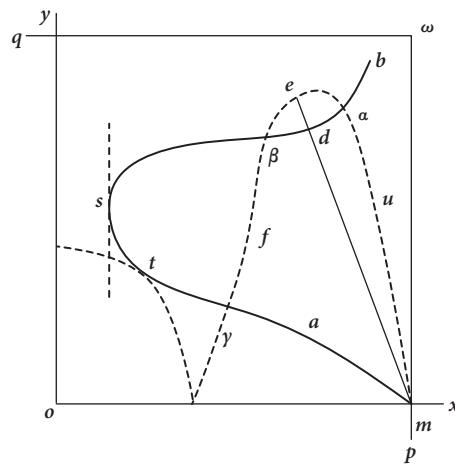


FIG 20

at which it is tangential to an indifference line of tastes t' ,³⁰ at which point equilibrium takes place.^[a] If, as is usually the case, this transformation line ab is such that the ophelimity index is higher at c than at neighboring points a, b , the equilibrium is stable. Indeed, the individual who happens to move away from c tends to return to it, because he always attempts as much as possible to move to points having higher ophelimity indices. For the same reason, if the transformation line had the shape $a'b'$, so that if the ophelimity indices of the points a', b' close to the equilibrium point c' were higher than the ophelimity index of c' , equilibrium would be unstable.

134. Maxima of ophelimity.

We now have to review the different maxima among the equilibrium points. First, there is one absolute maximum, at the highest point of the hill of pleasure, i.e., at its summit. At this point, the individual enjoys everything to satiety; there is little to be said about such a state of bliss.

Then there are a great many relative maxima. The point c'' in Figure 12 is the highest on the path mn ; it is a maximum subject to the condition that the individual move only along the path mn . The other points of tangency, c , are also maxima of a similar kind. Among these, one point may be higher than all the others: it is a *maximum maximorum*. A terminal point also indicates a maximum; it is the highest point on a segment of a path, but it is not so high as the next tangency point.

In Figure 20 the point t is for the second individual, the highest one on the common line of equilibrium.

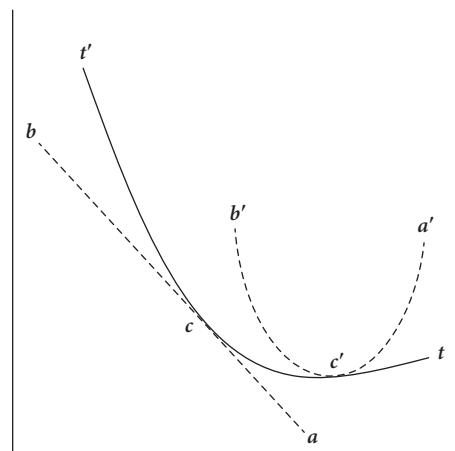


FIG 21

As for the point s , it indicates a maximum of a kind different from the preceding ones, since it is no longer a maximum of ophelimity, but a maximum amount of commodity A.

135. Modes and forms of equilibrium in production. If we assume that the line hk in Figure 18 is that of maximum profit for the producer or producers,^[a] we need only repeat the reasoning set out above in the case of exchange. The producer tends to stay on that line, just as the consumer stays on the exchange line.

136. There is, however, a difference in the case of paths that do not intersect the line hk of maximum profit (Figure 22). If the producer follows the path mk , it is

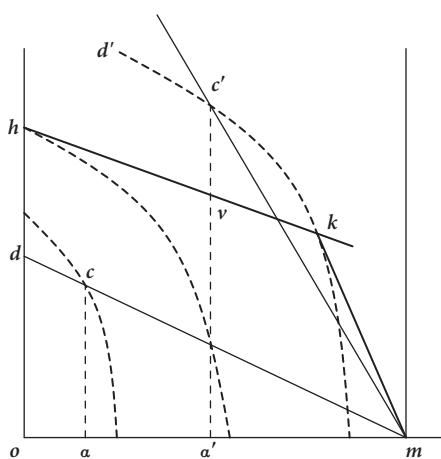


FIG 22

understandable that he will stop at k , because he would be worse off on one side or on the other; but, if he follows the path mc , which is not tangential to any indifference curve of obstacles, why should he not continue further along this path up to the point allowed by his customers' tastes?

137. This is where competition comes in. The line hk divides the plane into two regions; in the one below ftk , with respect to m , it is in the producer's interest to increase, along the linear path mc , the amount ma of commodity A transformed; in the region on the other side of hk , with respect to m , the producer's interest is to restrict, along a linear path mc' , the quantity ma' of commodity B transformed. Thus, the situation is not the same for producers who are at c and those who are at c' .

138. The producer who is at c may be tempted, even if he is alone, to increase the amount he transforms, and he will do so if he is assumed to follow rigorously the principles of phenomena of type (I). He will compare his situation at different points along path mcd , and he will see that he would be better off beyond c . Consequently, if the consumer does not wish to go beyond point c on this path, the producer will agree to give up a larger quantity of B per unit of A , i.e., he will increase slightly the slope of the path mc with respect to mo .^[a] Moreover, if he is alone, he will finally realize that although he hopes thus to gain, he is in fact losing, and he will cease acting according to type (I); and will instead behave according to type (II).

If there are several competitors, the one who raises the slope of the path mc will in fact profit for a short while. Moreover, if he did not take such action someone else would. In this way the slope of mc with respect to mo is gradually raised and we come closer to the line hk . When we are there, there is no longer any advantage in increasing the quantity of A transformed; the effect consequently vanishes after the cause.

139. If the producer finds himself at c' , he soon realizes that he would benefit by decreasing the amount ma' of A to be transformed. To increase this quantity, he had to fight his competitors; but, to decrease it, he can act alone without bothering about the others. He thus lowers the slope of mc' on mo , and he comes closer to the line of maximum profit hk , without giving the slightest thought to whether or not his competitors are following suit. It will be noted that this movement may take place entirely along the path mc' ; consequently by acting in strict accordance with the principles of type (I), he settles down at v , because he is better off there than at c' . He will not move beyond v toward m , because that would make him worse off.

140. In conclusion, therefore, the producer who finds himself beyond hk , with respect to m , is led back to hk by his personal interest. The producer who is on this side of hk ,

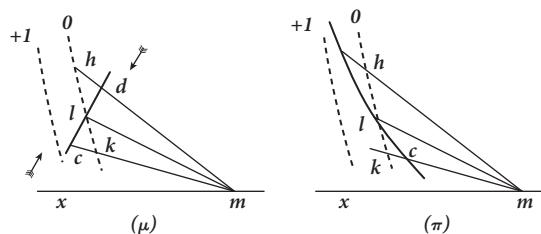


FIG 23

relatively to m , tends to return to hk , perhaps of his own accord, but certainly under the pressure of competition. He would certainly move there even of his own accord, if he were assumed to behave in strict accordance with type (I).

141. We still have to examine the case in which this line of maximum profit does not exist.

Let cd be the exchange line and hk the producer's line of complete transformations. The area of positive indices lies beyond hk , with respect to m . Two cases arise; they are indicated respectively by (μ) and (π) .^[a]

142. Let us first examine case (μ) . At c , the consumer is in equilibrium, for he finds himself on his exchange line; the producer is satisfied, since he is in the region of positive indices; this state of affairs could thus last for a long time.

But if the producer wishes to be still better off, and acts strictly according to the principles of type (I) phenomena, he will go further and move along the path mc ; he is prevented from doing so by consumers' tastes, and will therefore try to give up a larger quantity of B to such a consumer for the same amount of A, i.e., he will raise the slope of the path with respect to the A-axis, and will thus come closer to the line hk .

Moreover, if the producer were alone, he would soon realize that it is foolish to behave in this way; the result obtained is the exact opposite of what he is aiming at. He would thus cease to act according to the principles of type (I), and would apply those of phenomena type (II).

143. When a large number of producers compete among themselves, any one of them who raises the slope of the path $mcty$ by a slight amount, attains, for a short while at least, the desired result; he takes customers away from his competitors and he advances more or less into the region of positive indices. He could even stay there, if his competitors were not inclined to follow his example. If they do, and if competition is effective, they will in their turn raise the slope of the path with respect to mn , and so on. Thus, producers and consumers will gradually move in the direction of the arrow, and will come closer to point l , where the line hk of complete transformations intersects the exchange line cd . The producers cannot move beyond this line, because they would enter the region of negative indices, following the exchange line cd ; and they cannot go along lh , because the consumers would not follow them. Thus, they cannot but stay at l , which is a point of equilibrium, and of stable equilibrium.

144. Alternatively, we could say that lc is the only line of possible equilibrium. This is not the case with ld , since it is located in the region of negative indices. On the line cl , competition among producers has the effect of moving the equilibrium point toward l .

145. Let us now examine the case (π) . We shall see that, as before, ld is the only possible equilibrium line, since lc lies in the region of negative indices. If the producers are at d , they are well off, for they are in the region of positive indices; but competition among themselves makes the slope of md increase with respect to mx ,³¹ whence we move away from l . It is precisely at l that equilibrium could take place, because, there, the consumers as well as the producers are satisfied; but, as soon as we move away from l , on the side of h , instead of being drawn back to it, we are driven further and further away from it. On the side of k , we are drawn back to l . We have here a special kind of equilibrium; it is stable on one side, and unstable on the other.

No example of such an equilibrium will be found in Figure 18. If we compare case (β) of Figure 18 with case (μ) of Figure 23, we shall see that the conditions for stability of the equilibrium are precisely the opposite in the case (β), i.e., for exchange and production under incomplete competition, and in case (μ) i.e., for exchange and production under complete competition. This is so because in case (β) since the line hk is the exchange line (or maximum profit line), the individuals to whom the line refers stay there voluntarily; whereas in cases (μ) and (π), since the line hk is one of complete transformations, the individuals to whom it refers are driven toward it by competition alone.

146. In case (β), those who were at h stayed there because being well off they had no reason to move; movement was only caused by the consumer who wanted to reach c on his exchange line cd . In case (μ), on the other hand, this movement is due to the fact that those who are at k would like to improve their position by advancing along the path kc . In case (β), equilibrium is possible at d , and we move away from it because of those who wish to move to k ; in case (μ), it is not possible to stay at d , because at this point the producers incur losses, are ruined and disappear; we are thus led back to point l .

We have described the phenomenon as it evolves in the long run. It is always possible for producers to sustain losses for a short period of time.

147. Let us now see what happens when the number of producers acts upon the obstacles.

Let mo, mn be the axes for the producers, s, s', \dots ,^[a] the indifference lines, and cd , the consumers' exchange line. If there is only one producer, he will stop at the intersection l of the exchange line and the line hk of maximum profit. It will be the same if there are several producers on condition that their number does not affect the obstacles. Consequently, whether their number be small or large, they will all obtain maximum profit when the total quantity am of A is transformed into al of B.

148. Let us suppose instead that the line hk refers to the case of a single producer, and that other producers may emerge in the same conditions. If there are two producers, for each to obtain the maximum profit all the quantities must be doubled; if there are three, the quantities must be trebled, etc. If the line hk refers to total output, it will thus be shifted according to the number of producers. It would also be shifted, if, in a general way, output, instead of being doubled or trebled, had only to be increased in a certain proportion. The line s of complete transformations would also be shifted.

If by some extraordinary chance the lines thus shifted, when there are for example two producers, should intersect precisely at the point g of the exchange line cd , equilibrium will take place at g . For one of the producers cannot stay at l , because the other, to attract customers, will change the slope of path ml until it coincides with the path mg . He cannot go beyond, because he would then enter the region of negative indices, and there is no place for a third producer.^[a]

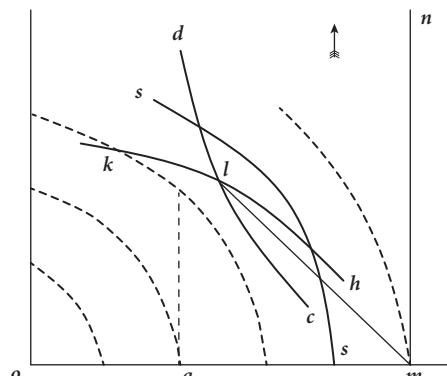


FIG 24

149. It could hardly ever happen that the lines of maximum profit and of complete transformations thus shifted, could intersect precisely on the exchange line. As long as the latter line intersects the line of maximum profit at a different point from which it intersects the line of complete transformations, equilibrium may take place at the point of intersection of the exchange line and the maximum profit line. But, since the producers are making a profit at this point, new competitors will emerge, provided of course this is possible, until the line of maximum profit no longer intersects the exchange line. When this takes place we find ourselves back in the case already dealt with (§141), and equilibrium will take place at the intersection of the exchange line and the line of complete transformations.^[a]

The same reasoning is applicable to the commodities of the second category (§102).

150. In conclusion, therefore, equilibrium takes place at the point where the line of maximum profit intersects the exchange line; but, when it is possible for new producers to emerge, and when the line of maximum profit is thus shifted so that it no longer intersects the exchange line, equilibrium takes place where the exchange line cuts the line of complete transformations. The first case occurs when competition is incomplete (§105); the second, when it is complete.

151. In the case of phenomena of type (II), if the producer acts according to this type, he will advance as far as possible into the region of positive indices and consequently the equilibrium point will be the point of tangency between the exchange line and an indifference line, in the case of complete competition (Figure 14); it will be the point of tangency between the exchange line and the line of maximum profit^[a] in the case of incomplete competition (Figure 13). This holds only, of course, when these points lie within the bounds of the phenomenon considered.

If the consumer operates according to type (II), he will force the producers to stay on the line of complete transformations. If the paths have to be straight lines starting from m , equilibrium in the case of complete competition will not differ from the equilibrium which obtains in the case of phenomena of type (I); but it could be different if it were in the consumer's power to change the shape of the paths (VI, 17, 18).

152. Prices. So far, we have been reasoning in a general way, trying not to make use of prices; we have had to mention them, however, when we wanted to cite concrete examples; even in the general theories, we have considered them more or less implicitly; in effect we have employed them without mentioning them explicitly. We shall now do well to accept them and discuss them without further ado; but it was useful to show that economic theories do not proceed directly from the consideration of a market where certain prices prevail, but rather from that of the equilibrium that arises from the opposition between tastes and obstacles. Prices appear as auxiliary unknowns, which are extremely useful for solving economic problems, but which must ultimately be eliminated leaving only tastes and obstacles facing each other.

153. The PRICE of Y in terms of X is the amount of X which must be given up to obtain one unit of Y.

When the price is constant, one may compare any amounts of X and Y, find the ratio between the amount of X given and the amount of Y received, and thus obtain the price. When prices are variable, one must compare infinitesimal amounts.

154. It follows from our definition of the price that, if we move from point c to point d by exchanging ac of A for ad of B, the price of B in terms B, the price of B in terms of

A will be equal to the slope of the line dcm referred to the oB -axis, and the price of A in terms of B is expressed by the slope of this same straight line referred to the oA axis.

155. In the preceding paragraphs, we have often spoken of increasing or decreasing the slope of mn with respect to one of the axes, e.g., oB ; this is synonymous with raising or lowering the price of B in terms of A.

156. [The value in exchange of the economists corresponds to the price just defined. Moreover it may be noted that the concept of price is more precise (§226). Besides, the economists distinguished between the *value* which was some fraction, for example $\frac{6}{3}$, and the *price*, which was a fraction whose denominator was unity, such as $\frac{2}{1}$.]³²

If 6 units of wine were exchanged for 3 units of bread, the *value in exchange* of bread in terms of wine would be $\frac{6}{3}$, and since, in this case, one has to give up 2 units of wine to obtain 1 unit of bread, the *price* of bread in terms of wine would be 2. There is no point in having two words to express things that differ as little as $\frac{6}{3}$ and $\frac{2}{1}$, especially since political economy has ceased to be a kind of literature and has become a positive science.

157. The economists used the concept of *value in exchange* to establish the theorem that a general increase in values was impossible, whereas a general increase in prices was possible. In the preceding example the value of bread in terms of wine was $6/3$, and that of wine in terms of bread was $3/6$. One need only have a smattering of arithmetic to understand that when one of these fractions increases, the other decreases, their product being always equal to one. Thus, if 12 units of wine are exchanged for 3 units of bread, the value of bread in terms of wine increases and becomes $12/3$, but the value of wine in terms of bread decreases, and becomes $3/12$. As for the price of bread in terms of wine, it increases and becomes 4 instead of 2.

158. The general concept of the price of a commodity in terms of another is useful in economics, since it abstracts from money. In practice, in civilized nations, the price of every commodity is expressed in terms of only one of them, which is called money; this is why, in dealing with concrete phenomena, it is very difficult to avoid speaking of price in that sense. Even in theory, it is very useful to introduce this concept at the very beginning. This has the drawback, it is true, of thus anticipating the theory of money, which should come after the general theory of economic equilibrium, but there is no great harm in that, especially if we compare it with the advantage in clarity and ease of exposition to which this concept leads.

159. Let us recapitulate the conclusions so far obtained, but use the general concept of price.

160. Phenomena of type (I) are those in which the individual accepts the prices he finds on the market, and in which he tries to satisfy his tastes at these prices. By doing

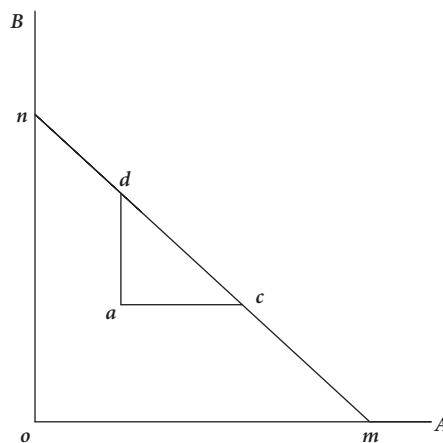


FIG 25

so, he contributes, without intending to, to altering these prices, but he does not act directly with the intention of changing them. At a certain price, he buys (or sells) a certain quantity of a commodity [; if the person with whom he transacts were to accept another price, he would buy (or sell) a different quantity of the commodity. Or, what amounts to the same thing, in order to make him buy (or sell) a certain quantity of a commodity, a certain price must be quoted.

161. Type II, on the other hand, consists of phenomena in which the individual's principal aim is to alter prices in order to derive a certain advantage. He does not leave the choice of different prices to the person with whom he bargains; he simply imposes a price, and leaves him only the choice of the quantity that he buys (or sells) at that price. The choice of price is no longer bilateral, as in type (I), but has become unilateral.

162. We have already seen that in the real world, type (I) corresponds to free competition (§46) and type (II) to monopoly.

163. Where there is free competition and nobody is privileged, the choice of price is bilateral. Individual 1 cannot impose his price on 2, or individual 2 on 1. In such a case the trader faces the following problem: "Given such a price, what quantity shall I buy (or sell)?" Or, what amounts to the same thing: "To induce me to buy (or sell) such and such a quantity, what would the price have to be?"

164. Where there is monopoly in any form, someone is privileged. He takes advantage of his privilege to fix the price the choice of which becomes unilateral. His problem is thus the following: "What price should I impose on the market, in order to achieve my purpose?"

165. Type (III) also corresponds to monopoly, but it is distinct from type (II) in that its aim is different. The problem which a socialist state will have to face is this: "How should prices be determined so that the people I administer can enjoy the maximum welfare that is consistent with their existing conditions, or with those that I feel should rightly be imposed upon them?"

166. It should be noted that, even if a socialist state were to suppress any possibility of exchange by prohibiting sales and purchases, prices would not vanish on that account; they would remain if only as an accounting device for the distribution and transformation of commodities. The use of prices is the simplest and easiest way to solve the equilibrium equations; if one persisted in not using them, one would probably end up by resorting to them under a different name. There would be a mere change in language, but none in substance.

167. Prices and the second kind of obstacles. We have seen that the data of the problem must include the rates at which successive portions of the commodities are transformed. If prices are introduced, this can be expressed by saying that we must specify the way in which the prices of successive portions vary. For instance, we may ascertain that all these portions have the same price, which, incidentally, may be unknown; or that their prices increase (or decrease) according to a certain law.

168. Several^[a] authors have fallen into error on this point, and it therefore deserves special attention. A fundamental distinction must be made with regard to price variations. There can be variation in the prices of successive portions purchased in order to reach an equilibrium position; and there can be variations in prices as between two entire processes leading to the [equilibrium position.]³³

(a) For instance, today an individual purchases 100 grams of bread at 60 centesimi per kilogram, another 100 grams at 50 centesimi per kilogram, and a further 100 grams at 40 centesimi per kilogram. He thus reaches an equilibrium position after having bought 300 grams of bread at different prices. Tomorrow, he repeats the identical process. In this case, the prices vary for the successive portions purchased to arrive at the equilibrium position, but they do not change when the process is repeated.

(β) Instead, tomorrow the same individual buys 100 grams of bread at 70 centesimi per kilogram, 100 grams at 65 centesimi, and 100 grams at 58 centesimi. The prices vary not only with respect to successive portions but also as between one process leading to equilibrium and another.^[b]

(γ) The same individual buys 300 grams of bread all at the same price of 60 centesimi per kilogram, and thus reaches the equilibrium position. Tomorrow, he repeats the same process. In this case the prices of the successive portions do not vary; and neither does it vary from one process leading to equilibrium to another.

(δ) Finally, this individual purchases 300 grams of bread today all at the same price of 60 centesimi per kilogram, and thus arrives at the equilibrium position. Tomorrow, to reach that position, he buys 400 grams of bread by paying a constant price of 50 centesimi for all the successive portions. The prices of the successive portions are constant in this case too, but the prices vary from one position leading to equilibrium to another.

169. The foregoing will be more easily understood by means of graphs.

In all the diagrams, *ab* and *ac* indicate the paths followed in the various purchases i.e., the prices paid for the various portions. In:

(a) Prices vary with successive portions, but their variations are repeated identically in the successive processes leading to equilibrium.

(β) Prices vary with successive portions, as well as with successive processes leading to equilibrium.

(γ) Prices are constant in successive portions, and in successive processes leading to equilibrium.

(δ) Prices are constant in successive portions, but they vary with the successive processes leading to equilibrium.

In the present state of the science, the general cases to be considered are (γ) and (δ), but there may come a time when it is worthwhile to consider cases (a) and (β) as well.^[a]

170. When a large number of persons come together in a market and act independently of one another and in competition, it is obvious that at a given moment some will buy the first portions, others the second portions, etc., and thus reach a state of equilibrium; and since there is only one price on a given market at a given moment, it will be seen that the prices of these various portions must be the same. Strictly speaking, this would not prevent the same individual from paying different prices for different portions; but such an assumption would lead to unusual consequences that would be entirely at variance from the real world, whence the hypothesis that best fits reality is that of equal prices for successive portions. This does not of course, prevent successive prices from being different, as in (δ), Figure 26.

This is especially true of consumption. If an individual purchases 10 kilograms of sugar, coffee, bread, meat, cotton, wool, nails, lead, paint, etc., he does not buy the first kilogram at one price, the second at another, and so on. This would not be impossible but as a rule this is not what happens. It should be noted, moreover, that it is quite possible

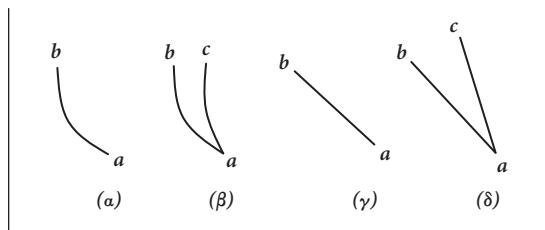


FIG 26

for this individual to buy 10 kilograms of onions at one price today and 10 kilograms at another price tomorrow, which corresponds to case (δ) Figure 26. In large cities it often happens that in a market, fish is more expensive in the early morning than just before noon when the market is about to close. The cook of a first-class restaurant may come early in the morning in order to have a wider choice, and purchase 20 kilograms of fish at a certain price. The cook of a second-class restaurant will arrive later and buy what is left over at a lower price. We are still in case (δ),^[a] Figure 26. Moreover, in the case we are considering we could, without serious error, reckon in terms of an average price. We should never forget that our aim is only to arrive at a general idea of the phenomenon.

171. In the field of speculative trading, it is nearly always necessary to assume that different portions are bought at different prices. If, for example, some bankers should wish to corner the copper market, they would have to take account of the advisability of buying the metal at a rising price; in such a case, to consider only an average price might lead to very serious mistakes.⁹ Similarly, if one were to try to make a special study of the auction processes of certain commodities, e.g., fish, one would have to take price variations into account. But all this would constitute a separate study of secondary phenomena. These modify the principal phenomenon which, in the last analysis, consists in the adaptation of consumption to production.

Moreover, the problem of speculation we have just discussed pertains more to dynamics than to statics. There are then several successive equilibrium positions to be considered. Apart from some exceptional cases, prices in large markets vary only from one day to the other, at least to a significant extent; and usually it is legitimate, without risk of serious error, to substitute an average price for the various actual prices.^[a]

172. When the price of the successive portions being traded is constant, the ratio of these quantities is also constant, i.e., if the first unit of bread is exchanged for two units of wine, the second unit of bread will also be exchanged for two units of wine, and so on. This phenomenon is depicted graphically by a straight line, whose slope with respect to one of the axes is the price (§153). Thus, by specifying the condition that the price is constant, this determines only that the path followed by the individual must be a straight line, but does not state which straight line it is. Let us imagine an individual possessing 20 kilograms of bread, which he wants to exchange for wine; if it is determined that the price is the same for successive portions exchanged, this determines only that the path to

⁹ This is why the attempt to corner the copper market in 1887–88 came to grief.

be followed is a straight line. On the axis measuring the quantities of bread, let the length om be equal to 20; the individual will then be able to follow any path chosen among the straight lines ma , ma' , ma'' , etc. If it should then be determined that the price of bread in terms of wine must be equal to 2, i.e., that two units of wine must be given up for one unit of bread, the straight line to be followed would then be fully determined. If we take oc to be equal to 40, this line will be mc ; it is only by following along this straight line, starting of course from m , that one unit of bread can be exchanged for two units of wine.

173. The angles oma , oma' , oma'' , ..., must all be acute, because the price is intrinsically positive; i.e., if an individual is to obtain something in an exchange he has to give up something else; consequently, in order that his stock of one commodity can be increased, he has to reduce his holdings of another commodity. If one of the angles oma , oma' , ... were obtuse, both quantities would increase simultaneously. If the angle oma were equal to zero, the price would be zero; no matter how much bread one had, no wine could be obtained. If oma were a right angle, the price would be infinite. For an only slightly smaller angle one would have price such that the very least quantity of bread could be exchanged for a huge quantity of wine. The angles oma , oma' , ... in the diagram stand for prices between these two extremes.

174. When the path to be followed is not given directly, but only in terms of the prices of successive portions, a calculation must be made to know the total quantities^[a] of commodities transformed.

Let us suppose that there are only two commodities A and B; that the price of B is expressed in terms of A, and that, for example, 1 kilogram of A is exchanged for a certain quantity of B at a price of $1/2$, then 2 kilograms of A for another quantity of B at a price of $1/3$, then 1 kilogram of A for another quantity of B at a price $1/4$. The quantities of B thus successively obtained will be 2, 6, and 4. Thus, in all, 12 kilograms of B will have been obtained at various prices in exchange for a total of 4 kilograms of A.

If there are more than two commodities, and if the prices of B, C, D, ... are expressed in terms of A, the total amount of A that has been transformed must obviously be equal to the sum of each portion of B, C, D, ..., multiplied by its respective price. These equalities indicate the point that is reached by following a certain path.

175. The budget of the individual. By selling things in his possession the individual obtains a certain amount of money; we shall call it his *income*. By purchasing objects for his use, he spends a certain sum of money; we shall call it his *expenditure*.

If one considers, for instance, the transformation of 8 units of A into 4 units of B, and if A is money, the price of B in terms of A is 2. Income is 8 units of A; the expenditure in terms of money is 4 units of B multiplied by the price 2 of B i.e., 8 units. Income is equal

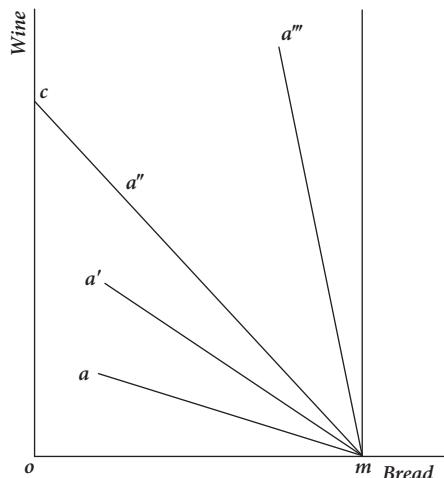


FIG 27

to expenditure, which means in this case that 8 units of A have been transformed into 4 units of B.

If there are more than two commodities, it is easy to see that the income must still be equal to expenditure, because otherwise it would mean that the individual received or spent money in another way than through the transformation of commodities. This equality between income and expenditure is called the **BUDGET OF THE INDIVIDUAL**.

176. The budget of the producer. The producer also has his budget. We have already referred to it, without mentioning it explicitly, when we discussed the transformation of one commodity into another.^[a] We saw that this transformation could leave a positive or a negative residual, which is actually a credit or debit item in the "profit and loss" account.

This is true of any transformation. The producer buys certain commodities, and incurs certain expenditures; this is the expenditure side of the budget. He sells the commodities he has produced; this is the income side of the budget. The locus of complete transformations is the one at which the budget balances without profit or loss.

177. The cost of production. By taking into account all the expenditures necessary to produce a commodity, and by dividing this total by the amount produced, one obtains the **COST OF PRODUCTION** of this commodity.^[a]

178. This cost of production is expressed in terms of money. Some authors have considered a cost of production expressed in terms of ophelimity. This is useless and only leads to ambiguities. We shall therefore never attach such a meaning to the term "cost of production."

[If a certain object A can be consumed directly, and one transforms it into another object B, the sacrifice made by forgoing the direct consumption of A may be considered as the cost in terms of ophelimity of B. But there are a great many cases in which A cannot be consumed directly; there are, then, strictly speaking, no direct sacrifices made in transforming A into B. To determine a cost in terms of ophelimity, one must change the meaning of that expression, and we say that if A can be transformed into B or C, the cost of production of B in terms of ophelimity is the pleasure forgone by transforming A into B instead of transforming it into C, and conversely.]

[One should never quarrel about words, and we can give any meaning we like to the expression: cost of production in terms of ophelimity. But it should be observed that the first meaning is essentially different from the second. The first distinguishes production from exchange; the second confounds them. The first really reveals a certain cost in terms of ophelimity; the second gives only one of the conditions which, with others, can determine that cost.¹⁰]

[For instance, suppose an individual possesses some flour and wants to transform it into bread. He may, by ignoring the costs of this transformation, consider the cost of bread in terms of ophelimity as equal to the pleasure forgone by not eating his flour in the form of dough. But, if he has to take into account every indirect use to which the flour can be put, it is impossible for him to have a unique object to which he can attach the name cost of production. This flour could be transformed into meat by feeding it to rabbits, turkeys, or chickens; it could be given as food to workers building a house

¹⁰ This is another of the innumerable and vain attempts to circumvent the necessity of solving a system of simultaneous equations (§219), to take vague account of the interdependence of economic phenomena and to conceal beneath a terminology lacking in precision one's ignorance of the solutions of the problems tackled.

or manufacturing a hat, gloves, and so on indefinitely. The consideration of this pseudo cost of production then leads simply to the recognition of the equality of the weighted ophelimities of the commodities consumed by the individual (§198).]³⁴

179. Not every commodity necessarily has its own cost of production. There are commodities which have to be produced jointly, for instance wheat and straw; they consequently have a joint cost of production.

180. Supply and demand. In political economy, it is customary to distinguish between the quantity of a commodity that an individual gives up on reaching an equilibrium point and the quantity he receives: the first is called his supply, the second his demand.

181. These two terms, like all terms in non-mathematical economics, have been used in a nonrigorous, equivocal, and ambiguous way, and it is truly incredible how many pointless, senseless, and jumbled discussions there have been about them. Even today, among non-mathematical economists, it is not easy to find authors who know³⁵ the meaning of these terms, which they nevertheless use at every turn.

182. Let us start by considering two commodities, and let us return to Figure 12.^[a] An individual possesses a quantity om of A and none of B; by following a certain path mn he arrives at the equilibrium point c'' by exchanging qm of A for qc'' of B; we shall say that, along this path, and at the equilibrium point $c''^{[b]}$, the individual's supply of A is qm , and his demand for B is qc'' .

183. It should be noted at once that these quantities would change if the shape of the path were to change, i.e., they depend upon obstacles of the second kind. Even when the shape of the path remains the same, e.g., when the path is a straight line, these quantities vary with the slope of the line, i.e., with the price.

184. Let us again refer to Figure 12. Given any price of A in terms of B, i.e., given the slope of mn with respect to om , the intersection of this straight line with the exchange line cc''' indicates the demand qc'' for B and the supply qm of A. The exchange curve may thus also be called the SUPPLY CURVE and the DEMAND CURVE. In Figure 20 the curve $masb$ is, for the first individual, the *demand* curve for B; and this demand is generally put into relation with the price of B in terms of A, as expressed by the slope of a path (e.g., me) on the oy -axis. It is also, again for the first individual, the *supply* curve of A; and this supply is generally put into relation with the price of A in terms of B (no longer to the price of B in terms of A), i.e., to the slope of a path (e.g., me) on the mo -axis.

185. In the case of two commodities, if the path is *assumed* to be a straight line, the demand for B thus depends solely on the price of B and the supply of A solely on the price of A.

186. One should be very careful not to extend this conclusion to the case of several commodities. The supply of a commodity depends upon the prices of every other commodity exchanged; the same is true of the demand for a commodity.

187. This is not all. We assumed the point of equilibrium to be at c , in Figure 7;^[a] it might instead be the terminal point a . In that case the quantity of A supplied would be rm and the quantity of B demanded would be ra ;^[b] these quantities would depend upon the position of the point a , i.e., upon the obstacles.

In general, supply and demand depend upon all the circumstances of economic equilibrium.

188. When only two trading individuals are considered, one of them supplies A and demands B; the other supplies B and demands A. We saw (§117) that an equilibrium

point in the exchange between two individuals is reached at the point at which the two individuals' exchange curves intersect. In terms of the new names we have just given these curves, we may then say that an equilibrium point is one at which the first individual's demand curve for B intersects the second individual's supply curve for B; or, which comes to the same thing, an equilibrium point is one at which the first individual's supply curve for A intersects the second individual's demand curve for A. Or, to put it in still another way, a point of equilibrium is one at which the demand for one of the commodities is equal to the supply.

189. This proposition was known in non-mathematical economics, but without there being a precise idea of it, and in particular, without the knowledge of the conditions under which alone the theorem is valid, nor of the restrictions it implies. Even today, most of those who call themselves economists are unaware of these questions.

Then there are also people who assert that "the mathematical method has so far failed to formulate any new truth;" this is true in a certain sense, because something of which an ignoramus has not the slightest idea can be neither true nor new to him. When one is not even aware of the existence of certain problems, one hardly feels the need to look for their solution.

190. For the producer, supply and demand have no meaning unless an additional condition is stated which determines in which part of the region of possible equilibrium one wishes to settle. For the preceding theorem to remain valid in the case of production, in the case of phenomena of type (I) under complete competition, one can impose the condition that supply and demand are restricted to being on the line of complete transformations.

191. Should one then want the theorem of equilibrium, on account of the equality of supply and demand, to apply also to commodities for which there is a line of maximum profit, as in § 105, one would have to give a different meaning to supply and demand, and relate them to that line.

192. In the case of several individuals and several commodities, it is clear that by summing the demand for each commodity over all the individuals, one obtains the total demand for each commodity; similarly for supply.

193. The mode of variation of supply and demand has been called the "law" of supply and demand. This will be discussed in another chapter. For the moment, all we need to know is that in the case of two commodities, when the price of one commodity increases, the demand decreases, whereas the supply increases at first and may then decline.

194. When we consider a path mc' in Figure 15^[a] which leads to a point c' on the line of complete transformations, the slope of the straight line mc' with respect to the mb -axis, on which the quantities of commodity B are measured, is equal to the cost of production of commodity B, obtained through the complete transformation at c' . And if c' also lies on the exchange line, that slope measures the selling price. It follows from this that at the points of intersection c, c' of the exchange curve with the curve of complete transformations, i.e., at the equilibrium points, the cost of production is equal to the selling price.^[b]

195. We have seen that equilibrium could be stable or unstable. We now give an explanation of this, in terms of the concepts of price and of supply and demand.

Let individuals who exchange be at a point of equilibrium; let us suppose that the price of B rises, and see what happens.

The first individual, who sells A and buys B, reduces his demand for B; the second individual may increase or decrease his supply of B. Two cases must be distinguished; (1) the supply of B increases, or decreases but in such a way as to remain in excess of the demand for B. The phenomenon then proceeds as at the two points (α) and (γ) in Figure 18.^[a] (2) The supply decreases in such a way as to become less than the demand. This is the case with the point (β) in Figure 10. Essentially, one needs only check whether the supply is larger or smaller than the demand at the new price. In the first case, equilibrium is stable, for the seller is induced to lower his price in order to adjust his supply to the demand. In the second case equilibrium is unstable, because the buyer is not satisfied and he must be content with the smaller supply available; consequently, he will raise his price in order to obtain a larger quantity of the commodity, but he is mistaken, since he ends up getting less.

Similar observations may be made in the case of production; it would be very easy to translate what was set forth in §§140, 141, 142, into the new language.

196. Equilibrium in the general case. So far, we have discussed mainly the case of two individuals and two commodities; we now have to deal with the equilibrium of any number of individuals and any number of commodities.

In this chapter, we shall restrict ourselves to considering the general case of equilibrium for phenomena of type (I), with complete competition.

Let us assume that a state of equilibrium has been reached, i.e., a point where certain quantities of commodities are transformed, through exchange or otherwise, indefinitely, at certain prices. Let us try to determine these quantities and prices. This case is represented graphically by (γ) in Figure 26, and we assume that the operation described by (γ) is repeated indefinitely. A certain individual exchanges, for instance, 10 kilograms of bread for 5 kilograms of wine, thus reaching an equilibrium position; and he repeats this process indefinitely.

In type (I), the individual is guided only by his own tastes, and accepts the market prices as he finds them. For his tastes to be satisfied by the exchange just mentioned, it should not be to his advantage to move farther out nor to stay closer in. Suppose the price of wine in terms of bread is 2. If the individual continues with the exchange and gives up another 10 grams of bread, he will receive 5 grams of wine. If the ophelimity (or ophelimity index) of these 10 grams of bread were less than that of the 5 grams of wine, it would be to the individual's advantage to add this exchange to the one already carried out [of 10 kilograms of bread for 5 kilograms of wine].³⁶ Were the ophelimity of these 10 grams of bread greater than the ophelimity of the 5 grams of wine, it would be to his advantage not to exchange all 10 kilograms of bread for the 5 kilograms of wine, but to exchange only 9.990 kilograms of bread for 4.995 kilograms of wine. Thus, if the ophelimity of these 10 grams of bread at the equilibrium point is not to be greater or smaller than that of the ophelimity of the 5 grams of wine, it can only be equal to it.

197. It must be added that for this reasoning to be rigorous the quantities would have to be infinitesimal. When they are finite it cannot be said that the ophelimity of 10 grams of bread added to 10 kilograms of bread is equal to the ophelimity of 10 grams of bread. One might, however, simply reason by approximation and consider an average. But we need not dwell further on this; in one way or another, we have an idea of the phenomenon.

198. For very small quantities, we may assume that the ophelimity is proportional to the quantities. The ophelimity of the 5 grams of wine will thus be about half that

of the 10 grams of wine (it would be exactly half if we were considering infinitesimal amounts). It may thus be said that equilibrium requires the ophelimity of a very small quantity of bread to be equal to half that of the same very small quantity of wine. The elementary ophelimity (§33) of bread should therefore be equal to half the elementary ophelimity of wine. That is, remembering that the price of wine is two, we may again say that the weighted elementary ophelimities (§34) of bread and wine should be equal.

In this form, the proposition is general for type (I) and applies to any number of individuals who are guided directly by their own tastes (§41), the pleasure obtained from the consumption of each commodity is independent of the amounts consumed of the others (IV, 10, 11). In that case, each individual compares one of the commodities, say A, to the others B, C, D, . . . ; and he stops undertaking his transformations when the weighted ophelimities of all these commodities are equal for him. Thus, for each individual, there are as many conditions as there are commodities, minus one. If, for instance, there are three commodities; A, B, and C, we say that the weighted elementary ophelimity of A is equal to that of B, and also to that of C, which gives us precisely two conditions.

199. This category of conditions expresses the fact that each individual satisfies his tastes DIRECTLY (§41) as far as the obstacles allow him to do so. To distinguish them from the others we shall call it the category (A) of conditions.

200. Another category of conditions, which we shall denote by (B), is obtained by drawing up the budget of each individual (§175). The number of conditions in this category is thus equal to the number of individuals.

By summing up all the individual budgets we obtain the budget of the community, which is made up of the residuals which remain after the sales and purchases of each commodity are offset against one another. Thus, some individuals have sold a total amount of 100 kilograms of oil, and if the others have bought 60 kilograms, the community has in all sold 40 kilograms of oil. All these residuals, multiplied by their respective prices, must balance. For instance, if the community has sold 20 kilograms of wine at 1.20 lire per kilogram, and 60 kilograms of wheat at .20 lire per kilogram, it will have obtained 36 lire from its sales; and if it has bought only oil, since its income must equal its expenditure it must have spent 36 lire on the oil. Consequently, if the prices and quantities of all but one of the commodities bought and sold by the community are known, conditions (B) will enable us to determine the quantity of the commodity omitted.

201. Let us enumerate the conditions we have obtained. If, for instance, there are 100 individuals and 700 commodities, category (A) will yield 699 conditions for each individual and 69,900 conditions for 100 individuals. Category (B) will yield 100 additional conditions. In all, we shall thus have 70,000 conditions. In general this total is equal to the number of individuals multiplied by the number of commodities.

Let us enumerate the unknowns. If one of the commodities serves as money, there are 699 prices for the other commodities. For each individual, there are the quantities of each commodity he receives (or gives up); we thus have a total of 70,000 quantities. Including the prices, we have 70,699 unknowns.

By comparing the number of conditions, 70,000, with the number of unknowns, 70,699, we shall soon see that, in order for the problem to be fully determinate (§38), 699 conditions are needed; in general, as many as there are commodities, minus one.

202. These must be obtained by considering the obstacles. In exchange, the obstacles consist, apart from the conflict among individuals' tastes which has already been taken into account in conditions (A), simply in the fact that the total quantities of commodities are constant, since what is given up by one individual is received by the others; and in all, for each commodity, the sales of the community exactly offset the purchases. But conditions (B) furnish the total quantity of a commodity that is sold, or bought, by the community, when the corresponding amounts of the other commodities are known (§200); it will therefore be sufficient to impose the condition for all the commodities but one, i.e., for 699 commodities that the excess of the community purchases over its sales be equal to zero, since from conditions (B) we know that this excess is also zero for the last commodity.

We thus have a new category, which will be called (C), of conditions relating to the obstacles.

203. Six hundred and ninety-nine conditions were missing; category (C) consists of precisely 699 conditions. The number of conditions is now equal to that of the unknowns, and the problem is completely determinate.

204. We might have said of the 700 commodities that, for the community, the quantities sold were equal to the quantities bought, so that there was a zero residual for each of the 700 commodities. We would thus have had one additional condition in category (C); but, as against this, we would have had one less in category (B). Indeed, when the quantities of all the commodities are known, it is sufficient to have the budget of all but one of the individuals in order to have that of this omitted one as well. What he receives is, of course, equal to what the others together give up; and what he gives up is equal to what the others together receive.

205. Let us consider production. Suppose that, of the 700 commodities, 200 are transformed into 500 others whose cost of production we shall compute. In the case of complete competition, equilibrium can only take place where the cost of production is equal to the selling price. For if the cost is higher, the producer incurs losses and has to drop out; if the cost is lower the producer makes a profit and others will enter to share in it. We thus have a category of conditions which we shall call (D), requiring that the cost of production of each of the 500 commodities produced be equal to the selling price.

206. In the case of exchange, we had to require that the total quantities of all but one of the 700 commodities remain constant. In the case of production, this is no longer necessary, but instead we must require that 200 commodities have been transformed into 500 other ones, i.e., that the quantities of the former that have been used up have been substituted for the quantities of the latter that have been produced. For reasons analogous to those just mentioned, it is sufficient to require this condition for the 200 commodities, less one. We thus have a new category of conditions (E); [and in the present case these consist of 199 conditions.]³⁷

The conditions in this category require equilibrium to take place on the line of complete transformations.

207. Summing up the number of conditions (D) and of conditions (E), we have altogether 699 conditions, i.e., precisely the number we were lacking, and the problem is fully determined.

208. In the case of phenomena of type (I), with complete competition and constant prices for successive portions in a single process, we may therefore state the following theorem:

A point of equilibrium is one such that the following conditions are fulfilled: (A) for each individual, equality of the weighted ophelimites; (B) for each individual, equality of income and expenditure. Furthermore, in the case of exchange: (C) for every commodity, equality between the quantities available before and after the exchange. On the other hand, in the case of production the above conditions are replaced by the following: (D) equality of the cost of production and the selling price for every commodity produced; (E) equality between the quantities of commodities demanded for the transformation and the quantities of these commodities actually transformed. (1906 Appendix 26, 28; 1909 Appendix, 24, 63, 80, 83).^[a]

209. Moreover, one among the conditions (B) and (C) is superfluous, and the same is true with respect to conditions (B), (D), and (E).

210. Let us choose at random a commodity A, to be used as money; the prices of all commodities will thus be expressed in terms of A. Furthermore, as we have done previously (§198), let us compare the other commodities with A one by one, and suppose that, for each individual, we have the indifference lines as between A and B, A and C, etc. The possible equilibrium points are those where the indifference curve as between A and B has a tangent whose slope with respect to the *ob*-axis is equal to the price of B in terms of A; similarly the slope with respect to the *oc*-axis of the tangent to the indifference lines as between C and A must be equal to the price of C in terms of A; etc.^[a]

211. We thus have conditions analogous to those obtained in the case of two commodities. But, in the latter case we know *a priori* the distance *om* in Figure 12, i.e., the quantity of A initially possessed by the individual. In the case of several commodities, on the other hand, *om* is unknown; it is that part of A which the individual transforms into another commodity, say B. The category (A) of conditions thus simply expresses the fact that equilibrium is possible at the points where the tangent to the indifference curve as between an arbitrary commodity and commodity A has a slope, on the axis measuring the quantity of that arbitrary commodity, equal to the price of that commodity.

212. Category (B) of conditions indicates, in the case of two commodities, the path followed by each individual. If there are three commodities, one can still draw a geometrical representation of conditions (B) by measuring the quantities of these commodities along three orthogonal axes. One of the budgets (B) represents a plane on which the exchange or the transformation takes place. Similarly it may be said in the case of more than three commodities that each budget (B) indicates the locus of transformations performed by the individual whose budget is referred to.

213. Conditions (C), in the case of two commodities and two individuals, reduce to one, namely that the quantity of A given up by one individual is received by the other. It is by virtue of this condition that if we draw the indifference curves of both individuals as in Figure 16, the path followed by each of them is represented by a unique straight line *[mc]*.³⁸

214. Let us examine the correspondence between the conditions relating to obstacles and those relating to producers. In the case of two commodities, conditions (D) reduce to a single one which indicates that the price of the commodity is equal to its cost of

production. Conditions (E) also reduce to one, which is that there is no residual of A, i.e., that equilibrium takes place on the line of complete transformations.

215. Equilibrium may be stable or unstable. Suppose we drop the equations of category (A) referring to the first individual, i.e., we no longer pay attention to whether this individual's tastes are satisfied; his budget continues to be balanced since conditions (B) all continue to hold. The number of equations that have been dropped from category (A) is equal to the number of commodities, minus one (\$198); this is also the number of prices. It follows from this that, when we allow the tastes of one individual in the community not to be satisfied, we can fix the prices arbitrarily.

216. This demonstration was necessary to show that the operation we are about to perform is possible. Let us suppose that an equilibrium position has been attained for all the members of the community; let us modify the prices slightly and restore the equilibrium for every individual, except the first one. This is possible, as has been demonstrated above.

After this operation, every individual is satisfied, except the first one. It should now be noted that the latter individual successively compares all the commodities with one of them—A in our case; and that, since we are considering phenomena of type (I), he compares only the ophelimity he enjoys at the different points on each path. For A and B, for A and C, etc., we shall thus have phenomena like those of (α), (β) and (γ) in Figure 18 which we have so often discussed, and similar cases of stable and unstable equilibrium. In other words, the individual considered receives and gives up, at the new prices, certain quantities of commodities which are larger or smaller than those which, for him, correspond to equilibrium. He will thus strive to return to his equilibrium position, which he will not be able to do except by changing the prices at which he buys and sells. By so doing, he may get closer to the equilibrium position from which we supposed him to have been displaced; or he may move away from it. The first case is one of stable equilibrium; the second, one of unstable equilibrium. For equilibrium to be stable for the community, it must of course be so for every individual member of it.

217. The conditions for economic equilibrium we have enumerated provide a general idea of this equilibrium. In order to know what certain phenomena were, we have had to study how they came about; in order to know what economic equilibrium is, we have tried to see how it was determined. It should be noted, moreover, that the purpose of such a determination is not in the least to arrive at a numerical computation of prices. Let us make the most favorable hypothesis for such a computation; let us suppose that all difficulties regarding knowledge of the data of the problem have been overcome, and that the ophelimities of every commodity for each individual are known, as well as all the conditions of production of the commodities, etc. This is already an absurd hypothesis; and yet it is not enough to give us the practical possibility of solving the problem. We have seen that, in the case of 100 individuals and 700 commodities, there would be 70,699 conditions (in fact, a large number of conditions, so far disregarded, would increase that number still further); we would thus have to solve a system of 70,699 equations. That would practically exceed the power of algebraic analysis, and it would do so still more if one were to consider the incredible number of equations that would be needed for a population of forty million individuals and some thousands of commodities. In such a case, the roles would be reversed; it would not be mathematics that would come to the aid of political economy, but political economy that would come to the aid of mathematics.

In other words, if all these equations could really be known, the only humanly possible way to solve them would be to observe the practical solution brought about by the market [by means of certain quantities and certain prices].³⁹

218. But if the conditions we have just enumerated cannot be of practical use in numerical computations of quantities and prices, they are the only means known so far for arriving at a conception of how these quantities and prices vary, and better still, of how, in general, economic equilibrium comes about.^[a]

219. Under the pressure of facts, even those economists who were unaware of these conditions have had to take them into account. We may summarize their work by saying that they tried to solve a system of equations without using mathematics; and since this is impossible, they had no other means of escape than to resort to expedients, some of them very ingenious indeed. In general, they proceeded as follows: they supposed, more or less implicitly, that all the conditions (equations) except one were satisfied, and thus only one unknown remained to be determined, by means of known quantities. This problem was not beyond the power of ordinary logic to solve.¹¹

Instead of one single condition, one may also consider only a single category of conditions (equations) that determine equilibrium, because these conditions being similar, ordinary logic is able to deal with them, though not very rigorously, as if they were a single equation.

[The following is an example of nonsensical phrasing, of a kind that is still used in literary economics: "If one imagines a condition of full and free competition, the degree of limitation—as well as the cost of substitution and the degree of marginal utility—will be identical with the degree of qualificative^[a] limitation, i.e., with the cost of production."]

[This seems to mean something, but it makes no sense. The author does not say exactly what he means by degree of limitation. Moreover, he has a very vague idea of something he calls *cost of production*, and which is not at all the monetary cost. He has a glimpse of something else: *marginal utility*. And by association of ideas, he establishes an identity which exists only in his imagination.]

[Of course, such a mode of reasoning can only lead to error. In fact, what he tells us is this: "If one considers the value of a good in a single exchange, one can only say that the price of this good is determined by its degree of quantitative limitation."]

[Let us apply this theory to a concrete case. Let us imagine a traveler in the center of Africa, having in his possession a score of *La Traviata*, which is the only one in that area; its "degree of quantitative limitation," if this expression means anything, must thus be very high. Nevertheless, its price is zero, for the negroes with whom our traveler is in contact do not appreciate this commodity in the slightest.]

[We have retrogressed. Phaedrus and La Fontaine were better economists. The cock that found the pearl already knew that in addition to the consideration of "quantitative limitation," there is a question of taste:]

*Ego quod te inveni, potior cui multo est cibus,
Nec tibi prodesse, nec mihi quidquam potest.*⁴¹

¹¹ This is what I argued for the first time in the *Giornale degli Economisti*, September, 1901.⁴⁰ See also *Systèmes*, II, p. 288 ff.

[As to the ignoramus in La Fontaine, it may be that the manuscript he inherited had a high degree of "quantitative limitation," and that it was the only one of its kind; but, if nobody was interested in the manuscript, this ignoramus would not have obtained a ducatoon for it.

[Some have tried, at least, to find a limit to prices, and it has been said that "nobody would agree to pay for a commodity more than it would cost him to produce it by himself."

[If this proposition is interpreted literally, there can only be a question of cost in money [numéraire], since one cannot compare two heterogeneous quantities: a price and sacrifices. Let us leave aside until later (§224) the error involved in assuming a cost of production that is independent of prices. We shall merely note that even if this proposition were true, it would be useless in most cases, for among the commodities we consume there are almost none that we could produce by ourselves, and those few that we could produce would cost us vastly more than the price at which we buy them. How would you set to work to produce by yourself, directly, the coffee you drink, the cloth you wear, the newspaper you read? And what would be the price of one of these commodities if, however implausibly, you were to produce it directly?

[Literary economists, seeking at all costs to avoid the study of the totality of conditions determining economic equilibrium, have attempted to simplify the problem by changing the meaning of the expression "cost of production," and by substituting for the cost of production in terms of money [numéraire] a cost in terms of sacrifices, the meaning of which is vague and indeterminate and lends itself to all kinds of interpretations.

[Let us imagine an individual with a garden where he can grow strawberries; it is said that he will obviously not be willing to pay a price for strawberries that involves a greater sacrifice for himself than he would make by producing them directly. This proposition, which is intended to avoid the complexity of the economic phenomenon, is simple only in appearance; if one wants to make it more precise, the complexity that one tried to evade reappears. How can we evaluate the "sacrifices" made by the individual growing his strawberries? Will it be the pain he undergoes, plus the expenses incurred? I do not know how one would go about aggregating these heterogeneous quantities. But let that pass. Let us assume that this sum can somehow be calculated. We have in this way isolated the production of strawberries for our individual from the rest of the economic phenomenon. But in this sense the proposition is false. If the owner of the garden is a talented painter, in one day's work he will earn enough to buy more strawberries than he could produce by working six months in his garden; he is thus better off if he paints, and buys his strawberries for much more than they would "cost" him (IX, §§42ff.).

[To make our proposition true, one would have to change the meaning of the term *cost*, and say that our individual should consider not the pain directly undergone in producing strawberries, but the advantages he forgoes by devoting his time to growing strawberries instead of using it otherwise. But, in that case, the production of strawberries is no longer isolated from the rest of the economic phenomenon. The proposition we have set forth is no longer sufficient to determine the price of strawberries; it means only that any individual tries to make the most advantageous use of his labor and of the other factors of production available to him. This leads simply, in this case, to setting down some of the conditions (equations) of economic equilibrium and precisely the conditions we have labelled (A) (§199).

[We could continue in this way, and try to remove the difficulties pointed out at the beginning. Some may object that a man is unable to produce most of the commodities he consumes. Very well. Let us carry out the same operation for the commodities consumed by the individual as for the factors of production at his disposal. Let us not expect him to produce his watch directly; the poor fellow would never be able to manage. Let us call "cost of production" the pleasure he forgoes in other directions when he spends his money on a watch instead of buying something else. Provided one is sufficiently fair to the reader to warn him clearly that the term "cost of production" is used in this strange sense, one may then say that the price paid for a watch is such that it represents a pleasure equal to the "cost of production" of the watch. But we shall thus obtain simply the equations that were lacking to complete the set of equations (A), some of which were already obtained in considering the factors of production. We will thus have constructed a theory of exchange, whereas we were apparently trying to construct a theory of production; and it is with the purpose of putting the reader off the track that one has unconsciously altered the meaning of the expression "cost of production" in such a strange way.

[We have dwelt at some length on this proposition of literary economics, not because it is worse than others, but only to show an instance, chosen at random, of the deplorably vague and erroneous way in which such questions are still treated, and of the absurdities that are commonly taught in the name of economic science.]⁴²

220. Let us consider only the category (A) set out in §208, and let us assume that all the other categories of conditions are automatically satisfied. In this case, we may say that prices are determined by ophelimity, since it is precisely category (A) which establishes the equality of the weighted ophelimities. Or by using the language of the economists who approach the problem in this way, we could say that the *values* are determined by the *utilities*, or, again that utility is the *cause* of value.

221. Let us, instead, consider only the category (D) conditions of §208, and assume that every other set of conditions is automatically satisfied. In that case, we shall be able to say that prices are determined by the equality between the cost of production and the selling price of each commodity.¹²

If we wish to take into account the fact that the commodities considered are those that can be produced at this cost at the moment when equilibrium is established, we shall speak of the cost of *reproduction* instead of the cost of production.

Ferrara^[a] went further; he considered the cost of producing, not a commodity, but a sensation;¹³ this led him to take into account, albeit imperfectly, not only conditions (D), but also conditions (A). When one realizes that he succeeded in this without the valuable help of mathematical considerations, which make the problem so simple, one must admire the truly extraordinary power of his intellect. No other non-mathematical economist has gone farther than he.

222. Let us consider categories (A) and (B); they enable us to infer the quantities of commodities as determined by their prices (the quantities as a function of the prices), i.e., what economists have called the *laws of supply and demand*. And if, as above, we assume that the other categories of conditions are automatically satisfied, we can say that the quantities are determined by the prices, through the laws of supply and demand.

¹² *Cours*, I, 80.

¹³ *Ibid.*

Non-mathematical economists have never had a clear idea of these laws. They have often spoken of the supply and demand of a commodity as if they depended on the price of that commodity alone.¹⁴ When they realized their mistake, they corrected it by speaking of the *purchasing power* of money, without even knowing exactly what that entity was.

223. Moreover, since they did not have a clear idea of the fact that demand and supply were precisely the result of conditions (A) and (B), they considered demand and supply as quantities independent of these conditions; whence arose the problem of whether the desire that an individual has for something he cannot afford is to be considered as part of the demand, or of whether the quantity of a commodity available on a market which its owner does not wish to sell should be considered as part of the supply.

Thornton¹⁵ considers a case in which there are a certain number of gloves to be sold, and assumes that they are being sold at successively lower prices until all of them are disposed of. He admits that the quantity supplied is the total number of gloves, and observes that only the last portion is sold at a price which makes supply and demand equal, "by far the largest part being sold at prices at which supply and demand would be unequal."⁴³ He confuses the equilibrium point at which supply and demand are equal with the path followed in order to arrive at this point, a path along which supply and demand are not equal (§182).

224. The cost of production was also conceived by [literary]⁴⁴ economists as a certain normal price around which the prices determined by demand and supply had to gravitate; in this way they came to take account, albeit very imperfectly, of the three categories of conditions (A), (B), and (D). But they considered them as independent of one another, and it seemed as if the cost of production of a commodity was unrelated to the prices of this and other commodities. It is easy to see what a glaring error this was. For instance, the cost of production of mineral coal depends on the price of machinery, and the cost of production of machinery depends on the price of this very same coal. And it depends on it even more directly, when one considers the consumption of coal by the machines used in the mine.^[a]

225. Price or *value in exchange* is determined at the same time as economic equilibrium, and this arises from the conflict between tastes and obstacles. Anyone looking at one side only and considering only tastes, believes that these alone determine prices, and sees the *cause* of value in *utility* (ophelimity). Anyone looking at the other side and considering only the obstacles believes that only these determine prices, and he finds the cause of value in the cost of production. And if, among the obstacles, he considers only labor, he finds the *cause* of value exclusively in labor.

If, in the system of conditions (equations) which, as we have seen, determine equilibrium, we assume that every condition is satisfied automatically except those regarding labor, we shall be able to say that value (price) depends upon labor alone; this theory will not be false, but incomplete. It will be true provided the assumptions are fulfilled.^[a]

¹⁴ Cairnes, *Some Leading Principles of Political Economy*, chapter II. "Supply and demand, when spoken of with reference to particular commodities, must[, if our statements are to be significant,] be understood to mean Supply and Demand at a given price . . ."^{42[a]}

¹⁵ *On Labour* [, p. 70 of the Italian translation].^{43[a]}

226. The conditions that were, often unconsciously, ignored or shunted to one side, came back by themselves, because after the problem had been solved, it was often intuitively felt that they had to be taken into account. Thus Marx, in his theory of value, tried to eliminate—by using averages or other means—conditions that he had had to disregard in order to make value depend on labor alone.¹⁶ Thus, to many economists, the term *value in exchange* means not only a relation, the ratio of exchange of two commodities, but in addition, and somewhat imprecisely, certain ideas of purchasing power, of equivalence of commodities, and of obstacles to be overcome. The outcome of all this is a somewhat badly defined entity, which for that very reason may include certain ideas of conditions that had been ignored but which it is nevertheless felt should be taken into account.

[All this is obscured by the vagueness and the lack of precision of the definitions, by a babble of words which appear to mean something, but which in fact have no content.]⁴⁶

So many vague and sometimes contradictory meanings have thus been given to the term *value*, that it is better not to use it in the study of political economy.¹⁸ That is what Jevons did by using the expression *exchange ratio*; and it would be better still as did Walras, to use the concept of the price of a commodity B in terms of a commodity A (§153).

[A certain exchange takes place: 1 unit of A is exchanged for 2 units of B. In this exchange, the price of A in terms of B is 2. This is a fact; and it is out of these facts that economic science sets out to construct a theory.

[Several authors put a good deal more into the notion of what they call *value* than is included in the notion of price; that is, to the facts of the past, they add a forecast of the future. They say that the value is 2 if one can normally exchange 2 units of B for 1 unit of A.

[They express themselves less clearly, because all these theories have to remain vague in order to conceal the errors they contain; but this is what lies at the basis of their thinking.

[It should first of all be noted that, given the above concept of value, commodities in wholesale markets would hardly ever have any “value,” for their price varies from one purchase to another; the opening price on the market often is very different from the closing price.

[Some authors try to conjure away this difficulty by distinguishing between *value* and its magnitude! As if a quantity could exist independently of its size! Besides, even if this were admitted, such a metaphysical entity would be a perfectly useless thing to consider. In fact, in this way the conditions one is unable to take account of in the determination of economic equilibrium are thrust aside into the vagueness of a definition.

[Furthermore, in constructing a theory one should never confuse the facts that this theory is to explain and the predictions that can be based on it. The wholesale prices

¹⁶ In a recently published book, we read that “price is the concrete manifestation of value.” After the incarnations of Buddha, we now have the incarnations of value!

[What can this mysterious entity really be? It is, so it appears, “the capacity of a good to be exchanged for other goods.” This amounts to defining something unknown in terms of something still less well known; for what can this “capacity” mean? And—what is even more important—how is it measured? Of this “capacity” and its homonym “value,” we know only its “concrete manifestation,” that is to say, its price. There is therefore really no point in cluttering ourselves up with these metaphysical entities; let us content ourselves with prices.]^{45[a]}

¹⁷ *Systèmes* I, pp. 338; II, p. 121 ff.

¹⁸ *Systèmes* II, ch. XIII.

of copper on the London exchange are facts; a theory incorporating them has to be constructed before there can be the slightest hope of predicting what they will be in the future; and, as yet, such a forecast is absolutely impossible. There is nothing real, apart from these prices, that could be considered as the "value" of copper. If those who have no scientific conception of economics think otherwise, it is because they have a vague glimmering that, if certain prices have been quoted in London for copper, and if it is likely that other unspecified prices will be quoted in future, this is because copper is used indirectly to satisfy human tastes, and because there are obstacles in the way of obtaining it. These concepts, which science makes precise, have for such persons only a vague and indeterminate meaning, which they associate with the word *value*, in order to give them a name.

[There is no real entity corresponding to what literary economists call *value* which depends objectively on the thing considered, such as the density or any other physical property of that thing. Neither does such an entity exist in the form of an "estimate" of the thing that one or several individuals might make of it. Nor, to give it substance, is it sufficient to consider certain obstacles to its production.

[If the vague and indeterminate thing that literary economists call *value* bears any relation to prices, we say that it depends upon *all* the circumstances, without exception, which influence the determination of economic equilibrium.

[What is the *value* of diamonds? You cannot answer that particular question either by considering the desires which they arouse in men and women, or in considering the obstacles encountered in their production, or the valuations derived from these desires and obstacles, or the "quantitative limitations," or the cost of production, or the cost of reproduction, etc. All these circumstances influence the price of diamonds, but taken in isolation or even in a group, they are not sufficient to determine it.

[For instance, toward the end of 1907, no notable change had occurred in the circumstances just enumerated; nevertheless, the price of diamonds dropped, and would have fallen further still had it not been supported by the syndicate which held the monopoly. The crisis was so acute that the main diamond producers, the De Beers and the Premier Companies, suspended the distribution of dividends. What circumstance had so suddenly changed the value of diamonds? Merely the financial crisis in the United States and in Germany. These countries, which were large buyers of diamonds, almost completely suspended their purchases.

[To explain and foresee such phenomena, the literary economists' metaphysical theories are of no use, whereas the theories of scientific economics are perfectly adapted to these facts.]⁴⁷

227. The thing indicated by such names as value in exchange, exchange ratio, price, does not have *one* cause; and we may henceforth affirm that any economist looking for the cause of value shows thereby that he understands nothing of the synthetic nature of economic equilibrium.

Formerly, it was generally believed that there had to be *one* cause of value; the discussion was concerned only with what that cause was.

It is interesting to note that the power of the opinion according to which there had to be one cause of value was so strong that not even Mr. Walras was able to escape its influence entirely, although he himself had helped to demonstrate the error of this opinion by providing the conditions of equilibrium in a particular case. He expresses two

contradictory ideas, On the one hand, he tells us that “all the unknowns in the economic problem depend upon all the equations of economic equilibrium,” and this is a sound theory. On the other hand, he asserts that “it is certain that *rareté* (ophelimity) is the cause of value in exchange;” this is a reminiscence of faded theories that do not correspond to the real world.¹⁹

These blemishes may be excused, indeed they are natural in a period when new and better theories are being substituted for inaccurate ones; but they could not be excused now that these new theories have been constructed and improved.

228. In short, theories that relate value (price) to *the final degree of utility* (ophelimity) alone, are not very useful in political economy. The most useful theories are those that consider economic equilibrium in general, and that investigate the way it arises from the conflict of tastes and obstacles.

[It is the mutual dependence of economic phenomena that makes the use of mathematics indispensable in the study of these phenomena; ordinary logic may suffice for the study of relations of cause and effect but it soon becomes powerless in dealing with relations of mutual dependence. These require the use of mathematics, in pure economics as well as in rational mechanics.]⁴⁹

The main benefit to be drawn from the theories of pure economics is that they give us a synthetic conception of economic equilibrium; at present there is no other way open to us to achieve this end. But the phenomenon investigated by pure economics sometimes diverges slightly and sometimes widely from the concrete phenomenon. It is the task of applied economics to study these divergences. However, it would be [unrealistic and]⁵⁰ unreasonable to claim to be able to settle concrete phenomena solely with the theories of pure economics.

¹⁹ *Éléments d'économie politique pure*, Lausanne, 1900. “Theoretically, all the unknowns of the economic problem depend upon all the equations of equilibrium” (p. 289). “It is certain that *rareté* is the cause of value in exchange” (p. 102).

Mr. Walras was probably misled by the secondary meanings of the word *rareté*. In his formulae, as he himself concedes, it means the *Grenznutzen* of the Germans, *the final degree of utility* of the English, or our elementary ophelimity; but here and there in the text we find the additional imprecise idea that a commodity is scarce in comparison with the wants to be satisfied, because of the obstacles that have to be overcome to obtain it. There is also a vague glimpse of a notion of obstacles, and the proposition that “*rareté* is the cause of value in exchange” thereby becomes less inaccurate. [The responsibility for this confusion is not to be imputed to this eminent scientist; it is entirely due to the mode of reasoning that is current in economics; a mode of reasoning which Mr. Walras’ works have contributed precisely to rectifying.]⁴⁸

CHAPTER IV

Tastes

1. In the preceding chapter we attempted to obtain a very general and thus somewhat superficial idea of the economic phenomenon; and in so doing we avoided a great many of the difficulties encountered instead of solving them. We must now occupy ourselves with these, take a closer look at the details we passed over, and complete the theories we have so far only outlined.

2. **Tastes and ophelimity.** We have tried to reduce the phenomenon of individual tastes to the pleasure experienced by a person when he consumes certain things, or at least uses them in some manner.

Here, we at once face a difficulty. Should we consider the use, or the consumption [of a commodity], merely as optional, or also as obligatory? In other words, are the quantities of commodities that appear in the formulae of pure economics to be understood as being consumed only to the extent that the individual pleases, or as necessarily consumed even if instead of pleasure they cause discomfort? In the first case the ophelimities are always positive; they cannot go below zero, since the individual will stop consuming or using something as soon as he is satiated with it; in the second case, the ophelimities may become negative and represent pain instead of pleasure.

Both cases are theoretically possible; to solve the question thus posed, we must turn our attention to the real world and see which of them political economy should be concerned with.

3. It is not hard to see that it is for the first category that a theory is required. If somebody has more water than is necessary to satisfy his thirst, he is not bound to drink it all; he drinks what he likes and leaves the rest. If a lady has ten dresses, she need not wear them all at the same time; and at present people do not usually go around wearing all the shirts they own. In short, people use the goods they possess only insofar as it suits them.

4. But, this being granted, the meaning of the quantities of commodities appearing in the formulae of pure economics changes slightly. These are no longer quantities consumed, but the quantities that are at the individual's disposal. Hence, the concrete phenomenon departs somewhat from the theoretical one. For the sensation of present consumption we substitute, as a motive of the individual's actions, the present sensation of the future consumption of the goods at his disposal.^[a]

5. Moreover, in the case in which the individual possesses a sufficient quantity of goods to be satiated, we ignore the trouble he may have in getting rid of the superfluous quantities. Usually, it is true, this is insignificant; as the [French]¹ proverb puts it: *abondance de biens ne nuit jamais^[a]* (store is no sore). But there may be exceptional cases in which the trouble is important and must be taken into account.

6. As for the sensation of potential consumption, which replaces that of actual consumption, if one considers actions that are repeated, which is precisely what political

economy does, these two sensations are ultimately in a constant ratio, so that the one can replace the other without serious error. In exceptional cases, for very improvident and thoughtless individuals, it may be useful to take the difference between these two sensations into account; but for the moment we shall not dwell on this point.

7. There is another advantage to considering the quantities at an individual's disposal; it saves us from having to take the order of consumption into account, and to suppose that this order is the one that suits the individual best. Obviously, one does not experience the same enjoyment if one eats the soup at the beginning of the meal and the dessert at the end as one does if one begins with the dessert and ends with the soup. The order of consumption would thus have to be taken into account, which would increase the difficulties of the theory enormously; and it is as well to get rid of this thorn in the flesh.^[a]

8. That is not all. The consumption of commodities may be independent: the ophe-limity provided by the consumption of one commodity may be the same whatever the consumption of the others, i.e., it may be independent of them. But this is not generally the case, and it often happens that the consumption of commodities is dependent, i.e., the ophe-limity provided by the consumption of one commodity depends upon the consumption of other commodities.

Two kinds of dependence should be distinguished:

(1) that which arises when the pleasure derived from the consumption of one thing is related to the pleasures derived from the consumption of others;

(2) that which is due to the fact that we may substitute one thing for another in order to produce sensations that are approximately equivalent for an individual, if not identical.

9. Let us examine the first kind of dependence. In reality the pleasure we derive from the consumption of one thing depends upon our consumption of others. Moreover, in order that certain things may yield us pleasure, they have to be consumed jointly with others; for instance, unsalted soup is not very tasty, and a suit without buttons is [not very agreeable].^[2]

Fundamentally, the cases we have just considered differ only quantitatively; the first displays in lesser degree those characteristics that are more pronounced in the second, and one passes from the one to the other by imperceptible degrees. Moreover, it may be useful to distinguish the following extreme cases: (a) the [dependence of consumption]^[3] may arise from the fact that we appreciate the use and the consumption of something more or less according to our situation at the time; (β) this dependence may arise from the fact that some things have to be combined to give us pleasure; they are therefore called COMPLEMENTARY GOODS.

10. (α) The first kind of dependence^[a] is very general, and cannot be ignored when extensive variations in the quantities of commodities are being considered; it is only when the variations are minor that we may assume, as a first approximation, that the consumption of certain things is independent. It is certain that anyone suffering from extreme cold cannot enjoy a delicately flavored dish; a starving man does not derive great pleasure from looking at a fine painting or from listening to a well-told story, and if he were given some food, it would hardly matter to him whether it were served in coarse earthenware or in fine china. On the other hand, in this kind of dependence, at least for small variations in quantities consumed, most of the change in ophe-limity of a commodity comes from the variation in the quantity of that commodity. One prefers to eat chicken from a fine plate; but in the end, if the plate is only relatively fine, the

pleasure would not be very different. Conversely, the pleasure derived from using a fine plate depends mainly upon the plate, and does not vary much if the chicken is more or less plump, or more or less delicious.

11. A good many of the authors who developed pure economics were led, in an attempt to simplify the problems they wished to study, to assume that the ophelimity of a commodity⁴ depends only on the quantity of that commodity at an individual's disposal. They cannot be blamed for this, because the difficulties have to be solved one at a time, and to proceed soundly it is better to proceed slowly. But it is now time to take a step forward and to consider also the case in which the ophelimity of one commodity depends upon the consumption of all the others.

As to the kind of dependence we are now examining, we shall, as a first approximation and only for small variations, assume that the ophelimity of a commodity depends exclusively upon the quantities of that commodity. But we shall have to take other kinds of dependence into account.

12. (β) The concept of complementary goods can be extended to a greater or smaller degree. To obtain light, one needs a lamp and also kerosene; but a glass is not necessary in order to drink wine: one can also drink from the bottle.

By extending the idea of complementary goods we could take account of this dependence by considering as separate commodities all the combinations of commodities that the individual directly uses or consumes. For instance, instead of considering the coffee, sugar, cup, and spoon separately, we could consider them together as a single commodity comprising the elements needed to drink a cup of coffee. But one thus avoids one difficulty only by falling into other, more serious ones. First, in forming this ideal commodity, why stop at the spoon? We should also take account of the table, the chair, the rug, the house containing all these fine things, and so on ad infinitum. We thus multiply the number of commodities beyond all reasonable bounds because every possible combination of real commodities gives us one of these ideal commodities.

Of two evils, it is therefore better to choose the lesser one, and take such composite commodities into consideration only in cases in which they depend so closely upon one another that it would be very difficult to consider them apart. In the other cases, it would be better to consider them separately, and we are thus brought back to the previous case. But it must never be forgotten, when we proceed in this way, that the ophelimity of one of the commodities depends not only on the quantities of this commodity, but also on the quantities of the other commodities that are linked to it in its use or in its consumption, so that it is surely a mistake to consider it as dependent on the quantity of this commodity alone. This error may be negligible when only small variations in the quantities of commodities [are allowed],⁵ because then it may be approximately assumed that the consumption of the commodity in question takes place under certain average conditions with regard to the secondary commodities.

Reverting to the preceding example, if we had to consider the extreme case in which there were no cup for the coffee, we could not, without serious error, assume the ophelimity of coffee to be independent [of the cup];⁶ but if we instead consider a situation not very different from the existing one, say one in which the variations consist simply in having a cup that is a little better or a little worse, it is possible without serious error to consider the ophelimity of coffee as being independent of the cup. Strictly speaking, the ophelimity of coffee for an individual varies with the sugar, the cup, the spoon, etc.

at his disposal; but if we assume a certain average situation with regard to all these things, we can as a rough approximation consider the ophelimity of coffee to depend only on the quantity of coffee at a given individual's disposal. Likewise the ophelimity of sugar will depend only on the quantity of sugar, etc. That would no longer be true if we considered large variations in the quantities or prices. Whether the price of sugar be 40 or 50 centesimi per kilogram does not make much difference to the ophelimity of coffee; but if sugar became unobtainable, the ophelimity of coffee would be very much affected; and even a mere increase in the price of sugar from 50 centesimi to 2 lire per kilogram would cause a by no means negligible variation in the ophelimity of coffee.

13. We may therefore conclude that if one wishes to allow for very extensive variations, it is necessary, at least for most commodities, to consider the ophelimity of one commodity as dependent not only on the quantity of that commodity used or consumed, but also on the quantity of many other commodities that are used or consumed at the same time. If these are not considered, and if one is content to consider the ophelimity of one commodity as dependent only on the quantity of this commodity, it becomes necessary to discuss only very small variations, and consequently to examine the phenomenon in the neighborhood of a given equilibrium point only.

14. Let us now move on to the second kind of dependence. A man may appease his hunger with bread or potatoes, he may drink wine or beer, he may wear woolen or cotton clothes, and for light he may use a kerosene lamp or tallow candles. It is conceivable that a certain equivalence can be established between quantities consumed that correspond to a given want. But we should distinguish clearly whether the equivalence is relative to the individual's tastes or to his wants.

15. If the relation of equivalence refers strictly to the individual's tastes, it is nothing but the relation given by an indifference curve for equivalent commodities; it is therefore useless to make a separate study of it. To say that a man regards the substitution in his diet of two kilograms of potatoes for one kilogram of beans as being equivalent for his tastes, is to say that the indifference curve for beans and potatoes passes through the point 1 kilogram of beans and 0 kilograms of potatoes and the point 2 kilograms of potatoes and 0 kilograms of beans.

16. Sometimes, on the other hand, the equivalence relates not to tastes but to wants. In that case, there would no longer be an identity between the relation of equivalence and that of the indifference curve. For example, a man may appease his hunger by eating either 2 kilograms of polenta⁷ or 1 kilogram of bread; a lady may adorn herself with a necklace made of either imitation pearls or fine pearls. Relative to tastes, there is not the slightest equivalence between these things; the man prefers the bread and the lady the fine pearls; it is only under the pressure of necessity that they substitute polenta⁷ and imitation pearls for the genuine articles.

17. When the man makes use of bread and polenta⁸ at the same time, and the lady imitation and fine pearls, it can no longer be assumed that the ophelimity of polenta⁸ is independent of that of bread, or that the ophelimity of imitation pearls is independent of that of fine pearls; but we have to consider the ophelimity of a certain combination of bread and polenta,⁸ and of imitation and fine pearls, or take account in some other way of the mutual dependence of the consumption of the various commodities.

18. This dependence is a very extensive phenomenon. A very large number of commodities exist in different qualities, and these qualities go on being substituted for one another as the individual's income increases. The word "shirt" covers a large number of very different objects, from the coarse smock of the peasant to the fine batiste chemise of the elegant lady. There are a great many qualities of wine, cheese, meat, etc. The man who has nothing else eats plenty of polenta;⁷ [if he has plenty of bread, he will leave the polenta aside, eat only a little of it and a lot of bread; if later he happens to have some meat, he will use it and cut down on his consumption of bread.]⁹ One cannot tell what pleasure someone may derive from a certain quantity of polenta⁷ if one does not know what other foods are available to him. What pleasure does an individual obtain from a cloak of coarse wool? To answer such a question one needs to know what other clothes he already has at his disposal.

19. These phenomena reveal a certain hierarchy of commodities. If, for instance, commodities [A, B, C, D, etc.]¹⁰ are capable of satisfying a certain want, an individual will use commodity A because he is unable to use the others, which are too expensive for him. If he becomes better off, he will use A and B together; if he is still better off, he will use only B; then he will use B and C, then C alone; then C and D; etc. This is of course only the main part of the phenomenon, since as the man who uses D may still occasionally make use of small quantities [of A, of B, etc.]¹¹

To be precise, we may say that any commodity in such a series is SUPERIOR to the preceding ones and INFERIOR to the succeeding ones. For instance, let us take the series: polenta,⁸ bread, second-rate meat, prime meat. A very poor man eats plenty of polenta,⁸ little bread, and scarcely any meat. When he becomes better off, he will eat more bread and less polenta;⁸ if his situation improves still more, he will eat bread and second-rate meat, and only now and then a little polenta;⁸ if he becomes still more affluent, he will eat prime meat and other quality foods, but very little polenta,⁸ and little bread—and that will be of a higher quality than the kind that he ate previously.

We see how extensive the kind of dependence is that we are discussing, and how we cannot proceed without taking it into account. To do this, two paths are open to us, as before.

20. We may consider this kind of dependence only in cases where it is very marked, [in which the dependence of the consumption of the commodities is considerable,]¹² and where the individual's preference is neither slight nor negligible; and consider the other commodities consumed as independent.

21. But, still in this field of approximations, we could proceed differently and widen the consideration of this kind of interdependence instead of narrowing it down. We could, for instance, consider a more or less large number of man's tastes or wants, and postulate with respect to these that certain quantities of commodities that can be substituted for each other are equivalent. In the case of food, for instance, certain equivalences could be established among quantities of bread, potatoes, beans, meat, etc. In this case, we would only have to take account of the total ophelimity of these equivalent quantities.

22. Since such substitution equivalences are only approximate, we must not, even when considering the second kind of dependence, depart too far from a certain average state for which these equivalences have been approximately established.

23. The difficulties encountered here are not peculiar to this question. We have already seen (§18) that they are generally encountered in the case of very complex phenomena. In civilized societies there are enormous quantities of various commodities capable of satisfying innumerable tastes. To have a general idea of the phenomenon, one must necessarily ignore many details, and this can be done in several ways.

24. We have considered the main kinds of dependence; there are others, and the phenomenon is very varied and complex. In short, the ophelimity of a commodity depends upon all the circumstances under which its consumption takes place. But if we wish to consider the phenomenon in its full range, any theory becomes impossible for reasons already set out and repeated many times; it is thus absolutely necessary to separate out the main parts of the phenomenon, and to extract from the complete and complex phenomenon ideal and simple elements that can be the object of theories.

This aim can be attained in several ways; we have indicated two of them, but others are also possible. Each of these ways has certain merits and, according to the circumstances, may be preferred to the others.

25. As in all the concrete sciences in which one phenomenon is substituted as an approximation for another, the theory must not be extended beyond the limits for which it was intended; and, whichever of the above methods is followed, our conclusions must not be extended—at least not without further research—beyond the [boundaries of a small]¹³ region surrounding the equilibrium point observed in practice.

26. Moreover, there are other very important facts that compel us to proceed in this way. It must in fact be borne in mind that when conditions change, people's tastes change, too. We can ask a lady who already possesses diamonds: "If diamonds should cost a little more, by how much would you reduce your purchases of them?" and expect a reasonable reply. But if we were to ask a peasant woman who had never seen a diamond: "If you were a millionaire, how many diamonds would you buy at a particular price?" we should obtain a random and worthless answer. In one of his epigrams, Martial writes: "Priscus, you often ask me what I would do if I were rich and powerful. Do you think that one can know one's future feelings? Tell me, what would you be like if you became a lion?"¹⁴

Strictly speaking, it is not even necessary for the conditions of the phenomenon to change radically in order for tastes to change; but tastes may also change in some other way and on account of small variations in external conditions. It should be added that an individual is not exactly the same from one day to another.

27. This observation leads us to a proposition of no small importance. Let us start with an example. In Italy, people drink coffee, and not tea. If the price of coffee should rise very much, and the price of tea should drop considerably, the immediate effect would be a reduction in the consumption of coffee, whereas the consumption of tea would hardly increase if at all. But little by little, after a time which will certainly be very long, because people's tastes are very persistent, Italians may substitute tea for coffee; whence

¹ XII, 92.¹⁴

the ultimate result of the considerable reduction in the price of tea will be a significant increase in its consumption.

In general, we must always distinguish between short-term and long-term changes. In economic statics¹⁵ we should study the first type exclusively, except in rare cases. Let us suppose that the indifference curves as between a commodity B and another one A (which could be money) were today those indicated by the solid lines s in Figure 28; and that, after a century, they became the dashed lines t . Suppose further that the individual has an amount oa of money. Today, whatever may be the price of B (within certain limits), this individual will spend almost the same amount, ah , of A; in a century, he will still spend an amount ak , which as the price varies will be almost the same, but which will be different from ah .^[a]

28. A long time must elapse before the indifference curves s turn into indifference curves t ; we may thus assume, without significant error, that in a short span of time, e.g., one, two, or even four or five years, they will remain equal to s .

29. We have assumed that a man is able to compare two sensations; but, when they are not simultaneous—and, indeed, it does not seem that they can be—he can only compare one sensation with his conception of another one. Hence the real phenomenon again diverges from the theoretical one, and it may be useful in some cases to take account of this divergence in a subsequent approximation. It may often be admitted, on the contrary, that the idea of a future sensation is not too misleading, particularly because economics is concerned only with average and repeated phenomena, and hence if that idea diverges too far from the future sensation in the first trial it will be rectified in subsequent trials.

30. It thus appears that if the theoretical phenomenon being studied departs significantly in some cases from the concrete phenomenon, in most of the usual cases it represents it with a more or less rough approximation, always provided the following conditions are fulfilled: (1) We can study only what happens in a small region, at the center of which lies the concrete phenomenon which supplies us with the factual data that are necessary to construct a theory. In the real world, we are roughly at a position of equilibrium of the economic system: we are able to know how the system behaves in the neighborhood of this position, but we lack the data to know what would happen if the factual conditions of the system were substantially modified. (2) We consider only average phenomena, which repeat themselves, in such a way as to eliminate most accidental variations.

If someone thinks this is too little, he has only to show us how to do better. The path is open and the progress of science is continuous. But in the meantime, a little is better

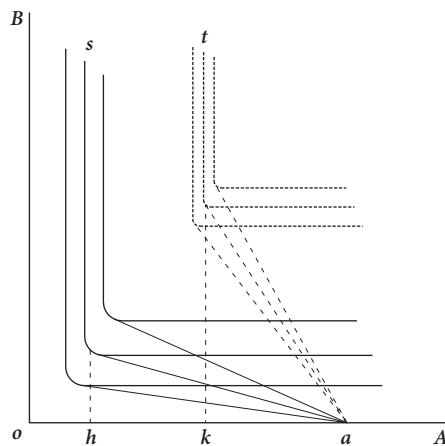


FIG 28

than nothing; the more so because experience shows that in every science a little is always a necessary step on the way to more.

31. Some have thought that, simply because it uses mathematics, political economy has acquired, in its deductions, the rigor and certainty of the deductions of celestial mechanics. This is a very grave mistake. In celestial mechanics, all the conclusions that are drawn from a hypothesis have been verified by the facts; whence it appears extremely likely that this hypothesis is sufficient to give us a precise idea of the concrete phenomenon. In political economy we cannot hope to have such a favorable result, because we know without any doubt that our hypotheses diverge in part from reality; consequently, the conclusions that will be inferred from them will fit the facts only within certain limits. The same is true, moreover, in most arts or concrete sciences, e.g., in engineering. It may thus be said that theory is more often a method of research than of proof, and we should never neglect to verify whether its deductions correspond to reality.

32. Ophelimity and its indices. In speaking of ophelimity, we should not forget to distinguish between TOTAL OPHELIMITY (or its index) and elementary ophelimity (or its index). The first is the pleasure (or the index of pleasure) provided by the total quantity of commodity A possessed; the second is the quotient of the pleasure (or the index of pleasure) procured by a very small additional amount of A, divided by this amount (III, 33).

An individual assumed to be standing at a point on the hill of pleasure (III, 58) enjoys a total ophelimity represented by the height of that point above a horizontal plane. If one cuts the hill of pleasure with a vertical plane parallel to the *oa*-axis on which the quantities of commodity A are measured, a certain curve is obtained; the slope, with respect to a horizontal line, of the tangent to this curve, at the point where the individual is standing, is equal to the elementary ophelimity (§§60, 69).

A man may know whether the pleasure provided by a certain combination I of commodities is equal to the pleasure he derives from another combination II, or whether it is greater or smaller. We have made use of this fact (III, 55) to determine the ophelimity indices, i.e., the indices that indicate whether the pleasure provided by a given combination is equal to that derived from any other combination, or whether it is greater or smaller.

Moreover, a man may know roughly whether, in passing from combination I to combination II, he experiences more pleasure than in passing from combination II to some other combination III. If this judgment could be made sufficiently precise, we could know in the limit whether by passing from I to II this man experiences a pleasure equal to that which he experiences in passing from II to III; consequently, in passing from I to III, he would experience a pleasure that would be double the one he enjoys in passing from I to II. This would be sufficient to allow us to consider pleasure or ophelimity as a quantity.^[a]

But we are unable to attain such precision. A man may know that the third glass of wine provides less pleasure than the second; but he cannot possibly know how much wine he must drink after the second glass to obtain a pleasure equal to that derived from this second glass. This is the reason for the difficulty in considering ophelimity as a quantity, except as a mere hypothesis.

Among the infinite number of conceivable systems of indices, the only ones worth retaining are those with the property that if more pleasure is experienced in passing from I to II than from II to III, the difference between the indices of I and II should be greater than that between the indices of II and III. In this manner the indices will always provide a better representation of ophelimity.

[Ophelimity, or its index, for one individual, and ophelimity, or its index, for another individual, are heterogeneous quantities. They may neither be added together, nor compared: *No bridge*, as the English say. A sum of ophelimities enjoyed by different individuals does not exist; such an expression has no meaning.]^{16[b]}

33. Characteristics of ophelimity. In what follows, we shall assume that ophelimity for an individual is a quantity; it would be easy however, to modify the reasoning so as to make it valid only for ophelimity indices.

By virtue of our hypothesis concerning the quantities of commodities—and by these quantities are meant only those that are at an individual's disposal^[a] (§3)—ophelimity is always positive; this is [a first characteristic of ophelimity].¹⁷

A¹⁸ second characteristic, which was recognized by the first economists who studied this subject, would consist in the fact that if the ophelimity of a commodity is considered to depend solely on the quantity of that commodity, the elementary ophelimity (III, 33) decreases when the amount consumed increases. Attempts have been made to make this property depend on Fechner's law;² but this entails the assumption that the commodity is necessarily consumed; and we have already seen (§3) that this would draw us into numerous difficulties. Moreover, of the great variety of economic uses [of commodities], several differ too widely from the phenomena to which Fechner's law applies.

It is better to have recourse directly to experience, and this shows that in fact, in many uses or [acts of consumption],²⁰ the law applies that elementary ophelimity decreases as the quantity consumed increases.

34. Finally, it is a very general fact that the more we have of something, the less valuable each unit of that thing becomes for us. There are exceptions. For instance, a collector becomes more attached to his collection the more complete it becomes; it is a well-known fact that certain peasant proprietors become more and more avid of further purchases as their land holdings increase; finally, everybody knows that the miser's eagerness to increase his hoard increases the more he possesses. In general, saving has a certain ophelimity of its own, independently of the benefit it yields in the form of interest; and this ophelimity increases with the quantity of saving up to a certain limit, after which it decreases, except in the case of the miser.

35. There are, further, commodities whose ophelimities are not independent (§9). As to the interdependence of case (a), we may—at least in general—consider elementary ophelimity to decline as the quantity increases; indeed it often decreases faster than when ophelimity is independent. As to interdependence in case (β), elementary ophelimity may rise and then fall as the quantity increases. For example, if a shirt lacks a single button, the ophelimity of this button is greater than that of the others; and that of yet

² Fechner, *Revision der Hauptpunkte der Psychophysik*, Leipzig, 1888. Wundt, *Grundzüge der physiologischen Psychologie*.¹⁹

another button is very small. But this phenomenon is partly analogous to those of the discontinuous variations we have already studied (III, 65). It must be recalled that we do not study individual phenomena, but collective and average ones. Shirts are not sold with one button missing; hence the abstract case just noted is never met with in practice. We must consider the consumption of thousands of shirts²¹ and of thousands of buttons, and in that case it may be assumed without significant error that elementary ophelimity decreases as the quantity increases.

36. As to interdependence of the second kind (§8), it may be observed that in general the elementary ophelimity of a commodity decreases as the quantity of the commodity increases, until it becomes zero. This elementary ophelimity remains zero until the commodity to which it refers is eliminated from consumption, or until only an insignificant amount of it remains and is replaced by some other superior commodity.

37. In conclusion, and except for part of the phenomenon in the case of complementary goods, the elementary ophelimity of most commodities decreases when the amount consumed increases. The first glass of water provides more pleasure to the thirsty man than the second one; the first portion of food gives more pleasure to the hungry man than the second one, and so on.

38. We can go further in this direction and find a third characteristic of the ophelimity of a very large number of commodities. Not only does the second glass of wine yield less pleasure than the first one, and the third less than the second, but the difference between the pleasure obtained from the third and that obtained from the second is less than the difference between the pleasure provided by the first and that obtained from the second. In other words, as the amount consumed increases, not only does the pleasure derived from additional small equal quantities decrease, but also the pleasures provided by these small quantities tend to become equal. For someone who has 100 handkerchiefs, not only is the pleasure derived from the 101st handkerchief very small, but it is also nearly equal to that provided by the 102nd one.

39. We now have to examine what happens when it is not the quantity of the commodity whose elementary ophelimity is being considered that varies, but the quantities of other commodities with which it is dependent.

In the case of dependence of type (α) (§9), the pleasure obtained from a small quantity of commodity A added to the amount consumed is usually greater when we suffer less from the lack of other commodities; consequently, the elementary ophelimity of A increases when the amounts of B, C, . . . increase. This happens also in the case of dependence of type (β), at least within certain limits. The pleasure provided by a lamp, added to other lamps, is greater when plenty of oil is available, so that this additional lamp can also be used. Conversely, what would be the purpose of having plenty of oil, if one had no lamps to burn it? We shall thus conclude by saying that, in general, as regards the first kind of dependence, the elementary ophelimity of A²² increases when the quantities of other commodities B, C, . . . increase.

40. In the case of the second kind of dependence^[a], the situation is just the reverse. If a commodity A can replace a commodity B, the elementary ophelimity of A will be all the smaller the more plentiful is its substitute B.

41. To gain a better understanding of how all this takes place, let us draw up a table, with numbers chosen at random and whose only aim is to give tangible form to the foregoing considerations.

Quantities of		Pleasure provided by	Quantities of		Pleasure provided by
A	B	AB	A	B	AB

Dependence of the first kind (α) (§9)

10	10	5.0	10	11	5.2
11	10	5.4	11	11	6.1
Pleasure provided by 1 unit of A ...		0.4			
	Difference between these pleasures				0.9
					+0.5

Dependence of the first kind (β) (§9)

10	10	5.0	10	11	5.15
11	10	5.1	11	11	7
Pleasure provided by 1 unit of A ...		0.1			
	Difference between these pleasures				1.85
					+1.75

Dependence of the second kind (§14)

10	10	5.0	10	11	6.0
11	10	5.9	11	11	6.1
Pleasure provided by 1 unit of A ...		0.9			
	Difference between these pleasures				0.1
					-0.8

The reader will note that the difference in pleasure provided by 1 unit of A is positive for dependence of the first kind, and negative for dependence of the second kind. This difference is always equal to that which obtains in comparing pleasures provided by 1 unit of B. This is so because we have implicitly assumed that the pleasure derived from the combination AB is independent of the order of consumption.^[a]

42. Let us make up a commodity A out of proportional doses of two other commodities B and C, e.g., 1 unit of bread and 2 units of wine. If B and C are independent, or if there exists between them a dependence of the first kind, we may repeat the above reasoning, and we shall see that, in general, the ophelimity of A decreases as the amount of A increases. The exceptions may be disregarded, for the reasons indicated in §35.

43. **Characteristics of indifference curves.** Economists began by seeking to obtain the characteristics of ophelimity from experience; they then inferred the properties of indifference lines from them.

The process can be reversed. Where the elementary ophelimity of a commodity depends solely on the quantity of this commodity, both procedures are equivalent. But it is worth noting that in the general case, namely that in which the consumption of the

commodities is dependent, the study of indifference lines gives results which could be less easily reached for the time being, at least—by calling on experience alone to determine the properties of ophelimity.

44. One of the first properties of indifference lines is obtained by observing that the quantity of one commodity must be increased to compensate for a decrease in the quantity of another. From this it follows that the angle oe is always acute. This property corresponds precisely to the property the elementary ophelimities have of always being positive.

45. Furthermore, if we except the small number of instances indicated in §34, it can be observed that in order to offset the loss, by equal amounts, of a small quantity of a given commodity, one will need so much less of another commodity, the more one possesses of the first commodity. The result is that indifference lines are always convex to the axes, that their shapes are similar to t , and never to s, s' (Figure 29). To have these latter shapes, they would have to refer to a commodity each unit of which becomes more valuable as the amount of this commodity at an individual's disposal increases. Obviously this case would be quite exceptional.

46. When we consider several commodities A, B, C, . . . , one can no longer properly speak of indifference lines; but there are then properties analogous to those pointed out above which are very useful for the theory.

Any one of these commodities, A for instance, may be chosen as money. Of the others, some will be sold, and some will be bought; we may consider separately the amounts of money needed for these purchases, or received from these sales; by deducting the sum of the expenditures from the sum of the proceeds, we obtain the quantity of A yielded by the set of these operations, and conversely.

By comparing A successively with each of the commodities B, C, . . . we obtain indifference lines, with properties that are identical with those already indicated.

47. Furthermore: (1) if there is a certain expenditure on balance, this means that the purchases have exceeded the sales, i.e., the decrease in A has been offset by an increase in some of the commodities B, C, . . . ; (2) whatever may be the dependence of consumption, if we suppose that in order to offset the expenditure of one lire, a certain fraction of a certain combination of B, C, D, . . . , would be required, then the more the individual's income decreases, the higher the fraction will become and conversely.

If an individual incurs a certain expenditure to buy a lamp, a wick, and some oil (dependence of the first kind (β)), and to lodge, dress, and feed himself (dependence of the first kind (α)²³ with respect to the lamp), and if the expenditure incurred and the enjoyment provided exactly offset each other, it is obvious that this offset would no longer hold if all these expenditures were doubled, because, on the one hand, money becomes more valuable to the individual as he possesses less of it, and, on the other, lamps, etc. would become less valuable, since he has more of them.

As a rule, when a large number of individuals are considered, discontinuous variations are transformed, with little error, into continuous ones.

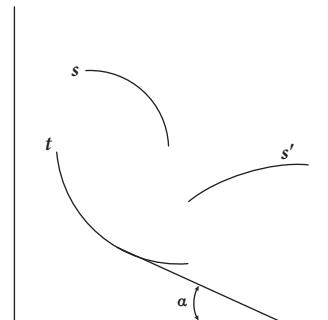


FIG 29

48. Relation between ophelimity or indifference curves and supply and demand. The properties of ophelimity and of indifference curves are closely related to certain characteristics of the laws of supply and demand. We shall set forth a number of these relations here; but for their proof, the reader is referred to the Appendix.

49. Let us consider the supply and demand on the part of an individual who has two or more commodities at his disposal. If the consumption of these commodities is independent, or if there is a dependence of the first kind between them, the demand for a commodity always falls as the price of this commodity rises; the supply increases at first, and then it can decline as the price rises.

As for commodities between which there is a dependence of the second kind, when the price rises demand may increase and then decrease; the supply may decrease and then increase.

In the real world, the difference occurs especially on the demand side. It is more striking in certain circumstances. Let us imagine an individual with a given income, which he distributes over the purchase of several commodities. If consumption of these commodities is independent, or if there is a dependence of the first kind between them, the demand for each of these commodities always increases when income increases. If, on the contrary, the dependence is of the second kind, demand may increase and then decrease as income increases.

50. This proposition^[a] is enough to convince us of the need to study the second kind of dependence. Let us see what correspondence there can be between theoretical deductions and concrete facts. If we assume that the ophelimity of a commodity depends only on the amount of this commodity consumed or possessed by the individual, the theoretical conclusion is that the consumption of such commodities increases when income increases; or in the limit that it ends up being constant when income is above a certain level. Consequently, if a peasant eats only polenta,⁸ he will eat more of it as he becomes richer, or at least as much as he did when he was poor. The man who uses up one pair of wooden clogs per year and no shoes because they are too expensive will be able to make use of a hundred pairs of clogs when he becomes rich; but, in any case, he will use at least one pair. All this clearly flies in the face of the facts: our hypothesis must thus be rejected, unless it is assumed that these facts are not significant.

51. Such is not the case; indeed, as we have already noted (§19),^[a] we are in the presence of a very wide-ranging phenomenon, because, for a great many commodities there are several qualities; and as income increases, the superior qualities take the place of inferior ones; consequently, the demand for the latter first increases with the rise in income, but thereafter it declines until it becomes insignificant, or even zero.

52. This conclusion would no longer hold if instead of considering actual commodities we had considered large categories of ideal commodities (§21); for instance, if we were to consider food, housing, clothing, decorative objects, and entertainment. In that case it would not be absurd to say that the expenditure for each category of commodities increases with an increase in income; and one could suppose without serious error that the ophelimities are independent, or rather, that there is a dependence of the first kind among them.

53. In the real world an individual usually demands a great variety of commodities, and supplies only one or a few. A great many people supply only labor; others, the use

of their saving; others, certain commodities they produce. The case of simple barter of two commodities, connected by a dependence of the second kind, is quite exceptional; a laborer sells his labor and buys polenta⁸ and bread, but we do not observe the barter of bread for polenta.⁸ The conclusions of the theory can thus not be verified directly in this case, and another procedure to test them would be needed which can be obtained by considering the distribution of income.

54. Various shapes of indifference lines and of exchange lines. It will be helpful to represent the properties of ophelimity graphically. Suppose an individual has two commodities A and B, of which only one, A, is useful [ophélimé] to him. In this case the indifference lines are straight lines parallel to the oB axis. The hill of ophelimity is a cylindrical surface of which any section, running parallel to oA , is indicated by bgh .²⁴ If the quantity oa^{25} of A suffices to reach satiety, the cylindrical surface leads into a high plateau G indicated by $gh^{26[a]}$ on this section. The property the elementary ophelimity has of decreasing when the quantity of A increases explains the decrease in the slope of the hill from oB to G ,²⁷ i.e., on the section, from b^{28} to f and to g (§32).^[b]

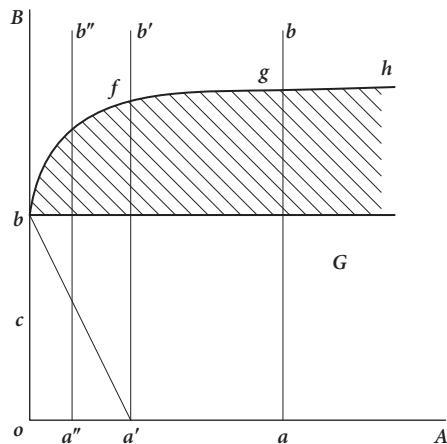


FIG 30

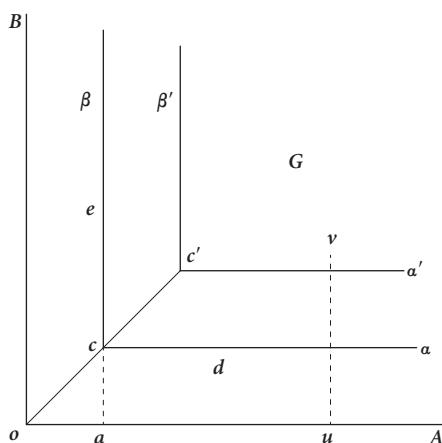


FIG 31

straight line ba^{33} with respect to oB . In the case of Figure 30,³⁴ we can say that *the entire quantity of B available is supplied*.

The individual never demands B, since for him this commodity has no ophelimity, but he may supply it if he possesses some of it, e.g., ob . This is the case we have noted (III, 98). No linear path starting from b can be tangential to an indifference line, and we have in a, a', a'', \dots as many terminal points; the oA -axis is thus part of the contract line.^{29[c]} It is obvious that even bo is part of it [whence that line is boA .]³⁰ If the contract curve²⁹ of some other individual cuts bo at c , the amount of B given in exchange is bc , and the price is zero. If this contract curve²⁹ cuts oA at a' ,^{31[d]} or at any other similar point, the quantity given up in exchange is still the entire amount bo ; the price varies according to the position of the point a' ,³² being equal to the slope of the

55. If A and B are two complementary goods, which can be enjoyed only when combined in strictly determined proportions, the indifference lines are right-angled straight-line segments βca , $\beta'c'a'$. The hill of ophelimity is made up of two cylindrical surfaces, and there may be a plateau in G^{35} indicating satiety. The pleasure enjoyed by an individual at c is the same as that experienced at d or at e , because since the commodities have to be combined in strictly determined proportions, the amounts cd of A, or ce of B, are superfluous.

56. When the hill of ophelimity has a continuous surface, a section drawn along uv (Figure 32) has a shape similar to (I). In reality, in the case of many complementary goods, we instead have a staircase, as in (II). For instance, the handle and the blade of a knife are complementary, and it is not possible to use a handle and a tenth of a blade; consequently, we shall have so many steps exactly one unit wide. As has already been repeatedly observed, one can for large numbers, and subject to only slight error, replace these steps by a continuous surface whose section will be like section (I), and be bordered by a continuous curve (III, 65).

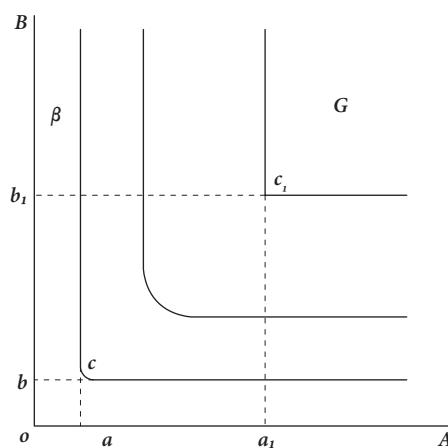


FIG 33

minimum amount of water he needs in order not to die of thirst. It is obvious that he would not relinquish a small quantity of bread even in order to obtain plenty of water, or conversely; consequently, the indifference lines will be ca , $c\beta$ with a very slightly curved angle at c . For larger quantities of bread and water, the angle can be more curved, but it will hardly be so or not at all at c_1 , when the individual has the amounts oa_1 of bread and ob_1 of water, with which he is completely satiated. Beyond this lies the plateau G .

58. The reader should never forget that political economy, like any other concrete science, proceeds only through approximations. For simplicity, theory examines extreme

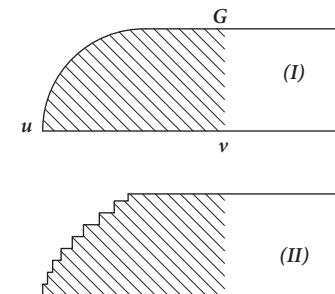


FIG 32

57. When the goods are only approximately complementary, the angles $a, a', \dots^{[a]}$ are more or less curved. Let us consider an individual who has only bread, A, and water, B, or, if one prefers, food and drink. Without bread, he will starve to death, whatever the quantity of water at his disposal; consequently, along oB , total ophelimity is zero, and the elementary ophelimity of a small portion is infinite, i.e., the hill rises perpendicularly. Without water, he will die of thirst, whatever the amount of bread at his disposal, and consequently, along oA , total ophelimity or pleasure is also zero and elementary ophelimity is again infinite. Let oa be the minimum amount of bread the individual needs in order not to starve, and ob the

cases, but concrete cases only approximate these extreme cases. Thus, when an architect wants to know for how many cubic meters of masonry he has to pay the contractor, he will consider the wall as a right-angled parallelepiped, and it would be ridiculous to object that the wall is not a geometrically perfect parallelepiped, and then foolishly show off with remarks about mathematical rigor. [This is what often happens in economic science.]³⁶

59. The [exchange line]^{37[a]} is obtained by joining the points c, c', \dots ^[b], in Figure 31, or the points $[c, c_1, \dots]$ ³⁸ in Figure 33, in which the linear paths starting from a point similar to point a in Figure 28 are tangential to the small curves that replace the angles, or by joining the analogous points that we would obtain if the paths were to start from a point lying on the oB axis.

60. Let us suppose that the elementary ophelimities of A or of B are independent, i.e., that the elementary ophelimity of A depends only on the quantity of A , and the elementary ophelimity of B only on the quantity of B . This property can be expressed graphically in the following way. Let us draw any arbitrary straight line uv parallel to oB , and let us draw lines $bh, b'h', \dots$, parallel to oA . The hill of ophelimity will be cut by as many curves $bc, b'c', \dots$; the slope with respect to the horizontal lines $bh, b'h', \dots$, of the tangents $bt, b't', \dots$, to these curves, at the points b, b', \dots , is equal to the elementary ophelimity of A corresponding to the quantity ou of A (§32). Since this elementary ophelimity does not vary with the quantity of B , the slopes of the tangents $bt, b't', \dots$, are all equal. Analogous properties would obtain with respect to a straight line parallel to oA .

61. As a consequence of this, the lines of Figure 31 cannot represent the indifference lines of two commodities whose ophelimities are independent, since the slopes just mentioned, although constant between β and c , decrease all at once, or rapidly, at c , and become zero between c and a . This confirms the need to consider the consumption of certain commodities as dependent.

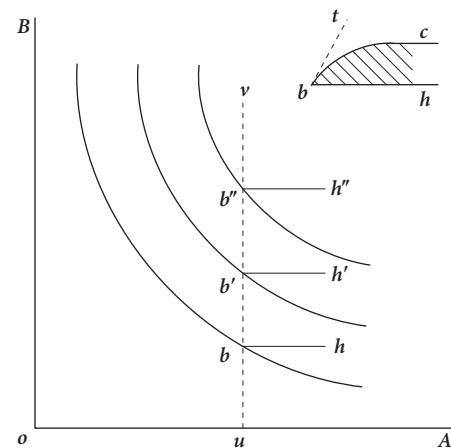


FIG 34

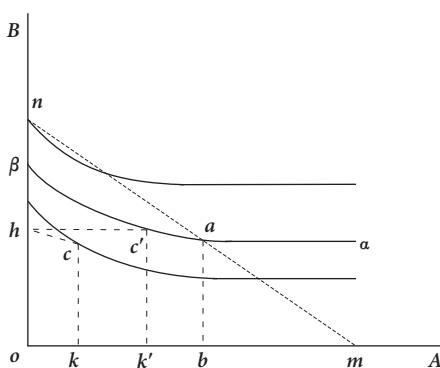


FIG 35

62. To get an idea of the indifference curves in the case of interdependence of the second kind, let us consider two commodities A and B , such that A is inferior to B (§19), and such that they may be substituted for one another. As examples of such goods, let us take polenta³⁷ and bread^[a]. An individual may satisfy his hunger by eating only polenta³⁹ or only

bread, or some of each of these foods; he prefers bread to polenta,³⁹ at least in certain proportions.

Let us suppose for simplicity that 3 units of A can replace 2 units of B; the reasoning would, it may be added, be the same whatever the law of substitution. Let om be equal to 3 and on to 2, and let us draw the straight line mn . On this line, the material need of the individual is satisfied. For instance, at m he satisfies his hunger with 3 units of polenta;⁴⁰ at n , with 2 units of bread; at a with ba of bread and ob of polenta;⁴⁰ but his satisfaction is not the same at these points. When he is at a , any additional amount of A is superfluous; consequently, aa , parallel to om , is an indifference line. This line is then extended from a to β . At n the individual would be satiated with B; at β he has a little less of it. This difference in pleasure obtained from the use of on and $o\beta$ is the same as that experienced by the individual when he can only use B and must be satisfied with ab of B and ob of A.

If the individual has oh of B, which he exchanges for A at the price of A in terms of B given by the slope of hc on oA , he demands ok of A; and at a lower price, given by the slope of hc' , he will demand a larger quantity ok' .

63. In the extreme case of two commodities A and B, one of which can always be substituted for the other at a constant rate, e.g., 4 units of A are always equivalent to 3 units of B, the indifference lines are straight lines whose slope is such that oa is to ob as 3 to 4.⁴¹ Starting from a , the contract line is the same straight line ab .

64. If we have a certain number of commodities A, B, C, ..., we may suppose for a moment that the prices of B, C, ... are fixed, and that a certain amount of money is allocated among these commodities.

This sum of money becomes, in that case, a commodity which may be compared with A; we can thus extend the use of graphs to a large number of commodities.

65. The indifference curves between this sum of money and commodity A will very often be shaped as in Figure 37. The quantities of money are measured along oQ , and the quantities of commodity A along the oA -axis. From the points q', q, q'' ^{42[a]}, tangents $q'm', qm, q''m''$ are drawn to the indifference curves. These are such that the slopes of these tangents with respect to oA keep increasing as one moves from o in the direction of Q .

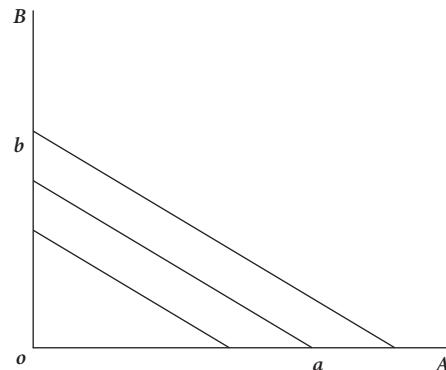


FIG 36

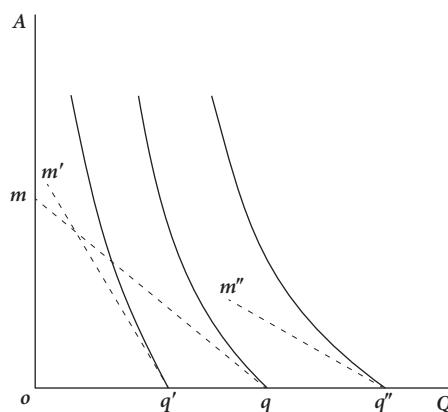


FIG 37

The slope of qm on oA gives the price of commodity A. It will be noticed that a person standing at q cannot be in equilibrium on a straight line with a steeper slope [than qm]⁴³ on oA , i.e., with a higher price. Thus, if the minimum price of A is given by the slope of qm on oA , a person who has an amount oq of income can barely start buying some of commodity A; the person with an income of only oq' could buy none of it, because the tangent $q'm'$ is less steep on oA than qm . Someone standing at q'' ⁴⁴ can, on the contrary, buy some quantity of commodity A, because $q''m''$ ⁴⁵ has a steeper slope with respect to oA than qm . Consequently, when a commodity has a minimum price below which it cannot be procured, only someone with an income above a certain level will buy it. As we know, this is how things happen in the real world.

66. Thus, provided we also take account of the hierarchy of commodities, (§19)^[a], we have a reasonable representation of the concrete phenomenon. Let us suppose that we have different series A, B, ..., of these commodities which can be substituted for one another.

A	B	C	D	E	F	G
A'	B'	C'	D'	E'	F'	G'
A''	B''	C''	D''	E''	F''	G''
A'''	B'''	C'''	D'''	E'''	F'''	G'''

With a certain income, an individual uses⁴⁶ the commodities enclosed by the solid rectangle; if his income increases, he will use⁴⁶ the commodities enclosed by the dotted rectangle; as his income increases, he will pay no attention to certain less expensive inferior commodities of lesser quality, and will use⁴⁶ others that are more expensive and of better quality.

67. Indifference curves shaped like those in Figure 38 do not correspond to the most common commodities, because, according to these curves, even someone with a very small income would purchase very expensive commodities, though in small amounts.

Nevertheless, if we considered the indifference curves within a small region G, we could employ these, or others, according to convenience. The real curves are certainly very complicated; it will be sufficient for the theoretical curves to fit the real curves closely within the small area considered. It may further happen that curves which approximate the real curves better than others within this small area may diverge considerably from those outside it, and conversely.

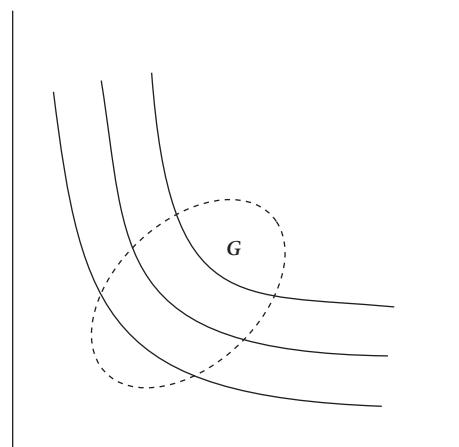


FIG 38

68. The case in which there are many commodities is very complex; it is therefore useful to have several ways of simplifying it. To pass from a certain combination of commodities A, B, C, ..., to another A', B', C', ..., we may subdivide the operation into two parts: (1) the proportions within the combination are maintained, and all the quantities are increased (or decreased) proportionately; (2) the proportions are changed, we then finally reach the combination A', B', To take a concrete case, let us imagine, for instance, an individual with an annual income of 1,200 lire; his income increases and becomes 2,400 lire. The allocation will be as follows:

Expenditure for:	First real situation		Theoretical intermediate situation		Second real situation	
	lire	% of income	lire	% of income	lire	% of income
Food...	720	60	1440	60	1200	50
Housing...	360	30	720	30	600	25
Clothing, etc....	120	10	240	10	600	25
Income...	1200	100	2400	100	2400	100

It should be noted that the first operation is generally much more important than the second, particularly in the case of moderate increases in income. When income increases, the proportionate expenditures on the main categories of expenditure: food, housing, clothing, amusements, will admittedly change; but this is a secondary phenomenon, as compared with the principal one, which is the increase in all of these expenditures.

69. **The hill of ophelimity.** From the fact that the elementary ophelimity of a commodity decreases as the quantity at an individual's disposal increases, it follows that the slope of the hill of ophelimity is steeper at the base, and that it decreases as the height increases (§32). [It resembles the hill in Dante's *Purgatory*:

Questa montagna è tale
Che sempre al cominciar disotto è grave,
E quanto più va su, e men fa male.
Purg., IV, 88–90

[For basic necessities, the analogy is complete.

E la costa superba più assai
Che da mezzo quadrante a centro lista.
Purg., IV, 41–42.]⁴⁷

70. The following property is very important for the theory. If, in traveling along a linear path in a certain direction, one starts descending, one will continue to descend if one keeps following the same direction. If on the contrary one starts ascending, one may later descend.

This will be proved in the Appendix; here, we can only give an intuitive insight into it.

For paths like ab , it is obvious that one always ascends in the direction of the arrow, and that one descends in the opposite direction. With paths like $mc^{[a]}$, one ascends in the direction of the arrow, as far as c , and then one descends. From c to m , in the direction opposite to that of the arrow, one always descends. To ascend, it would have to be possible to cross an indifference line from below, at some point like c'' , instead of from above, as at $c^{[b]}$. But if this happens, the curve that passes through c'' , as it must always have a tangent that makes an acute angle a , as shown in Figure 39,⁴⁸ cannot shoot off from c'' to e , but must necessarily bend around toward f .

Now this concavity at h is contrary to the property of indifference lines noted in §45; our hypothesis can thus not be accepted.

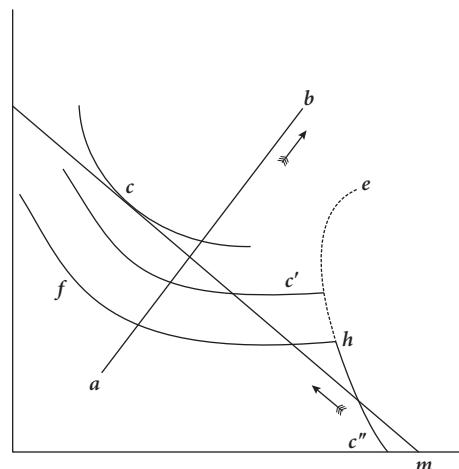


FIG 39

CHAPTER V

Obstacles

1. The study of how obstacles are overcome—i.e., the study of production—will be more elaborate than that of the action of individual tastes, because of the great complexity of the production process in civilized nations.

2. **Division of labor and the firm.** Among all civilized nations we can observe a phenomenon known by the classical term DIVISION OF LABOR. It consists essentially in the fact that production requires the combining and using of a large number of elements.¹ As was well observed by [our]² Ferrara, if we consider each of these elements and the role it plays in production, what we observe is a *division of labor*; if these elements are considered as a whole, and if the purpose for which they work together is envisioned, what we observe is *cooperation*.¹ The same phenomenon thus has two different names, depending on the point of view from which we look at it.

3. If we take the division of labor in its most restricted sense, which is etymologically also the [most appropriate]⁷ one—that of sharing of work among several individuals—we see that it has the effect on the one hand of separating the functions, and on the other of making the individuals depend on one another. With the extension of the division of labor, there is an increase in the number of parts making up the whole production process; and, since these parts depend upon one another, there is a widening of cooperation among individuals.

4. The *firm* is the organization that combines the various elements of production and puts them to use for the purpose of production. It is an abstract concept, like that of *homo economicus*, and bears the same relation to the actual firm as does *homo economicus* to the man of flesh and blood. The consideration of the firm is only a means of studying separately the different functions performed by the producer. The firm may appear in different forms: it may be entrusted to private individuals, or be managed by the state, the local authorities, etc.; but this does not change its [nature in the least].⁸

5. A concrete picture of the firm may be obtained by thinking of it as a vessel into which numerous streams converge, representing the elements of production, and out of which flows a river that represents the output.

6. These elements are provided partly by individuals, such as, for example, labor and certain products; and partly by other firms, such as, for example, certain products that are needed to obtain other products.

¹ Ferrara [uses the word]³ *association*. In the preface, entitled [*L'agricoltura e la divisione del lavoro*],⁴ XIV, after having recalled the fact that several individuals contribute to the production process, rather than [a single]⁵ one, he adds: “When we consider this fact—this *participation*—from the point of view of the goal and the common result, what we see is *association*; when we consider it from the point of view of the [participating]⁶ individuals, what stands out is *division*.”

Economic circulation may roughly be represented in the following way. A, A', A'', ..., are the firms; m, m', m'', m''', ..., n, n', n'', n''', ... are the individuals.⁹ Some of these individuals, for example, m, m', m'', n, n', n'', supply certain things to firm A (e.g., labor, saving, etc.); and we can imagine a number of streams which start from these individuals and flow into firm A, as do products from other firms. It may happen that A's products are not suitable for direct consumption; in such a case there is a flow of products from A which is divided among other firms A', A''. Individuals m, m', ..., [n, n', ...]¹⁰ obtain the products they consume, either from firms A', A'', or generally¹¹ from other firms A''', These flows are interwoven in an almost inconceivable number of ways, so great is the variety they present. A workman usually sells his labor to a single firm, and he obtains products from a large number of other firms which need not have the slightest direct connection with the first one. These complex relationships have to be disentangled and the phenomenon reduced to its elements.

7. With this purpose in mind, let us consider one firm by itself, its inputs and outputs, its receipts and expenditures, and the way in which production is organized.

8. **The objective of the firm.** We have to make a distinction similar to the one we made in the case of the individual (III, 40). We have [, that is,]¹² two types of phenomena: (I) The firm accepts market prices without trying to change them directly, although it contributes—unknowingly and unintentionally—to influence them indirectly.² It is guided solely by the intention of attaining a certain objective. For the individual, this was the satisfaction of his own tastes; for the firm, the objective has to be stated, and this we shall do later. (II) The firm may, on the contrary, set out to change market prices directly, in order to derive some advantage, or for any other purpose.

9. What has already been said about types (I) and (II) in the case of an individual applies equally to the firm, and may be taken as repeated. For the firm as for the individual, type (I) is that of free competition and type (II) that of monopoly.

We may conceive of a large number of objectives for the firm; but we should obviously limit ourselves to those that can be observed in the real world.

10. Firms very often try to obtain the maximum advantage for themselves, and this advantage is nearly always—one might even say always—measured in terms of money. The other cases may be considered as exceptions.

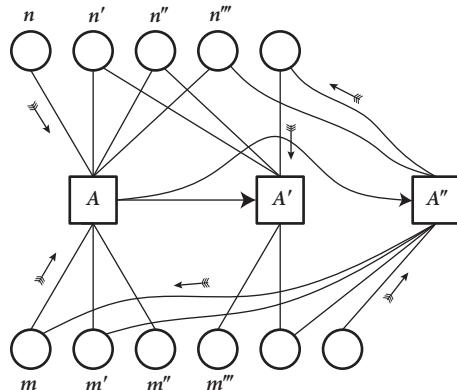


FIG 40

² [This stipulation is essential. If it is omitted, what was a true proposition becomes false.]

[It is not true that market prices are determined independently of the firm's actions. It is true the firm makes its calculations as if this were the case, and that it modifies prices unintentionally and often without even realizing it. The phenomenon is of the kind represented by pursuit curves (§ 11).]

[For another similar error, cf. § 70.]¹³

To achieve the greatest money profit, direct and indirect means are used. Directly, each firm tries to pay as little as possible for its purchases, and to charge as much as possible for what it sells. Further, when there are several ways of obtaining a commodity, it will choose the one with the least cost. This is true for type (I) as well as for type (II); the difference between these two types consists only in that in type (I) the firm accepts the market conditions as they are, whereas in type (II) it sets out to modify them.

Indirectly, the firm, when it has the power to do so, i.e., when it is of type (II), will try to bring about any alteration in market and production conditions which can, or which it thinks can, yield it some pecuniary profit. We have already mentioned some of the means that are used when we dealt with exchange (III, 47); we shall now examine some others.

11. It may be noted that the objective that the firm has in view may fail to be achieved—and this in different ways. First, the firm may be completely mistaken, and in the expectation of making a pecuniary profit it may employ means that actually inflict damage on itself. It may also happen that this money profit corresponds to a loss of ophelimity for the persons enjoying it. Finally, in a less obvious and subtler case, the objective itself may be modified as a result of the very means used to achieve it, and the firm may follow one of the curves that are called^[a] *pursuit curves*.¹⁴ For instance, suppose the firm is at *a*, and wishes to move toward *m* by following the path *am*; in so doing it shifts the position of *m*, and by the time it reaches *b*, *m* has shifted to *m'*. Again, the firm moves toward *m'*, and accordingly follows the path^[b] *bm'*;¹⁵ but once it has arrived at *c* it finds that the target has moved to *m''*; it will then follow the path *cm''*, and so on. Thus, although it left *a* in the direction of *m*, it finally settles at *M*, which represents an objective it did not have in view at all. We shall see later how this occurs in a very important case—that of free competition (§74).

12. As in the case of exchange (III, 49) we must, in the case of production, distinguish from type (II) a class of phenomena characterized by the fact that the firm seeks to procure the maximum welfare of all those who participate in the economic activity;¹⁶ we thus have the same type (III) as that mentioned in the study of exchange.

13. The various courses open to the firm. First of all, when a firm goes on the market to buy or sell, it may follow the various paths already studied in relation to exchange (III, 97, 98); it also, as a rule, has several courses open to it for obtaining the commodity it wants to produce. Some elements of production are fixed; whereas others are variable. To produce wheat flour, one of course needs wheat, but wheat may be ground in a mill driven by hand, or by an animal, or by wind, water, or steam. One can use millstones or reinforced iron cylinders. One may use more or less adequate means of separating the bran from the flour, etc.

14. Furthermore, the quantities of these elements are themselves variable within more or less narrow limits. Here the classic example is that of extensive or intensive cultivation of land. The same quantity of wheat can be obtained from a large or a small area of land

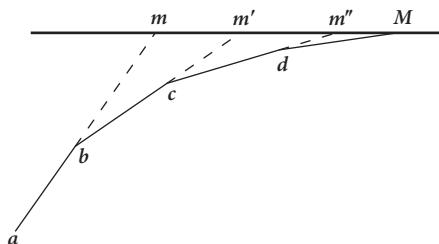


FIG 41

by varying the other elements of cultivation. But the same phenomenon is observed in every other line of production. Some elements vary only slightly; for example, from the same quantity of wheat one can obtain a little more or a little less flour. Other elements vary markedly: there is a vast difference between a millstone driven by a mule, and one of those big steam mills that are used nowadays to transform wheat into flour. There is also a vast difference between the crew that rowed the ancient galleys and the crew of a [transatlantic liner];¹⁷ [and there is, consequently, an equally large difference between the ratios of labor to the vessel's value in these two means of transport];¹⁸ [and an infinite number of other similar examples could be produced].¹⁹

The firm must make a choice between these different courses, whether it operates according to type (I) or type (II).

15. Here we find one of the most serious errors in political economy. It has been assumed that this choice is imposed by the technical conditions of production, i.e., that it is determined exclusively by the state of technical progress. This is not correct. Technical progress is only one of the elements of choice. Naturally, before railways were invented they could obviously not be used for the transport of commodities; but now that they are known, they have not replaced all other means of transport. In some circumstances, carts drawn by draft animals are still used; in others, wheelbarrows, and in others, still different means. Now that the sewing machine is known, one can sew by machine, but sewing by hand has not disappeared. For lighting, one still uses a combination of candles, oil, kerosene, gas, and electricity.³

16. In each case, it is necessary to determine which means of production to use. Suppose an entrepreneur has to transport gravel from a quarry to some other place; depending on the circumstances, it will be to his advantage to use horse-drawn carts or to construct a small railway. Another entrepreneur may have to saw lumber; he will either have the lumber sawed by men, or he will install a mechanical sawmill. In these cases and in other analogous ones the entrepreneur's decision will be determined not only by technical considerations, but also by economic ones.

To be able to choose among the various means, it is necessary to know what they are. Let us therefore start by selecting one of them and let us study it.

17. Capital goods.⁴ Suppose we wish to draw up the accounts of a mill that is driven by a water wheel.

Flour and bran are being produced. The principal elements of production are the mill stream, the mill shed, the water wheel, the shafting, the millstones,²² etc.; the tools, the lighting equipment, etc.; the oil for the machines; other materials for lighting, cleaning, and many other purposes; the labor of the miller and his assistants, the working capital for expenditures; and the wheat to be ground.

18. Some order must be put into these varied and different elements, and they must be classified, though this classification, like every other one, will be partly arbitrary.

³ *Systèmes*, II, pp. 372ff. 20

⁴ On the diverse meanings this expression can have, see Irving Fisher, "What is Capital?" *Economic Journal*, December 1896; "Senses of Capital," *ibid.*, June, 1897; "Precedents for Defining Capital," *Quarterly Journal of Economics*, May, 1904. The *Nature of Capital and Income; The Rate of Interest*. [These last two books are of capital importance.]²¹

See also my *Systèmes*, I, pp. 158, 357–62.

In reality, it is energy—the mechanical power of the water flow—that is transformed in the process of production; but, in the economic phenomenon, this element of production appears in different forms, i.e., in the form of the employment—or use—of the mill stream.

Similarly the building is also transformed, although only gradually, in the process of production. [This building cannot be conceived of as hanging in the air, without the ground it stands on. We can, however, separate by abstraction the building and the ground on which it is built.]²³ In that case, since the latter is in no wise [used up],²⁴ we have a factor of production that is being made use of without being consumed.

19. We may then extend this concept approximately to other objects, and distinguish two large classes of productive elements: the first comprises things that are not consumed, or that are [used up]²⁴ only slowly; the other comprises things that are consumed rapidly.

20. This classification is arbitrary and not very rigorous, as are the words: *slowly*, *rapidly*; but experience tells us that it is very useful in [treating the economic material].²⁵ Similarly, in speaking of men it would be very difficult to do without expressions such as *young* and *old*, although nobody can say at what precise moment youth ends [and at what precise moment]²⁶ old age begins. Common language is forced in this way to substitute arbitrary qualitative differences for real quantitative ones.

21. A name has been given to objects that are not consumed, or are consumed only slowly, in the process of production; they are called **CAPITAL GOODS**.

The precise boundary between the class of capital goods and that of the other elements of production is no better determined than that between youth and middle age.

Moreover the same object may, depending on the point of view considered, be classified as a consumption good or a capital good. In the preceding example, the mechanical energy of the water that drives the mill is consumed, so that from this point of view it may be said that energy is consumed in the production of flour; and we can enter in the firm's accounts so many units of horsepower consumed, at such and such a price per unit. But the same thing may be expressed in an altogether different way. To produce flour one uses the mill stream, which is not consumed but remains intact; and, in the firm's accounts one may enter so much for expenditures, not for the consumption, but for the use of the mill stream. In the final analysis, nothing is changed in the accounts.

22. If we want to use the concept of capital goods, we shall have no difficulty in including in this class the lade used to drive the mill as well as the mill shed. The water wheel may also be included. But what are we to say about the millstones? If we consider them as being consumed only slowly, we classify them as capital goods; but if we think of them as being consumed much faster than the building or the water wheel, we shall put them in the category of consumption goods.

23. If it were used incautiously, such a fluid classification might easily lead to meaningless conclusions; in fact, economists who have used similar qualitative classifications without reservations have often been given to veritable verbal quibbles.

However helpful, therefore, it may be for us to use common language, we should not hesitate to abandon it if there is no way to amend it and bring it back to quantitative reality.

24. Now there is such a way; it is enough to enter in the firm's budget certain expenditures incurred for the purpose of reconstituting the things that we consider to be capital

goods. We will then be justified in saying, in rigorous terms, that they are used without being consumed.

Let us imagine that our miller consumes precisely two pairs of millstones a year. He starts the year with one pair of new millstones, and he ends it after having consumed a second pair. If he wishes to consider the millstones as consumption goods, he will include the following among his expenditures: on January 1, the purchase of the first pair of millstones; on July 1, the purchase of the second pair. If he wishes to consider them as capital goods, he will include among his expenditures: on July 1, the cost of the first pair of millstones, to reconstitute the capital stock; and on December 31, the cost of a second pair of millstones, to reconstitute the stock of capital once again.

The expenditures are thus the same, in whatever way the millstones are considered. It is true that there is a difference in the dates at which they are incurred—we shall come back to this when we deal with transformations in time—but neglecting this for the moment, we can see that no matter how the millstones are classified, the end result in the budget is the same (and we shall see that it is still the same when we deal with transformations in time (§47)); and since it is only the end result in the budget that matters, we may adhere to the qualitative classification of capital goods, and include certain objects and exclude others, as we please^[a].

Likewise, it is of little concern to an insurance company with precise mortality tables whether it classifies a thirty-year-old man as young or middle-aged; in either case the death rate for him will be the same.

25. The theory of economic equilibrium with and without the concept of capital. Since economic equilibrium arises out of the conflict between man's tastes and the difficulties encountered in obtaining things that can satisfy them, one need consider only things that will be consumed directly or the use of which will be consumed. In the production of these things, one can consider exclusively the elements [of production] consumed, and in that case one abstracts from the concept of capital; or one can consider the consumption of certain commodities and the use of certain capital goods. In the end, the result will be the same. In either case, it is necessary to take account of transformations over time (§47).

Both these ways of considering the phenomenon tend to be used in practice. To obtain bread and satisfy one's hunger, one meets the obstacle consisting in the need for an oven to bake the bread. The oven appears here as a capital good; provided certain expenditures are incurred, it will last indefinitely and continue to produce bread. Or the obstacle consists in acquiring the things (bricks, limestone, etc.) which, after being consumed and transformed, will constitute the oven. In this form, there is no longer any capital; there is only consumption [of elements of production] spread over a more or less large amount of bread produced. There will be, in addition, the expenditures for transformations in time, with which we are not for the moment concerned.

In civilized countries, the oven and everything needed for its construction are considered as equivalent to their value in money; i.e., capital goods as well as consumption goods may be replaced by their money value. The obstacle then appears in a third form: the expenditures that have to be incurred.

26. Consequently, one of the obstacles to obtaining bread appears in one of the three following forms: the availability of the oven; the availability of the land, labor, bricks,

limestone, etc. needed to build the oven; and the availability of the funds to buy the oven, or to buy the things that are needed for its construction.

27. We have said that these funds have to be available, not that they have to be in one's material possession in the form of money. Indeed, owing to certain institutional arrangements that are common in civilized nations, considerable expenditure can be incurred with only a small amount of money, which circulates.

Many writers have fallen into a peculiar error as a result of overlooking this very obvious point. It was thought that the obstacle, in this third form, consisted in the material possession of the entire sum of money equal to the value of the object—i.e., the oven in this example. Then, reverting to the concept of capital and to its first form, it was concluded that capital consisted only of money.

The element of truth in this observation is that any capital good can be evaluated in terms of money. Similarly, all consumption can be evaluated in terms of money. [When we say that someone had a five-lire dinner, we do not mean that he ate a silver scudo;²⁷ or when we say that to produce bread a thousand lire's worth of something is needed, we do not mean that two hundred scudi or fifty marenghi²⁸ must be used materially to produce the bread. In either case, to incur a total expenditure of one thousand lire, the material use of ten marenghi may be needed; and then it is only these 10 marenghi, i.e., 200 lire, that can be considered to be capital.]²⁹

The study of economic equilibrium when only consumption is considered gives us an idea of the phenomenon as a whole, and makes us disregard its component parts. This may be useful in some cases, but as a rule we ought not neglect the study of these parts. It is true that obstacles in the way of travelling by train can be reduced, in the final analysis—apart from transformations in time, which will be dealt with later—to the labor and materials needed to build the railway, supply the rolling stock, and run it. There is thus no doubt that equilibrium will ultimately result from the confrontation between these obstacles and men's tastes for travel. But the jump from tastes to obstacles is really too great, and we must pause a little to consider what are the intermediate links in such a long chain. We shall have to consider separately at least the construction and running of the railway; we shall thus study the phenomenon in its first form and, if it is desired, in its third form.

28. Similar considerations hold for commodities that are used up in the production process. It is not clear in the above analysis why we have stopped at the stage of the bricks, limestone, etc., needed for the construction of the oven, and not gone further back—to the clay of which bricks are made, to the consumption [of elements of production] needed to make the kiln in which the bricks were baked, and so on; but we should thus be led to a concept of the phenomenon that is too general and too remote from reality. In the real world, there are different firms; and the one that produces bread does not generally produce bricks. They thus have to be considered separately.

[Some economists have attempted to reduce production in the last analysis to sacrifices of ophelimity. It is true that if production transforms only commodities that could be consumed directly—or the use of which, at least, could be so consumed—such an analysis is possible. But it is no longer so for the very large number of things that acquire ophelimity only after having been transformed. Thus, for instance, a copper mine has no use other than that of producing copper. The high cost of producing gold is not due to the fact that by working the gold mines, one makes the sacrifice of renouncing the

pleasure that could be derived from the direct use of these mines, because that pleasure does not exist. By parting with one's savings it is true that one forgoes the pleasure of contemplating those savings in the form of gold coins; but this has only a very distant connection with the rate of interest.

[One thus takes the wrong path, and one that cannot lead to any satisfactory result. One should, on the contrary, consider the totality of things at one's disposal and compare the results that will be obtained by allocating these things in alternative ways for production. These results can be evaluated in terms of money or in terms of the various pleasures and sacrifices they yield. These will produce conformities and differences, agreements and oppositions that will have to be studied.]³⁰

29. The firm's budget will therefore be drawn up in the following way. It receives from other firms certain commodities that it [uses up],³¹ [and enters their values under consumption];³² it possesses certain things called capital goods, which by means of accounting devices will be considered as remaining always identical in themselves. In its budget, these capital goods show up as the expenditures needed to reconstitute them, together with a certain sum paid for their use. In the example of the millstones, this sum will serve precisely to bridge the gap between the two phenomena that we discussed in §24. In the first of these—i.e., when the millstones are considered as consumption goods—they are entered in the accounts on January 1 and on July 1 as expenditures incurred in buying a pair of millstones; in the second—i.e., when the millstones are considered as capital goods—these expenditures appear in the accounts on July 1 and on December 31. We shall come back to all this when we consider transformations in time; we must now examine a little more closely the expenditures incurred to reconstitute the objects classed as capital goods.

30. Depreciation and insurance. Things may deteriorate^[a] slowly, because they wear out, or they may be destroyed completely or partly, for some fortuitous reason.

Maintenance and depreciation make the reconstitution of capital possible in the first case; insurance makes it possible in the second.

A machine is maintained in good order by repairs; nevertheless, it becomes old, and some day it will be better to buy another one than to go on spending money on its upkeep. A ship can be maintained in good condition by repairs, but not indefinitely. Depreciation should provide not only for physical wear and tear, but also for what is called economic obsolescence. For a day comes when a machine or ship, etc., may still be in perfect physical condition but has grown old, and the time has come to replace it with another machine, another ship, etc., of a more modern and improved type. In the budget the expenditure for repairs generally appears among the current expenditures; depreciation is accounted for separately and serves to reconstitute the capital stock.

Insurance³³ is the sum that must be saved every year and accumulated to provide for contingencies. A firm may self-insure the objects in its possession that are subject to contingencies. This actually happens sometimes in the case of large shipping companies, which self-insure their own ships. In this case insurance³⁴ shows up in the balance sheet in the same way as depreciation, and is a sum which constitutes a special fund administered by the company. In most cases, however,³⁵ the insurance is undertaken by another firm, which is concerned exclusively with this kind of operation. In such a case the firm that wants to insure some objects pays an *insurance premium* to one of these companies^[b], which will refund the value of the object if it comes to be totally or partly

destroyed as a result of some of the contingencies [specified in the contract].³⁶ [There is an infinite variety of possible contracts;]³⁷ but in essence, they are all the same, in that their object is always to reconstitute the capital stock.

31. Industrial companies usually have a third special fund—called the reserve fund—which serves various purposes, among which, however, that of insuring the corporate capital and reconstituting it when necessary is frequently³⁸ the most important. In fact, fortuitous circumstances result not only in the loss of material objects. A war, an epidemic, a commercial crisis, by changing the conditions under which an industry operates, may cause momentary and transitory losses. Some of the company's capital is then lost, and is reconstituted precisely by means of the reserve fund.

These brief explanations serve only to give some idea of the means with which one can provide for the reconstitution of capital, rather than to exhaust the subject (§68, VIII, 12 et seq.). It is enough for us to know that in one way or another we should provide for the reconstitution of capital, or else take its variations into account in some other manner.

32. Let us consider a house in a city where population is declining and real estate values are falling. This fact must be taken into account in its depreciation. Another house is located in a thriving city where real estate values are rising. This phenomenon is the opposite of the preceding one, and in order not to multiply terms we shall consider as negative depreciation the sum needed to keep the value of the capital constant. Likewise there can be a negative insurance premium when the contingency is advantageous rather than harmful to the object's owner.

Bonds provide a good example of all these phenomena. Suppose an individual pays 120 lire for bonds with a nominal value of 100 lire, redeemable in ten years by the company that will pay the bondholder 100 lire. The owner of these bonds possesses something which [decays in such a way that it]³⁹ is worth 120 lire today and will be worth only 100 lire in ten years' time. If these bonds are to be considered as capital, one will therefore have to provide for depreciation to make up the difference.

If these bonds were worth 80 lire today instead of 120 lire, there would still be a difference between this price and their price in ten years' time; but this difference would be to the bondholder's advantage, and this would be taken account of by negative depreciation.

If the bonds under discussion are redeemed by annual withdrawals instead of all being redeemable in ten years' time, the owner of a bond purchased for 120 lire loses 20 lire in that year when his particular bond comes up for withdrawal. He would gain 20 lire, if he had bought the bond for 80 lire. The first case corresponds to a positive insurance premium, the second case to a negative one.

[One should also take account of dynamic variations, of *appreciation* or *depreciation* of gold; but we shall abstract from this type of phenomenon here.]⁴⁰

33. The services of capital. Since, owing to a fiction that brings us closer to reality—and coincides with reality when depreciation and insurance are taken into account—capital is assumed to remain always in its original state, it cannot be said that it is being transformed into the product. Only its use serves to obtain this product and we may say that it is the service of capital that is transformed into the product.

It will be noted that this is only a formal question. Actually it is the energy—the mechanical power of the mill streams—which breaks the wheat down to produce flour; thus it is actually the energy of the mill stream which, combined with the grain, is transformed into flour. Fundamentally we express the same thing, though in

different ways, when we say that the use of the mill stream serves to produce flour, or that it is the SERVICE of the mill stream which, with the wheat, is transformed into flour.

34. Material and nonmaterial goods. Economists at the start of the 19th century engaged in long discussions as to whether every economic good was material, or whether there were also nonmaterial goods; and the outcome of the discussion was pure quibbling. The last word was surely spoken by [our]² Ferrara, who has shown clearly that "every product is material if we consider the way in which it manifests itself; and every product is nonmaterial if we consider the effect that it is destined to produce." It should immediately be added that the material identity of two things does not imply their economic identity. But this observation [takes us into another field].⁴¹

35. Production coefficients. To obtain a unit of output, one uses certain amounts of other products and capital services. These quantities are called PRODUCTION COEFFICIENTS.

36. If, instead of considering one unit of output one considers any amount of output, the quantities of the other products and of the capital services employed to obtain this amount of output are called the FACTORS OF PRODUCTION.

It is really useless to have two names for things that differ only by a simple proportion, and we shall, as a rule, use the term production coefficient. We mentioned the other one only because some authors use it.^[a]

37.⁴² The production coefficients may vary in several ways (§15, 76), and they are determined by the firms in different ways, depending on whether the economic phenomena correspond to type (I) or type (II).

38. TRANSFORMATIONS IN SPACE (III, 72). There is not a great deal to be said about these transformations. We must simply note that they provide us with a first example of things which, although materially identical, are economically different. One ton of wheat in New York and one ton of the same wheat in Genoa are materially identical, but economically different, [and hence they may have different prices];⁴³ the difference in their prices is not necessarily equal to the cost of transport from one of these localities to the other. This method of accounting for the price differential is founded on an inaccurate theory of economic equilibrium (III, 224).

There is never any lack of transformations in space: they are sometimes insignificant, and sometimes of major importance. Some firms—the transportation companies—concern themselves with them exclusively. The ease of transformation in space widens market areas and makes competition more extensive; these transformations are thus of great social importance. The 19th century will be remembered [in history]⁴⁴ as [the]⁴⁵ one in which great improvements in these kinds of transformations took place, resulting in extensive social changes.

39. Transformations in time (III, 72). These are entirely analogous to the preceding ones; but, whereas [the existence of] transformations in space has always been acknowledged, transformations in time have been and very often are denied. The reasons for this are numerous; I shall mention only two of them.

Transformations in space are accompanied by labor and a visible cost; hence the recognition of these does not jolt the prejudices of those who believe that the difference in the prices of two commodities can depend only on the difference in the amounts of labor needed to produce them, or more generally, on the difference in their costs of

production. With respect to transformations in time, on the other hand, the material dependencies in these transformations cannot be detected with the erroneous theory just referred to.

But there is another reason—the most important of all—why transformations in time have failed to be recognized. This is that the subject has been treated with sentiment instead of with reason, and that these sentiments are based on cupidity. Nobody—or hardly anybody—studies the question of transformations in time in a spirit free from all prejudice. Everybody knows, even before studying the question, how it should be settled, and argues like a lawyer handling the case of a client.

40. If we adopt a purely scientific attitude, we shall at once see that, just as two materially identical objects differ economically according to the place where they are available, they differ economically according to the time at which they are available. A meal today and the same meal tomorrow are not at all the same thing; when an individual suffers from cold, he needs an overcoat immediately; and this same overcoat, if made available in a day, in a month, or in a year, would certainly not render him the same service. It is thus obvious that two economic goods that are materially identical but are available at different times, can have different prices, just as goods that are not materially identical may have different prices. It is difficult to understand why people consider it natural for the price of wine to be different from that of bread, or for the price of wine in one place to be different from the price of the same wine in another place, and then are amazed when the price of this wine available today is not the same as that of the same wine available in a year's time.

41. But the craze of premature practical applications causes people who study the subject not to stick to the scientific problem we have just posed, but to run off at once to examine whether it would not be possible by some means to see to it that the price of wine available today should become precisely equal to that of wine available next year.

I do not wish to discuss this now, any more than I wish to investigate whether there are any technical means capable of making the price of wine equal to that of bread, or the price of wheat in New York equal to that of wheat in Genoa. It is enough to have brought out the fact that commodities available at different times are economically different commodities, and that they can therefore have different prices.

42. We shall learn from the theory of economic equilibrium how these prices are determined. One must thus be very careful not to fall into the error of saying that the *cause* of the difference in these prices lies in the difference in the times at which the goods are available. For there is not *one* cause of this difference, but a very large number; and it is these that in all circumstances, without a single exception, determine economic equilibrium. The sole purpose of considering time is to differentiate from one another two goods that are not available at the same moment. Similarly, chemical composition differentiates copper ore from metallic copper, but it is not the *cause* of the difference in their respective prices. This difference does not have a single cause, but a very large number of causes; or to express the matter more rigorously, it stands in relation with many other facts, that are precisely those that determine economic equilibrium.

43. **The budget of the firm and transformations in time.**⁴⁶ We saw in §26 that production can be considered in three different ways that essentially lead to the same result.

44. I. *Consumption activities are considered exclusively, without making use of the concept of capital.* In this case transformation in time consists in substituting a good that is available at a certain time for a good available at another time. To produce wheat, seed has to be used. The sowing may be considered as a consumption activity that takes place at seed time. This quantity of wheat is not economically identical with another equal amount of wheat available only at the time of the next crop. The two economic combinations for production—(A) 100 kilograms of wheat to be consumed at seed time; (B) 100 kilograms of wheat to be consumed at the time of the next crop—are not identical; they are different *commodities*; consequently, (A) may have a price that is different from that of (B); in general, this price is higher (exceptionally, it could be lower). The difference between the price of (A) and that of (B) is the price of a transformation in time, and shows up in the firm's expenditures. For instance, a person who sows wheat for the first time certainly cannot use wheat from his *last* crop for this purpose, since this does not exist; only the *next* crop will be available to him, in due course. In his budget he must thus enter a certain expenditure on the debit side for this transformation.

45. II. *One makes use of the concept of capital.* In this case, the transformation in time results from the need to have, or to produce, the capital *before* being able to produce the commodity. The price of the transformation in time will be part of the cost of the use of capital.

The seed needed to produce wheat may be considered as a capital good. It is consumed at the time it is sown, and it is reconstituted at harvest time; so that from the standpoint of the agricultural concern it remains intact, and it is only its use during a certain period of time that serves in the production of wheat. In 1895, an agricultural concern had, say, 100 kilograms of wheat; they were used as seed. In the 1896 harvest, 100 kilograms of wheat were set aside; they were again employed as seed in the same year. In the 1897 harvest, 100 kilograms of wheat were set aside. Let us stop here and draw up the balance of the operation. The farm started with 100 kilograms of wheat at its disposal, and it ends up with 100 kilograms of wheat. In fact, [therefore, in the aggregate]⁴⁷ it has not consumed any of it; it has simply enjoyed the *use* of this quantity. The transformation in time consists in this use, and the price of that transformation is part of the price of this use. If the firm is alone, the price of this use will be paid by the firm itself, and it will be in equilibrium with the sacrifices necessary to produce the object it uses. If the firm buys this object from some other firm, it will have to weigh the inconvenience of having to pay the price of the object in advance and against the benefit that it obtains from its use, and see whether they offset and balance each other. Finally, the firm, instead of producing or buying the object, may purchase only its use; and the price of this use will appear as an expenditure in the firm's budget.

46. III. *The money value of the factors of production is considered.* In this case the transformation in time involves money, and consists in exchanging a sum available at a certain time for an identical sum available at some other time.

Let us suppose that the 100 kilograms of wheat are worth 20 lire. For the agricultural concern, having these 20 lire in hand means having available the 100 kilograms of wheat needed for seeding. It is not necessary for the concern to have a marengo⁴⁸ physically available; half a marengo, for example, may be enough. With this money, it purchases 50 kilograms of wheat; then it sells cheese to obtain another half marengo, with which it buys another 50 kilograms of wheat; it will thus have a total of 100 kilograms of wheat.

Transformation in time thus consists in the fact that the farm needs to have, in 1895, twenty lire at its disposal, which it will repay only in 1897. It must therefore enter in its budget the expenditure necessary to have this sum available regardless of whether the payment is made to the concern itself or to others.

47. Let us go back to the example given in §24. If the miller considers his millstones as consumption goods, we have in his budget, on the expenditure side:

(A)			
January 1	100	lire	
July 1	100	"	
Total year	<u>200</u>	"	

If he considers them as capital good, the expenditures will be:

(B) ⁴⁹			
July 1	100	lire	
December 31	100	"	
Total	<u>200</u>	"	

Combination (A) involves the same expenditure as combination (B), but at a different time.

The millstones must be paid for with the flour produced. In combination (A) the millstones must be bought on January 1, and they will be paid for with the flour produced between January 1 and June 30; a transformation in time is thus needed in order to make available on January 1 what would [otherwise] be available only on June 30 in the same year. If one makes use of the concept of money, a sum of 100 lire which would [otherwise] be available only on June 30 must be made available on January 1. Let us suppose that 2 lire must be paid for this. The same operation will have to be repeated for the period from July 1 to December 31. In all, four lire will have to be paid, and the total expenditure for combination (A) will be 204 lire.

In combination (B), the millstones are paid for only on July 1 by which time a sufficient quantity of flour will have been produced, between January 1 and June 30, to permit this expenditure. But on the other hand, to be able to make use of combination (B), the use of this capital is needed. Exactly as in combination (A), the use of the millstones is required as from January 1. If this capital is evaluated in terms of money, the use of 100 lire is needed for one year; and, if 4 lire are spent for this use, the total expenditure on combination (B) will be 204 lire, which is the same as that for combination (A).

48. *Interest on capital.*⁵⁰ The obstacle that takes the form of the cost of the use of a capital good is partly independent of the social system of organization and has its origin in transformation in time. No matter how society is organized, clearly a meal that can be eaten today is not identical with a meal that can be eaten only tomorrow, and 10 kilograms of strawberries available in January are not the same as 10 kilograms of strawberries available in [May or]⁵¹ June. The social system of organization determines the form in which this obstacle will appear, and partly modifies its substance. Exactly the same holds for material transformations and transformations in space (VIII, 18, et seq.).

The same object can be produced by any of these three transformations. For instance, an individual in Geneva, in July, uses a piece of ice to cool his drink. This piece of ice may have been produced in an artificial ice plant (material transformation); it may

have been transported from a glacier (transformation in space); or it may have been collected during the winter and kept until summer (transformation in time).⁵ These transformations are bought at the price of certain inconveniences and costs, which partly depend on the social system of organization, and which are partly independent of it. For example, if the individuals in a community collect ice in January and gather firewood in July of the same year, they will enjoy cool drinks in July but will have endured the cold in January. If they could have gathered firewood in January and ice in the following July, the effort would have been exactly the same, and they would have been warm in winter and cool in summer. Having to advance the work of collecting the ice costs them the cold they endured in January, and this is obviously independent of the social system of organization.

If there is a second community which lends firewood to the first one in January, to be returned to it in July, the first community will no longer endure the cold; thanks to this loan, it will consume in January—not materially, but economically—the firewood that it will not gather until six months later, and will enjoy this transformation in time. The second community makes precisely the opposite transformation in time.

49. When capital goods are private property, the lender—who grants the use of it to others—usually receives a certain sum which we shall call the **GROSS INTEREST** on this capital^[a].

50. This interest is the price of the *use* of capital goods; it pays for their *services* (§33). Again this is a question of form and not of substance. If an individual pays 10 lire for a certain quality of cherries, he buys a commodity. Let us suppose that this quantity is precisely the output of a cherry tree in one year; if the individual, for 10 lire, buys the use of the cherry tree for one year, this amounts essentially to having the same quantity of cherries as before, for the same price. Only the form of the operation is different; he has now bought the *service* of a capital good (§33).

51. It should be noted that if the person who eats the cherries is the same as the one who owns the cherry tree, there is no longer anyone to whom the 10 lire can be paid; but the fact remains that this person enjoys the cherries. And this fact may be considered from two standpoints: (1) directly, as the enjoyment of a commodity; and (2) indirectly, as the enjoyment of the *service* of a capital good.

52. When the phenomenon is studied under the form of services of capital goods, we have to investigate how the prices of these services are determined, i.e., what is the amount of this gross interest. It would be easy to be convinced that it should be equal to the sum of all the expenditures necessary to reconstitute the capital, i.e., maintenance, depreciation, and insurance. But usually, gross interest is larger than this sum; and the difference, which we shall call **NET INTEREST**, is something whose origin is not so obvious.

53. To say that this net interest is payment for transformation in time is to put off the difficulty rather than resolve it; indeed, we would then ask why transformation in time has a price, and how this price is determined.

⁵ [These are the principal transformations in the three cases considered; but, in each of these cases, the principal transformation is accompanied by the two others, which are secondary. The manufacture of artificial ice does not produce the ice at the precise moment it is consumed; and a certain time is required to carry the ice from the glacier to the place where it is consumed. Transformation in time is thus not absent in these two cases, even though it is secondary. Likewise, transformation in space is not absent in the first and third cases. Finally, the material transformation, if only for supplying the ice cut in pieces, is not absent in the second and third cases either.]⁵²

54. It is tempting to try and see a causal relation between the existence of net interest and the ownership of capital. These are indeed concomitant facts; besides, it is obvious that if there were no owners of capital, there would be nobody to whom net interest could be paid; only the expenditures for reconstituting the capital would remain, and these would have to be incurred in any case. In other words, the obstacles that become apparent with the existence of net interest are due exclusively to the fact that capital is owned.

55. Such an assertion is far from being absurd a priori, and it could very well be true. The facts have thus to be examined in order to see whether or not they confirm this statement.

In Italy the obstacles encountered in obtaining sea water, if we disregard the labor and other expenses necessary to draw it out, are due exclusively to the fact that the government, having a monopoly of the sale of salt, forbids private persons from exploiting sea water. These obstacles therefore depend exclusively on the social system of organization; if the government left everybody free access to the water, all the obstacles that prevent Italians from obtaining sea water would vanish, except, of course, those to which we have referred, i.e., labor and the other expenses that must be incurred to transport the sea water to the place where it is to be used. Here then we have an example which seems to support the thesis that net interest on capital has its origin in the social system of organization.

The obstacles we encounter in obtaining cherries are apparent to us in the form of the price charged by the cherry merchant. This new instance seems then to be similar to the preceding one, and one is led to believe that it would be sufficient to eliminate the cherry merchants in order to abolish the obstacles in the way of obtaining cherries. But a little reflection will show us that this is not the case. Behind the merchant stands the producer; and behind the producer, there is the fact that cherries are not available in superabundant quantities, relative to our tastes, as in the case of seawater. Can we then say that the social system of organization, by virtue of which the cherry merchant exists, does not contribute in any way to the obstacles in the way of obtaining cherries? Certainly not; but it may be said that it forms only a part of the obstacles; and careful observation of the facts will also cause us to add that it usually forms a very small part of them, in comparison with the rest of the obstacles.

The obstacle we face in acquiring cherries—or, which amounts to the same thing, in having the *use* of a cherry tree—arises therefore from the fact that the cherries at our disposal are in smaller quantities than we would want to satisfy our tastes completely. And it is in the conflict between this obstacle and our tastes that the phenomenon of the price of the use of the cherry tree originates.

56. In general, the obstacle encountered in the use of capital goods—or in the corresponding transformation in time—arises from the fact that capital goods—or the means of effecting that transformation in time—are available in smaller amounts than we would want in order to satisfy our tastes. And it is in this conflict between the obstacle and our tastes that the phenomenon of net interest on capital—or of the price of transformation in time—originates.

Thus we are simply brought back to the general theory of the price of any object, which always originates in the conflict between tastes and obstacles; a conflict which can exist only when the object considered is available in a smaller quantity than would be desired to satisfy our tastes completely (III, 19).

57. Net interest is thus governed by identically the same laws that govern any price; and the cost of transformation in time obeys the same laws as the cost of transformation in space, or the cost of any [material]⁵³ transformation.

This cost of transformation in time cannot be determined independently of the other prices and of all the circumstances on which economic equilibrium depends; it is determined simultaneously with all the other unknowns, by the conditions of economic equilibrium.⁶

58. **Net interest on different capital goods.** It does not follow in the least from the preceding analysis that there is a single net yield on all capital goods, i.e., that the price of transformation in time does not vary with the circumstances in which it takes place. In fact, different capital goods yield different rates of net interest. Different net rates of interests are paid: for the use of a horse; for the sum of money this horse is worth; for the same sum lent on a mortgage, or on a bill of exchange, or on a simple I.O.U., etc.

The theory of economic equilibrium will show us that [we can approximately determine different classes of capital goods, and that in most of these classes the] net yields tend to become equal; and it will teach us also under what conditions this will occur. But it is essential not to confuse the properties characteristic of certain phenomena and the properties they acquire only in a state of economic equilibrium.

59. **The balance sheet of the firm and yields on capital goods.** The balance sheet of a firm must be drawn up at a particular time; and all the receipts and expenditures of the firm must be carried back to this date, adding or subtracting a certain sum depending on the net interest rate. This is usually done with simple interest for short periods, and by compound interest for longer periods.

In insurance computations, one often considers the *present* value of a *future* sum. Let us suppose, for instance, that a company is to pay 100 lire at the end of each year to an individual who is thirty years old, for the rest of his life. Let us take the experimental data gathered by the English insurance companies. By certain devices, which there is no point in going into here, these data are modified in such a way as to smooth out certain irregularities that are assumed to be accidental. We thus know that, out of 89,865 living individuals who are thirty years old, 89,171 will still be alive at the age of thirty-one; 88,465 at the age of thirty-two, etc. Consequently, if 100 lire were to be paid to each of these individuals, at the end of the first year 8,917,100 lire would have to be paid out; at the end of the second year 8,846,500 lire, etc. It is assumed—and this is hypothetical—that the future will be similar to the past; moreover, for each individual one uses numbers that are proportional to those we have just cited; i.e., it is assumed that on the average the amount to be paid to each individual will be

$$\frac{8,917,100}{89,865} = 99.228 \quad \text{at the end of the first year};$$

$$\frac{8,846,500}{89,865} = 98.442 \quad \text{at the end of the second year};$$

and so on.

⁶ *Systèmes*, II, pp. 288 ff.

One then computes the amounts which, with interest compounded yearly, reproduce the above sums. Here, an assumption must be made regarding interest. Let us suppose it to be 5 percent. A sum of 94,503, yielding 5 percent, becomes 99,228, after one year; a sum of 89,290 becomes 93,754.5,⁵⁵ after one year, and 98,442 after two years. We may thus say that the *present* value of 99,228, payable in one year, is 94,503; and the present value of 98,442, payable in two years, is 89,290.

60. Industrial accounts are drawn up more simply. Most interest is simple, and it is taken into account only in an approximate way.

Briefly, each balance sheet, to be accurate, must be drawn up at a particular time, and every expenditure and receipt must be assessed at that date. Let us suppose that the balance sheet is prepared on January 1, 1903, and that the interest on capital is 5 percent. An expenditure of 1,000 lire incurred on June 30, 1902 should be recorded as 1,025 lire in the balance sheet. The same is true for the receipts. In ordinary accounting, this expenditure, or this receipt, would appear as 1,000 lire on June 30; but in the case of expenditure, a 25 lire item is recorded for interest expenditure, and in the case of receipts an equivalent amount is recorded as interest received. In the end, this comes to the same thing.

61. The budget of the firm, and the entrepreneur's labor and capital. In the firm's accounts every expenditure must be recorded; if the entrepreneur [contributes anything to or]⁵⁶ performs any service for the firm, he must assess its value and enter the corresponding amount in the expenditures.

A man may be the manager of a corporation, or of a firm owned by some other individual, and in this case he receives a salary; or he may be the manager of his own business, in which case his salary is lumped together with the profit he makes from the business; but we must put an end to this confusion if we want to know the exact cost of the firm's output and the results of its operations. Likewise the capital this man brings to his own business must be considered as having been lent to it, and the interest on it should figure among the expenditures. Take a man who has been earning 8,000 lire a year by managing someone else's business, then takes over and runs it on his own, and invests 100,000 lire in the plant. The apparent⁵⁷ profit of this firm, without taking account of the owner's labor and capital, is, say, 10,000 lire. In fact, there is a loss of 2,000 lire, since 8,000 lire should have been entered among the expenditures for the manager's salary as well as 4,000 lire for interest on capital. If the man had continued to work as someone else's manager and had bought bonds yielding 4 percent, he would have had an annual income of 12,000 lire; [as it is],⁵⁸ he obtains only 10,000 lire, and thus loses 2,000 lire.

[This is only one of the ways of drawing up the profit and loss statements, on given assumptions. Any other accounting method would do, provided it keeps an exact account of the facts. If a salaried manager wants to know whether it would be profitable for him to resign and establish his own business, he will obtain an answer from his accounts if they are properly kept.]⁵⁹

62. The firm and the owner of economic goods. The firm, as we already remarked in §4, is only an abstraction by which one part of the production process is isolated.

The producer is a complex being, in whom the entrepreneur, the manager, and the capitalist are combined; we have now separated them but that is not enough; we have yet to consider the owner of certain economic goods which the firm makes use of.

Let us take a landowner who produces wheat on his property; he may be considered as a producer (III, 102) who obtains a commodity at a cost that increases with output. But two aspects must be distinguished in this individual: (1) the owner of the land; (2) the entrepreneur who employs the land and other economic goods to produce wheat. To give a concrete form to the abstraction, let us consider an entrepreneur who rents this piece of land and produces wheat.

63. If the producer finds himself on the side of the positive indices, he makes a profit. To whom will this profit go, now that the landowner and entrepreneur have become separated?

The problem may be resolved by means of the general principles already set forth. Suppose that for the landowner, land—of which the quantity that he owns is represented by oh —has no direct ophelimity. Along the oa -axis let us measure the sum in money that the landowner draws from his land. This is case (IV, 54); the exchange line for the landowner is hoa . For the entrepreneurs the axes will be hn , ho . Let hk be a line such that, for an arbitrary amount hb of land, the entrepreneur breaks even if he pays bd ; for him hk will be an indifference line, and precisely the one with index zero, i.e., that of complete transformations. If kk' is made equal to 1, the curve $k'h'$, parallel to kh , will be another indifference curve, i.e., the one with index 1, and along it the entrepreneur will make a profit of 1. Beyond hk lie the curves with negative indices.

64. If the entrepreneur has a monopoly, he will obtain the maximum profit by moving onto the indifference curve $h''k''$ which passes through o . He will keep all of the profit from production, and the owner will have none. If there is competition among the entrepreneurs, he will end up on line hk , for the reasons already explained so many times. The equilibrium point lies at k , at the intersection of hk and the landowners exchange line oa . The landowner takes all the profit from production, and the entrepreneur takes none. The same would obviously be true if the land, or any other commodity of this kind, had an ophelimity for the landowner.

65. It may be concluded that, when there is competition among firms, they must stay on the line of complete transformations; consequently they make neither profit nor loss.

The indifference curves of the obstacles do not, and cannot, change; but the curve of maximum profit for the landowner becomes the curve of complete transformations for the firm.

We now have to find out how, and to what extent, this theoretical proposition holds for actual firms, which differ in greater or lesser degree from theoretical ones.

66. Actual firms^[a] and their profits and losses. It is, first of all, obvious that the theoretical proposition can be true only on the average with respect to actual firms. Indeed, these differ from abstract firms in that they [are at the same time owners of]^[61] a certain

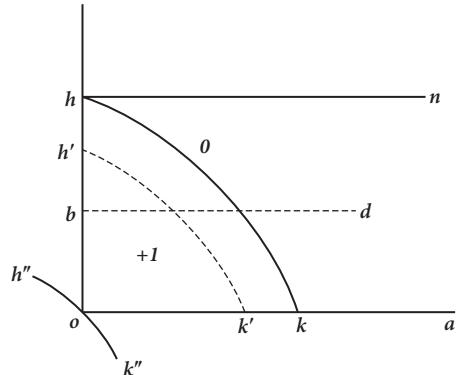


FIG 42⁶⁰

organization, a certain reputation which attracts customers, certain pieces of land, mines, workshops, which they have purchased, etc. The abstract character of entrepreneurship is always combined in a greater or lesser degree with that of ownership.

67. If one reasons objectively, it is easy to see that for actual firms—[or at least, for a very extensive class of them, and] ⁶² on the average—there can be neither profit nor loss, provided of course that all expenditures are taken into account, including interest on the firm's capital. Nowadays, a large number of these firms are of corporate form, and their shares are sold on the Stock Exchange; from those remaining, new corporations spring up every day. Consequently, any person who has some money, even a small amount, can take part in these companies by purchasing one or more shares. It would therefore really be inconceivable for these to have any advantage over government or other bonds on which fixed interest is paid. If such an advantage existed, everybody would buy corporate shares. We have said that all circumstances must be taken into account; one must therefore take account of the uncertainty of dividend payments, of the fact that corporations have a relatively short life, etc. Thus, while their shares may seem to have higher yields, if appropriate deductions are made the yield becomes equal, on the average, to that of fixed-interest [government] ⁶³ bonds. For instance, in Germany coal mining stocks, which yield about 6 percent, are approximately equivalent to Prussian bonds yielding $3\frac{1}{3}$ percent.

68. It may be observed, moreover, that this equivalence is partly subjective,⁶⁴ that is, it is a fact that the Germans believe in this equivalence—otherwise, they would sell their Prussian consols to buy mining or other stocks—but the real situation could differ somewhat from the idea that people have of it.^[a]

The concrete phenomenon thus differs from the theoretical one. For short-term, frequently repeated operations, which may be subject to numerous adjustments and readjustments, it seems that this differential should be slight; but we cannot assert a priori that it is equal to zero; it seems rather that, although slight, it will always exist.

Take, for instance, two uses of savings that yield an equal net interest—taking account of insurance and depreciation premiums—but where the first has a probability of large profits and large losses, which is absent in the second (VIII, 12).

A venturesome people will prefer the first investment, and a conservative people, the second; consequently, as a result of the difference in demand for these uses of capital, the net yields need no longer be equal. A venturesome people will prefer to buy shares in industrial companies rather than government bonds; a thrifty and economically timid people will do the opposite. It may thus happen that in the real world, industrial firms will have a small profit or a small loss on their own account.⁶⁵

69. Only experience can provide us with further relevant information; fortunately, statistics elaborated with great care by the *Moniteur des intérêts matériels* offer a way of obtaining an empirical idea of the phenomenon.

This excellent newspaper has patiently carried out research, using official documents, on the fortunes that have befallen Belgian corporations that were founded between 1873 and 1887. In all there were 1,088 of them with a total capital of 1,605.7 million. Deducting 112.6 million which had not been paid up, there remains an initial total capital of 1,493.1.

Of these companies, 251, with a capital of 256.2 million have gone out of business, and it is not possible to find any trace of them; probably their whole capital was lost.

Another 94, with a capital of 376.5 million were liquidated, having apparently lost their entire capital. Also liquidated were 340 companies, with a capital of 462.4 million—which repaid approximately 337 million; 132 companies, with a capital of 166.8 million which were liquidated with a profit, having repaid 177.5 millions. The total repayments amounted to 514.5 million. Of the capital invested in the companies, partly lost and partly still active in 1901, there remains 978.6 million. This totals to the above original amount of 1,493.1.

The total income⁶⁶ earned by the surviving companies was about 55.9 million a year. When this is related to the initial capital we see that in the final analysis the capital has yielded 5.7 percent.

This is not very different from the yield of an ordinary money loan.

The net yield has to be less than the figure arrived at because from this income of 55.9 million one would still have to deduct depreciation and insurance premiums, whose precise amount is not known. But, basing ourselves on the 5.7 percent rate, we may note that from 1873 to 1886 there were plenty of opportunities to buy government bonds of perfectly solvent states yielding 4 or 5 percent. We thus see that, in Belgium, the income earned from savings invested in corporations is just about equal to what one could have obtained by buying government bonds from creditworthy states.

It should also be noted that the profits⁶⁷ of some of these companies, for example the mining companies, include the profits⁶⁷ of the landowners.⁶⁸

Even if—to allow for the unreliability of these statistics—we assume that the 251 companies that disappeared without leaving any trace had repaid half their capital—and all those who are familiar with the stock market know how ill founded this assumption is—net yield would have been less than 6.6 percent; consequently, the difference from the average interest on an ordinary loan is not large, if any exists.

These results are confirmed by other statistics published in the same newspaper on January 31, 1904.

Between 1888 and 1892, 522 corporations were founded in Belgium, with a capital of 631 million francs, according to the most recent accounts. Still to be paid are 37.3 million; the true capital thus amounts to 593.8 million.

There are 98 companies—with a capital of 114.3 million, about which there is no longer any information. Let us suppose that they repaid half of their capital, i.e., 57.6 million. Thirty-eight companies, with a capital of 51.7 million, on which 4 million remained to be paid, were liquidated, with a profit of 3.6 million; they thus repaid 51.3 million. Ninety-five companies, with a capital of 94.7 million, on which 3.1 million remained to be paid, were liquidated at a loss of 18.6 million; they thus repaid 73 million. Five other companies were liquidated with a minimal loss, and they repaid 35.5 million. The total repayments amount to 216.4 million. Thus, a capital of 377.4 million is left over.

The annual profit was 12.5 million, hence the yield was 5.9 percent.

Naturally, if the firms that incur losses and are liquidated are not taken into account, the yield becomes higher; and this is what gives rise to the preconceived opinion that where there is free competition, firms earn a profit considerably in excess of the net current yield of capital. This prejudice is further strengthened by the practice of confusing the profit of the firm⁶⁹ with the profit of the owner,⁷⁰ or with the returns⁷¹ from certain exclusive rights, from patents, etc.

The average yield is obtained by summing the high and low yields. The newspaper referred to has calculated these yields for various firms in its March 31, 1901 issue. For banks, they vary between 10.7 and 1.8 percent; for railways, between 20.4 and 1.6 percent; for tramways, between 9.6 and 0.8 percent; for coal mines, between 17.8 percent (leaving out an exceptional case with 38.3 percent) and 0.86 percent; for ironworks and mechanical industries, between 12.9 and 2.10 percent; for zinc products, between 30.9 percent (Vieille Montagne) and 11.8 percent; for flax manufactures, between 16.5 and 0.66 percent, for glass factories, between 13 and 3.1 percent. All these yields were calculated in relation to the nominal capital.

In short, leaving all theory aside and making generous allowances for the imperfections and unreliability of the statistics, the facts show that—at least in Belgium—where free competition prevails, firms obtain on the average a net yield on their capital that differs only slightly from the current yield on loans, if indeed these two kinds of yields are not very nearly equal.

The facts thus correspond very closely to the theoretical deductions.

70. Variability in production coefficients. [We have already noted (§15) the error in the belief that the production coefficients depend solely on the technical conditions of production.

[Another completely erroneous theory is the so-called theory of *definite proportions*. This expression is particularly ill chosen, since it is borrowed from chemistry which has, indeed, recognized that the elements are combined in strictly defined proportions; whereas, on the contrary, the factors of production in political economy may, within certain limits, combine in any proportions. Two parts of hydrogen combined with one part of oxygen produce water; but it is impossible to obtain combinations consisting of two and one-tenth parts, two and two-tenths parts, etc., of hydrogen with one part of oxygen. On the contrary, if, in a certain industry, 20 units of labor are combined with 10 units of capital,⁷² in the same industry one will find slightly different proportions, such as 21, 22, etc., units of labor for 10 units of capital.⁷²

[But we shall not belabor this point. The names of things do not matter; it is the things themselves that must be studied.

[Now most of the economists who have adopted the theory of *definite proportions* seem to believe that there are certain proportions in which factors of production must be combined, independently of the prices of these factors. This is wrong. Where labor is cheap and capital is expensive, labor will take the place of machines, and vice versa. There is no objective property of the factors of production corresponding to the fixed proportions in which these factors are supposed to be combined; there are only proportions which vary with prices and which correspond to certain maxima, either of money profit or of ophelimity.

[This is not all; these ratios vary not only with the prices of the factors of production but also with all the circumstances of economic equilibrium.

[Ask a chemist in what proportions hydrogen combines with chlorine, and he will have no hesitation in replying. Ask an entrepreneur in what proportions labor and capital must be combined to transport a load of merchandise; he will not be able to answer, unless you first mention the prices of labor and of capital. That will not be enough. He will also want to know the quantity of merchandise to be transported, the distance to which it is to be moved, and a host of other similar circumstances.

[These considerations are general and apply to any kind of production. Save in exceptional cases, there are no fixed proportions that should be assigned to the production coefficients in order to obtain the maximum money profit; rather, these proportions vary not only with prices, but also with all the circumstances of production and consumption.

[Naturally, there are limits beyond which the variability of the production coefficients cannot extend. For instance, however perfect an extraction process may be, it will never be possible to extract more metal from a piece of ore than the ore actually contains. One can, by means of very advanced methods, obtain 40 hectoliters of wheat from a hectare of land which formerly yielded only 10, but, at least in present circumstances, it is certainly not possible to obtain 100 hectoliters.

[Technical conditions set limits within which the determination of the production coefficients is an economic problem.

[In short, these coefficients cannot be determined independently of the other unknowns of economic equilibrium; they are in a relation of mutual dependence with the other quantities that determine economic equilibrium.]⁷³

The firm's main task in the production process is to determine the production coefficients in relation to all the other [technical and]⁷⁵ economic conditions.^[b]

71. At this stage, two types of phenomena must be distinguished, exactly as was done in the case of the consumer and the producer (III, 40). At the present time firms generally follow type (I). They make their calculations according to the prices prevailing in the market, without paying attention to anything else; and it would be impossible to act otherwise. A firm sees that at the market prices it can reduce its costs of production by decreasing the quantity of labor and increasing the quantity of capital⁷² (machinery, etc.). It follows this course without further ado. In fact, the increased demand for savings may cause the interest rate to rise; and the reduction in demand for labor may cause its price to fall; but the firm has absolutely no criterion for estimating these effects, even in a roughly approximate way, hence it will abstain from doing so. Moreover, whatever the causes of a phenomenon, one need merely observe how any firm makes its calculations to see that this is so. If a day comes when the *trusts* take over a large segment of production, this state of affairs may change, and numerous industries will behave according to type (II) in the determination of the production coefficients; this stage has not yet been reached,

⁷ [As the literary economists are not only incapable of solving a system of simultaneous equations, which is the only way to obtain an idea of the mutual dependence of economic phenomena, but even unable to understand what it is, they make superhuman efforts to treat phenomena in isolation which they are unable to consider in their relations of mutual dependence. It is to this end that they have thought up vaguely metaphysical theories of *value*; it is to this end that they have attempted to *determine* the selling price by the cost of production; it is to this end that they have elaborated the theory of *definite proportions*; and it is again to the same end that they continue to spout a stream of erroneous propositions.

[We are speaking here only of those who try to deal with questions of pure economics without having the indispensable knowledge for doing so. Nothing is further from my intention than to belittle the work of economists who deal with questions of applied economics by means of practical considerations. One can be a distinguished engineer and have only a very superficial idea of the calculus; but one should then be wise enough to abstain from writing a treatise on the calculus.

[It should be added that there are mathematicians⁷⁴ who have undertaken to deal with questions of pure economics without having the necessary economic knowledge^[a], and have fallen into errors comparable with those of the literary economists.]⁷³

but this does not prevent many enterprises from following type (II) in the sale of their products.

72. We must have a good understanding of the way in which the firm operates. It bases its calculations on market prices, and modifies its demand for economic goods and labor accordingly. But these modifications in demand change the prices, whence the previous calculations are no longer correct; the firm repeats its calculations on the basis of the new prices; once again, the modifications in demand on the part of the firm and of other firms which act in a similar manner, change the prices; whence the firm must again redo its calculations; and so on, until, after successive trials, it finds the position where its production cost is a minimum.⁸

73. As we have seen in similar cases (III, 122), competition compels even the producer who would not like to, to follow type (I). It could happen that a firm will refrain from increasing, say, the labor it employs, for fear of causing its price to rise; but what this firm tries to refrain from doing, another competing firm will do; and the first firm will be forced to follow suit if it does not want to find itself at a disadvantage and be dragged down to ruin.

74. It should further be noted that as competition pushes the firm on to the line of complete transformations, the result is—if the phenomenon is considered on the average and in the fairly long run—that it is actually the consumers who ultimately derive the larger part of the benefit resulting from all this effort on the part of the firms^[a].

In this way the competing firms end up at a point where they did not have the least intention of going (§11). Each one of them was seeking only its own advantage, and was concerned with the consumers only to the extent that it could exploit them; and, on the contrary, as a result of all the successive adjustments and readjustments imposed by competition, all this striving on the part of the firms turns out to be to the benefit of the consumers.

75. If no firm were to gain anything from this process, the game would not last very long. But in fact it happens that the fastest and shrewdest make a profit for a time, until the equilibrium point has been reached;^[a] whereas those that are slower and less alert will lose and be ruined.^[b]

76. There are certain relations among production coefficients which make it possible to offset a reduction in some of them by an increase in others; but this is not true of all the coefficients. For instance, in agriculture a reduction in the area cultivated may be offset, within certain limits, by an increase in capital⁷² and labor, so as always to obtain the same output. But it is obvious that one could not maintain the same output of wheat by enlarging the granaries and reducing the area cultivated. A goldsmith may increase labor as much as he wants, but from a kilogram of gold he will never be able to extract more than a kilogram of gold jewelry of the same fineness.

77. There are also cases where such substitution would be theoretically possible but not economically so; there is no point in considering all the relations between the production coefficients that do not fall within the scope of practically possible limits. For instance, it is pointless to investigate the possibility of reducing the amount of labor needed to tinplate copper pans by using gold pans instead. But if the price of silver were

⁸ *Cours*, §718.

to fall further, one might envision the substitution of silver pans, or of silverplated copper pans for the copper ones.

78. Distribution of output. The cost of production depends not only on the quantities⁷⁶ transformed, but also on the number of producers or firms. For each of these there are overhead costs that must be allocated over its output; and moreover, the technical and economic conditions of production change according to whether the size of the firm is large or small.

79. It has been believed that firms operate under more favorable conditions the larger their output; and this view has given rise to a theory according to which competition⁷⁷ must lead to the formation of a small number of large monopolies.

The facts do not bear out this theory. It has long been known that, in agriculture, for each kind of productive activity there are certain limits to the size of an enterprise that it does not pay to exceed. For example, olive growing in Tuscany and cattle breeding in Lombardy are two entirely different kinds of enterprise. It would not be at all in their interests for the great Lombard tenant farmers to rent olive groves in Tuscany, where instead sharecropping continues to flourish.

It has been shown by numerous facts that, in industry and trade, concentration of firms beyond certain limits is more harmful than useful. It used to be said that the department stores in Paris would ultimately be concentrated in a single one; instead they have multiplied, and their number continues to increase. As for American *trusts*, some have prospered, but others have failed badly, with heavy losses.

80. It may in general be assumed that for each kind of productive activity there is a certain size of firm which corresponds to the minimum cost of production; consequently, left to itself, production tends to be distributed among firms of this kind.^[a]

81. General equilibrium of production. We have already seen (III, 208) that for phenomena of type (I) equilibrium is determined by certain categories of conditions⁹ which we denoted by (D) and (E). The first category of conditions, (D), establishes that the costs of production are equal to the selling prices; the second establishes that the quantities demanded for the purpose of transformation are the quantities actually transformed.

Nothing fundamental is changed in these conditions when capital is taken into consideration; only the form differs, in that the transformation not only of commodities but of commodities and capital services is taken into account.

It will be noted that not every commodity need have its own cost of production. For instance, wheat and straw are produced at the same time and have a joint cost of production. In such cases there are certain relations that determine the proportions in which the commodities are thus combined; for example, we know how much straw is obtained [per unit of wheat].⁷⁹ Such relations form part of the category (D) of conditions.

82. We now have to take the variability of production coefficients into account. We begin by assuming that the entire quantity of a commodity Y is produced by a single firm. In the phenomena of type (1) which we have just been considering, the firm accepts the market prices and makes its calculations with these prices in order to determine how best to adjust its production coefficients to them.

⁹ [Some authors misinterpret these conditions as theorems. One must indeed be very ignorant not to be able to distinguish things that are so different.]⁷⁸

Suppose that it is possible at market prices—for example at a cost of 5 lire per man day—to produce the same quantity of Y by cutting down labor outlays by 50 lire per day, provided expenditures on machinery are increased by 40 lire per day; it will obviously be in the firm's interest to perform this operation.

But when, [as a result of this choice],⁸⁰ the demand for labor has fallen and that for machines has risen, the prices will change; and so will the total quantity of commodity Y produced by the firm, because at the new price of Y a different amount will be sold.

Once again, with these new prices and the new total output, the firm will repeat its calculations. And this will continue until, for certain prices and certain quantities, the saving in labor expenses is equal to the extra expenditure on machinery; at that point the process will stop.

83. In the case of phenomena of type (II), the path followed is different. When it is possible in practice, which is very often not the case, the changes in the prices and quantities are taken into account immediately. Consequently, in the preceding example, the firm will not base its calculations on the assumption that the workman's daily wage is 5 lire, but will estimate it at, say, 4.80 lire, to take account of the fact that the wage rate may⁸¹ fall with the fall in the demand for labor; it will do the same for machinery, and likewise for the quantity produced.

To act in this way, one obviously has to know how to calculate the variations in prices and in quantities; in practice this is rarely the case, and even then only in cases of monopoly. A farmer can easily calculate, on the basis of market prices, whether it is more to his advantage to employ the power of a horse or that of an engine to drive a pump; but neither he nor anybody in the world can possibly know what effect the substitution of the engine for the horse will have on the prices of horses and engines; or what extra quantity of vegetables will be consumed once consumers enjoy the savings resulting from this substitution.

84. Let us come back to the case of phenomena of type (I). In general, there are several producers. Production is distributed among them, as described in §§78–80; and each of them then determines its production coefficients, as if he were the sole producer. If this alters the distribution, calculations are repeated with the new distribution, and so forth.

85. The conditions thus obtained for the distribution [of output among firms] and for the determination of the production coefficients, will make up a category that we shall call (E)^[a].

To determine the production coefficients, there are first of all the relations holding between these coefficients, and the relations indicating which coefficients are constant; then come the conditions by virtue of which the values of the coefficients are determined so as to obtain the minimum cost of production (§82).

It can be proved in much the same way as before that conditions (F) are equal in number to the unknowns to be determined [(Appendix 27)].⁸²

86. For phenomena of type (II), some of the conditions (D)—that is, those corresponding to firms which follow type (II)—are replaced by other conditions which indicate that these firms derive the maximum profit from their monopoly position. This profit is generally expressed in money. Conditions (E) remain the same. Conditions (F) will change, either because, as we saw in §83, the path followed is different, or because there may be a monopoly of certain factors of production, or of certain firms.

87. In general, when an entire community is considered, [and if one limits oneself to studying economic phenomena without taking other social phenomena into account, it can be said that]⁸³ the sum, in money, of the firms' sales is equal to the sum spent by consumers [saving being considered as a commodity]⁸³ and the sum of the firms' purchases is equal to the [sum of the]⁸³ incomes of the individuals [comprising]⁸⁴ the community.

88. Production of capital goods. The principles we have just established are general and apply to any kind of productive activity, but among these there are some kinds that deserve separate consideration.

Capital goods are often produced by the firms that use them, but often by other firms, too. These are commodities that provide a return only by the interest they yield; hence, whoever produces or purchases them thus has to pay a price for them equivalent to this interest,⁸⁵ once equilibrium is achieved and⁸⁶ if one operates according to type (I).

But under these conditions the selling price is equal to the cost of production; and on the other hand there is only one price in the market for the same commodity. It follows from this that, under the above conditions, the net yields (§52) on all capital goods must be equal.

This conclusion is strictly subject to the assumption made that all these capital goods are produced at the same time.

[Moreover, this is only given as the main part of the phenomena, in general, as when we say that the earth is a sphere.

[As a second approximation, one should establish broad classes of capital goods, and take into account the restrictions of the kind previously set out (§58 et seq.).]⁸⁷

89. Successive equilibrium positions. Let us consider a number of successive and equal periods of time. In general, the equilibrium position changes from one such period to another. Let us suppose that the price of a certain commodity A is 100 in the first period and 120 in the second. If, in each period, precisely the quantity of A produced in this period is consumed, then we can only say that the first portion of A will be consumed at the price of 100, and the second at the price of 120. But if, in the first period, some (or all) of A is left over, the phenomenon becomes much more complex and gives rise to very important considerations.

The portion of A that is left over had a price of 100; but it is now assimilated to the new portion of A whose price is 120, which will consequently have this price, too. In this way the owner of this portion of A, whether an individual or the community, makes a profit equal to the difference between the prices, i.e., 20, multiplied by the amount left over. The owner would incur a similar loss if the second price were less than the first.

Moreover, this gain would be only nominal, if the prices of all other commodities had increased in the same proportion; and, for the possession of A to have some advantage over the possession of B, C, . . . , these proportions would have to be different.

90. Rent. When the idea of capital is introduced, the phenomenon, although remaining substantially the same, takes on another form.

Let A represent a capital good. As we saw in §24, the accounts are drawn up in such a way that it can be assumed that A is employed without being consumed, that only its use is enjoyed. Consequently, it is not a portion of A, but the whole amount of A, that remains after the first period, [and shows up again in the second].⁸⁸

Let us start by assuming that the net yield on capital is the same in the first and second periods and that it is, say, 5 percent. This means that A, which had a price of 100 in the first period, then yielded a net interest of 5 units; and that, in the second period, having a price of 120, it yields a net interest of 6 units.

Conversely, prices can be derived from yields. Let A be a non-produced capital good, e.g., land. In the first period it yielded a net interest of 5 units; from this, one may infer that its price must be 100; in the second period, it yields a net interest of 6 units; one may conclude that its price has become 120 units.

This represents a gain for the owner of this capital good A; but, if every other capital good has risen in price in the same proportion, there is no advantage in having A rather than B, C, . . . If, on the contrary, the capital goods have not all risen in price in the same proportion, the possession of one of them may be more or less advantageous than that of another.

91. Let us suppose that, on the average, the price of every capital good has increased by 10 percent; the price of A, instead of being 100, should then be 110, and at 5 percent it should yield a net interest of 5.50 units; consequently, compared with other capital goods, A yields an additional .50 of net interest. We shall call this the *acquired rent*^{89[a]} from passing from one position to another.¹⁰

92. Let us next suppose that the change also affects the net rate of interest; it was 5 percent in the first situation, and it becomes 6 percent in the second. In this case A, which was worth 100 in the first situation, yielded a new income of 5 units; and since it is worth 120 in the second situation, it will yield a net interest of 7.20 units. But suppose that on the average the prices of all capital goods have risen by 10 percent. If A had been subject to the same conditions as that average, its price would have been 110, and it would yield, at 6 percent, a net interest of 6.60 units; instead it yields a net interest of 7.20 units; the difference, that is, .60, measures the advantage to the possessor of A; this is the *acquired rent* in passing from the first situation to the second.¹¹

93. The rent of land, or Ricardo's rent, is a special case of the general phenomenon we have just examined.¹² It has given rise to innumerable discussions, most of them pointless. It was investigated whether landed property alone had this privilege, and acknowledged by some that the phenomenon was more general. Others denied the existence of rent, with the aim of defending the landlords; still others, on the other hand, in opposing them, saw in rent the origin of all kinds of social woes.

94. Ricardo asserted that rent does not form part of the cost of production. First of all we have here the common mistake of believing that the cost of producing a commodity is independent of the remainder of the economic phenomenon. But leaving this aside, and turning to the reasoning that purports to prove that rent is not part of the cost of production, we see that it boils down to the following propositions: (1) It is assumed that a commodity, e.g., wheat, is produced on pieces of land of decreasing fertility; (2) it is assumed that the last portion of the commodity is produced on land that yields zero rent. Since a commodity has only one price, this is determined by the cost of production, equal to the selling price of the last portion; and this price will of course not vary if, for

¹⁰ *Cours*, §§ 746ff.

¹¹ The general concept, with algebraic symbols, is set forth in my *Cours*, § 747, footnote.

¹² *Cours*, § 753.

the first portions, the rent, instead of being collected by the landlord, is collected by the entrepreneur; it will simply be a gift to the latter.

95. It should be noted that the second assumption is often incorrect, and that there may be a rent on all the properties.⁹⁰ Furthermore, even granting that these assumptions are true, it should be observed that if the owner were at one and the same time entrepreneur and consumer, rent would necessarily have to be deducted from the cost of production. Take, for example, two pieces of land which, with an expenditure of 100 on each, yield 6 and 5 units of wheat respectively; and let the price of wheat be 20 lire. The first piece of land has a rent of 20, the second, zero. In a system in which there is one landowner, one entrepreneur, and one consumer, the consumer pays 220 lire for 11 units of wheat; of this amount, 20 lire go to the owner as rent, and 200 lire as expenses. The cost of production for the entrepreneur, which is equal to the selling price, is 20 lire.

If there is a single person who is landowner, entrepreneur, and consumer, this quantity of 11 units of wheat is produced with an expenditure of 200 lire, and each unit will cost 18.18. The cost of production is thus different from what it was previously.

96. We must examine the relation between this special case and the general theory of production (III, 100).

Along oy , let us measure the prices of the quantities of wheat, and on ox the quantities of money representing the expenditures. Set oa equal to ab , equal to 100; ah , equal to 120, is the value of the quantity of wheat produced on the first piece of land; lk , equal to 100, is the value of the quantity of wheat produced on the second piece of land; and ohk is the line of complete transformations. If we draw the line ost parallel to hk , hs will be equal to 20; the line vst ⁹¹ is the indifference line of obstacles, with index 20. It is the only one for which a linear path starting from the origin can be tangential to an indifference line, above hl (it coincides with this line from s to t).

There is a line of maximum profit, which is precisely st . Equilibrium must take place on this line. We need only repeat what was said in the preceding paragraphs.^[a]

97. When the landowner is the same person as the entrepreneur and the consumer, he no longer consumes his wheat at the same price for all the portions; he follows the line of complete transformations, ohk , instead of following the line of constant prices ost ; equilibrium takes place at a point on hk , instead of at a point on st .

This phenomenon occurs in cases that are much more general than the one we have just discussed, and we shall study it further in the next chapter.^[a]

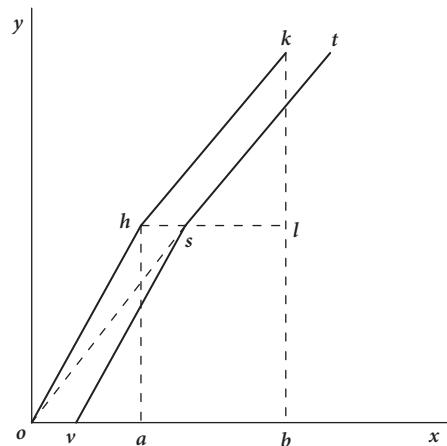


FIG 43

CHAPTER VI

Economic Equilibrium

1. Examples of equilibrium. Let us start by studying some special cases, choosing them to be simple as far as possible.

Let us imagine an individual who transforms wine into vinegar at the rate of one unit of wine for one unit of vinegar. Let us ignore all the other production expenses. Let t, t', t'', \dots , be the indifference curves of the individual's tastes for wine and vinegar, and let om be the quantity of wine available to him every month, which we shall assume to be equal to 40 liters. We seek the point of equilibrium.

The problem is extremely simple, and can be solved at once. From m , let the straight line mn be drawn with a slope of 45° on the ox -axis; the point c , where it is tangential to an indifference line, is the equilibrium point. The quantity of wine transformed is indicated by am , which is equal to ac , which is the quantity of vinegar obtained.

The cost of production of vinegar, in terms of wine, is 1; and, when we draw the line mn^1 with a slope of 45° on the ox -axis, we assume that the price of vinegar, expressed in terms of wine, is also 1.

2. Let us now see what becomes of the general theories in the various special cases we are studying.

The indifference lines of obstacles are parallel straight lines with a slope of 45° on the ox -axis. Indeed, whatever the quantity of wine available, it is always possible to transform part of it, in small or large quantities, into vinegar at the rate of one unit of wine for one of vinegar. The indifference line oh has index zero; it is the line of complete transformations. If we set oa equal to 1, the straight line ah' , parallel to oh , will be the indifference line with positive index +1. Indeed, if we have the quantity oa' of wine, equal to 2, and if, in the course of the transformation, we stop at c , on the straight line ah' , we shall have transformed one unit of wine into one of vinegar, leaving a positive residual of one unit of wine. If $k''b$, parallel to ox , is set equal to 1,² the straight line $k''h''$, parallel to oh , will be an indifference line with index -1.² Thus if, having 2 units of wine, we stop at d on this line, we shall obtain 3 units of vinegar, and we are short of one unit of wine in order to obtain this quantity.

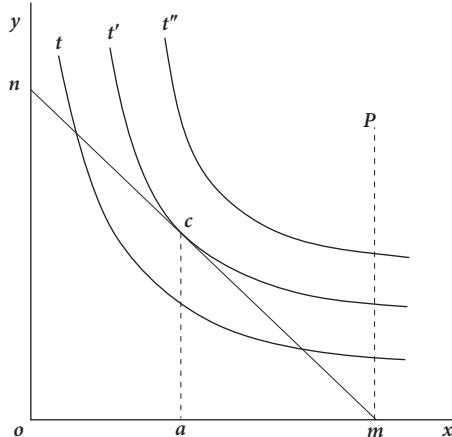


FIG 44

3. The case just examined is a limiting one. If the straight line oh were shifted³ a little to the left, we would have the case of commodities with increasing cost of production (III, 102); if it were shifted³ to the right, we would have the case of commodities with decreasing cost of production. In the case examined, the cost of production is constant, neither increasing nor decreasing.^[a] The straight line oh is not only that of the complete transformations, but is also its own tangent. Moreover, if we transpose Figure 44 onto Figure 45, making the point o of Figure 45 coincide with the point m of Figure 44, the straight line oh of Figure 45 will coincide with the straight line mn of Figure 44, and will indicate the unique path followed in production and in consumption.

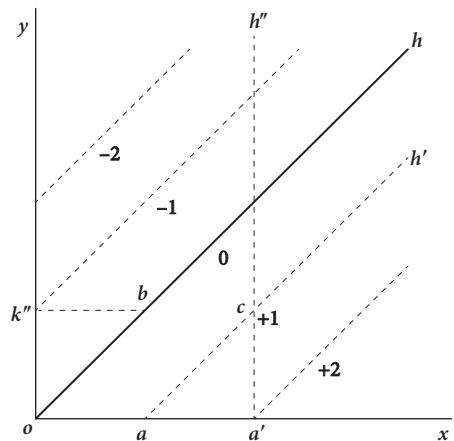


FIG 45

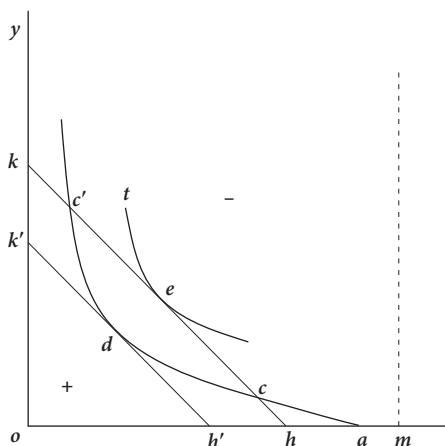


FIG 46

figure for consumption [Figure 44], we shall make om equal to 40 liters of wine and mh equal to 14^[a] [see Figure 46], and we shall draw the straight line hk with a slope of 45° on mo ;¹ this will be the indifference line of index zero, i.e., the line of complete transformations. If the individual's exchange line is $acc'd$,⁵ its intersections c and c' with the line of complete transformations will be points of equilibrium.^{6[b]}

¹ Owing to lack of space, the point e has been placed in the figure between c and c' ; but in fact, it should be placed beyond c' , on the straight line hk , moving from c toward c' .⁴

4. Let us modify the conditions of the problem slightly. Let us suppose that the ratio of the quantity of wine to the quantity of vinegar obtained (the price of vinegar in terms of wine) is not constant. For instance, we may take account of the expenses of the transformation, so far disregarded. Each week, 14 liters of wine are given to a man who supplies the barrel and the tools, and provides the work to carry out production. In this way, up to 60 liters of wine can be transformed into vinegar. Furthermore, let us separate the producer from the consumer. There will be one man who produces the vinegar and sells it to the consumer, receiving payment in wine.

Graphically, by transposing the figure for production [Figure 45] directly onto the

graph, by transposing the figure for production [Figure 45] directly onto the

5. If there is only one producer and if he can act according to type (II), he will try to obtain maximum profit, and the equilibrium point will be the point *d* where the exchange line is tangential to the straight line *h'k'*, parallel to *hk*.

6. If there is competition, the producer will not be able to stay at *d* and will be pushed back onto the line *hk*.

7. If the consumer is the same person as the producer, and if he has not decided on the path to follow in advance (Appendix [33]),⁷ he will follow the line of complete transformations without bothering about anything else, and will stop at the point *a*, Figure 46, where this line is tangential to an indifference curve of tastes, *t*. The fact that the point *e* differs from the points *c* and *c'* results from the kinds of paths followed being different.

In exchange with constant prices, the paths followed are *mc* and *mc'*; when the producer and consumer are the same person, the path followed is the broken line *mhe* (V, 97).

8. This path could also be followed in the case of exchange. For example, an innkeeper makes his customers pay: (1) a fixed amount for his overhead and his profit; and (2) the bare cost^[a] of the food he serves them. In this case the customer will follow a path similar to *mhk*.

9. It will be noted that the point *e* lies higher than the points *c* and *c'*; i.e., the customer enjoys more ophelimity at *e* [than at *c* or *c'*].⁸

This is what can be observed in practice, without the need of theories. An innkeeper charges 4 lire for a bottle of wine, of which 2 lire cover his overhead and his profit, 2 lire being the price of the wine. A customer buys only one of these bottles, because for a second one he would be prepared to pay 2 lire but not 4. Now the innkeeper changes his method of settling accounts. He first charges each customer 2 lire, then sells them as many bottles as they want at 2 lire each. The customer considered will drink two bottles. As a result, he will obtain more pleasure, while the innkeeper will make as much money as before.^[a]

10. Let us come back to the case of a producer who has the power to compel the consumers to come down to the point *d*. Suppose there is a syndicate which forbids the producers to accept a price less than that corresponding to the point *d* or to another point between *d* and *c*. Competition can no longer operate as indicated above, but must operate in a different way. The profit made by the producers at *d* induces other producers to take part in it; the number of producers therefore increases;^[a] and as each one of them must earn his own living out of production, the cost of production necessarily rises. In other words, the line *hk* of complete transformations shifts and will finally pass through the point where the producers had stopped. Such a phenomenon has become widespread in some countries where, owing to the syndicates, a large number of people live off production like parasites.

11. The case just considered is the simplified type of a very frequent phenomenon which occurs when overhead expenses are spread over the output in such a way that the cost per unit of output decreases as output increases—within certain limits, of course.^[a]

12. Let us see what happens with respect to another category of commodities whose cost of production increases as output increases.

Let us suppose, for instance, that with one unit of A, two units of B can be obtained at first, and thereafter one unit of B for each unit of A. The costs will be as follows:

Quantity of A transformed	Quantity of B produced	Cost of B in terms of A
1	2	0.500
2	3	0.667
3	4	0.750
4	5	0.800

Graphically, if we make mh equal to 1⁹ and hl equal to 2,⁹ and then draw the line lk , with a slope of 45° on mo , the broken line $h'lk'$ will be the line of complete transformations; the other indifference lines will be given by line segments parallel to $h'lk'$. If we round the angle^[a] at l a little, we shall have at l itself the point of tangency of the path ml and an indifference line. Joining these points of tangency, we shall have the line ll' . Then if $k'l'$ passes through m , the linear path starting from m and tangential to the indifference curve $h'l'k'$ will coincide with the same line $l'k'$. Consequently, the locus of points of tangency, i.e., the line of maximum profit (III, 105), will be the broken line $ll'k'$. Its intersection c with the exchange line mcd will give a point of equilibrium.

The producer would of course like to go a little farther on the side of the positive indices. For instance, he would be better off at the point c'' ; but he is driven from it by competition, as we already saw (III, 137).

13. Even in this case^[a] competition may have another effect, as we have already shown for commodities with decreasing production costs (§10); it may, without changing prices, cause the number of competitors to increase and thus raise the cost of production. In this way the line of maximum profit shifts and ends up by passing through the point where the producers stood [when they were selling at the price fixed either by their syndicate or in some other way].¹⁰

Equilibrium will again take place on this line. The producers will come closer to the line if competition acts on the prices; the line will come closer to the producers if competition has the effect of increasing the number of producers and the expenses of production.

14. All this corresponds to the real world. Given the economic conditions in a country, there is a certain output of wheat per hectare which, for a given piece of land, corresponds to maximum profit, and it is at this output that the farmer will settle. The price is determined by the equality of the cost of production, including this profit, and the price the consumer is willing to pay for the amount produced in these conditions. Of course, the farmer would prefer to obtain a still higher price, but he is precluded from obtaining it by competition.

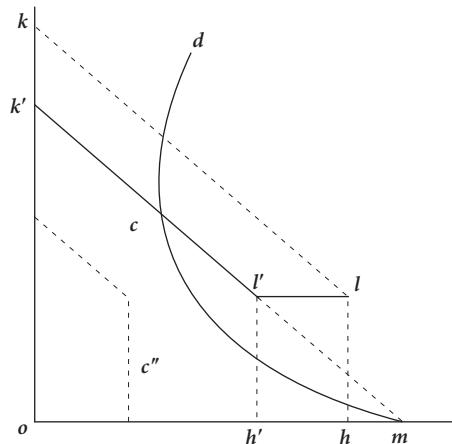


FIG 47

15. Ordinary economics had some inkling of the difference between the cases we have examined, but it never succeeded in having a precise idea of this difference, and it could not even explain the different ways in which competition operates.

16. If, in the hypothetical case we just considered, some persons operate according to phenomena of type (II), the point of equilibrium will be l'' , where the exchange line mcd is tangential to a producer's indifference curve, because this is the point of maximum profit.^[a] If the form of mcd were a little different, this point could lie in the neighborhood of l' .

17. If the consumer is also the producer,^[a] he will follow the line of complete transformations htk , and the equilibrium point will be given by the point of tangency of this line and an indifference line of tastes.

18. There could also be consumers who [have the power to]^[11] compel producers to follow linear paths which, starting from m , end up on the line of complete transformations. In this case the equilibrium point would be e (§§43–47).

19. **Usual forms of exchange and production.** One can think up shapes as unusual as one might wish for the indifference curves of tastes and of obstacles, and it would be difficult to prove that they have never existed or will never occur. We shall of course restrict ourselves to the most usual ones.

20. Among the commodities in common use, it is only in the case of labor that one can observe in practice that, beyond a certain limit, the supply, instead of increasing, decreases with the price. A rise in wages has resulted in every civilized country in a fall in working hours. For other commodities we observe in the real world that supply nearly always increases with the price; but this is perhaps because what we observe is not the law of supply in simple exchange but the law of supply in production.

21. In any case, always excepting for labor, we cannot assert that we shall observe exchange curves in the real world with shapes like those of Figure 17 (III, 120); the curves instead seem to have shapes similar to those of Figure 48. The exchange curve, referred to the ox - and oy -axes, is mcd ; similarly this curve for another individual, and referred to the axes ωm and ωn , is mcr . This is true within the admittedly restricted limits of the observations. We do not know what happens to these curves beyond d and r .

22. In these circumstances, there is only one equilibrium point, at c , and it is a point of stable equilibrium.^[a]

23. In production, many examples can be observed of commodities with decreasing costs and of others with increasing costs; but it appears that the cost, although initially decreasing, is ultimately always increasing beyond certain limits. For these commodities, we have points of tangency of the linear paths starting from m , and thus a line $l'l''l'''$ of maximum profit.^{[12][a]} If we observed only the phenomena in the shaded part of the diagram

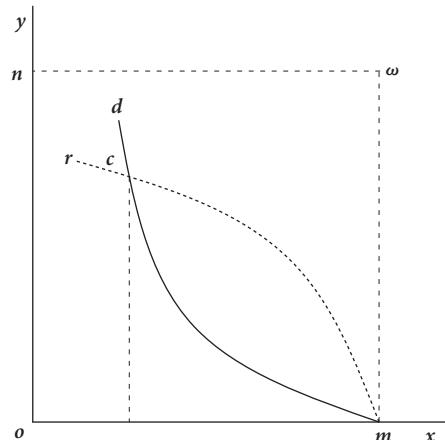


FIG 48

where costs are always decreasing¹³ with an increase in the quantity transformed, this line $l'l''$ would not exist.

24. For commodities produced under decreasing costs, we can observe in the real world the two equilibrium points given by the theory—Figure 46 (§4)—but powerful frictions may sometimes allow unstable equilibrium to last for a fairly long time.

A railway may cover its expenses by having high fares and transporting small amounts, or by having low fares and transporting large amounts. We thus have the two points c and c' of Figure 46 (§4). Small shopkeepers stay at the point c by selling little at high prices; the big department stores have [made their appearance and have]¹⁴ shifted the equilibrium point over to c' , by selling large quantities at low prices; and now the shopkeepers request the intervention of the law to bring the equilibrium point back to c .

25. There are also numerous instances of the line of maximum profit for commodities produced under increasing cost. The extensive cultivation in the vicinity of Rome cannot be explained otherwise. In England, following the removal of duties on wheat and as a result of competition from foreign wheat, the shapes of the indifference curves of obstacles for wheat cultivation have changed; and within certain limits the cost of production of wheat has declined, instead of rising, with the quantity produced—thus the change in wheat cultivation, which became still more intensive.

26. **Equilibrium of tastes and production.** Let us consider an isolated community, and assume that an individual's expenditures are confined to the commodities he purchases, and that his income is derived solely from the sale of his labor, of other capital services, or of other commodities.

Under these conditions, economic equilibrium is determined by the conditions we have already laid down (III, 96 et seq.) concerning tastes and obstacles. We have seen that tastes, and the consideration of the existing quantities of certain goods, determine the relations between prices and quantities sold or bought. On the other hand the theory of production has taught us that, given these relations, quantities and prices can be determined. The problem of equilibrium is thus completely solved.

27. **Equilibrium in general.** The preceding theoretical case departs considerably from reality in one respect. In the real world, individuals' incomes are far from originating only in the goods they contribute for the purposes of production. The public debt of civilized nations is enormous; only a very small part of [the money from]¹⁵ this debt has been devoted to productive purposes, and this often badly. The individuals who enjoy the interest from this debt can thus not be regarded in any way as having contributed economic goods to production. Similar observations may be made concerning the salaries of the steadily rising bureaucracy of modern states, concerning expenditures for war, the navy, and many public works. We are not in the least inquiring here whether—and

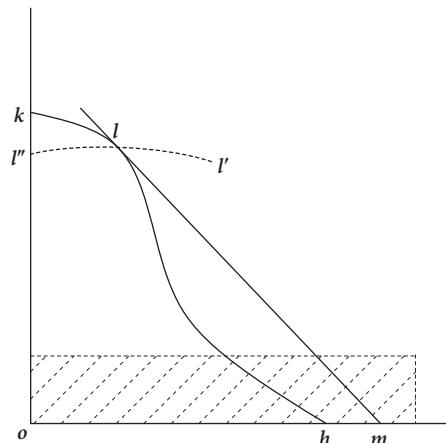


FIG 49

to what extent—these expenditures are useful to society, and in which cases they are indispensable to it; we simply state that their utility [when it exists,]¹⁶ is of a different kind than that which is the direct result of economic production.^[a]

28. On the other hand, individuals' expenditures are far from being restricted to the economic goods they buy. Taxes are a notable part of these expenditures.

By a very rough calculation, which is perhaps not too far from the truth, it can be estimated that in several European countries about 25 percent of individuals' income is taken in taxes. The theory set forth above would thus only be valid for at most ¾ of the amounts that make up the total income of a nation.

29. It is easy to modify this theory so as to take account of the phenomena we have just indicated. For this, it is sufficient to distinguish in individuals' income the part derived from economic phenomena from the part which is extraneous to them; and to proceed in the same way for expenditures.

30. The part of income that is left to individuals is spent by them according to their own tastes; and, as for its allocation among different expenditures, this comes within the scope of the theory, already set forth, of equilibrium with respect to tastes. The part of income levied by the public authority is spent according to other criteria which are not the concern of economic science to investigate. Economic science must thus consider them as forming part of the data of the problem to be solved. The laws of demand and supply will follow from the consideration of these two categories of expenditures. If only one of them were to be considered, the discrepancy with respect to the concrete phenomenon could be considerable. For instance, in the case of iron and steel the demand on the part of governments takes up a significant part of output.^[a]

31. As regards equilibrium with respect to obstacles, we must take account of the fact that firms' expenditures are not, as before, equal to the individuals' total income, but form only a part of it, since the rest originates elsewhere (public debt, salaries, etc.).^{17[a]} The allocation of the part devoted to the purchase of goods to be transformed in the process of production is determined by the theory of equilibrium with respect to the obstacles. The allocation of the other part of income is determined by criteria which, as in the preceding, analogous case, lie beyond the scope of economic research, and which must therefore be borrowed from other sciences and be considered as part of the data of the problem.

32. Properties of equilibrium. Equilibrium has certain properties associated with the conditions under which it obtains, which deserve to be known.

33. We shall start by defining a term which it is convenient to use in order to avoid long-drawn-out sentences. We shall say that the members of a community enjoy, in a certain situation, MAXIMUM OPHELIMITY when it is impossible to move slightly away from this position [in such a way that the ophelimity enjoyed by each member of the community increases or decreases].^{18[a]} [That is to say, every small displacement from this position must necessarily have the effect of increasing the ophelimity enjoyed by some individuals and decreasing that enjoyed by others, i.e., of being agreeable to some and disagreeable to others.]¹⁹

34. Equilibrium of exchange. We have the following theorem:

For phenomena of type (I), when equilibrium takes place at a point where the traders' indifference curves are tangential to each other, the members of the community enjoy maximum ophelimity.

It should be noted that this equilibrium position may be reached either along a linear path, i.e., with constant prices, or along an arbitrary path.

35. This theorem can be rigorously proved only with the aid of mathematics (Appendix, [§34]);²⁰ I shall confine myself here to providing [just a sketch [of the proof].]²¹

Let us start by considering an exchange between two individuals. For the first, the axes are ox and oy , and for the second, $\omega\alpha$ and $\omega\beta$, and they are positioned in such a way that the paths followed by the two individuals form a single line in Figure 16 (III, 116). The indifference lines are t, t', t'', \dots , for the first individual and s, s', s'', \dots , for the second. For the first individual, the hill of pleasure rises from o toward ω , and for the second it rises instead from ω toward o .^[a]

For phenomena of type (I), we know that the equilibrium point is to be found at a point of tangency of the two individuals' indifference curves. Let c be one of these points. If we move away from it along the path cc' , we ascend the hill of pleasure of the first individual and descend that of the second; and conversely if we follow the path cc'' . It is thus not possible to move away from c so as to benefit, or harm, both individuals at the same time; but necessarily, if one of them benefits the other is harmed.

36. For phenomena of type (I), equilibrium takes place at a point such as c ; for phenomena of type (II), it takes place at a point such as d ; we thus see the difference between these two kinds of phenomena as regards maximum ophelimity.^[a]

37. Going back to Figure 49,^[a] we see intuitively that, by prolonging the path cc' toward h , one keeps descending the second individual's hill of pleasure; whereas, on the other hand, while one starts by climbing the first individual's hill of pleasure, one then descends, after passing beyond the point where $cc'h$ is tangential to an indifference line. Consequently, if we move along a straight line a finite distance away from the equilibrium position, the ophelimities enjoyed by the two individuals may vary in such a way that the one increases while the other decreases, or that they both decrease; but they cannot both increase. This, however, is true only for commodities whose ophelimities are independent, or in the case in which these commodities are linked by a dependence of the first kind (IV, 42).

The rigorous proof, not only in this case but in the general case of several commodities and several individuals, can be provided only with mathematics (Appendix, [§§39, 40]).²²

38. If experiments could be carried out on human societies, as the chemist does in his laboratory, the preceding theorem would give us a means of solving the following problem:

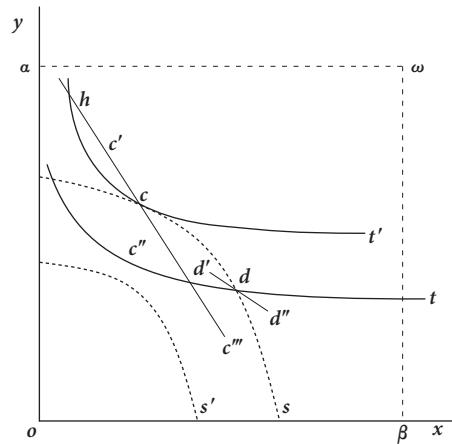


FIG 50

A given community is considered; we do not know the ophelimity indices of its members; we know that an equilibrium exists in the exchange of certain quantities; we ask whether it is obtained under the same conditions as would hold under free competition.

It is necessary to perform an experiment to see whether, if the exchanges remain fixed in the form in which they are being carried out, other exchanges, at constant prices, can be added (note: added, not substituted), that would satisfy every individual. If so, the equilibrium does not take place as it would under free competition; if not, it does take place under these conditions.

39. Equilibrium of production. Several cases must be distinguished here:

(1) *Constant selling prices.* (a) Production coefficients that vary with total output, i.e., commodities whose cost of production varies with the quantity; (β) production coefficients that are constant with respect to the quantity, i.e., commodities whose cost of production is constant. (2) *Variable selling prices.*

40. (1) (a) This case is depicted in Figure 46 (§4). The equilibrium points c, c' are not those that give maximum ophelimity in transformation (Appendix I, §34]).²² Consequently, there may be a point which does not lie on the line of complete transformations and at which the firm which carries out the transformation would make a profit, while the consumers would be better off than at c, c' . This case is sometimes met with in the real world in connection with trusts.^[a]

41. (1) (β) This is the case of Figure 44 (§1). The point of equilibrium c provides maximum ophelimity from the transformations (Appendix I, §34]).²²

42. (2) The variable prices may be such as to bring about a phenomenon similar to that of case (1) (a).

But if these prices can be used to obtain maximum ophelimity in the transformations, the point e of Figure 51 at which the maximum is achieved can thus be reached (Appendix I, §34]).²²

43. If one follows the path amu^{23} of complete transformations, this point will certainly be reached; this is also true if one follows a path avu , which follows that line only along the part veu ; or finally, if one follows a path $all'e^{24}$ tangential at e to the line of complete transformations, and to the indifference line t .^[a]

In practice, this last path is very difficult to follow, because one must guess the precise location of the point e .²⁵ The first two paths, on the other hand, may be followed without knowing the precise location of e .

44. Probably the larger part of production is of the type in which the cost of production varies with the quantity produced; we may therefore assert that the system of constant²⁶ prices generally used in our society does not yield maximum ophelimity; and considering the large number of products to which this conclusion applies, it appears that the loss of ophelimity must be very great.

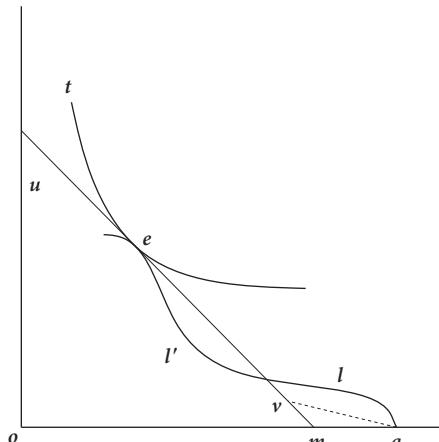


FIG 51

45. It is precisely for this reason that, even in our social system of organization, producers would benefit from charging variable²⁷ prices; but being unable to do this directly, they try to do it indirectly by means of expedients that will bring about only a very rough approximation to the solution that would provide maximum ophelimity.

Generally, prices can be made variable by distinguishing consumers by groups; and this expedient is better than nothing, but it is far from the solution which would make prices variable for all consumers.

46. The grave error of judging economic facts by moral standards leads many people, more or less consciously, to believe that the producer's profit can only be the consumer's loss, and vice versa. Consequently, if the producer makes no profit, i.e., if he is on the line of complete transformations, it is firmly believed that the consumer can suffer no detriment.^[a]

Without dwelling on the fact that, as has been already explained, the line of complete transformations can be obtained with excessive production costs, one should not forget the very frequent case indicated in §39 (1)(a).

47. Suppose, for example, that a country consumes 100 units of a commodity X and that this commodity is produced in national factories at a cost of 5 per unit. The total cost is 500; and if the total selling price is also 500, the national producers will make no profit.

It now comes to pass that they produce 200 units, which makes the cost of production fall to 3. They sell 120 units in the country at 3.50, and 80 units abroad at a price of 2.50. In all, they receive 6.20²⁸ for a commodity that cost them 6.00,²⁸ and they thus make a profit. The consumers in the country complain that they are paying more dearly for the commodity than it is being sold for to foreigners; but in essence they pay less for it than they were paying before, whence it is really to their advantage, not to their detriment.

Possibly, but this is not certain, something similar has sometimes occurred in Germany, where producers sell abroad at a price that is lower than that prevailing in the country; in this way they can increase output considerably and reduce the cost of production.

48. The phenomena we have just studied suggest, in an abstract way and leaving practical difficulties aside, an argument of great weight in favor of collectivist production. Much better than the present mixed system of production, partly competitive and partly monopolistic, collectivist production could set non-uniform prices which would make it possible to follow the line of complete transformations and thus to reach the point *e* in Figure 46 (§4), whereas we now have to stop at the point *c'*, or worse still, at the point *c*. The benefits to society might be so great that they would offset the detriment inevitable in a system of production of this kind. But for this to be true, collectivist production would have to have as its own purpose the achievement of maximum ophelimity in production, and not the provision of monopoly profits to the workers, or the pursuit of humanitarian ideals.² As the older economists already realized, the pursuit of the greatest advantage for society is mainly a problem of production.^[a]

² Among the socialists, G. Sorel has the great merit of having understood that the problem that collectivism has to solve is mainly a production problem.²⁹

Even cooperative societies could lead us on to the line of complete transformations, but this does not happen because they, too, allow themselves to be led astray by ethical, philanthropic, and humanitarian fantasies. [You can't course two hares at once.]³⁰

If one looks at the phenomenon exclusively from the point of view of economic theory, it is a very poor way to administer the private railway system to charge the companies running them, as is done in Italy, a fixed levy on gross receipts (or even on net receipts) for the benefit of the state; because of this the companies, instead of being pushed toward the line of complete transformations, are kept away from it.

49. The coefficients of production are determined by free competition in such a way as to ensure maximum ophelimity (Appendix [§ 31]).³¹ Free competition tends to equalize the net yields of the capital goods that can be produced with savings; for savings are, of course, transformed into the capital goods that yield the highest returns, until the abundance of these capital goods causes their net yields to fall to the general level. This equality of net yields is also a condition for obtaining maximum ophelimity from the use of these capital goods. Even in this case, the rigorous proof can only be given with the aid of mathematics;³ here, we can only provide a rough indication of how the process works out.

50. As regards the yield on capital, it may be observed that if savings obtain a higher yield in one use than in another, this means that the first use is more "productive" than the second. Consequently, there is an advantage for "society" in decreasing the first use of savings and increasing the second; thus one arrives at an equalization of the net yields in the two cases. But this reasoning is not very precise and rigorous: and consequently, it can by itself prove absolutely nothing.

51. A better—but only slightly better—reasoning is one that, without using mathematics, introduces the coefficients of production.

Firms determine them so as to minimize costs; but competition drives them onto the line of complete transformations; consequently, [it is their customers—buyers and sellers—who ultimately benefit from their efforts].³²

The defect of this kind of proof lies not only in its lack of precision, but also—and mainly—in that it does not provide a clear idea of the conditions necessary for the theorem to be true.

52. Equilibrium in the collectivist society. We now have to deal with phenomena of type (III), which so far have only been barely alluded to (III, 49).

To give them a concrete form, with an abstraction analogous to that of *homo economicus*, let us consider a collectivist society whose object is to procure maximum³³ ophelimity for its members.

53. The problem can be subdivided into two others, which are completely different and which cannot be solved with the same criteria: (1) We have a problem of distribution: how should the goods owned or produced by society be distributed among its members? (III, 12, 16). Ethical and social considerations of many kinds, comparisons of different individuals' ophelimities, etc., have to be introduced. This is not the place to deal with them.^[a] We assume, then, that this problem has been solved. (2) We have a production problem: how should economic goods be produced so that—when these are

³ *Cours*, §724.

then distributed according to the rules derived from the solution of the first problem—the members of society obtain maximum ophelimity?

54. From what has been said, the solution of this problem is simple.

Prices, net yields on capital, may disappear—if that is possible—as real entities, but they will remain as accounting entities; without them, the Ministry of Production would be groping in the dark, and would not know how to organize production. Clearly, if the state is the owner of all capital goods, it will receive all the net interest.

55. To obtain maximum ophelimity, the collectivist state will equalize the different net yields and determine the coefficients of production as they are determined by free competition. In addition, after having carried out the distribution according to the rules of the first problem, it will have to allow a new distribution which the members of the community will bring about themselves, or which the socialist state will put into effect; but which in any case will have to take place as if it were carried out under free competition.

56. The difference between phenomena of type (I) and those of type (III) thus lies essentially in the distribution of incomes. In phenomena of type (I), this distribution is the result of all the historical and economic contingencies in which society has developed; in phenomena of type (III), it is settled as the consequence of certain socio-ethical principles.

57. We must, furthermore, investigate whether certain forms of production are more easily workable in the real world with phenomena of type (I) or with those of type (III). In theory, nothing prevents us from assuming that with free competition the line of complete transformations, for instance, is being followed; but in practice this may be more difficult with free competition than with collectivist production (§48).

58. The socialist state seems to be better able than free competition to bring the point of equilibrium on to the line of complete transformations. Indeed, it is difficult for a private company to follow exactly the line of complete transformations in its sales. For this, it would first have to charge its overhead to its customers, and then sell them at cost price less the overhead. Except in special cases, it is not clear how this could be done. The socialist state, on the other hand, can make the consumers of a commodity pay for its overhead expenses in taxes, and then sell the commodity at cost price; it is thus able to follow the line of complete transformations.

59. The socialist state may allow the consumers of a commodity the enjoyment of the rent^[a] (V, 95) derived from its production. When the line of maximum profit cuts the exchange line, i.e., when competition is incomplete, and with simple competition among private producers, equilibrium takes place at this point of intersection. The socialist state can move this equilibrium point onto the line of complete transformations, as if competition were complete.

60. In an economic state based on private ownership, production is controlled by the entrepreneurs and owners; there is consequently a certain expense which forms part of the obstacles. In a collectivist state, production would be controlled by employees of the state; the expenses to maintain them might be greater, and their work less efficient; in that case, the advantages pointed out might be offset and turn into losses.

61. In short, pure economics provides us with no truly decisive criterion with which to choose between a system of organization based on private ownership [or competition]³⁴

and a socialist system. Such criteria can be obtained only by taking account of other characteristics of the phenomenon.^[a]

62. Maxima of ophelimity for sectoral communities. Phenomena of type (III) may refer, not to the entire community, but to a more or less restricted part of it. If a single individual is considered, type (III) coincides with type (II).

For a certain number of individuals considered collectively, there exist values of the production coefficients that provide this community with such quantities of economic goods that, if distributed according to the rules established by the problem of distribution, they provide the maximum ophelimity to the members of this community.⁴

The proof of this proposition is similar to that given when we considered the total community.

63. In the real world, trade unions, producers enjoying tariff protection, and trade associations which exploit consumers, provide us with numerous instances in which the production coefficients are determined with the aim of benefiting certain sectoral communities.

64. It should be observed that, except for certain quite exceptional cases, these values of the coefficients differ—often greatly—from those that would yield maximum ophelimity to the entire community.

65. International trade. Up to now, except for the preceding case, we have considered only isolated communities. To come closer to reality we now have to consider interrelated communities. [This theory has as a special case the theory of international trade, and we may therefore distinguish it by this name.]³⁵

The preceding case differs from the present one. In the former case it was assumed that certain coefficients of fabrication could be imposed upon a whole community consisting of the sectoral communities A, B, C, . . . ; and we looked for those values of the coefficients that would provide the maximum ophelimity to the members of community A. In the latter case it is not assumed that community A can impose certain production coefficients on the other communities B, C, . . . , directly but instead it is assumed that each of these communities is independent, and that consequently it can control its own production but not that of the others [, at least not directly].³⁶

Even in discussing a single community, one should take transport costs into account, but this necessity is still more obvious when one is dealing with communities separated in space. It is clear then that the price of the same commodity will be different in two different communities.

66. From what has been said about a single community, the conditions of equilibrium for several communities are easily obtained.

Let us consider a community, X, which has relations with other communities which we shall denote by Y, and which, for simplicity we shall consider as a single community. For each of these communities we have the conditions of equilibrium of tastes and obstacles already indicated; but these conditions are now not sufficient to solve the problem, because there are other unknowns, such as the number of economic goods exchanged between X and Y. Let us assume that there are 100 of them; we shall need 100 more conditions to determine them.

⁴ *Cours*, §727.

67. We have, first, the balance [of payments] of X in relation with Y, which must be drawn up so as to take account of every receipt and every expenditure, as was explained in §27 et seq. Y's balance [of payments] is superfluous, for the reasons already explained (III, 204).^[a] In the relations between X and Y, X's receipts are Y's expenditures, and conversely. Consequently, if receipts and expenditures balance for X, they also balance for Y. Thus the consideration of balances [of payments] gives us a single condition which we shall call (α).

68. Moreover, after allowing for transport and other accessory expenses (e.g., insurance, foreign exchange costs, etc.), the prices should be equal for the quantities traded, because there cannot be two prices in the same market. One of the commodities can be considered as an international currency; in that case there remain only 99 prices, and the equalization conditions, which we shall call β , will then be 99 in number.

Adding condition (α) to the 99 conditions (β), there are in all 100 conditions, which is exactly what we require to determine the 100 unknowns.

69.³⁷ But, in general, it cannot be assumed that there is only one currency, which would be the same for X and Y; one must assume that X and Y each have their own currencies, even when they are materially identical and coined in the same metal. In this case Y's currency is in a certain relation to X's currency, i.e., it has a certain price expressed in terms of X's currency, and this is another unknown. When this is added to the remaining 100, we have 101 unknowns. But as we now have 100 prices, conditions (β) now also are 100 in number; and adding condition (α), we obtain 101 conditions, i.e., as many as there are unknowns.

It remains to be seen how equilibrium is established, but we shall be able to do this only after we have studied money (VIII, 35 et seq.).

70. Equilibrium and prices.^[a] Hitherto, in all our reasonings we have taken one commodity as money; the ratios of exchange of this commodity in terms of the others, i.e., the prices, depend on the tastes and obstacles, and are determined once these tastes and obstacles are.

An initial modification must be made to this theory with regard to the quantity of money in circulation. This is that the commodity-money confers ophelimity not only in consumption, but also by its use as circulating medium. For every price to rise by 10 percent, say, it would thus be necessary not only for a corresponding adjustment to take place in the ophelimity of the commodity-money as compared with the ophelimities of the other commodities, but also for the quantity of money available to be sufficient for purposes of circulation at the new prices.

71. Quantity theory of money. Let us suppose that the quantity of money in circulation must vary in proportion to prices, which is roughly what can happen if, when prices change, the velocity of circulation does not change, and if the proportions of money substitutes do not change.^[a] This hypothesis is the basis of what has been called the quantity theory of money. Accepting this hypothesis, it would be necessary, in order for prices to rise by 10 percent, for the quantity of the commodity-money to increase, not only so that it can be consumed in larger amounts—whence its elementary ophelimity will diminish—but also so that the quantity of money in circulation will increase by 10 percent.

Prices would thus, finally, be determined by the ophelimity of the commodity-money and by the quantity of it in circulation.

72. If, instead of being a commodity, money were in the form of any kind of tally, e.g., of paper money, all prices would depend only on the quantity of this money in circulation.

73. The hypotheses we have just made are never completely confirmed. Not only do prices not all vary together in the same proportion, but furthermore, the velocity of circulation certainly varies, and the proportions of money surrogates vary, too. As a result, the quantity theory of money can never be more than approximately and roughly true.^[a]

74. In the case of paper money, it is thus possible to have two equilibrium positions for which all the circumstances are identical except the following: (1) All prices have increased, for example by 10 percent; (2) the velocity of circulation has increased, and the proportion of money substitutes may have increased as well, so that the same quantity of money is sufficient for purposes of circulation at the new prices.

75. In the case of a commodity-money it would be necessary for the velocity and the proportion of money substitutes to increase in such a way as to make the quantity in circulation excessive, so that the consumption of the commodity-money can increase, in order for its elementary ophelimity to fall.

76. The hypothesis made for paper money can be approximately confirmed; but the one made for commodity-money seems difficult to confirm in the real world in the proportions indicated, although it might often occur in smaller proportions. For this we may conclude that identical equilibrium positions with different prices would be possible in the first case and impossible in the second.

77. This last conclusion is perhaps too absolute. The conclusion could hardly be attacked if the consumption of the commodity-money were nearly as great as the sum of all the other types of consumption. Let us assume that in a community of farmers in which wheat, wine, oil, wool, and a few other commodities are consumed, wheat is chosen as commodity-money; the above conclusion would certainly hold. But would it still hold if, as in our societies, the commodity-money is gold, whose consumption is extremely small as compared to other types of consumption? It is not easy to grasp how all the prices must be precisely and strictly regulated by the consumption of gold for watch cases, jewelry, etc. The correspondence between these two phenomena cannot be perfect.

78. It should be pointed out that we are here leaving the realm of pure economics and entering that of applied economics. Similarly, rational mechanics teaches us that two equal and directly opposite forces are always in equilibrium, whatever their intensity; but applied mechanics tells us that if a solid body is interposed between these forces, it is necessary in addition to take the resistance of the materials into account.

79. Let us suppose that, all other circumstances remaining the same, all prices rise by 10 percent. For the equality of weighted ophelimities [to subsist, which ensures equilibrium,^[38]] the amount of gold available for consumption would have to increase; and it is because this amount cannot be increased that the prices must revert to their former level. But here the following points should be noted: (1) The equality of weighted ophelimities is attained approximately for the commodities in [extensive]^[39] and daily use, but not so well in the case of those commodities which are in more restricted use and which are bought only once in a while. Consequently, there is, in fact, for the ophelimity of gold a certain margin in the equality with which it is related to other goods. (2) If all

prices increase, the mining of gold ought to become less profitable and hence diminish. But gold mining is so spasmodic that it can be assumed to be governed by entirely different considerations; and within limits, variations in the prices of other commodities have little or no effect; (3) finally, a change in the conditions of circulation may also have some effect (§73). We may conclude that with a gold currency, identical equilibrium positions are possible, within certain limits, with different prices. Within these limits, they would no longer be completely and exclusively determined by the formulae of pure economics (§82).

80. Relations between equilibrium and the prices of factors of production. (1) Let us suppose that the prices of the factors of production all change, but that society's existing debts and credits (public debt, commercial credits, mortgages, etc.) do not change. For instance, if the prices of all the factors of production rise by 10 percent, product prices will also rise by 10 percent; therefore, in this respect nothing will be changed in the real conditions of the workers and the capitalists who cooperate in production. They receive 10 percent more, and they spend 10 percent more for their consumption. In another respect, their situation has changed, because since they continue to pay the same nominal amount to their creditors, they in fact give up 10 percent less than before in terms of commodities. Consequently, the assumed change favors those who take part in production, and harms those who have fixed incomes which are independent of production. It is superfluous to add that an opposite change would have the opposite effects.

81. For the change in prices to be possible, it must not be held back by money: we thus have to repeat the considerations indicated in §71 et seq. In the case assumed, and if gold is the currency, those taking part in production will perhaps (§79) consume a little more gold; those who have fixed incomes, a little less; in all, there will perhaps be a little more consumption, which will easily be supplied by the mines. As for circulation, its velocity may increase, and, if necessary, greater use can be made of surrogates. Nevertheless, prices could not rise beyond certain limits, because the amount of gold available would become too scarce.

82. In the real world, obstacles to price changes arise from the competition of independent communities, either in the same country or abroad (international trade), and from the difficulty of making prices all move together; consequently, those which do not move restrain the movement of others. These are the facts which, within the limits allowed by the forces which arise from the variation in the consumption and output of gold (§79), determine prices.

83. If the prices of most or all of a country's commodities rise, exports decline, imports increase, and gold flows out of the country and goes abroad; as a result, prices finally come down and return to their original state. The opposite effect can be observed in the case of a general fall in prices.

84. (2) Prices of the factors of production never all move together. Suppose that wages increase by 10 percent; the interest on new capital goods and on some of the older ones may also rise by 10 percent; but for some of these, it is possible that interest will not change, or not increase in proportion to the rise in wages, or it may even decline; since the capital goods cannot be withdrawn from production, they will have a negative rent. As a result, an increase in wages will benefit workers; the increase may be a matter of indifference to the owners of new capital goods and to the owners of some of the older

capital goods, but it will harm the owners of the rest of these capital goods, and all those who have fixed incomes.

85. Suppose now that it is the product prices which, as a result of certain measures, e.g., protective duties, are forced up, and let us consider the consequences. If it is assumed that the prices of all products increase, the prices of all the factors of production can increase in the same proportion if fixed incomes, debts, and credits are disregarded, and equilibrium will be established once again, as in § 71. Similarly, if account is taken of fixed incomes and of debts and credits, results analogous to those in § 80 will be obtained. As to the phenomena in § 84, it should be noted that when product prices rise, all capital goods—old and new—benefit, and for the most part positive rents appear.

86. The hypothesis we have just made is never confirmed in practice. It is not possible for the prices of all products to increase; consequently, some production is benefited⁴⁰ and some is injured.⁴⁰ New capital goods may flow toward the favored productive activities; old ones, which cannot be withdrawn from the injured productive activities will obtain negative rents.

87. Up to now we have considered successive equilibrium positions; we must also consider how the movement proceeds in passing from one position to another. A change brought about in one part of the economic organism does not spread instantaneously to the other parts; and during the time it takes for it to be propagated from one point to another, the phenomena differ from those that ensue once equilibrium is restored.

88. If wages increase, it is difficult for the entrepreneurs, except in special cases, to raise product prices correspondingly; consequently, until such an increase is obtained, they are worse off. In the meantime, the rise in wages brings workers greater benefit than they will ultimately obtain at the end of the process, because their incomes have risen whereas their consumption expenditures have not yet increased in proportion. Those with fixed incomes suffer less while the movement goes on than after it is completed.

89. Moreover, the movement can never be general. Wages and even product prices may rise in one branch of production but prices in other branches of production will rise little or not at all; and it is only after successive wage increases in many branches of production that the price increases corresponding to a general rise in wages cause is already forgotten.

90. The translation of these phenomena into subjective terms is remarkable. Man is impelled to act more by the sensations of the present than by forecasts of the future, and much more still by facts which act directly on him than by those which act only indirectly; consequently, in the case first considered, workers will be much more strongly impelled to ask for wage increases than they would be if they felt the effects of a general increase in wages; and likewise, entrepreneurs will be much more strongly impelled to resist the workers. As for those with fixed incomes, who in the end will have to bear the expense of the contest between workers and entrepreneurs, they show less good sense than the sheep which, upon being driven to the slaughter house, put up resistance when the smell of blood hits them. They imagine that strikes are directed against “capitalists,” whom they cannot even distinguish from entrepreneurs, and they do not understand that in the last analysis strikes hurt fixed-income earners and creditors much more than entrepreneurs and capitalists.^[a]

91. Entrepreneurs always attempt to raise the prices of the commodities they produce. They thus pursue their own advantage, since these price increases undoubtedly benefit

them during the relatively long time required to reach a new equilibrium position. Moreover, everyone expects to enjoy the full advantage from the price increase of his own commodity, paying no heed to the partial offset that will follow from the increased prices of other commodities. The same is true of landowners who strive to obtain positive rents. Workers are in general indifferent to these price movements, because they do not have immediate repercussions on their wages; they believe that only the "capitalists" have to worry about these price variations; consequently, they do not reject those which, in the final analysis, will be harmful to them, just as they do not support those which will ultimately be in their interest. However, contrary to this general observation, the workers in Germany have now come out against protective tariffs on foodstuffs, since they have realized that these duties would ultimately operate to their own detriment. This may be due in part to the education the socialists have given the workers of that country.^[a]

92. Economic circulation. In conclusion, production and consumption⁴¹ form a circle.^[a] Every alteration at one point of the phenomenon has repercussions, but not uniformly, on the other points. If product prices are made to rise, this causes factor prices to rise also, as a consequence. If, instead, factor prices are made to rise, this will as a consequence cause product prices to rise. So expressed, the two operations appear to be identical; but they are not, because the pressure exerted on product prices is not propagated to factor prices in the same way that the pressure exerted on factor prices is propagated to product prices. In short, in one way or another we arrive at a general increase in prices; but this increase is not the same for different economic goods, and these variations differ as between the first and second process. Different individuals are benefited or harmed, depending on whether one works with the first or second process.

93. False interpretations of the competition among entrepreneurs. Competition among entrepreneurs manifests itself in the tendency of entrepreneurs to supply more of a commodity at a given price than consumers demand; or, which comes to the same thing, in their *tendency* to supply a given quantity at a price that is lower than that paid by the consumers (IX, 94).^[a]

The observation of these facts, wrongly interpreted, has given rise to the erroneous belief that there is permanent overproduction. If such overproduction really existed, we should be able to observe a steadily increasing accumulation of commodities, thus; for instance, there would be constantly increasing world stocks of coal, iron, copper, cotton, silk, etc. This is not observed; thus this alleged overproduction can exist only as a tendency, not as a fact.

94. Assuming the existence of this overproduction, it has been asserted that it would be in the entrepreneurs' interest to raise the workers' wages, since, it is argued, this would increase workers' "purchasing power" and consequently their consumption as well.

95. The only element of truth in this proposition is that an entrepreneur who, for instance, pays double the former wages, double the interest on capital, and sells his products at double their former price, finds himself in the same situation as before, [and is neither better off nor worse off].⁴² But neither these doubled wages nor the doubled interest on capital will cause the total consumption of commodities to increase; they will only bring about a redistribution of the total—the larger part going to certain factors of production and a smaller part to those who have fixed incomes—and, moreover, the output of some commodities may increase, whereas that of others may decline.

96. Some writers, falling in a new and grosser error, have tried to deduce the cause of economic crises from this imaginary overproduction (IX, 92, 93).^[a]

97. Erroneous conceptions of production. It used to be said, and is still often said by many, that there are three factors of production; nature, labor, and capital—the latter term being understood to mean savings, or even capital goods proper.⁴³ This proposition makes little or no sense. It is not clear why nature is separated from labor and capital, as if labor and capital were not natural objects. What the assertion amounts to saying is that to produce one needs labor, capital, and something else, which one calls nature. This is not untrue, but is of little help in understanding what production is.

98. Others say that the factors of production are land, labor, and capital; others reduce everything to land and labor; still others to labor alone. This is the source of completely erroneous theories such as the one according to which the worker offers his services to the capitalist only when there is no more *free land*⁵ to cultivate, such as the theory that claims to measure value in units of “crystallized” labor.⁶

99. All these theories have one defect in common, which is that they forget that production is nothing but the transformation of certain things into others, and they would have us believe that all the individual products can be obtained by means of abstract and general things called land, labor, and capital. It is not such [things in general]⁴⁴ that are needed for production, but certain concrete and specific—often extremely specific—objects, depending on the product that is to be produced. For instance, to obtain Rhine wine, one does not need just any piece of land, but land on the banks of the Rhine; to make a statue, it is not just any labor, but a sculptor's labor, that is needed; to make a locomotive, one does not need just any capital good,⁴³ but the one that has the precise form of a locomotive.

100. Before their land was discovered by Europeans, the Australians did not know our domestic animals; they had as much *free land* as they wanted; but, however much labor they might devote to it, it is quite certain that they could not obtain a single sheep, or an ox, or a horse. Today, immense flocks of sheep graze in Australia; but they are not the result of *free land* in general, or of labor, or even of capital goods in general, but of a very special kind of capital good, namely the flocks of sheep that existed in Europe. If people who know how to cultivate land have a piece of land where wheat can be grown, if they have wheat seeds and also capital goods,⁴³ such as plows, buildings, etc., and finally enough savings to be able to wait for the next crop, they will be able to live and produce wheat. Nothing prevents us from saying that this wheat is produced by land, labor, and capital; but this is to name the genus by the species. All the land, all the labor, and all the capital in the world cannot produce a single grain of wheat, if we do not have that very special capital good which is the wheat seed.

101. These considerations would suffice to show the error in these theories; but what is more, these theories are in more than one respect inconsistent with historical and present facts. They are simply an outcome of feelings of rebellion against the “capitalists”⁴⁵ and they remain extraneous to the search for uniformities, which alone is the object of science.

⁵ *Systèmes*, II, pp. 285 ff.

⁶ *Ibid.*, pp. 342 ff.

CHAPTER VII

Population

1. Man as producer is the starting point of the economic phenomenon, which ends in man as consumer; and thus we have a flow that comes back full circle to its starting point.

2. **Social heterogeneity.** As we have already indicated¹ (II, 102), society is not homogeneous, and those who do not deliberately blind themselves to this fact must recognize that men differ markedly one from another, physically, morally, and intellectually.

To these differences inherent in the human species there correspond economic and social differences which can be observed in all nations, from the most ancient times to the present and in all parts of the globe, so that, since this characteristic is always present, human society may be defined as a community with a hierarchy.

As to whether or not the community could subsist if the hierarchy disappeared, this is something we shall not inquire into, if only because the data for such a study are not available. We shall limit ourselves to considering the facts as they have taken place up to now and can still be observed.

3. **The mean² and the distribution of the deviations.** The distribution of men's qualities is only a special case of a much wider phenomenon. Many things can be observed that have a certain mean;² a large number of observations deviate only slightly from the mean, and a very small number deviate substantially. If these deviations can be measured, a graph of the phenomenon can be drawn. We count the number of observations whose deviation from the mean is between zero and one; we set aa' equal to 1 and the area $abb'a'$ equal to the number of observations. Likewise we count the number of observations whose deviation from the mean is between 1 and 2 and set $a'a''$ equal to 1 and the area $a'b'b''a''$ ³ equal to that number of observations. Proceeding in this way for all positive deviations, which go from a to m , and doing the same for negative deviations, which go from a to n , we shall finally obtain a curve tbs .

4. Similar curves are obtained in many other cases, among which the following is noteworthy.

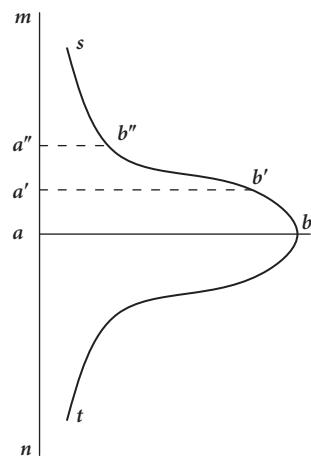


FIG 52

¹ On population, see R. Benini, *Principii di demografia*, Florence, 1901, a small book, but excellent from all points of view [and I cannot suggest a better one].¹

Suppose we have an urn containing 20 white balls and 30 black balls. We draw 10 balls out of the urn, replacing each ball, one at a time. A large number of similar drawings are made. The mean² will correspond to the drawing in which the 10 balls drawn from the urn consist of 4 white balls and 6 black balls. Many drawings will deviate very slightly from this mean; a very small number will deviate considerably. The phenomenon will yield a curve similar to that of Figure 52.

5. Starting from this observation, many authors jump to the conclusion that the two phenomena are identical. This is a very serious error. From the similarity of the two curves, we can only conclude that the two phenomena have a common property, which consists only in the fact that they have to do with observations that tend to cluster around a mean. To be justified in considering the two phenomena as equivalent, we must take the comparison of these two curves a step further in order to see whether they really coincide.

6. This has been done in a particular case. If the same quantity is measured a large number of times, the measurements will be different; the deviations from the true measure may be called the *errors*. The frequency of these errors yields a curve called the *curve of errors*, whose shape is similar to that of Figure 52. Observation [then]⁴ shows that this curve is the same as the one obtained in the case in which balls are drawn from an urn [in the manner indicated in §4²]^{6[a]}.

7. This result is really not all that correct,⁷ and it contains, basically, a *petitio principii*. In fact, the curve of errors does not always have the form indicated. When that is the case the discrepancy is said to result from “constant errors;” these are eliminated, and the curve in question is again obtained. From this it is concluded that the curve of errors has a certain shape when all the circumstances are eliminated that would give it another shape; such a proposition could not be more obvious, but all it does is to repeat the premises in the conclusions.

8. Leaving the theory of errors aside—since this is not the place to discuss it—let us observe only that in other cases it is impossible, for lack of data, to check whether the curve describing the phenomenon is identical with the curve corresponding to drawings from an urn; or else such a check may yield a negative result; and in neither case can the phenomena be regarded as equivalent.

9. It often happens that the natural phenomena show not just a single hump, as in Figure 52, but two humps, as in Figure 53, or even more. In that case the authors usually assume that the two humps of Figure 53⁸ result from the superposition of two curves of the kind shown in Figure 52, and they jump to the conclusion that the phenomenon shown in Figure 53 is equivalent to that of drawings from two urns of constant composition.

This is too hasty [and precipitous]⁹ a conclusion. We need merely observe that by suitably reproducing curves such as those of Figure 52 and superimposing them, we can obtain any curve whatsoever; consequently, the fact that a curve may be obtained by the superposition of several curves of the kind shown in Figure 52 tells us nothing about the nature of the resulting curve.

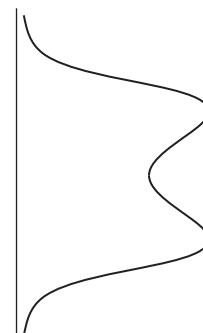


FIG 53

² [On this same problem, considered from another point of view, see Bertrand, *Calcul Des probabilités*, §§149, 150.]⁵

10. The study of the laws of wages supplies us in many cases with a certain average wage, with deviations spread out according to a curve similar to that of Figure 52, and which, moreover, is not symmetric around the line ab . But from this single analogy, it cannot by any means be concluded that these deviations follow the so-called law of *errors*.

11. **Distribution of incomes.**³ The analogy with other facts of the same kind leads one to believe that the income curve must have a shape similar to that of Figure 54. Setting mo equal to a certain income x , and mp equal to 1, the area $mnqp$ gives the number of individuals with an income between x and $x + 1$.^[a]

But for total incomes, statistics provide information only on the part cqb of the curve and possibly, in a very small number of cases, on the small stretch bb' ; therefore the part ab' , or rather ab , remains hypothetical.

12. The curve is not at all symmetric around sb ; the upper part sc is very elongated, and the part sa is very squashed.

From this alone it cannot be concluded that there is no symmetry between the qualities of the individuals who deviate on either side of the mean s .¹¹ Indeed, of two individuals who deviate equally from the mean of the qualities, the one who has an exceptional ability to earn money may obtain an extremely high income; and the one who has equal negative qualities cannot, without disappearing, fall below the minimum income sufficient for subsistence.

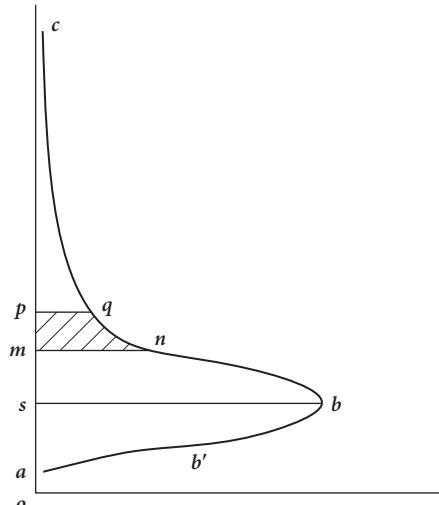


FIG 54

13. The curve $abnc$ is not the curve of human qualities, but is the curve of other phenomena that stand in relation to these qualities.

14. If we consider the marks¹² received by students in their examinations, we obtain a curve analogous to ABC [in Figure 55]. Suppose now that for any reason, the examiners never give less than 5 marks because a single mark below the average is enough to fail a candidate. In that case, for these same students, the curve will change its shape and take on that indicated by abC .¹³

Something similar happens with incomes. Above the average there is no limit to how high they can go, but below the average there is a limit to how far they can fall.

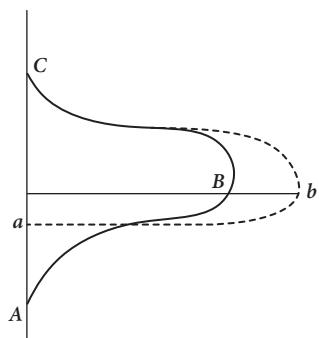


FIG 55

³ *Cours*, Book III, chap. I. To the facts set out in the *Cours* several others have been added; see the *Giornale degli Economisti*, Rome, January, 1897.¹⁰

15. The shape of the curve cqb of Figure 54, which is derived from statistics, does not correspond by any means to the curve of errors, i.e., to the shape the curve would have if the acquisition and preservation of wealth depended only on chance.⁴

16. Moreover, statistics reveal that the curve bcq ¹⁴ of Figure 54 varies very little in time and space; different nations at different times have very similar curves. There is thus a remarkable stability in the shape of this curve.

17. On the other hand it appears that there may be more diversity in the lower and lesser-known part of the curve. There is a certain minimum income oa below which men cannot descend without dying of poverty and hunger. The curve may collapse quite closely along the line ak indicating this income (Figure 56). Among nations in ancient times, among whom famines were frequent, the curve collapsed very much as in (I); among modern nations it [collapses less, and perhaps not at all, as in (II).]¹⁵

18. The area $ahbc$ in Figure 56 gives us a picture of society. The outward form varies little; inside, on the contrary, it is in constant motion: while some individuals rise to the upper regions, others fall down to the bottom. Those who reach ah [are destroyed and]¹⁶ disappear; on that side some elements are therefore eliminated. It is strange but certain that the phenomenon also occurs in the upper regions. Experience tells us that aristocracies do not last; there are many reasons for this phenomenon, and only a few of them are known, but there is no doubt about the reality of the phenomenon itself.

19. We have a region $ahkb'a'$ in which individuals are eliminated for lack of adequate incomes, whether they are good or bad; in this region selection operates very little, since poverty degrades and destroys the good elements along with the bad. Next comes a region $a'b'bla''$ where, instead, selection operates with maximum intensity. Incomes are not ample enough to save all elements regardless of whether or not they are fit in the struggle for existence, and they are not so meager as to depress the best elements. In this region, infant mortality is high, and it is probable that this mortality is a powerful means of selection.⁵ This region is the melting pot in which the future aristocracies (in the etymological sense: $\alpha\pi\sigma\tau\sigma\zeta$ = best) are fashioned; it is from this region that elements are drawn which rise into the upper region $a''lc$. Once they reach that region, their progeny declines; hence this region $a''lc$ subsists only on account of emigration from the lower region. As we have already said, the reasons for this fact are numerous and little known; the failure of the selection process may be one of the main reasons. Incomes are so high that they make it possible for even the weak, the poorly constituted, the dullwitted, and the dissolute, to be saved.

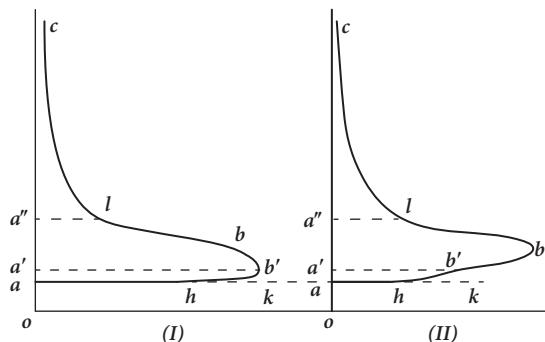


FIG 56

⁴ Cours, §962.⁵ Systèmes, II, Chapter IX.^{17[a]}

The lines $a'b'$, $a''l$ serve only to fix ideas; they have no real existence. The boundaries of the regions are not rigid lines, and one passes by imperceptible degrees from one region to the other.

20. The inferior elements in the region $a'b'la''$ fall into the region $ahb'a'$, where they are eliminated. If this region were to disappear, and if nothing else could play the same role, the inferior elements would contaminate the region $a'b'la''$, which would thus become less capable of producing the superior elements; these would go into the region $a''lc$,¹⁸ and the entire society would fall into a decline. This decay would be even more rapid if serious obstacles were put in the way of the selection process in the region $a'b'la''$. The future will reveal to our descendants whether such has been the effect of the humanitarian measures of our time.

21. It is not only the accumulation of inferior elements in a social stratum that is harmful to society, but also the accumulation in the lower strata of superior elements that are prevented from rising. When the upper strata are full of inferior elements and at the same time the lower ones are full of superior elements, social equilibrium becomes highly unstable, and a violent revolution is imminent. The social body can, in a way, be compared to the human body which promptly dies if the elimination of toxins is impeded.

22. The phenomenon is, it should be added, exceedingly complex. It is not enough to take account of incomes; one should also consider how they are used and how obtained. Among modern nations, incomes in the region $a'b'la''$ ¹⁹ have grown in a way that might seriously have impeded selection; but a substantial part of these incomes is now spent on alcoholic beverages, or squandered in other ways, so that the conditions that make selection possible nonetheless subsist. Moreover, alcoholism itself is a powerful agent of selection, wiping out individuals and races that cannot withstand it. The objection is usually raised that alcoholism is not only harmful to an individual, but also to his descendants. This objection is a very strong one from an ethical point of view, but it is worthless from the point of view of selection; it can even be turned against those who advance it. For it is obvious that a selective agent is all the more perfect the more it extends its action not only to individuals, but also to their descendants. Tuberculosis is also a powerful means of selection, since it destroys both a small number of strong people, and very large number of weak ones.

23. The data available to determine the shape of the curve blc refer mainly to the 19th century and to civilized nations; consequently, the conclusions drawn from these data do not hold beyond those limits. But it might be inferred with some probability that in other times and among other peoples, it has a shape somewhat similar to that found today.

Likewise, we cannot assert that this shape would not change if the social system were to be radically changed; if, for instance, collectivism were substituted for private property. It would appear difficult to do away with any hierarchy; and the form of this hierarchy could be similar to that now given by individual incomes, but would not correspond to money incomes.

24. If we confine ourselves to the limits indicated in §23, we see that, in the course of the 19th century, the curve blc slightly changed its shape in certain cases, the form remaining the same but the parameters varying; and this change has taken place [in a certain direction.

[To indicate this direction I made use, in the *Cours*, of the expression “decline in the inequality of incomes,” which was in common use. But this term has led to misunderstandings,⁶ as has the term *utility*, which I have had to replace by the term *ophelimity*. The same should be done with the term “inequality of incomes,” which ought to be replaced by some neologism, precisely defined. Political economy is unfortunately not yet advanced enough for us to feel free to use new terms, as is done without difficulty in chemistry, in physics, etc. I shall therefore use a still rather imperfect terminology, and designate by “decline in the inequality of relative incomes” a certain phenomenon to be defined.

[Let us imagine a community, A, consisting of one individual with an income of 10,000 francs and nine individuals each with an income of 1,000 francs; take another community, B, made up of nine individuals each with an income of 10,000 francs and one individual with an income of only 1,000 francs. Let us, for a moment, call individuals “rich” if they have an income of 10,000 francs, and “poor” those with an income of 1,000 francs. Community A contains one rich man and nine poor ones; community B contains nine rich men and one poor one.

[In everyday language, the difference between A and B is expressed by saying that the inequality of incomes is greater in A, where there is one rich individual out of ten, than in B, where instead there are nine rich individuals out of ten. To avoid any ambiguity, we shall say that in passing from A to B there is a decline in the relative inequality of incomes.

[“In general, when the number of people with incomes less than x decreases⁷ relative to the number of people with incomes greater than x , we shall say that the inequality of relative incomes declines.”⁸

[According to this definition, it may be said that the direction in which the curve of the distribution of incomes was changing slightly during the 19th century, in some countries, is that of a decline in the relative⁹ inequality of incomes.^{21]}

25. [This fact, which has been rigorously established by the mathematical study of the income curve, was ascertained earlier, both empirically and by induction, by Mr. Paul Leroy-Beaulieu,²³ who made it the subject of a famous work.^{24]} An attempt was made

⁶ See C. Bresciani, *Giornale degli Economisti*, January 1907.^[a]

⁷ [In the *Cours*, §964, this reads: increases. This is a printing error, which I pointed out immediately after the publication of the *Cours*.]²⁰

⁸ [This definition is precisely the one given in the *Cours*, §964, except that I now add the word *relative*.

[Following this definition, we read in the *Cours*: “But the reader is duly warned that by these terms I intend to indicate this thing and nothing else.” And in a footnote it is indicated that, if N_x is the number of individuals having an income of x and above, and N_h is the number of individuals having an income of h and above, and if we set

$$u_x = \frac{N_x}{N_h}$$

then “according to the definition I have given, the inequality of incomes will decrease as u increases.”^{21[b]}

[All this should really have been enough to clear up any misunderstanding].^{21[b]}

⁹ [The addition of this term to the denomination of the fact will not prevent new misunderstandings from arising, any more than did the substitution of the term *ophelimity* for that of *utility*, if one persists in trying to find out the meaning of terms from their etymology, instead of by adhering to rigorous definitions, especially to the mathematical definitions that are supplied. On this subject, see: “L’économie et la sociologie au point de vue scientifique,” *Rivista di Scienza*, 1907, No. 2.²²]²¹

to derive a general law from it, according to which the inequality of incomes was bound to continue to decrease. Such a conclusion truly goes far beyond what can be drawn from the premises. Empirical laws such as this one are of little or no value outside the boundaries within which they have been recognized as true.

26. Greater variations can be observed in some countries—for example in England—again in the 19th century, as regards the lower part *ahb* of the curve [in Figure 56]. It lies much less along the line *hk* indicating the minimum income required for subsistence.

27. If the shape of the curve in Figure 54 is changed in such a way that the very flattened²⁵ part is replaced by even a straight line, we have a curve *clb* that coincides with the one given by statistics; and the lower part *bka*, for which we have no data, will be replaced by the straight line *sb* which corresponds to a minimum income *os*, which is substituted for the minimum actual²⁶ incomes lying between *oa* and *os*.

28. This being granted, if one supposes that—as was the case with a number of nations in the 19th century—the form of the curve *blc* does not change and that only the parameters change, we arrive at the following proposition: (1) *An increase in the minimum income*, and (2) *a decline in the inequality of [relative] incomes*, (§24) *cannot take place, either separately or jointly, unless aggregate income increases faster than population*.

29. The converse proposition is valid save for a theoretical exception which is unlikely to be realized in practice,¹⁰ hence we may consider—barring this exception—²⁸ the following proposition to hold:

Whenever aggregate income rises more rapidly than population—i.e., when the average of all individuals' incomes increases—the following effects take place, either separately or jointly: (1) an increase in the minimum income; (2) a decline in the inequality of relative incomes (§24).

To prove these propositions it is necessary to have recourse to mathematics, and we therefore refer the reader to the *Cours*.

30. Given the tendency of the population to be distributed according to a certain form with regard to incomes, it follows that any modifications of some parts of the income curve have repercussions on the other parts; hence, finally, society resumes the usual pattern, just as a salt solution always yields similar crystals, be they large or small.

31. If, for example, one were to deprive the wealthiest citizens of all their incomes, thus removing the part *edc* from the figure for incomes, the figure would certainly not retain the form *abde*, but sooner or later would again take on a form *ats*, similar to the original one. Likewise, if a famine or any other similar disaster were to take off the lower stratum *akbf^[a]* of the population, the figure would certainly not retain the form *fb'bdc*²⁹ but would again take on a form *ats*, similar to the original one.

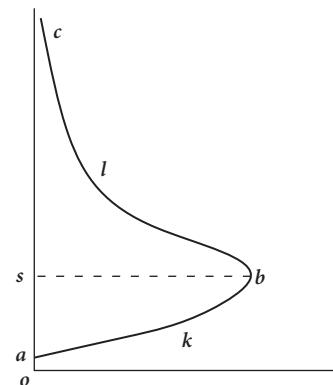


FIG 57

¹⁰ *Cours*, II, pp. 323–4.

32. Relations between economic conditions and population. It is obvious that man, like all sentient beings, tends to multiply according as the conditions of life tend to be favorable. Agricultural populations will be more dense where the soil is more fertile, and will become more sparse where the soil is less productive. Even the sub-soil, depending on how rich it is, will support greater or smaller numbers of people. The relationship is less simple in the case of industry and trade, since their connection with geological and geographical conditions are much more complex. Moreover, the population itself influences the very conditions that give it its livelihood; consequently, the density of population is the effect of certain economic conditions and is the cause of others. [Thus, actions and reactions follow each other in an endless chain.]³⁰

33. Countries where the density of population is highest are far from being the wealthiest. For instance, as is pointed out by Levasseur,³¹ Sicily has a density of 113 inhabitants per square kilometer, and France has one of only 72. Obviously, Sicily is not wealthier than France. The Ganges valley has a density twice as high as that of France.

34. But if density is not in direct relation to the wealth of different countries, it is in relation to variations in wealth within a single country. This is a first indication of a very general phenomenon. The reasons for this fact are as follows. The total number of individuals living in a given territory stands in relation to many other phenomena A, B, C, . . . which, for another territory, are partly different, for instance, A', B', C', . . . Let us assume that A denotes wealth; this varies from one territory to another, but the phenomena B, C, . . . also vary—for instance, the customs of the population, the greater ease of providing for one's needs in warm countries, etc. The effects of one of these phenomena may offset those of another, and the total effect differs from what it would be if only one of these phenomena had changed.

35. When we consider variations in wealth A, in a single country, we consider two situations: A, B, C, . . . , and A', B, C, . . . , in which the most important variation, if not the only one, is that of A. Thus the total effect, which is the only one we can observe, roughly coincides with the effect of the variation in A alone.

36. This is not all. If only variations in wealth are considered, it may—and does actually—happen that the absolute amount of wealth and the amount of the variations in wealth have opposite effects on the population.

37. For instance, in some countries the wealthiest part of the population has a lower birth rate than the poorest part¹¹ (§53); this does not prevent an increase in wealth from having as its primary effect: an increase in the number of marriages and births.

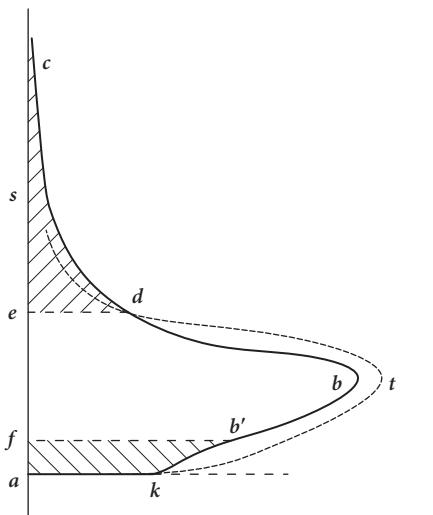


FIG 58

¹¹ *Systèmes*, II, p. 139.

38. In the 19th century, in civilized countries, a considerable increase can be observed in the average wealth per head. At the same time, the *marriage rate* (number of marriages per 1,000 inhabitants), the *birth rate*³² (number of births per 1,000 inhabitants), and the *death rate*³² (number of deaths per 1,000 inhabitants) have declined. The total population has increased, but its annual rate of increase has tended to decline.

39. These facts are mutually related. The increase in wealth has fostered the increase in population, and has very probably contributed to restraining the marriage rate and the birth rate; it has certainly operated to reduce mortality by making possible significant and costly health measures; by making people accustomed to a more affluent life, it very probably tends to reduce the annual rate of increase in population.

40. A decline in the marriage rate contributes directly to a fall in the birth rate, and consequently to a decrease in total mortality, which is considerably influenced by infant mortality. Cauderlier³³ even believes that variations in the birth rate are caused solely by variations in the marriage rate. The decrease in the marriage rate has thus, directly or indirectly, through the decrease in births, led to an increase in average wealth per head.

41. The decline in the birth rate is to a large extent the cause of the decline in mortality; and it has acted on wealth, as we have shown. It is, finally, a direct cause of the decline in the annual rate of increase in population.

42. The decline in mortality acts in the opposite direction; and as regards the size of the population, it has partly offset the decline in the birth rate. The decline in infant mortality is notable and certain; the decline in the death rate of adults is less marked and less certain.

43. Population is tending to remain almost stationary in France; it has risen considerably in England and in Germany, but even in both of these countries the rate of growth is tending to diminish. In the 19th century, the population of England increased in geometric progression, at a rate such that population doubled about every 54 years.¹² Since the average wealth per head has increased, substantially in fact, this means that in England wealth has increased at a faster rate than the geometric progression [of population] noted above.¹³

44. The improvement and the deterioration of the economic conditions of a country are related to demographic phenomena. To see this, it is necessary to have a criterion of the state of economic conditions. For agricultural nations in our part of the world, the price of wheat may serve that purpose; for industrial and trading nations, other facts have to be taken into consideration. According to Marshall,³⁴ the marriage rate in England in the first half of the 19th century depended mainly on agricultural output; in the second half of the 19th century, it instead depended mainly on the volume of trade.^[a] This change is due to the fact that England has become a mainly industrial country, from being mainly an agricultural one at the start of the 19th century.

45. At present, the marriage rate in England is related to the volume of external trade^[a] and to the total sums passing through the Clearing House; these are simply indices of industrial and commercial activity.

46. There are some general phenomena known as economic crises (IX, 73). Prosperous times are followed by times of economic depression, which are succeeded

¹² *Cours*, §211.

¹³ *Ibid.*, §212.

by other prosperous times, and so on. One can form a rough idea as to when prosperity reaches a maximum or a minimum, but the precise moment when the maximum and the minimum is reached cannot be determined; in making comparisons one must therefore proceed only by approximations.

47. If no account were taken of the preceding considerations, we could infer anything we wanted from statistical data. For example, if we wish to prove that the marriage rate is declining in England, we will compare the marriage rate of 17.6 in 1873, which is the year in which a period of prosperity comes to an end, with the marriage rate of 14.2 in 1886, at a time of severe depression. If instead we want to prove that the marriage rate is increasing, we will compare the marriage rate of 14.2 in 1886 with the marriage rate of 16.5 in 1899. Obviously we must avoid this kind of reasoning.

48. The mathematical theory of coincidence or of correlation helps us determine whether two events that are observed together a certain number of times are linked by chance or whether they occur together on account of some [causal]³⁵ relation. However, this theory is not easy to apply to the material we are dealing with. We do not deal with events that must coincide instantaneously, but rather with events that act on each other with a certain latitude,^{36[a]} whence the number of coincidences becomes truly a meaningless expression. Economic prosperity decreases, or increases, by degrees, and the available indicators of it depict the phenomenon only with a rough degree of approximation; moreover, a decrease or increase in this prosperity does not act at once on marriages, and acts still more slowly on births and deaths. If the curves of the phenomena one wishes to compare are represented graphically, one can see whether or not their oscillations stand in some relation to one another. This method, although very imperfect, is for the time being perhaps still the best one that can be used in practice.

49. The first and immediate effect of a rise in economic prosperity is to raise the marriage rate and the birth rate, and to bring about a decline in mortality. The first phenomenon is significant and is clearly visible. The second is less marked, and is perhaps, according to Cauderlier's theory, at least to a large extent merely a consequence of the first. The [presence of the] third is somewhat doubtful for civilized and rich nations; for poor nations, we have no very precise statistical data, but if account is taken of famines, which were frequent in former times, it can hardly be denied.

50. A rapid increase in the wealth of a country is favorable, in a way, to the process of selection, because it provides people with easy opportunities to get rich and to rise to the higher levels of society. A similar effect is obtained, with no increase in wealth, when the economic conditions of society are changing rapidly.

51. Up to now we have discussed only³⁷ variations in wealth; we must also consider no longer the variations but the state of this wealth, and thus compare two social situations that differ in that the average amount of wealth per head is larger in the one than in the other.

52. We already saw in §29 that this difference corresponds to another difference in the distribution of incomes and in minimum incomes; but the average amount of wealth per capita is related to many other very important facts.

53. Very rich nations have a very low birth rate, from which it may be concluded that the absolute amount of wealth acts in a way that is directly contrary to the variations in this wealth. A doubtful point remains, however. It may be that there is no causal relation between absolute wealth and³⁸ the birth rate, and that both these phenomena are instead

consequences of others, i.e., there are some causes which at the same time tend to increase wealth and to lower the birth rate.

54. Economic conditions act not only on the number of marriages, births, deaths, and on the total population, but also on all the characteristics of the population, on its customs, its laws, and its political constitution. Certain events are possible only if there is a significant increase in wealth. Among nations that can barely feed their adults, children are killed without scruple, and the old are systematically done away with; ¹⁴ [in our day, in wealthy nations],³⁹ pensions for the old and infirm are instituted. In very poor countries, women are treated [with less consideration]⁴⁰ than domestic animals; among civilized nations, among the extremely wealthy people of the United States of America, they have become luxuries who consume without producing.¹⁵ For such a state of affairs to be possible, the wealth of the country must obviously be very great. This status of women then reacts upon customs.

Feminism is a disease that can only affect a rich people, or the rich part of a poor people. With the increase of wealth in ancient Rome, immorality of women increased. If some modern girls did not have the necessary money to parade their idleness and lust, gynecologists would not be so busy. The stupid compassion for wrongdoers that has invaded certain modern nations can only subsist among wealthy nations, [who hardly suffer as a result of the destruction of a certain amount of wealth].⁴⁴ On the other hand, the increase in wealth, generally accompanied by a higher density of population and better means of communication, causes the disappearance of highway robbery in the countryside, since highway robbery no longer pays. This is not a result of an improvement of morality, because in the large cities precisely the opposite result can be observed: there, assaults are becoming extremely frequent.

With an increase in wealth, laws against [defaulting]⁴⁵ debtors may become much milder. We also know that socialist sentiments increase after a long period of peace and when there is an increase in wealth. In very poor countries, scarce capital goods are extremely expensive, and human labor is abundant and cheap; consequently, political power belongs to the capitalists, and very frequently to the landlords. As the wealth of the country increases, the value of capital decreases and that of labor increases; and the laborers acquire, little by little, the power and the privileges that used to belong to the

¹⁴ *Cours*, §247.

¹⁵ For a favorable view of American feminism, see T. Bentzon, *The Condition of Women in the United States*; for an opposite view, see an inquiry by Cleveland Moffet, of New York, reproduced in the *Mercure de France*, 1904. "Our country, some Americans say, is the one in which women receive most from man and give him the least. To them, men are only machines for making money. The wife hardly knows what her husband does, but only what he earns."

It should not be forgotten that writers always exaggerate, in one way or another.

[Mr. G. B. Baker, in an article published in the February issue of *Everybody's Magazine*, writes: "[The American society woman] is a creature of luxury and leisure. Her sole duty in life is to be amused and to be decorative. She has had time to acquire the accomplishments of society and the delicacies of refinement. Vastly superior in appearance to her mother, she is even superior to her father and brothers."⁴¹

[The situation was very different formerly, when wealth in America was far below its present level. For instance, Mistress Trollope⁴², who travelled in that country from 1827 to 1831, writes: "Except for dances... women are excluded from all the pleasures of men. These have numerous and frequent meetings... but women are never admitted to them. If such were not the constant custom, it would be impossible not to succeed in inventing some means to save the rich ladies and their daughters the trouble of fulfilling a thousand irksome housekeeping tasks which they nearly all must perform in their homes."]⁴³

capitalists. At the same time, one observes a change in customs, morality, sentiments, literature, and the arts. In poor countries, men of letters flatter the rich gentry; in rich ones, they flatter the common people.

Ancient writers were not unaware of the profound changes in the constitution of society that were brought about by the increase in wealth, but in general they described such changes, in the usual moralizing declamations, as "corruption." Sometimes, however, the facts are more accurately described. The author of *The Republic of the Athenians*, [who goes by the name of]⁴⁶ Xenophon, perceived the relation between the increase in wealth and the greater consideration shown to the lower classes of the population. He shows how the Athenians were led by reason of their business dealings to make quite a few concessions to slaves and aliens. Plato, in order to provide stability to the organization of his imagined Republic, takes great precautions to prevent its citizens from becoming too rich.

It is not by chance that the democratic system flourished in wealthy cities such as Athens and Rome; or that later on, in the Middle Ages, democracy revived where wealth reappeared, as in Provence, the Italian republics, and the free cities of Germany; or that democracy subsequently disappeared in these countries when wealth decreased. The Albigensian heresy appears to have been a purely religious event, whereas fundamentally it was largely a democratic movement that was destroyed by Crusaders from the north where wealth per head was very much less and consequently the social order was different.

The great plague which so savagely struck and scourged Europe around the middle of the 14th century, destroying many lives, caused the average amount of wealth per head to rise for a short period; whence the lower classes saw their condition improving, and as a consequence democratic movements arose in some regions such as, for instance, Wat Tyler's rebellion in England. This uprising was suppressed, but since it did not last very long, very little wealth was destroyed. Thus, since the causes remained, the effects continued to make themselves felt, and as Thorold Rogers remarks, "although the insurgent peasants had been beaten and dispersed, and their chiefs condemned or hanged, victory remained fundamentally theirs."⁴⁷

Villani observes⁴⁸ that, after the high death rate caused by the plague in Florence, "the men being few in number, abounding in the inheritance and succession of [worldly goods]⁴⁹ and forgetting the past events as if they had not taken place, gave themselves up to the most dissolute and disorderly lives, [such as they had not done before]⁵⁰ The common people, both men and women, as a result of the overwhelming abundance of everything, would not work in the usual crafts and demanded the most expensive and most delicate foods [for their livelihood]⁵⁰..."

The same thing happened in England. In Florence, which before the plague already had great wealth and democratic institutions, no attempt was made to oppose the workers' claims; in England—where, as a result of greater poverty, these institutions did not exist—an attempt was made by means of the famous *Statute of Labourers*⁵¹ to force workmen to be content with the wages they had had before the high mortality produced by the pestilence, but this attempt failed completely.

⁴⁶ *Cronice di Matteo Villani*, I, 4.⁴⁸

The best recent studies have shown how, in France and Germany, the years that preceded the birth of Protestantism were years of extraordinary economic prosperity, and this prosperity fostered the spread of religious reform and of the democratic movement that originally accompanied it. But, since the long wars that followed destroyed a large amount of wealth, the conditions that had given birth to the democratic movement faded away; hence, this movement completely, or almost completely, died out;¹⁷ but it revived later in England, France, and the rest of Europe with the new increase in wealth. And, if it is now more intense in France than elsewhere, it is not by chance that this circumstance coincides with the continued increase in wealth in that country, while the number of inhabitants remains almost constant and the average wealth per capita keeps increasing.

55. It should not be forgotten that the phenomena that we have seen follow a course parallel to the increase in wealth, act in their turn to modify the phenomenon of an increase in wealth itself, whence a certain balance is achieved between the latter phenomenon and the former ones.

It may also happen that this sequence of actions and reactions fosters the rhythmic movement that is peculiar to social phenomena. An increase in the average wealth per head is favorable to democracy; but democracy, at least insofar as one has been able to observe it up to now, [entails great destruction of]⁵² wealth and even succeeds in drying up its sources. Consequently, it digs its own grave and destroys what was giving life to it (§83).

History is full of instances that could be cited in support of this observation, and if it appears that this is not the case today, that is not only because the period of time during which the destruction of wealth has been going on has not been very long, but also because the marvelous technical improvements of our time have made it possible to produce a larger amount of wealth than has been squandered; but if the destruction of wealth were to continue to increase and if new improvements did not enable production to exceed or at least equal the destruction, the social phenomenon might change completely.

Objectively, the phenomena we have just studied stand simply in a relation of mutual dependence but subjectively they are usually interpreted as being in a relation of cause and effect; and even when, objectively, there may be something that comes close to this causal relation, it is noteworthy that the subjective interpretation often inverts the terms. Thus, it seems very likely—indeed almost certain—that humanitarian sentiments, legislative measures in favor of the poor, and other improvements in their condition contribute little or nothing to the increase in wealth, and sometimes even tend to reduce it. The relation of mutual dependence between these phenomena thus comes closer to a relation in which the increase in wealth is the *cause*, and in which the flourishing of humanitarian sentiments and the improvement in the condition of the poor are the *effects*. The subjective interpretation, on the contrary, considers humanitarian sentiments as a cause, as it is imagined that they are the cause of the improvement in the condition of the poor, i.e., of the increase in the portion of wealth they consume.

¹⁷ In Florence, the Medicis, through progressive taxation, got rid of their opponents and, at the same time, weakened democracy by removing the conditions that gave it its strength.

Some naive people imagine that if workers today eat meat every day, whereas a century ago they ate it only on holidays, this is due solely to the development of ethical and humanitarian sentiments—others say that it is because there has been a gradual recognition of the “great truths” preached by socialism—and they are unable to understand that the increase in wealth is an absolutely indispensable condition for an increase in the consumption of the common people, i.e., of the greatest number of people.¹⁸

The role played by the admirable humanitarians in bringing about an improvement in the economic conditions of the people is most often precisely that of the fly that alights on the ox’s horns [and says: “Let’s plow”].⁵⁵

56. From the preceding analysis it appears that the average amount of wealth per head is, in part at least, a reliable index of the economic, social, moral, and political conditions of a people. It stands to reason that other circumstances may intervene, so that this correspondence can only be approximate. Moreover, account should be taken of the fact that nations tend to imitate one another. Consequently, some institutions which, among a wealthy people, are directly related to its wealth, may be copied by another people, among whom these institutions would not have spontaneously arisen.

57. Production of personal capital goods. Like all capital, man has a certain cost of production; but this cost depends upon the way of life, the *standard of life* [as the English say]^{56[a]}.

58. If it is assumed that the cost of production of a man consists of the amount strictly necessary to keep him alive and bring him up, and that the equality between the cost of production and the price of the capital good obtained continues to hold for personal capital goods, the wage of labor being considered as interest (V, 88), it follows that men’s condition can never be improved in any way; any improvement for the workers’ benefit would simply result in lowering the cost of production. In this lies the essence of Lassalle’s so-called *iron law*,¹⁹ and it is the source of many errors on the part of other economists.

59. The two premises of the preceding reasoning are not borne out by the facts. We have already discussed the first one. As for the second, in its favor to be sure is the fact that the initial effect of an improvement in economic conditions is to increase the number of marriages and consequently that of births; but as against this there is the other fact that a permanent increase in wealth is associated with a decline in the number of births, and this second effect far outweighs the first.

60. [The increase in wealth does not proceed at a uniform pace; there are periods of rapid increase, others of stagnation, and even some of decline].⁵⁸ An increase in the number of marriages when the tide is rising is—in part, at least—offset by the decrease

¹⁸ I have been reproached⁵³ with the fact that, while explaining the succession of the *élites*, I did not mention the improvement of the condition of the poorer classes. I did not do so, because it does not appear to me, given the facts at my disposal, that this second phenomenon is a consequence of the first; it is a consequence of the increase in wealth, at least to a large extent. A boat comes down the river, carried along by the current, sometimes with one person in charge, sometimes with another; the two phenomena are simultaneous, not causally related.

Of course, this only allows us to see the main part of the phenomenon. The poorer classes may, as a secondary effect, derive some advantage from the struggle between the *élites*; for, as we know, when two fall out the third man wins].⁵⁴

¹⁹ *Systèmes*, II, p. 235.⁵⁷

in their number when the tide is ebbing; there remains the steady decline associated with the permanent increase in wealth.

61. The cost of production of an adult man obviously depends on infant mortality; but contrary to what one might believe, a decline in mortality in early infancy does not entail a proportionate decline in this cost.²⁰ This is because many of those who have been saved in their early childhood die a little later on, before they become adults.⁵⁹

62. Obstacles to reproductive power. An increase in population results from the opposition between reproductive power and the obstacles it may encounter. Two hypotheses are possible: It may be assumed that such obstacles do not exist and that the number of births is therefore always at a maximum, the number of deaths at a minimum and the increase in population at a maximum. Alternatively, it may be assumed that the reproductive power encounters obstacles that reduce the number of births, increase the number of deaths, and (leaving emigration out of account for the time being) limit the increase in population.

63. The first hypothesis is obviously contrary to the facts. It suffices to observe the oscillations in the number of marriages and births given by statistics; it is impossible to concede that they correspond precisely to variations in the reproductive instinct. Moreover, major oscillations can be observed in all nations. Famines, epidemics, and wars have considerably reduced the numbers of certain populations which, after a few years, have reverted to their original level.

64. We are thus left only with the second hypothesis, and it can be rigorously proved that it corresponds to the facts. Those authors who implicitly accept this hypothesis usually give it another form; they specify the obstacles, and declare that population is limited by the means of subsistence. This gives rise to discussions about how to augment the means of subsistence, whether by cutting out waste involved in using them, or by increasing them through measures considered to be useful for this purpose. And so the discussion is diverted. We should therefore cut short these considerations and consider, instead of an elastic limit such as that of subsistence, a fixed one, such as that of space.

65. In Norway, the difference between births and deaths from 1865⁶⁰ to 1880 yields an annual increase in population of 13.48 per thousand; for England, from 1861 to 1880, the corresponding figure is 13.4; for the German Empire, 12.3. Let us suppose that the population of these three countries, which was about 72,728,000 in 1880, continued to increase according to the smallest of these three observed rates,⁶¹ i.e., 12.3 per thousand per year. In 1,200 years, this would give a number of human beings equal to 1,707 followed by eleven zeros. The surface of the earth measuring 131 million square kilometers, there would thus be more than one inhabitant per square meter, which is absurd. It is therefore absolutely impossible for the population of these three countries to continue in the future to grow at the same rate as was actually observed from 1861 to 1880.

66. As for the past, it may be observed that, if the population of the earth had been only 50,000,000 at the beginning of the Christian era, and if it had grown at the rate actually observed in Norway, this would have resulted in 1891 in a number of human beings equal to 489 followed by sixteen zeros. Let us assume that in 1086 the population of England was about two million inhabitants; if it had grown at the rate observed at

²⁰ *Cours*, §255.

present, it would have had to be 84 billion in 1886. If the population of England were to continue to grow according to the law observed to hold from 1801 to 1891, in some six centuries and a half there would be one inhabitant per square meter in England.

All this is absurd; it is therefore certain that the population could not have grown in the past, nor will it be able to grow in the future, at the same rate as is observed now; it is therefore proved that there were and will be obstacles to this increase.

67. In looking for a proof of our proposition, we have incidentally also found another one. We see that the 19th century has been exceptional for the increase of population in Norway, England, and Germany (IX, 37), and that neither has there been in the past nor will there be in the future a similar growth in these countries for a long stretch of time.

68. Means of subsistence and population. Lack of means of subsistence can of course be an obstacle to an increase in population; this operates in different ways in the different social strata—see Figure 54 (§11). In the lower part, where the income curve⁶² lies very closely along the line of minimum income, lack of means of subsistence operates mainly by increasing mortality. This phenomenon is illustrated by many facts gathered by Malthus in his book. In the upper part the effect of lack of means of subsistence is only indirect. We have seen that the shape of the curve of income distribution varies only slightly; consequently, if a lower stratum is removed in Figure 54, all the upper strata come down somewhat, and the total area of the figure becomes smaller. It will be readily understood that, if workers disappear, the owners of the workshops where they worked will fall into poverty, as well as those among the so-called liberal professions who depend on these owners for their income. In the middle part of the social strata, a lack of subsistence felt directly by the lower strata often operates by bringing about a reduction in the number of marriages, by deferring the age of marriage and by causing a reduction in the number of births. The peasant who owns only a small farm does not wish to have too many children, in order not to divide this farm into too many parts. The middle-class person who is deprived of his usual sources of income, restricts his family's expenditure and the number of his children. In countries where a conspicuous part of the hereditary estate falls to the eldest son by right, the younger brothers often do not marry. These same effects can be observed in the highest strata of society, but in addition there is the very powerful phenomenon of the decadence of the élites, which causes all select races to disappear more or less rapidly.

69. Sismondi,⁶³ a fitting forerunner of our [modern]⁶⁴ *humanitarians*, believes he can prove the absurdity of the theory according to which the means of subsistence limit population, by taking the example of a family—that of the Montmorencys—which was on the verge of dying out in its day, whereas having always lived amidst plenty, it should, according to the theory criticized by Sismondi, have peopled the earth. With this great method of reasoning, anyone trying⁶⁵ to prove that the tortoise is an extremely swift animal could cite the example of the race horse.

70. It is not superfluous to note how lacking in precision is the term “means of subsistence.” It includes, to be sure, apart from food, which differs by country and races, also shelter from the weather, i.e., clothing and housing and, moreover, in cold countries, fuel for heating. [And all of these elements vary according to the circumstances. They are, certainly, not the same, for instance, for the European and the Chinese, or for the Englishman and the Spaniard.]⁶⁶

71. Nature of the obstacles.^[a] Following Malthus' example, one may divide the obstacles into PREVENTIVE ONES, which operate before and up to the moment of birth, and REPRESSIVE⁶⁷ ones, which operate after birth.

72. Preventive obstacles may operate in two ways: (α) by limiting the number of unions; (β) by limiting the number of births, whatever the number of unions. [These two ways may be combined.]⁶⁸ [Legitimate fertility may act according to (α), illegitimate fertility according to (β).]⁶⁹ Part of the population may live in celibacy; but this decrease in the number of unions (α) may be offset by an increase in the number of births from contracted unions (β).

73. (α) (1) Statistics show that among some modern civilized nations the number of marriages declines without illegitimate births increasing as a consequence. (2) Celibacy, when it is really observed, reduces the number of unions. The large numbers of harems of Eastern grandes, and polyandry in Tibet, have similar effects.

74. (β) (1) The custom of contracting marriage at a mature age reduces the number of births. This obstacle operates among certain civilized nations. Malthus exhorted people to have recourse to this means exclusively; he would have liked men and women to defer the age of marriage, and meanwhile live in strict chastity; this he called *moral restraint*.⁷⁰ (2) Marriages may be numerous and early, and the partners may use direct means of reducing the number of births. This is called *Malthusianism*, an improper term, since Malthus never came out in favor of such practices. (3) It is certain that, among many ancient nations, and among barbarous or savage nations—even in modern times—and probably among the inhabitants of certain large modern cities, abortion must be considered as an important preventive obstacle to births. (4) Incontinence and prostitution are also to be counted among preventive obstacles. (5) People believe that great intellectual activity is contrary to reproduction, but this is not certain. One could enumerate many other causes of [the decline in the number of births],⁷¹ but this is a subject that is beyond the scope of the present study.

75. Repressive obstacles may come from: (α) An increase in the number of deaths which are due directly to lack of food (poverty, famine), or indirectly to diseases fostered by poverty, or which are a consequence of lack of health measures—which cannot be put into practice, not only because of ignorance, but also because they are too costly; this cause acts continuously, and also discontinuously through epidemics. (β) An increase in the number of violent deaths, such as infanticides, murders, deaths caused by wars. (γ) Emigration.

76. Obstacles to population growth do not necessarily reduce the disproportion between population and wealth, because they may also reduce wealth. For instance, war may increase this disproportion, by destroying proportionately more wealth than men; emigration may impoverish a country less in men than in wealth.

77. The indirect effect of the obstacles may be different from the direct one (\$80).

It should be noted that a population A and a population B may have the same annual growth rate, resulting for A from a large number of births and a large number of deaths; and for B from a small number of births and a small number of deaths. The first type is that of barbarous nations and also, in part, of civilized nations until a century ago; in contemporary Europe, Russia, Hungary, and Spain approximate this type. The second type is that of the more wealthy and civilized nations; in contemporary Europe, France, Switzerland, Belgium approximate this type.

78. Even if the increase is the same for A and for B, their populations differ in composition. In A there are many children and fewer adults; the opposite is true for B.

79. Equilibrium between the number of births and deaths, which results in an increase in population, depends on an infinite number of economic and social causes; but once it is established, if a variation accidentally occurs in one direction, a variation in the opposite direction will immediately follow, which will restore the original equilibrium. As a matter of fact, this observation is a tautology,²¹ because it is this very fact that characterizes and defines equilibrium (III, 22). We should thus change the form of the observation just made and say that experience shows us that this equilibrium actually exists; but it can slowly change.

It is well known that after a war or an epidemic, marriages and births are more frequent, whence the population, decimated by war and epidemics, quickly reverts to its original level. Similarly a rise in emigration need not result in any fall in population, and may operate only as a stimulus to marriages and births. Conversely, a rise in the number of marriages and births may be rapidly offset by a rise in the number of deaths and in emigration.

80. An effect of a [totally]⁷⁴ different kind is generally produced by certain practices designed to reduce population, which may have the effect of permanently modifying customs and thus of altering the conditions of equilibrium and therefore changing the equilibrium itself. It is thus asserted that emigration, by providing an outlet to excess population, reduces prudence in generating offspring; and hence, in the last analysis emigration may in some cases be a cause not of a decrease, but of an increase in population. Similar observations have been made regarding abortion, the exposure of newborn children, and infanticide. Conclusive proofs are lacking, however, in support of this thesis.

81. Subjective view of phenomena related to an increase in population. The question of an increase in population and of the obstacles to it is one of those which [most]⁷⁵ people apparently cannot discuss without becoming emotional and getting upset; the cause of this lies in the fact that they are concerned not so much with scientific research as with defending a preconceived theory; and for those who contradict them they feel the wrath which assails believers against heretics.

Here we have a good example of the way economic factors combine with other factors to determine men's opinions. The ratio of wealth to population is a very powerful factor in determining social events; and it is these factors which, operating on people living in society, determine opinions. It is thus [only]⁷⁶ in this indirect way, and nearly always without the knowledge of those affected by this action, that the ratio of wealth to population operates (§54).

82. It is in the interest of the wealthy classes and political oligarchies for population to increase as much as possible, because abundance of manpower makes things easier

²¹ Some [among the best known]⁷² authors have seen in these facts an indication of a mysterious law, which they have called the "law of compensation." They will discover their alleged law in all cases where equilibrium exists.

Levasseur,⁷³ in *La population française*, II, p. 11, states: "When a demographic phenomenon suddenly deviates from the mean . . . usually a sudden reaction also takes place . . . ; the following year, sometimes even for several years in succession, this phenomenon still remains divergent from its mean and reverts to its level only after several oscillations, thus obeying a *law of compensation*."

for those who hire it, and because a larger number of subjects increases the power of the ruling class. If there were no other causes [intermingling with the effects along with these ones],⁷⁷ the phenomenon would thus be very simple: on the one hand, the wealthy and ruling classes would advocate an increase in [the birth rate];⁷⁸ on the other hand, the poorer classes would be favorable to restricting this increase. Such may be the theory, but in fact precisely the opposite might happen, and the rich could limit the number of their children in order to keep their estates intact, whereas the poor might have a large number of children in order to profit by them, or simply from improvidence. A phenomenon of this kind can be observed in France, and it is no accident that the *nationalists* and conservatives are active and enthusiastic supporters of measures designed to increase population (§86). The radical socialists, on the other hand, are not so shrewd, and their government shows itself ready to give approval to legislation aimed at [fostering the procreation of children]⁷⁹ (§86). It is true that in general such measures are usually ineffective; but if they were effective, they would destroy the very basis of the radical socialists' power.

83. The phenomenon is, it should be added, much more complex than it appears at first sight. First of all, without going outside the field in which the economic principle operates, we know that this principle may have differing effects by reason of men's ignorance or momentary wants.

Do revolutions occur more readily when the poorer classes suffer from poverty, or when they are relieved by [prosperity]?⁸⁰

84. If the first kind of answer to this question is accepted, there may be times when the wealthy and ruling classes will advocate the limitation of population for fear of seeing the power of their adversaries grow; and the popular leaders, on the contrary, will advocate an unrestricted increase in population, precisely in order to increase the number of their troops. This is what happened around the end of the 18th century and at the beginning of the 19th, and it was the basis of the polemic between Godwin⁸¹ and Malthus.

85. ⁸² If the second kind of answer to the question is accepted—and although this appears paradoxical at first sight, careful study shows it to be much more in agreement with the facts (§54)—the effects of the above-mentioned economic principle are entirely different. The ruling classes sometimes understand this, but sometimes they do not have a clear idea of it, and show an unawareness of the causes underlying observed events. So, although de Tocqueville⁸³ clearly pointed out the true solution of the problem in a special case, we see today many members of the ruling class acting in a way that will harm the future of their own class. Like blind men groping their way forward, they do not have any clear view of the road it would be to their advantage to follow, and they end up by bringing about their own ruin. Ethical factors also contribute to this outcome, as well as the physiological decadence of the élites. The leaders of the popular classes, i.e., essentially the new élites who emerge to dispossess the old élites, have frequently understood how an excess of poverty could lead simply to riots that are easily repressed by the ruling class; and how, on the contrary, an increase in well-being is more effective in preparing revolutions. This is why some of these leaders openly support the limitation of population, whereas others ignore this issue, or else give half-hearted support to measures aimed at increasing population (§82). But the leaders, who would sooner be disposed to limit it, have to contend with a serious obstacle in that they have to give satisfaction

to their followers' sentiments (§87). The common man is especially concerned with his present sensual wants, and wishes to eat, drink, and satisfy his sexual instincts; hence the leaders are induced to promise him that when "capitalism" is destroyed and the golden age has dawned, all these wants or desires can be satisfied without restraint.

86. [There are not only economic motives for actions;] ⁸⁴ there are also ethical, religious, metaphysical, and ascetic ones, etc. Religious conservatives are indignant at the very idea that, independently of any economic motive, man should seek to circumvent the divine precept: increase and multiply. Everything connected with sexual relations has in modern times been covered with a chaste, and also very hypocritical, veil. Hence the idea that man is bold enough to calculate the consequences of the sexual act—and in anticipation of them, to regulate them—appears to some people as something so monstrous that they become incapable of discussing it dispassionately. These are the motives, as well as others which it would take too long to enumerate, that impel many members of the upper classes of society to offer strong opposition to anything tending to limit population. Sometimes these motives reinforce the economic motives mentioned above, but sometimes, too, they are so strong that they may by themselves determine people's opinions. These doctrines are derived from sentiments only; and, instead of deriving [them] ⁸⁵ from the facts, their authors would have the facts be made to fit the theories. They already know the solution to the population problem before studying it; and if they have recourse to observation, it is not in order to look for the solution of the problem posed, but only to find arguments to justify their preconceived opinions.

87. Among the common people, other causes have similar effects, and we have already indicated them in §85. The promise of an extreme abundance of economic goods, thanks to a new social order, appears insufficient to some people, who wish to add to it the removal of all checks on the passions; some even go so far as to maintain that men will be allowed to give free play to their sexual instincts, because not the slightest troublesome consequence need be feared; and Fourier, ⁸⁶ more logical than the others, in the same manner gives satisfaction to all human instincts. These fantasies are sometimes dressed up in pseudoscientific garb, and it is stated that people will be able to indulge in their sexual instincts without fear because that instinct will diminish with the increase in intellectual [ability]. ⁸⁷ It may be noted that the effect remains exactly the same if few children are born, either because the sexual instinct is powerful but men do not let themselves be dominated by it, or because the instinct is weak but men by no means abhor it. All this excitement, then, is simply to find out whether, centuries hence, certain acts will or will not be voluntary. [Anyone with time to waste can go on arguing about it; we shall turn to another issue.] ⁸⁸

88. The facts we have just examined are psychological facts, facts concerning opinions and doctrines; it should immediately be added that these beliefs and opinions have had little or no effect on the actual growth of population; thus it seems that it is this increase which has operated on the psychological facts just indicated, rather than the other way around. In the first half of the 19th century, scientists and statesmen in France advocated the limitation of population growth, *Malthusianism*, and population was increasing; now, the necessity of stimulating the growth of population is advocated, and population remains stationary.

89. Malthus and his theories.²² In the approach still used in studying political economy, it is not considered acceptable to examine the population problem without mentioning Malthus; and although I do not approve of this custom, there is no point in flouting it so long as it subsists. Moreover, some good may come out of such a study, and Malthus' theories will provide us with an example of the errors in which one inevitably falls when theory is confused with practice and scientific research with moral preaching.

90. Malthus's work is [very]⁸⁹ confused: it is often difficult to understand precisely what questions he is dealing with. Essentially we may distinguish four parts in this work.

91. (1) A scientific part, i.e., a search for uniformities in phenomena. Malthus has the great merit of having set out and attempted to prove that reproductive power by itself would have led to an increase in population larger than that actually observed; from this it is concluded that this power is checked by certain obstacles. But, not contented with this general theory, Malthus has entered into some more questionable details. He has tried to establish that population tends to grow in geometric progression, and the means of subsistence in arithmetic progression; moreover he believed that this geometric progression was such that population would double every 25 years.

It is incredible how many controversies have raged over these two famous progressions of Malthus, and to how many stupid observations these have given rise.⁹⁰ In some cases, Malthus' ideas have been so badly understood by his detractors that their good faith may be questioned.

92. If we compare this theory of Malthus with the facts, we see that in a particular case—that of England in the 19th century—population grew in geometric progression, and doubled approximately every 54 years; but that wealth increased in a still more rapid progression; hence in this case the arithmetic progression does not correspond to reality at all (*Cours*, §§211, 212).

93. Similarly, Malthus goes far beyond the observation of facts when he asserts that the obstacles necessarily belong to one of the three following categories: *moral restraint*,⁹¹ vice, and miserable conditions of life (*misery*).⁹¹ The sole purpose of this classification is to convince people of the obligation to practice *moral restraint*.

94. (2) A descriptive and historical part, in which the author sets out to demonstrate the existence and the effects of the two latter kinds of obstacles. He says that the first kind “does not at present prevail much among the male part of society”, although abstention from marriage, when it is considered independently of its moral consequences, is a powerful factor among modern nations in reducing the number of births.

95. (3) A polemical part, in which the author seeks to prove that men's state of being well or badly off depends almost exclusively on their practicing greater or less restraint in the number of births; and that they depend very little, or even not at all, on the government's action or on the social order. This part is obviously in error.

96. (4) [A preceptive part.]⁹² The author has discovered the universal panacea, that is, *moral restraint*,⁷⁰ or, to use the current terminology, he has solved the “social question;”

²² For a view opposing *Malthusianism*, see the work of Professor Tullio Martello, *L'economia politica antimalthusiana e il socialismo*, Venice, 1894, which is full of penetrating observations and profound thoughts.

he mounts the pulpit and reveals [the great mysteries to the people].⁹³ This part may be ignored. One more sermon, over and above the endless number already delivered, to show how useful, beautiful, and noble chastity is [, makes no impression one way or the other].⁹⁴

97. Human society in general. As has already been noted (II, 102), society appears to us as a heterogeneous mass, hierarchically organized.²³ This hierarchy is never absent, except perhaps among savages, who live dispersed in the manner of animals. A consequence of this is that society is always governed by a small number of men, by an *élite*, even when it seems to have an absolutely democratic constitution; and this has been recognized since the most remote times. In the Athenian democracy, there were demagogues, i.e., the “leaders of the people;”²⁴ and Aristophanes, in *The Knights*, shows them lording it over the dazed people.²⁵

98. One could imagine a society with a stable hierarchy; but such a society would have nothing to do with the real world. In all human societies, even in those organized into castes, the hierarchy eventually changes; the main difference between societies is that this change may be more or less slow, or more or less rapid.

The fact, [already]¹⁰⁴ so often recalled, that aristocracies do not last, is stamped on the whole history of our society. This has also been known even since the remotest

²³ Professor R. Benini has carried out some excellent studies of these *social hierarchies*.

²⁴ The word δημαγωγός comes from δῆμος and ἀγω.

²⁵ Eq, 62: ὁ δ' αὐτὸνώς ὁρᾷ μεμακκοῦ κόρτα, “when he sees him in this state of stupidity.” See also *The Scholiast*. Incidentally, the whole comedy lays it on thick.^[a]

[Professor Mosca regrets and is very much upset that I failed to cite him when I recalled the fact that in society it is always a small number that governs, and he seems to believe that he discovered this himself. To satisfy him, I now give the titles of his works of which I know only the last one: *Teorica dei governi e governi parlamentari*, 1884; *Le costituzioni moderne*, 1887; *Elementi di scienza politica*, 1896.

[But the principle according to which it is the minority that rules has been known for a long time; and it is a commonplace that it is to be found not only in scientific works but even in exclusively literary works. Here is an example chosen at random; E. Fournier, in *L'esprit des autres*, 1856, p. 83, referring to the proverb: “Fools since Adam are in the majority” adds: “Alas! and whatever the parliamentary axiom may say, it is not the minority that governs.”⁹⁵ Balzac, in *Physiologie du mariage*, Chapter X, says: “Montesquieu, who had perhaps foreseen the constitutional régime says, somewhere or other, that good sense in assemblies was always to be found among the minority.” And, if we wanted, we could continue to provide other examples.

[In a very valuable scientific work by H. Sumner Maine on *Popular Government*, published in the *Quarterly Review*, April 1883 and April 1885,⁹⁶ the author recalls Strauss’s⁹⁷ opinion that “history is a sound aristocrat” and adds that “the progress of mankind has hitherto been effected by the rise and fall of aristocracies . . . There have been so-called democracies, which have rendered services beyond price to civilization, but they were only peculiar forms of aristocracy.”⁹⁸ Further on: “The modern enthusiasts for Democracy make one fundamental confusion. They mix up the theory, that the Demos is capable of volition, with the fact that it is capable of adopting the opinions of one man or of a limited number of men.”⁹⁹ The whole work would deserve quotation on this point. Similar opinions on the part of Renan¹⁰⁰ could be added.

[Tarde, too, and many other writers are of the opinion that, as Giusti puts it:

But the few, my dear friend, pull the many
If the many are held back by inertia or asininity.

[And to tell the truth, they usually are.

[Tarde has written entire volumes to show that civilization is exclusively the work of a few persons.]¹⁰¹ In our own day, French, English, United States, and other democracies are in fact governed by a small number of [petty]¹⁰² politicians. Similarly, absolute monarchies, except for the very rare cases in which the monarch is an extraordinary genius, are also governed by an *élite*, which is very often a bureaucracy.¹⁰³

times;²⁶ it has [now been newly]¹⁰⁶ confirmed scientifically by the inquiries of Jacoby and Ammon.²⁷ The history of human society is in large part the history of the succession of aristocracies.

99. No races of living beings can avoid decay in the absence of selection; and the human race is no exception to this law. The *humanitarians* may well close their eyes and willfully ignore this truth, but that does not alter the facts in any way. In every race a scum always develops that must be destroyed by selection. The pain caused by this destruction is the price paid for the maintenance and improvement of the race; it is one of those many cases in which the good of the individual is in opposition to that of the species (II, 30). Some kinds of selection may disappear; but they must be replaced by others if the decadence of the race is not to take place. Now there are some people who believe that henceforth the human race can do without the selection brought about by war. They may be right, but they may also be wrong. What is certain is that they do not provide any valid proof for their belief; certainly, [their]¹⁰⁷ declamations on the evils of war and the sufferings it brings on man [do not constitute such a proof].¹⁰⁸

100. Finally, there is a very important circumstance which, as has already been explained at some length, stands in relation to [the greatest number]¹⁰⁹ of social events and which determines many of them to a large extent. This circumstance is the relative amount of wealth—or rather of capital—per head in society. Where this ratio is higher, civilization is more developed. It should, however, be recalled that we are forced to evaluate [wealth]¹¹⁰ in monetary terms, and that the monetary unit is not at all fixed^[a]; consequently this [wealth per head]¹¹¹ is observed only as a very rough approximation.

Most people believe that variations in the distribution of wealth have a greater influence on new forms of society than variations in total wealth per inhabitant. But this point of view is entirely erroneous, since we have seen that the former variations are of slight importance (§16), whereas the latter variations can be very large (§92).

101. We have now mentioned four kinds of circumstances: hierarchy, the succession of aristocracies, selection, and the average amount of wealth or capital per head. These circumstances are by far the most important in determining the characteristics of society, i.e., of other social events. But the latter then operate in their turn on the former, so that what we have is a relation of mutual dependence[, and not one of cause and effect].¹¹²

102. Quantitative conditions for the utility of society and individuals. There does not appear for the time being to be any question of setting a limit on the amount of capital per head; but the day may come when this has to be considered.

²⁶ Dante, *Purgatorio*, VII, 121–122:

Rade volte risurge per li ram
L'umana probitate . . .

Paradiso, XVI, 76–78:

Udir come le schiatte si disfanno
Non ti parrà nuova cosa nè forte,
Poscia che le cittadi termine hanno.¹⁰⁵

²⁷ Paul Jacoby, *Etudes sur la sélection dans ses rapports avec l'héritage chez l'homme*. Paris, 1881; Otto Ammon, *Die Gesellschaftsordnung und ihre natürlichen Grundlagen*. [Jena, 1909]; G. Vacher de Lapouge, *Les sélections sociales*. [Paris, 1896].

103. On the other hand, for hierarchy, the succession of aristocracies, and selection, the problem of maximum utility is mainly quantitative. Human societies cannot subsist without a hierarchy; but it would be a very grave mistake to conclude from this that they will be the more prosperous the more rigid is this hierarchy. Similarly, the change in aristocracies is [beneficial]¹¹³ but neither should the need for some stability be disregarded. Selection must be maintained within such bounds that its effects [with respect to the utility of the species]¹¹⁴ are not bought at the cost of excessive suffering [on the part of individuals].¹¹⁵

These considerations raise many serious problems, which are not our concern here. It is enough to have indicated that they exist—which a great many people still do not realize, or question, or refuse to admit.

104. Stability and selection. One could imagine a human society in which every day each individual carried out his activity independently of the past; [changeability or]¹¹⁶ mutability in such a society would be at a maximum. In the absolute, such a state of affairs is impossible, because one cannot prevent an individual from depending—at least in part—on his own past activity and on the circumstances in which he has been living, if only because of the experience he has been able to acquire. The most wretched savage peoples only approximate such a state, for they generally have some hovel, some weapons—in short, some capital [goods].

105. At the other extreme, one can imagine a society in which every man has been assigned a role, from his birth to his death, and is not allowed to deviate from it; stability in such a society would be at a maximum, and society would be as though it were crystallized. Neither does this extreme case exist in the real world; societies organized in rigid castes come somewhat close to it.

106. Intermediate cases of all kinds can be found in past and present societies. In modern societies, the element of stability is furnished by private property and inheritance; the element of mutability and selection comes from the faculty that allows everybody to rise as high as possible in the social hierarchy. To tell the truth, there is nothing to indicate that this situation is perfect, or that it should last forever. If one could effectively take away some kinds of private property—for instance that of capital, and even, in part or wholly, inheritance—the element of stability would be greatly weakened, and the element of mutability and of selection would be strengthened. It is not possible to decide a priori whether this would be useful or harmful to society.

107. The reasoning that starts from the premise that in the past it was useful to reduce the force of one of these two elements and to increase that of the other, and concludes that it will also be useful to proceed thus in the future, is valueless, because in all quantitative problems of this kind there is a maximum [beyond which there is a decline].¹¹⁷ Reasoning in this way would be like concluding from the fact that the germination of a seed is promoted when the temperature rises from 0° ¹¹⁸ to 20° , that it would be promoted still more if the temperature were to rise, say, to 100° .¹¹⁹

108. Likewise, not the slightest value can be attached to arguments that start from the premise that a decline in one of these two elements and a rise in the other have been observed in the past, and conclude that this is also what will have to be observed in the future. Movements of human societies do not constantly occur in the same direction, but they are generally oscillatory.²⁸

²⁸ *Cours*, II, §258; *Systèmes*, I, p. 344.

109. The advantages of mutability, which is a cause of selection, and the drawbacks of stability, depend to a large extent on the fact that aristocracies do not last. Furthermore, because of man's [sentiment of]¹²⁰ misoneism and his reluctance to engage in excessive activity, it is good that the best individuals should be stimulated by competition on the part of those who are less able than they; so that even the mere possibility of change is useful. On the other hand, [mutability]¹²¹ pushed to extremes is very painful to man; it discourages him, it depresses him, and hence reduces his activity to a minimum. An individual who is worse off than someone else naturally wants to change his situation; but as soon as he has succeeded, he desires still more ardently to conserve what he has acquired and to stabilize his condition. Human societies have a very strong tendency to confer a certain rigidity on any new system of organization, and to crystallize in any new pattern. Accordingly it often happens that one passes from one pattern to another, not by a continuous motion, but by jumps: one pattern breaks down, and is replaced by another; the latter will break down in its turn, and so on. This is what can be observed in all forms of human activity, for instance, in language, law, [the arts,]¹²² etc. No living language is immutable; and on the other hand, a language composed exclusively of neologisms would not be understood; one must keep to a happy medium. The introduction of neologisms is not uniformly continuous, but occurs at intervals on the authority of famous writers, or of some literary authority such as [our own *Accademia della Crusca*]¹²³ or the French Academy. Similar phenomena can be observed in the field of legislation; and it is not only in countries where the law is codified that changes lead to a rigid new system, but even in those countries where legislation would appear to be much more malleable.²⁹

110. In social economy, mutability may take various forms, and these forms may be partially replaced by others. Change may act in a way opposite to selection; but we shall consider here only the type of mutability that promotes it. Violent revolutions often have this result. When active, energetic, and intelligent elements have accumulated in the lower strata and when, on the contrary, the upper strata are contaminated by an excessive proportion of decadent elements (§§ 20, 21), a revolution suddenly breaks out which replaces one aristocracy by another. The new social pattern then takes on a rigid form, and then it will in turn be shattered by some other similar revolution.

These violent revolutions may be replaced by infiltrations that cause the selected elements [—the fittest—]¹²⁶ to rise, and the decaying elements to fall. This movement nearly always exists, but may be of varying intensity; and it is this [diversity in]¹²⁷ intensity that gives rise to the accumulation, or lack of it, of decaying elements in the upper strata and select elements in the lower strata.

111. For the movement to be sufficient to prevent this accumulation from taking place, it is not enough for the law to allow it, or not put obstacles of any kind in its way, such as [the caste system]¹²⁸ for instance; it is also necessary that the circumstances be

²⁹ H. Sumner Maine, *Ancient Law*, London, 1861, Ch. III, compares the systems of equity in Rome and in England: “[In Rome as in England, jurisprudence based on Equity]¹²⁴ tended, and all such systems tend, to exactly the same state in which the old common law was when Equity first interfered with it. A time always comes at which the moral principles originally adopted have been carried out to all their legitimate consequences and then the system founded on them becomes as rigid, as unexpansive, and as liable to fall behind moral progress as the sternest code of rules avowedly legal.” [[French] translation, Courcelle-Seneuil, Paris, 1874, p. 66.]¹²⁵

such that the potential movement can be realized. For example, among warlike nations it is not enough for law and custom to allow a private to become a general; it is also necessary for war to provide him an opportunity for such a promotion. In trading and industrial nations, it is not enough for law and customs to allow the poorest citizen to become rich and to reach the highest positions in the state; it is also necessary for commercial and industrial activity to be intense enough for these ambitions to be realized for a sufficiently large number of citizens.

112. Measures that reduce debts directly or indirectly weaken the stable element, and therefore indirectly reinforce the element of mutability and of selection. Likewise, anything that brings about a general rise in prices has the same effect, but only so long as this increase lasts. If, for instance, all prices double, economic equilibrium ends, after a fairly long time, by being identical with what it was initially; but, in the passage from one state to another, debts are reduced, and mutability and selection are promoted. Debasement of coinage, the increases in the quantity of precious metals (for instance, after the discovery of America), issues of paper money, customs protection, the labor unions that push for wage increases, etc., have in part the effect of promoting mutability and selection; but they also have other effects, and it remains to be seen in each particular case whether the damage they cause does not exceed the potential advantages.^[a]

113. It has been noted that in Athens, after Solon's [reforms],¹²⁹ it was no longer necessary to resort to debt reduction; the currency was not debased, and no other devices were adopted to raise prices. The main reason for this must be sought in the intense commercial activity in Athens, which was alone sufficient to ensure the circulation of the aristocracies.

114. From the times of classical antiquity up to our own day, we can observe among the nations of Europe a succession of revolutions, legislative measures, deliberate or accidental facts, all of which help to strengthen the element of mutability and selection. It may with great probability be concluded from this that the element of stability, or even of change operating against selection, was extremely strong; consequently, as a reaction, there were developments that were designed to weaken it. For other societies the conclusion might be different. The need to provide for changes favorable to selection is also related to the proportion of selected elements produced from the lower social strata. It may be that the greater stability of some oriental nations is due, at least in part, to the fact that this proportion is lower there than among [Western]¹³⁰ nations.

115. If, among our [Western]¹³¹ nations the element of stability were exclusively the result of the institution of private property and of its consequence—inheritance, this would provide a very cogent demonstration of the need to reduce, or even to suppress, the institution of private property. It is strange that the socialists have not noticed the support that their doctrines could derive from this way of looking at things.

But the element of stability [that is opposed to change through]¹³² selection is far from being exclusively the consequence, in our society, of the institution of private property. Laws and customs have divided men into classes; and even where these classes have disappeared, as among the modern democratic nations, there remain appendages of wealth that enable some individuals to drive back competitors. In the United States of America, politicians and judges often sell themselves to the highest bidder. In France, the Panama case¹³³ and other similar cases have shown that European democracy does not differ essentially in this respect from American democracy. In general, from ancient

times up to our own day, the upper classes of society have used political power to despoil the poorer classes; at present, in some democratic countries, a diametrically opposite phenomenon seems to have begun. We have never been able to observe, for a very long time, a state of affairs in which the government remains neutral and does not assist the poor in despoiling the rich, or conversely. We thus cannot decide, purely empirically, whether the superior force of the element of stability which is opposed to the selection of elements from the lower strata, has its origin in the institution of private property or in the political domination of the upper classes. In order to draw correct conclusions, it would be necessary to disentangle these two kinds of facts and observe their effects separately.

116. Subjective interpretation of the preceding facts. So far, we have observed facts objectively; but they appear in a quite different light to men's consciousness and cognition. I have shown [elsewhere³⁰]¹³⁴ how the circulation of the *élites* was interpreted subjectively, hence I shall not dwell on this point. In general, men are inclined to give their particular demands the appearance of general demands. A new aristocracy trying to supplant another, older, one will usually join battle not in its own name, but in that of the majority of the population. A rising aristocracy [always]¹³⁵ wears the mask of democracy (II, 104).^[b]

The mental state induced by the accumulation of superior elements in the lower strata and of inferior elements in the upper strata has often been manifested in religious, moral, political, and pseudo-scientific theories about the equality of men. Hence the paradox that it is precisely the inequality of men that has impelled them to proclaim their equality.

117. In ancient times nations reduced debts and the interest on loans without theoretical discussions; governments in former times debased the currency without giving thought to economic theories, and took protectionist measures without even knowing what protection was. The facts were not the consequence of theories; quite the contrary, the theories were constructed in order to justify the facts. In our day, people want to give a theoretical foundation to all these facts. A religious justification has been provided for the reduction, or even for the abolition, of interest on money, and this has led to grave intellectual disputes, the practical effect of which is approximately nil, because they do not touch the real causes of the facts at all.

Let us suppose that it were possible to provide a rigorous proof that interest on money is not "legitimate," or, conversely, that it is perfectly "legitimate;" in neither case would the facts be changed, or else they would differ in quite a negligible way. It is the same with customs protection. All the theories for or against it have not had the slightest practical effect; studies or speeches on the subject have been able to have some effect, to be sure, but not on account of their scientific content, but because they aroused certain sentiments and [gave some people having certain interests the opportunity]¹³⁶ to unite.^[a] The theoretical disputes that took place some years ago on bimetallism were perfectly useless; they are now closed because the rise in prices has come from factors other than the free minting of silver. Today Marx's theory of value has [almost]¹³⁷ become a museum piece now that the socialist leaders have gradually risen to form part of the government of the state. The statement that value is crystallized labor is nothing but

³⁰ [*Systèmes*, I, p. 34.]^[a]

the expression of malaise of the select elements of the new aristocracy when forced to remain in the lower strata. It is consequently perfectly natural that, as they rise into the upper strata, there is a change in their sentiments, and consequently also in their mode of expression. This is especially true of the class as a whole, because sentiments may persist for some individuals even when the circumstances that gave rise to them have changed.

It should never be forgotten (II, 4) that men, as a rule, are not conscious of the origin of their sentiments; consequently it often happens that they believe that they submit the evidence to theoretical reasoning, whereas they act under the influence of quite different causes.

CHAPTER VIII

Landed Capital and Capital Goods Proper

1. Landed capital. Capital of this kind must be considered in the state in which it is found; and it is futile to try to separate land from the capital goods proper which are said to be "incorporated" in it.

Landed capital consists of agricultural land, mines, and industrial sites for industry, housing, and country cottages, etc.

2. Competition in landed capital takes place indirectly by means of its products or by means of the consumers who move to places where they find the landed capital that suits them. Thus, wheat from land in the United States of America is transported to Europe and competes with wheat from the land of this continent. Thus it is, too, that owing to the development of modern means of transportation, people working in the inner cities can live in the suburbs, so that suburban land in this way competes with land in the center of the city.

3. It is difficult and often impossible to produce new landed capital by saving; consequently, the phenomenon of *rent* shows up more clearly with this capital than in other cases.

4. Landed capital does not have any privileged economic position with respect to other capital goods; it is neither more nor less indispensable in production than the others. On the other hand, it often has a greater [social and]¹ political importance than other types of capital goods; and for a long time and among a large number of nations, political power has belonged to the owners of the land.

5. Ownership of land may assume many forms. In practice, there is a great variety of examples of the broad classes of ownership: collective, family, and individual.

6. Likewise, there is a great variety in the forms of relationship between owners of the land and those who work on it. Several of these forms may coexist and be more or less appropriate according to the circumstances. The search for the best form of ownership *in abstracto*² is thus an insoluble problem. In modern agriculture we find the following forms, which are very widespread: direct cultivation of the land by the owner and his family, cultivation by laborers under the owner's direction, tenant farming, and [sharecropping].³ Each one of these forms is better adapted than others to certain crops and certain economic and social contingencies.

7. It may be socially useful for land not to change ownership too easily; in general it is economically useful for ownership of land to be easily transferred to those who best know how to use it. It is equally useful for the nominal owner of the land to be also the actual owner. This is not the case when the land is mortgaged at a value nearly equal to that of the land itself. In that case, the nominal owner is in reality an agent of his creditors, and works on their behalf.

8. Capital goods proper.⁴ This category consists of all capital goods remaining after subtracting personal capital (men) and landed capital. Factories, houses, provisions^{5[a]} of various kinds, domestic animals, machines, means of transport, furniture, [metallic currency],⁶ etc. are among the principal capital goods proper. Most of them can be readily acquired through the transformation of savings. Several of these capital goods may be readily transported from one place to another, and consequently competition takes place directly between them. Cases of rent that can be observed for this type of capital are often less important than they are for landed capital.

9. Savings. Savings consist of economic goods that men abstain from consuming.^[a] Since these goods are usually valued in money, the illusion is easily born that savings consist of money.

10. The goods saved are not accumulated, but are promptly transformed; consequently, the total amount of available savings at a given moment in a country exist only to a small extent in the form of inventories; for the most part, they are in the form of capital goods proper, or in the form of improvements to landed capital, or they are incorporated in personal capital.

Care should be taken not to confuse [savings proper]⁷ with savings transformed into capital goods—i.e., transformed into things that are useful in production; or with *savings capital*,¹ which is that part of savings which, although not transformed into other capital goods, is nevertheless useful in production. For example, the wheat that is stored in a granary is savings proper; part of this wheat, when used to feed the laborers who till the land—a part which, though consumed in this process, will be reconstituted at harvest time—is *savings capital*; another part, which is used to purchase the oxen that plow the land, or the machine⁸ that threshes the wheat, ceases to exist in the form of savings and is transformed into *capital goods*.⁹

It should be kept in mind that this classification has the same peculiarities as those we recognized in the case of the concept of *capital goods*⁹ (V, 20); that is, it is unrigorous and somewhat arbitrary. It is nevertheless convenient in furnishing an idea of a large number of phenomena without making use of mathematics; and the lack of rigor does no harm, because no use is made of this classification in the formulae of pure economics, which alone provide us with rigorous proofs.

11. Savings are acquired only partly on account of the interest they yield; they also arise partly from man's desire to keep goods in reserve that can be consumed when the need arises; and they also originate in part from the same kind of instinct that operates with many animals. This is why men would not cease to save even if interest on savings should become equal to zero; it might even happen that some individuals would save more—at least within certain limits—or when the yield on savings declines.^[a] Let us imagine an individual who intends to stop working once he has saved enough to be able to enjoy an income of 2,000 lire for the rest of his life. If the interest rate on savings falls, he will have to work a larger number of years, or save more each year, or both, in order to obtain the same income. We may note that in civilized countries, from the beginning of the 19th century up to our own day, the yield on savings has been declining and at the same time the supply of savings has been increasing.

¹ *Cours*, §90.^[a]

In conclusion, within the limits—admittedly restricted—of our observations, we can by no means assert that the annual supply of savings depends exclusively or even mainly on (is a *function* of), the interest rate on savings; and still less can we assert that it increases with a rise in this interest rate, or conversely.

In the transformation of savings, man is moved by an enormous number of considerations; one of these is the gross interest he will draw from his savings. If all other considerations are the same as between two different transformations, he will choose the one that will yield the higher gross interest; but if the circumstances of these two uses are different, it can happen that he will choose the one that yields the lower gross interest, but offers other favorable opportunities.

We have already taken a number of these circumstances into account (V, 30), and we have eliminated them by subtracting from the gross interest certain amounts for insurance and depreciation of capital goods; what remains is [approximately]¹⁰ the net interest.²

We could proceed in the same way and likewise eliminate some other circumstances; but this would often be very difficult and of little utility.

12. It should, moreover, be remarked that the elimination just indicated is only approximate. It corresponds to objective considerations, whereas the use of savings is determined in large part by subjective considerations; we already noted this fact in discussing the profits that can be earned by firms (V, 68). Let us add an example. Consider two uses of a sum of 1,000 lire: (1) The probability of losing it in the course of the year is $1/4$, hence the insurance premium is 250 lire; gross interest is 300 lire, whence net interest is 50 lire. (2) The probability of loss of this sum is only $1/100$, consequently the insurance is only 10 lire; gross interest is 60 lire, whence net interest is 50 lire.

Net interest is thus the same in both cases; the two uses are therefore objectively equivalent. But the first will be preferred by some individuals, and the second by others; hence in reality each of these uses has a certain kind of savings directed towards it, and there is little or no competition between these two kinds of savings.

² [Literary economists, who have the unfortunate habit of speaking about things they do not understand, regard this as a tautology. It is natural, they say, that if from two gross rents one eliminates everything that makes them different, equal residuals will be obtained.]

[This is not the question. The rate of interest is related to a large number of circumstances: A, the insurance premium; B, the depreciation premium; C, the difference in price between a future good and a present good, or the net rate of interest; D, E, F, etc., an infinite number of other objective and subjective circumstances. The theorem consists in asserting that at a given time and place, and for certain types of employment of capital (e.g., purchase of securities on the stock market): (1) The set of circumstances D, E, F, . . . , although they may in exceptional cases have a preponderant influence, have in general, on the average, an influence which is much less than that of the circumstances A, B, and C; so that the first may often be disregarded in comparison with the second. (2) A and B are essentially variable, in any case much more so than C, which in the conditions indicated remains nearly constant. Thus, as a first and rough approximation, the residuals that are obtained by subtracting A and B from the gross rents are nearly equal.]

[These explanations are given here solely with a pedagogical intent, since any polemic with persons not used to scientific reasoning can only be a waste of time.]

[It is useless, for instance, to refute the assertion of Professor A. Graziani, who naively believes that “to include in one category of transformations that of species, like that of place and of time, amounts to making a verbal unification.” If the observation of the facts has not taught him that, far from being united only verbally, these transformations always exist together in concrete phenomena, so that most of the time they can be separated only by abstraction, he must be abandoned to his lucubrations, which have only a remote connection with scientific reality.]¹¹

[12 bis. There are infinitely many extremely varied circumstances which cause gross interest rates to vary.

[For instance, on the stock market, securities of a new issue, perfectly identical in every respect to the securities of former issues, may nevertheless, for a time—until they obtain a good rating—have a lower price than the securities of previous issues.

[Sometimes there are curious anomalies. For instance, the 5 percent 1906 Russian bond issue was, during the whole of 1907, quoted in London at 3 to 4 percent less than in Paris. Thus, on January 20, 1908, one could have bought these bonds in London³ at 91½ percent, whereas in Paris they were quoted at 96.10 percent. These bonds have absolutely the same guarantees, and in a short time will be negotiable equivalently in London and in Paris. The difference between the two prices might be explained at a pinch, at least in part, by speculation—by the fact that for a person who buys securities to resell them, Paris is a better place to trade because only a single price is quoted there. But it is difficult to understand why a Frenchman who wants to invest his capital in 5 percent Russian bonds should pay 96.10 percent for them in Paris, when he could have them for 91½ percent in London. This is probably due to psychological reasons, sheer habit, etc.

[But there is a still more curious anomaly. In Paris and Brussels, two series of 4 percent Argentine bonds are traded, with interest payable at the same time, namely in April and in October. It is absolutely impossible to establish the slightest difference in intrinsic value between the 4 percent Argentine bonds of 1897–1908 and the 4 percent Argentine bonds of 1900. Well, the second series was quoted in Paris and in Brussels, in 1907, at a few points above the first. On August 10, 1907, in Paris, the first series was worth 84.05 percent, and the second 91.70 percent. There is thus a commodity which is apparently unique¹² and which on the same market, at the same moment, has two prices. It may be that there is a question of rating of securities, but a complete explanation of these phenomena is yet to be provided.

[Many more anomalies can be observed in the sale of commodities at retail. It is not uncommon, for instance, to find two neighboring shops selling an identical commodity at different prices.

[The conclusion to be drawn from all these facts is the one we have already mentioned many times. Political economy, like many other sciences, is concerned only with general and average phenomena. Meteorology can tell us the average annual rainfall in a given locality; it is and will always be incapable of telling us anything about the fate of each and every raindrop.]¹³

13. Similarly, the various modes of employment of savings can give rise to different classes of savings, which are almost like commodities of different qualities.

Among the circumstances we may consider in this way there is that of the time during which savings must remain employed, i.e., the fact that lending of savings—or any other corresponding operation—is for a short or long term. In reality, savings do not constitute a homogeneous mass. A part may be employed only for a short time, another part for a relatively long period. All possible varieties can be found in the financial markets of our society; from the savings that may be lent for only a few days to those that may be lent for several years.

³ In London, two prices are quoted, one for securities the public wants to sell and the other for securities the public wants to buy. These two prices were 91 percent and 91½ percent.

14. The modern corporate form of organization in which stocks can be easily bought and sold has blunted the differences between savings that may be lent for a short term and those that may be lent for a very long term, because those who buy shares that have a wide market on the Stock Exchange are always sure of being able to resell them when they have need of their own savings. However, they are not sure of being able to obtain their purchase price. This explains why governments generally pay less interest on their treasury bills than on their bonds. For the former, one is certain to get back exactly the amount lent; for the latter, one may obtain more or less than that amount.

15. As with differences in time, various categories of savings may be distinguished according to differences in space. As a rule, a higher interest rate must be offered abroad than at home in order to induce savings to flow out of the country.

16. Many other psychological factors influence the yield on savings. In France, the alliance with Russia has had a beneficial effect on Russian government bonds, and consequently the yield on these is lower than it would have been without this goodwill on the part of French buyers.

Finally, psychological factors that accompany economic crises also have an effect on the determination of the yield on savings.

17. Interest on savings and the social system. Interest on savings arises out of the difference between something available today and something available after a certain lapse of time, just as the price difference between wine and oil arises from the difference in the qualities of these two commodities. But to determine the interest rate on savings quantitatively—as with the difference between the price of wine and that of oil—it is necessary to bring in all the conditions of economic equilibrium.

18. Consequently, whatever the social system of organization (V, 48), given that what man can enjoy today will never be equal to what he will be able to enjoy later—just as wine will never be equivalent to oil—there will always be interest on savings—just as there will always, at least in general, be a difference between the price of wine and the price of oil. But this interest and these prices will vary quantitatively in accordance with the social system, because that system has to be taken into account among the conditions of economic equilibrium (V, 48).

19. It is possible to conceive of a social state in which each person employs only the savings he himself produces and of which he is the owner; in such a social state one might say—to employ some modern jargon—that the producer is not separated from the means of production. Some people will have more savings than they can use, and for them the yield on savings will be approximately zero; others will be very short of savings, and for them the yield will be very high. When instead savings can be traded, the interest rate will have a value between these two extremes. This trading naturally entails certain expenses; nevertheless, the economic advantage to society is very considerable, and it is for this reason that in every society trading in savings finally comes about.

20. It is also possible to conceive of a social state in which the government has a monopoly of trade in savings, as there is at present a tobacco monopoly in certain countries. From a strictly economic point of view, it is hard to decide whether this monopoly in savings would cause the interest rate to rise or to fall; it can only be said that up to now state industries have in general had a higher cost of production than private industries, which is clearly demonstrated by the fact that state industries have never been able to withstand competition by private industry, and that the state has always had to

resort to force to eliminate this private competition. But it may be objected that what has not happened in the past may take place in the future, and there is nothing to stop us from believing that the organization of state industries could be improved. Moreover, the monopoly might be partial. For certain uses of savings, private trade may remain superior to monopoly; for others, there may be no great difference.

21. But two systems of organization for the use of savings may be equivalent from the economic point of view, and differ enormously from the social point of view. These two things must not be confused. The existence in a society of a class of shopkeepers gives this society a different imprint from what it would have if retail trade were carried out by department stores, cooperatives, or a state monopoly.^[a] Similarly, a society where there is private trading in the use of savings, and another society in which this commerce does not exist—because it is a state monopoly, or because everyone employs only his own savings—differ enormously from a social point of view, quite apart from any differences that may exist from an economic point of view.

22. **Subjective interpretation of the phenomena.** The obstacle closest to us that we find in procuring certain goods is the one that we find most striking. The child believes that the only obstacle to obtaining toys is the will of the shopkeeper who wants money. Likewise the grown-up may believe that the dealers' *greed* is the only obstacle to acquiring goods cheaply; it is this sentiment that has given rise to laws fixing *price ceilings* on merchandise. A person who wants to transform future goods into present goods believes there is no other obstacle than the usurer's dishonesty, or "exploitation by the capitalist."

23. In addition to these sentiments there are others that have their origin in the social system. Most people consider only the practical problem and hence the problem as a whole, and are absolutely incapable of splitting it up into its various parts.

24. The sentiments to which we have just referred are primitive; they arise directly in man as a reaction to the obstacles he encounters, and they will therefore always subsist, even if they are considerably weakened.

As we have repeatedly pointed out, man feels an overpowering need to give his sentiments a logical appearance, to represent as a result of reasoning what is the result of instinct, to provide a logical theory of his non-logical actions. The form of [this thinking]¹⁴ is that which agrees best with the times in which they occur, on the one hand, and with their authors' temperament, on the other.

Theories are more or less developed depending on the case. In the case of the obstacle created by the price that must be paid to a merchant selling a commodity, the theories do not seem ever to have been very complex; but when it comes to the obstacle created by the cost of transforming future goods into present ones, [there have been extensive and flourishing theories].¹⁵

25. There is an aura of mystery in the transformation of future goods into present goods; the subject therefore lends itself to subtle disquisitions. In part precisely because of this arcane characteristic, it has often been dominated by religious precepts, and has given rise to metaphysical, juridical, and economic theories. These theories can be studied by anyone who wishes to [be acquainted with the theory of human concepts]¹⁶ and the evolution of social psychology; but they teach us nothing about the objective phenomenon of interest on capital. The polemics to which they have given rise do not and cannot have the least effect in changing the objective phenomenon; or, to put it with strictest rigor, this effect is so slight that it may be considered negligible. For suppose, to

take an impossible hypothesis, that it can be proved that one of these theories is false, so clearly that the demonstration will be accepted by all; this will not undermine in the slightest the sentiments that have given rise to the theory, and consequently these sentiments will simply continue to give rise to other similar theories. And in the absence of any polemics, solely with the passage of time these sentiments will take on another form. It is thus that the Middle Ages produced theological and metaphysical theories, and that modern times give us economic theories, such as that of Marx's *surplus value*, that of *free land*,^[a] etc., without, however, causing the disappearance of juridical theories, such as that of Anton Menger who, with only a slender knowledge of political economy, deduces from it certain *fundamental rights*,⁴ which are really very amusing; but, after all, every tailor uses the cloth he has.

26. The alleged law of declining interest on capital. It is certain that, in the past, interest on capital has at times risen, and at other times declined, without it being possible to detect a general trend in the movement. It has been asserted that, starting from our own times, this movement was always bound to take place in the direction of a decline in interest rates. We should make note of this here, because it provides a good example of the confusion so often made between science and practical arts.

Leroy-Beaulieu maintains that there are three causes of the decline in the interest rate: (1) the security of the transactions [and the ease in negotiating credits],¹⁸ (2) the increase in the volume of savings, and the fact that all existing savings are now channeled into the market; (3) the decline, in a given state of technique, in the productivity of new capital goods. There are, he maintains on the other hand, three causes that are conducive to a higher interest rate: (1) great discoveries that are capable of being translated into practice; (2) the outflow of capital to the new countries; (3) wars and social revolutions.

He concludes that these last three causes have a lesser intensity than the first three, and that consequently there is bound to be a gradual decline in the yield on capital.⁵

27. There are two very different parts to this reasoning. The first is of a scientific character; and the second of a practical nature.

In the first part, the author establishes relationships between certain facts and interest on capital; and although there may be greater literary elegance than scientific rigor in these trinities^[a] of favorable and adverse causes in opposition to one another, this first part may be accepted.

In the second part, the author fixes his gaze into the future and divines what will happen. But how does he come to know that there will no longer be great discoveries similar to that of the railways? that prolonged wars do not threaten mankind? that we are safe from profound social upheavals? And yet, according to his own statements, if we are to accept his conclusion we must assume that none of this will happen. But even if his statement were correct, it would have been the result of an extraordinary intuition, a kind of second sight, and not of scientific reasoning; because, given our existing knowledge, no reasoning of this kind enables him to know whether in a few or many years from now there will or will not be prolonged wars, social upheavals, great discoveries, etc.

⁴ *Systèmes*, II, p. 107.¹⁷

⁵ "[But the result of all these movements is the normal tendency of the interest rate on capital to gradually diminish." *Traité théorique et pratique d'économie politique*, II, p. 165.¹⁹]²⁰

28. The facts already prove [at least in part,]²¹ that our author has not been a good prophet. He predicted that, some twenty or twenty-five years after 1880, and, consequently, from 1900 to 1905, interest on capital would decline in Western Europe to 1½ or 2 percent.⁶ On the contrary, in 1904 the 3 percent French loan, the 3 percent German loan, and the 2½ percent British loan were quoted below par, [and at the beginning of 1908 the German Empire issued a large loan at 4 percent.]²²

29. **Money.**⁷ A commodity that serves to express the prices of other commodities is an IDEAL MONEY ([the French]²⁴ *numéraire*), or a CONCRETE MONEY (or simply: money). [The latter enters into operation materially in exchanges; the former does not.]²⁵

We have a TRUE MONEY when the exchanges in which it enters are free. When a commodity is a true money, one kilogram of this commodity in nonmonetary form can be exchanged for one kilogram (a little more or a little less) of this commodity in monetary form. For instance, if 10 marenghi²⁶ are melted down in the crucible, the gold bullion obtained can be exchanged for very slightly less than 10 marenghi,²⁶ therefore the marenghi²⁶ constitute a true money. If 40 silver scudi²⁷ are melted down in the crucible, the silver bullion obtained can be exchanged only for much less than 40 scudi²⁷; at present it would be exchanged for about 20 scudi.²⁷ The silver scudo²⁷ is thus not at present a true money.^[a]

Any money that is not true is a fiduciary money, or else a false money. The former is willingly accepted by traders, without undergoing fraud or violence; the latter is accepted only because whoever receives it is legally obliged to do so, or is deceived.

[A borderline case]²⁸ between these two kinds of money is fiduciary money which is legal tender. For instance, Bank of England notes must be accepted by the public at their nominal value, but can be immediately exchanged for gold at the Bank of England. In the Latin Union, silver scudi²⁷ may in practice, but not legally, be exchanged for gold at little or no loss; they are thus a fiduciary money which is legal tender. Inconvertible paper currencies, when they cannot be exchanged for gold at par, are a false money.

30. Money fulfills two principal functions: (1) it facilitates the exchange of commodities; (2) it guarantees this exchange. The first function can be fulfilled just as well by a true money as by a false one; the second can be fulfilled only by a true money.⁸ It is by paying attention only to the first function that the error has been committed of considering money to be simply a token without intrinsic value.

31. **Foreign exchange.** One kilogram of gold in London and one kilogram of gold in New York are not identical objects; they are differentiated by space. Consequently, an individual may give up a bit more or a bit less than one kilogram of gold in London to obtain a kilogram of gold in New York. This little bit more or less is the EXCHANGE PREMIUM,²⁹ which is unfavorable to London in the first case, and favorable in the second.

32. Other circumstances of lesser importance help to differentiate these two equal weights of gold. It may be necessary for the gold to be coined; or the gold may already

⁶ [The author again made this prophecy in 1896 in his *Traité théorique et pratique d'économie politique*, II, p. 166.]¹⁸

⁷ I would like to refer to an excellent work by Professor Tullio Martello, *La Moneta*; unfortunately, it is out of print. [A new edition by some publisher is much to be desired.]²³

⁸ *Cours*, §§ 276 ff.

be in the form of [a certain currency].³⁰ Account should also be taken not only of the space but also of the time required for transportation, etc.

33. Taking all these circumstances into account, we can determine the expense of transporting to New York, and of making available in the form of the currencies³¹ in use there, one kilogram of gold now in the form [of bullion]³² in London. These expenses determine the GOLD POINT.

34. There are two ways in which a person in London can make a payment in New York. He may purchase a claim on New York (check, bill of exchange, etc.) and pay the exchange premium, or he may actually ship gold and pay the necessary expenses. He will obviously use the means most advantageous to him, and consequently he will buy claims as long as the exchange premium remains less or at most equal to the costs of transporting and transforming the gold.^[a] The gold point is therefore the point at which gold begins to be exported from the country to make payments abroad.

We have described the main lines of the phenomenon; many details should be added. The gold point may vary according to the circumstances: for instance, depending on whether gold is exported simply to settle a debt, or for speculative purposes, etc.

35. The exchange rate and international trade. If international trade is in a state of equilibrium, suppose this equilibrium is disturbed by an increase in commodity imports. This increase in imports will have to be paid for with the country's gold. The exchange rate will turn against the country, and the price of the home currency in terms of foreign currencies will fall; consequently, the prices of home commodities, which remain nominally the same, will decline when expressed in foreign currency. The result will be to stimulate exports and to discourage imports. We thus have two forces that tend to restore equilibrium. This is not all. To obtain gold from abroad, a higher interest rate will have to be paid; in practice, the banks of issue will have to raise their discount rate. This will create an obstacle to new transformations of savings into capital, and to new consumption; there will thus be a tendency via this route, too, to return to the equilibrium position.

If equilibrium is disturbed by an excess of exports, it is obvious that the phenomena are exactly opposite to those we have just described.

36. In a country with a paper currency, if equilibrium is disturbed by an excess of imports, the price of the paper currency declines in terms of gold. This stimulates exports and discourages imports, and these forces operate, as in the previous case, to re-establish equilibrium.

As for the discount rate,^[a] governments—to protect, as they say, trade and industry—usually try to keep it nearly constant. To succeed in this, they either restrict the amounts discounted—which in the end has an effect similar to that of a rise in the discount rate, since it tends to discourage new transformations of savings into capital goods, as well as consumption—^[b] or else they achieve their purpose by increasing the quantity of paper money in circulation, which depresses its price, and consequently increases the intensity of the forces that stimulate exports and depress imports.

37. It is essential not to confuse the dynamic effects that take place in passing from one equilibrium position to another with static effects in a given equilibrium position.

As a result of having fallen into this confusion, some authors have imagined that a depreciated currency favors exports and discourages imports. This is not correct; these effects occur only while the currency is depreciating.

Consider an equilibrium position in which a sum [of 100 units]³³ of paper money is worth 30 units in gold; and another position in which 100 units of paper money are worth 50 units in gold. These two positions may be identical, and even identical with the equilibrium position that would have been obtained with a gold circulation—except for secondary phenomena, depending on uncertainty as to the value of the currency. These positions are identical because prices have varied precisely in inverse proportion to the depreciation of the currency; that is, what was worth 100 in the equilibrium position with gold circulation is worth 125 when 100 units of paper money cost 80, and is worth 200 when 100 units of paper money cost 50.^[a] In these three equilibrium positions, exports are not stimulated more, and imports are not discouraged more, in one position than in another.

But while passing from the first position to the second, or from the second to the third, certain prices remain nominally the same, i.e., they decline when expressed in gold; and this is the reason why exports are stimulated and imports discouraged.

38. It is precisely because the equilibrium positions we have just indicated are identical that a country with a paper currency can return to a gold currency by changing the value of the monetary unit, assigning to it a nominal value equal to its real value.^[a] This was done in Russia and in Austria-Hungary.

39. On the other hand, if nothing were changed in the conditions of the country, and if gold were simply borrowed to restore convertibility, the effect would be nil: as soon as it entered the country, the gold would flow out again.^[a] If it were otherwise, this loan would have had the power to change all the economic conditions in the country and to bring it into a new equilibrium position.

Gold cannot be made to circulate in a country by introducing it in an artificial manner, but by attracting it by way of trade.

40. Gresham's law. This law states that "bad money drives out good;" but this is an elliptical expression. For bad money to be able to drive out good money, a sufficient quantity must be put in circulation; otherwise, both kinds of money can circulate at the same time; and this is what happens in fact with the low-value copper and nickel coins that circulate along with gold.

Gresham's law is nothing but a corollary of the principle of the stability of economic equilibrium. Since the quantity of money in circulation corresponding to equilibrium cannot be arbitrarily increased, if an additional quantity of money is put into circulation, an equal amount of money will go out of circulation and be exported or melted into bullion. It is obvious that the best currency³⁴—the one with the higher price—will thus be withdrawn from circulation and be replaced by the debased currency.³⁴

41. Bimetallism. It is possible, within narrow limits, to have two [true]³⁵ moneys in circulation—for instance, gold and silver. Let us now suppose that the price of silver in terms of gold declines; a greater quantity of silver will be minted, and the rise in the demand for this metal will cause the price of silver to rise—in fact it may cause it to rise enough to reach its former level. But the limits within which this phenomenon is possible are very narrow; and it will be readily understood that, if the output of silver exceeds these limits, the increase in the demand for silver for minting will not be sufficient to bring the price of silver back to its previous level; thus all the gold will go out of circulation, and the circulating medium will consist exclusively of silver. It is a fact that in France, bimetallism has always been unstable; sometimes, it has tended to turn into a gold monometallism,

sometimes a silver monometallism. At present, it would certainly have become a silver monometallism if the minting of silver had not been prohibited.^[a]

42. Money substitutes. Among civilized nations, money is used very little in exchange; it is replaced by [bank notes],³⁶ checks, bills of exchange, bankers' transfers, etc. In England, at the Clearing House, where the claims and debts and credits of certain bankers are settled, vast transactions take place that would be physically impossible to handle if a metallic money had to be used. The amount of metallic money in circulation in England has remained nearly constant, while commercial transactions have increased enormously; metallic money has been replaced by money substitutes.

Among modern civilized nations, money is coming to play a less and less important role in exchanges,^[a] which for the most part tend to be settled without having recourse to its use, as was the case when money did not yet exist and goods were obtained through direct barter.

43. Metallic money constitutes a very small part of the wealth of a country. For example, the wealth of England is valued at 251 billion francs, whereas metallic money is less than 3 billion. It can thus be seen how mistaken are those who consider gold to be wealth, or even only to be capital.

44. According to the estimates of the *United States Comptroller of the Currency*, there are about 26 billion francs of minted gold in the world. Needless to say this figure is very speculative.

45. The data for annual industrial consumption of gold and silver are even more tentative. Nevertheless, [since a little is better than nothing],³⁷ the following are the estimates of the *United States Comptroller of the Currency* for the year 1901:

Silver 1,370,685 kilograms

Gold 119,271 kilograms

Francs 411 millions

46.³⁸ Banks. Deposit banks receive deposits and make loans; they thus act as entrepreneurs who transform savings proper into savings capital, or sometimes into capital goods; they therefore play a very important part in production.

Banks of issue emit bank notes and keep metallic money which serves for the conversion of notes, in order that the notes can remain fiduciary money and not become false money. They thus fulfill the public function of assuring monetary circulation [of the metal],³⁹ and by saving on the use of metal and the resulting consumption of it that takes place when it circulates.

47. It is inaccurate to say that the gold lying in the vaults of the banks of issue serves as a *guaranty* of the notes. The one and only guaranty of the notes is for these to be always exchangeable without the slightest difficulty for gold. The gold metal the banks hold in their vaults is simply a means of effecting this exchange. The value⁴⁰ of the bank notes has no direct relation to the quantity of gold existing in the vaults of the bank, but only to the ease, or difficulty, of exchanging these notes for gold. If a bank has plenty of gold in its vaults and does not exchange it for its notes, the notes may be quoted below par; whereas another bank, which holds much less gold, but which exchanges [it for] its notes, will have its notes quoted at par. The Scottish banks, when they were free, were able for some time to ensure the exchange of their notes with a reserve of metallic money equal to approximately one seventh of the value of the notes.

48. The great banks of issue can, within certain limits, modify the conditions of the money market in their country through the rate of discount. But it is a mistake to believe that, where there is a true money, they are able to fix the discount rate at will; this rate must always be approximately equal to the one corresponding to equilibrium. When the Bank of England foresees future monetary difficulties and, in order to prevent them, decides to raise the discount rate, it borrows in the market against the security of English consols; it thus succeeds in reducing the amount of money available for loans.^[a]

49. When the metallic currencies⁴¹ in the vaults of a bank of issue are depleted, the bank, in order to correct this state of affairs, can only resort to raising the discount rate; any other means has little or no effectiveness, and may cause serious harm. Among the means to be avoided is the one that consists of borrowing in order to replenish the vaults with gold; if the causes that made gold flow out persist, the coffers will soon be empty again (§39).⁴²

50. A rise in the discount rate is harmful to entrepreneurs; this is why the latter exert pressure on the government, and the government in turn on the banks, to prevent it. And whenever this object is achieved, inconvertibility of bank notes is not long in coming.

CHAPTER IX

The Concrete Economic Phenomenon

1. When one wishes to study crystallography one starts with the study of geometry, not because it is believed that crystals are perfect geometrical bodies, but because the study of geometrical bodies provides indispensable ingredients for the study of crystals. In like manner we have started with the study of pure economics, not out of a belief that the abstract phenomena of this science were identical with the concrete phenomena, but simply because the first study was useful to us in undertaking the second one.

In Chapters VII and VIII we already turned to the study of concrete phenomena in investigating the properties of certain capital goods; we shall now concentrate on the concrete phenomena of the economy in general.

2. In consumption, the concrete phenomenon differs from the abstract one principally because some consumption is fixed by habit,^[a] and because for the rest, man is a very imperfect scale for weighing ophelimities. The equality of weighted ophelimities is thus achieved only as a fairly rough approximation.

3. Many commodities produced on a large scale must subsequently be sold at retail. Strangely enough, a good many economists almost disdain to concern themselves with retail prices, as if the subject were beneath the dignity of the science. They think they can discuss the wholesale¹ price of wine but not the price of a liter of wine sold by an innkeeper. And yet, nearly all the wine produced ends up being sold by innkeepers, restaurateurs, retailers, and producers for household use.

In retail sales, competition often operates little or not at all. Retailers are much more numerous and their capital adds up to a much vaster amount than would be necessary for the distribution of the commodities.^[a] It is to these circumstances that consumers' cooperatives and department stores owe their [prosperous]² success.

4. In the more civilized countries these retailers form syndicates and fix uniform prices, which are generally far higher than the wholesale costs of the commodities or than the costs of production; they are often two or three times as high, or even more.

5. The number of retailers, as well as their capital stock, increases up to the point where in spite of these high prices their occupation does not yield higher earnings than can be obtained in others.

6. It should be noted that the damage caused by such an imperfect organization of distribution is much greater than the damage that is attributable solely to the expense of maintaining this excessive number of retailers and of paying interest on the superfluous capital. Suppose that in a certain country these two sums were to amount to a total of 100 units a year; it would be greatly to the consumers' advantage to pay this sum of 100 directly to their parasites, provided that it were possible to have the kinds of prices for consumer goods that would result from a well-organized system of distribution. This observation is a general one and applies to all similar cases (VI, 8 et seq.).

Similarly, among the principal kinds of damage caused by the [overbearing behavior]³ of the labor unions, capitalist syndicates, and sellers' syndicates, one should include changes in the production coefficients, which take on values different from those that would assure maximum ophelimity. The wealth thus destroyed is often much greater than that appropriated by the syndicates.^[a]

7. Wholesale prices of many commodities vary daily; retail prices remain constant during fairly long periods. For instance, wholesale prices of flour, coffee, and cotton vary daily, whereas retail prices of bread, coffee, and cotton do not change. Consumers do not like prices to change too frequently, and retailers accommodate them by averaging out wholesale prices. In this respect, the concrete phenomenon differs again from the abstract one.

8. In wholesale production one finds phenomena that are closer to those studied by pure economics. The organization of this kind of production is [very good],⁴ and this explains why production cooperatives [have earned little or nothing in the way of a good yield].⁵ In wholesale production one also finds syndicates, trusts, and monopolies. Nevertheless, in Europe the damage suffered by consumers is perhaps less than that done them by shopkeepers' syndicates and trade unions.^[a] In the United States of America, it is perhaps equal or even greater.

9. Subjectively the phenomenon appears to be different because most people who [discuss]⁶ it are led by the contemporary humanitarian mania to excuse not only all the damage done by the workers and the less well-to-do, but even to excuse all the crimes that these worthy people commit; whereas hate blinds them when they speak of the well-to-do, and especially of the hated "capitalists," and still more when they discuss the "speculators."

Pantaleoni quite rightly remarks that "it is truly peculiar that this crusade against these alleged monopolies—and hence in favor of free competition, which is said to be threatened—is led by people who, when syndicates (trusts) are not involved, never tire of pointing out the damage, as serious as it is imaginary, done by this same competition, and of calling for legal measures against it that are no less drastic than those they would like to devise against the syndicates (trusts). It is equally peculiar that the same people who recognize a monopoly defined in an agreement concluded among producers to sell a commodity at one price rather than another, and again recognize this monopolistic character in the agreement if it has to do with the sale of certain services—for example transportation by rail or ship—no longer recognize this character in an agreement among individual sellers of personal services, such as bricklayers, [laborers],⁷ etc."

10. Trusts. Modern syndicates have two principal aims: (1) To allow firms to attain the size that corresponds to minimum cost of production. We have already discussed this in connection with the firm in general, and it is unnecessary to do so again. Pantaleoni adds as an aim the linking together of related firms and forming them into an economic unit. It cannot be denied that this is sometimes the case, but, at least for the present, it is very secondary in comparison with the other aim, which remains to be discussed: (2) To escape from the pressures of free competition, wholly or partially.

¹ *Giornale degli Economisti*, March, 1903, p. 240.⁸

11. Briefly, this second aim can scarcely be absent, but is often hidden. For example, it will be said that syndicates do not aim at raising prices, but at preventing them from falling so far as to become *ruinous*. But it is precisely those prices that are ruinous for producers which are advantageous to consumers, not only directly but also indirectly; because it is under the pressure of such prices that firms introduce improvements in their methods of production, and by removing this pressure from them the syndicate also removes the need for such innovations. It is true that such improvements are still encouraged by the desire to make greater profits; but it is in man's nature to act with greater energy to avoid imminent ruin than to acquire greater profits;^[a] and it is precisely for this reason that industries managed by the state, which in any case have their survival insured thanks to the taxpayers, do not make as much progress as private industries which have to struggle for their very existence.

12. In some countries, syndicates maintain that their only aim is to oppose *unfair* competition (against which they often request the assistance⁹ of the law); but one need merely take a closer look at things to realize that this competition which is described as *unfair* is simply plain competition, and nothing else. [On this, it is enough to cite the following example.]¹⁰ In May 1905, Swiss newspapers published the following announcement on behalf of the lithographers: "The general assembly of the Swiss society of master lithographers met on May 20 and 21 at Lucerne. Since unfair competition continues to be rampant, it has been decided to institute a committee of honor . . . whose task will be to evaluate unfair practices, *especially offers at ridiculously low prices* . . . The assembly has regretfully been obliged to exclude a firm which has been conspicuous, at different times, for its ridiculously low prices."

13. It cannot be denied that there have been some trusts that have prospered [successfully]¹¹ without enjoying any privileges, without the aid of customs protection [, and without the support of artifices];¹² but they are [of small importance]¹³ compared with the trusts which owe their origin and their [existence]¹⁴ to [measures]¹⁵ of this kind.

14. It should be noted that for the small syndicates, which are perhaps the most harmful to consumers because there are so many of them and because they charge excessive prices, the indulgence of the authorities and the [cowardly sloth]¹⁶ of consumers are often enough to make monopoly possible.² This is what partly causes the success of cooperative societies, a success that would be greater still if they had the courage to sell

² A cotton producer advertises his wares in the newspapers, adding, in order to ingratiate himself with the retailers' syndicate, that "he does not sell directly to consumers." If consumers also had a syndicate and responded by not purchasing this cotton, the producer would change his tune. Meanwhile, in some Swiss towns, darning cotton costs three times (*sic*) as much as in Italy.

[A Vevey milkman announced that he was selling milk at two centimes a liter less than the syndicate of the other milkmen. Municipal inspectors at once descended on him and made an analysis of the milk, on the basis of which they declared that the milk was watered. The milkman had another analysis made which showed that the milk was unadulterated. For the first analysis the municipal inspectors had let the milk settle and had taken the milk from the bottom of the receptacle—and as everybody knows, cream rises to the top; for the second analysis the milk was shaken and an average sample taken. Hence the difference between the two analyses.

[Meanwhile this competitor of the syndicate has suffered not inconsiderable losses; his example will serve to prevent anyone else from coming along and trying to sell milk to consumers at a lower price.]¹⁷

Similar instances could be cited ad infinitum. It may well be that all these syndicates have sublime virtues, but it is also certain—in fact utterly certain—that they charge consumers much more dearly for commodities than they would sell for under free competition.

at prices low enough to eliminate the economic parasites who keep prices high; this is precisely what the large department stores have done, and it is what they would do if in some countries they did not have to submit to the oppression of the law and of taxation, which intervene to prevent consumers from buying commodities at low prices.

15. In brief, there is no reason to believe that trade unions, industrial syndicates, [shopkeepers' syndicates]¹⁸ etc., need be intrinsically harmful to consumers; they become so only as a consequence of some of the practices they follow, and only to the extent that they [follow them].¹⁹

16. Collective agreements on production, labor, etc., can be of great value; it may therefore happen that in certain cases they may advantageously be substituted for individual contracts; this will depend mainly on their stipulations and on the certainty of their being executed. It is the lack of such certainty that constitutes the major obstacle encountered in collective labor contracts.

[There is at present a very marked tendency to place the worker above the civil and even the penal laws. These laws are binding only on the middle class. Thus, any worker may break his labor contract from one day to the next, on the pretext of striking. Employers who dare follow this example are invariably sentenced by the courts to pay damages plus interest. If a dispute between employers and workers is submitted to arbitration, the decision of the arbitrators is binding on the employers but not on the workers, who regard it as void if it does not suit them.]²⁰

17. In all periods of the history of our [Western] countries we find facts similar to those just indicated, [allowing certain people to employ artifices to appropriate other people's belongings].²¹ We may thus assert, as a uniformity revealed by this history, that men's activity takes two different directions: it tends to be directed first to the production or transformation of economic goods, and second to the appropriation of goods produced by others. [In classical antiquity, war was the principal means for appropriating the goods of others; today the operation takes place mainly to the detriment of one's fellow citizens.]²²

18. It is worth noting that the aforementioned division of human activity is not [peculiar to]²³ a distribution that would result from free competition, but applies generally. Let us imagine a society in which the goods are distributed according to any rule whatsoever; for instance, each member of the society receives an equal share. We would again find this division in men's activity: some of them will endeavor to produce the goods which are afterwards to be distributed equally; some of them will endeavor, not to produce, but to appropriate the goods produced by the others.

19. It is obvious that one cannot in this way obtain the maximum economic advantage for society. We cannot be quite so [definite in affirming that there is a loss in]²⁴ the social advantage, because the struggle for the appropriation of other people's property may be favorable to selection [(\$35)].²⁵

20. At the beginning of the 19th century, economists believed that this historical uniformity was about to come to an end. They considered it to be caused by ignorance, and were convinced that if the cause were removed through the spread of economic science, the effect would also disappear.³ Besides, this was at a time when it was being said, "Open

³ J.-B. Say's reasoning (*Cours complet d'économie politique pratique*, pp. 9–11) is typical: "Political economy, by revealing to us the laws according to which goods can be created, distributed, and consumed, thus tends to the effective preservation and well-being not only of individuals, but also of society, which otherwise would be a

a school, and you will close a prison." Quite the contrary: education has expanded, but delinquency has not diminished. In France, juvenile delinquency has increased along with education. All cultured persons have studied²⁶ political economy, but the society of which they form a part has not moved in the slightest degree in the direction J.-B. Say wished²⁷ for; on the contrary, it is now moving in the opposite direction. Theories have only a very limited influence on [men's]²⁸ actions; personal interest and passions [play a far larger part],²⁹ and some obliging theory is always on hand to justify them.^[b]

21. Among many instances, it will be enough to cite that of the *balance of trade* itself [to which Say has referred].³⁰ It is hardly possible to find a clearer and more rigorous proof, theoretically as well as practically, than the one which shows that a country does not become richer if the sum total of its exports exceeds that of its imports, and, conversely, that it does not become poorer if the sum total of its imports exceeds that of its exports. Nevertheless, even up to our day there are people who staunchly repeat this nonsense that the enrichment or the impoverishment of a country depends on whether it has a *favorable* or *unfavorable* balance of trade.

22. Say may be excused for having fallen into this error, because he could not know the facts—which for him lay in the future—which occurred in the second half of the 19th century, and which showed that the uniformity observed in the past continued to be verified in the present, and that it had not in the least been modified by the spread of education in general, nor by the knowledge of political economy in particular.

23. In past centuries high prices were considered an evil, and low prices a boon; today the opposite is true. In former times, governments strove to keep prices for their subjects down; today, they strive to increase them. Formerly, obstacles were raised to the exportation of wheat in order to keep home prices low; today, obstacles are raised to their importation in order to keep home prices high.

In France around the middle of the 16th century, so many complaints were heard about the rise in prices that the king became concerned about it and entrusted Malestroict³¹ to inquire into the matter. In the second half of the 19th century, since prices were declining slightly, or more exactly, stopped rising, there was a flurry among statesmen, academies, and scientists who set about to inquire into the cause of [such a great]³² calamity. The contrast between these two events, which are types of the same species, clearly throws into relief the difference between the two periods.

24. It is worthwhile searching for an explanation of this phenomenon. As usual we shall find it not in a single circumstance but in many. One of the most important of these is the difference in the social system. Formerly, [in France and in other countries,]³³

tale of unrelieved confusion and carnage . . . What a pitiful spectacle history presents! Nations with no industry, lacking everything, driven to war by want, and cutting each other's throats in order to survive . . . That was how society was among the ancients . . . I will say nothing of the barbarism of the Middle Ages, of feudal anarchy, of religious prescriptions . . . But the moment one is convinced that a state can grow and prosper without having to do so at someone else's expense . . . , nations can have recourse to the surest, the most rewarding and the least dangerous means; and each individual, instead of bewailing public misfortunes can enjoy his share of the progress of the body politic. This is what may be expected from a more widespread knowledge of the resources of civilization. Instead of basing public prosperity on the use of brute force, political economy takes as a basis a sound conception of man's interest. Consequently, people will cease looking for happiness where they cannot find it and will seek it where they are sure to find it . . . If nations had not been and did not still have the balance of trade on the brain, and the view that one nation cannot prosper except at the expense of another, fifty years of war would have been avoided in the last two centuries . . . It is therefore education we lack, especially education in the art of living in society."^[a]

persons with fixed or nearly fixed incomes were preponderant in the government administration, and a rise in prices was detrimental to them (VI, 80); nowadays it is the entrepreneurs and workers who preponderate, and rising prices are to their advantage. We must add special causes, which could operate either in the opposite direction to the general cause just noted or in the same direction. When the sovereign needed money, he levied [heavy]³⁴ taxes without worrying about whether this would cause a rise in prices, the direct benefit being in this case greater than the indirect harm; and in like manner he granted privileges which had the same effect. On the other hand, for wheat there was a particular cause which operated powerfully in making it advisable to keep the price as low as possible. Owing to the scant wealth among nations at that time, a high price for wheat was synonymous with famine, and riots and disorders of all kinds ensued. The government could thus hardly yield to the wishes of the landowners who [quite generally find it in their interest]³⁵ to have high wheat prices so as to earn their rents.^[a]

[As long as the rise in commodity prices is only partial, and as long as it is less than the rise in wages, workers do not suffer from it. But towards the end of 1906 the rise in prices became general in England, France, and Italy, and workers began to suffer from the high cost of living. The rise in commodity prices no longer appeared to everybody as a boon; but since no one wanted to tamper with the causes that had led to it, people were content with demanding subsidies for the workers from the government.

[In Italy, emigration from the countryside to the cities and the considerable rise in prices of building materials and in the wages of construction workers caused a considerable rise in house rents. Instead of letting this force act freely, which would slow down the emigration from the countryside to the cities and lower the wages of construction workers, subsidies are being demanded of the government and the banks to build new houses in the cities; and since this will be partly at the expense of the countryside, emigration to the cities will increase instead of diminishing. Agriculture in southern Italy is short of capital; instead of providing it with some and increasing output, this capital will be spent in the cities without any benefit to economic production.]³⁶

25. Around the middle of the 14th century a celebrated *Statute* was promulgated in England which remained in force until Elizabeth's reign and laid down that every able man without personal income had to work at a wage rate fixed by the *Statute*.⁴

In spite of this, agricultural wages increased, and continued attempts were made to resist this increase. A statute dating from the fifth year of Elizabeth's reign authorized judges of the peace at their quarterly sessions to fix the wages of craftsmen and agricultural laborers. These regulations remained in force until 1814, at which time competition was allowed to operate but workers' associations were prohibited. In 1825 these associations were allowed in part, but restrictions remained which were removed in 1875. There followed a very short period of freedom; and then the workers, from being oppressed, turned oppressors; they imposed their own conditions, and were supported by the law. In 1904 all the parties, in preparation for the coming elections, vied with each other in

⁴ No individual under 60, whether free or a slave, shall refuse to till the ground at the wage rates current in the twentieth year of the reign (1347). Only those who draw their incomes from trade, from some craft, or who have sufficient investment income, or who till their own land, . . . shall be allowed to refuse to do so . . . The old wages shall serve as the norm; those who ask for more will be prosecuted . . . Masters who pay higher wages shall be fined by an amount equal to triple the excess paid.³⁷

[crawling before and]³⁸ adulating the workers. The [so-called]³⁸ Liberal party, which has retained its name but which repudiated its principles, went over to socialism and promised, if it won, to put the whole power of the law at the service of the workers. The [so-called]³⁸ Conservative party, which is in power, is capable not only of promising but actually of introducing a law and having it passed by the House of Commons, to the effect that trade unions will no longer have any legal liability in strikes they have provoked, and strikers will be allowed to harass [strike breakers]³⁹ with impunity; and it is hinted that this is only the first installment of greater concessions to come.

26. In France the phenomenon is still more marked. Some years ago workers were not even allowed to organize; now unions enjoy extraordinary privileges; strikers can with impunity beat up workers who want to work, set fire to factories, and ransack banks and private dwellings.

The characteristics of the tax system—as well as, in part, those of the expenditures of the state and the municipalities—are among the most reliable symptoms of the economic and social state of a country; for the ruling class always makes the tax burden fall as much as possible on the subject class, and turns the expenditures to its advantage. Writers who discuss “fiscal equity” are mere dreamers; so far, no such thing has ever been seen in the whole wide world.

[The speech delivered by Mr. Ribot in the beginning of the year 1908 against the income tax contains some truly comical passages. After some profound reflection, this eminent politician has discovered that the income tax would divide citizens into two classes: those who vote for the taxes and those who pay them. That he did not discover this sooner is surely not the fault of his adversaries, for they have proclaimed from the rooftops that their aim was precisely what Mr. Ribot believes he has discovered; and it is to the extent that the income tax achieves this aim that it will give the country a little more “social justice.”]⁴⁰

Mr. Paul Leroy-Beaulieu⁵ very aptly describes the present evolution in France [in the following way]:⁴¹ “Thus the principal indirect taxes have been the object of considerable reductions in the past twenty years or so, especially in the past ten years; nevertheless, exemptions of small- and middle-income earners from the [occupancy tax]⁴² have been retained in full.

“Moreover, some years ago, small estates were exempted either totally or partially from the state land tax . . . Let us now turn to the middle- and high-income taxpayers. For twenty years, legislation has done nothing but to raise the absolute and relative share of taxes of this group through changes in rates and new taxes, and also through the introduction of the principle of progression in certain taxes. The progressive character of the [occupancy tax]⁴² in the cities has been strongly accentuated; a recent law has made the higher rents pay for the tax from which low and medium rents were exempted. The intermediate and especially the higher license fees have been continually raised, whereas the smaller ones have been continually exempted. Succession duties have been subjected to a markedly progressive scale, which, for large collateral inheritances, amounts to veritable extortion, to a kind of confiscation . . .⁴³ In times past, and even recently, the state budget devoted little or nothing to relief and philanthropy . . . The budgets of the

⁵ *Journal des Débats*, July 1904.

municipalities had, it is true, some endowments of a humanitarian character, but these were rather restricted. Primary education was not yet free, or only exceptionally; today, not only is it free everywhere, but the schools grant [subsidies].

"The budget of the state, and especially the budget of the municipalities, swarms with subsidies and aid of all kinds of a philanthropic and humanitarian nature. The result is that every day a larger share of public resources is devoted, not to the general public utilities, but to the particular benefit of the less well-to-do part of the population."

[A highly talented author, who is poles apart from Mr. Leroy-Beaulieu from the point of view of economic theory, has the following to say: "The party aims, in all countries and at all times, to conquer the State and use it in the best interest of the party and its allies. Up to the past few years Marxists taught on the contrary that they wanted to suppress the State . . . Things naturally changed when success at the polls made the socialist leaders realize that the possession of power offers great advantages . . ."⁶ "To have a proper understanding of the transformation that has taken place in socialist thinking, one should examine the composition of the modern state. It is a body of intellectuals in whom privileges are vested and which possesses so-called political means to defend itself against attacks from other groups of intellectuals eager to reap the profits of public office. Parties are organized to lay hands on these jobs and they are analogous to the State. The thesis that Marx posed in his *Communist Manifesto* that 'All social movements have up to now been carried out by minorities for the benefit of minorities' could thus be made more precise: we would prefer to say that all our political crises consist in the substitution of intellectuals for other intellectuals . . ."⁷

[The point could not be better expressed, and Georges Sorel provides a very accurate description of what is going on in our societies.⁸]⁴⁴

27. [Objectively,]⁴⁵ all these facts can be summarized in a general way and without going into particular details by saying that [the economic phenomenon always]⁴⁶ tends to be governed in the interest of those classes of society which predominate in the government.

[Subjectively, it is in the name of "justice," of "morality," and nowadays of "progress," that the adversaries appear to fight each other. But among those who set out to storm the bastion of the old society, only the masses sincerely believe in this new religion; the leaders who form the new élite are fully aware of the vanity of it all. The blind faith of the masses and the skepticism of the leaders are one of the most important causes of victory. On the side of the decadent élite, on the contrary, the leaders themselves more or less believe in this "justice," in this "morality," in this "progress;" they are thus hampered in their movements and drag their followers down to certain defeat.⁹]⁴⁸

⁶ [Georges Sorel, *La décomposition du marxisme*, Marcel Rivière, 1908, p. 48.]

⁷ [Georges Sorel, *loc. cit.*, p. 50.] ⁸ [See also the numerous works of G. de Molinari.]

⁹ [This is what happened to Marshal Mac-Mahon and his ministers. There is hardly an instance in history where such an eminently favorable situation has been so badly bungled. If only these people had the slightest energy, and steadiness of character, their victory and that of the bourgeoisie would have been assured. But they were good, honest humanitarians, incapable of deriving the slightest advantage from the circumstances.]

[After its defeat, the bourgeoisie could only whine and wail, invoking the "justice" of its victors, whose *vae victis*⁴⁷ resounded in their ears. It thus justified the social utility of its opponents' victory. The world belongs to the strong.]

28. If real man were only a *homo economicus*, the appearance of the phenomenon would differ much less from [the reality just described]⁴⁹; and all those who [knowingly and]⁵⁰ deliberately pursue a definite goal, might often frankly confess that they are setting about to do so because they find it to their advantage; but real man is also a *homo ethicus*, so that this special interest tends to masquerade in the guise of a general interest (II, 105,⁵¹ 106).

[There are some typical facts in this field. At a certain moment of evolution, the people who wish to change the social system modify certain laws, but dare not alter others yet, for fear of running too counter to the dominant prejudices. This is what, from a political point of view, happened in Rome at the advent of the Empire; it is what, from an economic point of view, can be observed in our own day.

[Our laws and our codes are still based on certain principles which it is desired to change. Their provisions are replaced by others as much as is possible; but when this cannot be done, judges are requested at least to decide according to their consciences and not according to the letter of the law. This consideration has given rise to some very fine theories in France, Italy, and Germany. The two latter countries have not yet gone beyond this stage of evolution; France is in the process of doing so, and the same people who enthusiastically applauded the judgments of the "good judge" who deliberately flouts the law in order to follow his conscience—as he put it—now applaud Minister Briand with the same enthusiasm when he declares that the judge must not be concerned about his conscience, but only with the letter of the law.

[If we consider only the way in which these two points of view are expressed, there seems to be a manifest contradiction between them; and this contradiction really exists for people whose thinking is not different from the form which it assumes; but such people are, in general, the faithful of the new religion, and the intensity of their faith prevents them from discerning the contradiction between two absolutely contrary propositions (II, 48). But, for the leaders, this contradiction does not exist at all since for them the question is one of means and not ends, and it is perfectly natural that the means should change when circumstances do. As for the theorists, it is not for nothing that casuistry was invented; and when their masters desire it they will revert from "free law" to literal law with the same effortlessness with which they at present seek to substitute the former for the latter. Law has always had, and will always have, theories calculated to foster the interests of the ruling class.]⁵²

29. [Innumerable facts of a similar nature could be cited; and to tell the truth they constitute the fabric of the history of societies. But it would be a mistake to believe that only the interest of the ruling class is at stake;⁵³ [still other facts contribute]⁵⁴ to change the form of the phenomenon, and this is because these actions are nonlogical ones, and because they are in part performed under the pressure of circumstances external to man, who does not clearly perceive their purpose.

30. All⁵⁵ this can be readily understood if we study the transformation now beginning, which is giving rise to a new privileged class. History provides us with other instances of similar transformations, whose general pattern is familiar to us but its details less so; whereas we are more familiar with the details of the transformation now under way, we can see only dimly the general trends as these are in the future. Thus, the study of the past and that of the present are of mutual assistance to each other [, each benefiting the other as its complement].⁵⁶

[The facts indicated in §28 are only one particular instance of much more general phenomena.

[Societies are continually undergoing transformation, and in the case of civilized societies this transformation is particularly rapid in our time; it is often more of form than of substance, but it nonetheless takes place. Everything changes, even the form of novels and plays; conceptions of ethics and law take on new forms.]⁵⁷

Slight changes in society can be brought about in a single day, such as the day the law is modified. For other more important changes, one passes to be sure from a legal state A to a legal state B [in one day],⁵⁸ but already under system A the judgments of the courts are leaning toward system B and constitute a transition between A and B. Finally, for profound social changes, there is a transitional state which often lasts for many years during which the law is still nominally A, but where little by little it ends up by no longer having any value, and state B exists in fact when the law finally acknowledges it.¹⁰

This phenomenon is so well known in Roman law, English law, as well as other legal systems, that there is no point in indicating the transformations these systems have undergone in this way. We shall simply recall a recent fact, because it throws light on a transformation that is now taking place in our societies.

A judgment¹¹ of the Swiss Federal Court runs as follows: "As has already been recognized by the Federal Court in several judgments, the guaranty of property, as set out in article 12 of the Fribourg constitution,¹² as well as—under this form or another—in the constitution of all the other cantons (with one sole exception),¹³ is not absolute: In court, it has always been admitted that constitutional dispositions of the kind contained in article 12, aforementioned, guarantee the inviolability of property only to the extent that this property is determined and defined in the internal legislation of the cantons [so far so good, but now comes the priceless part];⁶² in other words, the legislation of a canton may, without infringing upon the above-mentioned constitutional principle, restrict the contents of property rights, determine the special rights implied in these, modify, extend, or restrict the system of property rights, on the sole condition [(note well, the sole one)]⁶³ that it be done in general terms, applying equally to all."

This being the sole condition, property rights may be restricted up to the point of their abolition. Hence⁶⁴ according to this mode of reasoning, a law declaring that

¹⁰ Aristotle had already noted an analogous phenomenon of which the great laboratory of the Greek republics provided him with examples (*Politics*, IV, 5, 2): *ῳστὸν οἱ μὲν νόμοι διαμένουσιν οἱ προϋπάρχοντες, κρατοῦσι δ' οἱ μεταβάλλοντες τὴν πολιτείαν*. "So that the laws previously established, be maintained as they were before, whereas the power already belongs to those who changed the government of the city."

¹¹ Mourlevat v. Conseil d'Etat de Fribourg, June 1, 1904 (*Journal des tribunaux et Revue judiciaire*, Lausanne, 1905).

¹² The following is article 12 of the Fribourg Constitution of May 7, 1907: "Property is inviolable. A departure from this principle is permissible only in cases of the public interest as determined by the law and on the prior condition that fair and complete compensation be paid or guaranteed."

In 1857, socialist principles had not yet been introduced into the legislation. It is very easy to change the constitutions of the cantons; this article could thus be easily eliminated or modified, but that would perhaps be premature, and it could not happen without inconvenience, since one would thus encourage the resistance of those who are not yet entirely converted to socialism; hence, [as a better solution,]⁵⁹ pending the time when it will be possible to change [the meaning of]⁵⁹ this article explicitly, [it is advisable]⁶⁰ to change it implicitly by [twisting]⁶¹ the meaning of the words.

¹³ The exception is that of the canton of Ticino, whose constitution does not contain an article analogous to the one we have just cited.

private property is abolished, without any compensation and applying equally to all citizens, would not be in the slightest contradiction to an article of the constitution which lays down that the right to property is inviolable and cannot be restricted without compensation.

The reason for this contradiction is obvious. We are in a state of transition, in which private property is already being penalized, but people do not dare attack it too openly yet.¹⁴

“Although the Carolingian State,” writes Pertile,¹⁵ “is not yet a feudal one, nevertheless⁶⁷ elements are already developing under its domination that will lead to the fief of private law and political feudalism.”

Although, as the historian of the future will say, the situation of France at the beginning of the 20th century is not yet one that is dominated by a privileged caste which has come up from the working class, elements are nevertheless already developing from which the domination of this caste will emerge.

“Between the 6th and the 9th century,” writes [Numa Denis] Fustel de Coulanges,¹⁶ “between the time of Clovis and that of Charlemagne, the history of political institutions is very obscure. This is not for lack of documents. We have some chronicles . . . Conditions of life in those times are described there in clear and precise terms. They tell us how men lived, spoke, and thought. In spite of all this, it is still very hard to know how the people were governed. This is because these documents do not agree among themselves . . .”

This same lack of agreement is now apparent in France. Legally, a privileged caste does not yet exist;¹⁷ and if we study only the law,⁶⁸ we have to say that the worker is as much subject to the law as the bourgeois, the striker as much as the worker who wants to work, and indeed that the law punishes those who [aim to deprive others of their]⁶⁹ freedom to work. But, if we [start to]⁷⁰ study the facts directly, we are led to entirely opposite⁷¹ conclusions.¹⁸ We also⁷³ see that until very recently [these conclusions],⁷⁴

¹⁴ [In 1908, G. Sorel published a book entitled *Réflexions sur la violence*.]⁶⁵ It is the most remarkable scientific work that has appeared for many years in the field of sociology. I am therefore happy to find in it the confirmation of many of the theories of the Italian edition of this Manual, published in 1906. G. Sorel arrives at these conclusions after following a path that is independent of and different from the one I followed; and this circumstance makes it more probable that the theories in question correspond exactly to the facts.

[See also, by the same author, *Insegnamenti sociali della economia contemporanea*, Palermo, 1906.]

¹⁵ *Storia del diritto italiano*, I, p. 191.⁶⁶

¹⁶ *Etude sur les origines du système féodal du VIe au VIIIe siècle (Académie des sciences morales et politiques)*.

¹⁷ Fustel de Coulanges, *Les origines du système féodal; le bénéfice et le patronat pendant l'époque mérovingienne*, Paris, 1890, p. 429. “The feudal system thus existed as early as the 7th century with its characteristic features and its complete structure. However, it did not exist alone . . . Legally, it was the monarchical institutions that governed men. Feudalism was outside the regular order. The laws no longer combated it as they did at the time of the emperors. At all events, they did not yet give it their official blessing. Vassalage already held an important place in the customs and in people's interests, but hardly any in public law.”

¹⁸ [G. Sorel, *Réflexions sur la violence*, p. 41: “On the degeneration of the capitalist economy is grafted the ideology of a bourgeois class that is timorous and humanitarian and which claims to have freed its thinking from the conditions of its existence; the breed of daring leaders responsible for the great achievements of modern industry is vanishing, and is being replaced by an extremely politicized aristocracy which asks to be left in peace. This degeneration fills our parliamentary socialists with joy. Their role would vanish if they were to confront a bourgeoisie energetically launching out on the road of capitalist progress, a bourgeoisie that was ashamed of timorousness and proud to pursue its class interests. Their power is vast in the face of a bourgeoisie that has become nearly as stupid as the eighteenth century nobility. If the haute bourgeoisie

which were [applied]⁷⁵ in practice, were not accepted—and were even rejected—in theory, and that it is only now that they begin to be approved in theory. We are thus approaching the last period of the evolution in which they will acquire a legal form. When this stage is reached, precisely who will make up the privileged caste will be determined at the same time. This is still uncertain today. It does not consist and probably never will consist of all the workers, but only of workers organized in trade unions, or rather only of those belonging to the particular unions looked upon favorably by the government.¹⁹

31. These workers are above the law, because the police do not oppose their [overbearing behavior],⁷⁶ or—what amounts to the same thing—oppose it ineffectively; because if they commit crimes, they are not prosecuted; or if they are prosecuted, the government forces the judges to acquit them. Moreover, no witness will give evidence for the prosecution, because those who could do so know that they would have no protection against the revenge of the accused; and if, by any chance, they should be convicted in court, they will soon be pardoned. Finally, very frequent amnesties contribute to ensure complete impunity.²⁰

At Lorient, in 1903, a striker was sentenced in court for grave acts of violence; his comrades besieged the court, broke the windows, and injured a judge. Twenty-seven of them were judged and sentenced, but the sub-prefect intervened immediately, and threatened the president of the court to “hold him responsible” for any disorders this sentence might provoke; thereupon the president reopened the session that had been closed and, in agreement with the judges, modified the sentence. At Armentières, most of the men guilty of looting, arson, and assault were not even tried; only persons not belonging to the privileged caste were prosecuted, and the public prosecutor himself, in his indictment, was forced to admit that “the preliminary investigation found it nearly impossible to gather useful testimony, since most of those whose houses had been overrun and ransacked had fled or had gone into hiding, a prey to terror, and the others hesitated or refused to speak for fear of reprisals.”²¹

continues to degenerate regularly at the speed of the last few years, our official socialists may reasonably expect to achieve the object of their dreams and sleep in sumptuous mansions.”⁷²

¹⁹ *Immunity*, which is precisely one of the main circumstances that gave rise to the feudal system, was granted by the sovereign to whomever he wished, and there was no fixed rule for determining the persons who were to enjoy it. Fustel de Coulanges (*loc. cit.*, §30), p. 424, writes: “For many centuries, [immunity] was one of those acts repeated over and over again which imperceptibly modify and in the end transform the institutions of a nation. By changing the nature of obedience among the great, and by shifting obedience among the small and the weak, it changed the structure of the social body.”

²⁰ J.G. Sorel, *Réflexions sur la violence*, p. 28, discusses France, but what he says also applies perfectly to Italy: “Nearly all union leaders know how to make the most of this situation, and they teach the workers that it is not a question of asking for favors, but that one must take advantage of *bourgeois cowardice* to impose the will of the proletariat. These tactics are supported by too many cases not to take root in the laboring class.”

²¹ P. 30. “A social policy based on bourgeois cowardice, which consists in always yielding to the threat of violence, cannot but give rise to the idea that the bourgeoisie is sentenced to death and that its disappearance is only a matter of time. Each conflict that gives rise to violence thus becomes a vanguard fight . . .”⁷⁷

²² Pertile, *loc. cit.*, p. 259: “. . . the right of appeal was often illusory. And this is either because of the difficulty in making use of it resulting from distance or from obstacles interposed by the baron; or because of the procedure applied in certain countries, which seems deliberately calculated to discourage even the boldest; or, finally, because even if a better judgment were obtained, the king often lacked the means to have it executed, when [even]⁷⁸ the baron did not take [harsh]⁷⁸ vengeance for his vassal’s audacity.”

32. A large number of facts of this kind could be cited; here finally is one that may be regarded as typical. In June 1904, there was a strike of the tramwaymen at Nice which was accompanied by the usual violence; we take from a French newspaper the account of how the matter ended:

Nice, June 28. "This evening, at seven o'clock the prefect and the public attorney were informed by telegram that the reprieve of the five demonstrators sentenced at the beginning of the strike of the tramway men had just been signed by the President of the Republic. It will be recalled that it was on the basis of the formal promise that the pardon of the convicts would be granted within forty-eight hours that the strikers had agreed to resume work. At a meeting held last night, the tramway men, dissatisfied by the delays in the signing of the act of clemency, had decided to leave their work again today, Tuesday, if by that time their imprisoned comrades had not been freed. This would have entailed a general strike, since all the labor organizations had declared their solidarity with the tramway men. The prisoners were freed tonight, at half past nine. The public attorney himself went to the prison to discharge the formalities of their release. The news that the prisoners had been set free caused the greatest joy among the workers."

It is, it should be added, a common practice on the part of strikers to return to work only after having imposed as a condition the release of those arrested and convicted in court; the government humbly obeys.

In May of 1905, at Limoges, with the consent of the police who looked on, passive and indulgent, the strikers for several days besieged [the plant of the Beaulieu firm]⁷⁹ in which eleven persons, among them four small children, were suffering from starvation. The mayor, a socialist deputy, intervened, but cautiously, begging the [worthy]⁸⁰ strikers [and criminals]⁸¹ to at least let some bread into the besieged house in order to give the besieged something to eat; but the besiegers, adding insult to injury, allowed only one loaf to be brought in for the eleven starving persons. These poor people, made wise by the events at Cluses, did not even try to defend themselves; had they made the slightest attempt to do so, the police would have acted immediately, and put them under arrest, and they would have been convicted by the court, as had happened to the unfortunate industrialists at Cluses (II, 92). A child, the janitor's son, was driven by hunger to go in search for some milk. He was hit by the strikers who were protected by the authorities, and had two ribs broken; not satisfied with this, the strikers, using violence, repulsed the doctor who sought to treat the unfortunate child.

The *humanitarians* naturally side with these worthy strikers [and criminals].⁸² The Minister Etienne replied to the delegates of the Limoges merchants, who had come to Paris to request that citizens' persons and property be protected, "that they were the elder sons of democracy and that they should display feelings of goodwill and affection to their younger brothers, the workers, in order to restore their misguided minds to equanimity and reason."

Notwithstanding, the French government was obliged to use force to defend itself against these "younger brothers;" but the Assembly then voted relief for the injured aggressors and for the injured among the police who had been attacked, thus putting on the same footing—without making any distinction between them—the criminals and those who had upheld the law.

In Italy, the [“railwaymen”]⁸³ abused to their hearts’ content the public who pay and feed⁸⁴ them.²² They were not punished, [nor did they suffer the slightest harm,]⁸⁶ and were even praised by members of the upper class; and the citizens have to resign themselves to enduring those nice people’s whims.²³

In Italy, in France, and in Russia,²⁴ [the revered]⁸⁸ strikers have come forth with another [splendid]⁸⁸ claim, namely that they should be paid for days on strike as if they had been working, and in some cases they have found people weak and cowardly enough to yield on this point. If this rule should become general, there seems no reason why the workers, on various pretexts, should not remain on strike for the whole year; they would go off and enjoy themselves and would nevertheless obtain their pay. What is astonishing is not that they desire to do so, since after all everyone tries to obtain as much as he can; what is astonishing is instead the [craven stupidity]⁸⁹ of the humanitarian gentlemen who seek out one sophism after another to justify these claims.

[The best is yet to come. Where the evolution is further advanced, which is to say in France and in Italy, a doctrine is coming to be developed according to which the police must allow themselves to be insulted, scoffed at, struck, stoned, knocked down by rioting strikers, without making use of their weapons. Up to now, such behavior has been considered more appropriate for an anchoretic saint than for a soldier, but the 20th century is witnessing the rise of a contrary opinion. Soldiers and gendarmes who dare defend themselves and repel their aggressors with their weapons are accused of “losing their cool”—this is the stock phrase; if, on the other hand, upon being wounded by bricks, paving stones, iron bolts, or other projectiles, streaming with blood, they suffer it all without hitting back blow for blow, their conduct is said to be “admirable.” Where the evolution is not yet so advanced—for instance, in Germany—quite a different meaning is given to the expression “admirable conduct” as applied to soldiers and to the police. The sport of throwing stones at them would be extremely dangerous for anyone tempted to practice it; even the strikers and rioters carefully refrain from such activity, whereas they practice it widely in countries where they may indulge in this pleasure almost with impunity.

[The decadent bourgeoisie of our time want two contradictory things. On the one hand, they expect their property and their persons to be protected by the police; on the other hand, they demand that the police abstain from any act that would shock the exquisite sensitiveness of the bourgeois nerves, and above all that the police do not, for any reason whatsoever, shed the blood of the adversaries whom it is their job to contain and combat. Such a state of affairs is unstable and cannot last. If, one day, someone energetic and ambitious is found in the army, he will, with his comrades, join

²² Among Italian politicians, Mr. Napoleone Colajanni had the great merit of acknowledging how exaggerated were the ferrovieri’s [railwaymen’s] claims and the by no means small merit of saying so bluntly.⁸⁵

²³ [New facts have come to light that confirm the observations contained in these lines, published in 1906.]⁸⁷

²⁴ Writing about Russia, the *Moniteur des intérêts matériels* of June 7, 1905 has this to say: “People have gone so far as to express this unjustified claim to be paid during the days while on strike, and some employers in the north having been weak enough to yield, this almost ridiculous demand is now being raised in every quarter.”

[At the time of the agricultural strikes in northern Italy in 1908, among the “demands” of the rural workers, there is the following one: the workers will have the right to abstain from work every time they are ordered to do so by the leaders of their leagues, and the landowners will be obliged to pay the wages of these workers, as if they had been working.]⁹⁰

the opponents of the bourgeoisie; they, at least, have no such absurdly contradictory pretensions, nor do they have such sensitive nerves, and if they want the ends they have sense enough to want the means.

[The keenness to make money still shown by the bourgeoisie in our day has been cited as proof of their energy; but this keenness for gain is no substitute for the fighting spirit, which they tend to lack more and more. Among innumerable proofs one need only recall the fate of the usurers—the Jews and Lombards in the Middle Ages. More than once the sword stripped them of their gold; a similar fate awaits those who find themselves in similar circumstances.]⁹¹

[32 bis.]⁹² The humanitarian religion is so widespread in our time, it so completely permeates all theory and reasoning, that to report acts of violence is synonymous with censuring them. Now there are, on the contrary, many cases in which one does not wish to censure or to approve such acts, but simply to take account of them in the study of the direction of a certain social evolution.

[Two classes now confront one another: let us call them A and B. The struggle between them becomes more intense every day, and cannot but end in a decisive battle. The members of the first class try to get richer, each for himself, without bothering too much about the common interest of their class; the members of the second class put this interest ahead of everything, and expect, rightly or wrongly, a betterment of their particular situation to result from a common victory of the whole class. Among the A's, treason, far from being censured, is praised and admired. The A's consider that the best among them is the one who is best able to promote the B's interests,²⁵ and that this holds for every branch of human activity. The politician who wants to please the A's has to be concerned above all with gratifying the B's; the magistrate, the historian, the man of letters, the playwright, will enjoy success with the A's only by scoffing at them and extolling the merits of the B's. Among the B's, on the contrary, anyone who betrays the interests of his class is despised, spurned—struck if need be—and punished by all licit or illicit means at their disposal. Any public or private activity is judged by the B's from the point of view of the interests of their class, and anyone who displeases them is condemned with a sentence far more formidable than those of the bourgeois judges.

[The A's live from day to day, taking care only to avoid trouble and to postpone as much as possible a battle that those among them who have not lost all common sense recognize as inevitable; for this reason they keep yielding and yield everything; they make themselves humble and inconspicuous, they stoop to the basest flattery and willingly allow themselves to be trampled on by their opponents. They whine and complain that their good intentions are not recognized; they declare, more or less hypocritically, that they live only to make the B's happy; for this they were created and brought into the world, it is their "social obligation," their religion. Those among them who call themselves Christians no longer believe in the divinity of Christ, but they believe that he was the first of the socialists, and this satisfies their need for religiosity; they declare that "religion is a way of life," and "a way of life" is a kind of socialism. If, among the humanitarians, there are some shrewd fellows who, while declaiming about "Progress, Science, and Justice,"

²⁵ The humanitarian bourgeoisie has made a saint out of the politician Waldeck-Rousseau, who betrayed it and handed it over to its enemies.

do not fail to fill their pockets, there are others among them who take these speeches seriously and who aspire to become good ascetics.²⁶ The B's have a long-term and firmly fixed design; they want to replace the A's. They accept everything from the A's and yield them nothing in return; they feel proud and arrogant in their power, in the indomitable energy they can expend to achieve their aim. They never stoop so low as to flatter their opponents, and still less to say that they live only for the their opponents' good. Asceticism holds no attraction for them at all, and the dreamings of the humanitarians leave them cold. It is in vain that ingenious "intellectuals" have conceived the marvelous theory of "solidarity;" this prodigious effort of their minds has been completely lost, and the B's have not even deigned to be concerned with it. The A's may feel themselves "in solidarity" with the B's as much as they like; but the B's do not feel themselves the least bit "in solidarity" with the A's.

[From a superficial point of view, the philanthropic activity of the A's carries us into a strange world, where everyone seems to be especially concerned with other people's interests and to neglect his own interests; but a closer look shows that this is so only in appearance. In reality, this activity of the A's is often aimed at satisfying some of their immediate interests, some of their passions, sentiments, and prejudices. But the philanthropic form is not unimportant; it influences the substance and prevents the A's from realizing exactly the results which their activity will have in the end.²⁷

[Some among the A's, having completely forgotten the great lesson of 1789, spend their time "extinguishing the lights of heaven,"⁹⁵ and while waiting until this activity has produced much the same effects as before, they obtain as a reward, smiles and praise from Mr. Viviani. Others have devoted themselves to destroying those institutions such as the army²⁸ and the magistracy which can prevent social disintegration; they are paving the

²⁶ There are very many facts to prove that *humanitarianism* is a religion; we have cited a few of them; the following may be added. The humanitarians' need for proselytism is similar to that of the exalted believers in other religions, such as, for instance, the Christians in the Middle Ages.

It is understandable that the revolutionaries should have an interest in ensuring that, even outside their country, there be no centers of resistance to the revolution, or even any social organizations that escape their dominance. But what possible interest can a good Italian, French, or English bourgeois have, for instance, in the existence or otherwise of a parliament in Russia or in Persia? Nevertheless, these worthy bourgeois approach this subject with a passion comparable only to that of a Catholic missionary bent on converting the infidels. The bourgeois press has been seen, with pious deceit, systematically to pass in silence over the crimes of the Russian revolutionaries and to dwell at great length on the repressive measures that these crimes drew upon themselves. The Shah of Persia has incurred the censure and wrath of our worthy humanitarians for having brutally dissolved his revolutionary parliament. They even rewrite history. Up till now, the general belief was that Julius Caesar was [quite a good]⁹³ commander. It now appears that he was nothing of the kind, and that we have been entirely wrong about him. This story is not without some analogy with the story that the Christians in the Middle Ages considered the Moslems as idolators and turned Virgil into a famous magician.

Hypocrisy, which is the plague of all religions, is not lacking in certain manifestations of the humanitarian religion. The same English statesmen who cry out: "The Duma is dead, long live the Duma,"⁹⁴ take great care not to grant a parliament to Egypt or to India. There is a holy compromise with heaven.

²⁷ A mere glimmer of common sense would have enabled the Russian ruling classes to understand that *Tolstoyism* could only lead to the defeats in the Russo-Japanese war and to the revolutionary Saturnalia that followed. But among the rulers, some got richer thanks to customs protection and embezzlement, and the others were dazed by their humanitarian faith.

²⁸ G. Sorel, *Réflexions sur la violence*, p. 82: "Trade unionism in France is at present engaged in antimilitarist propaganda which clearly shows the immense distance which separates it from parliamentarian socialism on this question of state. Many newspapers believe that this is only an exaggerated humanitarian movement . . . this

way with all their power for the triumph of the B's. Others devote themselves to secondary chores; they feel an urge to protect everything and everybody. They protect children and the young, adults and the elderly, men and women, decent workers, strikers, criminals, malefactors of all kinds, prostitutes, pimps, *Apaches*, swindlers—in short, all kinds of people, except themselves.

[Every year, in July and August, there is a swarm of flies and humanitarian congresses, quite apart from the insects and congresses which infest the other months of the year. The B's take no part in these Saturnalia, but the A's are full of solicitude about social assistance and the national and international protection of workers; with the prohibition⁹⁶ of night work for women in industry; with preventing people from drinking what they please, and preventing sweethearts from corresponding, and with innumerable other similar matters. A fairly⁹⁷ large number of the A's live in terror of microbes and in respectful fear of the sawbone.⁹⁸ The latter has taken the place of the [director of conscience]⁹⁹ in former times; he prescribes the rules for drinking, eating, and even love making for his subjects, who dream only of imposing these rules by force on others, exactly in the same way as the believer of past centuries tried to impose his faith on unbelievers by force.

[But it is above all for the evildoers that all the tender solicitude of the A's is assured.²⁹ By means of the so-called laws of reprieve, of pardon, etc., we shall soon include among the rights of man and the citizenry the right to commit at least a first offense with impunity. To let the delinquents off lightly, their sentences are in certain cases not entered in their penal records; they are thus allowed to take advantage of the good faith of their would-be employers, to deceive them, and to commit new crimes.

[All this degenerate nonsense has no hold on the B's. If they agree with the A's to "extinguish the lights of heaven," it is only because, rightly or wrongly, they believe the operation to be advantageous to their class. They have never been seen holding meetings for the "social protection" of the A's, or, indeed, for any other kind of protection for their adversaries. They are energetic and robust males who want to eat and drink their fill, and make love when it suits them, and who laugh at Monsieur Purgon.¹⁰⁰ They leave it to their opponents to *drink the slaves' water*.³⁰ They will show indulgence to the Apaches as

is a great mistake. It should not be believed that this is a protest against the harshness of discipline, or the length of military service, or the presence in the upper ranks of officers hostile to the present institutions; those reasons have led many bourgeois to applaud declamations against the army . . . but these are not the trade unionists' reasons. The army is the clearest manifestation . . . that one can have of the state. The trade unionists do not propose to reform the state . . . they wish to destroy it."

²⁹ A most remarkable case, which may be considered as typical of its kind, is that of Jeanne Weber which took place in France. This woman was accused of having killed several children. Excellent doctors returned a verdict of natural death. France possesses two Courts of Cassation: the one that bears this name and the *League of the rights of man and of the citizen*. . . . The latter body naturally took up Jeanne Weber's cause, and the magistracy had to bow to this pressure and release the woman. She was thus allowed to go on with her exploits, but she made the mistake of being caught red-handed. Rochefort had the courage to single out the woman's protectors as being responsible for the latter crime; but this is only the sally of a brilliant writer, and the serene inanity of the humanitarians has not been ruffled in the slightest by it.

It should be noted that the learned Faculty, which is not even able to distinguish whether a child has died from strangulation or from natural causes, knows, on the contrary, practically to a man, the exact number of deaths which in a given country are *indirectly* due to the use of alcoholic beverages! Humanitarian *Science*—a far cry from experimental science, which purports only to discover the uniformities in the facts—has such amazingly unfathomable aspects as this.

³⁰ Ovid, *Amores*, [I], vi, 25–26:

. . . SIC UNQUAM LONGA RELEVERE CATENA,
NEC TIBI PERPETUO SERVA BIBATUR AQUA.¹⁰¹

long as these remain on their side; just as Julius Caesar protected Claudius and his gangs as long as he found some advantage in it. It is interest that governs the conduct of the B's, not sentimental twaddle. When the leaders of the B's are the masters, they will not hesitate to get rid of anyone who might disturb the order they have established, and will destroy them without the least scruple.³¹ The evildoers will do well to make the most of the earthly paradise provided by the naive imbecility of a decadent bourgeoisie, for this state of affairs will soon come to an end and will not recur for a long time.

[Finally—and this fact alone is as important as all the others—whereas the A's have a phobia about bloodshed, the B's are ready to shed as much blood as is required to achieve their aims, and they will certainly not forego victory even if it can be achieved only by advancing over heaps of dead bodies.

[In Europe the A's comprise the larger part of the bourgeoisie and win over the majority of law-abiding socialists, parliamentarians, or others of that kind. The B's today call themselves trade unionists, tomorrow they will have another name, and probably still others, until the day of victory. These questions of denomination are of no importance, and the separation between the A's and the B's is established not by words, but by facts.

[The facts we have just mentioned suffice to predict which side will probably be victorious. The course of events could, it is true, be changed by extensive wars or—which is infinitely less probable—by a change that will take place in the character and sentiments of the bourgeoisie.³²

[All known historical facts confirm that no social class can retain its property and its power in the long run if it does not have the strength and energy necessary to defend them. In the long run, only force determines social forms; to have forgotten this principle will be regarded as the great mistake of the 19th century.]¹⁰²

33. [At the present time, we are witnessing the passage from one equilibrium position to another. A degenerate bourgeoisie no longer has the courage to defend the property it still holds. Its situation may be compared to that of the Carthaginians on the eve of the third Punic war when new strips of territory were taken from them every day by Masinissa, and Rome prevented them from defending themselves,³³ until finally their city was ruined and they themselves were reduced to slavery. Their humble submission to the enemy availed them nothing.

³¹ The numerous cases in which mobs went to lynch criminals clearly show that the popular classes still retain the energy of the race, an energy that has been lost in the upper classes.

In the same vein, it should be noted that when the religious communities were robbed in France, only a man of the common people gave his life for his faith. In Italy, the socialists, who risk their lives in riots, are all people of the lower class.

³² G. Sorel, *Réflexions sur la violence*, p. 35. "Agitation, if properly channeled, is extremely useful to socialist parliamentarians, who boast to the government and the wealthy bourgeoisie that they know how to moderate the revolution; they are thus able to make a success of the financial affairs in which they have an interest and obtain small favors for a number of important voters . . ." pp. 36-37: "The great bulk of voters understand nothing about what happens in politics and have no grasp of economic history; they are on the side that appears to hold power; and it is possible to obtain anything from them that one wishes if it can be shown that one is strong enough to force the government to capitulate. One should not, however, go too far, because the bourgeoisie might wake up and the country might throw itself into the arms of a resolutely conservative statesman."

³³ In June 1908, at the time of the agricultural strike at Parma, the government prevented the bourgeois from defending themselves, whereas it left their opponents free to attack them. The opponents went so far as to stop the trains and make the strike breakers get out.

[The submission of the bourgeoisie to its enemies and its cowardly compliance will be just as useless to it.^{34]}¹⁰³

All this clearly reveals the existence of a privileged caste which is alone in imposing its will on the government, whereas the government does not and would certainly not tolerate this on the part of the bourgeois or of any other social class. And it also reveals the change in opinions that precedes and prepares for a change in the laws, since these developments, instead of meeting with outrage,¹⁰⁴ are accepted with supine resignation by the bourgeois class itself. The French Court of Cassation has decided that strikes are in violation of the labor contracts, so that the present law is still in force; but public opinion already demands its amendment, and Jaurès proposes that it be amended so that a labor contract should not be violated by a strike. Once this concession has been obtained, a very important privilege will have been established for the workers. They will be allowed to abandon a factory for [days, months, and years],¹⁰⁵ whereas the employer will remain bound by the labor contract; but if, for instance, the innkeeper with whom certain workers board were to cease to give them their meals, it would be considered, and rightly so, that the contract had been broken, and that these workers could provide for themselves elsewhere.

^{34.}¹⁰⁶ [1906] [The progressive change of opinion is even more evident from the writings of a certain Michel Augé-Laribé in the *Revue politique et littéraire* on 10 June 1904 concerning the agricultural strikes in the south of France: "The strikers were blocking the paths at the exits or¹⁰⁷ entrances of the properties in order to obtain a complete stoppage of work. But it must be pointed out that in general these stoppages have not given rise to very strong protests.³⁵ The fact that some small landowners have armed themselves to break through these lines is correct, but exceptional . . . there have been some regrettable incidents;³⁶ this has rarely come to blows,³⁷ but there have been frequent threats. If one cannot hesitate to condemn them,³⁸ should one not nevertheless remark that there is some injustice³⁹ in proclaiming the sacred right to strike⁴⁰ without allowing the workers

³⁴ G. Sorel, *Refléxions sur la violence*, p. 169: "What a fine piece of reasoning it is on the part of these gentlemen—the pundits of social duty—which assumes that violence will no longer increase or will even decrease if only the intellectuals will display more politeness, fawning, and cringing in honor of the unity of the classes. Unfortunately for these great thinkers, things are taking quite a different course. Violence continues to increase, whereas it was supposed to diminish according to the principles of high sociology."

³⁵ To know why, one should read the words cited above (§31) by the public attorney at Armentières. A friend of mine, who is a landowner in the places where the agricultural strike took place, and whom I had asked for news, sent me a great deal of precise information, but forbade me from publishing it since, he said, "My position here is already difficult enough; it would become untenable if some friend of the [prefect]¹⁰⁸ were to report your article to him."

[Our author is also quite amusing in reckoning that a deed is not a crime if it does not give rise to many strong protests!]

³⁶ Note the euphemism: the privileged caste does not and cannot commit any crime; it can only give rise to some "regrettable incidents."

³⁷ For those at the receiving end of these blows it will perhaps be of very little comfort to know that they did not have many companions in misfortune.

³⁸ There is an ambiguity here. The reader should not think that what is being discussed is condemnation in the courts of law, which in fact humbly bowed to those people who dealt *a few blows* and made many threats; the condemnation being discussed here is simply verbal, which make no difference at all.

³⁹ One wonders what idea of *justice* this author can have.

⁴⁰ Our author seems unable to understand the difference that exists between the right not to work and the right to hit and threaten those who work.

any legal means to ensure its exercise?⁴¹ How can one honestly rejoice at an increase in wages and condemn the procedures that have been absolutely necessary to obtain it?⁴² It is indeed certain that the striking activity could not have succeeded had the workers not been able to use these illegal tactics."⁴³

[Very little sense can be found in the form of this excerpt; but in its substance it is extremely clear. The author simply means to say that the new privileged caste must have the power, in certain cases—which he does not define precisely—to impose its own will on the rest of society. As long as the law fails to recognize this right, it is necessary to obtain it by illegal violence. Similarly, as long as the law did not repress homicide, private vendettas made up for this deficiency.]

34. [1909] Among the changes under way, not the least remarkable is that of an agreement between entrepreneurs and workers. The entrepreneur is not the capitalist; he hires the services of capital on the market just as he does those of the workers. What does it matter to him to pay dearly for these services, if he can sell his products at a price that ensures him a profit? Of course, he would prefer to sell the commodities he produces at a high price, and pay low wages; but since this is not possible, he makes a virtue out of necessity, and follows the line of least resistance. Now, the latter is, incontestably, at least in our time, on the side of the consumers. The pusillanimity of some of these consumers is unbelievable. They have had the idea of setting up so-called *buyers'* associations. To judge by this name one might imagine that these worthy people associate in order to obtain commodities of the best possible quality, at the lowest price. Not at all! Their only purpose is to obtain preferential treatment from the entrepreneurs for the workers and the clerks; all the rest is hardly their concern.

A concrete example of the possibility of an agreement between entrepreneurs and workers at the consumers' expense is provided by a recent Australian law. Manufacturers of agricultural machinery have been granted a prohibitive tariff against the import of American machinery, but on the condition that if they do not pay "fair and reasonable" wages to their workers, they will have to pay an amount on their products equal to one-half the duties imposed by the customs tariff.

It should be noted that these laws are effective only insofar as they create privileges. If they could be general, they would only lead to a nominal rise in prices, the consumers then becoming indistinguishable from the producers. It is true that there would be a transitional period in which the creditors and the rentiers would be despoiled, but afterwards one would revert approximately to the initial equilibrium.

⁴¹ The strikers had to take the law into their own hands because the law was not coming to their aid. If the police had arrested those who wanted to work, the landowners who were prepared to employ them, and all those who were speaking in defense of such "crimes," the strikers would not have had to concern themselves with hitting and threatening!

⁴² Thus, whoever rejoices about something must accept any means to obtain it! Here is a little dialogue that the author did not consider: "I rejoice in seeing you with that beautiful diamond." — "I stole it from a jeweler." — "I no longer rejoice." — "But that was absolutely the only means to obtain it." — "I do not think that the end justifies the means."

[There might even be some heretics of the new orthodoxy who do not rejoice at the increase in wages.

⁴³ Therefore, anything that the law forbids becomes legitimate provided it is necessary, or deemed to be so, in order for a strike to succeed. The right to strike is the right to use any means that are necessary to impose the will of the strikers on the bourgeois.

35.¹⁰⁹ Great European wars or other events of this kind may stop the course of the evolution now going on; but if these events do not occur, and if this evolution comes to an end, the result will be an economic state that will not be very different in substance from the present state of affairs, although it may differ [in form and]¹¹⁰ in name. We shall still have an economic state in which the monopolies of certain privileged individuals will exist side by side with free competition among the other citizens. The main change in substance will be in the individuals who are privileged; in the end, we shall have a new bourgeoisie, under another name.

[Georges Sorel believes that a complete change is possible. He says: "The Marxist conception is that revolution is made by the producers who, accustomed to the factory system in large-scale industry, reduce the role of the intellectuals to that of clerks carrying out as few jobs as possible. For everybody knows that a business is regarded as being better managed the smaller is its administrative staff"]⁴⁴

[I would not dare to assert that such a system is impossible; the future may be rich in surprises. Who in Aristotle's day could have foreseen the parliamentary system that presently governs almost all civilized nations? But what we know of history and of contemporary facts appears to rule out the possibility of a change of this kind, at least in the near future.]

[I agree with Sorel, on the other hand, with respect to the means that could bring such a development about. "By studying the ways in which people's minds have been prepared for revolutions, we can easily recognize that they have always had recourse to social myths, whose formulas have varied according to the times."⁴⁵ One must expect to be confronted with numerous deviations that will appear to call everything in question; there will be times when one will give up for lost everything that had been regarded as definitely won . . . It is precisely because of this character of the new revolutionary movement that one should beware of providing formulas other than mythical ones: discouragement could result from the disillusionment produced by a disproportion between the actual and expected state . . ."]⁴⁶

[These are really the characteristics of a faith and a religion, i.e., of forces that may act effectively to bring about social change; and I must here recall what I have said above (I, 43) and, in general, the observations I have often repeated about sentiments and nonlogical actions.]

[On the other hand, the humanitarians' mistake is not that they have a religion, for it is by means of a religion that one acts upon society; but that they have chosen a religion that suits only [spiritless individuals,]¹¹¹ totally deprived of energy and courage, and which, could it triumph, would cause European society to fall below the level of Peruvian society at the time of the Incas.]¹¹²

35 [bis].¹¹³ Let us try to extend the considerations developed above (III, 11, 12) and (VI, 33) to the concrete phenomenon and even to the social phenomenon.

Any economic or social state is generally neither absolutely rigid nor such as to allow any movement whatsoever. Conditions that we shall call *constraints*¹¹⁴ impede certain movements and permit others. For instance, a caste society permits certain movements

⁴⁴ G. Sorel, *La décomposition du Marxisme*, p. 51.

⁴⁶ G. Sorel, *La décomposition du Marxisme*, p. 63.

⁴⁵ G. Sorel, *La décomposition du Marxisme*, p. 55.

within the same caste and impedes them between different castes. A society with private property and inheritance impedes certain movements that would be possible in a society where these institutions do not exist.

There are two kinds of problems: (1) choice of constraints in order to achieve a certain aim; (2) the constraints being given, choice of movements to be adopted among those that are possible.

The aim one wishes to pursue may of course be of any kind. One could aim at the largest possible population, or the happiest one, the most moral one, etc.; but these terms, except the first one, are vague and need to be defined if one wishes to use them in a scientific analysis.

It may be observed that the first problem may, if need be, be included in the second; for this, it would be enough to consider a society without constraints, and the constraints to be determined by the first problem would then be given by the second one.

But in general, there are constraints admitted by everyone, and it would be vain to call them into question again. For instance, it would be quite absurd to consider cannibalism as something possible in our time and in our civilized societies.

Let us then suppose that certain constraints are given, and let us consider the second problem.

Two quite distinct kinds of movements must be studied. (1) Some movements may be in the interest of all members of society, or of some of them, without being detrimental to others. (2) Certain movements may be in the interest of certain members of society only by being detrimental to others.

As long as movements of the first kind are possible, one can increase material well-being, or the moral, religious, or other qualities of all members of society, or at least of some of these members, without harming the others. This cannot be done when only movements of the second kind are possible.

The situation in which movements of the first kind cease to be possible may thus be considered as a certain maximum situation. This is how we considered it in the case of ophelimity (VI, 33); and in that case the reasoning has all the desirable rigor (*Appendix*). It may still have this property when one is concerned with something measurable; it can only be somewhat vague when one is concerned with things that cannot be measured.

Nevertheless, even in the latter case, many lines of argument that have long been pursued and that continue to be pursued concerning the social order really reduce to relatively imperfect considerations about this maximum. Other arguments neglect it in part—these are the metaphysically and religiously based arguments; nevertheless, even these nearly always try to reconcile the two approaches and claim to attain this maximum.

Let us consider slavery. If the masters were simply and brutally to say: "We like it this way, and we shall impose our will," it is clear that there is no concern for any maximum for society as a whole.

Aristotle begins by basing slavery on metaphysical considerations, but he hastens to add that it is useful for the slave to serve and for the master to command (*Polit.*, I, 2, 20). In doing so, he resolves precisely one case of our general problem.

Some authors have held that slavery was useful because it was the only means of providing spare time to men who, by their discoveries, enabled civilization to develop. This is again a particular case of our problem; but here the aim is different from the

previous one; it is not a question of the present utility of masters and slaves, but of the future utility of society.

From the economic point of view, any monopoly that is effective, any faulty determination of the production coefficients, any misuse of the economic goods at the society's disposal, carries us farther away from the position of maximum ophelimity. This fact may be expressed in several forms. For instance, we can say that the social order could be changed in such a way that all the members of society might enjoy greater well-being, or that at least some of them might enjoy greater well-being without harming others. Or we can say that the persons suffering from a social order that departs from maximum ophelimity could, if they were allowed to reach this maximum position, pay a sum such that the new social order would be to everybody's advantage.¹¹⁵ Thus, in former times the repurchase of some seigniorial rights may have been to the advantage of both the villeins and the lords. We can also say that, in a situation that departs from maximum ophelimity, there is a contrast between the interests of persons who derive advantage from the situation and the general interest. It may be added that, so long as the position of maximum ophelimity is not reached, a development or benefit to a given branch of economic activity is not necessarily a benefit to society. It becomes so only when the position of maximum ophelimity is reached.

All these propositions, and still others, relate essentially to the same facts and consider different aspects of the problem of maximum ophelimity.

36. Thus we have at present an economic state which is similar in part to the preceding one, and which will probably not change very much, at least in the near future. It is a state consisting of free competition with monopolies, [constraints,]¹¹⁶ privileges, and restrictions. What varies are the proportions in which these various elements combine.

37. At the beginning of the 19th century, large-scale industry developed, and it has progressed more rapidly than the restrictive legislation that is presently hitting it. It is this circumstance that is in part responsible for the extraordinary increase in the wealth and in the population of civilized states in this century (VII, 67).

But the movement has now slowed down considerably, and there is a strong tendency for part of the present form of society to crystallize by means of constraints of all kinds, and we are thus coming closer to the rigid types of regulations⁴⁷ that were shattered at the

⁴⁷ Man is restricted even in his slightest actions. For instance, the law prescribes that workers must rest on Sundays. In Switzerland, the members of a religious sect, the [so-called]¹¹⁷ *Adventists*, asked to be allowed to rest on Saturdays but this was denied them by the authorities. Not only does the law lay down when and how man must rest, but it also concerns itself with his eating and drinking habits, often on the pretext of hygiene, and sometimes without even that pretext. In some wine-producing countries it is forbidden to make wine with dried grapes. In this case, there is no hygienic pretext; the sole purpose of the law is to [serve the interests of]¹¹⁸ the wine producers. If such a course is to be followed, why not compel women to wear silk dresses instead of woolen ones in order to benefit the silk spinners and weavers?

The teetotalers,¹¹⁹ who have little sense, keep calling for new laws to prevent people from drinking what they like. Then again, other fanatics condemn tea, meat, and even milk!

[On the pretext of combating pornography, one tries to prevent the publication of any book that cannot be placed in the hands of children without danger! New Congregations of the Index intervene mysteriously with the State Railways and forbid the sale in stations of newspapers and books which do not appear sufficiently moral to these worthy inquisitors!

[The laws supposedly directed against the "white slave traffic" are most of the time nothing but a means of protecting national prostitutes. A woman is allowed to sell her charms in her homeland, but she is forbidden to offer them on a more profitable market.

end of the 18th and at the beginning of the 19th centuries. Thus, the theory that assumes that man may act freely by following his own tastes applies only to an ever more restricted field, since the constraints that are imposed on men, and which rigidly determine their actions, increase every day.

38. Because of [the encroachments of]¹²² these restrictive regulations, [it also follows that it would be]¹²³ a great mistake to try to anticipate practical results—even in strictly economic matters—in the light of economic theories alone. Restrictive measures are coming to occupy the principal role, and this matter belongs properly to the theory of nonlogical actions.

39. **International trade.** This subject is highly complex, and an adequate treatment would require [almost]¹²⁴ an entire volume [such as the present one].¹²⁴ [For reasons of space]¹²⁴ we must thus confine ourselves to a very brief outline.

40. **Economic theory.** Let us consider two communities, each possessing certain capital goods which, at least within certain limits, cannot be transported to the other to compete with its capital goods.

Between these two communities, exchanges of commodities and of certain capital services may take place, as well as imports and exports of government and corporate securities, etc.^[a]

41. Let us begin by considering only exchanges of commodities and imports and exports of money. We have already seen that in civilized countries the amount of gold in circulation is a very small part of the national wealth, and that [this quantity]¹²⁵ does not vary much. Imports and exports of gold serve to bring about equilibrium when it is disturbed, but in the long run they approximately offset each other and may be disregarded in comparison with exchanges of commodities and of capital services. Herein lies the essence of J.-B. Say's *law of markets*.^[a]

42. Each community will put its own capital to the most advantageous use. Let us suppose that there are only two commodities, A and B. The first community produces A, for instance, and obtains B in exchange; the second community produces only B and obtains A in exchange. From this fact, it can only be concluded that the first community has a greater interest in producing A—for its own consumption and in order to obtain B in exchange—than in producing A and B for its own consumption; and the same holds, *mutatis mutandis*, for the second community. But we can not conclude from this that B is produced more easily by the second community than by the first, and that A is produced more easily by the first community than by the second. Herein lies the essence of Ricardo's theory of comparative costs.

[A campaign has been launched against the poste restante on the pretext that it is used by lovers! A horde of fanatics is on the lookout night and day for ways of depriving their fellow men and women of every remaining freedom and of reducing society to one vast monastery of which these marvelous specimens of the human race would be the superiors!

[The law on the weekly day of rest is degenerating, in France and Italy, into a meddling and all-embracing tyranny!

[After having regulated factory work, people now want to regulate work at home as well, and in England a law is being put forward to fix a minimum wage for such work! An individual will be allowed to starve if he wants to; but he will not be allowed to work for less than that wage. For some people, the ideal social order appears to be one in which each citizen will be flanked by an inspector who will regulate his work, his relaxation . . . , and his pleasure!]¹²⁰

Let us bear in mind that every year, without exception, a large number of laws are enacted with the object of depriving man of the right to do things that were previously allowed. [If it continues like this,¹²¹ one will end up by controlling all the acts of man from the moment of birth until his death.

All that has just been said is, however, not very precise; nor is it very clear what is meant by saying that one thing is produced *more easily* than another. Professor Bastable warns us that the comparison between the costs of A and of B must be made not with respect to the prices, but to the *sacrifices*; but he does not—and cannot possibly—tell us precisely what these *sacrifices* consist of. In reality, this theory cannot be rigorously set forth without the aid of mathematics.^[a]

43. ^[a] Ricardo provides a very simple example, in which the communities are each reduced to a single individual.¹²⁶ “Two men can both make shoes and hats, and one is superior to the other in both employments; but in making hats, he can only exceed his competitor by one-fifth or 20 percent, and in making shoes he can excel him by one-third or 33 percent; will it not be for the interest of both, that the superior man should employ himself exclusively in making shoes, and the inferior man in making hats?”⁴⁸

Professor Bastable, who cites this example, adds: “It is merely a matter of calculation to see that both parties would gain by such an arrangement.”¹²⁸

44. But this is not true. It is strange that he did not see that it is true in some cases only, and not in others. Ricardo’s reasoning serves only to indicate a possible case, not a necessary one.

45. Let A and B be the two commodities discussed by Ricardo, and let us suppose that the less able worker makes, in one day, 1 unit of A or 1 of B. To keep to Ricardo’s example,¹²⁹ the abler worker will in one day make six-fifths of A or four-thirds of B. This is indicated in the following table, in which I and II indicate workers.

	I	II
A ...	$\frac{6}{5}$	1
B ...	$\frac{4}{3}$	1

Let us assume that the two workers each need respectively 30 days to produce A and 30 days to produce B, and that their tastes are thereby satisfied; we shall have:

	I	II	Total amounts
(a) {	A 36	30	66
	B 40	30	70

Still according to Ricardo, let us suppose that I produces only B and II only A; we shall have:

	I	II	Total amounts
(β) {	A 60	60	
	B 40	80	

⁴⁸ In this example we see immediately of what the *sacrifices* alluded to in Ricardo’s theory consist, because we are considering not two communities, but two men, and because we suppose that the commodities are produced only by labor. But the real world is far more diverse¹²⁷ and complex.

The total amount to be shared between the two is larger for B, but smaller for A, and we do not know whether, taking the individuals' tastes into account, these offset each other or not. If they do (§51), Ricardo's proposition is true; if not, the proposition is false (§52). For example, if A consists of bread and B of coral ornaments, it might very well happen that the decrease of 6 units of bread is not offset by the increase by 10 units in the quantity of coral.

46. For Ricardo's conclusion to be definitely true, when I produces only A and II only B, the total amounts produced between them must both be greater than in the case in which I and II each produce A and B for the direct satisfaction of their own tastes.⁴⁹

47.¹³⁰ For example, let us suppose that I still works 30 days to make A and 30 days to make B but that II works 22 days to make A and 38 days to make B. Moreover, and this is the main point, let us suppose that tastes are satisfied by the quantities so produced; we shall have:

	I	II	Total amounts
(γ)	A 36	22	58
	B 40	38	78

The total amounts produced when I makes only B and II makes only A are greater than the total amounts above; consequently, it is certain that there is some way to distribute the amounts so as to make each of the two individuals better off. For instance, they could be distributed in the following way:

	I	II	Total amounts
(δ)	A 37	23	60
	B 41	39	80

It is obvious that the combination (δ) is definitely better for each individual than the combination (γ).

48. Let us now perform a computation that will be useful later on (§55).^[a] Let us assume that in combination (β) the price of A is 1, as is also the price of B. In the combination (δ), I exchanges 37 of A for 39 of B, and consequently for him the price of A in terms of B is 39/37; individual II exchanges 39 of B for 37 of A, and consequently

⁴⁹ While II makes 1 unit of A, suppose that I makes x ; and while II makes 1 unit of B, I makes y .

Let (μ) be a combination in which for a length of time t , I makes only B and II makes only A; and (π) another combination in which I makes A during a time $t - \theta$ and makes B during a time θ ; II makes A during a time $t - \theta'$ and makes B during a time θ' .

If we want the quantities of A and B produced in the combination (μ) to be greater than those produced in the combination (π), we shall have to have

$$\begin{aligned} t &> (t - \theta)x + t - \theta', \\ ty &> \theta y + \theta', \end{aligned}$$

or

$$\theta' < (t - \theta)y, \theta' > (t - \theta)x.$$

These formulas^[a] serve to draw up the tables in the text. It should be observed that in order to make them possible we must have

$$y > x.$$

for him the price of B in terms of A (i.e., supposing the price of A to be unity) is $37/39$. But the price of A must be equal in both markets (there are no transport costs), and the same holds true for the price of B; II's prices must therefore be multiplied by $39/37$, and we shall have the following prices:

(δ)		
	I	II
A ...	$\frac{39}{37}$	$\frac{39}{37}$
B ...	1	1

In combination (a), if the price of B is 1, as we have assumed, the price of A, for I, will be $10/9$; and if II protects himself against the import of B by means of a customs duty of $1/9$, we shall have the following prices:

(a)		
	I	II
A ...	$\frac{10}{9}$	$\frac{10}{9}$
B ...	1	$\frac{10}{9}$

The ratio $10/9$ is greater than the ratio $39/37$; consequently, in our example, and still as a mere possibility, the prices in combination (a), which is the combination holding under protection, are higher than in combination (δ), which is that holding under free trade.

49. In practice, the prices do not refer to the protected¹³¹ commodity B but rather to the commodity A (money), which circulates freely. Under this hypothesis the prices of the free-trade combination (δ) are:

(δ')		
	I	II
A ...	1	1
B ...	$\frac{37}{39}$	$\frac{37}{39}$

The prices of combination (a) under protection are:

(a')		
	I	II
A ...	1	1
B ...	$\frac{9}{10}$	1

Consequently, the protective duty of II on commodity B causes the price of B to rise for II, and to fall for I.

50. Let us come back to case (β), and assume that the individuals' tastes are satisfied in such a way that these individuals are better off than in case (α) when the following distribution is carried out:

	I	II	Total amounts
(ϵ)	A 29	31	60
	B 49	31	80

This is to say that for I the decrease in A is more than offset by the increase in B; as for II, he obtains larger amounts of both commodities, i.e., he is certainly better off than before.

In this case, but only thanks to the assumption regarding the tastes of I, Ricardo's conclusion still holds.

It should be observed that if the two communities are not in communication, and community I still wants to obtain 49 of B, it will have only 27.9 of A, whereas community II will have only 30 of A and 30 of B; consequently, in conclusion, they will [both]¹³² be worse off than before.

51. By reasoning as in §49, we begin by observing that the prices are proportional to the following values:

	I	II
A	... $\frac{31}{29}$	1
B	... 1 $\frac{29}{31}$	

But the prices of A on both markets must be equal (transport costs are assumed equal to zero), and likewise those of B; so, we must multiply II's prices by $31/29$, to obtain the following prices:

	I	II
A	... $\frac{31}{29}$	$\frac{31}{29}$
B	... 1 1	

Thus the conclusion of §49 continues to hold. But we should note that this is only a possibility, and that if we were to choose other values the conclusion would no longer hold.

52. For instance, suppose that tastes were to be satisfied not by combination (ϵ) but by the following:

	I	II	Total amounts
(θ)	A 28	32	60
	B 45	35	80

The prices expressed in terms of B in the combination (*a*), which is the one holding under protection, would be lower than the prices of combination (*θ*), which is the one under free trade; and if the prices are expressed in terms of A, II's protective duty on commodity B would cause the price of B to rise, not only for II but also for I. However, it will turn out in fact that, even under free trade, it will be combination (*a*) that will come about. Indeed, if I, to satisfy his tastes, starts by producing 45 of B, he will have [enough]¹³³ time left to produce 31.5 of A; thus, it is more to his advantage to produce A and B than to produce only B, and to obtain A through exchange with II. We have here a¹³⁴ case in which Ricardo's proposition cannot be accepted (§45).

Everything we have just said should be considered only as a faint allusion [to the phenomenon,]¹³⁵ by way of examples, to enable us to discern certain possibilities by induction. Rigorous demonstration can be obtained only by recourse to the formulae of pure economics and by the use of mathematics.

53. If a community has a monopoly of a commodity, and if the members of the community compete for the sale of this commodity, [it is obviously to the community's advantage to replace competitive prices by monopoly prices],¹³⁶ and this can be done by imposing a duty on exports.

54. A duty on imports is essentially different from the preceding kind. When this duty effectively reduces the importation of the foreign commodity—replacing it partially or totally in consumption by the national commodity, which is produced in a larger amount—there is, in general, a destruction of wealth.⁵⁰

The exceptions are unimportant; such would not be the case, in general, with the combination we have indicated above (VI, 47), i.e., when instead of having constant prices for [successive]¹³⁷ portions of the commodity, one can set different prices within the country and abroad, and when this can lead to an [appropriate]¹³⁸ reduction in the cost of the commodity; for under this combination the price falls when one proceeds from the first state to the second, which is precisely the opposite of the effect of a protective duty.^[a]

The case studied above (VI, 47)¹³⁹ can be modified; that is, it can be assumed that under free trade 100 units of a commodity X will be produced at a cost of 5, and which will be sold at this price of 5. The entrepreneurs thus make no profit. Then, by imposing a protective duty, they sell 90 units inside the country at a price of 6, and 60 units abroad at a price of 4. They thus sell altogether 150 units, and obtain 780 for them.

The cost of production of these 150 units must be higher than 4.67; otherwise the entrepreneurs would not need a protective duty, and could sell 100 units at home at a price of 5, and 50 units abroad at a price of 4, obtaining a total of 700, or a sum equal to the cost. Let us thus suppose that the cost is 4.80. The 150 units will cost the entrepreneurs 720; and since they receive 780, they make a profit of 60. But the consumers lose 90, and this sum is higher than what the producers gain. This conclusion is general.⁵¹

⁵⁰ *Cours*, §864 ff.

⁵¹ [b] Let *a* be the quantity produced under free trade, at price *p*; then, when as a result of a protective duty the price at home is *p'*, let *b* be the quantity sold in the country, and *c* the quantity sold abroad, at price *p''*. Finally, let *q* be the unit cost of production when *b + c* is produced.

In order for the producers to derive a profit from the protective duty, one must have

$$p'b + p''c > (b + c)q.$$

We may thus, *grosso modo* and as a first approximation, conclude that any protective duty causes a destruction of wealth in the country that imposes it. This conclusion will still hold if, in addition to the exchange of commodities, we consider the numerous other facts determining the balance of debts and credits between the two countries under consideration.

Among the [worst]¹⁴⁰ evils caused by protection we must include the alteration in the values of the production coefficients which would provide maximum ophelimity. For instance, in England free trade has encouraged the intensive cultivation of wheat; in some states on the European continent, protection promoted the extensive cultivation of wheat.

Trade unions and producers' syndicates have similar effects.

55. Indirect economic effects. One of these effects which, if it does not actually exist is at least assumed to exist, is well known. It has been said that protection might be useful to protect infant industries which, when they grow up, will no longer need the protection. It cannot be denied a priori that this could sometimes be the case, but we have no examples of it. All the industries to which protection has given birth have kept asking for more, and the day has never come when they have declared themselves ready to do without it.

The theoretical possibility indicated in §§49, 51 appears in many cases really to exist, and a large number of facts incline people to believe that in a good many countries protective duties have caused a large number of prices of protected commodities to increase, in such a way as to lead to a general rise in the cost of living. We have already spoken of the effects of a¹⁴¹ general rise in prices (VI, 80), and it is not necessary to dwell again on this point.

If a country produces certain commodities, and if other countries levy protective duties on them, the prices of these commodities will decline in the country where they are produced (§49). However, an experimental verification of this theoretical deduction is much less easy to make than that of the preceding one.

Finally, the destruction of wealth that is caused by protection has, in its turn, many¹⁴² economic and social effects (VI, 54 and *passim*), which appear therefore as indirect effects of protection.

56.¹⁴³ Distributive effects. Protection obviously changes the distribution [of incomes] among certain individuals. There are infinitely many possible combinations; speaking very generally and roughly we can say that agricultural protection particularly favors the landlords, whose *rents* it increases. Industrial protection is permanently favorable to the owners of industrial land, and temporarily to the entrepreneurs who at first obtain temporary *rents* which, however, are more or less rapidly [reduced and]¹⁴⁴ eliminated by the competition from other entrepreneurs. Industrial protection favors

In order that they should derive no profit from this combination under free trade, one must have

$$pa + (b + c - a)p'' < (b + c)q.$$

Finally, in order for the producers' profit to be larger than the consumers' loss, one would have to have

$$p'b + p''c - (b + c)q > (p' - p)b.$$

From these inequalities we obtain

$$b > a;$$

which is impossible, since the protective duty, by causing the price to rise, must lower the quantity sold in the country and, consequently, b must be smaller than a .

skilled workers who obtain higher wages than they would have obtained if the protected industries had not emerged, but it is to the detriment of the workers remaining in the non-protected industries or in agriculture. Finally, a part of the bourgeoisie belonging to the liberal professions is also favored; industries have greater need than agriculture of engineers, lawyers, notaries, etc.

These effects vary according to the conditions of production in the various countries. In Russia, for instance, industry is protected entirely¹⁴⁵ at the expense of agriculture. In Germany, industry and agriculture can be and are both protected; as a result, agriculture is favored by protection of its own without suffering too much from the industrial protection.

57. Social effects. Industrial protection in an agricultural country and free trade in an industrial country both have the effect of fostering industry; and thus these opposite measures may, because they are used in different countries, have similar effects, which consist above all in giving power to or increasing the power of the working class and of democracy, [to say nothing of]¹⁴⁶ socialism. Protection in Russia has effects of this kind, as does free trade in England.

Where there is landed aristocracy, as in Germany, agricultural protection strengthens it and helps to prevent it from being overthrown¹⁴⁷ by other aristocracies. This is why agricultural protection is perhaps indispensable in Germany in maintaining the present social order.^[a]

Industrial protection in countries that are essentially agricultural, and free trade in essentially industrial countries, by favoring industry constitute a powerful means of selection of the working class and also of the bourgeoisie, which provide clerical workers, engineers, etc., for industry.

Protection is also, in general, a means of selection for those who obtain protective tariffs by various devices, such as buying support of voters, journalists and politicians. But, in fact, this selection yields a very decadent¹⁴⁸ aristocracy, which is inferior even to the one that would be created by brigandage, for that would at least produce men of courage.

58. Fiscal effects. Among modern nations, protection does not present itself in pure form, but is always bound up with fiscal measures. All modern states which resort to protection extract vast sums from customs duties for their budgets: in the United States of America and in the Swiss Confederation, customs duties constitute the main [, indeed almost the only]¹⁴⁹ source of government revenues.

59. Within each country, modern democracies tend to replace indirect taxes by direct ones; it is only by means of customs duties that one can tap the citizens comprising the great bulk of the population, whereas direct taxes—especially progressive taxes—exploit the well-to-do classes, which always form a small fraction of the total population. In some cases, protection restores to some of the well-to-do a fraction of what has been taken away from them by progressive taxation, or even by other taxes, the proceeds from which are spent [in part]¹⁵⁰ on measures of state socialism.

60. From all the preceding discussion it can be seen how complex is the practical and synthetical problem of knowing whether or not protection is more advantageous than free trade. Indeed, in this general form the problem is insoluble because it lacks a precise meaning. We should instead consider a particular problem which may be enunciated in the following way: Given all the economic and social conditions in a country at a

certain time, what is more advantageous for that country at that time: free trade or protection?

61. The following reasoning is erroneous, because it ignores some essential conditions of the problem:^[a] Protection¹⁵¹ entails a destruction of wealth, hence for all countries and at all times protection is harmful and free trade is beneficial.⁵²

62. Causes of protection. These causes definitely do not include the theoretical solution of the economic problem of [protection].¹⁵⁴ Even if it could be proved to be manifestly obvious that protection always entails a destruction of wealth, and even if this were taught to all citizens as they are taught their *abcs*, protection would lose such a small number of adherents, and free trade would gain so few, that the effect could be virtually or perhaps entirely ignored. The causes underlying men's actions are quite different from these:⁵³

[Auro suadente, nil potest oratio.]¹⁵⁶

63. Protection is generally brought about by a league whose principal participants are: (1) Those who expect to receive a direct [and]¹⁵⁷ substantial benefit from protection, i.e., landowners who will obtain a permanent rent from it; entrepreneurs who will get just a temporary rent but this does not bother them, provided it lasts long enough for them to make some money; and those who have professions that can be protected. (2) [Petty]¹⁵⁸ politicians who count on the tariff proceeds (§58) to swell state revenues, of which they will then dispose. All those who hope to benefit from the state's expenditures, and who are intelligent enough to understand that to increase the expenditures one must first increase the receipts. (3) Those in whom one succeeds in awakening *nationalist* sentiments, thus leading them to believe that protection is useful to defend the fatherland against foreign countries. [In a small number for customs protection, in a larger number for other restrictionist measures,¹⁵⁸ one should also take account of the "ethically minded"—a handful as regards customs protection, a larger number where other restrictive measures are concerned; they imagine or seem to believe that these measures are favorable to their ethics. These are often a strange breed; when they are sincere, one can pull the wool over their eyes; and when they are not, it is they who pull the wool over other people's eyes. (4) Finally there are those—still a small number at present—who are educated, intelligent and far-sighted enough to see that democracy tends more and more to destroy the wealthy, and who, having neither the will, the courage, nor the power to

⁵² The present writer has made the mistake of expressing himself at times in polemical works—which, it should be added, have no scientific value—in a way that could have given rise to the belief, at least implicitly, that he employed this kind of argument. [And this fault is all the greater in that,]¹⁵² as early as 1887, he wrote: "In the end, the consideration of the social effects and of their economic consequences, which might be called the doubly indirect effects of protection, constitutes the most original part of the question, and in my opinion it is the only one that can sometimes raise serious doubts about the greater or lesser utility of free trade in certain special cases." (*Sulla recrudescenza della protezione doganale*; memorandum read before the Academy of the *Georgofili*, May 28, 1887.)¹⁵³

⁵³ Bourdeau, who is an acute observer of the evolution of socialism, writes: "How unsatisfactory is the work of all of us who are concerned with socialist questions when we confine ourselves to setting forth and refuting abstract theories about which most workers know nothing and care less! Ideas among the common people come from their sentiments, the sentiments come from their feelings, and their feelings in turn spring from their way of life and from the nature, duration and profitability of their work." *Socialistes et sociologues*, p. 164.¹⁵⁵

resist directly, choose this covert means to win back a little of what has been taken from them and, in any case, of not being the only ones to pay taxes.

64. It is a league of this kind that established a monopoly of alcohol in Switzerland; however, category 4 was missing and category 3 was slightly different. The monopoly was formed of: (1) Those from whom the administration of the monopoly buys the alcohol at a price much higher than the [usual]¹⁵⁹ market price.⁵⁴ The farmers, who may freely distill their own produce and who then sell this alcohol at a much higher price than they would obtain if the monopoly did not exist. (2) The public authorities, whose revenues increase with the monopoly receipts. (3) The teetotalers who, as good sectarians, approve of anything that may strike down, or which they imagine could strike down, their enemy: alcohol.

[No one of these three categories of persons may have had enough power to institute an alcohol monopoly by itself; they succeeded because they were united.]¹⁶⁰

65. In England the present protectionist movement is the product of category 4 of §62.¹⁶¹ Category 3 is very numerous, and the nationalist sentiment finds an outlet in the pursuit of a closer union with the colonies. In fact, categories 1 and 2 conceal themselves partly behind category 3, which is the standard bearer of the league.

66. To explain how the defenders of protection obtain such an easy hearing, we must add some remarks of a general nature concerning social movements. The intensity of an individual's work is not proportional to the advantage that this work may procure him, or to the damage it may help him avoid; [it increases more than proportionately to that advantage or that damage].¹⁶² If a certain measure A causes the loss of one lira to each of 1,000 individuals and a gain of 1,000 lire to a single individual, that individual will pursue his course with considerable energy, whereas the 1,000 will defend themselves listlessly; hence it is very likely that in the end success will go to the man^[a] who, by means of measure A, aims at appropriating the 1,000 lire.⁵⁵

Protectionist measures yield large benefits to a small number of individuals, and cause slight harm to each of a very large number of consumers. This circumstance makes it easier to put contemplated protective measures into practice.

It should moreover be observed that the impression that a total sum makes on people is generally stronger than the totality of impressions made by each of the parts of this sum. A hundred lire are arithmetically equal to 100 times one lira; but this equality does not hold with respect to an individual's feelings; a total sum of 100 lire may make a much stronger impression than 100 times one lira. This is truer still if the 100 lire are received directly, and if the separate lire comprised in the total of 100 are obtained indirectly; the difference is greater still, if there is some doubt about the circumstances that bring forth these individual lire.

A producer can make a fairly accurate assessment of what he stands to gain from a tariff imposed on the commodity he produces; suppose he estimates that it will bring him 100

⁵⁴ Numa Droz, *Essais économiques. Le monopole de l'alcool en Suisse*, p. 577: "As it is the Confederation that negotiates the contracts for the supply of alcohol, it is to it that one turns, particularly in election years, to request it . . . to improve the terms of the contracts and pay a higher price for home-grown potatoes; otherwise the elections might go the wrong way. Thus we are reduced to having the *electoral potato*."

The ethical teetotalers do not see or pretend not to see these things.

⁵⁵ *Systèmes*, I, p. 128; *Cours*, II, §1046 ff.

lire. Little does it matter that the commodity is sold bit by bit; in relation to the tariff it is a single operation, and the sum of 100 lire is considered as a whole. As a consumer, he will have to bear the costs of the protection granted to the other commodities. Let us assume that he buys 100 of these commodities for his own consumption; each one will cost him, owing to the tariff, one lira more. And again it matters little or not at all that the purchase of each of these commodities is made at one time or more. In all, the individual in question will lose 100 lire, i.e., exactly what he will gain by protection; and yet, the impression made on him by one of these facts will be quite different from the impression made by the other. Not only will the 100 lire he gains as a whole on his commodity make a stronger impression on him than the 100 lire he loses bit by bit; but in addition, the first [calculation]¹⁶³ is far more reliable—or, if one prefers, less unreliable—than the second. Protection almost certainly causes the price of the [protected]¹⁶⁴ commodity to rise; but on the other hand it is not at all certain that the price of non-protected commodities will not rise for quite different reasons. In short, what the individual's additional receipts will be is almost certain, but what his additional expenditures will be is very doubtful.

This is not all. Frequently the hypothesis we have just formulated does not apply, and a producer gains more from the protection granted to his commodity than he loses from the protection granted to the producers of other commodities.

Let us take an economic state in which the causes A, B, C, . . . of a destruction of wealth are operative and another in which all these causes are removed. There can be no doubt that in the second economic state (distribution remaining the same) everybody will be better off than in the first. But if instead we compare a state in which the causes A, B, C, . . . of a destruction of wealth exist, with another state in which only the causes B, C, . . . are present, we can no longer assert that in this second state all individuals are better off than in the first, because the destruction of wealth brought about by B, C, . . . may increase so much as to offset, and even to exceed, the destruction caused by A in the first state.

The opinion of liberal economists that protective duties are imposed on a country by a league of [petty]¹⁶⁵ politicians and a small number of producers, cannot be generally accepted, because we have at least one particular case in which it is contradicted by experience. In Switzerland, protective tariffs have been approved by a popular referendum, i.e., by the majority of the voters at the polls.

Equally mistaken is the opinion that protective duties are tolerated only by reason of the ignorance of the public; for those who benefit from these duties often show [keen perception and a true sense of opportunity],¹⁶⁶ and the fault of those who bear the costs of protection is not so much ignorance as lack of courage and [of fitness for vigorous work].¹⁶⁷

This can be seen even more clearly by observing how consumers fail to act any differently in similar cases where the excuse of ignorance cannot be invoked. For example, when a league such as that of the lithographers makes it known to all that it will excommunicate and persecute any producer found guilty of charging prices favorable to consumers (§12), the latter could instead come to his support, and combat those who take such pains to harm [them].¹⁶⁸ If they do not even do this, [if they do not have enough spirit to roll up their sleeves for really easy work],¹⁶⁹ how could they [ever]¹⁷⁰ take upon

themselves the much more arduous task of changing the laws and of avoiding the burden of protective duties?^[b] In short, the world is for those who will lay hands on it.^[c]

67. ¹⁷¹ It is not enough to condemn protection to observe that it is established by those who have a direct advantage from it and to a large extent by those who aim to appropriate other people's wealth; for if these are the reasons, the ends these people achieve could well be for the good of the country. We have seen that in the determination of the production coefficients, entrepreneurs have only their own advantage in view; and nonetheless they end up by organizing production for the greatest good of the consumers. Something analogous might be the case with protection [§35].¹⁷²

68. It is not possible to judge the effects of protection or of free trade by comparing the countries where these exist, since these countries differ in many other respects. Such a comparison can only be made, with due caution, for a single country and for a span of time not exceeding two or three years, when the country passes from protection to free trade, or conversely. In such cases, the other circumstances vary little in comparison with the variation as between protection and free trade, whence one may, with some probability, attribute the variation of the effects [at least in part] to the variation of the circumstance that [has changed]¹⁷³ the most.

69. We thus obtain practical confirmation of the statement that protection, by reducing imports, reduces exports too. This phenomenon has been observed in a large number of cases and for many countries.⁵⁶

70. As a result of what has been said in §68, it is a mistake to cite the prosperity of the United States as a proof of the utility of protection, or the prosperity of England as a proof of the utility of free trade.⁵⁷

Neither can one compare England and Germany as if the only difference between these countries was that England practices free trade and Germany protection.

71. It may also be noted that if in England free trade operates [on the one hand]¹⁷⁵ to increase wealth, the power of the trade unions operates [on the other hand]¹⁷⁵ to destroy it. The great protracted strike of the mechanical workers was the result of their determination not to allow the employers to introduce improved machinery, except with the [illustrious]¹⁷⁶ workers' permission, and on condition that the profits from the new machinery would go to the workers; in practice, this would have amounted to preventing the introduction of the machinery, which would have had to be paid for by the employers without yielding them any profit.

There is a contradiction between the behavior on this occasion of the humanitarian and decadent bourgeoisie,⁵⁸ who sided with the workers, and the wailing of the very same

⁵⁶ *Cours*, §881.

⁵⁷ The following proposition from the *Cours*, §891, is therefore erroneous: "England, thanks to its fidelity to the principles of liberal political economy, continues to see its prosperity increase..." The author was wrong in adopting, without putting it to a severe enough examination, a proposition that was current among liberal economists and which appeared to them as axiomatic. Moreover, he expressed himself poorly because while it is true that, at the time he was writing, England was practicing free trade and had a monetary system true to liberal economic principles, nevertheless municipal socialism, which has made great progress since, was already raising its head, and the system of humanitarian constraints was also flourishing. To be precise, the author should not have spoken in such absolute terms of England's fidelity to liberal economic principles.

⁵⁸ Among the latter, there are bishops and archbishops who would have done better to concern themselves with theology than with political economy.

bourgeoisie over the fact that German industry is getting the better of English industry. When one wants something, one should not bewail the necessary consequences.

If English industry has made less progress in the past few years than German industry, this is certainly due in part to the negligence of the industrialists, who have been resting on their former reputation without bothering to move forward, but still more is it due to the tyranny exerted [in England]¹⁷⁷ by the trade unions over the employers; on the other hand, German industry remains free from this evil for the moment, or at least does not experience it with the same intensity.

72. [If it turns out that protectionist measures take hold]¹⁷⁸ in England, this will certainly result in some destruction of wealth; but if, on the other hand, the new social order which will result from this policy makes it possible to put a brake on municipal socialism, on humanitarian constraints, or even only to curb the power of the trade unions somewhat, an enormous amount of wealth will be saved, which may make it possible to compensate, or even to overcompensate, for the loss due to protection. The final and total result might thus be an increase in prosperity.

73. **Economic crises.** The economic complex composed of molecules that are in continuous vibration; this is due to the very nature of men and to the economic problems they have to solve. These movements may take place in different directions, so that they partly offset one another. At times, we see some industries and some trades prospering, while other industries and other trades, languish; on balance, there is compensation, and we cannot say that there is a general state of prosperity or a state of economic depression.

But every now and then, for whatever reasons, these movements of the elements of the economic complex go in the same direction. We see then that nearly all industries, commerce, and professions prosper; or that they become stagnant and suffer: thus there is a general state of prosperity or a general state of economic depression.^[a]

74. It is this latter state, when it is sufficiently pronounced, that has been called a CRISIS. But, since observation shows that a state of depression is always preceded by a state of extraordinary activity, the meaning of the word "crisis" should be extended to these two phenomena as a whole; the period of extraordinary activity should be called the *upward phase* of the crisis, and the period of depression, the *downward phase* of the crisis.

This definition of the crisis is, however, not very precise.^[a] Movements of the elements of the economic complex are continually taking place. We withhold the name "crisis" from minor movements and accord it to the major ones; but how can we distinguish one from the other? We should at least have some standard of measurement. Since this is not possible, there remain the extreme cases about which there is no doubt; but we cannot safely use this terminology for intermediate cases. The same holds for the words young and old, which are used to indicate [different ages].¹⁷⁹

75. The crisis is just one particular instance of the great law of rhythm, which regulates all social phenomena.⁵⁹ The social order determines the form of the crisis, but does not act on its substance, which depends on the nature of man and on the economic problems. There are crises not only in commerce and in private industry, but also in public undertakings. Municipalities go through periods during which they transform whole cities, and periods during which they [contract and]¹⁸⁰ undertake no new works.

⁵⁹ *Systèmes*, I, p. 30.

States have never built railways at a uniform pace; in some periods they have built a great many and in others, very few. From time to time, England goes through a *naval panic*: the nation dreads a foreign invasion, and all the expenditures for the navy are hastily approved; these panics are followed by periods of quiet, in which construction of new vessels is slowed down.^[a]

76. To produce commodities takes time, and often considerable time prior to consumption. For production to be perfectly adjusted to consumption, one should: (1) Be able to forecast consumption. (2) Be able to forecast the outcome of the productive process exactly. Neither the one nor the other can be done with any certainty.

77. In the present social order it is private producers and merchants who try to make these forecasts. As a reward if they guess right, they become rich; as a penalty if they guess wrong, they are ruined. Under a socialist regime, civil servants would have to do this job; it is probable that they would be more grievously and more often mistaken than private persons. To be convinced of this, we need merely recall, among other facts, what a poor performance governments make of supplying their field armies with food, whereas private commerce shows amazing skill in meeting the much more varied and complex consumer needs of large cities like Paris, London, and Berlin.

Production, in its efforts to adjust to consumption, sometimes runs ahead and sometimes lags behind, and the oscillation in the one direction is often the cause of the oscillation in the other. When French vineyards were plagued with phylloxera, production lagged behind consumption, and the price of wine rose. There was plenty to be gained by increasing output; everybody undertook to restore vineyards with American vines, and therefore production, because it had lagged behind, overtook consumption. Production of wine is now in excess of demand at the ruling prices; hence another oscillation is beginning in the opposite direction.^[a]

78. Crises arise mainly from two kinds of causes: (α) Any objective change in the conditions of production may give rise to a crisis, if it is sufficiently extensive. This was the cause of famines in former times. (β) The subjective synchronization of economic movements transforms into intense crises movements which would otherwise have given rise to minor alterations of economic equilibrium.

79. The subjective cause has a powerful effect: in some periods men are full of confidence; at other times, they are completely discouraged. Such states of mind have undergone a change, as a result of experience. The memory of the declining phase of past crises tempers exaggerated faith in flourishing success in the upward phase of a new crisis; the recollection of the upward phase of past crises moderates excessive discouragement in the downward phase of a new crisis.^[a]

All those authors who have studied crises carefully have noticed the role played in them by people's imagination. Montesquieu has some very apt comments on the crisis that occurred in Law's time.⁶⁰ But in general, they have taken as being an effect of the crisis what is, on the contrary, one of its principal causes.

⁶⁰ *Lettres persanes*, CXLII. He imagines Law speaking in the following terms: "People of Bétique, do you want to be rich? Imagine that I am very rich and that *you are too*; repeat to yourself every morning that your fortune has doubled during the night, and if you have creditors, go and pay them as much as you imagined, and now tell them to put their own imaginations to work also."

80. During the upward phase, everybody is contented, and there is no talk of crisis. Nonetheless this period prepares inevitably for the downward phase, which makes everybody discontented, and which alone has been given the name of crisis. The upward phase usually lasts longer than the downward phase. The upward climb is gradual; the drop is precipitous.

81. Much greater damage is imputed to crises than they actually cause; this is so because man is very sensitive to his ills, whereas he easily forgets the benefits he has enjoyed. He feels that the benefits are owed to him, and that the ills are undeserved blows. The distresses of the downward phase of the crisis act strongly on man's imagination, and he forgets the advantages obtained during the upward phase.

In the last analysis it has not at all been proved that the oscillatory movement called a crisis causes nothing but harm to human society. It could well be the case instead that the movement is more to its advantage than to its detriment.

82. The facts that are attendant on crises have been considered as the causes of the crises.

During the upward phase, when everything prospers, *consumption* increases, and entrepreneurs increase *production*; to this end, they transform *savings* into goods-in-process¹⁸¹ and fixed capital, and they have extensive recourse to *credit*; *circulation* is more rapid.

Each of these facts has been considered to be the exclusive cause of the downward phase which has been called the crisis. The truth of the matter is simply that these facts have been observed in the upward phase, which always precedes the downward phase.

83. What is called overconsumption in the upward phase is simply an increased consumption due to the economic prosperity in this period which will be transformed into underconsumption, i.e., a decreased consumption, when in the downward phase economic prosperity declines.

Likewise, production increases in the upward phase to satisfy the growing demands of consumption, and there is then *underproduction*; for instance, in the upward phase coal "shortages" can nearly always be observed. In the downward phase that follows, consumption falls off and output becomes superabundant; for a time, i.e., until steps have been taken to curtail production as well, there is "overproduction."

It is fantasy to speak of permanent overproduction. If such a thing existed, somewhere there would have to be, as we have already noted, ever-increasing stocks of commodities for which the production exceeds consumption; but no such thing has ever been observed.

Similar remarks could be made about overinvestment and the recourse to credit.

When one speaks of a "circulation crisis," the effect is usually taken for the cause. [Monetary] circulation is sometimes rapid (in the upward phase), and sometimes slow (in the downward phase) as a result of the crisis; and it is not, on the contrary, the crisis which is caused by these variations in the velocity of circulation.

84. Likewise there are phenomena which are independent of crises and which, when wrongly interpreted, may have given rise to the errors just pointed out.

The permanent phenomenon called overconsumption is nothing but man's tendency to consume as much as he can to satisfy his tastes: it is the force that stimulates production.

What is generally called overproduction is the entrepreneur's tendency to supply, at a certain price, more of a commodity than is demanded for consumption; it is the force that stimulates consumption.

Since consumption and production are not ever, and never can be exactly equal, there is, from time to time, actually an excess of the one or the other, which is soon made up for by a corresponding deficiency.

As an example, suppose there are some producers who have stocks of a commodity, and who produce 100 units of this commodity in one year. Suppose consumption is 120 units, so that the 20 additional units will be withdrawn from stock. In the following year the producers, attracted by this excess consumption, will raise their prices and produce 110 units, whereas their customers, held back by this very increase in prices, will consume only 90; there will then be an excess of production of 20 units, which will replenish the stock. There is thus at one time an excess of consumption and a deficiency of output and at another time a deficiency of consumption and an excess of output.

Analogous phenomena can be observed in the production and consumption of coal, cast iron, and many other commodities; but the oscillations generally last more than a year.

85. Symptoms of a crisis. Mr. Clément Juglar¹⁸² finds symptoms of crises in the balance sheets of banks of issue; Mt. Pierre des Essars, in the velocity of circulation of current accounts in banks of issue.

The amount of available savings^[a] bears a relation to the oscillatory movements [called]¹⁸³ crises. In the upward phase, this amount decreases; in the downward phase it increases.

Just as a small river basin connected with the sea may indicate the sea level, the quantities of money available in the coffers of the banks of issue may give an idea of the amount of savings available in the country.

We should be careful not to confuse the effect with the cause and not to imagine that a crisis can be prevented by artificially retaining gold in the banks' coffers. To reason in such a way would be similar to breaking one's thermometer to prevent the temperature from rising.

[At the beginning of the downward phase of a crisis, there are always people who maintain that the crisis is due to a lack of circulating medium, and one sees all kinds of projects hatched for reforming the banks and even the monetary system. Now, what appears as a lack of circulating medium is precisely the force which acts to restore the equilibrium that has been disturbed. Let us suppose that just before the downward phase begins, the amount of circulating medium is considerably increased. The result will simply be to prolong the upward phase; this will have the effect of taking the economic aggregate still further away from its equilibrium position and consequently of worsening the crisis which must inevitably take place. There is only one way of stopping speculators, producers, and consumers, who are moving further and further away from the equilibrium position: it is to cut off their supplies; in other words, to make them short of the circulating medium with which they could continue their operations.

[The fantastic conceptions of President [Theodore] Roosevelt, who has accused the trusts and the speculators on the stock exchange of being responsible for the crisis in the United States at the end of 1907, belong to the world of fiction. The crisis was general;

it hit countries like England where trusts are the exception, and countries like Germany, where extremely severe legislation has reduced speculation on the stock exchange to a minimum. In Paris, speculation is much more widespread than in Berlin; and, if France has been relatively immune to the crisis, it is because, not having taken part in the upward phase, it has by this very fact avoided the downward phase. Where there is no flood tide, there is no ebb tide, either.

[A financial crisis observed on the stock exchange must be clearly distinguished from an economic crisis that hits production.

[A financial crisis occurs all at once, at the beginning of the downward phase. It is profound, but it passes quickly. In such circumstances, the discount rate of the Bank of England is suddenly raised to a high level, but a few months are enough for it to come back to its normal level.

[An economic crisis starts slowly, develops during many years, and ends only gradually, when a new upward phase begins.

[On the stock exchange, during the downward phase, fixed-income securities—mainly gilt-edge government bonds—increase in price, whereas shares of industrial firms are depressed. The opposite is found in the upward phase.

[When the downward phase begins, some governments ascribe it to reprehensible manipulations on the part of stock market speculators. Thus, in 1907 and 1908, the Italian government took police action against people who made the mistake of selling instead of buying industrial shares. Such measures generally have the opposite effect from the one intended, for, far from restoring confidence, they contribute to increasing the distrust.

[It should, moreover, be clearly understood that the bears are not in reality very harmful, except to other speculators, namely the bulls. If the bears sell shares below their value, this provides an excellent opportunity to buy them for people who have money available.

[As for persons who keep their stocks in their safes and draw their dividends, they are quite indifferent to the struggle between the speculators on the stock exchange.

[Far from being always harmful, bear speculation may be very useful in some cases. It prevents the upward phase from going on for too long, and the economic aggregate from moving further and further away from the equilibrium position; and when the upward phase arrives, it prepares for the downward phase. It is a well-known fact on the stock exchange that *short-covering purchases* are one of the most powerful factors making for a rising market.

[Finally, it should be remarked that *bull speculation* and *bear speculation* are often only two terms for one and the same thing. How could the bears sell if nobody was buying? How could the bulls buy if nobody was selling?¹⁸⁴]

86. Clément Juglar has noticed that during the upward phase the quantity of money in the vaults of banks of issue decreases, and that the portfolio increases; in the downward phase the opposite effects are observed. That author made a special study of the maxima and the minima of the receipts¹⁸⁵ and of the portfolio, and succeeded in establishing definite relationships between [crises and]¹⁸⁶ these phenomena.

87. Pierre des Essars¹⁸⁷ has computed, for a period of 85 years, the velocity of circulation of current accounts in the Bank of France, and (for a somewhat shorter period) in the Bank of Italy, and was able to verify that the [velocity of] circulation is at a maximum

when the upward phase ends and when the downward phase begins, and at a minimum in the liquidation period of the crisis.

88. W. Stanley Jevons believed he could determine the approximate length of the phases of crises. According to him, there would be three years of commercial depression, three years of commercial activity, two years of intense commercial activity, one year of maximal activity, and one year for the crash; and after that, other periods identical with the preceding ones would set in again. In this way, about ten years would elapse between one crisis and the next.

The real phenomenon does not occur with such regularity, and the periods are not all of the same length; thus, Jevons's description can serve only to provide a broad idea of the [phenomenon].^{188[a]}

Appendix to the Italian (1906) Edition

1. This appendix is by no means a treatise on mathematical economics—if only because there would not be enough space for such an undertaking. It is only a compendium designed to provide some idea about this part of political economy, and to make possible a better understanding of what I have set out in the present Manual.

2. Let x and y be the amounts of economic goods X and Y possessed by an individual.¹ Assume that we need not take account of the order in which X and Y are consumed (IV, 7), that is, let us consider the arrangements $x y$ and $y x$ as being identical. Starting from a given combination $x_1 y_1$, let us look for all the other combinations $x_2 y_2, x_3 y_3 \dots$, which are equivalent to the first one for the individual. Among these the choice is, for him, a *matter of indifference* (III, 52). By interpolation, we shall obtain an equation:

$$(1) \quad f_1(x, y) = 0$$

such that if x is given the values

$$x_1, x_2, \dots$$

we shall have for y the values

$$y_1, y_2, \dots$$

Equation (1) is that of an *indifference line*¹ (III, 54). Starting from another combination x'_1, y'_1 , not included among the preceding ones, we shall have the equation of another indifference line, and so on. Let us give each of these indifference lines an index I , as is explained above (III, 55); we shall find that corresponding to the indices

$$I_1, I_2, \text{ etc.}$$

we have

$$f_1, f_2, \text{ etc.};$$

¹ The notions of *lines of indifference* and of *lines of preference* were introduced into the science by Professor F. Y. Edgeworth. He started out from the concept of *utility* (ophelimity), which he assumed to be a known quantity, and from it deduced the definition of these lines. I have inverted the problem. I have shown that, starting from indifference lines, given directly by experience, we can immediately obtain the determinateness of economic equilibrium and work back to certain functions, including ophelimity if it exists; or at any rate we can deduce the ophelimity indices.

and, interpolating the coefficients we shall obtain a function that will reproduce the preceding ones for various values of I ; hence, the equation

$$(2) \quad f_1(x, y, I) = 0$$

will give us, for suitable values of I , all the indifference curves.²

3. If we consider equation (2) as that of a surface, the projections of its level curves on the $x y$ plane will be the indifference lines. This surface is partly arbitrary, since I is partly arbitrary itself; that is, it is any one among the surfaces that have as projections of their level curves the indifference curves given by the equations

$$f_1 = 0, \quad f_2 = 0, \quad \dots,$$

and of those intermediate equations that can be deduced by interpolation, or by giving suitable values to I in equation (2).

It will help for simplicity to put equation (2) in the form

$$(3) \quad I = \Psi(x, y).$$

Giving I a constant value, we obtain an indifference curve.

4. If we assume that pleasure is a quantity, that is, that ophelimity exists, and if we designate the ophelimity of the combination xy by

$$(4) \quad I = \Phi(x, y),$$

it is clear that the choice between two combinations will be a matter of indifference if they yield the same value of I ; that is,

$$(5) \quad \Phi(x, y) = \text{constant}$$

will be the equation of an indifference curve. This equation would therefore have to be included among equations (3) or among the intermediate ones deduced by interpolation.

5. The surface that has I as ordinate may be called the surface of pleasure or of ophelimity (III, 58). It is one of the surfaces that has (3) as its contour lines, but we do not know which one. This is why, in general, we cannot deduce the measure of ophelimity from the experience of static economics; this experience gives us only equations (3), or those that are intermediate among these.

6. Differentiating equation (2), holding I constant, and then eliminating I by use of (2), or else differentiating (3) directly, we obtain

$$(6) \quad \Psi_x dx + \Psi_y dy = 0,$$

which is therefore also the differential equation of the level curves of (4).

² For further elucidations see P. BONINSEGNI, "I fondamenti dell' economia pura," in the *Giornale degli Economisti*, Rome, February 1902.

Equation (6) is independent of the system of indices adopted. Integrating it must give all these systems of indices, and we shall have

$$I = F(\Psi),$$

where F is an arbitrary function and

$$\Psi = \text{constant}$$

is an integral of (6).

7. The elementary ophelimity (III, 33) of X is²

$$(7) \quad \frac{\partial I}{\partial x} = \varphi_x(x, y)$$

and a similar expression holds for the elementary ophelimity of Y.

We have therefore³

$$(8) \quad \varphi_x = \Psi_x F', \quad \varphi_y = \Psi_y F'.$$

8. In the case in which the elementary ophelimity of X depends only on x , and that of Y depends only on y (IV, 8), it is immediately seen from (8) that equation (6) must have an integrating factor F' such that φ_x is a function only of x and φ_y is a function only of y .

The general integral of the equation

$$(9) \quad \varphi_x dx + \varphi_y dy = 0$$

would then be

$$I = F(\Phi),$$

which would give

$$\frac{\partial I}{\partial x} = \varphi_x F', \quad \frac{\partial I}{\partial y} = \varphi_y F';$$

but since [the elementary ophelimity of X is a function only of x , and that of Y is a function only of y ,]⁴ in the preceding equations F' must be a constant, that is,

$$F' = A,$$

and therefore

$$I = A\Phi.$$

In that case the elementary ophelimity may therefore be determined from experience, only a constant remaining indeterminate, which represents the unit of measurement.

9. It worth noting that, in static economics, two objects X and Y having the elementary ophelimities

$$(10) \quad \varphi_x, \quad \varphi_y,$$

behave precisely as if they had the ophelimities

$$(11) \quad \varphi_x F'(\Phi), \quad \varphi_y F'(\Phi);$$

whence if we denote two ideal objects having these ophelimities by X',Y', the objects X and Y are completely equivalent to X' and Y'.

It follows from this that: (1) Objects X and Y having elementary ophelimities that depend respectively on x and y alone can always be considered as equivalent to ideal objects X',Y', each of which has an elementary ophelimity depending on both x and y . (2) Conversely, in some cases, two objects X and Y each having an elementary ophelimity depending on x and y may be regarded as equivalent to an ideal object X' whose elementary ophelimity depends only on x and to another ideal object Y⁵ whose elementary ophelimity depends only on y .

10. Properties of indifference curves. (1) Since, along an indifference curve, a decrease in the quantity of X must be offset by an increase in the quantity of Y, and vice versa, we must have

$$(12) \quad \frac{dy}{dx} < 0;$$

and this is the first characteristic of the indifference lines.

(2) In general, then, and leaving exceptional cases aside, the variable amount dy that a person is willing to give up, along an indifference curve, in return for a constant amount dx , decreases as x increases; hence we have the second characteristic

$$(13) \quad \frac{d^2y}{dx^2} > 0.$$

Further, dy decreases less the greater is x , whence—still in general—we have

$$(14) \quad \frac{d^3y}{dx^3} < 0.$$

Hitherto we have considered the variations of the coordinates along a given indifference line; let us now see what follows as we pass from one line to another. Let us designate by δ_x the variations that take place in passing from one line to another along a line parallel to the x axis; and by δ_y the same variations along a line parallel to the y axis.

Considerations analogous to the preceding ones allow us to ascertain that

$$(15) \quad \delta_x \frac{dy}{dx} > 0, \quad \delta_y \frac{dy}{dx} < 0,$$

which, incidentally, can be obtained directly from first principles of the calculus of variations.

In other words, if $a b c$ represents the elements of an indifference line, and $a' b'$ those of another, the slope α' of $a' b'$ referred to ox is greater than the slope α of $a b$ and less than the slope β of $b c$.

11. Characteristics of the indices deduced from those of the indifference lines. Suppose if necessary that the equation already has an integrating factor. Then

$$\frac{\partial \Psi}{\partial x} = \psi_x, \quad \frac{\partial \Psi}{\partial y} = \psi_y;$$

and

$$I = \Psi$$

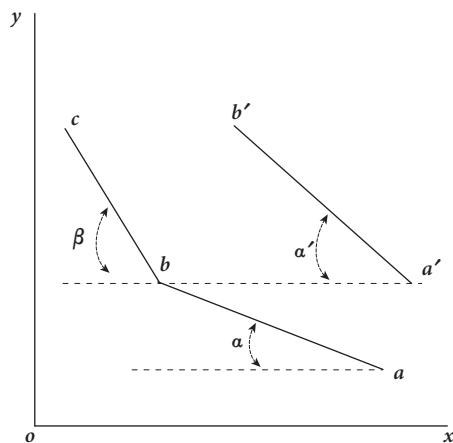


FIG 59

furnishes a system of indices.

(1) According to the first property of the indifference lines, dx and dy must have opposite signs; hence, by virtue of (6), ψ_x and ψ_y must have the same sign, which can be chosen to be positive. The first property of the indices (IV, 32), which corresponds to the first property of the indifference lines, is thus given by

$$(16) \quad \frac{\partial I}{\partial x} > 0, \quad \frac{\partial I}{\partial y} > 0.$$

(2) The first of the inequalities (15) may be written as

$$-\frac{\partial}{\partial x} \frac{\psi_x}{\psi_y} > 0;$$

or, denoting the second derivatives of Ψ by ψ_{xx} , etc., we obtain the first of the following inequalities, and the second one is obtained similarly from the second inequality of (15):

$$(17) \quad \begin{cases} \psi_{xx}\psi_y - \psi_{xy}\psi_x < 0, \\ \psi_{yy}\psi_x - \psi_{xy}\psi_y < 0. \end{cases}$$

In the case in which the system of indices is such that

$$\psi_{xy} = 0,$$

the preceding inequalities become

$$(18) \quad \psi_{xx} < 0, \quad \psi_{yy} < 0,$$

yielding the second property of these indices (IV, 33).

(3) Let us see what the second property of the indifference curves corresponds to. Putting

$$y' = \frac{dy}{dx}, \quad y'' = \frac{d^2y}{dx^2}, \quad \dots,$$

and differentiating along an indifference curve, the equation

$$(19) \quad \psi_x + y'\psi_y = 0;$$

yields

$$y''\psi_y = -\psi_{xx} - 2\psi_{xy}y' - \psi_{yy}y'^2,$$

or

$$y''\psi_y^3 = -\psi_{xx}\psi_y^2 + 2\psi_{xy}\psi_x\psi_y - \psi_{yy}\psi_x^2.$$

Hence, by virtue of (13),

$$(20) \quad -\psi_{xx}\psi_y^2 + 2\psi_{xy}\psi_x\psi_y - \psi_{yy}\psi_x^2 > 0.$$

But this does not furnish a new property of the indices, because this inequality follows simply from (17).

12. Another characteristic of ophelimity. We have seen (IV, 42) that when the ophelimities of X and Y are independent—or even, in general and for large numbers, when there is a dependence of the first kind between them—it may be considered that the elementary ophelimity of a commodity composed of X and Y in arbitrary proportions decreases as the total quantity increases.

This means that if we set

$$y = ax,$$

and if we consider a commodity obtained with x of X and y of Y, the quantity

$$d^2\Phi = \varphi_{xx}dx^2 + 2\varphi_{xy}dxdy + \varphi_{yy}dy^2$$

will always have to be negative for all positive values of x ; that is, we shall have to have

$$\varphi_{xx} + 2a\varphi_{xy} + a^2\varphi_{yy} < 0.$$

Hence, as is well known, we must have

$$(21) \quad \varphi_{xx}\varphi_{yy} - \varphi_{xy}^2 > 0.$$

When the ophelimities of X and Y are independent, so that

$$\varphi_{xy} = 0, \quad \varphi_{xx} < 0, \quad \varphi_{yy} < 0,$$

the inequality (21) is satisfied, and we therefore do not have a new characteristic of ophelimity.

But in the case of dependence of the first kind, in which

$$\varphi_{xy} > 0,$$

the inequality (21) does give us a new characteristic of ophelimity (§49).

As is well known, this inequality tells us that the *indicatrix* of the ophelimity surface is an ellipse. Besides, in the limit this surface can be a plane.

In the case of dependence of the second kind (IV, 41), if we have

$$\varphi_{xy} < 0,$$

it will be seen that, when dx and dy have the same sign, we always have

$$d^2\Phi < 0;$$

but, when they have opposite signs, we are unable to draw any conclusions.

13. Characteristics of the indifference curves deduced from those of ophelimity.
Let us now carry out the operation inverse to the preceding one. Let us assume that we know the ophelimity

$$I = \Phi,$$

and let us derive from it the characteristics of the indifference curves

$$(22) \quad \varphi_x dx + \varphi_y dy = 0.$$

(1) From the first property of ophelimity (IV, 32) we can at once derive the first property of the indifference curves.

(2) Equation (22), if treated like equation (19), yields an equation similar to the one thereby obtained in which the ψ 's are replaced by the φ 's, that is,

$$(23) \quad y''\varphi_y^3 = -\varphi_{xx}\varphi_y^2 + 2\varphi_{xy}\varphi_x\varphi_y - \varphi_{yy}\varphi_x^2.$$

When the consumption of the commodities is independent, we have

$$(24) \quad \varphi_{xy} = 0;$$

and since the second property of ophelimity (IV, 33) yields

$$(25) \quad \varphi_{xx} < 0, \quad \varphi_{yy} < 0,$$

we see from (23) that we have

$$(26) \quad y'' > 0;$$

which yields the second property of the indifference curves (§10).

But, if the consumptions of the commodities are not independent, equation (24) no longer holds, and we have to take account of the value of φ_{xy} .

(1) *First kind of dependence* (IV, 9). In both cases (α) and (β) we have, according to what was said above (IV, 40),

$$\varphi_{xy} > 0;$$

hence this inequality⁶ combined with (25) and (23) shows that (26) still holds.

This case and the preceding one can be treated together, and we can at once deduce the inequality⁶ (26) from (23) and (21).

(2) *Second kind of dependence* (IV, 14). According to what was said above (IV, 41), we have

$$\varphi_{xy} < 0;$$

hence nothing can be concluded from equation (23) concerning the sign of y'' , and we must have recourse to direct observation of the indifference lines.

14. When there are several economic goods X, Y, Z, ..., the consumption of which is independent, we compare Y, Z ... with X and obtain indifference curves precisely as in the case of two goods.

The ophelimity index is

$$(28) \quad I = \Phi(x, y, z \dots),$$

x, y, z , being independent variables. The elementary ophelimity of X depends solely on x ; that of Y on y , etc.; that is,

$$(29) \quad \frac{\partial I}{\partial x} = \varphi_x(x), \quad \frac{\partial I}{\partial y} = \varphi_y(y), \quad \dots$$

For the first kind of dependence (α) (IV, 10), we may still compare Y, Z ... to X, and we have an equation like (28); but, instead of equations (29), we have

$$\frac{\partial I}{\partial x} = \varphi_x(x, y, z, \dots), \quad \frac{\partial I}{\partial y} = \varphi_y(x, y, z, \dots).$$

In many cases, we observe that the elementary ophelimity of X varies considerably with x and slightly with y, z, \dots ; and similarly for those of Y, Z, We may therefore approximately, and within certain limits, put

$$\frac{dI}{dx} = \varphi_x(x_0, y_0, z_0, \dots), \quad \frac{dI}{dY} = \varphi_y(x_0, y_0, z_0, \dots),$$

x_0, y_0, \dots being constant values; and so we fall back again to the previous case (IV, 11).

15. For dependence of the first kind (β), we have goods that are strictly complementary (IV, 12). If, for example, X cannot be used except with a double quantity of Y, we shall have

$$y = 2x;$$

one of the variables in (28) therefore vanishes. Similarly, with the relations that may subsist among x, y, z, \dots , some of those variables are eliminated in (28); the remaining ones may be considered as independent, and we come back to the preceding cases.

When the goods are not strictly complementary, we may approximately, within certain limits, consider them as having a dependence of the first kind (α) (IV, 12).

Let us study the dependence of the second kind (IV, 16). It differs from the dependence of goods that are strictly complementary in that for these latter goods, given the quantity of one of the goods the quantities of the others are also determined; whereas for the dependence of the second kind these quantities are determined only within certain limits. For example, given the number of table knives x , the number of handles y is also determined, that is, $x = y$. On the other hand, if we assume that a man is to subsist by eating a certain amount x of bread, and a certain amount y of cornmeal, we shall find, say, that x can vary between zero and one, while y varies between two and zero, all intermediate combinations being possible. The considerations advanced for the first kind of dependence of the first type (α) hold for those combinations. At the limits of the combinations we must consider the hierarchies of commodities (IV, 19).

16. From what has been said so far it will be seen how difficult it is to apply mathematical analysis to the problem of ophelimities in general.³ The difficulty is also due to the fact that analysis does not lend itself to the treatment of discontinuous functions of the kind needed to represent ophelimity.

Let us look at a very simple case such as that of (IV, 55); the algebraic representation of the broken line $a c \beta$ in Figure 31 can be obtained formally, but this is not of much help because the case is difficult to handle by means of mathematical analysis; all the more so since we must take account of the fact that we can only use the straight line segments $a c$ and $c \beta$, and not their prolongations. Even more difficult would be the analytical

³ In the *Giornale degli Economisti*, Rome, September 1904, Professor Boninsegni has published an excellent study in which he investigates "the functions of demand and supply in the case of barter, assuming the elementary ophelimities to be linear."

representation of the case of Figure 31 (IV, 57) which nevertheless appears to be so simple geometrically.

The conclusion from all this is that there is no point in trying to consider the problem in its full extent; the problem can be studied only in a small region round a certain point (IV, 67), and it is necessary to substitute approximate functions for those that would strictly represent the ophelimities, and which, it should be added, are absolutely unknown to us.

17. Instead of indifference lines, we can use other lines to represent the individual's tastes. Let us assume that we have a certain combination $x_1 y_1$, and let us consider all those adjacent to it given by $x_1 + dx_1, y_1 + dy_1$, such that

$$dx_1^2 + dy_1^2 = d\rho^2.$$

Let us try to find the combination the individual prefers. Then from that combination let us pass to another one, and so on. We will thus have a *preference* line. These lines are those with the maximum gradient on the ophelimity surface and are normal to the indifference lines (§19).

18. We speak about the obstacles of the second kind (III, 73). Let us assume that we are given the line that is followed in the transformations (III, 74) whose equation will be

$$(30) \quad f(x_1, y_1, \mu) = 0,$$

μ being a parameter which, if made to vary, yields some kind of curve. Let us assume that the obstacles of the second kind constrain us to follow this kind of curve. Let us choose one of these curves at random, for example, the one for which $\mu = \mu_1$; equilibrium will take place at the point of tangency of one of these curves and an indifference curve (III, 94), that is, it will be determined by the equations

$$(31) \quad f_x dx + f_y dy = 0, \quad \varphi_x dx + \varphi_y dy = 0;$$

in which, as usual,

$$f_x = \frac{\partial f}{\partial x}, \quad f_y = \frac{\partial f}{\partial y}.$$

From equation (31) we obtain

$$(32) \quad \varphi_x f_y - \varphi_y f_x = 0.$$

This equation and equation (30) determine the coordinates x, y of the equilibrium point.

19. If only the kind of transformation line is specified, it remains still to determine μ , and therefore another equation is needed. But in any case great care must be taken in deriving equation (30) to obtain equation (31) to see that μ is treated as a constant, since equilibrium takes place along one of those lines, which is then determined by the other conditions of the problem.

If μ remains variable, equation (32) gives us a class of curves that could serve in place of the indifference lines or preference lines to determine an individual's tastes.

20. In the case of exchange⁷ between two individuals 1 and 2, if we designate by the indices 1 and 2 the amounts referring to these individuals, the condition that what is received by one is given up by the other is expressed by

$$(33) \quad x_1 + x_2 = X, \quad y_1 + y_2 = Y;$$

X and Y being constants. Equation (30) becomes

$$(34) \quad f_1(x_1, y_1, \mu) = 0,$$

for the first individual, and

$$f_1(X - x_2, Y - y_2, \mu) = 0$$

for the second. This is not a new equation, since it is simply a consequence of (33) and (34). Let us put

$$f_1(X - x_2, Y - y_2, \mu) = f_2(x_2, y_2, \mu)$$

and denote the left side of (34) by f_1 . If each individual follows along the line (30) without bothering about anything else except to arrive at a point of equilibrium *on that line*—that is, in the case of free competition—we shall have for each individual an equation similar to (32), that is

$$(35) \quad \varphi_{1x}f_{1y} - \varphi_{1y}f_{1x} = 0, \quad \varphi_{2x}f_{2y} - \varphi_{2y}f_{2x} = 0.$$

The problem is solved by these equations, (34), and (33); in all, there are five equations that determine the five unknowns: x_1, y_1, x_2, y_2, μ .

21. Equation (30) is generally given in another form, that is, we are given a relation involving the price p of X in terms of Y ,

$$(36) \quad p = -\frac{dx}{dy}.$$

The price must be the same for the two individuals; hence, going back to the previous example, we must have

$$\frac{dx_1}{dy_1} = \frac{dx_2}{dy_2};$$

this equation must hold not only at the point of equilibrium but also along the path followed in the exchange.⁸

Let us assume that the price has the form

$$(37) \quad p = \mu_1 + h_1 y_1 = \mu_2 + h_2 y_2.$$

Taking account of (33), we obtain

$$\mu_1 + h_1 y_1 = \mu_2 + h_2 (Y - y_1) = 0;$$

and since this equation must hold whatever the value of y_1 , it splits up into the following two equations:

$$(38) \quad h_2 = -h_1, \quad \mu_2 = \mu_1 + Y h_1;$$

it therefore remains to determine only the two constants μ_1, h_1 .

Let us denote by $x_{10}, x_{20}, y_{10}, y_{20}$, the initial values of x_1, x_2, y_1, y_2 , and integrate the equations for the price; in order to obtain the functions previously denoted by f_1, f_2 , we shall have

$$\begin{aligned} f_1 &= x_1 - x_{10} + \mu_1 (y_1 - y_{10}) + \frac{h_1}{2} (y_1^2 - y_{10}^2) = 0 \\ f_2 &= x_2 - x_{20} + \mu_2 (y_2 - y_{20}) + \frac{h_2}{2} (y_2^2 - y_{20}^2) = 0. \end{aligned}$$

These two equations define a single path for the exchange,⁸ by virtue of equations (37), (33), and (38). One of these equations is the consequence of the others and must be suppressed.

The equilibrium will therefore be determined by the equations

$$(39) \quad \begin{cases} \varphi_{1x} f_{1y} - \varphi_{1y} f_{1x} = 0, & \varphi_{2x} f_{2y} - \varphi_{2y} f_{2x} = 0, \\ f_1 = 0, & f_2 = 0, \quad y_1 + y_2 = Y. \end{cases}$$

These are five equations which determine the four quantities x_1, x_2, y_1, y_2 , and one of the two constants μ_1, h_1 . The other constant must therefore be given or be determined by some new condition. As for the constants μ_2, h_2 , they are determined by (38).

If the prices are constant, equations (37) become

$$p = \mu_1 = \mu_2;$$

and the equilibrium is determined by the system (39). The five equations of that system determine the quantity and the price.

No account has been taken of the first equation of (33) because, as will be seen in §28, it is a consequence of the others.

Should we wish to retain it, we would have to suppress one of the equations of which it is a consequence, whence for example the equilibrium would also be determined by the system

$$(40) \quad \begin{cases} \varphi_{1x}f_{1y} - \varphi_{1y}f_{1x} = 0, & \varphi_{2x}f_{2y} - \varphi_{2y}f_{2x} = 0, \\ f_1 = 0, & x_1 + x_2 = X, \quad y_1 + y_2 = Y. \end{cases}$$

22. It should be noted that the first equation of (31), and equation (36), give

$$p = f_{1y} : f_{1x} = f_{2y} : f_{2x},$$

whence (32) becomes

$$(41) \quad \varphi_{1x} = \frac{1}{p}\varphi_{1y}, \quad \varphi_{2x} = \frac{1}{p}\varphi_{2y};$$

and this is the form in which it is generally given. We could find it directly, by observing that, in equilibrium, the price along the [exchange line]⁹ at the point of equilibrium must also be the price along the indifference line which passes through that point.

Equations (41) express the equality of the weighted ophelimities at the point of equilibrium.

23. It should be noted that equations (41) hold only at the point of equilibrium. Hence: (1) The price appearing in these equations is that subsisting at the equilibrium point. If the price is constant, it is obviously not different at the point of equilibrium and at an arbitrary point on the path followed in the exchange;¹⁰ but, if it is variable, it can be and generally is different. (2) We cannot consider equations (41) as partial differential equations in $\varphi_{1x}, \varphi_{1y}, \varphi_{2x}, \varphi_{2y}$, and integrate them, since they hold only for particular values of the variables. Such observations are so elementary that they would really be superfluous had they not been forgotten by quite a few authors.

Equations (33) and (34) are of a different nature from equations (41), because they hold for any values of the variables along the path followed in the exchange and not just for the particular values of the equilibrium point alone, as in (41). Hence, these equations can be differentiated with respect to x_1, y_1, x_2, y_2 . We must distinguish carefully whether they are differentiated along the [exchange path]¹¹ or in passing from one of these paths to another. In the case in which the exchange path is given by a constant price, when we differentiate in passing from one path to another, the price must be made to vary. This observation, too, is so elementary that it would be absolutely superfluous had it not been forgotten by a certain author¹² who even imagined that in the *Cours* I made a mistake in differentiating along the [exchange path],¹¹ assuming p to be constant; and in order to lend credence to this and similar observations and discoveries, he dug up Weierstrass¹³ who, poor soul, has really not the slightest connection with all this business and who never committed such a blunder in his whole life.

24. In the very general case in which we consider prices that do not vary with the quantities x, y , we have now seen that the equation

$$f = 0$$

took the form

$$(42) \quad x - x_0 + \mu(y - y_0) = 0.$$

This represents a straight line and the parameter μ is equal to p .¹⁴

25. Let us return to the case in §20 and assume that individual 1 continues to follow one of the lines (30) without caring about anything else except to arrive at the equilibrium point, but that individual 2 has the power to change the value of the coefficients that determine the path (the case of monopoly), and that this possibility is used in order to obtain certain advantages. In the case of constant prices which we shall now consider, the coefficient that determines the path is the price.

In the system (40), we have to suppress the second equation, which holds only when individual 2 does not have the power to impose the path to be followed, and we thus have four equations

$$(43) \quad \begin{cases} \Phi_{1x}f_{1y} - \Phi_{1y}f_{1x} = 0, \\ f_1 = 0, \quad x_1 + x_2 = X, \quad y_1 + y_2 = Y. \end{cases}$$

with which we shall be able to determine the four quantities when the price is known.

To determine the price, we must see what advantages individual 2 wishes to obtain. Let us continue to denote by d the variations along one of the curves (30), while μ remains constant; and let us denote by δ the variations [that take place] when we pass from one curve to the other, that is, by making p vary. From equations (43), we obtain the values of $\delta x_2, \delta y_2$, as functions of $\delta\mu$, and we can put

$$(44) \quad \delta x_2 = m_x \delta\mu, \quad \delta y_2 = m_y \delta\mu.$$

(1) If [individual] 2 wishes to obtain maximum ophelimity, we must put

$$\varphi_{2x}\delta x_2 + \varphi_{2y}\delta y_2 = 0;$$

and, by virtue of equations (44), we have

$$(45) \quad \varphi_{2x}m_x + \varphi_{2y}m_y = 0.$$

This equation replaces the second one of (40). Equations (43) and (45) are five in number and determine the five unknowns.

(2) If individual 2, in selling some Y , wishes to procure the maximum quantity of X , we must put $\delta x_2 = 0$, or, by virtue of (41),

$$(46) \quad m_x = 0.$$

This equation replaces the second one of (35), and, combined with (43), gives five equations to determine the five unknowns x_1, y_1, x_2, y_2, μ .

26. Let us assume that we have several individuals: 1, 2, 3 ... and several commodities: $X, Y, Z \dots$ and, to treat the most general case, let us assume that the prices p_y, p_z, \dots of the commodities are variable, still remaining the same for the various individuals.

Let the number of individuals be θ and that of the commodities be m .

For individual 1, the quantities at any moment of the exchange are

$$x_1, y_1, z_1, \dots;$$

at the beginning they are

$$x_{10}, y_{10}, z_{10}, \dots;$$

at the end they are

$$x'_1, y'_1, z'_1, \dots;$$

and similarly for the other individuals.

Let us put

$$(47) \quad X = x_1 + x_2 + \dots, \quad Y = y_1 + y_2 + \dots;$$

since the total quantities remain constant in the exchange, we must have

$$(48) \quad X = X_0 = X', \quad Y = Y_0 = Y', \quad \dots.$$

These are the equations that characterize exchange, and they may also be written in the following way

$$(49) \quad \left\{ \begin{array}{l} x_1 - x_{10} + x_2 - x_{20} + \dots = 0, \\ y_1 - y_{10} + y_2 - y_{20} + \dots = 0, \\ \dots \end{array} \right.$$

27. Let

$$f_1 = 0$$

be the equation of the path followed by individual 1 in the exchange. In the usual conditions of exchange, f_1 must be a function only of x_1, y_1, z_1, \dots , and not of x_2, y_2, z_2, \dots ; since, if it were a function of these variables, it would vary with them, and hence the quantity of X possessed by [individual] 1 would vary not only for the exchanges carried out by him but also for those carried out by others. This would be the case, for example, in which [individual] 1 levied a tribute on the exchanges of others. Hence, in the case of pure exchange the path followed by each individual must be given by an equation between the quantities that refer exclusively to that individual. In addition, the prices, although variable at successive moments of the exchange, are, at the same moment, the same for all individuals.

The prices of Y for the first, second, ... individual are

$$-\frac{\partial x_1}{\partial y_1} = f_{1y} : f_{1x}, \quad -\frac{\partial x_2}{\partial y_2} = f_{2y} : f_{2x}, \quad \dots;$$

these must all be equal; and similarly for the prices of commodities Z, U, ...; whence we have

$$(50) \quad \begin{cases} f_{1y} : f_{1x} = f_{2y} : f_{2x} = \dots \\ f_{1z} : f_{1x} = f_{2z} : f_{2x} = \dots \end{cases}$$

These equations and (47) must hold for all values of the variables, and this condition will serve to determine some of the coefficients.

28. We have in general

$$f_{1x} \frac{\partial x_1}{\partial y_1} dy_1 + f_{1y} dy_1 = 0, \quad f_{2x} \frac{\partial x_2}{\partial y_2} dy_2 + f_{2y} dy_2 = 0.$$

Summing, and taking account of equations (50), we obtain

$$(51) \quad \frac{dx_1}{dy_1} dy_1 + \frac{dx_2}{dy_2} dy_2 + \dots + \frac{f_{1y}}{f_{1x}} (dy_1 + dy_2 + \dots) = 0.$$

But, differentiating equations (49) we obtain

$$\begin{aligned} \frac{dx_1}{dy_1} dy_1 + \frac{dx_2}{dy_2} dy_2 + \dots &= 0, \\ dy_1 + dy_2 + \dots &= 0. \end{aligned}$$

.....

When the second of these equations is satisfied, so is the first one by virtue of (51). It follows from this that the first equation of (49) is a consequence of the others and of the equations of the paths followed,

$$(52) \quad f_1 = 0, \quad f_2 = 0, \quad \dots;$$

and hence it must be suppressed.

If the price of X, for example, depends on the quantities consumed of Y, Z, ... ; and the price of Y depends on the quantities of X, Z, ... ; the price of X will vary according to whether one begins by consuming X and then Y or vice versa. Since consumption is independent of the order in which it is effected, this must not take place. It is therefore appropriate to assume that p_x is a function only of the quantity of X consumed, p_y a function only of the quantity of Y, and so on. Otherwise, we must provide the conditions that fix the order of consumption.

29. So as not to digress too far, let us assume that the equations of the paths followed have the form

$$x_1 - x_{10} + \mu_1(y_1 - y_{10}) + \frac{h_1}{2}(y_1^2 - y_{10}^2) + v_1(z_1 - z_{10}) + \frac{k_1}{2}(z_1^2 - z_{10}^2) + \dots = 0$$

so that equations (50) become

$$(53) \quad \left\{ \begin{array}{l} \mu_1 + h_1 y_1 = \mu_2 + h_2 y_2 = \dots \\ v_1 + k_1 z_1 = v_2 + k_2 z_2 = \dots \\ \dots \end{array} \right.$$

For these equations and (47) to determine the variables, we must have

$$(54) \quad \left\{ \begin{array}{l} \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} + \dots = 0, \\ \frac{\mu_1}{h_1} + \frac{\mu_2}{h_2} + \dots = -Y, \\ \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots = 0, \\ \frac{v_1}{k_1} + \frac{v_2}{k_2} + \dots = -Z, \\ \dots \end{array} \right.$$

There are in all $2(m-1)$ equations; and $2\theta(m-1)$ coefficients. It remains therefore to determine $2(\theta-1)(m-1)$ coefficients.

Owing to equations (54), the second equation of (47) and the first row of equations in (53) furnish a system of equations of which one is the consequence of the others; one of these equations must therefore be suppressed. Similar observations must be made for the third equation of (47) and the second row of equations (53), etc. The equilibrium will therefore be determined by the system

$$(a) \quad \left\{ \begin{array}{l} \varphi_{1x} = \frac{f_{1x}}{f_{1y}} \varphi_{1y} = \frac{f_{1x}}{f_{1z}} \varphi_{1z} = \dots, \\ \varphi_{2x} = \frac{f_{1x}}{f_{1y}} \varphi_{2y} = \frac{f_{1x}}{f_{1z}} \varphi_{2z} = \dots, \\ \dots \end{array} \right.$$

$$(\beta) \quad f_1 = 0, \quad f_2 = 0, \quad \dots,$$

$$(\gamma) \quad \left\{ \begin{array}{l} y'_1 - y_{10} + y'_2 - y_{20} + \dots = 0 \\ z'_1 - z_{10} + z'_2 - z_{20} + \dots = 0 \\ \dots \end{array} \right.$$

$$(d) \quad \begin{cases} \mu_2 + h_2 y'_2 = \mu_3 + h_3 y'_3 = \dots \\ v_2 + k_2 z'_2 = v_3^{15} + k_3 z'_3 = \dots \\ \dots \dots \dots \dots \dots \end{cases}$$

or by the other system consisting of equations (α) , (β) , to which are added equations (53): equations (γ) remain suppressed.

One way or another, the number of equations in the system is

$$2(\theta - 1)(m - 1) + \theta + m - 1.$$

There remain to be determined $2(\theta - 1)(m - 1)$ coefficients and $m\theta$ quantities; hence in all we have

$$2(\theta - 1)(m - 1) + m\theta$$

unknowns. Therefore, with the equations available we can determine the $m\theta$ quantities, and there will remain

$$m\theta - \theta - m + 1 = (\theta - 1)(m - 1)$$

coefficients that must be given, or that will have to be determined with other conditions.

30. Constant prices. When the prices are constant, equations (53) indicate only that there is a constant price p_y which is the same for every individual, and similarly for p_z, p_u, \dots . Equations (52) become

$$\begin{aligned} x_1 - x_{10} + p_y(y_1 - y_{10}) + p_z(z_1 - z_{10}) + \dots &= 0 \\ x_2 - x_{10} + p_y(y_2 - y_{20}) + p_z(z_2 - z_{20}) + \dots &= 0 \\ &\vdots \end{aligned}$$

¹⁶ and equilibrium is determined by the following system of equations¹⁶

$$(A) \quad \left\{ \begin{array}{l} \varphi_{1x} = \frac{1}{p_y} \varphi_{1y} = \frac{1}{p_z} \varphi_{1z} = \dots \\ \varphi_{2x} = \frac{1}{p_y} \varphi_{2y} = \frac{1}{p_z} \varphi_{2z} = \dots \\ \dots \end{array} \right.$$

$$(B) \quad \begin{cases} x'_1 - x_{10} + p_y(y'_1 - y_{10}) + \dots = 0 \\ x'_2 - x_{20} + p_y(y'_2 - y_{20}) + \dots = 0 \\ \dots \end{cases}$$

$$(C) \quad \begin{cases} y'_1 - y_{10} + y'_2 - y_{20} + \dots = 0 \\ z'_1 - z_{10} + z'_2 - z_{20} + \dots = 0 \\ \dots \end{cases}$$

There are in all $m\theta + m - 1$ equations that serve to determine the $m\theta$ quantities $x'_1, x'_2, \dots, y'_1, y'_2, \dots$, and the $m - 1$ prices $p_y, p_z, \dots; p_y$ being the price of Y, and p_z the price of Z, etc.

These equations correspond to the categories (A), (B), (C) of conditions indicated in chapter III, §199 et seq. If to (C) for the sake of symmetry we add the equation relating to the x 's, we have in the system (A), (B), (C) one equation too many, which is a consequence of the others.

31. Production. Let us assume that certain commodities (or capital services) A, B, C,...are transformed into commodities X, Y,

If consumption had to be equal to production at every moment, it would not be possible to follow any other path but the one given by production. On the other hand, in order to come closer to reality, we shall assume that the producer has a certain stock of commodities which neither increases nor decreases at the end of the process being considered, and by virtue of which the path followed can be arbitrary, for example, that of constant prices.

The number of individuals is θ , the number of commodities X, Y, ..., is m , and that of commodities A, B, ..., is n .

The quantities $x_1, x_{10}, \dots, x'_1, \dots, a_1, a_{10}, a'_1, \dots$, have a meaning analogous to that set out in §26.

In the case of monopoly, the final quantities $X', Y' \dots$ consumed by the firm's customers can be smaller than the quantities transformed, the surplus being consumed by the firm. In the case of free competition, these quantities must be equal.

In one case or the other, the intermediate amounts produced need not correspond to the amounts consumed. They would so correspond if the path of complete transformations were followed. Otherwise we have to distinguish between the intermediate amounts consumed and the amounts produced. For the former we shall reserve the notation X, Y, \dots ; we shall denote the latter by x, y, \dots . Arriving at the equilibrium position, in the case of free competition, these quantities will have to be equal, and hence for the equilibrium position but not for the intermediate positions along the paths followed to reach them, we shall have

$$(55) \quad X = x = X', \quad Y = y = Y' \dots$$

32. In the case of exchange,⁷ the quantities of commodities remain constant; in the case of production, they vary by means of the transformations. We shall have

$$(56) \quad \left\{ \begin{array}{l} x_1 + x_2 + \dots = X \\ y_1 + y_2 + \dots = Y \\ \dots \\ a_1 + a_2 + \dots = A \\ \dots \end{array} \right.$$

and

$$(57) \quad \left\{ \begin{array}{l} x_{10} + x_{20} + \dots = X_0 \\ y_{10} + y_{20} + \dots = Y_0 \\ \dots \dots \dots \dots \dots \dots \\ a_{10} + a_{20} + \dots = A_0 \\ \dots \dots \dots \dots \dots \dots \end{array} \right.$$

It will be helpful if we take the price of A to be equal to unity in order to avoid discussing two different problems at the same time, that is, the problem of production and in addition the way in which prices are affected by variations in the cost of production of a commodity X which has been chosen as money.

Since production is a transformation of A, B, \dots into X, Y, \dots , the first quantities decrease and the second increase when production is completed, that is,

$$a_1 < a_{10}, \quad b_1 < b_{10}, \quad \dots, \quad x_1 > x_{10}, \quad \dots$$

In terms of A, the prices are

$$(58) \quad \left\{ \begin{array}{l} p_x = -\frac{\partial a_1}{\partial x_1} = -\frac{\partial a_2}{\partial x_2} = \dots \\ \dots \dots \dots \dots \dots \dots \\ p_b = -\frac{\partial a_1}{\partial b_1} = -\frac{\partial a_2}{\partial b_2} = \dots \end{array} \right.$$

For greater simplicity, we shall write the equations of the paths in the form

$$(59) \quad a_1 - a_{10} = f_1, \quad a_2 - a_{20} = f_2, \dots$$

Equations (58) become

$$(60) \quad \left\{ \begin{array}{l} f_{1x} = f_{2x} = f_{3x} = \dots = -p_x, \\ \dots \dots \dots \dots \dots \dots \\ f_{1b} = f_{2b} = f_{3b} = \dots = -p_b. \end{array} \right.$$

For the reasons indicated in §28, f_{1x} must be a function solely of x_1 , f_{2x} must be a function solely of x_2 , and so on. The first equation of (55) and the first set of equations of (60) therefore make it possible to express x_1, x_2, \dots as functions of X ; and similarly we shall have y_1, y_2, \dots as functions of Y , and so on. Substituting these values in equations (60), f_{1x}, f_{2x}, \dots will be functions of X ; and so on.

Differentiating (59), we have

$$\frac{\partial a_1}{\partial X} = f_{1x} \frac{\partial x_1}{\partial X}, \quad \frac{\partial a_2}{\partial X} = f_{2x} \frac{\partial x_2}{\partial X}, \quad \dots$$

.....

Summing the equations of each row and taking account of (56), we obtain

$$\frac{\partial A}{\partial X} = f_{1x} = -p_x, \quad \frac{\partial A}{\partial Y} = f_{1y} = -p_y, \quad \dots;$$

as *a priori* it could be seen had to be the case.

Differentiating (59) totally, we shall have

$$da_1 - f_{1b}db_1 - \dots = f_{1x}dx_1 + \dots,$$

$$da_2 - f_{2b}db_2 - \dots = f_{2x}dx_2 + \dots,$$

.....

Summing and taking account of (60) and of (56), we shall have

$$(61) \quad dA - f_{1b}dB - \dots = f_{1x}dX + \dots,$$

or

$$(61 \text{ bis}) \quad dA + p_bdB + \dots + p_xdX + \dots = 0.$$

Since $-f_{1b}$ or p_b is a function solely of B , $-f_{1x}$ or p_x is a function solely of X , etc., we shall have

$$(62) \quad A - A_0 - \int_{B_0}^B f_{1b}dB - \dots = \int_{X_0}^X f_{1x}dX + \dots,$$

$$(62 \text{ bis}) \quad A - A_0 + \int_{B_0}^B p_bdB + \dots + \int_{X_0}^X p_xdX + \dots = 0.$$

The latter formula expresses the budget of the consumers, that is, it expresses the fact that total income is equal to total expenditure.

For a reason already explained in §32, we have

$$A < A_0, \quad B < B_0, \quad \dots, \quad X > X_0, \quad \dots.$$

Usually, the quantities X_0, Y_0, \dots are equal to zero; and such we shall assume them to be for the sake of simplicity.

33. The technical conditions [of production] will enable us to determine the quantities of A, B, ... that are needed to produce the quantities of commodities x, y, \dots ; that is, we shall have

$$(63) \quad \begin{cases} A'' = F(x, y, \dots), & B'' = G(x, y, \dots), \dots \\ A'' = A_0 - A, & B'' = B_0 - B, \dots \end{cases}$$

If a_x, b_x, \dots are the production coefficients of commodity X, a_y, b_y, \dots those of commodity Y, and so on, we shall have

$$(64) \quad \left\{ \begin{array}{l} a_x = \frac{\partial F}{\partial x}, \quad b_x = \frac{\partial G}{\partial x}, \quad \dots \\ a_y = \frac{\partial F}{\partial y}, \quad b_y = \frac{\partial G}{\partial y}, \quad \dots \\ \dots \end{array} \right.$$

In the real world, there are many commodities whose production is independent, that is, each of them is produced by a distinct firm. If X, Y, \dots are of this kind, it is essential to establish how these commodities are produced independently of each other. In fact, when for example p_b depends on B , the cost of X will be different depending on whether commodity X is produced before or after commodity Y ; if it is produced before, B can be bought at a lower price than if it is produced afterwards.

Among the various ways of looking at the way production proceeds, the one that approximates real conditions most closely is that of considering p_b, p_c, \dots as depending no longer on B, C, \dots , but only on B', C', \dots ; by this means, those prices are assumed to be constant during the operation of production that leads to the equilibrium point, and varying only with the total quantities of B', C', \dots corresponding to the equilibrium point.

Similarly, we shall assume that a_x, b_x, \dots are functions solely of x ; a_y, b_y, \dots are functions solely of y , and so on.

34. We have assumed that the quantities X, Y, \dots may differ from the quantities x, y, \dots ; on the other hand, the quantities A, B, \dots that the consumers sell are equal to the same quantities that the firm uses up; and, if we do not want to discuss buying and selling—if for example the producer and the consumer are one and the same individual—we assume that he does not consume every particle of the commodity the moment it is produced; but we consider that from his stocks of A, B, C, \dots he draws precisely what he needs for production.

It follows from this that the values (63) must satisfy equations (61) and (61 bis). Substituting them in the first equation, we have

$$(65) \quad a_x dx + a_y dy + \dots - f_{lb}(b_x dx + b_y dy + \dots) - \dots = -f_{lx} dX + \dots$$

Substituting them in the second one, we obtain

$$(65 \text{ bis}) \quad a_x dx + a_y dy + \dots + p_h(b_x dx + b_y dy + \dots) + \dots = p_x dX + \dots$$

The left side of this equation shows us how much the consumers receive, and the right side how much they spend. This corresponds to the case in which the consumer's income is equal to his expenditure at every moment in the production process. If instead this equality holds only over the entire production span, a consumer who has a certain quantity of savings of which he avails himself to offset a possible reduction in earnings at a certain moment by a rise in earnings at another moment, equation (65 bis) no longer holds, but only the equation that is deduced from it by integration. We shall see later on that it is precisely these integrals that appear among the equations that determine the equilibrium; this therefore corresponds to the case that best approximates the real world in which consumers, thanks to their savings, avoid the need to equalize their incomes and expenditures at every single moment of the production process.

Let us now turn to the producers. The sum total that they spend is equal to the amount, just written out, that the consumers receive; and, if

$$\pi_x dx, \pi_y dy, \dots$$

are the costs of production of the quantities of commodities $dx, dy \dots$ we shall have

$$a_x dx + a_y dy + \dots + p_b(b_x dx + b_y dy + \dots) + \dots = \pi_x dx + \pi_y dy + \dots$$

Substituting for $p_b, p_c \dots$ their values, we shall also have

$$a_x dx + a_y dy + \dots - f_{lb}(b_x dx + b_y dy + \dots) - \dots = \pi_x dx + \pi_y dy + \dots$$

And, since the production of the commodities is assumed to be independent, the equalities just written are split up into as many others as there are commodities, that is

$$(66) \quad \left\{ \begin{array}{l} (a_x - f_{lb} b_x - \dots) dx = \pi_x dx, \\ (a_y - f_{lb} b_y - \dots) dy = \pi_y dy, \\ \dots \end{array} \right.$$

$$(66 \text{ bis}) \quad \left\{ \begin{array}{l} (a_x + p_b b_x + \dots) dx = \pi_x dx, \\ (a_y + p_b b_y + \dots) dy = \pi_y dy, \\ \dots \end{array} \right.$$

The sums that the producers draw in from the sale of dX, dY, \dots are equal to those that the consumers spend, and hence they are

$$-f_{lx} dX, -f_{ly} dY, \dots$$

or

$$p_x dX, p_y dY, \dots$$

35. Equilibrium of production. We must relate what the firm does to what consumers do; alternatively, we must relate production to consumption.

In other words, we must determine what relations must hold between x and X , y and Y , etc.

There can be various relations of this kind, and to each of them there correspond certain economic phenomena.

In the case of free competition, as in that of individual production, the initial and final values of x, X, y, Y, \dots must be the same. In the case of monopoly, they may be different.

The same point is expressed in a different way by saying that the firm spends as much on production as it receives in the case of free competition. On the other hand, it receives more than it spends in the case of monopoly. This difference can be maximal or fixed arbitrarily, or again, determined arbitrarily by other conditions.

If production is to be regulated so as to procure maximum ophelimity for consumers, it will first be necessary for the amounts x, X, y, Y, \dots to have the same initial and the same final values, and in addition certain conditions will have to be satisfied for the maximization of ophelimity when the kind of path followed can vary. In other words, such conditions are necessary in addition to those that establish the equality of income and expenditure for production.

In all cases, the conditions can have two forms, that is: (1) They can be expressed by putting the point reached by consumption into relation with the point reached in production. This is accomplished by equations (61), (62) and others of a similar form. (2) They can be expressed by relating the expenditures for consumption to those for production. This is accomplished by equations (61 bis), (62 bis) and others of a similar form.

36. Free competition and variable prices. The condition that the initial values and final values of x, y, \dots and of X, Y, \dots should be equal, and the other condition that the cost of production of each commodity should be equal to the proceeds from the sale of that commodity, are equivalent. The former is expressed by taking account of equations (66), and we have

$$(67) \quad \left\{ \begin{array}{l} \int_{X_0}^{X'} (a_x - f_{1b} b_x - \dots) dx = - \int_{X_0}^{X'} f_{1x} dX, \\ \int_{Y_0}^{Y'} (a_y - f_{1b} b_y - \dots) dx = - \int_{Y_0}^{Y'} f_{1y} dY, \\ \dots \end{array} \right.$$

The second is expressed by taking account of equations (66 bis), and we have

$$(67 \text{ bis}) \quad \int_{X_0}^{X'} (a_x + p_b b_x + \dots) dx = \int_{X_0}^{X'} p_x dX \dots$$

By integrating from X_0 to X' to obtain the cost of production of the commodity, we have implicitly assumed that for X_0 the cost of production is zero, i.e., that nothing has been spent before producing the first small portion of the commodity.

This is not usually the case. There are overhead costs that have to be incurred independently of the amount produced; and we have seen an extremely simple example of this (VI, 4); we must therefore also take account of this case.

As already indicated in §32, we shall assume

$$X_0 = 0, \quad Y_0 = 0, \dots$$

We shall then denote by α_x, β_x, \dots the amounts of A, B, ... that must be used up before producing the first small portion of commodity X; and we shall use similar notations for Y, Z,

In this way, instead of the equations (67 bis) we have the following ones:

$$(67 \text{ ter}) \quad \left\{ \begin{array}{l} \alpha_x + p_b \beta_x + \dots + \int_0^{X'} (\alpha_x + p_b b_x + \dots) dx = \int_0^{X'} p_x dX, \\ \dots \end{array} \right.$$

Summing the [equations] (67), we recover the [equations] (62), and summing the (67 bis) we recover the (62 bis). Hence, if we keep all the equations of (67), we must suppress (62); and if we keep (62), we must suppress one of the equations of (67). The same may be said for (67 bis), (67 ter), and (62 bis).

The equilibrium will be determined: (1) By the equations that express the equality of the weighted ophelimities (equations (A) of §30 in which are included those referring to A, B, ...). (2) By equations (59) which specify the paths followed. (3) By the equations (67 bis) or (67 ter) that equate the costs of production to the sales proceeds. (4) By the equations (63) that yield the amounts of A, B, ... needed for manufacture. (5) By the equations (56) that indicate the sums of the individual amounts. (6) By the equations (60) by virtue of which the price of a commodity is the same for different individuals.

37. Free competition and constant prices. When prices are constant, the last of the categories just noted—which simply indicates that p_x , for example, is the price of X for all individuals—disappears. The other categories give rise to the following equations.

$$(A) \quad \left\{ \begin{array}{l} \frac{1}{p_x} \varphi_{1x} = \frac{1}{p_y} \varphi_{1y} = \dots = \varphi_{1a} = \dots, \\ \frac{1}{p_x} \varphi_{2x} = \frac{1}{p_y} \varphi_{2y} = \dots = \varphi_{2a} = \dots, \\ \dots \end{array} \right.$$

$$(B) \quad \left\{ \begin{array}{l} a'_1 - a_{10} + p_b(b'_1 - b_{10}) + \dots p_x(x'_1 - x_{10}) + \dots = 0, \\ a'_2 - a_{20} + p_b(b'_2 - b_{20}) + \dots p_x(x'_2 - x_{20}) + \dots = 0, \\ \dots \end{array} \right.$$

$$(D) \quad \left\{ \begin{array}{l} p_x(X' - X_0) = \int_{X_0}^{X'} a_x dx + p_b \int_{X_0}^{X'} b_x dx + \dots \\ \dots \end{array} \right.$$

$$(E) \quad A_0 - A' = F, \quad B_0 - B' = G, \quad \dots$$

$$(M) \quad x'_1 + x'_2 + \dots = X', \quad \dots$$

In the case to which the equations (67 ter) correspond, equations (D) become

$$(D \text{ bis}) \quad \left\{ \begin{array}{l} p_x X' = a_x + p_b \beta_x + \dots + \int_0^{X'} a_x dx + p_b \int_0^{X'} b_x dx + \dots, \\ \dots \end{array} \right.$$

If the production coefficients are constant, the systems (D), (E) become

$$(D') \quad \left\{ \begin{array}{l} p_x - a_x + p_b b_x + \dots, \\ p_y - a_y + p_b b_y + \dots, \\ \dots \end{array} \right.$$

$$(E') \quad \left\{ \begin{array}{l} A_0 - A' = a_x(X' - X_0) + a_y(Y' - Y_0) + \dots, \\ B_0 - B' = b_x(X' - X_0) + b_y(Y' - Y_0) + \dots, \\ \dots \end{array} \right.$$

Equations (D') express the equality of the cost of production of a unit of the commodity and the selling price of that unit.

It is easy to verify that if we retain in (D') all the equations for each commodity, we have one too many. In fact, multiplying the second equation of (E') by p_b , the third by p_c , and so on, then summing and taking account of equations (D'), we have

$$A' - A_0 + p_b(B' - B_0) + \dots + p_x(X' - X_0) + \dots = 0.$$

But the same equation is obtained by summing equations (B).

We must never forget that the system of equations just written out holds only for the equilibrium position, and not for the intermediate positions.

38. Equations (A), (B), (D), and (E) correspond to the categories of conditions indicated by these letters in Chapter III, §205 et seq.

If the commodities A, B, ... provide ophelimity to the individuals directly, they appear in equations (A); if they do not, they do not appear in these equations, but in that case the quantities a_1, a_2, \dots instead of being unknowns, are given. In any case it can be verified that the number of distinct equations is equal to the number of unknowns.

Equations	(A)	number	$(m + n - 1)\theta$,
"	(B)	"	θ ,
"	(D)	"	m ,
"	(E)	"	n .

In all we have

$$(m + n)\theta + m + n$$

equations, but one is the consequence of the others, whence there are only

$$(m+n)\theta + m + n - 1$$

equations; and the number of unknowns is the same; these are the $(m+n)\theta$ quantities and the $m+n-1$ prices.

39. Let us turn to the production coefficients. If they are constant, they simply appear among the given quantities. If they vary only as functions of the total quantities produced X, Y, \dots , they are given as functions of those quantities. Finally, if they vary in such a way that an increase in some of them can be offset by a decrease in others, they have to be determined; and this task is the job of the firm. Let us consider the case of competition, that is, the case in which the firm accepts market prices without trying to modify them directly (III, 89; V, 8). Let us assume that the technical conditions of production give us a relation

$$(68) \quad f(a_y, b_y, \dots, e_y) = 0$$

among the coefficients a_y, b_y, \dots, e_y ; the others are assumed to be constant. If they were not, there would be analogous relations to the preceding one, and the reasoning would be the same.

We have seen that for phenomena of type (I) the entrepreneur accepts market prices as they are; he seeks to reduce the cost of production to the minimum, basing his calculations on market prices and the quantities produced (V, 82); which means that in differentiating to obtain the conditions for a minimum, it is necessary to treat the prices and the quantities produced as constants; the condition for minimum cost of production will be

$$(69) \quad 0 = p_a d' a_y + p_b d' b_y + \dots;$$

where d' denotes the variations in the production coefficients.

By virtue of equations (68), a_y may be considered as a function of the independent variables b_y, c_y, \dots ; whence equation (69) yields

$$(70) \quad p_a \frac{\partial a_y}{\partial b_y} + p_b = 0, \quad p_a \frac{\partial a_y}{\partial c_y} + p_c = 0.$$

Differentiating (68) and substituting in the preceding equations, we have

$$(71) \quad p_a \frac{\partial f}{\partial b_y} - p_b \frac{\partial f}{\partial a_y} = 0, \quad \dots$$

These equations, combined with (68), determine the coefficients of production a_y, b_y, \dots . These form part of the category (F) of conditions (V, 82).

In addition, that category contains the conditions for the distribution of the output among firms (V, 78). If, for example, a firm produces q_z of Z, and if that output increases

by dq_z , the cost of production will increase by a certain amount which, when equated to zero, will give the condition of the minimum of that cost; hence

$$(72) \quad 0 = p_a \frac{\partial a_z}{\partial q_z} + p_b \frac{\partial b_z}{\partial q_z} + \dots$$

This and other similar equations determine the quantities q_z, \dots and hence the distribution.

40. Individual production and constant prices. Let us consider an isolated man who consumes what he produces. There is no market. He need only try to make the best possible use of the A, B, ... in order to produce X, Y,

Let us begin by assuming that this man likes to fix the path of consumption in such a way that the f_{1x}, f_{1y}, \dots are constant (constant prices).

It is necessary for him to arrive at the same point for consumption and for production. Hence, equations (61), (62), (68) and other analogous equations hold for him. But we know that they are equivalent to (61 bis), (62 bis), (68 bis) and other analogous equations; hence, for this man the equilibrium will be determined precisely by the same equations that hold in the case of free competition.

41. Individual production with maximum ophelimity. Let us assume that the path to be followed in consumption has not been determined; the individual seeks only to attain the maximum ophelimity.

While the symbols

$$\frac{\partial A}{\partial X}, \dots$$

denote, as before, derivatives taken along any equilibrium path, we shall use the symbols

$$\frac{dA}{dX}, \dots$$

to denote the derivatives taken in passing from one path to another.

Considering successively the transformations that give X, Y, ..., we shall have for the equilibrium position

$$\varphi_x dX' + \varphi_a \frac{dA'}{dX'} dX' + \varphi_b \frac{dB'}{dX'} dX' + \dots = 0$$

.....

or

$$(73) \quad \begin{cases} \varphi_x + \varphi_a \frac{dA'}{dX'} + \varphi_b \frac{dB'}{dX'} + \dots = 0, \\ \varphi_y + \varphi_a \frac{dA'}{dY'} + \varphi_b \frac{dB'}{dY'} + \dots = 0, \\ \dots \end{cases}$$

But also in passing from one consumption path to another, equations (63) always have to be satisfied; and from these we obtain

$$(74) \quad \left\{ \begin{array}{l} \frac{dA'}{dX'} = -a'_x, \quad \frac{dB'}{dX'} = -b'_x, \dots \\ \dots \end{array} \right.$$

a'_x, b'_x, \dots being the values of the production coefficients at the equilibrium point.

Substituting the values (74) in equations (73), we shall obtain

$$(75) \quad \left\{ \begin{array}{l} \varphi_x = a'_x \varphi_a + b'_x \varphi_b + \dots, \\ \varphi_y = a'_y \varphi_a + b'_y \varphi_b + \dots, \\ \dots \end{array} \right.$$

Whatever the path f followed in consumption, the weighted ophelimities will always be equal at the equilibrium point; that is,

$$\frac{1}{p_x} \varphi_x = \frac{1}{p_y} \varphi_y = \dots = \varphi_a = \dots;$$

and hence equations (75) become

$$(76) \quad \left\{ \begin{array}{l} p_x = a'_x + p_b b'_x + \dots \\ p_y = a'_y + p_b b'_y + \dots \\ \dots \end{array} \right.$$

These are the equations that must be satisfied for the path followed to yield maximum ophelimity. If we compare them to (67 bis) or (67 ter), we see that they constitute the equality of the integrands in (67 bis) or (67 ter) respectively at the upper limit of integration. The path to be followed to obtain the maximum ophelimity therefore remains subject to this sole condition. We can then express ourselves in the following two, equivalent, ways.

To obtain equilibrium with maximum ophelimity, not only must the integrals on the left of (67 bis) and (67 ter) satisfy these equations, but the integrand at the upper limit must be equal to $p_x dx^{17}$ and so on.

Or, to put the matter in another way, not only must the cost of production of the entire output of the commodity be equal to the consumption expenditure, but also the cost of production of the last small portion must be equal to the selling price of that last portion.

42. Let us see if the path just determined is compatible with constant prices.

If prices are constant, equations (67 ter) give

$$a'_x X' + p_b b'_x X' + \dots + a_x + p_b \beta_x + \dots$$

$$- \int_0^{X'} x \left(\frac{\partial a_x}{\partial x} + p_b \frac{\partial b_x}{\partial x} + \dots \right) dx = p_x X',$$

.....;

or, taking account of (76), we shall have

$$(77) \quad \begin{cases} \int_0^{X'} x \left(\frac{\partial a_x}{\partial x} + p_b \frac{\partial b_x}{\partial x} + \dots \right) dx = a_x + p_b \beta_x + \dots \\ \int_0^{Y'} y \left(\frac{\partial a_y}{\partial y} + p_b \frac{\partial b_y}{\partial y} + \dots \right) dy = a_y + p_b \beta_y + \dots \\ \dots \end{cases}$$

If these equations are not satisfied, it is not possible to have at one and the same time constant prices and maximum ophelimity in transformations.

If the production coefficients are fixed, so that the quantities $a_x, \beta_x, \dots, a_y, \dots$ are zero, equations (77) are satisfied; and it follows from this that when the production coefficients do not vary with the quantities produced, while the variation studied in §38 and §39 still remains permissible, we obtain, at one and the same time in equilibrium, constant prices and maximum ophelimity in transformations.⁴

If the production coefficients are variable, to obtain maximum ophelimity it would be necessary for equations (77) to be satisfied; or in general, they should depend on the values of X', Y', \dots ; hence, maximum ophelimity would be obtained only for an equilibrium point with these values X', Y', \dots and in general not for any equilibrium point. We can therefore say that, except in singular cases, constant prices yield maximum ophelimity only when the production coefficients are constant.

43. Collective production. Let us imagine a community that wishes to regulate production in the best possible way for its members. Distribution will be carried out according to the rules considered appropriate, distributing certain initial quantities $x_{10}, y_{10}, a_{10}, \dots$ to the members of the community.

44. Let us first of all assume that the consumption paths are given. In that case, reasoning as we did in §40, we shall see that equilibrium is determined precisely as in the case of free competition.

45. Collective production with maximum ophelimity. When the transformations that yield X are made to vary in passing from one path to another, as was done in §41, retaining the notation of that section and denoting by

$$\delta\Phi_1, \delta\Phi_2, \dots$$

⁴ For equilibrium in general, Professor Walras has considered exclusively the case corresponding to this.

the variations in ophelimity for each individual, we shall have

$$(78) \quad \left\{ \begin{array}{l} \delta\Phi_1 = \varphi_{1x}\delta x'_1 + \varphi_{1a}\delta a'_1 + \varphi_{1b}\delta b'_1 + \dots \\ \delta\Phi_2 = \varphi_{2x}\delta x'_2 + \varphi_{2a}\delta a'_2 + \varphi_{2b}\delta b'_2 + \dots \\ \dots \end{array} \right.$$

As usual, equality of the weighted ophelimities yields

$$p_x = \varphi_{1x} : \varphi_{1a} = \varphi_{2x} : \varphi_{2a} = \dots$$

Hence we obtain from (78)

$$(79) \quad \frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots = p_x \delta X' + \delta A' + p_b \delta B' \dots$$

Equations (63) must always be satisfied in passing from one consumption path to another; hence when X' varies by $\delta X'$, the other variations are:

$$\delta A' = -a'_x \delta X', \quad \delta B' = -b'_x \delta X', \quad \dots$$

Substituting in equation (79) we obtain

$$(80) \quad \frac{1}{\varphi_{1a}} \delta \Phi_1 + \frac{1}{\varphi_{2a}} \delta \Phi_2 + \dots = (p_x - a'_x - p_b b' x - \dots) \delta X'.$$

Since the quantities $\varphi_{1a}, \varphi_{2a}, \dots$ are intrinsically positive, it follows that, if the expression

$$\frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots$$

is zero, some of the $\delta\Phi_1, \delta\Phi_2, \dots$ must be positive and some negative; they cannot all be positive or all be negative.

Recalling the definition given (VI, 33) of maximum ophelimity for a community, we see that it is expressed algebraically by the equation

$$(81) \quad \frac{1}{\varphi_{1a}} \delta \Phi_1 + \frac{1}{\varphi_{2a}} \delta \Phi_2 + \dots = 0.$$

Introducing this condition into equation (80) and the other analogous equations for Y, Z, \dots , we revert to equations (76). Hence the conclusions of §41 also hold for a community, with the caution that the maximum ophelimity for that community is defined in the manner referred to (VI, 33).

46. In the case of exchange,⁷ taking commodity X to be money as usual, while for production we assume that money is commodity A, we have

$$\begin{aligned} \frac{1}{\varphi_{1x}} \delta\Phi_1 &= \delta x'_1 + p_y \delta y'_1 + p_z \delta z'_1 + \dots \\ \frac{1}{\varphi_{2x}} \delta\Phi_2 &= \delta x'_2 + p_y \delta y'_2 + p_z \delta z'_2 + \dots \\ &\dots \end{aligned}$$

hence, summing, we have

$$(82) \quad \frac{1}{\varphi_{1x}} \delta\Phi_1 + \frac{1}{\varphi_{2x}} \delta\Phi_2 + \dots = \delta X' + p_y \delta Y' + \dots$$

But, by virtue of the equations (48),

$$\delta X' = 0, \quad \delta Y' = 0, \quad \dots,$$

hence we have

$$\frac{1}{\varphi_{1x}} \delta\Phi_1 + \frac{1}{\varphi_{2x}} \delta\Phi_2 + \dots = 0;$$

and, in exchange with free competition, maximum ophelimity is always attained.⁵

47. Properties of equilibrium. Let us look at the same things in a different way, and confine ourselves to constant prices for the successive portions.

We have seen in §37 that by summing equations (B) we obtain the equation

$$(83) \quad A' - A_0 + p_b(B' - B_0) + \dots + p_x(X' - X_0) + \dots = 0.$$

If in this equation, for the prices p_x, p_y, \dots , which are constant for successive portions, we substitute the new prices

$$p_x + \delta p_x, \quad p_y + \delta p_y, \quad \dots,$$

also constant for successive portions, we shall have

$$(84) \quad \delta U + \delta V = 0,$$

$$(85) \quad \left\{ \begin{array}{l} \delta U = p_x \delta X' + p_y \delta Y' + \dots \\ \quad + \delta A' + p_b \delta B' + \dots, \\ \delta V = (X' - X_0) \delta p_x + \dots \\ \quad + (B' - B_0) \delta p_b + \dots \end{array} \right.$$

⁵ *Giornale degli Economisti*, November 1903, Rome, pp. 410-413.¹⁸

Reasoning as in §45 it is easy to see that we have

$$(86) \quad \delta U = \frac{1}{\varphi_{1a}} \delta \Phi_1 + \frac{1}{\varphi_{2a}} \delta \Phi_2 + \dots$$

The $\delta \Phi_1, \delta \Phi_2, \dots$ of this section are different from the $\delta \Phi_1, \delta \Phi_2, \dots$ of §45.

For the maximum ophelimity to be attained, the expression (86) must be zero; and that will follow by virtue of equation (84) when

$$(87) \quad \delta V = 0.$$

This is then the condition that must be met in order to obtain maximum ophelimity.

Equation (87) is always verified for exchange and free competition, since we have

$$X' - X_0 = 0, \quad Y' - Y_0 = 0, \quad \dots$$

For production, the equations (D) of §37 give

$$(X' - X_0) \delta p_x + p_x \delta X' = (a'_x + p_b b'_x + \dots) \delta X' + \delta p_b \int_{X_0}^{X'} b_x dx + \dots$$

.....

Summing all these equations and taking account of the fact that

$$B_0 - B' = \int_{X_0}^{X'} b_x dx + \int_{Y_0}^{Y'} b_y dy + \dots,$$

we shall have

$$\begin{aligned} (X' - X_0) \delta p_x + (Y' - Y_0) \delta p_y + \dots + (B' - B_0) \delta p_b + \dots + p_x \delta X' p_y \delta Y' + \dots \\ = (a'_x + p_b b'_x + \dots) \delta X' + (a'_y + p_b b'_y + \dots) \delta Y' + \dots; \end{aligned}$$

or

$$\delta V + p_x \delta X' + p_y \delta Y' + \dots = (a'_x + p_b b'_x + \dots) \delta X' + (a'_y + p_b b'_y + \dots) \delta Y' + \dots$$

We have assumed that the production of the X, Y, \dots is carried out independently, that is, that the $\delta X', \delta Y', \dots$ must be regarded as independent; and hence, for the equation (87) to be satisfied, we must have

$$p_x = a'_x + p_b b'_x + \dots;$$

.....

and these equations are precisely (76).

The case to which the equations (D bis) in §37 correspond gives an identical result, since the quantity

$$a_x + p_b \beta_x + \dots,$$

which is not dependent on X' , vanishes when we take the variation δ ; and we have the equations

$$X' \delta p_x + p_x \delta X' = (a'_x + p_b b'_x + \dots) dX' + \delta p_b \int_0^{X'} b_x dx + \dots;$$

.....

which are identical with the equations just obtained, when in these equations we set $X_0 = 0, \dots$

In the case in which the production coefficients are fixed, equations (D') of §37 give

$$\delta p_x = b_x \delta p_b + c_x \delta p_c + \dots,$$

$$\delta p_y = b_y \delta p_b + c_y \delta p_c + \dots,$$

.....

Multiplying the first of these equations by $X' - X_0$, the second by $Y' - Y_0$, and so on, summing and taking account of (E'), we shall obtain

$$(X' - X_0) \delta p_x + (Y' - Y_0) \delta p_y + \dots \\ + (B' - B_0) \delta p_b + (C' - C_0) \delta p_c + \dots = 0.¹⁹$$

This equation is precisely (87), whence we conclude that, when the production coefficients do not vary with the amounts produced, the paths followed with constant prices yield maximum ophelimity. I first obtained the proof of this theorem in the present form; subsequent studies led me to the proof in §42 and §45.

48. Finite variations in the case of exchange. Let us consider a position of equilibrium which we shall call I, and for which we have the quantities

$$x'_1, y'_1, \dots, x'_2, \dots;$$

let us consider another position of equilibrium for which we shall have

$$x''_1, y''_1, \dots, x''_2, \dots;$$

the intermediate values will be

$$x_1, y_1, \dots, x_2, \dots;$$

Let us assume that we pass from I to II not by any arbitrary paths but by paths for which we have

$$(88) \quad \begin{cases} x_1 = x'_1 + a_1 t, & y_1 = y'_1 + \beta_1 t, \dots, \\ x_2 = x'_2 + a_2 t, \dots, \end{cases}$$

$a_1, \beta_1, \dots, a_2, \dots$ being constants, and t a new variable. We can also put

$$p''_y = p'_y + \sigma_y t, \quad \dots, \quad a''_x = a_x + \omega_x t, \quad \dots$$

but the quantities $\sigma_y, \dots, \omega_x, \dots$ are no longer constant; they are quantities that result from the equations that must be satisfied by the prices and production coefficients.

The second variation of ophelimity for an individual is

$$\delta^2 \Phi_1 = \varphi_{xx} \delta x_1^2 + \varphi_{yy} \delta y_1^2 + \dots + 2\varphi_{xy} \delta x_1 \delta y_1 + \dots;$$

and, by virtue of equations (88),

$$(89) \quad \frac{\delta^2 \Phi_1}{\delta t^2} = \varphi_{xx} a_1^2 + \varphi_{yy} \beta_1^2 + \dots + 2\varphi_{xy} a_1 \beta_1 + \dots$$

Let us assume that this quantity is always negative. In that case, $\delta \Phi_1$ will keep decreasing as t increases, that is, in passing from position I to position II. We have seen in §45 that at the position of equilibrium I, some of the $\delta \Phi$ must be positive and some negative as one continues to depart from the equilibrium position, and when finite variations are substituted for infinitesimal variations, all these $\delta \Phi$ will decrease. Hence, in position II, some of the $\delta \Phi$ may still be positive and some negative, or all may be negative, but they cannot all be positive. Thus, it is not possible to depart from equilibrium position I to the advantage of all the members of the community; but some or all of them will necessarily be harmed.

49. It remains to be seen what conditions make the expression (89) negative. As is well known, these are obtained by forming the successive Hessians and putting

$$(90) \quad \varphi_{xx} < 0, \quad \begin{vmatrix} \varphi_{xx} & \varphi_{xy} \\ \varphi_{xy} & \varphi_{yy} \end{vmatrix} > 0, \quad \begin{vmatrix} \varphi_{xx} & \varphi_{xy} & \varphi_{xz} \\ \varphi_{xy} & \varphi_{yy} & \varphi_{yz} \\ \varphi_{xz} & \varphi_{yz} & \varphi_{zz} \end{vmatrix} < 0, \quad \dots$$

In the case in which the ophelimities of the commodities are independent,⁶ these conditions are verified; and more simply we see at once that (89) is always negative since

$$\varphi_{xx} < 0, \quad \varphi_{yy} < 0, \quad \dots, \quad \varphi_{xy} = 0, \quad \varphi_{xz} = 0, \quad \dots$$

⁶ This case was studied by me for the first time in the *Giornale degli Economisti*, Rome, November 1903.¹⁸ The proof in substance is the same as the one given here, but is simpler in form because, instead of the general case, I considered directly the simplest case of commodities whose consumption is independent.

In the case of commodities having a dependence of the first kind, we know that a commodity composed of such commodities has an elementary ophelimity which decreases as the quantity increases; and a consequence of this is that the right side of equation (89) must be negative when the quantities α_1, β_1, \dots are positive.

In fact, reverting to the considerations set out in §12, let us imagine a commodity composed of $\alpha_1 t$ of X, $\beta_1 t$ of Y, and so on, t being a new variable. We shall have

$$dx_1 = \alpha_1 dt, \quad dy_1 = \beta_1 dt, \quad \dots;$$

and hence

$$d^2\Phi_1 = (\varphi_{xx}\alpha_1^2 + \varphi_{yy}\beta_1^2 + \dots + 2\varphi_{xy}\alpha_1\beta_1 + \dots)dt^2.$$

Since the elementary ophelimity decreases as the quantity increases, the right side must always be negative, and thus the expression (89) will also be negative when the quantities are positive.

For the dependence of the first kind, we have

$$\varphi_{xx} < 0, \quad \dots, \quad \varphi_{xy} > 0, \quad \varphi_{yz} > 0, \quad \dots$$

In the expression (89), there are two categories of terms. The first category consists of terms containing the squares of the quantities α_1, β_1, \dots ; and, whatever be the signs of these quantities, these terms always remain negative. The second category consists of terms containing products of the quantities α_1, β_1, \dots taken two at a time; and, when these quantities are positive, those terms are positive. In this case the sum obtained by adding them to the terms of the first category is negative, according to what we have just seen; this will therefore still be the case, with greater reason, when some of the terms of the second category are negative instead of positive. This happens precisely when some of the quantities α_1, β_1, \dots become negative.

It follows from these considerations that in each case and whatever be the signs of the quantities α_1, β_1, \dots , the expression (89) remains negative.

It should be added that the inequalities (90) will therefore also be verified, which gives us, for the case of dependence of the first kind, certain conditions that must be satisfied by the second derivatives of total ophelimity. In §12, we confined ourselves to considering the particular case of only two commodities.

We may therefore conclude that in the case in which the ophelimities of the commodities are independent, and in which they have a dependence of the first kind (IV, 12), the expression (89) is always negative, and hence the theorem in §48 is verified.

Appendix to the French (1909) Edition^[1]

1. This appendix has as its sole aim an elucidation of the theories set forth in the text. It is by no means a treatise on mathematical economics, for which much more space would be needed than is available here.¹

2. Let x and y be the quantities of economic goods X and Y possessed by an individual. Let us suppose that the order in which these goods are consumed need not be taken into account (IV, 7), i.e., let us consider the arrangements xy and yx as being identical.

Let us choose any combination, $x_1 y_1$, and let us look for all the other combinations $x_2 y_2, x_3 y_3, \dots$ that are equivalent for this individual, i.e., for which the choice is a matter of indifference to him (III, 52). By interpolating, we will be able to obtain an equation

$$(1) \quad f_1(x, y) = 0$$

such that, if x is assigned the values

$$x_1, x_2, x_3, \dots,$$

then y takes on the values

$$y_1, y_2, y_3, \dots$$

Equation (1) is that of an *indifference line*² (III, 54). Starting from another combination, $x'_1 y'_1$ which is not one of the former, the equation of another indifference line will be obtained, and so on. Let us attach an index, I , to each of these indifference lines, as we have already indicated (III, 55). To the indices I_1, I_2, I_3, \dots there will correspond the functions f_1, f_2, f_3, \dots .

By interpolating the parameters that occur in these functions, we shall obtain a function f that will reproduce the functions f_1, f_2, \dots for the different values of I . By assigning suitable values to I , the equation

¹ I present here the results—which are new in part—of my latest studies on the economic problem. This appendix should thus take the place of my previous work on the subject.

² The concepts of indifference lines and preference lines were introduced into the science by Professor F. Y. Edgeworth. He took as his starting point the concept of *utility* (ophelimity), which he assumed to be a known quantity, and he deduced from it the definition of these lines. I have inverted the problem. I have shown that by starting from the notion of indifference lines—a concept that is given directly by experience—one can succeed in determining economic equilibrium, and work back to certain functions, one of which is ophelimity, if it exists. In any case, one will obtain *indices* of ophelimity.

$$(2) \quad f(x, y, I) = 0$$

will furnish all the indifference lines.³

3. If equation (2) is considered as that of a surface, the projections on the xy plane of the contour lines of this surface will be the indifference lines. This surface is partly arbitrary, since the indices I are partly arbitrary; i.e., it is any one of the surfaces that have as projections of their contour lines the indifference curves defined by the equations

$$f_1 = 0, \quad f_2 = 0, \quad \dots,$$

and by¹ those that are intermediate among these.

In short, we know only the projections of the contour lines, and this is not sufficient to determine the surface from which these contour lines are derived; this surface remains partly arbitrary.

To simplify matters, equation (2) may be expressed in the form

$$(3) \quad I = \Psi(x, y).$$

By assigning a constant value to I , we obtain an indifference line.

The same considerations apply, of course, to any number of goods, so that we obtain

$$(4) \quad I = \Psi(x, y, z, \dots)^{[2]}.$$

4. When a system of indices (3) or (4) has been obtained, an infinity of others is given by the equation

$$(5) \quad I = F(\Psi),$$

F being an arbitrary function.^[3]

In passing from a combination x, y, z, \dots , to the combination $x + dx, y, z, \dots$, the index I increases by

$$(6) \quad \frac{\partial I}{\partial x} dx = F' \cdot \Psi_x dx,$$

Ψ_x being the partial derivative of Ψ with respect to x . The second combination will be preferred by the individual to the first, since he will have more of X and as much of every other good. If we wish a higher index to designate a combination that is preferred to one with a lower index, the increment of I given by (6) should be positive when dx is positive. The arbitrary choice of F should thus be somewhat restricted, so that the right side of (6) and the right sides of the analogous equations in y, z, \dots , are positive. This is what will always be assumed.^[4]

³ For more details, see P. Boninsegni, "I fondamenti dell'economia pura," in the *Giornale degli Economisti*, Rome, February 1902.

5. If we differentiate equation (5)^[5] holding I constant, we obtain

$$(7) \quad 0 = \Psi_x F' dx + \Psi_y F' dy + \Psi_z F' dz + \dots$$

or

$$(8) \quad 0 = \Psi_x dx + \Psi_y dy + \Psi_z dz + \dots$$

An equation equivalent to this one could be obtained directly by experience. We shall therefore seek to determine by what positive amount, $\Delta_1 x$, the variable x would have to increase in order to offset the decrease represented by the negative amount Δy ; in the same way, we shall determine the $\Delta_2 x$ that corresponds to Δz , etc.^[6] Then by setting

$$\Delta x = \Delta_1 x + \Delta_2 x + \dots$$

we shall obtain an equation of the form

$$0 = p'_x \Delta x + q'_y \Delta y + \dots,$$

and passing to the limit, we shall have

$$(9) \quad 0 = q_x dx + q_y dy + q_z dz + \dots^{[7]}$$

This equation is equivalent to equation (7) or equation (8). It must therefore have an integrating factor in the case considered, but not in other cases.^[8]

6. Equation (9) is the only one we need, strictly speaking, to construct the theory of economic equilibrium; now, this equation contains nothing that corresponds to ophelimity, or to the ophelimity indices; the whole theory of economic equilibrium is thus independent of the concepts of (economic) *utility*, of value in use, or of ophelimity;⁴ it needs only one thing, namely knowledge of the limits of the ratios

$$\frac{\Delta_1 x}{\Delta y}, \frac{\Delta_2 x}{\Delta z}, \dots;$$

the quantities $\Delta_1 x, \Delta y; \Delta_2 x, \Delta z; \dots$, being such that the choice among the combinations

$$x, y, z, \dots; \quad x + \Delta_1 x, y + \Delta y, z, \dots, \quad x + \Delta_2 x, y, z + \Delta z, \dots$$

etc., is a matter of indifference.

⁴ I, too, started by constructing the theory of economic equilibrium on the basis of these concepts, as did all other economists. But I then realized that one could do without these concepts, and I have developed the theory of choice, which gives more rigor and clarity to the whole theory of economic equilibrium.

One could thus write an entire treatise of pure economics starting from equation (9) and from other analogous ones, and, perhaps, one day there will be some point in doing so.⁵

Upon integrating equation (9), we would obtain equation (4) or equation (5). One might well consider it appropriate, then, for the sake of brevity, to attach an arbitrary name to the quantity I . It is thus that in mechanics it was considered appropriate to attach the name *kinetic energy* to a certain integral, and in thermodynamics, the name *entropy* to another. But one might well—if only as a minor convenience—prefer not to give any name to the function (5) at all, and designate it simply by the letter I ; nothing in economic theories would be changed.⁶

7. But, just as in mechanics—after having defined mathematically the work of a force, kinetic energy, potential, energy, etc., one is justified in studying the relations between these quantities and empirical facts—so in studying economic science one is led to investigate how the quantity I is related to empirical facts.

This is what we are now going to do; but the reader must not forget that this is a digression, that the study we are now undertaking is not at all necessary in order to construct the theory of economic equilibrium, and that it even lies beyond its scope.^[9]

8. The differential equation (9) has an integral (§5). Assuming it to have been put in the form (5), the arbitrary function having been chosen as indicated in §4, it has the following two properties: (1) to two combinations as between which the individual's choice is indifferent, there correspond equal values of I ; (2) if a certain combination (α) is preferred to another one (β), to (α) there will correspond a value of I that is higher than that corresponding to (β) (§134).

9. If we consider the pleasure derived from a combination x, y , we may say that the individual is indifferent between the choice of one or another of two combinations giving rise to the same amount of pleasure, and that, of two combinations causing different amounts of pleasure, he will choose the one that gives him more pleasure.

A correspondence is thus established between the quantity I and pleasure. The former may serve as an index of the latter.

But this correspondence is not one-to-one, for, to a given combination x, y , there may correspond an infinity of values of I , depending on the form of F adopted. If the correspondence were one-to-one, we might adopt I as a measure of pleasure, in the sense that to a given pleasure there would correspond only one value of I (abstracting from the unit of measurement); that to two equal pleasures, there would correspond two equal values of I ; and that to a pleasure that is greater than another, there would correspond a value of I that is greater than that corresponding to the latter.

⁵ This is one of the many reasons why my theories are absolutely distinct from those of what is called the Austrian School.

⁶ This is quite unthinkable for literary and metaphysical economists. One of them, a professor of political economy in an Italian university, refers to another extremely learned professor who engaged in profound researches into the etymology of *ophelimity*, without succeeding, alas, in discovering what this quantity means.

Could one imagine a professor of thermodynamics undertaking etymological research into ancient Greek authors to discover the meaning of *entropy*?

This remark is enough to make one aware of the present backward state of political economy as compared with sciences such as mechanics, astronomy, physics, chemistry, etc.

10. Let us suppose that an integrating factor can be found such that Ψ_x will be a function only of x , Ψ_y only of y , etc. In that case, among the infinite number of systems of indices there is one such that the partial derivative Ψ_x of Ψ with respect to x is a function only of x , the partial derivative of Ψ_y is a function only of y , etc. This system is obtained by assuming that in equation (6) and the other analogous equations, F' is equal to a constant, A .^[10] Then,

$$(10) \quad \begin{cases} \frac{\partial I}{\partial x} = A\Psi_x, & \frac{\partial I}{\partial y} = A\Psi_y, \dots, \\ I = A\Psi, & \Psi = \int \Psi_x dx + \int \Psi_y dy + \dots \end{cases}$$

If, for these goods, the pleasure derived from consuming dx depends only on x , that derived from consuming dy depends only on y , etc., then among all the values given by equation (5), only those given by equations (10) correspond to the pleasure provided by the consumption of x, y, z, \dots . This correspondence is one-to-one except for the value of A , which fixes the unit of measurement. In this case, one may thus take the quantity I given by equation (10) as the measure of the pleasure provided by the combination x, y, z, \dots , or, if one wishes, as the measure of the value in use, the utility, the *rareté* (Walras), the ophelimity of this consumption.

11. But if Ψ_x is not a function only of x , Ψ_y of y , etc., the correspondence between I and pleasure is no longer one-to-one; the quantity I can no longer be taken as the measure of pleasure; it is only an index of pleasure.

It should not be forgotten that we are dealing here solely with goods whose order of consumption is a matter of indifference. In the opposite case the conclusion we have just reached would be different.

12. When there are only two economic goods, the function I always exists, whether or not the order of consumption is a matter of indifference.

“The passage from the case of only two goods to that of three or more goods deserves a more detailed treatment than is found in the *Manuale*. Indeed, it is well known that a differential expression with two terms

$$Xdx + Ydy$$

always admits an infinite number of integrating factors, whereas an expression with three or more terms need not have any.”⁷

We shall now take up this question.

Let us first remark that if we assume that the individual is allowed to choose the order of his consumption (IV, 7), he will choose the order that is most agreeable to him. Then

⁷ This is what Professor Vito Volterra wrote concerning the Italian edition of this book, in the *Giornale degli Economisti*, April 1906.^{2[11]}

Literary economists' criticisms are devoid of value; but the remarks and the criticisms of a scholar like Mr. Volterra are of great value, and are helpful for the progress of science.

Following this remark, I published in the *Giornale degli Economisti*, July 1906, an article³ in which I tried to clarify the point to which Mr. Volterra rightly drew my attention. It is this article that I am now summarizing in the text, but lack of space compels me to limit myself to the main findings, and to leave out the elaboration of the arguments; as against this I am adding some new considerations.

any multiple-term differential expression is integrable, because the path of integration is given. This case is thus analogous to the preceding one. We need only deal with the case in which the individual, for whatever reason, may consume the goods in any order whatsoever, without keeping to the one that is most agreeable.

13. Let us suppose that the individual starts at the point x, y, z, \dots, t , and that he consumes the amounts $\Delta x, \Delta y$, determined in such a way that he is indifferent between the choice of the preceding combination and the combination $x + \Delta x, y + \Delta y, z, \dots, t$. We shall, by experience, find the equation

$$\Delta x + b'_y \Delta y = 0.$$

In the following, we shall assume that a'_x, b_y , and the other analogous quantities,^[12] depend only on the coordinates x, y, z, \dots of the point to which they refer, and that they do not depend at all on the order of consumption.

If we pass to the limit and put

$$\Delta x = \frac{\partial x}{\partial y} \Delta y,$$

we shall have

$$(11) \quad \frac{\partial x}{\partial y} dy + b_y dy = 0.$$

Other analogous equations will be obtained by varying x and z , x and u, \dots , and x and t . Summing these equations, and making use of

$$dx = \frac{\partial x}{\partial y} dy + \frac{\partial x}{\partial z} dz + \dots + \frac{\partial x}{\partial t} dt,$$

we shall obtain:

$$(12) \quad 0 = dx + b_y dy + c_z dz + \dots + n_t dt.$$

If we multiply this equation by an arbitrary factor, it will take on the form:

$$(12 \text{ bis}) \quad 0 = A_x dx + B_y dy + \dots + M_s ds + N_t dt.$$

The quantities $b_y, c_z, \dots, m_s, n_t$ are given by experience; the quantities A_x, B_y, \dots, N_t are therefore given by experience, up to a factor of proportionality.

When the order of consumption does not influence the choice the individual makes among the consumer goods, equation (12) has an integrating factor; when the order of consumption influences the choice the individual makes, equation (12) has no integrating factor.

14. Let us suppose that choice is influenced by the order of consumption. Let us consider some definite order of consumption, say x, y, z, \dots, s, t . Let us determine

by experience an indifference variety (in hyperspace), and let us write its equation in the form (5). Equation (5) nevertheless differs from the equation of the same form that we now obtain, in that equation (5) is valid whatever the order of consumption, whereas the one we obtain now is valid only for the definite order we are considering.^[13]

We thus see that in the following two cases—(1) if the order of consumption is a matter of indifference; (2) if the order of consumption influences choice, but this order is fixed in advance—one obtains an equation of the form (5), or the corresponding differential equation,^[14] which may be written

$$(13) \quad 0 = \varphi_x dx + \varphi_y dy + \varphi_z dz + \dots + \varphi_t dt.$$

Experience does not provide precisely the functions $\varphi_x, \varphi_y, \dots, \varphi_t$, but only their ratios with respect to one of them; for example:

$$\frac{\varphi_y}{\varphi_x}, \frac{\varphi_z}{\varphi_x}, \dots, \frac{\varphi_t}{\varphi_x}.$$

Following the predetermined order, the individual starts from the point $0, 0, \dots, 0$, and reaches the point x, y, \dots, s, t , after following the path

$$(a) \quad 0, 0, \dots, 0; \quad x, 0, \dots, 0; \quad \dots; \quad x, y, \dots, 0; \quad \dots \quad x, y, \dots, s, t.$$

If he thereafter follows the new path

$$(b) \quad 0, 0, \dots, 0; \quad x + dx, \dots, 0; \quad \dots; \quad x + dx, y + dy, \dots, t + dt,$$

he will still find himself on the indifference variety passing through the point x, y, \dots, s, t , provided that equation (7), obtained by differentiating (5), is fulfilled.

The paths $0, 0, \dots, 0; x, 0, \dots, 0$; and $0, 0, \dots, 0; x + dx, dy, \dots, 0$, are special cases of the preceding paths. We must therefore have

$$(14) \quad 0 = \Psi_x(x, 0, \dots, 0)F'(\Psi(x, 0, \dots, 0))dx + \\ \Psi_y(x, 0, \dots, 0)F'(\Psi(x, 0, \dots, 0))dy.$$

But on the other hand, since the choice between the combinations $x, 0$ and $x + dx, dy$ is a matter of indifference, we must have an equation that differs only by a factor from the following:

$$0 = A_x(x, 0, \dots, 0)dx + B_y(x, 0, \dots, 0)dy.$$

Since this equation and the preceding one must hold simultaneously, we must have

$$(15) \quad \begin{cases} \Psi_x(x, 0, \dots, 0)F'(\Psi(x, 0, \dots, 0)) = A_x(x, 0, \dots, 0)G(x), \\ \Psi_y(x, 0, \dots, 0)F'(\Psi(x, 0, \dots, 0)) = B_y(x, 0, \dots, 0)G(x), \end{cases}$$

where G is an arbitrary function.

By a similar argument with respect to the paths $0, \dots, 0; x, \dots, 0; x, y, \dots$, and $0, \dots, 0; x, 0, \dots, 0; x, y + dy, dz, \dots, 0$, we shall have

$$(16) \quad \begin{cases} \Psi_y(x, y, \dots, 0)F'(\Psi(x, y, \dots, 0)) = B_y(x, y, \dots, 0)G'(x, y), \\ \Psi_z(x, y, \dots, 0)F'(\Psi(x, y, \dots, 0)) = C_z(x, y, \dots, 0)G'(x, y), \end{cases}$$

G' being an arbitrary function.

But if we set $y = 0$ in the first equation of (16), we obtain an equation that differs from the second one of (15) only in that $G'(x, 0)$ is replaced by $G(x)$; we must therefore have

$$G'(x, 0) = G(x)$$

and, in general, the functions G, G', G'', \dots may be replaced by $G(x, 0, \dots, 0)$, $G(x, y, \dots, 0), \dots, G(x, y, \dots, t)$. But since A_x, B_y are known only up to a proportionality factor, the functions G may be understood to include this factor. We thus have

Such are the relations that must subsist among the quantities A_x, B_y, \dots , and Ψ_x, Ψ_y, \dots , given by experience.^[15]

15. Let us assume that pleasure can be measured,^[16] and see whether it is possible to establish a correspondence between this pleasure and the quantities appearing in equations (17).

When the individual starts out at the point x, y, \dots, s, t , let $P_x dx, Q_y dy, \dots, S_s ds, T_t dt$, be the pleasures provided by consuming dx, dy, \dots, dt , respectively.

If the individual is indifferent as between choosing the combinations x, y, \dots, t , and $x + dx, y + dy, \dots, t + dt$, we must have:

$$(18) \quad 0 = P_x dx + Q_y dy + \dots + T_t dt.$$

Comparing this equation with equation (12 bis), we have

$$(19) \quad P_x = A_x H, \quad Q_y = B_y H, \quad \dots, T_t = N_t H,$$

where H is a function of x, y, \dots, t .

The pleasure the individual will obtain by following the path (a) of §14 will be

$$G = \int_0^x P_x(x, 0, \dots, 0) dx + \int_0^y Q_y(x, y, \dots, 0) dy + \dots + \int_0^t T_t(x, y, \dots, t) dt$$

or again:

$$G = P(x, \dots, 0) - P(0, 0, \dots, 0) + Q(x, y, \dots, 0) - Q(x, 0, \dots, 0) + \dots \\ + T(x, y, \dots, t) - T(x, y, \dots, s, 0).$$

Differentiating, and comparing the result with equation (7), we obtain^[17]

Each of the terms $\Psi_t, \Psi_s, \dots, F'$, on the left side of these equations is a function of all the variables x, y, \dots, s, t .⁴

A comparison of the first equation of (20) with the last equation of (17) shows that

$$H = 1;$$

this also follows from the fact that F' , being arbitrary, may always be supposed to include H .^[19]

Equations (20) and (17) will be satisfied if we set

$$(21) \quad T_t = \Psi_t F', \quad S_s = \Psi_s F' + \chi_s, \quad \dots, \quad P_x = \Psi_x F' + \chi_x.$$

Each of the functions occurring in these equations is a function of all the variables x, y, \dots, s, t . Furthermore, χ_s is zero for $t = 0$, χ_u is zero for $t = 0, s = 0; \dots$ and χ_x is zero for $t = 0, s = 0, \dots, y = 0$.

As a matter of fact, the first equation of (21) is the same as the last equation of (17); the second one of (21)—if it is assumed that $t = 0$ —becomes the second-to-last one of (17), etc.

Taking equations (19) into account, equations (17) become, through integration:

$$\begin{aligned} T(x, \dots, t) - T(x, \dots, s, 0) &= F(\Psi(x, \dots, t)) - F(\Psi(x, \dots, s, 0)) \\ S(x, \dots, s, 0) - S(x, \dots, 0, 0) &= F(\Psi(x, \dots, s, 0)) - F(\Psi(x, \dots, 0, 0)) \\ \dots & \\ P(x, 0, \dots, 0) - P(0, \dots, 0) &= F(\Psi(x, 0, \dots, 0)) - F(\Psi(0, \dots, 0)) \end{aligned}$$

and these values satisfy equations (20).

16. Equations (21) show that as long as we have no further data from experience, we cannot establish a one-to-one correspondence between the ophelimities P_x, Q_y, \dots, T_t and the quantities $\Psi_x, \Psi_y, \dots, \Psi_t$ given by experience. The latter may certainly serve as indices of the former, but they do not measure them.

17. The values (21) may be divided into two categories.

First Category. The terms $\chi_s, \chi_u, \dots, \chi_x$ are all zero. In this case the quantities P_x, Q_y, \dots, T_t happen to be the partial derivatives of one and the same function. But then these quantities may represent the pleasure derived from consuming commodities when this pleasure is independent of the order of consumption. Thus, the commodities X, Y, ..., T, whose consumption yields a pleasure that depends on the order of consumption, may—fictitiously, *when they are consumed in a given order*—be considered as equivalent to commodities whose consumption provides pleasure independently of the order of their consumption. But precisely for this reason, these fictitious pleasures are different from the real pleasures.^[20]

Second category. All the terms $\chi_s, \chi_u, \dots, \chi_x$, or some of them, are different from zero. In this case the pleasures, P_x, Q_y, \dots, T_t , vary according to the order of consumption.^[21] It is therefore in this second category that one must try to find the expressions for the real pleasures.

18. For this, one needs to find a way to get rid of the arbitrary function, as we did in §10.

Let us suppose that the individual follows the path

$$(y) \quad \begin{aligned} 0, 0, \dots, 0; & \quad h, 0, \dots, 0; \quad h, k, \dots, 0; \quad \dots; \quad h, k, \dots, m, n; \\ x, k, \dots, n; & \quad x, y, \dots, n; \quad \dots; \quad x, y, \dots, t. \end{aligned}$$

Let us determine by experience the indifference variety that corresponds to paths of this kind;^[22] we shall as usual have an equation of the form

$$(22) \quad I = F(\varphi)$$

or

$$(23) \quad 0 = \varphi_x F' dx + \varphi_y F' dy + \dots + \varphi_t F' dt.^[23]$$

The quantities $\varphi_x, \varphi_y, \dots$, are given by experience.

The pleasure, or ophelimity, that the individual will thus enjoy, will be

$$(24) \quad \begin{aligned} G = & \int_0^h P_x(x, 0, \dots, 0) dx + \int_0^k Q_y(h, y, \dots, 0) dy + \dots \\ & + \int_0^n T_t(h, k, \dots, m, t) dt + \int_h^x P_x(x, k, \dots, n) dx \\ & + \int_k^y Q_y(x, y, l, \dots, n) dy^5 + \int_n^t T_t(x, y, \dots, t) dt. \end{aligned}$$

Differentiating this equation, we shall obtain an equation

$$(25) \quad 0 = \frac{\partial G}{\partial x} dx + \frac{\partial G}{\partial y} dy + \dots + \frac{\partial G}{\partial t} dt,$$

which must be equivalent to equation (23); but we have, according to (24),

$$\frac{\partial G}{\partial t} = T_t(x, y, \dots, t)$$

and this value is independent of h, k, \dots, n . Equation (23), or an equivalent one obtained from experience, must therefore have an integrating factor, such that the last term of this equation is independent of h, k, \dots, n .^[24]

There is, moreover, only one such factor, for, denoting it by Γ , we know that the others will have the form

$$\Gamma F(\varphi),$$

where F is an arbitrary function. Now, φ depends on h, k, \dots, n , and hence so does $F(\varphi)$; it is therefore only the factor Γ that is independent of these quantities. Multiplying the equation given by experience by this factor Γ , we shall have a value—not containing an arbitrary function—for T_t multiplied by a constant a .

It should be recalled that we do not know the functions A_x, B_y, \dots but only the ratios of each to one of the rest, because they contain an arbitrary factor (§13). Equations (19) give

$$(26) \quad P_x = \frac{A_x}{N_t} T_t, \quad Q_y = \frac{B_y}{N_t} T_t, \quad \dots,$$

or

$$(26 \text{ bis}) \quad P_x = \frac{1}{n_t} T_t, \quad Q_y = \frac{b_y}{n_t} T_t, \quad \dots,$$

and, since the quantity T_t is determined by experience, up to a constant, all the remaining quantities P_x, Q_y, \dots , are too.

There is thus established a one-to-one correspondence between the pleasures, or ophe-limities, P_x, Q_y, \dots, T_t , and the quantities given by experience. The latter may thus serve to measure the former.

19. Let us summarize the results obtained. Abstracting from a constant, which fixes the unit of measurement, there are two cases in which it is possible to obtain a one-to-one correspondence between the quantities given by experience that determine the indifference lines, or *varieties* (in hyperspace), and the pleasures (ophelimities) enjoyed by the individual who consumes dx, dy, \dots, dt after having reached the point x, y, \dots, t : (1) if the order of consumption is a matter of indifference, and if we know that the pleasure derived from consuming dx depends only on x , the pleasure from consuming dy only on y , etc; (2) if the pleasure differs according to the order of consumption, provided it is assumed that the necessary experiments can be made.

The case that remains excluded is thus the one in which the order of consumption is a matter of indifference, and in which the pleasure derived from consuming dx depends on x, y, \dots, t , or the pleasure derived from consuming dy depends on x, y, \dots, t , etc.⁸

⁸ These results were published for the first time in my article in the *Giornale degli Economisti*, July 1906.⁶

In the case in which the order of consumption is a matter of indifference, there exists a function of x, y, \dots, t , such that the partial derivatives of this unique function represent the indices of pleasure, or the pleasures of consuming dx, dy, \dots, dt , starting from the point x, y, \dots, t .

In the case in which the order of consumption influences pleasure, such a unique function does not exist, so long as the path to be followed is not determined.

20. It may be convenient to give names to the quantities we have considered.

The quantity I may, in any case, serve as an index of pleasure; we shall call it the *index of ophelimity*.⁹ Whenever this quantity can be used as a measure of pleasure, it is the *ophelimity*. If it corresponds to the consumption of a finite amount of goods, we will call it the *total ophelimity*. Its partial derivatives I_x, I_y, \dots , with respect to the variables x, y, \dots , will be called the elementary ophelimities of the goods X, Y,

If we consider a consumption path starting from a point x, y, \dots, t , and returning to it, we shall say that a *closed cycle* has been followed if we return to that point with the same ophelimity index as the one with which we had set out. This case corresponds to indifference as to the order of consumption.

We shall say that an *open cycle* has been followed, if we come back to the starting point with an ophelimity index different from the one from which we had set out. This case corresponds to the case in which the order of consumption influences the pleasure obtained from the consumption.

21. By making use of this terminology we may express the results of §19 in the following way.

Except for a constant, which determines the unit of measurement, ophelimity may be determined by means of the experiments that yield the indifference varieties, in two cases: (1) if the cycle is closed, and each ophelimity depends only on the variable to which it has reference; (2) if the cycle is open.^[26]

The case that remains excluded is that of closed cycles, when the elementary ophelimities are functions of two or more variables.

Total ophelimity always exists in the case of closed cycles. It also exists in open cycles, provided the path is followed in a determinate order. It does not exist in open cycles when the path is not determined.¹⁰

This concludes the digression announced in §7, and we shall now take up the basic concepts of economic equilibrium.

22. Equilibrium in the case of one individual and two goods.¹¹ Let us suppose that the individual starts from the point x_0, y_0 and that he is obliged to follow a certain path

⁹ Mr. Gide has suggested the term *desirability*. There is nothing to prevent us from adopting this term. But it is somewhat awkward to speak of the *desirability* of something one has already consumed. Usually one's desires are directed towards what one has not yet consumed.

All these names are of little importance. What is important is to have a good grasp of the thing to which a name has been given, and that there should be no misunderstanding on that score.^[25]

¹⁰ A remarkable study on the generalization of the concept of ophelimity was published while this book was in press. See V. Furlan, "Cenni su una generalizzazione del concetto d'ofelimità," *Giornale degli Economisti* (Rome), September 1908.^[27]

¹¹ This case is worth considering only as a first step in the study of the general case of economic equilibrium.

I disassociate myself completely not only from the economists of the so-called *Austrian School*, but also from other economists such as Professor Marshall, in that, in my opinion, only the need to consider systems of simultaneous equations that determine equilibrium in the general case justifies the use of mathematics in political economy.

whose projection on the xy plane is (III, 74)

$$(27) \quad f(x, y) = 0.$$

Let us also suppose that, starting from the point x_0, y_0 , the indices given by equation (3) begin to rise. Since a combination with a higher index is preferred to one with a lower index, the individual will start moving along the indicated path, and will continue up to the point at which the indices cease to rise, and begin to decline. But this is the point where the path is tangential to an indifference line, that is, where the curve (27) is tangential to the projection of an indifference curve.^[28]

This point will therefore be determined by the two equations

$$(28) \quad f_x dx + f_y dy = 0, \quad \varphi_x dx + \varphi_y dy = 0;$$

and by equation (27). To determine the two unknowns, x, y , we thus have the two equations^[29]

$$(29) \quad f = 0, \quad f_x \varphi_y - f_y \varphi_x = 0.$$

Here, φ_x, φ_y denote the partial derivatives of the index-function.

It is worth noting that the equilibrium has just been determined without using the concepts of *utility* (ophelimity), prices, etc.

23. Suppose we have a concave surface with contour lines whose height below the horizontal xy plane is given by (3). On this surface, let us trace out a line whose projection is (27). Let us put a heavy ball on this line. The point at which it will be in equilibrium is precisely the one given by equations (29). The equilibrium of this ball and the economic equilibrium will be two similar phenomena.^[30]

24. The case of several economic goods. Suppose we have any number of goods. The individual must move on the *variety* (in hyperspace)

$$(30) \quad f(x, y, z, \dots) = 0;$$

he will stop when he is indifferent as to the choices he could make by moving further.

We have seen in §14 that when the sequence of choices is a matter of indifference or, alternatively, fixed in advance, one obtains the differential equation (13) of an indifference variety. This equation is equivalent to the following ones:

$$\frac{\partial x}{\partial y} = -\frac{\varphi_y}{\varphi_x}, \quad \frac{\partial x}{\partial z} = -\frac{\varphi_z}{\varphi_x}, \quad \dots;$$

and experience supplies us with the values of the terms on the right.

I consider that the use of mathematics for problems involving one individual and two or even several goods does not yield results comparable in importance with those obtained in cases of general economic equilibrium.

It is my opinion that it is the interdependence of economic phenomena that compels us to resort to mathematical logic.

This approach may be good or bad; but in any case it should not be confused with those of economists who construct theories while neglecting this very interdependence.

On the other hand, equation (30) yields

$$(31) \quad f_x \frac{\partial x}{\partial y} + f_y = 0, \quad f_x \frac{\partial x}{\partial z} + f_z = 0, \quad \dots$$

By combining these equations with the preceding ones, we thus have:

$$(32) \quad \varphi_x = \frac{f_x}{f_y} \varphi_y = \frac{f_x}{f_z} \varphi_z = \dots$$

If the number of goods is m , equations (32) are $m - 1$ in number;⁷ and together with equation (30) we have the m equations necessary to determine the m unknowns x, y, \dots

25. If the order of consumption influences choice, this order must be fixed before the equilibrium point can be determined. When it is fixed, we have a function of x, y, \dots , that can serve as an index for the choices, and we are back to the preceding case.

26. Equations (30) and (32) are fundamental for the theory of economic equilibrium. Equation (30) is that of the obstacles; it is by specifying it that we shall find the innumerable cases of this equilibrium.

We have assumed that an obstacle can be described by the equation of a curve, surface, or variety. It will often be described by families of curves, surfaces, or varieties; equation (30) will then be replaced by

$$f_1(x, y, \dots, \mu_1, \mu_2, \dots) = 0, \quad f_2(x, y, \dots, \mu_1, \mu_2, \dots) = 0, \quad \dots;$$

where μ_1, μ_2, \dots are parameters to be determined. For this, other equations are needed.

27. Let us consider a case of equilibrium analogous to the one examined in (VI, 4).

The individual transforms X into Y.

He possesses x_0 of X; he starts by consuming an amount a of X, without producing anything; then, to produce each unit of Y, b units of X are required.^[31] We then have

$$x_0 - x = a + by,$$

or

$$(33) \quad a + by - x_0 + x = 0.$$

This corresponds to equation (27). Equation (29) becomes

$$(34) \quad \varphi_y - b\varphi_x = 0.$$

Equations (33) and (34) determine the amount of X that will be transformed into Y.^[32]

28. We have just dealt with a problem of an *individual economy*. Let us next suppose that there are several individuals. If one of them has the power to fix the path that the others have to follow, the others face simply the kind of problem we have just solved. Another problem exists for the individual whom we have supposed to have control over

the economic phenomenon, and whom we shall call individual 2. To begin with, we shall assume that he has only to deal with one other individual¹² whom we shall call individual 1.

29. The quantities of goods possessed by the first individual are x_{10}, y_{10} before any exchange takes place, and x_1, y_1 at the equilibrium point; the partial derivatives of the index that determines the choices are $\varphi_{1x}, \varphi_{1y}$. For the second individual, these quantities are $x_{20}, y_{20}, x_2, y_2, \varphi_{2x}, \varphi_{2y}$.

The total quantity of each good remaining constant in the course of the exchange, we shall have

$$(35) \quad x_{10} + x_{20} = x_1 + x_2, \quad y_{10} + y_{20} = y_1 + y_2.$$

30. If both individuals' tastes must be satisfied, the equilibrium points can only be points of tangency of an indifference curve of the first individual and an indifference curve of the second individual. But there is an infinity of such points, and more conditions are required to resolve the problem.

If individual 1 is free to follow the path that has been assigned to him, he will move along it only so long as it remains above the indifference line passing through x_{10}, y_{10} and at most, at the limit, he will move along this line. It will thus be at the point of tangency of this line and of an indifference line of individual 2 that the equilibrium point that is most advantageous to individual 2 will be found.

We shall have

$$(36) \quad \varphi_1(x_1, y_1) = \varphi_1(x_{10}, y_{10}), \quad \varphi_{1x}\varphi_{2y} - \varphi_{1y}\varphi_{2x} = 0;$$

which, with the two equations (35), give us four equations; and we can thus determine the four unknowns, x_1, y_1, x_2, y_2 .^[33]

31. It could happen that individual 2 simply aims at getting the largest possible amount of X. In that case, he will still force individual 1 to move along an indifference line, but he will let him continue the exchanges along this line as long as possible. If this line cuts the x -axis, that is the point at which equilibrium will take place.^[34]

32. Individual 2 may have the power to compel individual 1 to follow not just any path determined so as to please individual 2, but only a path chosen by individual 2 out of the family of curves

$$(37) \quad f(x_1, y_1, \mu) = 0.^[35]$$

That is to say, individual 2 can only determine μ .^[36]

For equilibrium, we must first have equations (29), that is,

$$(38) \quad f_1(x_1, y_1, \mu) = 0, \quad f_{1x}\varphi_{1y} - f_{1y}\varphi_{1x} = 0;$$

and thence μ must be determined according to the conditions fixed by individual 2.

¹² Once again this is a problem that is worth studying only as a first step in the study of the general case of economic equilibrium. The case of only two traders does not, moreover, arise in reality; it is only one example of the real case of several traders and several commodities.

33. (1) If individual 2 fixes the condition so as to arrive at the most favorable combination among all those he can obtain, we will have to express the fact that, for him, the index reaches a maximum upon varying μ ; we will therefore have

$$(39) \quad \varphi_{2x} \frac{dx_2}{d\mu} + \varphi_{2y} \frac{dy_2}{d\mu} = 0;$$

and, by virtue of equations (35),

$$(39 \text{ bis}) \quad \varphi_{2x} \frac{dx_1}{d\mu} + \varphi_{2y} \frac{dy_1}{d\mu} = 0.$$

From this equation and the equation obtained by differentiating the first equation of (38) with respect to μ , we eliminate

$$\frac{dx_1}{d\mu}, \quad \frac{dy_1}{d\mu},$$

so as to obtain, along with equations (35) and (38), the five equations needed to determine the five unknowns, x_1, y_1, x_2, y_2, μ .^[37]

(2) If individual 2 fixes the condition so as to obtain the maximum amount of Y, we will have to express the fact that the value of y_2 given by equations (35) and (37) reaches a maximum upon varying μ .

When y_2 is a maximum, y_1 is a minimum, by virtue of equations (35).^[38] We must therefore differentiate equation (37) with respect to μ , then set

$$\frac{dy_1}{d\mu} = 0,$$

and eliminate $dx_1/d\mu$; we shall thus have the fifth equation that is required to determine μ .^[39]

34. Finally, it may be assumed that neither of the two individuals has the power to impose a particular value of μ on the other.^[40] Each partner in the exchange is concerned only with making the choice that is most advantageous to him, without it entering his mind to alter *directly* the value of μ . This is the case of free competition (III, 41, 46).

For individual 1, we still have equations (38). If we substitute in the first of these equations the values of x_1, y_1 , given by (35), we shall obtain the equation of the path followed by individual 2, and it is this path that must be tangential to an indifference curve of individual 2. We shall thus have

$$(40) \quad \varphi_{2x} dx_2 + \varphi_{2y} dy_2 = 0;$$

and, by virtue of equations (35),

$$\varphi_{2x} dx_1 + \varphi_{2y} dy_1 = 0.$$

Consequently:

$$f_{1x}\varphi_{2y} - f_{1y}\varphi_{2x} = 0.$$

Since this case is very important, we shall write the relevant equations together:

$$(41) \quad \begin{cases} x_{10} + x_{20} = x_1 + x_2, \quad y_{10} + y_{20} = y_1 + y_2, \\ f_{1x}\varphi_{1y} - f_{1y}\varphi_{1x} = 0, \quad f_{1x}\varphi_{2y} - f_{1y}\varphi_{2x} = 0, \\ f_1(x_1, y_1, \mu) = 0. \end{cases}$$

These five equations serve to determine the five unknowns x_1, y_1, x_2, y_2, μ .

35. The following remarks will be useful.

We considered two cases in which individual 2 acts with absolute power. He imposes on individual 1 the path to be followed (§32).^[41]

We then considered two cases in which the powers of individual 2 are less extensive. He can only determine a parameter of the family of curves that must be followed by individual 1. These are cases of monopoly (§33).^[42]

Finally, individual 2 has no power over individual 1, any more than individual 1 has any power over individual 2. This is the case of free competition (§34).

The parameter μ is determined by the will of individual 2 in the cases of monopoly; it is indirectly determined by the action of individuals 1 and 2 in the case of free competition.

If we compare equation (39) and equation (40) we see that the former assumes that one passes from one curve of the family (37) to another, and the latter assumes that one always stays on one and the same curve in this family (III, 41–42).

It should be noted that in differentiating equation (37) to determine the point of tangency with an indifference line, one must take care not to vary μ , because in doing so one would shift from one indifference line to another. This remark is so elementary that it may appear to be superfluous; I mention it only because a certain author⁸ has fallen into the gross error of varying μ .¹³

Equations (35) and (37) hold for any values of x_1, y_1 , while the equation

$$f_x\varphi_y - f_y\varphi_x = 0^{[44]}$$

holds only for those values of x_1, y_1 that correspond to the point of equilibrium. The same is true, in general, of equations (32). Some authors have fallen into serious errors through their having neglected this very elementary observation.

If we suppress the third⁹ equation of (41), which concerns individual 2, the other equations give the amounts of commodities exchanged by individual 1, as a function of μ . These functions may be considered as expressing the *law of supply and demand*, for any value of μ .^[45]

36. In the case of three goods, we need not resort to the consideration of hyperspaces.

¹³ Or more precisely, the price that corresponds to μ . Better still, this author imagined that in these circumstances, I was mistaken in always holding the price constant while differentiating!^[43]

For an individual, there are indifference surfaces instead of indifference curves. The obstacles give rise to the equation of a surface

$$f(x, y, z) = 0,$$

instead of that of a curve (27).

Equilibrium takes place where this surface is tangential to an indifference surface. When the order of consumption is a matter of indifference, any line traced out on the surface representing the obstacles, and reaching a point at which this surface is tangential to an indifference surface, leads to an equilibrium point.

37. Differentiating the equation

$$(42) \quad f(x, y, z, \dots) = 0$$

partially, we obtain

$$-\frac{\partial x}{\partial y} dy = \frac{f_y}{f_x} dy, \quad -\frac{\partial x}{\partial z} dz = \frac{f_z}{f_x} dz, \quad \dots$$

The left sides of these equations represent the amounts of X that the individual must give up, while satisfying equation (42), in order to receive dy of Y, or dz of Z, etc.; and vice versa. It is convenient to give a name to the quantities

$$(43) \quad \frac{f_y}{f_x}, \quad \frac{f_z}{f_x}, \quad \dots;$$

they are called the *prices* of Y, Z, ..., in terms of X, and we write

$$(43 \text{ bis}) \quad p_y = \frac{f_y}{f_x}, \quad p_z = \frac{f_z}{f_x}, \quad \dots$$

When X stands for money,¹⁰ the quantities (43) are also called *prices* in ordinary language.

In the case of exchange, they are the prices observed on the market; it is thus the quantities (43) that are furnished by observations, and it is from these quantities that equation (42) must be deduced. If we designate by p_y the price of Y in terms of X, by p_z the price of Z in terms of X, etc., we have

$$(44) \quad \frac{\partial x}{\partial y} = -p_y, \quad \frac{\partial x}{\partial z} = -p_z, \quad \dots;$$

and these are the equations that must be integrated to obtain equation (42).^[46]

38. The prices often vary with the quantities x, y, z, \dots . In the study of certain phenomena, such as cornering, this circumstance cannot be ignored. But in a very large number of other extremely important phenomena, prices may be considered as constants.

When prices are constants, equations (44) can immediately be integrated, yielding for equation (42)

$$x + p_y y + p_z z + \dots = c,$$

where c is a constant. But since x_0, y_0, z_0, \dots , are the initial values of x, y, \dots , we must also have

$$x_0 + p_y y_0 + p_z z_0 + \dots = c,$$

and consequently equation (42) becomes

$$(45) \quad x - x_0 + p_y(y - y_0) + p_z(z - z_0) + \dots = 0.$$

This equation has a special significance in political economy. It expresses the balance of receipts and expenditures for the individual under consideration (III, 175).

Whether the prices are constant or variable, the individual's budget [constraint] for the exchanges dx, dy, \dots , is always given by

$$(46) \quad dx + p_y dy + p_z dz + \dots = 0.$$

39. When prices are variable, this equation need not be integrable. In such a case the individual's budget, for finite amounts x, y, \dots , depends on the order of consumption. There is no longer a function such as (30) to express the effects of the obstacles; rather, these effects are expressed by (46). The order of consumption must be fixed in advance if one wants to ascertain the individual's budget [constraint]. Once this order is fixed, equation (46) becomes integrable, and one has an equation of the form (30), but one that is valid for this order of consumption only.^[47]

40. In order not to dwell too long on this, let us confine ourselves to some special cases. Let us suppose we have three goods, let a and b denote two constants, and let us set

$$p_y = \frac{ay}{x}, \quad p_z = \frac{bz}{x}.$$

The equation

$$dx + \frac{ay}{x} dy + \frac{az}{x} dz = 0$$

is integrable, and yields

$$(47) \quad x^{2[48]} - x_0^2 + a(y^2 - y_0^2) + b(z^2 - z_0^2) = 0.$$

Let us now choose values for which it is not integrable, for example:

$$p_y = \frac{ay + cz}{x}, \quad p_z = \frac{bz}{x}.$$

Let us suppose that one starts buying some Y, and that afterwards one buys some Z. The path of integration is thus determined, and we shall have^[49]

$$(48) \quad x^2 - x_0^2 + a(y^2 - y_0^2) + 2cz_0(y - y_0) + b(z^2 - z_0^2) = 0.$$

Conversely, if one starts buying some Z, and afterwards some Y, we shall have

$$(49) \quad x^2 - x_0^2 + b(z^2 - z_0^2) + a(y^2 - y_0^2) + 2cz(y - y_0) = 0.$$

If we have $y_0 = 0, z_0 = 0$, the two equations (47) and (48) become identical, and take on the form

$$x^2 - x_0^2 + ay^2 + bz^2 = 0.$$

But they are identical in appearance only, for in equation (47) the path of integration is arbitrary, while it is fixed in equation (48). If this path is changed, and if one buys some Z before buying some Y, the form of equation (47) does not change, whereas instead of equation (48) we have, in the case considered,

$$x^2 - x_0^2 + bz^2 + ay^2 + 2cxy = 0.$$

These values of p_y, p_z, \dots , indicate only the law of successive purchases. They should not be confused with the values taken on by the prices at the equilibrium point, which are expressed as functions of the coordinates of this point (III, 169).

For example, at the equilibrium point we have

$$p_y^0 = f(x', y', z', \dots);^{[50]}$$

where x', y', z', \dots , are the values of x, y, z, \dots , that correspond to this point. This price may remain the same during the whole series of purchases leading to the equilibrium (III, 169, γ); and it is in this sense that we say that it is a constant. Or it may vary in the course of successive purchases (III, 169, α) according to a law expressed by

$$p_y = F(x, y, z, \dots);$$

and it is in this sense that we say that the price is variable. At the equilibrium point we should of course have

$$(50) \quad F(x', y', z', \dots) = f(x', y', z', \dots).$$

These principles are very simple, but their neglect may lead, and has indeed led, to serious errors.^[51]

41. Equilibrium in the case of one individual, any number of goods, and constant prices. Equilibrium is determined by equations (45) and (32); and taking the equations (43 bis) into account, we may write the following system of equations:

$$(51) \quad \begin{cases} \varphi_x = \frac{1}{p_y} \varphi_y = \frac{1}{p_z} \varphi_z = \dots, \\ 0 = x - x_0 + p_y(y - y_0) + p_z(z - z_0) + \dots. \end{cases}$$

There are in all m equations that determine m quantities, x, z, \dots , at the equilibrium point.

The equations of the first row of this system may be written

$$(52) \quad p_y = \frac{\varphi_y}{\varphi_x}, \quad p_z = \frac{\varphi_z}{\varphi_x}, \quad \dots$$

There is an essential difference between these equations and equations (43 bis), which also furnish the values of p_y, p_z, \dots . Equations (43 bis) hold for any values of the variables, whereas equations (52) are valid only for the values of x, y, z, \dots , that correspond to the equilibrium point. From equations (43 bis) one can compute the derivatives of the prices with respect to the variables x, y, \dots ; these derivatives cannot be computed from equations (52). This remark is similar to the one we already made in §35.

To simplify the notation, we do not always make a notational distinction, as we did in §40, between the values x, y, z, \dots , assumed at any point on the path followed, and those values x', y', z', \dots , that refer to the point of equilibrium. But this is a distinction that should never be lost sight of.

If Y stands for a commodity that the individual sells, y_0 cannot of course be zero. If it is a commodity that is being purchased, y_0 is, on the contrary, generally zero.

42. Let us make another digression, to indicate another way to determine ophelimity.

Instead of undertaking experiments to determine the indifference lines or varieties, let us undertake experiments to ascertain what amounts of commodities the individual will buy at certain given prices.

Let us set

$$y_0 = 0, \quad z_0 = 0, \quad \dots,$$

and assign a certain value to x_0 ; by experiment we can determine what quantities y, z, u, \dots , an individual will actually purchase in exchange for a part of the amount x_0 of X that he possesses. Let us repeat these experiments, by varying x_0 ; then we shall obtain the values of y, z, u, \dots , as functions of x_0, p_y, p_z, \dots . If we eliminate x_0 , by means of equation (45), the values of y, z, \dots , will be given as functions of x, p_y, p_z, \dots . As a result of these operations we shall have $m - 1$ equations in the $2m - 1$ quantities and prices: $x, y, z, \dots, p_y, p_z, \dots$; it may thus be assumed that these equations determine the values of the $m - 1$ prices as functions of the m quantities; that is to say, experience provides us with p_y, p_z, \dots , as functions of x, y, z, \dots .^[52] Substituting these values into equations (52) we shall thus have obtained, by experiment, the ratios of each of the quantities $\varphi_x, \varphi_y, \varphi_z, \dots$, to one of them. This is precisely what we obtained (§14) by considering indifference varieties.^[53]

From here on, the reasoning proceeds just as before.

The fact that such experiments may be more or less difficult, or even impossible to carry out in practice, is of little importance; their mere theoretical possibility suffices to

prove the existence of ophelimity indices in the cases examined, and to reveal some of their characteristics.

43. The theory of economic equilibrium could be derived directly from the experiments just described. Indeed, these experiments yield

$$p_y = a_y, \quad p_z^{\text{II}} = b_z, \quad \dots,$$

where a_y, b_z are known functions. These equations take the place of those in the first row of the system (51), and the equilibrium point is determined. But if we proceed in this manner, so long as the experiments are not actually performed we do not have the small amount of information on the quantities a_y, b_z, \dots , that can at least be obtained from the consideration of choices.^[54]¹⁴

44. Properties of indifference lines. Let us try to see what precisely can be learned from everyday experience about this subject.

Let

$$(53) \quad \varphi(x, y) = 0$$

be the equation of an indifference curve.

(1) First of all, we know that a decrease in x must be compensated for by an increase in y , and conversely. We must therefore have

$$(54) \quad \frac{dy}{dx} < 0.$$

(2) In general, and if we leave aside certain exceptional cases, the variable amount dy that one is prepared to give up along an indifference line for a constant amount dx , decreases as x increases; we thus obtain the second characteristic of indifference curves, expressed by

$$(55) \quad \frac{d^2y}{dx^2} > 0.$$

(3) However, the larger is x , the less dy decreases, so that, save as always in exceptional cases, we must have

$$(56) \quad \frac{d^3y}{dx^3} < 0.$$

Some qualifications should be made in the case of goods that have a dependence of the second kind, as we shall see more precisely in the following paragraph.^[56]

45. Suppose now that we pass from one indifference line to another. Let δ_x denote the variation from one line to another in a direction x parallel to the x -axis, and δ_y the variation parallel to the y -axis.

¹⁴ This point of view is developed in a learned article by E. Barone. See E. Barone, "Il Ministro della produzione nello stato collettivista," *Giornale degli Economisti*, September 1908.^[55]

Reasoning as before, we shall have^[57]

$$(57) \quad \delta_x \frac{dy}{dx} > 0, \quad \delta_y \frac{dy}{dx} < 0.$$

Let $a b c$ represent constituent points of an indifference line, and $a' b'$ constituent points of another. The slope α' of $a'b'$ on ox is greater than the slope α of ab , and less than the slope β of bc .^{[58][59]}

This characteristic seems surely to hold for goods that are independent in consumption. For example, if one has 5 units of X and 5 units of Y, and, passing to another indifference line, one still has 5 of Y, but^[12] 10 of X, it certainly appears, from all we know about consumption, that in this second position one will be willing to give up more of X for 1 unit of Y than in the first situation. The same conclusion is reached in the case of goods that have a dependence of the first kind. But it is doubtful whether it also holds in the case of goods having a dependence of the second kind. If Y is an *inferior* good and X a *superior* good (IV, 19), then when X and Y are consumed simultaneously by an individual, it is conceivable that this individual may exchange a certain amount of X for a certain amount of Y—for instance, 1 unit of X for 3 of Y; but when the individual has an abundance of X, whereas Y is on the verge of disappearing from his consumption, he may well refuse to give up one unit of X for even a very large amount of Y, which is contrary to the assumption expressed by^[13]

$$\delta_x \frac{dy}{dx} > 0.$$

Indeed, the quantity dy/dx being negative, this inequality indicates that dy decreases in absolute value as [the quantity of] X increases.^[14]

On the other hand, it is difficult to accept the assumption that, in general, for values of x ^[15] between zero and that obtained when X replaces Y completely in consumption, there will not arrive a point where, with X becoming less useful as it becomes more abundant, the individual is willing to content himself with decreasing amounts of Y as the quantity of X increases.

New observations are thus necessary to throw light on this question. They will probably lead to the setting up of several categories of goods having a dependence of the second kind.

It is not so much direct observations that can be useful, as indirect ones. Following the example practiced in the other physical sciences, we should formulate different

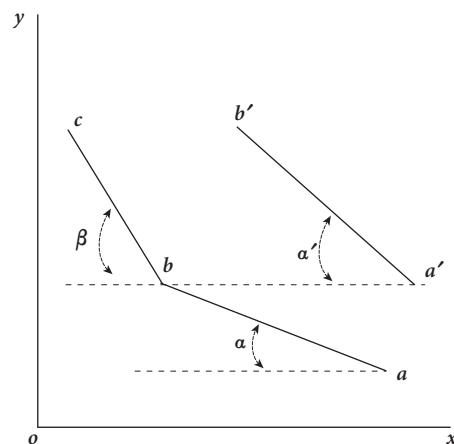


FIG 59

hypotheses concerning the possible values in (57), and compare the consequences of these hypotheses with reality.

46. Characteristics of the indices. Let the index be

$$I = \varphi(x, y)$$

Along an indifference curve we will have

$$\frac{dy}{dx} = -\frac{\varphi_x}{\varphi_y};$$

and since dy and dx must have opposite signs, φ_y and φ_x must have the same sign. We can choose the sign to be positive, which corresponds to the condition that a combination that is preferred to another will have a higher index. If dx is positive, the combination $x + dx, y$ will be preferred to x, y , and consequently $\varphi_x dx$ must be a positive quantity (§134).

The first characteristic of the indices (IV, 32) is thus given by

$$\varphi_x > 0, \quad \varphi_y > 0.$$

The inequalities (57) may be expressed as

$$-\frac{\partial}{\partial x} \frac{\varphi_x}{\varphi_y} > 0, \quad -\frac{\partial}{\partial y} \frac{\varphi_x}{\varphi_y} < 0;$$

and consequently

$$(58) \quad \begin{cases} \varphi_{xx}\varphi_y - \varphi_{xy}\varphi_x < 0, \\ \varphi_{yy}\varphi_x - \varphi_{xy}\varphi_y < 0; \end{cases}$$

where the $\varphi_{xx}, \varphi_{xy}, \varphi_{yy}$ are the second-order partial derivatives.

When the system of indices is such that

$$\varphi_{xy} = 0,$$

the inequalities (58) become

$$(59) \quad \varphi_{xx} < 0, \quad \varphi_{yy} < 0,$$

and we thus obtain the second characteristic of the indices (IV, 33).

Under the same hypothesis, the third characteristic^[60] of the indifference lines yields for the indices

$$\varphi_{xxx} > 0, \quad \varphi_{yyy} > 0, \quad \dots$$

47. Let us suppose that the quantities dx, dy, dz, \dots , are all positive; the combination $x + dx, y + dy, z + dz, \dots$, will be preferred to the combination x, y, z, \dots , and consequently, we must have

$$d\varphi > 0.$$

But

$$(60) \quad d\varphi = \varphi_x dx + \varphi_y dy + \varphi_z dz + \dots,$$

and

$$(61) \quad d^2\varphi = \varphi_{xx} dx^2 + \varphi_{yy} dy^2 + \dots + 2\varphi_{xy} dxdy + \dots$$

Three cases must be examined.

Case 1. φ_x depends only on x , φ_y only on y , etc. (IV, 8). We then have

$$(62) \quad \varphi_{xy} = 0, \quad \varphi_{xz} = 0, \quad \dots, \quad \varphi_{yz} = 0, \quad \dots$$

Case 2. The goods have a dependence of the first kind (IV, 9). Save for a few exceptions in the sub-case (β) indicated in (IV, 35), we have in general (IV, 39)^[61]

$$(63) \quad \varphi_{xy} < 0, \quad \varphi_{xz} < 0, \quad \dots, \quad \varphi_{yz} < 0, \quad \dots$$

Case 3. The goods have a dependence of the second kind (IV, 14). Then, we have in general (IV, 40)

$$(64) \quad \varphi_{xy} > 0, \quad \varphi_{xz} > 0, \quad \dots, \quad \varphi_{yz} > 0, \quad \dots$$

In the three cases, the indices have the characteristic indicated by equations (59); and we have^[62]

$$(65) \quad \varphi_{xx} < 0, \quad \varphi_{yy} < 0, \quad \varphi_{zz} < 0, \quad \dots$$

If the quantities dx, dy, \dots , are all assumed to be positive, or at least of the same sign, we have in the first and the third cases

$$(64)^{[63]} \quad d^2\varphi < 0.$$

This inequality might cease to hold if the quantities dx, dy, \dots , were not all of the same sign. This case, which is very important, will be examined later on (§124).

48. In the second of the cases just considered, the preceding analysis does not suffice to determine the sign of $d^2\varphi$, when the quantities dx, dy, \dots , all have the same sign. We must have recourse to other considerations. We have seen (IV, 42) that in this case a commodity composed of X, Y, Z... in fixed proportions¹⁶ may be treated as if it

were a single commodity, and consequently the inequality (64) holds. This has certain implications with respect to the partial derivatives of φ .

To have a commodity composed of x, y, z, \dots , we must set

$$(65) \quad y = ax, \quad z = \beta x, \quad \dots,$$

where a, β, \dots are positive constants. Then, the inequality (64) becomes

$$(66) \quad \varphi_{xx} + a^2\varphi_{yy} + \dots + 2a\varphi_{xy} + \dots + 2a\beta\varphi_{yz} + \dots < 0.$$

It is known that this entails

$$(67) \quad \varphi_{xx} < 0, \quad \begin{vmatrix} \varphi_{xx} & \varphi_{xy} \\ \varphi_{xy} & \varphi_{yy} \end{vmatrix} > 0, \quad \begin{vmatrix} \varphi_{xx} & \varphi_{xy} & \varphi_{xz} \\ \varphi_{xy} & \varphi_{yy} & \varphi_{yz} \\ \varphi_{xz} & \varphi_{yz} & \varphi_{zz} \end{vmatrix} < 0, \quad \dots;$$

in the case under consideration this provides a new characteristic of the indices.

In the inequalities¹⁷ (67) the variables x, y, z, \dots , must be permuted in every possible way, yielding other inequalities¹⁷ similar to (67).

In the case of two goods, the inequalities¹⁷ (67) become

$$\varphi_{xx} < 0, \quad \varphi_{xx}\varphi_{yy} - \varphi_{xy}^2 > 0.$$

We know that the second inequality is the condition for the *indicatrix* of the surface

$$I = \varphi$$

to be an ellipse.

49. Let us suppose for a moment that the inequalities (58) could be assumed to hold for goods having a dependence of the second kind.

The product of two negative quantities is a positive quantity; hence by multiplying together the two expressions of (58) we obtain¹⁸

$$(\varphi_{xx}\varphi_{yy} - \varphi_{xy}^2)\varphi_x\varphi_y - (\varphi_{xx}\varphi_y^2 + \varphi_{yy}\varphi_x^2 - 2\varphi_{xy}\varphi_x\varphi_y)\varphi_{xy} > 0.^{[64]}$$

For goods having a dependence of the second kind, we have

$$\varphi_{xy} < 0;$$

consequently, it is a positive quantity that must be subtracted from the first term in the above expression, and the result must be positive; we must therefore have

$$\varphi_{xx}\varphi_{yy} - \varphi_{xy}^2 > 0.$$

But this is precisely the condition for

$$\delta^2\Phi = \varphi_{xx}dx^2 + \varphi_{yy}dy^2 + 2\varphi_{yx}dxdy$$

always to have the same sign, which in this case is negative.

If we could, therefore, assume that the inequalities (58) hold for goods having a dependence of the second kind, then we see that in the case of two goods, the second variation of ophelimity would be negative (§124).

50. It is possible to proceed in a manner opposite to the one we have adopted, and start out from the properties of ophelimity in order to deduce the properties of indifference lines from them.¹⁵

51. The shapes of indifference lines are certainly very complex, and the examples we have furnished in the text show how difficult it is to submit them to algebraic analysis, apart from some special cases.¹⁶ The difficulties arise from the fact that analysis cannot easily be used to represent discontinuous functions such as, for example, those of Figure 31 (IV, 55) or of Figure 33 (IV, 57).

It would thus be futile to try to deal with the problem in its full extent; we must content ourselves with studying it in a small region around the point under consideration (IV, 67). Moreover it will be necessary to replace functions that could represent the ophelimities exactly by other functions that will often be only very rough approximations.

52. **The general laws of supply and demand.** Equations (51) determine the equilibrium point for an individual.^[65]

Let us set

$$\varphi_x = m.$$

Then, m is the [elementary index]²⁰ of the commodity whose price is equal to one—i.e., money.¹⁰

Let us write the first row of equations (51) in the form

$$(68) \quad \varphi_x = m, \quad \varphi_y = p_y m, \quad \varphi_z = p_z m, \quad \dots$$

Taking the derivatives with respect to p_y in all these equations, we obtain

$$(69) \quad \left\{ \begin{array}{l} \varphi_{xx} \frac{\partial x}{\partial p_y} + \varphi_{xy} \frac{\partial y}{\partial p_y} + \dots = \frac{\partial m}{\partial p_y} \\ \varphi_{xy} \frac{\partial x}{\partial p_y} + \varphi_{yy} \frac{\partial y}{\partial p_y} + \dots = p_y \frac{\partial m}{\partial p_y} + m \\ \varphi_{xz}^{[66]} \frac{\partial x}{\partial p_y} + \varphi_{yz} \frac{\partial y}{\partial p_y} + \dots = p_z \frac{\partial m}{\partial p_y} \\ \dots \end{array} \right.$$

¹⁵ For details, see the Italian edition.¹⁹

¹⁶ In the *Giornale degli Economisti*, Rome, September 1904, Professor Boninsegni published an excellent study on supply and demand functions for the case in which the elementary ophelimities are linear.

Let us denote by R the Hessian

$$(70) \quad R = \begin{vmatrix} \varphi_{xx} & \varphi_{xy} & \varphi_{xz} & \dots \\ \varphi_{xy} & \varphi_{yy} & \varphi_{yz} & \dots \\ \varphi_{xz} & \varphi_{yz} & \varphi_{zz} & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}$$

Let us denote by R_1, R_2, \dots , the determinants obtained by replacing the elements of the first column, second column, etc., of this determinant by the elements 1, p_y, p_z, \dots . Further, let $H_{i,n}$ denote the minor obtained by deleting the element in the i th row and n th column of R , this minor having the sign it must have in the development of $R^{[67]}$, so that

$$R = \varphi_{xx}H_{1,1} + \varphi_{xy}H_{2,1} + \dots$$

Owing to the form of R ,

$$H_{i,n} = H_{n,i}.$$

We shall have

$$(71) \quad \begin{cases} R \frac{\partial x}{\partial p_y} = mH_{2,1} + R_1 \frac{\partial m}{\partial p_y} \\ R \frac{\partial y}{\partial p_y} = mH_{2,2} + R_2 \frac{\partial m}{\partial p_y} \\ R \frac{\partial z}{\partial p_y} = mH_{2,3} + R_3 \frac{\partial m}{\partial p_y} \\ \dots \end{cases}$$

Taking the partial derivative of the last equation of (51), we shall have

$$(72) \quad 0 = \frac{\partial x}{\partial p_y} + y - y_0 + p_y \frac{\partial y}{\partial p_y} + p_z \frac{\partial z}{\partial p_y} + \dots$$

Let us form the determinant

$$M = - \begin{vmatrix} 0 & 1 & p_y & p_z & \dots \\ 1 & \varphi_{xx} & \varphi_{xy} & \varphi_{xz} & \dots \\ p_y & \varphi_{xy} & \varphi_{yy} & \varphi_{yz} & \dots \\ p_z & \varphi_{xz} & \varphi_{yz} & \varphi_{zz} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{vmatrix}$$

As before, let $M_{i,n}$ denote the minors of this determinant, each one with the appropriate sign in the development, so that

$$M = M_{2,1} + p_y M_{3,1} + p_z M_{4,1} + \dots$$

We shall have

$$(73) \quad \begin{cases} M = R_1 + p_y R_2 + p_z R_3 + \dots, \\ M_{3,1} = H_{2,1} + p_y H_{2,2} + p_z H_{2,3} + \dots. \end{cases}$$

Substituting the values (71) in equation (72) we shall obtain

$$0 = (y - y_0)R + m M_{3,1} + \frac{\partial m}{\partial p_y} M;$$

or

$$(74) \quad \frac{\partial m}{\partial p_y} = - \frac{(y - y_0)R + m M_{3,1}}{M};$$

and consequently,

$$(75) \quad \frac{\partial y}{\partial p_y} = \frac{-(y - y_0) + m \left(\frac{MH_{2,2}}{RR_2} - \frac{M_{3,1}}{R} \right)}{M} R_2.$$

This formula^{[68][69]} shows how the demand (or the supply) of a commodity, Y, varies with the price p_y , of this commodity, in the most general case in which the elementary indices are functions of all the variables x, y, z, \dots ¹⁷

For another good, e.g., Z, we shall have

$$(76) \quad \frac{\partial z}{\partial p_y} = \frac{-(y - y_0) + m \left(\frac{MH_2}{RR_3} - \frac{M_{3,1}}{R} \right)^{[70]}}{M} R_3.$$

53. In the first case of §47, i.e., when we have

$$\varphi_{xy} = 0, \quad \varphi_{xz} = 0, \quad \dots, \quad \varphi_{yz} = 0, \quad \dots;$$

we obtain

$$(77) \quad \begin{aligned} R &= \varphi_{xx} \varphi_{yy} \varphi_{zz} \dots, \quad H_{1,1} = \frac{R}{\varphi_{xx}}, \quad H_{2,2} = \frac{R}{\varphi_{yy}}, \quad \dots \\ H_{2,3} &= 0, \quad \dots, \quad R_1 = R \frac{1}{\varphi_{xx}}, \quad R_2 = R \frac{p_y}{\varphi_{yy}}, \quad \dots \\ M_{3,1} &= p_y H_{2,2}, \quad \dots \\ \frac{M}{R} &= \frac{1}{\varphi_{xx}} + \frac{p_y^2}{\varphi_{yy}} + \frac{p_z^2}{\varphi_{zz}} + \dots \end{aligned}$$

¹⁷ I derived these formulae for the first time in the *Giornale degli Economisti*, August 1892.²²

For brevity, let us put

$$T = \frac{M}{R};$$

then we shall have

$$(78) \quad \begin{cases} \frac{\partial m}{\partial p_y} = -\frac{y - y_0 + m \frac{p_y}{\varphi_{yy}}}{T} = -\frac{y - y_0 + \frac{\varphi_y}{\varphi_{yy}}}{T}, \\ \frac{\partial y}{\partial p_y} = \frac{-(y - y_0)p_y + m \left(T - \frac{p_y^2}{\varphi_{yy}} \right)}{T\varphi_{yy}}, \\ \frac{\partial z}{\partial p_y} = \frac{\partial m}{\partial p_y} \frac{p_z}{\varphi_{zz}}, \quad \dots \end{cases}$$

Further,

$$(79) \quad \frac{\partial p_y(y - y_0)}{\partial p_y} = -\frac{\partial m}{\partial p_y} \left(T - \frac{p_y^2}{\varphi_{yy}} \right).$$

54. In these formulae, m is an intrinsically positive quantity, as are the prices p_y, p_z, \dots . The quantities $\varphi_{xx}, \varphi_{yy}, \dots$ being negative,^[71] T is a negative quantity, and $T\varphi_{yy}, T\varphi_{zz}$, are positive quantities. Finally, by virtue of formula (77), the quantity

$$T - \frac{p_y^2}{\varphi_{yy}} \quad [72]$$

is negative.

If commodity Y is *demanded* by the individual, the quantity $y - y_0$ is positive; it is negative if the commodity is *supplied*.

As a result of this fact, the following conclusions may be deduced from formulae (78).

(1) If commodity Y is demanded, then always

$$\frac{\partial y}{\partial p_y} < 0.$$

*The demand decreases when the price increases.*¹⁸

Since the numerator of $\partial m / \partial p_y$ consists of one positive and one negative term, nothing can be concluded with respect to its sign. But equation (79) shows that this sign is the

¹⁸ One should not confuse this general proposition obtained in the case in which the price of a commodity depends upon all the quantities exchanged, and, conversely, the quantity of a commodity exchanged depends on all prices, with apparently similar propositions obtained by assuming that the price of a commodity depends only on the amount of this commodity bought or sold. A table showing the price of a commodity in one column and the amount of this commodity an individual buys or sells in the other, with other commodities not taken into account, does not correspond to reality; it is only a hypothetical case.

same as that of the term on the left in (79). Now, this term represents the variation in the expenditure

$$p_y(y - y_0)$$

incurred by the individual in providing himself with some Y.

When the price of Y increases, three cases are possible: (a) The individual reduces his expenditure on Y; then he has more money at his disposal for other purchases; the ophelimity index of money must therefore decrease. This is what is indicated in our formulae by a negative value of $\partial m / \partial p_y$. The third row of formulae (78) shows that the amounts of Z, U, . . . , demanded all increase.

(β) The expenditure on Y remains the same. We then have

$$\frac{\partial m}{\partial p_y} = 0$$

and the amounts of Z, U, . . . , demanded all remain the same.

(γ) The expenditure on Y increases. Then the individual has less money at his disposal. He reduces his expenditure on other commodities and, as our formulae indicate, the elementary index of the ophelimity of money increases.

(2) If commodity Y is supplied, the numerator of the second formula of (78) has one positive and one negative term.^[73] Nothing may be concluded about the sign of $\partial y / \partial p_y$. On the contrary, we always have

$$\frac{\partial m}{\partial p_y} < 0, \quad \frac{\partial z}{\partial p_y} > 0, \quad \dots;$$

and by changing the sign of $y - y_0$ in formula (79), we obtain

$$\frac{\partial p_y(y_0 - y)}{\partial p_y} > 0.$$

The individual thus obtains a larger total sum from his sale of commodity Y. We cannot say whether he will sell more or less of it. The ophelimity index of money decreases.

55. These results have been obtained for the case in which

$$\varphi_{xy} = 0, \quad \varphi_{xz} = 0, \quad \dots, \quad \varphi_{yz} = 0, \quad \dots,$$

but when these quantities are sufficiently small—which corresponds in practice to a very large number of cases—equations (78) and (79) still hold approximately; consequently, the results do not differ from those we have just indicated.

But it should not be forgotten that there are other cases in which the values of the φ_{xy}, \dots , may be large enough to modify these results.

56. Following Professor Marshall, several economists have believed it possible, in general, to treat the ophelimity index m of money as constant, for small variations in the prices and quantities. This amounts to setting

$$\frac{\partial m}{\partial p_y} = 0.$$

The formulae we have just presented show that this proposition cannot be accepted.¹⁹
Even when the quantity

$$\frac{\partial m}{\partial p_y}$$

is very small, it cannot be disregarded unless one can prove that the error thus committed does not affect the results obtained.

57. When information is available concerning the variations of demands and supplies, formulae (74), (75), (76), and (78) enable us to obtain information about the ophelimity indices; and conversely.

58. Let us suppose that in the case of equations (78) we have

$$(80) \quad \frac{\partial m}{\partial p_y} = 0.$$

For this equation to hold, either the denominator of this expression, which is given by formulae (78), must be infinite, or the numerator must be zero.

The denominator T can be quite large—even very large—when a very large number of goods are involved, but it is not infinite, at least in general. If one assumes that $\partial m / \partial p_y$ can be disregarded, because T is very large, it will follow that all the $\partial z / \partial p_y, \partial u / \partial p_y, \dots$ can also be disregarded as well. As for $\partial y / \partial p_y$, we have

$$\frac{\partial y}{\partial p_y} = \frac{m}{\varphi_{yy}}.$$

The hypothesis just made amounts to assuming that when p_y varies, only the quantity y varies, whereas z, u, \dots , remain constant. This assumption can be accepted in certain cases^[74] but in general it cannot.

The equation

$$0 = \frac{\partial x}{\partial p_y} + \frac{\partial(p_y y)}{\partial p_y} + p_z \frac{\partial z}{\partial p_y} + \dots$$

indicates the variation of the budget when p_y varies. If the hypothesis made above is accepted, all the terms vanish except one, and we obtain

$$\frac{\partial(p_y y)}{\partial p_y} = 0,^{[75]}$$

but this is impossible,²⁴ because if on the one hand the terms supposed to be zero are effectively very small, they are on the other hand very numerous, so that their sum is not negligible.

¹⁹ I emphasized this point in the *Giornale degli Economisti*, March 1892, April 1895, and in the *Cours*, §84.²³

In every problem relating to exchange, the budget equation and the consideration of its variations are essential, at least in general. It follows that in these problems, one cannot, except in very special cases that have to be justified, assume the ophelimity index m to be constant.

59. Let us now consider the other hypothesis, that is, that the numerator vanishes.

One then obtains

$$y - y_0 + \frac{\varphi_y}{\varphi_{yy}} = 0,$$

and consequently,

$$\varphi_y = \frac{B}{y - y_0};$$

B being an arbitrary constant. Now, it cannot in general be assumed that the ophelimity index of the consumption of the amount y depends on the initial amount y_0 held by the individual. In the cases of the commodities supplied, y_0 cannot be zero, and consequently the form we have just found for φ_y , as well as the assumption that led up to it, must be rejected.

In the cases of commodities demanded, y_0 can be zero, and the form

$$\varphi_y = \frac{B}{y}$$

becomes admissible. But let us look at its consequences.

Suppose an individual supplies some of X, and buys all the other commodities Y, Z, . . . , for which we assume

$$(81) \quad \varphi_y^{25} = \frac{B}{y}, \quad \varphi_z = \frac{C}{z}, \quad \dots$$

We shall have

$$x_0 - x = p_y y + p_z z + \dots,$$

$$\varphi_x = \frac{B}{p_y y} = \frac{C}{p_z z} = \dots;$$

and consequently,

$$(82) \quad (x_0 - x)\varphi_x = B + C + \dots$$

This equation shows that the expenditure $x_0 - x$ incurred by the individual does not vary when the prices of the commodities he purchases vary. This is not admissible, in general.

Moreover, there remains φ_x ; and if the form

$$\varphi_x = \frac{A}{x_0 - x},$$

is not admitted, the ophelimity index is no longer constant. If we assume for a moment that φ_x has this inadmissible form,^[76] we are led to still less acceptable consequences.

In fact, equation (82) then becomes

$$A = B + C + \dots$$

If this relation among the constants of the ophelimity indices does not hold, equilibrium is impossible. If, by pure chance, it were to hold, equilibrium would be indeterminate. One might arbitrarily choose a value for x , and then set

$$y = \frac{B(x_0 - x)}{Ap_y}, \quad z = \frac{C(x_0 - x)}{Ap_y}, \quad \dots;$$

equilibrium would then subsist.

In any case, we are thus led to consequences that compel us to reject—at least in general—the assumption of a constant ophelimity index m .

60.^[77] Let us suppose that $\alpha, \beta, \gamma, \dots$, are very small quantities, and set

$$\varphi_x = \frac{A}{x^{1+\alpha}}, \quad \varphi_y = \frac{B}{y^{1+\beta}}, \quad \varphi_z = \frac{C}{z^{1+\gamma}}, \quad \dots$$

We obtain

$$p_y y = \left(\frac{B}{A} \right)^{\frac{1}{1+\beta}} x^{\frac{1+\alpha}{1+\beta}} p_y^{\frac{\beta}{1+\beta}};$$

and then,

$$p_y y = \frac{B}{A} x (1 + \epsilon_y).$$

In similar fashion we obtain²⁶

$$p_z z = \frac{C}{A} x (1 + \epsilon_z), \quad \dots;$$

where $\epsilon_y, \epsilon_z, \dots$, are very small quantities.^[78]

In general, if we put

$$h_0 = x_0 + p_y y_0 + p_z z_0 + \dots;$$

the last equation of (51) becomes

$$h_0 = x_0 + p_y y + p_z z + \dots$$

We shall have

$$\begin{aligned} h_0 &= x(H + K), \\ H &= 1 + \frac{B}{A} + \frac{C}{A} + \dots, \quad K = \frac{B}{A}\epsilon_y + \frac{C}{A}\epsilon_z + \dots \end{aligned}$$

Consequently,

$$x = \frac{h_0}{H + K} = \frac{h_0}{H}(1 + l).$$

Since the quantity K is very small, the quantity l will be also. Then, we will obtain

$$(83) \quad \left\{ \begin{array}{l} p_y y = \frac{B}{A} \frac{h_0}{H} (1 + l)(1 + \epsilon_y), \\ p_z z = \frac{C}{A} \frac{h_0}{H} (1 + l)(1 + \epsilon_z), \\ \dots \end{array} \right.$$

61. The value of T , given in §53, becomes

$$\begin{aligned} -T &= \frac{x^{2+a}}{(1+a)A} + \frac{p_y^2 y^{2+\beta}}{(1+\beta)B} + \dots \\ &= \frac{x^{2+a}}{(1+a)A} + \frac{p_y^2 B x^2 (1 + \epsilon_y)^2 y^\beta}{(1+\beta)A^2} + \dots \\ &= \frac{x^2}{A} (H + q), \end{aligned}$$

where q is a very small quantity.

Now, equations (78) yield

$$(84) \quad \left\{ \begin{array}{l} \frac{\partial m}{\partial p_y} = -\frac{y_0 A H}{h_0^2} (1 + m_y) \\ \frac{\partial y}{\partial p_y} = \frac{B}{A} \frac{p_y y_0 - h_0}{H p_y^2} (1 + n_y) \\ \frac{\partial z}{\partial p_y} = \frac{C y_0}{A H p_z} (1 + r_y) \\ \dots \end{array} \right.$$

where m_y, n_y, r_y, \dots , are all very small quantities.

These formulae may also be obtained directly from equations (82) and (83).

62. If Y is a commodity that is demanded, at least one other commodity must be supplied. Suppose it is Z; z_0 must not be zero, but it must be a positive quantity; consequently,^[80]

$$p_y y_0 - h_0 < 0;$$

and we verify again, by virtue of equations (84), that

$$\frac{\partial y}{\partial p_y} < 0.$$

If Y is supplied, at least one commodity must certainly be demanded; but the initial amounts of the commodities demanded may be zero. If Y is the only commodity supplied, and all the others are demanded, with zero initial amounts, we have

$$\frac{\partial y}{\partial p_y} = 0.$$

If another commodity, say, U, were to be supplied, u_0 could not be zero, and consequently

$$\frac{\partial y}{\partial p_y} < 0.$$

When y decreases, the amount $y_0 - y$ supplied increases. The increase in price will therefore always cause the supply to increase. For the supply to start decreasing after having increased, the factor $1 + n_y$ will have to change sign; but this is not possible as long as n_y remains a very small quantity. The hypothesis just made is therefore not compatible with the hypothesis that a, β, \dots , are very small quantities, unless, to make up for this, some other quantities become very large.

When Y is demanded, and $y_0 = 0$, the principal part of $p_y y$ in the formulae (83) is independent of p_y ; it therefore remains constant when p_y varies. The variation can then only come from the terms in l and in ϵ_y , which may be neglected when the ophelimity indices are assumed to have the form (81).

63. General case of exchange with constant prices. Suppose we have θ individuals, indicated by $1, 2, \dots, \theta$, and m commodities X, Y, Z, ...

Let us assume that all the individuals behave, in their exchanges, according to type I, i.e., the system of free competition (III, 4).^[81] This means that each one of them accepts the market prices, even though in reality these prices are *indirectly* modified by the exchanges carried out by these individuals.²⁰ For each individual, we shall therefore

²⁰ As we have already observed, this condition must never be forgotten. Its omission would invalidate the proposition of which it forms an essential part.

I often repeat certain considerations because they are constantly overlooked, forgotten, or passed over by persons who write on the subject of economic theory.

have equations similar to equations (51). Let the subscript i be attached to each symbol referring to individual i . Equations (51) and those that express the fact that the total quantities of goods do not vary in the exchange, yield

$$(A) \quad \left\{ \begin{array}{l} \varphi_{1x} = \frac{1}{p_y} \varphi_{1y} = \frac{1}{p_z} \varphi_{1z} = \dots, \\ \varphi_{2x} = \frac{1}{p_y} \varphi_{2y} = \frac{1}{p_z} \varphi_{2z} = \dots, \\ \dots \end{array} \right.$$

$$(B) \quad \left\{ \begin{array}{l} x_1 - x_{10} + p_y(y_1 - y_{10}) + p_z(z_1 - z_{10}) + \dots = 0, \\ x_2 - x_{20} + p_y(y_2 - y_{20}) + p_z(z_2 - z_{20}) + \dots = 0, \\ \dots \\ x_{\theta-1} - x_{\theta-1,0} + p_y(y_{\theta-1} - y_{\theta-1,0}) + \dots = 0. \end{array} \right.$$

$$(C) \quad \left\{ \begin{array}{l} x_1 - x_{10} + x_2 - x_{20} + \dots = 0, \\ y_1 - y_{10} + y_2 - y_{20} + \dots = 0, \\ \dots \end{array} \right.$$

It will be noted that, in the system of equations (B), we do not write down the equation corresponding to the subscript θ , for this equation is the consequence of the other equations (B) and of equations (C). If it was desired to write down this equation, some other one would have to be eliminated.

The equations just written correspond to the categories (A), (B), (C) of the conditions indicated in (III, 199 et seq.)

The unknowns are: (1) the $m - 1$ prices; (2) the $m\theta$ quantities $x_1, x_2, \dots, y_1, y_2, \dots$; in all, $m\theta + m - 1$.

The equations are: (1) the $(m - 1)\theta$ equations (A); (2) the $\theta - 1$ equations (B); (3) the m equations (C); or, in all, $m\theta + m - 1$.

The equations are equal in number to the unknowns, and the problem is thus well determined.^[82]

As has already been noted (§35) in an analogous case, it must not be forgotten that equations (B) and (C) hold for all values of the variables $x_1, x_2, \dots, y_1, y_2, \dots$; whereas equations (A) hold only for the values of these variables corresponding to the point of equilibrium.

In the system (A), $\varphi_{1x}, \varphi_{2x}, \dots$, may be functions of all the variables x, y, z, \dots , and so may $\varphi_{1y}, \dots, \varphi_{1z}, \dots$, etc.

64. If the equation missing from (B) is reintroduced, and one of the equations of (C) is removed to make way for it, equations (A) and (B) can be handled just as we have handled the system (51) in order to derive the laws of supply and demand. The $m\theta$ quantities $x_1, x_2, \dots, y_1, y_2, \dots$ will be functions of the $m - 1$ unknowns p_y, p_z, \dots , and the $m - 1$ equations (C) will enable us to determine these unknowns.

65. It may happen that the holder of a commodity being supplied, say Y, does not use it to satisfy his tastes; we then say that *he supplies the entire amount at his disposal*. If we

indicate this by $y_{10} = y_1$,^[83] where individual 1 is the holder of this commodity, there will be one less unknown. On the other hand, the equation of the system (A) containing the quantity φ_{1y} must be removed. The number of equations thus still remains equal to the number of unknowns.

66. Since money is a commodity, it must have its own ophelimity for some individuals, but it may have none for others. Suppose that X has no ophelimity for individual 1. Then in system (A), the equation in φ_{1x} must be removed; we thus lose one equation.

But on the other hand, since X has no ophelimity for individual 1, he will not consume any of it. He will use the entire amount of it that he receives to acquire goods Y, Z, . . . , among which there is a good that represents saving. We shall then have

$$x_1 - x_{10} = 0;^{[84]}$$

this determines x_1 . We therefore have one less unknown, and the number of equations is again equal to the number of unknowns.

67. Operations according to type II. Let us assume that individual 1 does not accept the prices as he finds them in the market, but that he attempts to modify them in order to achieve a certain goal.

This case includes the one commonly known by the name of monopoly. The individual sells some Y and purchases other commodities. He does not take the ophelimity index of Y into account either because Y has no ophelimity for him, or because an excess of Y is of no concern to him, provided he attains his other goals.^[85]

Among these goals we shall consider two principal ones: (a) the individual tries to obtain the maximum yield, expressed in money, from his monopoly; (b) the individual tries to obtain the maximum ophelimity.

68. Monopoly of one individual and one commodity. Since the individual does not take the ophelimity index of Y into account, the equation in φ_{1y} is missing from the system (A). To restore the equality between the number of equations and the number of unknowns, one of the unknowns must be given. Suppose it is p_y ; we shall then have^[86]

$$y_{10} - y_1 = f(p_y).$$

(a) If individual's object is to extract the largest possible amount of money from his monopoly, he will have to maximize

$$(y_{10} - y_1)p_y = p_y f(p_y),$$

and this requires

$$(85) \quad \frac{d(p_y f)}{dp_y} = 0.$$

This equation serves to determine p_y , and the problem is solved.^[87]

If there were a terminal point^[88] preceding the value of y thus determined, the individual would have to stop at that point. This is the case in which he does not possess an amount of Y corresponding to the maximum given by equation (85).

(β) If the individual's object is to obtain the maximum ophelimity, he must set^[89]

$$\frac{d\varphi_1}{dp_y} = 0;$$

or

$$(86) \quad 0 = \varphi_{1x} \frac{dx_1}{dp_y} + \varphi_{1y} \frac{dy_1}{dp_y} + \varphi_{1z} \frac{dz_1}{dp_y} + \dots$$

We know x_1, y_1, \dots , as functions of p_y ; equation (86) thus contains only known quantities, and it solves the problem.

To return from the case of monopoly to the case of free competition, it is necessary to express the fact that $d\varphi_1$ is zero, not when p_y varies but, on the contrary, when y varies while p_y remains constant. Then instead of equation (86) we have

$$0 = \varphi_{1x} dx_1 + \varphi_{1y} dy_1,$$

and, recalling the definition of the price, we shall have

$$0 = \varphi_{1x} - \frac{1}{p_y} \varphi_{1y};$$

this is precisely the equation that was missing because it had been suppressed.

The case in which individual 1 also has a monopoly in Z can be treated in much the same way as the one we have just indicated.

69. Monopoly of two individuals and one commodity.^{21[90]} Suppose that individuals 1 and 2 sell Y, by operating according to type II, and that they purchase the other commodities.

Two equations are now missing in the system (A), and consequently, two unknowns must be given. Let us take p_y and y_2 as given; all the remaining unknowns will be expressed as functions of the latter two, and if we put

$$s_1 = (y_{10} - y_1)p_y, \quad s_2 = (y_{20} - y_2)p_y,$$

we shall have an equation of the form

$$(87) \quad F(s_1, s_2, p_y) = 0.^[91]$$

(a) To obtain the maximum yield in money, s_1 and s_2 must be maximized.

²¹ Professor F. Y. Edgeworth was the first to deal with a special case of this problem, by making certain assumptions. See the *Giornale degli Economisti*, July 1897.²⁷

Let an arbitrary value be given to s_2 . The condition for s_1 to be maximized is^[92]

$$(88) \quad \frac{\partial F}{\partial p_y} = 0.$$

If we eliminate p_y in equations (87) and (88), we shall have

$$(89) \quad f(s_1, s_2) = 0.$$

If an arbitrary value had been given to s_1 , and if we had sought the condition for a maximum of s_2 , we would still have had equation (88), and consequently we would have fallen back on equation (89). The latter therefore furnishes the maximum value of s_1 for an arbitrary value of s_2 , and vice versa.

Geometrically, equation (89) represents the visible outline of the surface (87) on the $s_1 s_2$ plane.

We have taken s_2 to be arbitrary, and have found equation (89) in order to determine the maximum s_1 when p_y varies. Now let s_2 vary and let us determine the maximum of s_1 ;^[93] we shall have

$$(90) \quad \frac{\partial f}{\partial s_2} = 0.$$

Conversely, if we wanted to determine the maximum of s_2 when s_1 varies, we would have

$$(91) \quad \frac{\partial f}{\partial s_1} = 0.$$

We would thus have three equations to determine our two unknowns. The assumptions that have led us to this result are therefore not generally compatible; and it may not be assumed that both individuals act according to type II.

Geometrically, the two equations (90) and (91) can hold only at singular points of the curve (89). Equations (89) and (90) determine the point (α) where the curve qr , whose equation is (89), has a tangent parallel to the s_2 -axis. Equations (89) and (91) determine the point (β) where this curve has a tangent parallel to the s_1 -axis. These two points are generally different; and, as a consequence, the three equations (89), (90), (91) are not compatible.

From the mathematical point of view it is inaccurate to say, as is often done, that in the case of two monopolists and one commodity the problem of equilibrium is indeterminate. On the contrary it is overdetermined, since conditions imposed that are incompatible.^[94]

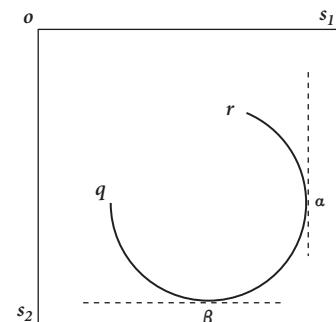


FIG 60

70. (β). Suppose that it is desired to make the ophelimity index a maximum. Let us replace the two equations that are missing in the system (A) by the equations

$$(92) \quad t_1 = \varphi_1, \quad t_2 = \varphi_2;$$

t_1 and t_2 being new variables. The expressions for φ_1 and φ_2 are known to us as functions of $x_1, y_1, \dots, x_2, y_2, \dots$; we shall thus obtain the expressions of all the unknowns as functions of t_1 and t_2 ; and we shall have an equation of the form

$$F(t_1, t_2, p_y) = 0.^{[95]}$$

The remainder of the reasoning is now similar to that set out above, and leads to the same consequences.

71. **Monopoly of two individuals and two commodities.** Let us suppose that individual 1 sells some of Y and purchases all the other commodities, and that individual 2 sells some of Z and purchases all the other commodities.

Two equations are still missing in the system (A) and we must, as before, take two unknowns as given. Let us take p_y, p_z . All the other quantities will become functions of p_y, p_z ; we shall thus have^[96]

$$(93) \quad F_1(s_1, p_y, p_z) = 0, \quad F_2(s_2, p_y, p_z) = 0,$$

or

$$(94) \quad f_1(\varphi_1, p_y, p_z) = 0, \quad f_2(\varphi_2, p_y, p_z) = 0.$$

We must now make s_1 a maximum, *when p_y varies*; and s_2 must also be made a maximum—not, as previously when p_y varies, but *when p_z varies*. That is the essential difference from the previous problem, and this difference is the cause of the difference in the conclusions.

When we consider the maximization of ophelimity, we must similarly make φ_1 a maximum *when p_y varies*, and make φ_2 a maximum *when p_z varies*.

In the case in which the monopolists wish to obtain the greatest yield in money from their monopoly, we will therefore have to set

$$\frac{\partial F_1}{\partial p_y} = 0, \quad \frac{\partial F_2}{\partial p_z} = 0.$$

These two equations, when added to the two equations (94),^[97] enable us to determine the four unknowns s_1, s_2, p_y, p_z . The problem is therefore solved.

In the case in which monopolists seek the maximum ophelimity, we must set

$$\frac{\partial f_1}{\partial p_y} = 0, \quad \frac{\partial f_2}{\partial p_z} = 0.^{28}$$

These two equations, when added to the two equations (94), solve the problem.

The difference between this problem and the preceding one lies essentially in the fact that in the preceding problem it was desired to make s_1 a maximum when p_y and s_2 varied, and also s_2 a maximum when s_1 and p_y varied. This is impossible.

In the present problem it is proposed to make s_1 a maximum when p_y and p_z vary, and s_2 a maximum when p_y and p_z vary. There is nothing impossible in this—at least in general—and the problem is well determined.

The same conclusion is reached by seeking the maximum of φ_1 and the maximum of φ_2 .^[98]

72. The following considerations are so elementary that they are probably superfluous.

In the problem of §69,^[99] the unknowns p_y and p_z could be taken as given. This would, of course, change nothing in the conclusions.

Let us suppose that in the case of the present problem, we had the two equations

$$(95) \quad F_1(s_1, p_y, p_z) = 0, \quad F_2(s_2, p_y, p_z) = 0.$$

If we arbitrarily fix the amount s_2 that individual 2 reaps from his monopoly, the second of the equations we have just written down determines p_z as a function of s_2, p_y . We shall then have

$$\frac{\partial F_2}{\partial p_y} + \frac{\partial F_2}{\partial p_z} \frac{\partial p_z}{\partial p_y} = 0.$$

The condition that s_1 be a maximum when p_y is made to vary yields

$$\frac{\partial F_1}{\partial p_y} + \frac{\partial F_1}{\partial p_z} \frac{\partial p_z}{\partial p_y} = 0.$$

These two equations lead to the following one:

$$\frac{\partial F_1}{\partial p_y} \frac{\partial F_2}{\partial p_z} - \frac{\partial F_2}{\partial p_y} \frac{\partial F_1}{\partial p_z} = 0.$$

Let us eliminate p_y, p_z from this equation and the two equations (95); we shall obtain an equation of the form (89). The remainder of the reasoning is identical with the reasoning that was applied to this equation, and yields the same results.

73. From the economic point of view, it may be observed in the case of the problem of §69 that by assuming there to be a position in which one of the monopolists obtains s_1 from his monopoly, and the other obtains s_2 , it is sufficient for the first one to lower his price by an insignificant amount in order to increase his gain and reduce his competitor's share to zero; and vice versa. It is therefore impossible to obtain a solution of the problem we have posed, since no position s_1, s_2 is an equilibrium position.^[100]

74. By reasoning in this manner, we are tempted to believe that the problem is indeterminate, contrary to what was said in §69.

The source of this contradiction is to be found in the way the problem is posed. We must distinguish between the *power* an individual has to act as a monopolist, and the *fact* that he exercises this power by behaving according to type II.

In the problem of §69, we assume that two individuals behave *in fact* according to type II, in order to sell the same commodity, Y, and we arrive at the conclusion that our hypothesis cannot be realized.

In the problem of §73, we assume that two individuals have the *power* to behave according to type II, in selling a same commodity Y, and we arrive at the conclusion that the problem is indeterminate, because we do not know what use each of the two individuals will make of his *power*.

This conclusion is identical with the preceding one. If both individuals could use their monopoly, we would not need to know what use each of them would make of it in order for the problem to be determinate.

75. It is idle to ask pure economics what will happen when two individuals who have the power to act as monopolists in the sale of one and the same commodity confront each other. Pure economics, by telling us that it is impossible for these two individuals to use their monopoly in fact, for them both to behave according to type II, has answered as fully as it can. It is up to the observation of facts to teach us the rest.

Pure economics cannot even tell us that the two individuals will go back and forth indefinitely between two extreme positions of equilibrium. This in no way results from the fact that equilibrium is determined by two incompatible equations.

76. Still less should it be imagined that the observation of facts will lead us to a unique solution. On the contrary, there is an infinity of solutions.

First of all there are a great many and a great variety of cases in which the two *potential* monopolists reduce to one monopolist in fact. If the two monopolists reach an agreement, there is only one left. *Cartels, trusts, etc.*, reveal many ways in which such an agreement can come about. Likewise, there is only one monopolist left if the second [potential] monopolist accepts the prices fixed by the first one, who is then acting according to type II.

This last case arises frequently in the real world. When a company “controls” (this is the technical term) a significant fraction of output—for instance, 80/100—it often happens that it is this company that fixes the prices; the producers of the remaining 20/100 accept them as given.

Next, there are the very numerous cases in which a commodity, Y, which appears to be a single commodity, is in fact divided into several commodities. Thus, a lady of fashion does not buy her clothes in department stores; she goes to a dressmaker. Accessory circumstances, too, such as credit, special regard for customers, etc., may differentiate commodities that are otherwise identical.

Finally, the aim of monopolist 1 may be to ruin his rival, monopolist 2; or alternatively, to enable him barely to subsist, so as not to drive him into taking the risk of engaging in a cutthroat struggle. There are an infinity of other circumstances of this kind, all of which change the nature of the problem in question.

Moreover, such a change may, in exceptional cases, arise out of the problem itself. Suppose, for instance, that the sum $y_{10} + y_{20}$ of the amounts of Y possessed by individual 1 and individual 2 is less than the amount which, in the case in which 1 and 2 formed a single monopoly, corresponded to the maximum amount of X that they would be able to

obtain from their monopoly. In this case individuals 1 and 2 both have terminal points (III, 62); it will suit them each to offer the total amount available. We are no longer in the case of individuals behaving according to type II, but in the case of individuals behaving according to type I (§65).

It is well to remember that most cases observed in reality are cases of production monopoly rather than of exchange monopoly.

76 bis. An example may be useful to clear up some points.

Suppose we have two monopolists, who sell commodity Y and buy commodities X and Z. To simplify matters, we consider a single consumer; the case in which there are several consumers is, moreover, similar.^[101] The consumer buys y of commodity Y, and sells $x_0 - x$ of X and $z_0 - z$ of Z. We assume that the ophelimity indices have the forms implied in the following equations.

For the two monopolists we have the equations

$$\frac{1}{x_1} = \frac{1}{p_z} \frac{c'}{z_1^2}, \quad \frac{1}{x_2} = \frac{1}{p_z^{[102]}} \frac{c''}{z_2^2},$$

$$p_y y_1 = s_1 = x_1 + p_z z_1, \quad p_y y_2 = s_2 = x_2 + p_z z_2. \quad [103]$$

From these equations we obtain

$$s_1 = p_z \left(z_1 + \frac{z_1^2}{c'} \right), \quad s_2 = p_z \left(z_2 + \frac{z_2^2}{c''} \right).$$

For the consumer, we have

$$\frac{1}{x^2} = \frac{a^2}{p_y y^4} = \frac{b}{p_z z},$$

$$x_0 - x = x_1 + x_2, \quad z_0 - z = z_1 + z_2. \quad [104]$$

We shall thus have

$$p_z z = \frac{b}{a^2} p_y y^4, \quad x = \frac{y^2}{a} \sqrt{p_y}.$$

Setting^[105]

$$s = s_1 + s_2,$$

the monopolists' equations give

$$s = x_1 + x_2 + p_z(z_1 + z_2) = x_0 - \frac{y^2}{a} \sqrt{p_y} + p_z z_0 - \frac{b}{a^2} p_y y^4;$$

whence

$$p_z = \frac{1}{z_0} \left(s - x_0 + \frac{y^2}{a} \sqrt{p_y} + \frac{b}{a^2} p_y y^4 \right).$$

Now,

$$y = y_1 + y_2 = \frac{s_1 + s_2}{p_y} = \frac{s}{p_y};$$

substituting this value in the preceding equation, we obtain p_z as a function of p_y and s . Consequently, the equations we obtained for the monopolists,

$$z_1 + \frac{z_1^2}{c'} = \frac{1}{p_z} s_1, \quad z_2 + \frac{z_2^2}{c''} = \frac{1}{p_z} s_2,$$

have as their right sides s_1 and s_2 multiplied by a function of p_y and s ; that is to say, these expressions are functions of p_y, s_1, s_2 . One must, of course, choose the positive roots of these equations, yielding

$$\begin{aligned} z_1 &= -\frac{c'}{2} + \sqrt{\frac{c'^2}{4} + \frac{c's_1}{p_z}}, \\ z_2 &= -\frac{c''}{2} + \sqrt{\frac{c''^2}{4} + \frac{c''s_2}{p_z}}. \end{aligned}$$

But

$$z = \frac{bp_y y^4}{a^2 p_z} = \frac{bs^4}{a^2 p_z p_y^3} = z_0 - (z_1 + z_2);$$

whence, finally,^[106]

$$z_0 - \frac{bs^4}{a^2 p_z p_y^3} + \frac{c' + c''}{2} - \sqrt{\frac{c'^2}{4} + \frac{c's_1}{p_z}} - \sqrt{\frac{c''^2}{4} + \frac{c''s_2}{p_z}} = 0.$$

This is equation (87)²⁹ of §69.

If there are only two commodities, X and Y, this equation takes on a particular form, which is interesting to examine.

For the consumer, we have simply

$$\frac{1}{x^2} = \frac{a^2}{p_y y^4}, \quad x_0 - x = x_1 + x_2;$$

and for the monopolists

$$s_1 = x_1, \quad s_2 = x_2.$$

We therefore have

$$\frac{y^2 \sqrt{p_y}}{a} = x_0 - s;$$

and

$$x_0 - s - \frac{s^2}{a} p_y^{-\frac{3}{2}} = 0.$$

This is equation (87)²⁹ of §69, which thus takes the form^[107]

$$F(s_1 + s_2, p_y) = 0.$$

Eliminating p_y from this equation and equation (88),^[108] we obtain for equation (89)

$$(89 \text{ bis}) \quad f(s_1 + s_2) = 0,$$

and the two equations (90) and (91)^[109] become identical, namely

$$(90 \text{ bis}) \quad f'(s_1 + s_2) = 0.$$

But in this case, it is equations (89 bis) and (90 bis) that are in general incompatible.²² The remainder of the reasoning is the same as that of §69, and the conclusions are identical.^[110]

77. [111] Production. Let us suppose that certain goods, A, B, C, . . . , or³⁰ certain capital services, are transformed into other goods, X, Y, Z,

Let θ = number of individuals;

n = number of commodities, or capital services, A, B, . . . ;

m = number of commodities, X, Y, . . . ;

π_x, π_y, \dots = cost of production for the producer of commodities X, Y, . . . ;

p_x, p_y, \dots = selling prices of the latter;

p_a, p_b, \dots = prices of commodities A, B, . . . ; we shall take A as money, and set

$$p_a = 1.$$

$x_1, y_1, \dots, x_2, y_2, \dots$ = quantities of output that are being consumed,^[112] up to some intermediate position;^[113]

$x'_1, y'_1, \dots, x'_2, y'_2, \dots$ = these same quantities at the equilibrium position.

²² With the givens of the problem.

$a_1, b_1, \dots, a_2, b_2, \dots$ and
 $a'_1, b'_1, \dots, a'_2, b'_2, \dots$ will have analogous meanings for A, B,

To simplify, we shall suppose that the initial quantities of commodities X, Y, ..., are zero. As for the initial quantities of A, B, ..., we shall indicate them by $a_{10}, b_{10}, \dots, a_{20}, \dots$

We shall further adopt the following definitions.

Let x, y, \dots = total quantities of commodities *produced*, in an intermediate position, before reaching the equilibrium position;

X'', Y'', \dots = these same quantities, *produced* when the equilibrium position has been reached;

X, Y, \dots = total quantities *consumed*, in an intermediate position, before reaching the equilibrium position;

X', Y', \dots = these same quantities *consumed*, when the equilibrium position has been reached;

a, b, \dots = quantities *supplied* to the firm in an intermediate position;

A, B, \dots = quantities *transformed* by the firm in an intermediate position;

A', B', \dots = quantities *consumed*, when an equilibrium position has been reached;

A'', B'', \dots = quantities *supplied* to the firm, when an equilibrium position has been reached;

A''', B''', \dots = quantities *transformed* by the firm when an equilibrium position has been reached;

A'_0, B'_0, \dots = initial quantities of $A', B' \dots$.

We shall have

$$(96) \quad \begin{cases} a'_1 + a'_2 + \dots = A', b'_1 + b'_2 + \dots = B', \dots, \\ a'_{10} + a'_{20} + \dots = A'_0, b'_{10} + b'_{20} + \dots = B'_0, \dots; \end{cases}$$

$$(97) \quad A'' = A'_0 - A', \quad B'' = B'_0 - B', \quad \dots$$

$$(98) \quad \begin{cases} x_1 + x_2 + \dots = X, y_1 + y_2 + \dots = Y, \dots, \\ x'_1 + x'_2 + \dots = X', y'_1 + y'^{[114]}_2 + \dots = Y', \dots. \end{cases}$$

At the equilibrium position, we shall have

$$(99) \quad \begin{cases} x = X'', y = Y'', \dots, \\ X = X', Y = Y', \dots, \\ A = A''', B = B''', \dots. \end{cases}$$

But these equations do not hold in an intermediate position.

In the case of free competition (III, 44–46), we would have to have

$$(100) \quad X'' = X', \quad Y'' = Y', \quad \dots$$

In the case of monopoly (for example of Y), Y'' may be larger than Y' , the difference going to the monopolist as profit. Or again, some of the quantities A', B', \dots , will be different from the corresponding quantities A'', B'', \dots , and the difference will be the monopolist's profit.^[115]

78. Production coefficients.^[116] The technical conditions of production will determine the quantities A, B, \dots , as functions of x, y, \dots ; i.e.,^[117]

$$A = F(x, y, \dots), \quad B = G(x, y, \dots), \quad \dots$$

The partial derivatives

$$(101) \quad a_x = \frac{\partial F}{\partial x}, \quad b_x = \frac{\partial G}{\partial x}, \quad \dots, \quad a_y = \frac{\partial F}{\partial y}, \quad \dots,$$

are called *production coefficients*. $a_x dx$ is the amount of A that is necessary to produce dx of X, when x of X, y of Y, etc., have already been produced; $a_y, \dots, b_x, b_y, \dots$, have analogous meanings.

By assuming the existence of the integral functions F, G, \dots , we implicitly assume that the quantities of A, B, C, ..., used in production do not depend upon the path followed to reach the point under consideration. This is indeed how things take place in the real world.^[118]

Suppose that a_x, b_x, \dots are functions only of x , that a_y, b_y, \dots , are functions only of y , etc.^[119] Suppose also that there are overhead costs A''', B''', \dots , independent of x, y, \dots . In that case the integral functions F, G , certainly exist. We shall have

$$(102) \quad \begin{cases} A''' = F = A'''_0 + \int_0^{X''} a_x dx + \int_0^{Y''} a_y dy + \dots, \\ B''' = G = B'''_0 + \int_0^{X''} b_x dx + \int_0^{Y''[120]} b_y dy + \dots, \\ \dots \end{cases}$$

If we assume that the production coefficients are constant, and that there are no overhead costs independent of the quantities produced, we shall have

$$(103) \quad \begin{cases} A''' = a_x X'' + a_y Y'' + \dots, \\ B''' = b_x X'' + b_y Y'' + \dots, \\ \dots \end{cases}$$

If we assume that there are overhead costs A'''_0, B'''_0, \dots , we shall have

$$(103 \text{ bis}) \quad \begin{cases} A''' = A'''_0 + a_x X'' + a_y Y'' + \dots, \\ B''' = B'''_0 + b_x X'' + b_y Y'' + \dots, \\ \dots \end{cases}$$

79. Costs of production. We assume that the outputs X, Y, \dots , are produced independently of one another. The costs of production of dx, dy, \dots , when x, y, \dots , have already been produced, will be

$$(104) \quad \begin{cases} \pi_x dx = (a_x + p_b b_x + p_c c_x + \dots) dx, \\ \pi_y dy = (a_y + p_b b_y + p_c c_y + \dots) dy, \\ \dots \end{cases}$$

These expressions may or may not be the partial derivatives of one and the same function. If it is assumed that they are, it is thereby assumed that the same result will always be obtained whatever be the order, or layout, of the production processes. Otherwise the production costs would vary with that order. This matter deserves to be resolved by observing what happens in the real world.^[121] In the meantime one may assume the prices p_b, p_c, \dots to be constant without going too far astray from reality. With this hypothesis, and recalling that we assumed a_x, b_x, \dots to be functions only of x, a_y, b_y, \dots , only of y , etc., the integral function, of which the expressions (104) represent the partial derivatives, certainly exists. One may, moreover, integrate each of these equations and obtain the production costs of X'', Y'', \dots , separately, i.e.,

$$(105) \quad \Pi_x = \pi_{0x} + \int_0^{X''} \pi_x dx, \quad \Pi_y = \pi_{0y} + \int_0^{Y''} \pi_y dy, \quad \dots,$$

$\pi_{0x}, \pi_{0y}, \dots$, being the overhead costs independent of x, y, \dots . Taking equations (102) and (104) into account, we obtain

$$(106) \quad \pi_{0x} + \pi_{0y} + \dots = A_0''' + p_b B_0''' + \dots$$

It must not be forgotten that, by saying that p_b, p_c, \dots , are constant, we mean only to say that the prices of successive portions of B, C, ..., used in the same operation, do not vary. This is case (δ) indicated in (III, 169).

For some investigations, we shall, on the contrary, have to assume that the prices p_x, p_y, \dots , vary with successive portions.

80. Consumers' equilibrium. Let us begin by assuming all prices to be constant. Suppose further that consumers act according to type I (free competition). What we said about exchange yields immediately the following equilibrium equations:

$$(A) \quad \begin{cases} \frac{1}{p_x} \varphi_{1x}(x'_1) = \dots \varphi_{1a}(a'_1) = \frac{1}{p_b} \varphi_{1b}(b'_1) = \dots, \\ \frac{1}{p_x} \varphi_{2x}(x'_2) = \dots \varphi_{2a}(a'_2) = \frac{1}{p_b} \varphi_{2b}(b'_2) = \dots, \\ \dots \end{cases};$$

$$(B) \quad \left\{ \begin{array}{l} a'_1 - a_{10} + p_b(b'_1 - b_{10}) + \dots + p_x x'_1 + p_y y'_1 + \dots = 0, \\ a'_2 [122] - a_{20} + p_b(b'_2 - b_{20}) + \dots + p_x x'_2 + p_y y'_2 + \dots = 0, \\ \dots \dots \dots \dots \dots \dots ; \end{array} \right.$$

$$(M) \quad \left\{ \begin{array}{l} x'_1 + x'_2 + \dots = X', \quad y'_1 + y'_2 + \dots = Y', \quad \dots, \\ a_{10} - a'_1 + a_{20} - a'_2 + \dots = A'', \quad b_{10} - b'_1 + b_{20} - b'_2 + \dots = B'', \quad \dots \end{array} \right.$$

There are $(m+n-1)\theta$ equations (A),
 " θ " (B),
 " $m+n$ " (M).
 Total $(m+n)\theta + m+n$ equations.

Summing equations (B) and taking account of equations (M) we shall have

$$(107) \quad A'' + p_b B'' + \dots = p_x X' + p_y Y' + \dots$$

If the prices p_x, p_y, \dots are variable, p_x being a function only of X , p_y of Y , etc., equation (107) will be replaced by

$$(107 \text{ bis}) \quad A'' + p_b B'' + \dots = \int_0^{X'} p_x dX + \int_0^{Y'} p_y dY + \dots$$

81. Equilibrium of the firms. We assume that the firms produce exactly the amounts of X, Y, \dots , that they sell, and that their gains or losses are expressed in terms of quantities of commodities A, B, \dots .

The quantities of A, B, \dots , that the firms need in order to produce the quantities x, y, \dots , have already been given in §78. We thus have the system

$$(108) \quad A''' = F, \quad B''' = G, \quad \dots$$

The total expenses Π_x, Π_y, \dots , necessary to produce x, y, \dots , are given by equations (105). Summing them, we have

$$(109) \quad A''' + p_b B''' + \dots = \Pi_x + \Pi_y + \dots$$

This equation could, moreover, have been written down directly, since each side represents the total expenses incurred in production.

82. Equilibrium of production. We now have to relate the firms and the consumers. Different economic states will result, according to the manner in which these relations between the firms and consumers are specified.

83. (a) Free competition. Entrepreneurs and consumers act according to type I. This state is characterized by the equality between the cost of production and the selling price of the commodities. We assume that this equality holds as between total receipts and total expenditures (§116). When prices are constant and there are no overhead costs,

this equality also entails the equality between the cost of production and the selling price of the last portion produced (§92).^[123]

We therefore have

$$(D) \quad p_x X' = \Pi_x, \quad p_y Y' = \Pi_y, \quad \dots$$

These equations added together give

$$p_x X' + p_y Y' + \dots = \Pi_x + \Pi_y + \dots;$$

and if we take equations (107) and (109) into account, this equation will become

$$A'' + p_b B'' + \dots = A''' + p_b B''' + \dots.$$

The quantities A'', B'', \dots , may well be larger than the quantities A''', B''', \dots , but they can not be smaller, since the firm cannot receive the commodities in question from anybody else but the consumers. The preceding equation thus entails the following ones:

$$(E) \quad A'' = A''', \quad B'' = B''', \quad \dots$$

In the case of variable prices, it suffices to replace $p_x X, p_y Y, \dots$, by

$$\int_0^X p_x dX, \quad \int_0^Y p_y dY, \dots$$

If the $\pi_{0x}, \pi_{0y}, \dots$, are zero, and the production coefficients are constant,²³ equations (D) become

$$(D') \quad \begin{cases} p_x = a_x + p_b b_x + p_c c_x + \dots, \\ p_y = a_y + p_b b_y + p_c c_y + \dots, \\ \dots \end{cases}$$

Owing to equations (103), equations (E) become

$$(E') \quad \begin{cases} A'' = a_x X'' + a_y Y'' + \dots, \\ B'' = b_x X'' + b_y Y'' + \dots, \\ \dots \end{cases}$$

Equation (107) is a consequence of the systems (B) and (M); equations (E) are a consequence of equations (D), (107), and (109).^[124] Consequently, in the system (B), (M), (109), (D) and (E), one equation follows from the others and must be dropped.

²³ This is the case studied by Mr. Walras. That author has the very great merit of having been the first to provide the general equations of economic equilibrium, in this particular case. He has thus opened up a very fruitful line of research.

This can also be seen directly from the systems (D'), (E'), which include the system (109). In fact, these systems give

$$A'' + B''p_b + \dots = p_x X'' + p_y Y'' + \dots;$$

or equivalently, since at the equilibrium point we have $X'' = X', Y'' = Y', \dots$,

$$A'' + B''p_b + \dots = p_x X' + p_y Y' + \dots.$$

Now, this equation is identical with equation (107), which follows from the systems (B) and (M).

Equations (109), (D), (E), one of which has been dropped, determine all but one—which remains unknown—of the values of $X, Y, [125] \dots, A'', B'', \dots$. The systems (A), (B), and (M) then comprise only that unknown, the $(m+n)\theta$ quantities $x_1, y_1, \dots, x_2, \dots, a_1, \dots$, and the $m+n-1$ prices—in all, therefore, $(m+n)\theta + m+n$ unknowns. But we have seen (§80) that the number of these equations is precisely $(m+n)\theta + m+n$. The problem of equilibrium is therefore solved, and determinate.

84. (β) Monopoly in production. Let us suppose that the producer of a commodity, Y, is able to act according to type II. One equation will be missing from the system (D), namely the equation

$$p_y Y' = \Pi_y.$$

Consequently, the complete system (E) no longer exists. Indeed this has to be the case, for if the entrepreneur makes a profit we need new data in order to know how he will use it. We may assume, in any way that we please, that he will use this profit to purchase some of X, of Y, ..., of A, of B, or of any other commodities. Moreover, all these cases can be treated in the same way. For simplicity, we shall assume that the entrepreneur's profit is realized in the form of commodity A, whose price is equal to unity.

85. Adopting this assumption, we once again obtain all the equations of the system (E), except the first, whose place is taken by an equation indicating that the difference $A'' - A'''$, instead of being zero, is equal to the entrepreneur's profit, i.e.,

$$A'' - A''' = p_y Y' - \Pi_y.$$

If we denote this profit by ξ , the systems (D) and (E) are replaced by the following:

$$(D'') \quad p_x X' = \Pi_x, \quad p_y Y' - \Pi_y = \xi, \quad p_z Z' = \Pi_z, \quad \dots,$$

$$(E'') \quad A'' - A''' = \xi, \quad B'' = B''', \quad \dots.$$

Here again, one of the equations is a consequence of the others and must be dropped.

Indeed, equations (D'') give

$$p_x X' + p_y Y' + \dots = \xi + \Pi_x + \Pi_y + \dots;$$

and, replacing Π_x, Π_y, \dots by their values, [we have]

$$p_x X' + p_y Y' + \dots = \xi + A''' + p_b B'' + \dots$$

On the other hand, the systems (B), (M) yield

$$p_x X' + p_y Y' + \dots = A'' + p_b B'' + \dots$$

These two equations, upon taking account of the equations

$$B'' = B''', \quad C'' = C''', \quad \dots,$$

of the system (E''), yield

$$A'' = \xi + A'''.$$

This is precisely the first equation of the system (E''), which is therefore a consequence of the others.

If we remove one equation from the systems (D'') and (E''), there remain $m + n - 1$. The systems (A), (B) and (M) furnish $(m + n)\theta + m + n$ equations. We thus have, in all,

$$(m + n)\theta + 2m + 2n - 1$$

equations.

The quantities A''', B''', \dots are always determined by equations (108). Next, we have for the unknowns:

the quantities $x_1, y_1, \dots, a_1, \dots$ numbering ...	$(m + n)\theta;$
the prices, numbering	$m + n - 1;$
the quantities $X, Y, \dots, A', B', \dots$, numbering	$m + n;$
the quantity ξ , numbering.....	1.
Total.....	<hr/> $(m + n)\theta + 2m + 2n.$

The number of unknowns therefore exceeds the number of equations by one; consequently, all but one of the unknowns can be determined as functions of one of them. The latter unknown may be chosen arbitrarily; let us choose it to be p_y .

All the other unknowns being expressed as functions of p_y ^[126] we shall have

$$\xi = f(p_y).$$

The monopolist usually attempts to maximize his profit, ξ , expressed in terms of money;³² we must therefore have

$$(111) \quad \frac{df}{dp_y} = 0.$$

This is the equation that was missing. The number of equations is now equal to the number of unknowns, and the problem is solved.

86. Let us suppose that the monopolist makes his calculations in terms of ophelimity. He uses his profit to purchase certain commodities X, Y, ..., A, B, ..., of which he procures for himself the amounts x''', y''', \dots

We shall have for him the equations^[127]

$$\frac{1}{p_x} \varphi_x(x''') = \dots \varphi_a(a''') = \frac{1}{p_b} \varphi(b''')^{[128]} = \dots; \\ p_x x''' + p_y y''' + \dots + a''' + p_b b''' + \dots = \xi.$$

These equations make it possible to determine the quantities as functions of the prices and of ξ . The total ophelimity φ enjoyed by the entrepreneur will therefore be a function of the prices and of ξ , and since the latter quantities are themselves functions of p_y , we shall have

$$\varphi = F(p_y).$$

For the entrepreneur to obtain maximum ophelimity, we must have

$$\frac{dF}{dp_y} = 0.$$

This is the equation that takes the place of equation (111) in this case.

87. As we have seen, the independent variable may be chosen arbitrarily. Thus, whether the monopolist acts to determine p_y or any other variable, the result will be the same, as far as the determination of the equilibrium point is concerned. But there could be differences with respect to other circumstances, among others with respect to the stability of equilibrium. This point will be clarified later on (§98).^[129]

87 [bis].^[130] If we were to suppose that, in the production of one and the same commodity, there were two individuals acting according to type II, the problem would be over-determined, and the hypothesis we have just made could not be realized. The proof is the same as that given in §69; and it gives rise to considerations that are analogous to those of §72, 73, 74, 75, 76.

88. If we suppose that there is one individual behaving with respect to a commodity Y according to type II, and another individual behaving with respect to another commodity Z, again according to type II, the problem is tractable; and it is solved by considerations analogous to those developed in §71.

It should not be forgotten that the firm usually makes its calculations in terms of money³² rather than ophelimity.

89. (γ) Maximum of ophelimity. This expression should first be exactly defined. As we have seen (VI, 53), two problems must be solved in order to procure maximum welfare for a community. Given the adoption of certain rules of distribution, we can search for the position which, while remaining always in conformity with those rules, furnishes the greatest possible welfare to the individuals of that community.

Let us consider any arbitrary position, and let us suppose that we deviate very slightly from it, in a manner consistent with the constraints.³³ If, by doing so, the welfare of every individual in the community is increased, the new position is obviously of greater advantage to each of them; and conversely, it will be of less advantage if the welfare of every individual is decreased. The welfare of some individuals may, moreover, remain constant without these conclusions being changed. But if, on the contrary, this slight movement causes the welfare of some individuals to increase and that of others to decrease, one can no longer affirm that it would be advantageous for the whole community to undertake it.

These considerations lead us to define a position of maximum ophelimity as one from which it is not possible to deviate slightly in such a way that the ophelimities enjoyed by the individuals, other than those that remain constant, all increase or all decrease (VI, 33).^{34[131]}

Let us indicate by δ any arbitrary variations, such as obtained, for instance, in passing from one path to another (§22);^[132] and by Φ_1, Φ_2, \dots , the total ophelimities for each individual. Let us consider the expression

$$(112) \quad \frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \frac{1}{\varphi_{3a}} \delta\Phi_3 + \dots,$$

If we exclude the case in which the $\delta\Phi_1, \delta\Phi_2, \dots$, are zero, we can see that since the quantities $\varphi_{1a}, \varphi_{2a}$, are essentially positive, this expression (112) can be equal to zero only if some of the $\delta\Phi$ are positive, and some others negative; some of them could, moreover, always be equal to zero. Consequently, if we set

$$(113) \quad 0 = \frac{1}{\varphi_{1a}} \delta\Phi_{1a} + \frac{1}{\varphi_{2a}} \delta\Phi_{2a} + \frac{1}{\varphi_{3a}} \delta\Phi_{3a} + \dots,$$

the case in which the variations are all positive or all negative will be excluded. Equation (113) thus characterizes the maximum of ophelimity for the community under consideration, according to our definition.^[133] The variations that occur in this equation must be all those that are compatible with the constraints of the system.

The definition of a maximum of ophelimity for a community should be chosen in such a way that it coincides with the one that would hold for a single individual, when the community reduces to this single individual. This is indeed the case with respect to the definition we have just given (§116).

90. Let us apply these considerations to production.

If there is a positive difference between the proceeds from the sale of a commodity X, and its cost, that is to say,

$$(114) \quad \int_0^{X'} p_x dX - \Pi_x > 0,^{[134]}$$

the commodities represented in this sum can obviously be made available and distributed to every member of the community, or to some of the members. In this way, all the terms in expression (112) become positive, or some of them become positive, the others being zero. A position of maximum ophelimity is therefore not attained. For it to be attained,

expression (114) must become zero; for then there are no longer any commodities available that can be used to make all the terms in (112) positive, or some of them positive, the rest being zero.

The condition we have just found is the same as that which characterizes free competition (83).

91. This condition is necessary,^[135] but in general is not sufficient. There are other variations to be considered.

The variations which, at the equilibrium point, take place for consumers along the path that leads to this point, simply reproduce equations (A). They have therefore already been taken account of; there is no use dwelling on them.

92. Let us suppose that the conditions of production and consumption of a commodity, X, vary. If the variation of the expression (114), which may be written

$$\delta \left(\int_0^{X'} p_x dx - \pi_{0x} - \int_0^{X''} \pi_x dx \right),$$

were positive, there would be a sum that could be distributed to the members of the community, and the terms in the expression (112) could all be made positive, always excepting those that are zero; and conversely, they could all be made negative, if the variation considered were negative. For maximum ophelimity, the variation must therefore be zero. Substituting for Π_x the value given by equations (109),^[136] we shall therefore have^[137]

$$\delta \left(\int_0^{X'} p_x dx - \pi_{0x} - \int_0^{X''} \pi_x dx \right) = 0.$$

Let us denote by p_x^0, π_x^0 the values of p_x, π_x at the equilibrium point. At this point, we have $X' = X''$. The preceding variation becomes:

$$(116) \quad \left(p_x^0 - \pi_x^0 + \int_0^{X'} \left(\frac{dp_x}{dX'} - \frac{d\pi_x}{dX'} \right) dx \right) \delta X' = 0$$

Instead of considering arbitrary variations, let us consider the variations that take place when the parameters (§26) in the expressions for p_x, π_x , remain constant. The δ 's then turn into d 's, and production is continued along the path leading to the equilibrium point. The parameters being assumed constant, the derivatives of p_x and π_x with respect to X' are zero, which gives

$$p_x^0 - \pi_x^0 = 0.^[138]$$

If this equation did not hold, it would mean that by causing a variation of $\delta X''$ of the amount produced, equal to the amount of X consumed, the production of X would leave a certain residual. And it is because this residual can be used to make the expression (112) positive or negative that the maximum of ophelimity is not attained.

We could arrive at the same result in another way.

At the equilibrium point we have

$$\begin{aligned}\delta\Phi_1 &= \varphi_{1x}\delta x_1 + \varphi_{1a}\delta a_1 + \varphi_{1b}\delta b_1 + \dots, \\ \delta\Phi_2 &= \varphi_{2x}\delta x_2 + \varphi_{2a}\delta a_2 + \varphi_{2b}\delta b_2 + \dots,\end{aligned}$$

.....

Substituting for $\varphi_{1x}, \varphi_{1a}, \dots$, the expressions for them given by equations (A) of §80, we obtain

$$\frac{1}{\varphi_{1a}} \delta\Phi_1 = p_x \delta x'_1 + \delta a'_1 + p_b \delta b'_1 + \dots,$$

Summing, and taking account of equations (96), (97), (98), and the fact that

$$X' = X'', \quad A'' = A''', \quad \dots,$$

at the equilibrium point, we shall have

$$\frac{1}{\varphi_{1a}}\delta\Phi_1 + \frac{1}{\varphi_{2a}}\delta\Phi_2 + \frac{1}{\varphi_{[139]}}\delta\Phi_3 + \dots = p_x^0\delta X'' - \delta A''' - p_b^0\delta B''' - \dots$$

If the movement takes place while still following the path along which production is carried out, the δ 's change into d 's, and we have

$$(117) \quad \frac{1}{\varphi_{1a}} d\Phi_1 + \frac{1}{\varphi_{2a}} d\Phi_2 + \dots = p_x^0 dX'' - dA''' - p_b^0 dB''' - \dots$$

Let us, on the other hand, look for the cost of production of dX'' . If the integral function exists of which the expressions (104) represent the partial derivatives, whether directly or whether because the path of integration is given, we obtain the cost of production of dX'' by substituting dX'' for dx in the first equation of (104), obtaining

$$\pi_x^0 dX'' = (a_x + p_h^0 b_x + \dots) dX''. [140]$$

The equations give

$$dA''' = a_x dX'', \quad dB''' = b_x dX'', \quad \dots;$$

consequently, the preceding equation becomes

$$\pi_x^0 dX'' = dA''' + p_b^0 dB''' + \dots$$

Let us substitute this value on the right side of equation (117). The left side is zero when maximum ophelimity is attained; the right side must then also be zero, and we have

$$0 = p_x^0 dX'' - \pi_x^0 \delta X'',$$

or

$$p_x^0 - \pi_x^0 = 0,$$

as before.

This theory is just a special case of a more general theory that will be set forth below (§109 and following).^[141]

For Y, Z, \dots , we of course have equations similar to those we have just found. Consequently, we could write

$$(118) \quad p_x^0 = \pi_x^0, \quad p_y^0 = \pi_y^0, \quad \dots$$

The values of π_x^0, π_y^0, \dots , are those that refer to the point of equilibrium.

92 [bis].³⁵ The results we have obtained may be expressed in the following way.

The necessary and sufficient conditions for a maximum of ophelimity to be attained are:

(1) Equality of the integrals:^[142]

$$\int_0^{X''} p_x dx = \pi_{0x} + \int_0^{X^{[143]}} \pi_x dx, \quad \dots,$$

(2) Equality of the last terms of these integrals:

$$p_x^0 = \pi_x^0, \quad p_y^0 = \pi_y^0, \quad \dots$$

When the π_{0x}, \dots , are zero and prices are constant, these two conditions reduce to a single one. The first one becomes

$$p_x X'' = \pi_x x, \quad \dots,^{[144]}$$

and the second one becomes

$$p_x = \pi_x, \quad \dots;$$

and since $X'' = x, \dots$, the first row of equations is identical with the second.

When the π_{0x}, \dots , are not zero and prices are constant, the first condition gives

$$p_x x = \pi_{0x} + \pi_x x, \dots, [145]$$

and the second gives

$$p_x = \pi_x, \dots$$

These equations are incompatible. Thus, in the case of phenomena of type (I), when there are overhead expenses π_{0x}, \dots , it is impossible, in general, to attain maximum ophelimity with constant prices (VI, 43).

This happens because one cannot continue to move with constant prices while maintaining equilibrium in the budget constraints.^[146]

In the case of free competition the two conditions mentioned tend to be fulfilled. When the first one holds, the producers will clearly tend to expand their output as long as

$$p_x^0 > \pi_x^0, \dots,$$

but they may be prevented from doing this by the state of the market.

On the other hand, if the second condition holds, competition acts to bring the first one about;^[147] but this may not be possible.

93. Numerical example. The preceding considerations will be clarified by a very simple numerical example.

Suppose there is a group of consumers, all identical, who sell A and B and buy X. Let us also suppose that there is a group of firms that transform A and B into X.

For any point of consumers' equilibrium the amounts consumed will be x, a, b .

To simplify the writing, we shall here change the notation. These quantities x, a, b , are those that were previously indicated by X', A', B' , at the point of equilibrium.

Let us put

$$\varphi_x = \frac{1}{\sqrt{x}} - \frac{1}{x + 0.5}.$$

The three characteristics of the indices

$$\varphi_x > 0, \quad \varphi_{xx} < 0, \quad \varphi_{xxx} > 0,$$

are verified for

$$x \geq 4.$$

These three characteristics are also verified for the functions

$$\varphi_a = \frac{M}{a^{0.4}}, \quad \varphi_b = \frac{N}{\sqrt{b}}.$$

Let us, further, set

$$a_0 = 17, \quad b_0 = 28.$$

The quantities supplied to the firm will be

$$A'' = 17 - a, \quad B'' = 28 - b.$$

The conditions of consumers' equilibrium are

$$(119) \quad \begin{cases} \frac{1}{p_x} \varphi_x = \varphi_a = \frac{1}{p_b} \varphi_b; \\ p_x x = A'' + p_b B''. \end{cases}$$

The amounts transformed by the firm are A''' and B''' , and we shall set

$$A''' = 3 + 0.5x, \quad B''' = 5 + x.$$

These are equations (108).

94. In the case of free competition, the systems (D) and (E) become

$$(120) \quad \begin{cases} p_x x = A''' + p_b B''' \\ A'' = A''', \quad B'' = B'''. \end{cases}^{[148]}$$

The first of these equations is identical with the last equation of (119), and must therefore be discarded, as we already knew to be the case.

Let us try to determine the parameters in such a way as to give rise to several points of equilibrium. We can have two such points. Let us assume that they correspond to the points given by $x = 4.2$ and $x = 12$. We shall have^[149]

$$\log M = \bar{1}.6413093, \quad \log N = \bar{1}.1872683.$$

Let us see what happens in the neighborhood of these points.^[150]

For the first, we have the following table.

x	$A'' - A'''$	$\log p_x$	$\log p_b$	B''
4	-0.08966	0.235354	1.339498	9
4.2	0	0.228533	1.339099	9.2
5	+0.29028	0.202422	1.338161	10

The firm cannot stay at a point below the point $x = 4.2$, because it would suffer a loss there, $A'' - A'''$ being a negative quantity. It can stay at the point $x = 4.2$, and at points for which $x > 4.2$.

On this side of the point $x = 5$, equilibrium is unstable, for the firm, upon reducing the price p_x , sells a larger amount of X and increases its gains. Even if it is alone, it will be

induced to move away from this side; it will be forced to do so if there are competitors. The movement can continue until a point of stable equilibrium is reached.

For the point $x = 12$, we have the following table.

x	$A'' - A'''$	$\log p_x$	$\log p_b$	B''
11	+ 0.25768	0.056649	1.372788	16
12	0	0.039397	1.386499	17
13	- 0.31643	0.023980	1.403162	18

The firm cannot go beyond the point $x = 12$ without entering a region where it would suffer a loss. On this side of that point, i.e., for $x < 12$, it is pushed towards the point $x = 12$ by competition. Hence, this is a point of stable equilibrium.

95. In this hypothetical case, there are some circumstances that deserve to be noted.

If a syndicate of suppliers of B compelled its members not to sell this commodity below a certain price, it could happen that the movement starting from the point of unstable equilibrium might be halted. Let us suppose, for instance, that the members of the syndicate are prohibited from selling their commodity below the price corresponding to $x = 4.2$. The equilibrium at this point would become stable, for to move away from it the firm needs to set a lower price for the commodity.

We shall see (§100) that the ophelimity enjoyed by the sellers of B is greater at the point $x = 12$ than at the point $x = 4.2$. Thus, the syndicate will have the effect of reducing the sellers' welfare, instead of increasing it.

This effect^[151] will come about as long as p_b decreases while B''^{36} is increasing. For

$$x = 4, \quad 5, \quad 6,$$

we have

$$\log p_b = 1.339498, \quad 1.338161, \quad 1.338845.$$

Hence, the effect indicated will continue up to a point located in the neighborhood of $x = 5$. Beyond this point, it will no longer take place.^[152]

96. Imagine now a syndicate of firms acting according to type II, in the production and sale of X.

Let us suppose that we still have

$$B'' = B''';$$

but that

$$A'' - A''' = \xi,$$

ξ being the firm's profit.

To facilitate the numerical computations, it will be convenient to take x as independent variable. The profit ξ is 0 for $x = 4.2$ and for $x = 12$; between these two values there is a maximum.

First giving x the values 5, 6, ..., we find

$$\begin{aligned} x &= 7, & 8, & 9, \\ \xi &= 0.63607, & 0.65367, & 0.58997. \end{aligned}$$

The maximum must therefore lie in the neighborhood of the point $x = 8$.

Substituting values of x that increase by tenths we have

$$\begin{aligned} x &= 7.6, & 7.7, & 7.8, \\ \xi &= 0.65709, & 0.65751, & 0.65706. \end{aligned}$$

A parabola could be drawn through these three points. If we set

$$x = 7.6 + u$$

we shall have

$$\xi = 0.65709 + u\Delta\xi + \frac{u(u-1)}{2}\Delta^2\xi;$$

or

$$\xi = 0.65709 + \left(\Delta\xi - \frac{1}{2}\Delta^2\xi \right) u + \frac{u^2}{2}\Delta^2\xi.$$

Taking the derivative and setting it equal to zero to attain the maximum, we obtain

$$0 = \left(\Delta\xi - \frac{1}{2}\Delta^2\xi \right) + u\Delta^2\xi.$$

This equation replaces equation (111) and may serve to find an approximate value of x . But it is pointless to ask for such precision in a hypothetical case, and we shall suppose simply that the maximum corresponds to $x = 7.7$.

We shall have the following table:^[153]

x	ξ	$\log p_x$	$\log p_b$	B''
7.6	0.65709	1.129472	1.344071	12.6
7.7	0.65751	1.126992	1.344565	12.7
7.8	0.65706	1.124535	1.345120	12.8

The monopolists will therefore have to stop at the price p_x which corresponds to $x = 7.7$. This price is lower than the one that corresponds to $x = 7.6$.

97. If the syndicate acts as a single monopolist, it must fix the price and the distribution of the quantities among its members, so as to arrive precisely at the point $x = 7.7$.

If it fixed only one price, below which the members could not sell—for example, the price corresponding to $x = 7.7$ —the distribution of the quantities would remain

indeterminate. Let us therefore suppose that the syndicate determines the distribution but leaves some latitude for small oscillations.

The members of the syndicate cannot move in the region where $x > 7.7$, because the price p_x would fall below the predetermined limit. This limit does not prevent them from moving in the region where $x < 7.7$, but the competition of quantities leads them back to the point $x = 7.7$. This point is therefore a point of stable equilibrium.^[154]

98. The choice of the independent variable is immaterial. One may choose p_b . If the syndicate fixes this price p_b and the amounts of B that its members are allowed to purchase, there is no difference from the preceding case.

Nor does it make any difference either, at this point, if the syndicate fixes an upper limit to p_b —the one that corresponds to $x = 7.7$ —and leaves a little latitude for the quantities.

98 [bis].³⁷ It would no longer be the same if, for any reason, the syndicate considered it in its interest to stop at a point at which B'' increases when p_b decreases.

Let us suppose, for example, that the syndicate wishes to stop at the point $x = 4.2$. If it fixes the price p_x corresponding to $x = 4.2$, below which its members may not go, the latter will not be allowed to move in the direction $x > 4.2$. On the other hand, they would incur losses if they moved in the direction $x < 4.2$. The point $x = 4.2$ thus becomes a point of stable equilibrium.

But let us now suppose that the syndicate acts on p_b instead of on p_x . It fixes the price p_b corresponding to $x = 4.2$, and forbids its members to go below this limit. It also fixes the quantities, but with a little latitude.

The members of the syndicate cannot move in the region where $x < 4.2$ because either they would fall below the limit that was assigned to them with respect to p_b , or else they would incur losses. But they can move in the region where $x > 4.2$, and the competition of the quantities does not lead them back to this point.

Hence, if the syndicate acts on p_x , the point $x = 4.2$ is one of stable equilibrium; it is a point of unstable equilibrium if the syndicate acts on p_b .^[155]

99. Let us seek the point at which consumers obtain maximum ophelimity. We know that the prices of the commodities produced can no longer be constant. The equality of the last terms in the integrals, as indicated in §92, gives for the equilibrium point:

$$p_x = 0.5 + p_b.$$

Thus, the last portion, dx , is produced with $0.5dx$ of A, and dx of B.

The equilibrium will be determined by the following equations:

$$\frac{1}{p_x} \varphi_x = \varphi_a = \frac{1}{p_b} \varphi_b, \quad p_x = 0.5 + p_b, \quad A'' = A''', \quad B'' = B'''.$$

Eliminating the prices, we have

$$\varphi_x = 0.5\varphi_a + \varphi_b;$$

and expressing the quantities as functions of x , we obtain

$$\varphi_x(x) = 0.5\varphi_a(14 - 0.5x) + \varphi_b(23 - x).$$

This equation gives

$$x = 17.854;$$

and we then have

$$p_b^0 = 0.2967, \quad p_x^0 = 0.7967.$$

100. Let us now calculate the total ophelimities at these different points of equilibrium.

We have

$$\Phi = 2\sqrt{x} - \log(x + 0.5) + \frac{10}{6}Ma^{0.6} + 2N\sqrt{b};$$

the logarithm is Neperian.

Let us calculate the ophelimities from the point $x = 4.2$, that is to say, let us calculate

$$\Omega = \Phi(x) - \Phi(4.2).$$

We obtain

$$\begin{array}{cccc} x & 4.2 & 7.7 & 12 & 17.854 \\ \Omega & 0 & 0.355 & 0.854 & 1.062 \end{array}$$

From what we have seen (§92), maximum ophelimity is incompatible with constant prices; it is attained only for $x = 17.854$. Starting from the origin, one follows a broken line, no longer a straight line as would be the case with a constant p_x . The firm receives 3 of A and 5 of B without supplying anything; then it supplies some of X, in the proportion of 1 unit of this commodity for 0.5 of A and 1 of B. It suffices, moreover, for the last portions of the commodity to be supplied in this way.^[156]

101. Variability of the production coefficients. Among the production coefficients, some are constant, or nearly so, others vary with the quantity of output, and still others undergo a special kind of variation; they form a group such that an increase in some coefficients may be compensated for by a reduction in others. Finally, the cost of production for a firm may vary according to the total output of that firm.

102. Expressions (105) for the cost of production may be written

$$\pi_{0x} + \int_0^{X''} (a_x + p_b b_x + \dots) dx, \quad \dots;$$

and in these formulae, a_x, b_x, \dots , may be functions of x . We have therefore already taken account of its variability as a function of the quantities x, y, \dots ; hence we need no longer be concerned with it.

103. Let b_y, c_y, e_y , be a group of production coefficients such that variations in some of them are offset by variations in others.^[157] The technical conditions of production will reveal the law of these compensatory variations, which may be expressed by

$$(121) \quad f(b_y, c_y, \dots, e_y) = 0.$$

The firm must determine these coefficients subject to this law. For this, as in other economic phenomena, it may operate according to type I or type II.

104. Let us begin by assuming that the firm operates according to type I. It accepts the market prices, without trying to change them directly; it makes its calculations in terms of these prices, and determines the coefficients. But, without intending to do so, it has changed the market prices. It begins its calculations anew with the new prices, and so on indefinitely. The path followed by the firm is analogous to a pursuit curve.^[158]

In other words, the production coefficients under the integral sign must be considered as independent of the limits of integration. This is characteristic of phenomena of type I, both as regards prices and production coefficients.

If the coefficients b_y, c_y, \dots, e_y , are made to vary, the variation in the expenses incurred in producing Y will be

$$(122) \quad \delta\Pi_y = \int_0^{Y''} (p_b \delta b_y + p_c \delta c_y + \dots + p_y \delta p_b + \dots) dy.$$

In the present case, since the firm accepts the market prices and does not take their variations into account, it operates as if one had

$$\delta\Pi_y = \int_0^{Y''} (p_b \delta b_y + p_c \delta c_y + \dots) dy.$$

It is this expression that must be made equal to zero in order to minimize the expenses Π_y —a minimum that would be achieved if the prices remained constant; but it will not be achieved, because the prices vary, which will force the firm to begin its calculation afresh with the new prices.

We shall therefore have, in this, case,

$$(123) \quad 0 = \int_0^{X''} (p_b \delta b_y + p_c \delta c_y + \dots) dy.$$

Once this equation can be established in terms of existing market prices, the firm will no longer have to begin its calculations anew; it will stop there. Equilibrium will thus be attained when equation (123) subsists along with the other equilibrium equations.

105. If we have only equation (121) connecting the group of coefficients considered, one of them, e.g., b_y , may be assumed to be a function of the others, c_y, \dots, e_y , which are then independent variables. Consequently, equation (123) gives rise to the following equations:

$$\int_0^{Y''} \left(p_b \frac{\partial b_y}{\partial c_y} + p_c \right) \delta c_y dy = 0, \quad \dots$$

But the variations $\delta c_y, \dots$, are entirely arbitrary; consequently, the preceding equations can hold only if we have

$$(124) \quad p_b \frac{\partial b_y}{\partial c_y} + p_c = 0, \quad \dots, \quad p_b \frac{\partial b_y}{\partial e_y} + p_e = 0.$$

From equation (121) we can compute the partial derivatives of b_y , and substitute them in this system, which will contain $r - 1$ equations if the group b_y, c_y, \dots, e_y , is composed of r coefficients. Adding equation (121) to these equations, we shall thus have r equations, that is, as many as there are unknowns. The problem is therefore well determined.

These equations form part of the category (F) of conditions (V, 82).^[159]

If, instead of one equation (121), we had several of them, the reasoning would be similar and would lead to the same conclusions.

When the production coefficients are constant with respect to the variables x, y, \dots , equation (123) becomes

$$0 = p_b \delta b_y + p_c \delta c_y + \dots,$$

and we obtain equations (124) directly from these.

Substituting the values of the partial derivatives of b_y in the latter formulae, and setting as usual

$$f_b = \frac{\partial f}{\partial b_y}, \quad \dots, \quad f_e = \frac{\partial f}{\partial e_y},$$

we shall have

$$(125) \quad p_b f_c - p_c f_b = 0, \quad \dots, \quad p_b f_e - p_e f_b = 0.$$

106. If the firm acts according to type II, it will try to maximize its profits, either by simply reducing the cost of production to a minimum, or, if it is able to take account of the variations in the sales of Y, by trying to make the following expression

$$A'' - A''' = \int_0^{Y''} p_y dY^{[160]} - \Pi_y$$

a maximum.

The equation thus obtained will, as was explained in §§84, 85, take the place of the equation

$$A'' = A''',$$

which no longer holds.

In this case, not only the prices but also the production coefficients must, under the integral sign, be assumed to be functions of the limits of integration. The firm acts not

according to the values of the actual prices and production coefficients but according to the values that they will attain at the point of equilibrium.

This procedure assumes not only that the firm enjoys a monopoly position, but also that it knows how to operate so as to attain this maximum. Now, the latter condition is very hard, if not impossible, to realize—at least in general—in the present state of our knowledge. On the contrary, entrepreneurs have a fairly good knowledge—if not in theory, at least in practice—of the possible compensating variations among the production coefficients. They have, or acquire by fairly frequently repeated trials, a certain knowledge of the nature of equation (121), and use it to make their calculations and bring their production costs down as much as possible. Operations according to type I are commonplace and are performed by firms continually.

107. It remains to examine the question of the allocation of the quantities among firms (V, 78). If a firm produces q_z of Z, when its production is increased by δq_z the cost of production of Z will vary by a certain amount that will have to be made equal to zero if the firm wishes to minimize its production costs.^[161] We shall thus have the equation

$$(126) \quad 0 = \frac{\partial a_z}{\partial q_z} + p_b \frac{\partial b_z}{\partial q_z} + \dots$$

There will be other similar equations, one for each firm, and they will determine the distribution of output.

108. It is worth pointing out some very common errors on the subject of production coefficients.

Certain authors assume all production coefficients to be constant; others assume them all to be variable. Both ways of treating the phenomenon are equally erroneous. Some of these coefficients are constant, or nearly so, and some of them are variable.

Literary economists have a marked tendency to transform the properties of the relations among things into properties of the things themselves; this is due to their difficulty in dealing with problems where the mutual dependence of phenomena comes into play, not knowing the appropriate methods.

They have not failed to apply their erroneous method to the theory of production coefficients. They have imagined that there exist among the factors of production certain relations that make it possible to obtain maximum “utility” of production²⁴; and

²⁴ One author³⁸ provides the following definition of what he calls the law of definite proportions: “*in order to obtain a given useful result, the elements of production must be combined in a determinate ratio, or in other words: a useful result is in proportion to a qualitatively and quantitatively determinate combination of elements of production.*”

What this “useful result” can be we do not know; and further, what is in question is not any arbitrary “useful result;” it is a problem of maximum and minimum that has to be solved.

The way in which the proposition is expressed suggests that the “useful result” depends only on the ratios between the elements of production, whereas it depends on their prices, and since the prices depend on all the other circumstances that determine equilibrium, the “useful result” depends ultimately on all these circumstances.

governed by this conception, they believed they had discovered in political economy a law analogous to the law of definite proportions in chemistry.²⁵

All this is inaccurate and wrong. First, we ought to get rid of vague conceptions of utility of production, of useful effect, and others of this kind, and we should substitute precise notions for them, such that of the minimization of production costs or maximization of profits. Further, it should be clearly understood that the determination of the production coefficients is not only a technical operation, but that it depends on prices, on the state of the market, and in general on all the circumstances of economic equilibrium. It is a system of equations that has to be solved, and not a series of isolated and independent equations.

It is because literary economists have no clear ideas either about the solution of a system of simultaneous equations or even about the nature of such a problem, that they make such desperate efforts to substitute for this system of simultaneous equations a system of equations that can be solved one at a time; this kind of problem being the only one they are able to tackle.²⁶ This is what led them into error regarding the general theory of economic equilibrium, and it is what continues to lead them into error in this particular case.

Finally, there is a most important circumstance, which ought not to be neglected in the determination of the production coefficients; it is the consideration of the types of behavior according to which the firm acts. A determination made according to what we have called type I is essentially different from one made according to what we have designated

²⁵ One should never waste time quarreling about words. One can always accept this terminology if one wishes, provided it is clearly understood that the law of definite proportions in political economy has absolutely nothing to do with the law that bears the same name in chemistry.

The proportions between the elements of production may undergo infinitely small variations, which is not the case for the proportions between the elements in chemical compounds. In political economy, these proportions depend not only on the elements of production, but also on all the other circumstances that determine economic equilibrium. They are therefore in no wise definite when the elements are given, whereas in chemistry, they are; they are indefinite, and remain so as long as one does not take all the circumstances of economic equilibrium into account.

There are certainly some goods that combine in fixed proportions, either in consumption—e.g., the blade of a knife to its handle—or in production—e.g., the four wheels of a carriage. To these goods should be reserved the name of complementary goods; and in discussing these goods, we may quite properly speak of the law of definite proportions. But since, by hypothesis, the quantities in which these goods are combined are assumed to be *fixed*, it cannot be these goods that are alluded to when one proposes to determine the *variable* proportions between certain goods, with a view to ensuring certain properties of production: e.g., maximization of output, minimization of production costs, or maximization of profits, etc.

²⁶ Subsequently to the publication of the Italian edition of this *Manual*, there appeared in Italy a large succession of tracts, journal articles, and inaugural lectures, aimed at demonstrating that the theory of economic equilibrium was useless, to say the least, and that for the relations of mutual dependence, one should substitute relations of cause and effect. One author, by a stroke of genius, even discovered an infallible criterion for distinguishing these relations of cause and effect. Is it not obvious that if one fact precedes another one chronologically, the first is the cause, and the second the effect?

Already long ago, some people, applying this principle, used to say that since the hen lays the eggs, the hen is the cause and the egg is the effect; but other people objected that since the chicken comes from the egg, it is the egg that is the cause and the hen the effect. *Adhuc sub judice lis est.*³⁹

Truly, mother nature should have been kinder, and might have formed links between things better adapted to the intelligence and the knowledge of these authors; she would thus have spared them the unpleasantness of constructing theories that are a little too much like the reasoning of the fox which, in the fable, had lost its tail.⁴⁰

as type II. If this consideration is disregarded, we obtain an absolutely erroneous idea of the phenomenon.

109. Properties of economic equilibrium.^[162] The consumers of X, Y, are at the same time suppliers of A, B, When the amounts of the commodities increase by dX, \dots, da, \dots , the sum of the consumers' budgets may be written

$$(127) \quad p_x dX + p_y dY + \dots - da - p_b db \dots$$

Integrating along a given path we must obtain zero, for the receipts must balance the expenditures (including saving). The path being given, all the variables, X, Y, \dots, a, b, \dots , may be assumed to be functions of one of them, say X , and if we set

$$(128) \quad V = p_x + p_y \frac{dY}{dX} + \dots - \frac{da}{dX} - p_b \frac{db}{dX} - \dots,$$

we shall have

$$(129) \quad 0 = \int_0^{X'} V dX.$$

Let us vary the quantities X, Y, \dots, a, b, \dots ; these variations are not all independent, but must be compatible with the constraints of the system. If we take these constraints into account, there remain a certain number of variations that are independent. Let us suppose that the variation of expression (129) becomes identically equal to zero, without establishing *for the limits of integration* any new relations between the variations that have remained independent.

Let us expand the variation of the integral according to the usual method. We shall find^[163]

$$(130) \quad 0 = [\delta U]_0^{x'} + \delta R + \int_0^{X'} \delta T dX;$$

$$(131) \quad \left\{ \begin{array}{l} \delta U = V dX^{[165]} + p_y \omega_y + \dots - \omega_a - p_b \omega_b \dots, \\ \delta R = \delta X' \int_0^{X'} \frac{\partial V}{\partial X'} dX, \\ \delta T = \left(\frac{\partial V}{\partial Y} - \frac{dp_y}{dX} \right) \omega_y + \dots + \frac{\partial V}{\partial a} \omega_a \\ \qquad \qquad \qquad + \left(\frac{\partial V}{\partial b} - \frac{dp_b}{dX} \right) \omega_b + \dots, \\ \omega_b^{[166]} = \delta Y - \frac{dY}{dX} \delta X, \quad \dots, \quad \omega_b = \delta b - \frac{db}{dX} \delta X, \quad \dots. \end{array} \right.$$

We know that the sum of the first two terms, on the one hand, and the integral, on the other hand, in the expression (130), must vanish separately. As we have just said, we

assume that the first part vanishes, without establishing, for the limits of integration, any new relations among the variations. As for the second part, it may vanish whether or not any relations among the variations are established.

110. Expanding the value of δU , we shall have at the equilibrium point X'

$$(132) \quad \delta U = p_x \delta X' + p_y \delta Y' + \dots - \delta A'' - p_b \delta B'' \dots,$$

the value at the point zero being zero.

But at the equilibrium point, we have

and consequently,

$$\frac{1}{\varphi_{1a}} \delta \Phi_1 + \frac{1}{\varphi_{2a}} \delta \Phi_2 + \dots = p_x \delta X' + p_y \delta Y' + \dots - \delta A'' - p_b \delta B'' - \dots$$

Thus, at the equilibrium point,

$$(133) \quad \delta U = \frac{1}{\varphi_{1a}} \delta \Phi_1 + \frac{1}{\varphi_{2a}} \delta \Phi_2 + \dots$$

111. We have seen (§89) that by making the expression (133) equal to zero, one characterizes the points at which maximum ophelimity is attained.

When the value of δU vanishes at the equilibrium point without the introduction of any new relations among the variations that have remained independent, the right side of the expression (133) is also zero, and maximum ophelimity is attained, at least as far as this kind of variation is concerned.

If, on the other hand, δU could be made to vanish at the equilibrium point only by establishing new relations among the variations $\delta X'$, $\delta Y'$, ...it would suffice to assume that these relations are not satisfied in order to make the expression (133) positive or negative, and maximum ophelimity would no longer exist.

We now have to find the conditions under which the last two parts of the expression (130) become zero.

As for the integral,^[167] we know that it vanishes if the expression (127) is integrable, i.e., if the budget does not change no matter what path is followed to reach the point of equilibrium.^[168] This is the only case that will be studied here.

There remains δR to be considered. If we assume, as a constraint of the system, that at any point where production stops, the total expenses incurred for the production of the

commodities are equal to the total receipts derived from their sale, the variation of the difference between these two sums will have to be zero, and by reasoning as we did in §92 to obtain equation (116), we shall obtain^[169]

$$(134) \quad \delta P + \delta Q = 0,$$

$$\begin{aligned} \delta P &= \left(p_x^0 - \pi_x^0 + (p_y^0 - \pi_y^0) \left(\frac{dY}{dX} \right)^0 + \dots \right) \delta X', \\ \delta Q &= \delta X' \int_0^{X'} \left(\frac{dp_x}{dX'} - \frac{d\pi_x}{dX'} + \left(\frac{dp_y}{dX'} - \frac{d\pi_y}{dX'} \right) \frac{dY}{dX'} + \dots \right) dX. \end{aligned}$$

In this last expression, let us substitute for π_x, π_y, \dots , their expressions (104); note that the amounts supplied to the firm must be equal to the amounts transformed, i.e., we must have

$$da = a_x dX + a_y dY + \dots,$$

$$db = b_x dX + b_y dY + \dots,$$

.....;

we shall thus obtain

$$\delta Q = dR,^[170]$$

and consequently,

$$(135) \quad \delta P + \delta R = 0.$$

It follows from this that at the point at which economic equilibrium takes place, we have

$$\delta P = 0;$$

we shall also have

$$\delta R = 0,$$

and maximum ophelimity will be attained at this point.

When the production processes of the different commodities are independent, the equation $\delta P = 0$ yields equations (118) of §91.⁴¹

As long as the equation $\delta P = 0$ is not incompatible with the other conditions of the problem, it will be brought into effect by the operations of the entrepreneurs acting according to type I. And in that case, the equilibrium point reached has the property of ensuring maximum ophelimity.

Should the equation $\delta P = 0$ be incompatible with the other conditions of the problem, equilibrium might take place at a point where this equation does not hold, or at a

point where the total receipts of the entrepreneurs are not equal to the total production expenses (III, 100, 135); in these two cases, maximum ophelimity will not be achieved.

The condition that the receipts of the firm be equal to its expenditures by itself gives rise only to equation (134). But this is insufficient to ensure maximum ophelimity; it is also necessary for δP to vanish. This is an important result of the theory we have just developed.^{[171][172]}

112. We must next take into account the variability of the production coefficients. Let us suppose that the relation indicated by equation (121), whose meaning was explained in §103, holds among a set of these coefficients. As a result of equation (121), any one of the coefficients, say b_y , is a function of the other coefficients in the set. The variations of these coefficients remain arbitrary. The variables not included in the set remain constant. We shall denote these new variations by δ' .^[173]

Let us change the independent variable in the expression [(128)] for V , and take it to be Y . We shall have

$$V = p_x \frac{dX}{dY} + p_y + \dots - \frac{da}{dY} - p_b \frac{db}{dY} - \dots$$

Let us vary c_y . Since the quantities X, Y, \dots , do not vary, we have^[174]

$$(136) \quad \begin{cases} \delta' V = \delta' H - \delta' K; \\ \delta' H = \frac{dX}{dY} \delta' p_x + \delta' p_y + \dots - \frac{db}{dY} \delta' p_b \dots; \\ \delta' K = p_b \delta' \frac{db}{dY} + p_c \delta' \frac{dc}{dY}. \end{cases}$$

Since the quantities of A, B, ... supplied must be equal to the quantities transformed, we shall have

$$\frac{db}{dY} = b_y, \quad \frac{dc}{dY} = c_y;$$

whence

$$\delta' K = p_b \delta' b_y + p_c \delta' c_y.$$

On the other hand, equation (121) yields

$$\delta' b_y = \frac{\partial b_y}{\partial c_y} \delta' c_y;$$

we shall thus have

$$\delta' K = \left(p_b \frac{\partial b_y}{\partial c_y} + p_c \right) \delta' c_y.$$

If the firm acts according to type I—as was described in §104—we will have

$$(137) \quad \delta' K = 0.$$

This equation follows immediately from equations (124).

Moreover, equilibrium of the budgets requires that

$$\delta' \int_0^{Y'} V dy = 0;$$

and, reasoning as in §109, we obtain:^[175]

$$-\delta' K + \int_0^{Y'} \delta' H dY = 0.$$

Consequently, by virtue of equation (137), we have

$$\int_0^{Y'} \delta' H dY = 0.$$

But this quantity is none other than what we designated by δR in §109, where Y is now taken to be the independent variable; and since it vanishes, we can see, by repeating the argument of §109, that we also have

$$\delta' U = 0.$$

113. The variations that are a consequence of these operations also cause the expression (133) to vanish. In fact, since, b and c are the only quantities that vary, we have

$$\delta' \Phi_1 = -\varphi_{1b} \delta' b_1 - \varphi_{1c} \delta' c_1, \quad \delta' \Phi_2 = -\varphi_{2b} \delta' b_2 - \varphi_{2c} \delta' c_2, \quad \dots$$

Summing these equations and making use, as usual, of the system (A), we shall have

$$(138) \quad \left\{ \begin{array}{l} \delta' U = \frac{1}{\varphi_{1a}} \delta' \Phi_1 + \frac{1}{\varphi_{2a}} \delta' \Phi_2 + \dots \\ = -p_b \delta' B' - p_c \delta' C' = p_b \delta' B'' + p_c \delta' C''. \end{array} \right.$$

Since the quantities supplied must be equal to the quantities transformed, we shall have

$$\delta' B'' = \int_0^{Y''} \delta' b_y dy, \quad \delta' C'' = \int_0^{Y''} \delta' c_y dy.$$

The prices p_b and p_c are independent of y ; we can therefore write:

$$\delta' U = \int_0^{Y'} \left(p_b \frac{db_y}{dc_y} + p_c \right) \delta' c_y dy;$$

and by virtue of equations (124), we shall have

$$\delta' U = 0.$$

Maximum ophelimity is thus achieved.

114. In the same way, it could be shown that the operations outlined in §107 for the allocation of outputs are, on the one hand, compatible with equilibrium of the budgets, and on the other hand such as to assure maximum ophelimity.

115. We thus arrive at the conclusion that the operations performed according to type I, when they are possible, lead, in the cases we have just examined, to equilibrium points where maximum ophelimity is achieved.

This is one of the most important theorems in economic science,^[176] and the use of mathematics gives it [such] generality and rigor that, for the moment at least, it is difficult to see how it could have been obtained²⁷ in any other way.⁴³

115 [bis].⁴⁴ Expression (133) may take on a different form. At the equilibrium point, we have $X' = X'', Y' = Y'', \dots$,

$$\begin{aligned}\delta A'' &= \delta A''' = a_x \delta X'' + a_y \delta Y'' + \dots, \\ \delta B'' &= \delta B''' = b_x \delta X'' + b_y \delta Y'' + \dots,^{[177]} \\ &\dots \dots \dots\end{aligned}$$

Consequently, equation (133) becomes

$$\begin{aligned}\delta U &= (p_x - a_x - p_b b_x - \dots) \delta X'' \\ &\quad + (p_y - a_y - p_b b_y - \dots) \delta Y'' \\ &\quad + \dots \dots \dots\end{aligned}$$

or

$$\delta U = (p_x^0 - \pi_x^0) \delta X'' + (p_y^0 - \pi_y^0) \delta Y'' + \dots$$

For this expression to vanish, without the imposition of any new relations among the $\delta X'', \delta Y'', \dots$, we must have

$$p_x^0 = \pi_x^0, \quad p_y^0 = \pi_y^0, \quad \dots,$$

just as we already found in a particular case (§91).^{[178][179]}

²⁷ I provided proofs of this theorem for special cases in my *Cours* (1897). I subsequently provided increasingly general proofs in the *Giornale degli Economisti*, November 1903,⁴² and in the Italian edition of the *Manuale* (1906); and I present another one here.

These equations are a consequence of the condition that the budgets must remain in equilibrium when X'', Y'', \dots , are made to vary.

They are identical with the condition that production costs be equal to selling prices, when prices are constant and there is no occasion to take account of overhead expenses.

Finally, it must not be forgotten that the integral of the expression (130) must vanish. It is well known, moreover, that it will vanish identically when the expression (127) is integrable.

The conditions we have just found, which ensure the equilibrium of individuals' and firms' budgets, must be added to the condition that they operate according to type I, in order for maximum ophelimity to be achieved.

116. If the number of individuals in the community reduces to a single one, the condition by which we have defined maximum ophelimity for a community, namely:

$$0 = \frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots,$$

reduces to

$$\delta\Phi_1 = 0.$$

It then coincides with the definition of maximum ophelimity for an individual.

It follows from this that the conditions we have just obtained for maximum ophelimity for a community reduce, in the case of a single individual, to the conditions for maximum ophelimity for individual production.

If we denote by d the variations along a path—or a certain kind of path (§26; III, 74)—when the individual acts according to type I, the condition of equilibrium is given by

$$d\Phi_1 = 0.$$

This may or may not coincide with the preceding condition of maximum ophelimity. The aim of the study we have just undertaken has been precisely to look for conditions under which such a coincidence takes place.

It does not take place when the path is imposed by a person acting according to type II. Nor does it take place in some other cases, for instance, in the case in which prices must be constant while there are overhead expenses; for, in that case consumers may well act strictly according to type I. But producers cannot simultaneously fulfill both conditions of type I, namely the equality of the cost of production and selling price, not only for total output but also for the last portion produced when the equilibrium point is reached.

117. In general, for consumers operating according to type I, we always have, at the point of equilibrium:

$$d\Phi_1 = 0, \quad d\Phi_2 = 0, \quad \dots,$$

and consequently,

$$(139) \quad 0 = \frac{1}{\varphi_{1a}} d\Phi_1 + \frac{1}{\varphi_{2a}} d\Phi_2 + \dots;$$

the d 's being relative to the paths that have led to the point of equilibrium.^[180]

Further, at this point it may or may not be the case that

$$(140) \quad 0 = \frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots,$$

where the δ 's indicate arbitrary variations compatible with the constraints. The δ 's therefore include the d 's, but the d 's do not include the δ 's.

There are therefore two kinds of equilibrium points. For one kind, only equation (139) is satisfied, and these points do not give rise to maximum ophelimity. For the other kind, equation (140), which includes equation (139), is satisfied. These points do give rise to maximum ophelimity.

In some cases—as, for instance, exchange at constant prices (§119) according to type I—these two kinds of points reduce to a single one, and maximum ophelimity is always achieved.

118. According to the results just obtained, we see that the condition

$$0 = \frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots,$$

is the condition that must be fulfilled by an organization of production according to the type we have denoted by III (III, 49; VI, 53); in particular, it is the type which a socialist organization of production ought to follow.

119. In the case of exchange at constant prices, and when one operates according to type I, the proof of the conditions of maximum ophelimity become remarkably simple.²⁸

Taking the variations of equations (B) in §63, we shall have:

$$(141) \quad \left\{ \begin{array}{l} \delta x_1 + p_y \delta y_1 + \dots + (y_1 - y_{10}) \delta p_y + \dots = 0, \\ \delta x_2 + p_y \delta y_2 + \dots + (y_2 - y_{20}) \delta p_y + \dots = 0, \\ \dots \dots \dots \end{array} \right.$$

Summing and taking account of equations (C), we have:

$$(142) \quad \begin{aligned} \delta X + p_x \delta Y^{[182]} + p_z \delta Z + \dots &= 0 \\ X = x_1 + x_2 + \dots, \quad Y = y_1 + y_2 + \dots, \quad \dots \end{aligned}$$

Now, the first equation is none other than

$$\frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots = 0;$$

²⁸ I presented this proof for the first time in the *Giornale degli Economisti*, November 1903.⁴²^[181]

the conditions of maximum ophelimity are therefore always satisfied in the case of exchange, when individuals operate according to type I.

Similar proofs may be given, in analogous cases, for production.

120. It may be useful to relate this proof to the general proof we have just given.

The sum of equations (141) is composed of two parts. One is expressed by

$$(Y - Y_0)\delta p_y + (Z - Z_0)\delta p_z + \dots \\ Y_0 = y_{10} + y_{20} + \dots, \quad \dots;$$

and it vanishes identically because

$$Y - Y_0 = 0, \quad Z - Z_0 = 0, \quad \dots$$

The other part is the one given by equation (142). This is the part that corresponds to δU in expression (130). As for the part under the integral sign, it vanishes identically because

$$\frac{\partial V}{\partial Y} = 0, \quad \frac{dp_y}{dX} = 0, \quad \dots$$

121. Let us consider a point of equilibrium for which equation (140) holds:

$$(140) \quad 0 = \frac{1}{\varphi_{1a}} \delta \Phi_1 + \frac{1}{\varphi_{2a}} \delta \Phi_2 + \dots$$

Let I denote an equilibrium position, for which we have the quantities $x'_1, y'_1, \dots, x'_2, y'_2, \dots$. Let II denote another position, which may or may not be one of equilibrium. The quantities corresponding to this position will be denoted by $x''_1, y''_1, \dots, x''_2, \dots$. The intermediate values will be $x_1, y_1, \dots, x_2, \dots$.

Suppose we pass from I to II, not along just any arbitrary path, but along the paths defined by the following equations:

$$(143) \quad \begin{cases} x_1 = x'_1 + \alpha_1 t, & y_1 = y'_1 + \beta_1 t, \quad \dots, \\ x_2 = x'_2 + \alpha_2 t, & \dots \dots \dots \dots \dots \end{cases}$$

The $\alpha_1, \beta_1, \dots, \alpha_2, \dots$, are constants, and t is a new variable, assumed to be positive. We may also set

$$p''_y = p'_y + \sigma_y t, \quad \dots, \quad a''_x = a'_x^{[183]} + \omega_x t, \quad \dots,$$

but we must be mindful of the fact that $\sigma_y, \dots, \omega_x, \dots$, are not constants. They are functions of the variables, and they result from the equations that the prices and the production coefficients must satisfy.

It is obvious, moreover, that when the ophelimities at two given points are independent of the paths followed, another path having the same extremities as the path we have just considered will give identical results.

The second-order variation of the ophelimity, for an individual, is

$$\begin{aligned}\delta^2\Phi_1 &= \varphi_{xx}\delta x_1^2 + \varphi_{yy}\delta y_1^2 + \dots \\ &\quad + 2\varphi_{zy}^{[184]}\delta x_1\delta y_1 + \dots\end{aligned}$$

By virtue of equations (143), this expression becomes

$$\begin{aligned}(144) \quad \frac{\delta^2\Phi_1}{\delta t^2} &= \varphi_{xx}a_1^2 + \varphi_{yy}\beta_1^2 + \dots \\ &\quad + 2\varphi_{zy}^{[185]}a_1\beta_1 + 2\varphi_{xx}^{[186]}a_1\gamma_1 + \dots\end{aligned}$$

122. Let us suppose for a moment that this quantity is always negative. In that case $\delta\Phi_1$, will always decrease as t increases, i.e., as one passes from position I to position II. We have seen (§89) that at the point where maximum ophelimity is attained, some of the variations $\delta\Phi_1, \delta\Phi_2, \dots$, in expression (112) must be positive, and some negative. If one departs from this point with constantly negative values of $\delta^2\Phi_1, \delta^2\Phi_2, \dots$, the preceding negative variations will go on increasing in absolute value; the positive ones will go on decreasing in absolute value, and may even become negative; but none of the negative ones can decrease in absolute value and become positive. Consequently, it is impossible to pass from position I to position II, by following the paths indicated, and increase the welfare of all the individuals in the community. The ophelimity enjoyed by some individuals will increase, but for others the ophelimity will decrease.^[187]

This proposition is verified not only for infinitely small displacements, but also for finite ones.

If the movement takes place in the direction in which the $\delta\Phi_1, \delta\Phi_2$ are all zero, all the ophelimities will decrease. This is what happens when one follows the path along which the consumers' equilibrium is established.

123. The proposition expressed in the preceding paragraph is subject to three conditions: (1) Equation (140) must be verified. The point I must therefore be one of the equilibrium points for which maximum ophelimity is attained; (2) The finite displacements must take place along the paths (143); (3) Along these paths, the second-order variations of ophelimity, for each individual, must be negative.^[188]

124. The question now is to determine when this latter condition is fulfilled.

To this end let us now refer to the considerations set forth in §47 and 48. To simplify the notation, let us indicate the ophelimities φ_{1x}, \dots , simply by φ_x, \dots . We shall have for an individual

$$(145) \quad \delta^2\Phi_1 = \varphi_{xx}dx_1^2 + \varphi_{yy}dy_1^2 + \dots + 2\varphi_{xy}dx_1dy_1 + \dots$$

(1) If φ_x depends only on x_1 , φ_y only on y_1 , etc., we have

$$\varphi_{xx} < 0, \quad \dots, \quad \varphi_{xy} = 0, \quad \dots$$

and the expression (145) is always negative.

(2) If the goods have a dependence of the first kind, we have, in general:

$$\varphi_{xx} < 0, \quad \dots, \quad \varphi_{xy} > 0, \quad \dots$$

But in this case, as we have already seen in §48, we may consider a commodity that provides us with the inequality (66):

$$(66) \quad \varphi_{xx} + a^2\varphi_{yy} + \dots + 2a\beta\varphi_{yz} + \dots < 0,$$

where a, β , are positive constants.

The left side is composed of two kinds of terms. The first kind is composed of the ophelimities $\varphi_{xx}, \varphi_{yy}, \dots$, multiplied by the squares $1, a^2, \beta^2, \dots$. These terms are always negative, whatever the signs of a, β, \dots . The second kind of term is of the type $2a\beta\varphi_{yz}$. These terms are positive when a, β , are positive quantities; some of them are negative as long as a, β , do not all have the same sign. When these terms are all positive, their sum added to the sum of the negative terms of the first kind is negative, by virtue of inequality (66). This will be the case all the more, therefore, if some of these terms are negative. Consequently, the expression (66) is always negative, whatever be the quantities a, β, \dots . The expression (145) will therefore always be negative as well.

(3) If the goods have a dependence of the second kind (IV, 14), we have

$$\varphi_{xx} < 0, \quad \dots, \quad \varphi_{xy} < 0, \quad \dots,$$

and the expression (145) may be positive or negative.

Suppose for example that there are only two variables, and that we have

$$(146) \quad \delta^2\Phi_1 = \varphi_{xx}dx^2 + \varphi_{yy}dy^2 + 2\varphi_{xy}dxdy.$$

We know that the second-degree form which corresponds to the right side of this equation is defined if we have

$$\varphi_{xx}\varphi_{yy} - \varphi_{xy}^2 > 0,$$

and then the expression (146) will always be negative. But if we have

$$\varphi_{xx}\varphi_{yy} - \varphi_{xy}^2 < 0,$$

the form is indefinite, and the expression (146) may change its sign.

If for a category of these goods the inequalities (58) could be assumed to hold, it would follow, as was explained in §49, that the expression (146) is always negative. But the fulfillment of the inequalities (58) cannot be assumed, at least not without new observations.

125. It follows from this analysis that in the case in which the elementary ophelimities of the commodities are independent, and in the case of a dependence of the second kind,^[189] the second-order variation of the ophelimities is negative, and consequently the third condition indicated in §123 is fulfilled.

126. In the case of exchange, if all the traders operate according to type I, the first condition of §123 is always fulfilled. As a consequence, in this case and for the commodities just indicated, it is impossible to depart from a point of equilibrium by following the paths (143) except by decreasing all the ophelimities, or some of the ophelimities enjoyed by individuals in the community considered, it being possible for some other ophelimities to increase.

127. It is easy to recognize the economic meaning of expression (112), i.e.,

$$\frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \frac{1}{\varphi_{3a}} \delta\Phi_3 + \dots$$

If we set

$$\delta s_1 = \frac{1}{\varphi_{1a}} \delta\Phi_1$$

we have

$$\delta\Phi_1 = \varphi_{1a} \delta s_1;$$

this means that the consumption of the quantities $\delta x_1, \delta y_1, \dots$, of the commodities provides individual 1 with an increase in ophelimity (or in the ophelimity index) equal to that which this individual would have by consuming the amount δs_1 of commodity A.

We see therefore that the expressions $\delta s_1, \delta s_2, \dots$, all represent amounts of commodity A; and if we set

$$\delta S = \delta s_1 + \delta s_2 + \dots,$$

δS will be a quantity of commodity A, and we shall have:

$$(147) \quad \delta S = \frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots$$

128. The quantities

$$\delta\Phi_1, \delta\Phi_2, \delta\Phi_3, \dots,$$

cannot be added, since they are heterogeneous; but we can sum the quantities

$$\frac{1}{\varphi_{1a}} \delta\Phi_1, \frac{1}{\varphi_{2a}} \delta\Phi_2, \dots,$$

because they are homogeneous, since they all are quantities of the same commodity A.

129. The expression (112) or its equivalent, (147), therefore represents the amount of commodity A which, when distributed to the members of the community, would provide each of them the same pleasure as that provided by the actual consumption of the commodities $\delta x_1, \delta y_1, \delta x_2, \dots$.

It is clear that as long as, by following a certain path, the expression (147) has positive values, it is possible, by making available this positive quantity δS , to increase the welfare of all the members of the community; and if the values of δS were negative, it would be possible to reduce the welfare of all the members of the community. When the value of δS becomes zero, there is no longer any of the commodity left to distribute to all the members of the community. If we wish to increase the shares of some of them, this can only be done by decreasing the shares of some others, in such a way that the sum of all the shares remains constant, since

$$\delta S = 0.$$

This is why this equation can serve to define maximum ophelimity for a community.^[190]

The economic meaning of the expression (112) would enable us to write down directly certain equations obtained by other routes. But it was useful to show how these equations could be obtained merely by considering expression (112), and to show how they were thus related to the general theories of the calculus of variations.

130. A glance at all the theories we have just set forth will bring out more clearly the general framework that may sometimes be lost sight of in the study of the details.

Pure political economy, just like rational mechanics, began with the study of special cases, and then only gradually rose to the general study of economic systems. This is, however, by far the most important study in economic science; we possess as yet only the first elements of it, and an enormous amount remains to be done in this direction.

131. The economic system to be studied may be conceived of either as a theoretical model of concrete phenomena, or as an ideal model of phenomena which reformers would like to put into practice.

This first operation must be kept quite distinct from all the others. In the case of concrete phenomena, the aim of the operation is to obtain a theoretical phenomenon approximating as closely as possible the concrete phenomena one has in view. This leaves a certain latitude, and one representation does not necessarily exclude some other one. Thus, by a similar operation, the earth may be represented by a sphere, an ellipsoid, or a spheroid.

In the case of an ideal phenomenon, the difficulty of the operation lies in the fact that reformers' ideas are often completely lacking in precision; but this very difficulty is useful, in that it induces us either to correct these conceptions or to reject them as meaningless.

For example, there is no difficulty in building an ideal system in which the distribution of incomes would be carried out according to the principle: equal shares of certain things to each. But it is absolutely impossible to construct an ideal system on the principle: to each according to his merits; or: to each according to his wants. It is necessary, therefore, before pursuing the discussion further, to specify precisely what one means by these terms.

132. The foregoing study can be summarized in the knowledge of the constraints of the system and in the equations that express them.

If the number of these equations is equal to the number of unknowns, the system has a rigid form, everything in it being determined by the constraints; we need not be concerned with it further (III, 24).

The most interesting cases are those in which the number of equations expressing the constraints is less than the number of unknowns. The system is then changeable, and its position of equilibrium remains to be investigated.

133. The economic system can be separated into a number of further systems that are held together at certain points, generally at the points of equilibrium.

For example, we have a system of equations

$$(148) \quad F_1(x_1, y_1, \dots) = 0, \quad F_2(x_2, y_2, \dots) = 0, \quad \dots,$$

which leaves a number θ of unknowns undetermined. We have a second system

$$(149) \quad G_1(\xi_1, \eta_1, \dots) = 0, \quad G_2(\xi_2, \eta_2, \dots) = 0, \quad \dots,$$

which leaves a number θ' of unknowns undetermined. These systems can and must be considered independently of one another; but they are then linked by equations that hold only at the point of equilibrium. Let

$$(150) \quad x_1^0, y_1^0, \dots, x_2^0, \dots, \xi_1^0, \eta_1^0, \dots, \xi_2^0, \dots,$$

be the values of the variables at one such point. The systems (148) and (149) are bound together by equations containing the quantities (150), say:

$$(151) \quad H_1 = 0, \quad H_2 = 0, \quad \dots$$

In the preceding paragraphs, we considered the case in which the system (148) is that of the exchange constraints, and the system (149) is that of the production constraints. The system (151) then expresses the relations between the amounts produced and the amounts consumed (§82).

Among the constraints there may be noted those that separate the system under consideration from the rest of the economic world. We have generally assumed that our system receives nothing from outside; but we could equally well consider systems that do receive certain things.

The condition that the system receives nothing from outside is expressed, in the case of exchange, by the condition that the total amount of each commodity remains constant; and, in the case of production, by the condition that the amounts produced result from the amounts transformed.

134. To find the position of equilibrium of a system such as the system (148), it is necessary to know how the movements that the system is capable of are brought about.

Suppose we can discover certain functions

$$(152) \quad R_i(x_1, y_1, \dots), \quad i = 1, 2, 3, \dots,$$

which have the following property.

Let x_1 be one of the variables that can be considered as independent, and let it be given a positive increment dx_1 . If this results in a positive increment for R_1 , the movement will take place in the direction of the positive x_1 ; if it results in a negative increment, the

movement will take place in the opposite direction. Consequently, no movement will take place if this increment is zero.^[191]

At the equilibrium points, we must therefore have:

$$(153) \quad \left(\frac{\partial R_i}{\partial x_i} \right)^0 = 0, \quad \left(\frac{\partial R_i}{\partial y_i} \right)^0 = 0, \quad \dots, \\ i = 1, 2, 3, \dots$$

We denote by $()^0$ the values of

$$\frac{\partial R_1}{\partial x_1}, \dots,$$

when, after performing the differentiation, we substitute for x_1, y_1, \dots , the values x_1^0, y_1^0, \dots , corresponding to the points of equilibrium.

There will be similar equations for any system similar to the system (148); they will complete the number of equations necessary to determine the unknowns at the point of equilibrium.

135. These considerations are extremely general; they apply just as much to mechanical systems as to economic systems.

For the latter, there are several kinds of index functions, such as the functions (152). The easiest to find are those having to do with the budget, particularly in the case of monopoly; this is what explains why, among the first works on mathematical economics, we find those of Cournot,^[192] dealing precisely with analogous cases.

Let x' be the amount of a commodity sold by a monopolist, at the point of equilibrium, and let $F(x^0)$ be his gain per unit. His total gain will be:

$$x^0 F(x^0);$$

and this expression may be taken as one of the index-functions (152) for the case of a monopolist attempting to obtain the maximum possible gain expressed in money.¹⁰

The case of free competition is analogous, but a little more complicated (§140).

136. The problem of finding index-functions that determine the direction of movement of consumers is more difficult. But if we assume, for a moment, that we can measure the pleasure that is provided to a man by an act of consumption, and if we assume that this man tries to obtain the greatest possible pleasure, the functions that measure pleasure will be precisely the index-functions (152) that we are looking for.

This is how pure economics was born, thanks to the works of Jevons, Walras, Marshall, Edgeworth, Irving Fisher, etc.

137. There is something superfluous in the result just obtained, and this is precisely what makes the result doubtful. Strictly speaking, it is not necessary to measure pleasure in order to obtain the functions (152); it suffices to have functions that increase as pleasure increases, and conversely. On the other hand, nobody has succeeded in proving

that pleasure can be measured, that it is a quantity; nor—what is more—has anyone discovered how to go about measuring it.

It is out of these considerations that my theory of ophelimity indices was born.

138. We can take still another step forward in the direction of generalizing our theories. It is not necessary for the functions (152) to be ophelimity indices; it is sufficient for them to be indices of the direction in which we foresee that the individual will move. Find, for instance, indices of asceticism, and substitute them for the functions (152), and you will have a mathematical theory of asceticism. Try to discover the index-functions of altruism, and you will have a mathematical theory of altruism.

At the same time as I thus made the meaning of equations (153) more general, I also made the meaning of the equations of the constraints (148), (§133) more general. This move from the particular to the general is not at all peculiar to the theories of pure economics; we find it again in rational mechanics and in other sciences.

The consideration of obstacles of the *second kind* (III, 73 et seq.), and the distinction of the types of phenomena (III, 40, 89), provide the theories of mathematical economics with a high degree of generality.

139. If the index-functions (152) do not contain the quantities x_1^0, y_1^0, \dots , explicitly, we can substitute x_1^0, y_1^0, \dots for x_1, y_1, \dots , in these functions, and obtain

$$\left(\frac{\partial R_i}{\partial x_i} \right)^0 = \frac{\partial R_i^0}{\partial x_i^0}, \quad \dots$$

But these equations would no longer hold if the R_i contained the x_1^0, y_1^0, \dots , explicitly. In that case, the system

$$(154) \quad \frac{\partial R_i^0}{\partial x_i^0} = 0, \quad \frac{\partial R_i^0}{\partial y_i^0} = 0,^{45} \quad \dots, \\ i = 1, 2, 3, \dots$$

is different from the system (153).^{[193][194]}

140. A very important distinction in political economy is linked to this observation.

Some index-functions (168)^{46[195]} contain constants which, once we assume equilibrium to be established, turn out to be determined as functions of the quantities x_1^0, y_1^0, \dots . Now, from the exclusively mathematical point of view, two types of index-functions (152) need to be distinguished: (I) The constants are left in these functions, and one obtains the system of equations (153). (II) One substitutes for the constants their values as functions of the x_1^0, y_1^0, \dots , and one obtains the system of equations (154).

These types are the same as those from which we already started from the economic point of view (III, 39 and following) and (Appendix, §35, 63, 67, 83, etc.).

In a more general manner, type (I) will correspond to all the cases in which the individual to which the index-function refers cannot, or does not wish to, modify *directly* the values of certain constants in the index-function. Type (II) corresponds to the case in which he has this power and uses it.

This division of economic phenomena, already mentioned by me in 1896 in my *Cours*²⁹ has become much more general in my more recent works.

141. Let us imagine an individual who sells an amount dx of a commodity and makes a gain of

$$f(x)dx.$$

After he has sold the amount x^0 , his gain will be

$$(155) \quad \int_0^{x^0} f(x)dx.$$

Whether the case is one of free competition or of monopoly, every individual tries to obtain the greatest possible gain. The function (155) may thus be considered as one of the index-functions (152), which will indicate the direction in which the individual will move in these two cases. But in the case of free competition the individual is unable to vary the constants that are expressed as functions of x^0 when the equilibrium is established. Consequently, by differentiating equation (155) we shall obtain

$$(156) \quad f(x^0) = 0,$$

for one of the equations (153).

In the case of monopoly, the individual has the power to make these constants vary; it will therefore be necessary to substitute their values as functions of x^0 in the expression (155) and then differentiate. We shall thus obtain

$$(157) \quad f(x^0) + \int_0^{x^0} \frac{\partial f}{\partial x^0} dx = 0$$

for one of the equations (154).^[197]

The gain may be expressed in terms of ophelimity (ophelimity indices) or in money.

Let us suppose that one operates according to the type of free competition. If, for a trader, the gain is expressed in terms of ophelimity, equation (156) yields equations (A) of §80. If, for an entrepreneur, the gain is expressed in money, equation (156) yields equation (116) of §91.^[198]

Similar considerations apply to equation (157) in the case of monopoly.

²⁹ *Cours*, §59, footnote 1, and *passim*. After having set down (§591) the budget equation

$$r_a + p_b r_b + p_c r_c + \dots = 0,$$

I had added: "... if the trader is concerned only with the amounts of economic goods he enjoys, without trying by any maneuvers to influence prices—which is the case in free competition—it will have to be assumed that only the quantities r_b, r_c, \dots are variable, and p_b, p_c, \dots will have to be treated as constants in the differentiation. If, on the other hand, the trader aims at controlling his demand and his supply in such a way as to make prices vary, it will be necessary to consider not only r_b, r_c , but also p_b, p_c, \dots , as variables."^[196]

142. We should further consider the second derivatives of the index-functions, in the case in which the first derivatives vanish. This study is interesting for distinguishing the different kinds of equilibrium. This is a field of study that has as yet been only little explored.^[199]

143. Consideration of the constraints and of the index-functions leads ultimately to a certain system of equations

$$(\Omega)$$

that determines equilibrium.

What is the utility of having thus determined this theoretical equilibrium? It is, first of all, the knowledge of whether the conditions posed are mutually compatible; whether—the number of equations being equal to that of unknowns—equilibrium is well determined; and, finally, whether the equilibrium in the conditions assumed is possible or impossible.^[200] We also learn to distinguish stable from unstable points of equilibrium, and we find equilibria that are stable in some directions and unstable in others.^[201] All these considerations give rise to numerous and interesting studies.

Further, equations (Ω) enable us to study the effects of variations in the constants in these equations and their economic meaning. This is still another broad field of research which as yet has hardly been touched.^[202]

Progress in this direction is likely to depend mainly on the practical knowledge we shall be able to acquire about the nature and the values of the parameters in the index functions. This information may perhaps be obtained in an indirect way. Some assumptions will need to be made, and it will then be seen whether they provide results that correspond to reality (§45, 57, and 101).^[203]

It should be noted that the investigations into economic equilibrium made by literary economists necessarily assume the existence of equations (Ω); and the difference between these studies and those of pure economics lies mainly in the fact that the first are made in the dark, by very imperfect and often erroneous means, whereas the second are made in full knowledge of the situation.

Finally, it is again to equations (Ω) that we need to refer for any study of the properties of economic equilibrium.

144. Among these properties, there is a very important one; this is the one having to do with the maximum of ophelimity for a community (§89).

We should inquire whether a certain equilibrium position is or is not at the same time a position of maximum ophelimity. Economic antagonisms arise from the fact that it is in the interest of certain persons to establish equilibrium at a point that does not correspond to maximum ophelimity for the community as a whole. The most common case is that of monopoly; but there are an infinite number of other cases that are scarcely caught sight of by literary economists, and about which only the theories of pure economics can provide precise notions.

The theory of maximum ophelimity for a community, which was sketched in my *Cours*, has subsequently undergone considerable development in various papers I have published since the *Cours*, and it has thus become much more general.

145. For a community, maximization of ophelimity is determined by the equation:

$$(158) \quad 0 = \frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots,$$

which we can also write:

$$\delta U = 0.$$

The values of the variables correspond to the point of equilibrium.

If we assume that the quantitates a'_1, a'_2, a'_3, \dots , are constant, i.e., that $\delta a'_1, \delta a'_2$, are zero and that there are no other constraints, the variations $\delta x'_1, \delta y'_1, \dots, \delta x'_2$, will be independent, and equation (158) will result in:

$$\begin{aligned} \varphi_{1x}(x'_1) &= 0, & \varphi_{1y}(y'_1) &= 0, & \dots, \\ \varphi_{2x}(x'_2) &= 0, & \varphi_{2y}(y'_2)^{[204]} &= 0, & \dots, \\ &\vdots &&\vdots & \end{aligned}$$

This is the case in which each member of the community consumes commodities X, Y, ..., up to the satiation point and has a quantity of A that remains constant. This is clearly a position of maximum ophelimity.^[205]

146. When consumers operate according to type I, equation (158) is transformed into:

$$(159) \quad 0 = p_x \delta X' + p_y \delta Y' + \dots - \delta A'' - p_b \delta B'' \dots$$

as we already saw in §110.

In the case of exchange, the system is constrained by the condition that the values of $X', Y', [206] \dots, A'', \dots$, must remain constant.

Their variations are therefore zero, and equation (159) always holds.

147. A constraint that is generally imposed on economic systems is the condition that the sum of all the budgets be equal to zero; this means that the economic system considered does not receive anything from the outside and does not give up anything to the outside.

When prices are variable, the sum total of the budgets is given by equation (129) of §109. Not only is it zero at the equilibrium point, but its variation must also be zero, since with the δ 's one moves from a point where the sum is zero to another where it is also zero.

As we saw in §109, the variation in this sum total of the budgets may be expanded into three parts, the last of which is an integral which vanishes when—as we shall assume in the following—the value of this sum does not depend on the path followed to arrive at the equilibrium point; the first two parts are

$$\delta U + \delta R$$

and we have:^[207]

$$(160) \quad \delta R = \delta X' \int_0^X \left(\frac{dp_x}{dX'} + \frac{dp_y}{dX'} \frac{dY}{dX} + \dots \right) dX.$$

In the case of exchange, and when one acts according to type I, the prices p_x, p_y, \dots , under the integral sign must be assumed to be independent of the limit of integration X' ; consequently,

$$\delta R = 0.$$

We have already seen that $\delta U = 0$; consequently the condition given by the budget constraints is satisfied.

148. The case of production is a little more complicated.

The consumers' budget constraint always holds; that is to say, assuming as always that the integral of formula (130) vanishes, we must have:

$$(161) \quad 0 = \delta U + \delta R;$$

but as the quantities $X', Y', \dots, A'', B'', \dots$, are no longer constant, we do not know whether the two terms on the right side of equation (161) vanish separately.

The relations between the amounts consumed or supplied by the consumers: $X', Y', \dots, A'', B'', \dots$, and the amounts produced or transformed by the firm: $X'', Y'', \dots, A''', B''', \dots$, will furnish one of the principal constraints relating consumption and production (§82).

Exactly the same thing can be expressed by the following two conditions, which thus yield identical constraints.

(a) The total amounts sold to consumers are equal to the total amounts produced, and the total amounts supplied by the consumers are equal to the total amounts transformed:

$$(162) \quad X' = X'', \quad Y' = Y'', \quad \dots, \quad A'' = A''', \quad \dots$$

The infinitesimal amounts that are produced at the equilibrium point by following the production path are equal to the infinitesimal amounts consumed at this point by following the consumption path:

$$(163) \quad dX' = dX'', \quad dY' = dY'',^{[208]} \quad \dots$$

(β) The total revenue from the sale of commodities on the part of the firm is equal to the total cost of production:

$$(164) \quad \int_0^{X'} (p_y - \pi_x) dX = 0, \quad \int_0^{Y'} (p_y - \pi_y) dY = 0, \quad \dots$$

The revenue from the sale of the last portion of the commodity is equal to the cost of production of this portion:

$$(165) \quad p_x^0 - \pi_x^0 = 0, \quad \pi_y^0 - \pi_y^0 = 0, \quad \dots$$

149. These constraints are of type I, for the firm.

When they are satisfied, maximum ophelimity is attained.

The demonstration can be carried out in two ways, according as the constraints are expressed in the form (α), or in the form (β).

These two forms were employed in §91.^[209] Form (5) was employed in §115.

Either way, we prove that

$$(166) \quad \delta R = 0;$$

consequently, equation (161) yields

$$(167) \quad \delta U = 0,$$

which shows that maximum ophelimity is achieved.

Similarly, in the case of variation of the production coefficients, we again establish equation (166); and, by means of equation (161), we obtain equation (167) again. This procedure can be used in any other similar case.

150. Equations (162) need not be compatible with equations (163); or—what amounts to the same thing—equations (164) need not be compatible with equations (165). In that case, the firm cannot act strictly according to type I. Depending on the circumstances, it will be the one or the other system of incompatible equations that will not be satisfied (§111). Then maximum ophelimity is not achieved.

151. After having considered the infinitesimal variations of ophelimity (the ophelimity index), we have to consider the finite variations when one passes from one point to another, with certain constraints. The study of the second variation of ophelimity may serve in certain cases (§121 to 126) to show that under certain conditions it is impossible for ophelimity to increase for all the members of a community when one moves a finite distance away from the equilibrium point.^[210]

Of course, this is only one very special case of a class of studies that are as important as they are varied.

152. When, for an individual, there are several points of equilibrium, for each of which maximum ophelimity is achieved, one should look for the *maximum maximorum*. One could also inquire into the differences of ophelimities (ophelimity indices) at different points, for which maximum ophelimity need not be attained (§100). Such investigations are of the utmost interest from an economic point of view; they are likely to give us somewhat more precise ideas about phenomena of which we now have only a vague understanding.

Index of Subjects

Roman numerals indicate chapters; Arabic numbers indicate paragraphs. Arabic numbers shown in bold font indicate the principal paragraphs that pertain to the topic.^[*]

Abstraction. Capital. V, 33. How pure economy uses it. III, 1–5; VI, 78. Faculty of—different among men. II, 47. The firm. V, 4, 62, 65, 66. Models of—necessary to order a theory. I, 20, 21, **22**, 23–27; III, 37, 39–45, 110; IV, 31. Its use to order the actions of men. II, 2, 3.

Appropriation of goods of others. IX, 17–22, 25–27, 31–36.

Banks. VIII, 46–50.

Bimetallism. VIII, 41.

Birth rate. VII, 38–41, 53.

Budget of the firm. V, 7, 24, **29**, 43–47, 59–61, 66–69, 83, 87; *App.* 34. [*Fr. App.* 77–81].

Budget of the individual. III, **175**, 200, 204; *App.* 32. [*Fr. App.* 38].

Budget of the producer. III, 176, 205.

Capital. V, 17–20, **21**, 22–32, 45; VIII, 1–8, 10, 22, 25, 26–28.

Capital goods proper. VIII, 8.

Cause of price or value; non-existence of the—. III, 225–227; V, 42.

Circulation of *aristocracies* or élites. II, 103–107; VII, **19**–21, 55, 97, 101, 109–114, 116; IX, 30–35.

Class struggle. II, 103–109, 113–123; VII, 21; VIII, 25; IX, 23–27, 29–36.

Collective agreements on production. IX, 16.

Collectivist society. III, 49, 50, 165, 166; V, 12; VI, 48, **52**–61; *App.* 35, 43–47. [*Fr. App.* 89].

Competition. III, **46**, **82**, 86, 105, 116–127, 137–151, **162**, 163, 170, 196–216, **208**; V, 63, 73; VI, 6, 10, 13, 49–51; VIII, 2; *App.* 20–24, 26–35, **36**, 37. [*Fr. App.* 34, 83]. Complete and incomplete—. III, 105; VI, **150**.

Complementary goods. IV, 9, 12; *App.* 15. [*Fr. App.* 49].

Concrete phenomenon. Chapter IX. Divergences from the real phenomenon. III, 110; V, 27–31; IX, 2. Science is analytic, practice is synthetic. I, **26**–30. One cannot know—in every particular. I, 10. One cannot know—from a study of the concepts that we have *a-priori*. I, 15–17. Theories of concrete phenomena are always incomplete and only approximate. I, **11**, **19**, 20.

Constraints. IX, 37, 38.

[*] Numbers included between square brackets refer to the relevant paragraph number of the appendix to the French *Manuel*.]

- Continuous and discontinuous variations.** III, 65–67; IV, 47.
- Correlation.** VII, 48.
- Cost of production.** III, 177–179, 221, 224; V, 78–80, 94; *App.* 34. [*Fr. App.* 79]. Equality of the—and the sales price. III, 176, 205, 221; V, 66–69, 88; *App.* 35. [*Fr. App.* 83]. Cost of production increasing with the produced quantity. III, 102, 103; VI, 3, 12–14, 23. *Idem:* decreasing. III, 105; VI, 3, 4, 10, 11, 23, 24.
- Cost of reproduction.** III, 221.
- Curve of errors.** VII, 6.
- Debt reduction.** V, 112, 113, 117.
- Decline in the inequality of income.** VII, 24, 25.
- Demand.** See supply and demand.
- Dependence of goods.** IV, 8–24, 35–42, 49, 50, 55–57; *App.* 9, 12–15, 48, 49. [*Fr. App.* 46].
- Depreciation.** V, 30–32; VIII, 11, 12.
- Differences from the mean.** VII, 3–10.
- Discount rate of the banks of issue.** VIII, 49, 50.
- Distribution of incomes.** VII, 11–31; *App.* 39. Diminution in the inequality of income. VII, 24, 25.
- Division of labor.** V, 2, 3.
- Economic circulation.** V, 6; VI, 92.
- Economic crises.** VII, 46, 47; IX, 73–88.
- Economic dynamics.** III, 7, 8; see also: Economic crises.
- Economic equilibrium.** III, 14, 19, 22–24, 27, 90–228; VI, 26–31. *Appendix.* [*Fr. App.* 22–23]. Stable and unstable. III, 123, 124, 125, 126, 133, 215, 216. Successive positions of—. V, 89.
- Economic goods.** III, 16.
- Economic statics.** III, 7, 8, 9; *App.* 9.
- Entrepreneur. Firm.** V, 2, 4–7, 62–69; VIII, 50. Capital of the entrepreneur. V, 61. False interpretations of competition among entrepreneurs. VI, 93–96; IX, 84. The objective of the firm. V, 8–12. Courses open to the firm. V, 13–16.
- Evolution.** If it can be a source of moral precepts. I, 44. Of morals, II, 20.
- Exchange.** III, 40–51, 69, 70, 82, 83, 97–99; VI, 8. Equilibrium of—. III, 109–111, 112, 114–119, 120, 121–133, 196–204, 208; VI, 34–38; *App.* 20–30. [*Fr. App.* 63–67].
- Exchange ratio.** III, 226.
- Experimental truth.** I, 36. If the universal consent of men can substitute for experience. I, 46. Always contingent. I, 36.
- Factors of production.** V, 36.
- Feminism.** VII, 54.
- Final degree of utility.** III, 228.
- Foreign exchange.** VIII, 31–34, 35–39.
- Free exchange and protection.** IX, 42–72.
- Gold point.** VIII, 33, 34.
- Gresham's Law.** VIII, 40.
- Guarantee on bank bills.** VIII, 47.

- Hierarchy of commodities.** IV, 19, 51, 66.
- Hill of pleasure or ophelimity.** III, 58, 59, 60, 127, 134; IV, 69, 70; *App.* 5, 6, 48. [Fr. *App.* 3].
- Hill of profit.** III, 81, 84.
- History of economic phenomena.** I, 33.
- Human society in general.** VII, 97–101.
- Indices of ophelimity.** III, 35, 36, 55, 97; *App.* 2–11. [Fr. *App.* 46–50].
- Indices of profit of the producer.** III, 75, 76, 86.
- Indifference lines of obstacles.** III, 75–81, 100–105, 114, 133; V, 63–65; VI, 2–4.
- Indifference lines of the producer.** III, 76, 77, 100, 103–105.
- Indifference lines of tastes.** III, 52, 53, 54, 55–59, 77, 80, 97; IV, 27, 43–65, 67; VI, 19; *App.* 2–7, 10, 11, 13, 14, 16, 17. [Fr. *App.* 2–4].
- Insurance.** V, 30–32; VIII, 11, 12.
- Interest (yield) on capital.** V, 48–60, 66–69, 91, 92; VII, 117; VIII, 11, 48, 49, 50. Gross interest (yield). V, 49, 50. Net interest (yield). V, 52–58, 66–69, 88, 90; VI, 50, 51, 54. Interest rate on savings. VIII, 11–25. Alleged decline in the net interest on capital. VIII, 26–28.
- International trade.** III, 85; VI, 65–69; VII, 117; IX, 39–72.
- Intuition.** I, 45.
- Invention.** I, 51.
- Iron law.** VII, 58.
- Landed capital.** VIII, 1–7.
- Law of markets [Say's law].** IX, 41.
- Law of supply and demand.** III, 182–193, 198, 222, 223. [Fr. *App.* 52–55].
- Legal fictions.** II, 12.
- Line of complete transformations.** III, 75, 79, 108, 133, 141–151; V, 63, 65; VI, 4, 7, 10, 40, 43, 48, 58, 59.
- Line of maximum profit.** III, 105, 107, 113, 135–140, 147–150; VI, 12–25.
- Logical action and non-logical action.** II, 2, 3, 4, 5, 18, 22–26, 49–51, 91–93, 101, 108. People believe that non-logical relations are logical. I, 4, 9, 22; II, 108; VIII, 24, 25; IX, 20, 28, 29, 38, 62, 66, 79. Savings. VIII, 11.
- Marriage rate.** VII, 38–40, 45, 47, 49.
- Material and non-material goods.** V, 34.
- Maximum of collective utility.** VI, 33, 34, 37, 38, 44, 53–61; *App.* 45–48. [Fr. *App.* 89, 117]. For a partial collective. VI, 62–64.
- Maximum of ophelimity for the individual.** III, 130–134; VI, 9, 41–48; IX, 6, 19; *App.* 41, 42.
- Method.** I, 35, 36; II, 1; III, 3, 13, 189, 217, 218; *App.* 16.
- Money.** V, 27, 46, 86, 87; VII, 112; VIII, 29, 30.—is a rather small part of the wealth of a country . VIII, 43, 44. Surrogates for—. VIII, 42. Quantity theory of—. VII, 71–79.
- Monopoly.** III, 47, 48, 128–132, 151, 161, 162, 164; V, 64, 83; VI, 10; IX, 53, 54; *App.* 25. [Fr. *App.* 68–76 bis].
- Moral and religious sentiment.** II, 21, 22, 41–74, 81–87, 91, 110–123.
- Morales and customs.** II, 18–20, 23–40.

- Mortality.** VII, 38–40, 42, 49.
- Mutual dependence of phenomena.** III, 3, 219, 224, 225–228; V, 6; VII, 55.
- Objective relations and subjective relations.** II, 6, 7–17. The theories of those two genera are essentially different. II, 18.
- Obstacles.** III, 14, 15, 19, 23, 25, 27, 37, 68–74, 106, 118, 147, 202; Chapter V. First kind of—. III, 68, 69–72. Second kind of—. III, 68, 73, 74, 167, 168.
- Ophelimity.** III, 30, 32–36, 58, 78, 196–199, 228; Chapter IV; *Appendix*. Characteristics of—. IV, 33–47, 69, 70. Elementary ophelimity. III, 33; *App. 7*. Weighted elementary ophelimity. III, 34, 198; *App. 22*.
- Overconsumption.** IX, 82–84.
- Overproduction.** VI, 93–96; IX, 83, 84.
- Paper-money.** VI, 72, 74, 76, 112.
- Paths.** III, 60–63, 74–91, 94–98, 117–120, 212; *App. 18*, 24, 29, 36, 41. Linear paths. III, 96, 108, 133, 172; *App. 25*, 27, 30, 37, 40, 42. [Fr. *App. 38*].
- Political economy.** The scope and study of—. I, 1 footnote, 3; III, 1–5, 14. On taking account of morals. I, 24, 25, 26. Its evolution. III, 29–31, 157, 189, 219–228; IV, 11; V, 93; VI, 15, 97–101; VII, 69, 89–94; VIII, 21–27; IX, 20–22.
- Prices.** III, 152, 153–155, 158–174; VI, 7–9, 39–48, 54; *App. 21*. [Fr. *App. 37*, 39]. Relations between equilibrium and the prices of the factors of production. VI, 80–91. High prices and low prices. VI, 90, 91; IX, 23, 24.
- Private property.** VI, 60, 61; VII, 106, 115; VIII, 4–7.
- Production.** III, 71, 78, 79, 82, 84, 100–105, 135–151, 205–216; V, 81–97; VI, 39–51, 97–101; IX, 8, 37, 38. Equilibrium of—. V, 81–87, 96; VI, 39–61; *App. 31–46*. [Fr. *App. 77*, 82, 84, 88]. Distribution of—. V, 78–80; *App. 39*. [Fr. *App. 107*]. Production of capital goods. V, 88, 89.
- Production coefficients.** V, 35; *App. 33*. [Fr. *App. 78*]. How to determine the—. V, 16, 37, 82–86; VI, 49, 51, 55, 63; *App. 39*. [Fr. *App. 101–106*]. Variability of the—. V, 15, 70–77, 82–86; *App. 39*.
- Production of personal capital goods.** VII, 57–61.
- Purchasing power of money.** III, 222.
- Pure economy.** III, 3–7, 22, 228; VI, 61; IX, 38; *Appendix*.
- Quantitive theory.** I, 11, 31; II, 100, 110.
- Rareté.** III, 227.
- Real movement.** II, 97; III, 22.
- Reasoning by the method of elimination.** I, 18.
- Relations between economic conditions and the population.** VII, 32–56.
- Rent.** V, 62, 63, 69, 90–97; VI, 59; VIII, 3, 8; IX, 56, 63.
- Reproductive power.** Restrained by obstacles. VII, 62–67, 71–80.
- Rhythm.** II, 55, 56, 57, 58; VII, 55; IX, 73, 74, 75.
- Savings.** VIII, 9–21. Savings capital. VIII, 10.
- Science.** Pure science III, 5. No scientific proposition is true in an “absolute” sense; each proposition must be underpinned by the condition: *within the limits of the experiences known to us*. I, 36. Deals exclusively with experimental propositions. I, 36, 37, 38.

All that resonates precept is not scientific. I, 39. Science is different from faith. I, 36, 41, 101. Confusion between science and faith. I, 42, 43–48. Contrast between the conditions of action and knowledge. II, 101.

Scientific laws. See uniformity.

Selection. II, 30; VII, 50, 98, 99, 104–115; IX, 19, 57.

Sensations of a man, compared together, or with those of other men. III, 11, 12, 16, 17.

Services of capital. V, 33, 45, 50, 52.

Social heterogeneity. II, 102, 109, 110; VII, 2–21.

Social hierarchy. II, 102; VII, 97, 98, 103.

Social mutability. VII, 104, 106–115.

Sociology. [II], 75–80, 84.

Speculation. III, 171.

Stability. VII, 97, 98, 103, 105, 106, 109, 114, 115.

Standard of life. VII, 57.

Subjective view. II, 6–17, 91–96, 101, 104–109, 113–123; IX, 20, 22.—of price variations. VI, 90; IX, 23–27.—of population phenomena. VII, 55, 81–88, 116, 117.—of savings phenomena. VIII, 22–25.—of production phenomena. IX, 9, 12, 61, 62, 66, 79.

Subsistence and population. VII, 68–70.

Successive approximations. I, 13, 14; III, 6.

Supply and demand. III, 180–195, 222, 223, 224.

Syndicates. II, 50; VI, 63; IX, 4, 6, 10–16, 25, 26, 31.

Tastes. Chapter IV. III, 14, 19, 25, 27, 37, 78, 199. Direct and indirect effects of—. III, 39–42, 93–199; *App.* 23.

Terminal point. III, 62, 63, 94.

Theories. Character and criteria of truth. I, 20; II, 6. The—of concrete phenomena are only approximate. I, 11, 32.—are incomplete. I, 10, 19, 20; II, 7.—and direct experience and indirect experience. I, 20.—as opposed to practice. I, 28. Science is analytical, practice is synthetic. I, 26. Exclusively negative criticism is useless. I, 31.

Theory of comparative costs, Ricardo's—. IX, 42–52.

Theory of Malthus. VII, 89–96.

Trade. Wholesale—. IX, 7, 8. Retail—. IX, 3–6.

Transformation of economic goods. III, 70.—objective. III, 71.—material. III, 72; V, 48.—in space. III, 72; V, 38, 48.—in time. III, 72; V, 39–48, 53, 56.

Trusts. IX, 10–16.

Type of economic phenomena. III, 39, 40, 41, 42, 43–45, 49–51, 92, 110–115, 117–132, 138–151, 160, 161, 196–208; V, 8–12, 71–75, 81, 82–84, 86, 88; VI, 16, 52–61; *App.* 23, 25, 35, 39. [*Fr. App.* 35].

Uniformities or laws. I, 4, 5, 6. Conditions of—. I, 8, 9, 30. Exceptions to—. I, 7.

Utility. III, 29, 30, 35.—of the individual and of the species. II, 30, 34, 36, 37; VII, 99, 102, 103, 109.

Value in exchange. III, 156, 157.

Value in use. III, 29, 35.

Virtual movement. II, 97; III, 22.

Authors' Names Index

(Numbers indicate the page number)^[*]

- Aeschylus* 42
Ammon, Otto 214
Anaximander* 6
Anaximenes* 6
[Antigenes] 44
Aristophanes 30, 45, 46, 213
Aristotle 18, 34, 45, 58, 69, 241, 252, 253
Augé Laribé 250
Aynard 70

Baker* 202
[Balzac] 213
Barone* 330
Bastable, Prof. 256
Bayle 42
[Bazin, R.] 61, 62
Benini, Prof. R. 192, 213
Bentzon 202
Bernard*, Cl. 18
Bertrand, Prof. J. 193
[Boccaccio] 2, 35
[Boileau] xv
Boissier, G. 35, 43
Boninsegni, Prof. P. 274, 281,
 310, 335
Bourdeau, Jean 263
Bresciani* 197
Brunetière 17, 61, 62
Bulot 40

Cairnes 121
Cato 2
Cauderlier* 200, 201

Cicero 24, 30, 42, 47, 69
Colajanni, Prof. Dr. N. 69, 245
Combes 40, 55, 56, 59
Cournot* 77, 391
Croce* 4, 5, 7, 18

Dante 15, 38, 143, 214
Darwin* 47
Descartes* 18
[Dicaearchus] 6
Diogenes Laertes 18, 44, 45
Dionysius of Halicarnassus 69
Droz, Numa 264

Edgeworth, Prof. F. Y. 77, 83, 273, 309,
 347, 391
[Epictetus] 44
Essars, Pierre des 270, 271
Euclid* 4, 78
Euripides 42, 44

Fechner 133
Ferrara 120, 145, 154
Fisher, Prof. Irving 77, 78, 148, 391
[Fourier] 211
Fournier 213
[Fresnel] 24, 25
Funck-Brentano 40
Furlan* 320
Fustel de Coulanges 42, 242, 243

Gaius 34
Germain 70

[*Numbers refer to the page number for the present edition of the *Manuale-Manuel*. Names that are marked with an asterisk were not listed in the Authors' Names Index for the 1906 *Manuale*, but were listed in that index for the 1909 *Manuel*. Authors included within square brackets were not listed in either of these indexes (1906 and 1909), but they were cited by Pareto.]

- Gide* 320
 Giornale degli Economisti 8, 118, 194, 197,
 233, 274, 281, 304, 307, 310, 313, 319,
 320, 330, 335, 337, 340, 347, 382, 384
 Giusti 213
 Godwin* 210
 [Gohier, Urbain] 70
 Graziani* 122
 Greef, G. de. 1
 [Gresham] 229
 Grote* 37, 46, 57
 Guyot*, Yves 41
- Hegel 7, 8, 24
 Hesiod 37
 Horace 46
 Jacoby, Paul 214
 Jagemann, Prof. 71
 [Jaurès] 28, 68, 250
 Jevons 77, 122, 272, 391
 Juglar, Clément 270, 271
 Kant 18, 19, 34
 [La Bruyère] 62
 [La Fontaine] 118, 119
 [Laplace] 78
 Lapouge, Vacher de 214
 Lassalle* 205
 [Law] 268
 Lea 43
 Leibniz 18
 Leroy-Beaulieu, Paul 3, 197, 226,
 238, 239
 Levasseur 199, 209
 Lucas-Championnière* 44
 [Lucian] 44
- Machiavelli 43, 58, 59
 Magnaud* 34
 Maine, Sir Henry Sumner 49, 67, 213, 216
 [Malestroict] 236
 Malthus* 207, 208, 210, 212
 Marshall, Prof. 77, 200, 320, 339, 391
 Martello, Prof. Tullio 212, 227
- Martial* 130
 Marx, Karl 57, 66, 122, 218, 226, 239
 [Maxwell] 24, 25
 Menger*, Anton 226
 Mill, John Stuart 2, 29, 33, 34
 [Moffet] 202
 Molinari*, de 239
 Moniteur des intérêts matériels
 163, 245
 Montesquieu 27, 213, 268
 Mosca, Prof. G. 213
 Musset, Alfred de 28
- [Newton] 18, 78
 [Nietzsche] 14, 52
- Odyssey 29
 Ovid* 248
- Pantaleoni, Prof. Maffeo 233
 Pauthier 35
 Pertile 242, 243
 [Phaedrus] 118
 [Phidias] 44
 Pidoux 50
 Plato 8, 18, 27, 30, 35, 37, 203
 Pliny 2, 6
 Plutarch 44, 46
 Poincaré*, H 7, 22, 24
 Polybius 47
 Post 49
- Reinach*, S 41, 43
 [Renan] 213
 Ribot* 238
 Ricardo 171, 255, 256, 257, 259, 260
 Rogers*, Thorold 203
 Roosevelt 59, 60, 270
- [Sallust] 47
 [Sand] 51
 Say, J.B. 235, 236, 255
 Sextus Empiricus 30
 Sismondi 207
 Smith, Adam 2, 3
 [Socrates] 30, 35, 42, 44, 45, 46, 59

- Sorel, G. 10, 11, 57, 66, 69, 182, 239,
242, 243, 347, 249, 250, 252
- Spencer, Herbert 30, 31, 32, 41, 42,
60, 63
- Stahl 71
- [Strauss] 213
- Tannery, Paul 6, 19
- Tarde 52, 213
- Theophrastus* 35
- Teognis of Megara 49
- Tertullian 24
- Thornton 121
- Tocqueville*, de 43, 210
- Trollope* 202
- Thucydides 3, 46, 56
- [Ugo (Hugo), V.] 51
- [Vera] 7
- Villani 203
- Volterra, Prof. Vito 8, 313
- Waldeck-Rousseau* 246
- Walras 77, 122, 123, 124, 302, 313,
359, 391
- [Weierstrass] 285
- Wundt* 133
- Xenophon 30, 45, 203
- Zeller* 46

Annotations by John Chipman

PREFACE TO THE FIRST (1906) EDITION

[1] The dimensions of the original *Manuale* are $13^{\circ} \times 8^{circ}$ cm with thickness 2cm, covering 592 pages.

[2] The word “production” (*produzione*) is found in the original text, but it is clear from the context that this is a misprint for “protection” (*protezione*).

[3] The reference is to footnote (221)² on p. 120 of Vol. I of the *Cours d'économie politique*.

[4] Pareto is alluding to the scandal that was caused by the publication of his article “L’Italie économique” in the French periodical *Revue des Deux Mondes*, 61 (October 15, 1891), 904–944 (reprinted in V. Pareto, *Oeuvres Complètes*, Vol. II, *Le Marché Financier Italien*, edited by G. Busino, Geneva: Librairie Droz, 1965, pp. 1–36). The work contains a stinging indictment of the Italian political leaders of the day and their militarist and protectionist policies. It is recounted in Vol. I, pp. 134–9, of Gabriele de Rosa’s magnificent three-volume work, *Vilfredo Pareto, Lettere a Maffeo Pantaleoni* (Rome: Banca Nazionale del Lavoro, 1960), that Pareto had originally published the substance of his article in two issues of the newspaper *Secolo*, June 4–5, 1891, but that this had not attracted attention. He subsequently delivered two lectures on the subject in Milan, the second of which was broken up by the police. It was only after this that he decided to find a respectable outlet for his article abroad. No sooner was it published than he was denounced in the pro-government Italian press for his lack of patriotism. His friend Maffeo Pantaleoni, whom Pareto had cited approvingly in a footnote, lost his job as Professor of Statistics and Director of the Bari Business School as a consequence.

[5] An English translation of the verse is

“Whoso Cotin scorns does not esteem his king;
And, Cotin says, recks neither God, nor faith, nor law.”

CHAPTER I

I

[1] Italian *ricette*, French *recettes*. The word *recipe* in English, in addition to its meanings in medicine and cookery, has the more general meaning “a means (actual or suggested) for attaining or effecting some end” (Oxford English Dictionary), or “a means prescribed for producing a desired result” (Webster’s Collegiate Dictionary). The latter seems to reflect Pareto’s meaning perfectly.

[²] Passage omitted from the 1909 edition.

[³] This sentence from the 1906 edition was removed from the 1909 edition. Santa Lucia is the saint who is supposed to take care of eyesight.

[⁴] The indicated passage corresponds to the version in the 1909 edition. The corresponding passage in the 1906 edition reads: “choose one of them.”

[⁵] 1909 version. The 1906 version reads: “I beg the reader’s forgiveness in advance for any mistakes I may commit.”

3

[⁶] These two passages from the 1906 edition were removed in the 1909 edition.

[⁷] The passage quoted is the opening sentence of the Introduction to Book IV of Adam Smith’s *Wealth of Nations*, 1776.

[⁸] The quoted passage occurs in the second paragraph of the Preliminary Remarks in J. S. Mill’s *Principles of Political Economy*, London: John W. Parker, 1848, Vol. I, p. 1.

[⁹] Paul Leroy-Beaulieu, *Traité théorique et pratique d’économie politique* (in four volumes), 2nd ed., Paris: Librairie Guillaumin et C^{ie}, 1896. In the *Cours d’économie politique*, Vol. I (Lausanne: F. Rouge, 1896), Pareto criticized Leroy-Beaulieu’s approach in great detail (§572n, p. 419).

6

[¹⁰] 1909 version. The 1906 version reads: “Ariosto’s *Orlando furioso*.”

7

[¹¹] The term “social” is that used in the 1906 edition. The term used in the 1909 edition is “sociological.”

[¹²] This footnote was added in the 1909 edition. The reference is to Croce’s book review of Pareto’s *Manuale* published in his own journal, *La Critica: Rivista di letteratura, storia e filosofia*, 4 (February 1906), 129–134 (esp. p. 131), reprinted as Chapter XI, “Economia filosofica ed economia naturalistica,” of the second (1907) edition of Croce’s *Materialismo storico ed economia marxistica* [later editions, Bari: Gius. Laterza & Figli, 1921, pp. 259–268 (esp. p. 263) and 1961, pp. 265–275 (esp. 269)]. (There are English translations of this collection but they do not include this chapter.) See also the earlier interchange between Pareto and Croce published in the *Giornale degli Economisti* [2], 21 (July, August 1900), 15–26, 139–162; 22 (February 1901), 121–130, 131–138), also included in Croce’s above collection; an English translation appeared in the *International Economic Papers*, No. 3 (London: Macmillan, 1953), pp. 172–207.

[¹³] 1906 edition. Omitted in the 1909 edition.

9

[¹⁴] 1906 version. The 1909 edition replaces “theorem” by “phenomenon” in both places. This change must have been made by the French translator Alfred Bonnet.

10

[¹⁵] This footnote was added in the 1909 edition. $\theta\epsilonορία \varphiυσική$ = theoria physiki, or physical theory. (On Croce, see note 12 above.)

11

[¹⁶] Paragraph added in the 1909 edition.

12

[¹⁷] “To me this seems a questionable guess, as I know that some peaks of the Alps rise to a great height, not less than 50,000 paces.” Cf. Pliny, *Natural History*, in ten volumes, with an English translation by H. Rackham; Loeb Classical Library, London: William Heinemann Ltd., and Cambridge, Mass.: Harvard University Press, 1938, Vol. I, pp. 298–9.

13

[¹⁸] In the 1909 edition, “calculations” was replaced by “studies”.

16

[¹⁹] Added in the 1909 edition.

17

[²⁰] 1906 version. The 1909 version reads: “reasoning about”.

[²¹] Footnote 9 was added in the 1909 edition.

20

[²²] Paragraph added in the 1909 edition.

21

[²³] 1906 edition; omitted from the 1909 edition.

[²⁴] Vito Volterra, “Sui tentativi di applicazione delle matematiche alle scienze biologiche e sociali,” *Giornale degli Economisti* [2], 23 (November 1901), 436–458.

23

[²⁵] 1906 version. The 1909 version is “relations.” The Italian for “reactions” is *reazioni*; the translator must have misread this as *relazioni*.

25

[²⁶] 1906 version (*semplicemente*). The 1909 version is “implicitly” (*implicitement*), presumably another translator’s error.

28

[²⁷] Omitted from the 1909 edition.

[²⁸] Georges Sorel, *Introduction à l’économie moderne*, Paris: Librairie G. Jacques, n.d. (c. 1903). The original has a semicolon in place of the comma, and the remainder of the sentence reads: “he will not hesitate to impose customs duties.” The passage quoted here (as well as that of footnote 14) is part of a general criticism of the *Cours* which Pareto acknowledges in the Preface to be just.

33

[²⁹] The word “metaphysical” was added in the 1909 edition.

36

[³⁰] 1906 edition; passage removed from the 1909 edition.

38

[³¹] 1906 version, replaced in the 1909 version by its antonym, “assertions” (“*affirmations*”).

39

[³²] Footnotes 16 and 17 were added in the 1909 edition.

40

[³³] 1906 version (*utile*). The 1909 version is “utility” (*utilité*, corresponding to the Italian *utilità*). The change could well have been deliberate. The term “maximum utility for society” was later used by Pareto in his paper, “Il massimo di utilità per una collettività in Sociologia,” *Giornale degli Economisti* [3], 46 (April 1913), 337–341 (see also the *Trattato di Sociologia Generale*, Florence: Barbera, 1916, §§2128–2131), in a sense involving interpersonal utility comparisons, as opposed to the term “maximum ophelimity” which meant what we now call Pareto optimality.

41

[³⁴] 1909 version. The 1906 version was “pointless and foolish”.

42

[³⁵] “I believe because it is absurd.”

[³⁶] 1906 edition; removed in the 1909 edition.

[³⁷] The following is Longfellow’s translation of the verse in the text:

Mortals, remain contented at the *Quia*;
 For if ye had been able to see all,
 No need there were for Mary to give birth.

Cf. *The Divine Comedy of Dante Alighieri*, translated by Henry Wadsworth Longfellow, in three volumes (Boston and New York: Houghton, Mifflin and Company, 1867, 1895), Vol. II, *Purgatorio*, Canto III, p. 21. Charles Eliot Norton, in his textual translation (Boston and New York: Houghton, Mifflin and Company, 1893, Vol. II, *Purgatory*), explains (p. 15): “*Quia* is used here, as often in Mediæval Latin, for *quod*. The meaning is, Be content to know that the thing is, seek not to know *why* or *how*—*propter quid* it is as it is.”

[38] Longfellow’s rendering of the verse in footnote 18 is as follows:

There will be seen what we receive by faith,
 Not demonstrated, but self-evident
 In guise of the first truth that man believes.

(*Op cit.*, Vol. III, *Paradiso*, Canto II, p. 17.)

45

[39] 1906 version. The 1909 version replaces “inconclusive” by “without utility”.

[40] 1906 edition. The 1909 version is “facts of experience”.

[41] Added in the 1909 edition.

47

[42] “As Plato was conversing about Ideas and using the nouns ‘tablehood’ and ‘cuphood’, [Diogenes] said, ‘Table and cup I see; but your tablehood and cuphood, Plato, I can nowise see.’” Diogenes Laertius, *Lives of Eminent Philosophers*, with an English translation by R. D. Hicks, in two volumes, Loeb Classical Library, London: William Heinemann, and New York: G. P. Putnam’s Sons, 1925, Vol II, pp. 54–5. The transliterations are *τράπεζα* = trapeza, *τραπεζότης* = trapezotis, *κύαθος* = kyathos, *κυαθότης* = kyathotis. The French text initially misspells Plato’s name in Greek as Hlato.

[43] The indicated paragraphs in footnote 20 were added in the 1909 edition.

48

[44] Paragraph added in the 1909 edition.

49

[45] 1909 version. The 1906 version is “scientifically.”

CHAPTER II

6

^[1] 1906 version. The 1909 version reads: “scientific truth.” This change could have been deliberate on Pareto’s part, given that it immediately precedes the two new paragraphs (see note 2 below). However, the earlier version has been retained here since it is logically closer to the sentences that precede it, and more important, it is the statement referred to in Chapter I, §36.

^[2] The two indicated paragraphs (including footnote 3) were added to §6 in the 1909 edition. The reference in footnote 3 is actually to an article with a different title, namely “L’économie et la sociologie au point de vue scientifique,” in the *Rivista di Scienza* (also known as *Scientia*), Vol. I, No. II (1907), pp. 296–315. It is reprinted in V. Pareto, *Oeuvres Complètes*, Vol. IX, *Marxisme et Économie pure* (edited by G. Busino), Geneva: Librairie Droz, 1966, pp. 147–161.

9

^[3] The letter “S” was used in the 1909 edition, replacing the letter “T” used in the 1906 edition.

12

^[4] The letter “C” was omitted in the 1909 edition.

13

^[5] 1909 version. The 1906 version was “many things by”.

^[6] The words “a little” are found in the 1906 but not the 1909 edition.

^[7] 1906 version. The 1909 version is “animals.”

14

^[8] “My own opinion is that, if the kinds of divination which we have inherited from our forefathers and now practice are trustworthy, then there are gods and, conversely, if there are gods then there are men who have the power of divination.” Cf. Cicero, *De senectute*, *De amicitia*, and *De divinatione*, with an English translation by W. A. Falconer, Loeb Classical Library, London: William Heinemann, and New York: G. P. Putnam’s Sons, 1923, pp. 232–3.

^[9] “They have their abode in the air, the stars are their neighbors, their commerce is with the clouds, so they can learn what is preparing in the sky and promise the rain, which they feel already.” Cf. Tertullian, *Apologeticus* and *De Spectaculis*, with an English translation by T. R. Glover, Loeb Classical Library, London: William Heinemann Ltd., and New York: G. P. Putnam’s Sons, 1931, pp. 120–1. Pareto’s spelling and punctuation differ slightly from that given there.

^[10] The two additional paragraphs and footnote 10 were added in the 1909 edition.

16 and 18

[11] 1906 edition. Omitted from the 1909 edition.

18

[12] 1906 version. The 1909 version is: “a fair number of publications, it cannot”. The first part could simply be a variation in the translation, but the second part is certainly more emphatic in the 1909 edition.

[13] “in our society” was omitted in the 1909 edition.

[14] “and practice” was omitted in the 1909 edition.

[15] Passage added in the 1909 edition.

19

[16] *Oeuvres de Montesquieu*, Paris: Chez R. Leroux, 1828, Vol. VII, p. 226. The translation in the text is that of John Davidson, in Montesquieu, *Persian Letters*, in two volumes (London: privately printed, 1892), Vol. II, p. 18, where the letter is numbered LXXXIV. A literal translation of Montesquieu’s difficult phrase, *un rapport de convenance qui se trouve réellement entre deux choses*, which Davidson rendered as “a true relation existing between two things,” might be: “a relation of propriety which really exists between two things.” Pareto was probably poking fun not only at Montesquieu’s logic but also at his rather stilted way of expressing it.

20

[17] Jules Jaurès was a French socialist leader who opposed French intervention in the first World War, and organized a general strike in protest, whereupon he was assassinated by a fanatic. See Ploetz’ *Dictionary of Dates*, translated and edited by William H. Tillinghast, New York: Halycon House, Houghton Mifflin, 2nd revised edition, 1925. Company, 1925, p. 593.

[18] Passage omitted in the 1909 edition.

22

[19] The dots were omitted from the 1909 edition.

25

[20] The word “several” is used in the 1906 edition, “certain” in the 1909 edition.

[21] The complete passage reads as follows: “I merely declare my conviction, that the general principle to which all rules of practice ought to conform, and the test by which they should be tried, is that of conduciveness to the happiness of mankind, or rather, of all sentient beings: in other words, that the promotion of happiness is the ultimate principle of Teleology.” J. S. Mill, *A System of Logic; Ratiocinative and Inductive*, People’s edition, London: Longmans, Green, and Co., 1891, p. 621.

26

[22] “the aegis-bearing” was omitted from the 1909 edition.

27

[23] “Indeed Diogenes the Cynic used to say that Harpalus, a brigand of the day who passed as fortunate, was a standing witness against the gods, because he lived and prospered as he did for so long. Dionysius, whom I mentioned before, having plundered the temple of Proserpine at Locri, was sailing back to Syracuse, and as he ran before a very favorable wind, remarked with a smile, ‘See you, my friends, what a good crossing the immortal gods bestow on men guilty of sacrilege?’” Cf. Cicero, *De nature deorum* and *Academica*, with an English translation by H. Rackham, Loeb Classical Library, London: William Heinemann Ltd., and New York: G. P. Putnam’s Sons, 1933, pp. 368–9.

29

[24] The first passage, from the 1906 edition, becomes “is strange” in the 1909 edition. The second passage was omitted in the 1909 edition.

[25] The reference is to Herbert Spencer’s *The Data of Ethics*, London: Williams and Norgate, 1879, reprinted as Part I of *The Principles of Ethics*, in two volumes, New York: D. Appleton and Company, 1897, Vol. I, Part I, Ch. XII, §79, p. 212. Pareto must have been relying on a French or Italian translation, in view of the altered title. It could well be that he was misled by a poor translation of the quoted passage, which leaves out the words “conduce to,” and which when translated back into English reads: “actions carried out to please others procure us pleasure, because they create happiness in those around us.”

[26] *Op. cit.* (note 25), Part I, Ch. XII, §80, p. 213. Again, a literal translation of Pareto’s Italian quotation back into English reads: “whoever sets about to bring pleasure to others feels his own pleasures more intensely than is the case with someone who tends exclusively to his own pleasures.” While the words seem to say the same thing, Spencer’s own statement reads more like an empirical generalization, i.e., a sociologic law, about the consequences of “other-regarding actions”, rather than an exhortation to do good to others. Thus, Pareto apparently overlooked the possibility that Spencer was simply making a statement of fact—possibly erroneous, but refutable—of the kind he himself allowed as legitimate in his subsequent example concerning spinach and disease. It must be admitted, however, that Spencer did not contribute much, if anything, to the crucial problem of measuring differences in pitch or intensity of pleasure as between different individuals.

[27] 1906 edition. Omitted from the 1909 edition.

[28] 1906 edition. Replaced by “the” in the 1909 edition.

31

[29] *Principles of Ethics, op. cit.* (see note 25 above), Vol. I, Part II, Ch. II, §123, p. 337.

[30] This sentence was omitted from the 1909 edition.

[31] On *a priori* beliefs, cf. Spencer, *Principles of Ethics*, *op. cit.* (see note 25 above), Vol. II, Part IV, Ch. VI, §§278–80, pp. 54–61. Actually, Spencer defended *a priori* beliefs, and explicitly (p. 6) stated that his own doctrine was based on one. And while he criticized “men who profess Christianity and practice Paganism” (Vol. I, Part II, Ch. I, §118, p. 323—see also §§115–17), Pareto’s assertion that Spencer was critical of Christian ethics *per se* does not seem accurate.

35

[32] 1906 edition. Replaced by “But” in the 1909 edition.

37

[33] Literally, “composed of thin air”.

38

[34] The indicated words were omitted in the 1909 edition, where the phrase reads “implicitly with other principles.”

[35] “§9. And certainly the basic distinction in personal law is this, that all men are either free or slaves.”

“§10. Again, some of the free men are ingenuous, while others are freedmen.”

“§11. The ingenuous are those who are born free; the freedmen are those who are released from a legitimate slavery.”

39

[36] Omitted from the 1909 edition.

[37] This sarcastic term (*bella*) was omitted from the 1909 edition.

40

[38] 1906 edition. Omitted from the 1909 edition.

41

[39] 1906 edition; “important” in the 1906 edition.

42 and 43

[40] Omitted from the 1909 edition.

45

[41] 1906 version. The 1909 version reads: “opposite directions.” The explanation for this striking discrepancy is quite simple: the Italian is *nel senso esposto*, *esposto* being the past participle of *esporre* (to explain, expound), and *senso* meaning sense, manner, or

direction. Bonnet must have misread *esposto* as *opposto* (opposite). When this error is combined with the omission of the subsequent sentence (see note 42 below), the 1909 version becomes incomprehensible.

[42] Omitted from the 1909 edition.

48

[43] 1906 edition. “Greeks” in the 1909 edition.

[44] Added in the 1909 edition.

[45] Cf. George Grote, *Greece*. I. *Legendary Greece*. II. *Grecian History to the Reign of Peisistratus at Athens*, reprinted from the second London edition, in twelve volumes, New York: Peter Fenelon Collier, 1899, Vol. I, p. 439.

[46] Omitted from the 1909 edition.

[47] The translation in *terza rima* by Warwick Chipman, *The Inferno*, London: Oxford University Press, 1961, p. 116, is as follows:

“Ah but, my Guide, his death by violence
Still unavenged,” I said, “by any of those
Who share with him the shame of that offence,
Embittered him, and therefore, I suppose,
He went away from me with nothing said;
And therefore yet the more my pity grows.”

50

[48] The term used by Pareto, translated here as “scabs,” is *Crumiri* (Italian), or *Kroumirs* (French). The etymology is explained in Pareto’s *Trattato di Sociologia Generale* (Florence: Barbera, 1916, translated as *Mind and Society*, New York: Harcourt Brace and Company, 1935), §689. The Krumirs were a Tunisian tribe who were said to have betrayed Tunisia to the French invaders, at a time when Italy had a foothold in Tripoli and was vying with France for control of Tunis. Thereafter, strike-breakers in Italy were called disparagingly by the same name.

[49] The three indicated paragraphs were added in the 1909 edition.

[50] Removed from the 1909 edition.

[51] *Raison d’État* (literally, “reason of the State”), is the sacrifice of the interests of, and possibly also of justice towards, an individual in favor of the state.

[52] *Fait du prince* (literally, “fact of the prince”), is the arbitrary use of power by the King (of France).

[53] 1906 edition. Omitted from the 1909 edition.

[54] 1906 edition. Omitted from the 1909 edition.

[55] The last two paragraphs of footnote 31 were added in the 1909 edition.

[56] The reference is to Part II, *The Inductions of Ethics*, of Herbert Spencer's *Principles of Ethics* (*op. cit.*, note 25 above), Vol. I, Ch. I, §115, p. 318. Much of Spencer's work was first published in serial form in various English and foreign-language periodicals, which may account for the discrepancy in titles.

51

[57] 1906 version. The 1909 version is "sentiments of a certain category."

[58] The last two paragraphs of §51 were added in the 1909 edition.

53

[59] 1906 version. In the 1909 edition, "separately" was changed to "or thwarted". While the addition of "or thwarted" itself makes perfectly good sense, the omission of "separately" certainly removes the main point that Pareto is making. Probably, "or thwarted separately" would best express what Pareto meant to say.

[60] Herbert Spencer, *Facts and Comments*, New York: D. Appleton and Company, 1902. See the essay, "Feeling versus Intellect," pp. 35–43, where Spencer compares two tribes in the South Sea Islands, the Fijians and Arafuras, the first of whom were found to be highly intelligent but also cannibalistic, and the second very peaceful but unintelligent. The moral he draws is that persons should not be admired for having high intelligence, and that education tends to be overvalued.

56

[61] 1906 edition. The 1909 edition has "sentiment of patriotism" here.

57

[62] The explanation was removed from the 1909 edition.

58

[63] 1906 version. The 1909 version was "sentiment."

59

[64] The 1906 version is retained here. In the 1909 version, "latter" and "former" were interchanged.

61

[65] Omitted from the 1909 edition.

63

[66] 1906 version. The 1909 version reads "Aspasia carried on a prostitute trade," which seems to refer more appropriately to the immediately following quotation from

Athenaeus (see note 67 below) rather than to the Plutarch. The account in Plutarch's *Pericles* is as follows: "the business that supported her was neither honorable nor decent, for she kept a number of courtesans in her house." Cf. *Plutarch's Lives*, translated from the original Greek by John Langhorne and William Langhorne, Cincinnati: Applegate and Co., Publishers, 1853, p. 128.

[⁶⁷] The passage is from Athenaeus's *The Deipnosophists, or Banquet of the Learned*. Cf. Athenaeus, *The Deipnosophists*, with an English translation by C. B. Gulick, in seven volumes, Loeb Classical Library, London: William Heinemann Ltd., and Cambridge, Mass.: Harvard University Press, Vol. VI, 1937, p. 79.

[⁶⁸] For an alternative translation see Diogenes Laertius, *Lives of Eminent Philosophers*, with an English translation by R. D. Hicks, in two volumes, Loeb Classical Library, London: William Heinemann, and New York: G. P. Putnam's Sons, 1925, Vol. I, p. 137.

[⁶⁹] "Asked where he came from, he said, 'I am a citizen of the world.'" Cf. Diogenes Laertius, *op. cit.* (note 68 above), Vol. II, p. 65. The translator, Hicks, notes (p. 64): "If this answer is authentic, it apparently shows that the famous term 'cosmopolitan' originated with Diogenes." The Diogenes in question (404–323 B.C.) bears no relation to Diogenes Laertius the author, whose dates are unknown.

64

[⁷⁰] Cf. Plutarch's *Moralia*, in fifteen volumes, with an English translation by Edwin L. Minar, F. H. Sandbach, and W. C. Helmbold; Loeb Classical Editions, London: William Heinemann Ltd., and Cambridge, Mass.: Harvard University Press, 1961, Vol. IX, pp. 306–441, "The Dialogue on Love." The passages cited by Pareto occur on pp. 346–9.

65

[⁷¹] 1906 version. The 1909 version reads: instructive.

66

[⁷²] "He is also guilty of corrupting the youth." Cf. Diogenes Laertius, *op. cit.* (note 68 above), Vol. I, p. 171.

[⁷³] Omitted from the 1909 edition.

68

[⁷⁴] 1906 version. The 1909 version reads: "Vol. II, 2nd ed." Bonnet may have been correct in interpreting "2A" to mean *Zweite Auflage* (second edition), but "Vol. III" is incorrect. The passage will be found on p. 193 of the third edition (which presumably has the same pagination as the second edition) of Vol. II, Part 1 (*Zweiter Theil, Erste Abtheilung*)—which is a separate volume—of Edward Zeller, *Die Philosophie der Griechen*, Leipzig: Fues's Verlag (R. Reisland), 1879. The passage cited by Pareto occurs in section A of the first half (*Erster Abschnitt*) of this volume, and reads as follows: "Hatten nicht alle Gebildeten jener Zeit die Schule einer Aufklärung durchgemacht, welche den Glauben

und die Sitte der Väter gründlich zersetzt hatte?" The question mark was omitted from both the 1906 and the 1909 editions.

[⁷⁵] *Op. cit.* (note 74 above), p. 194. The German reads: "sie ist es auch nach dem Maßstab ihrer eigenen Zeit."

70

[⁷⁶] 1906 version. In the 1909 version, "years" (*anni*) was replaced by "army" (*armée*), which was presumably a misprint (for *année*). The 1906 version is certainly the correct one. In Greece at that time the years were named after the annually elected mayor, or magistrate, of Athens (called the *Archon Eponymous*) who served during that year. See, e.g., the Langhorne's edition of *Plutarch's Lives* (*op. cit.*, note 66 above), p. 573; also, *Plutarch's Lives* translated by W. R. Frazer, in three volumes (London: Swan Sonnenschein and Co., Lim., and New York: The Macmillan Co., 1907), Vol. III, p. 63; and *Plutarch's Lives*, Englished by Sir Thomas North [from the French translation], in ten volumes (London: J. M. Dent and Co., 1899), Vol. VIII, p. 272.

73

[⁷⁷] Omitted from the 1909 edition.

[⁷⁸] Cf. Polybius, *The Histories*, in six volumes, with an English translation by W. R. Paton (Loeb Classical Library, London: William Heinemann, and New York: G. P. Putnam's Sons, 1923), Vol. III, pp. 394-7: "But the quality in which the Roman commonwealth is most distinctly superior is in my opinion the nature of their religious convictions. I believe that it is the very thing which among other peoples is an object of reproach, I mean superstition, which maintains the cohesion of the Roman State. These matters are clothed in such pomp and introduced to such an extent into their public and private life that nothing could exceed it, a fact which will surprise many. My own opinion at least is that they have adopted this course for the sake of the common people. It is a course which perhaps would not have been necessary had it been possible to form a state composed of wise men, but as every multitude is fickle, full of lawless desires, unreasoned passion, and violent anger, the multitude must be held in by invisible terrors and suchlike pageantry. For this reason I think, not that the ancients acted rashly and at haphazard in introducing among the people notions concerning the gods and beliefs in the terrors of hell, but that the moderns are most rash and foolish in banishing such beliefs. The consequence is that among the Greeks, apart from other things, members of the government, if they are entrusted with no more than a talent, though they have ten copyists and as many seals and twice as many witnesses, cannot keep their faith; whereas among the Romans those who as magistrates and legates are dealing with large sums of money maintain correct conduct just because they have pledged their faith by oath. Whereas elsewhere it is a rare thing to find a man who keeps his hands off public money, and whose record is clean in this respect, among the Romans one rarely comes across a man who has been detected in such conduct."

[⁷⁹] This is, of course, simply the sentence translated in the text.

[80] Omitted in the 1909 edition.

[81] “We have excelled every race and every nation.” Cf. Cicero, *The Speeches*, with an English translation by N. H. Watts; Loeb Classical Library, London: William Heinemann, and New York: G. P. Putnam’s Sons, 1923, p. 341. The clause comes at the end of the following passage from the speech concerning the Response of the Soothsayers, pp. 338–341: “And indeed, who is so witless that, when he gazes up into heaven, he fails to see that gods exist, and imagines that chance is responsible for the creations of an intelligence so transcendent that scarce can the highest artistry do justice to the immutable dispositions of the universe? or who, once convinced that divinity does exist, can fail at the same time to be convinced that it is by its power that this great empire has been created, extended, and sustained? However good be our conceit of ourselves, conscript fathers, we have excelled neither Spain in population, nor Gaul in vigour, nor Carthage in versatility, nor Greece in art, nor indeed Italy and Latium itself in the innate sensibility characteristic of this land and its peoples; but in piety, in devotion to religion, and in that special wisdom which consists in the recognition of the truth that the world is swayed and directed by divine disposal, we have excelled every race and every nation.” An alternative translation of *superavimus* by “surmounted” rather than “excelled” would seem to come closer to Pareto’s interpretation of this passage.

77

[82] 1906 version. The 1909 version is “beyond”.

78

[83] Here and in the remainder of this section, “capital goods” corresponds to *capitali mobiliari* (literally: “mobile capital”). Later on, in the more technical sections, this term will be rendered by the more stilted expression “capital goods proper,” following tradition.

79

[84] Omitted from the 1909 edition. The omission is obviously a translator’s error, rather than deliberate, since the 1909 version makes little sense.

[85] 1906 version. The 1909 version is “Below,” which is of course absurd.

81

[86] The bracketed clause, which is not strictly relevant to the point Pareto is making, was expressly omitted by Pareto in the 1906 edition, but was included in the 1909 edition (translated from the original English in both cases). The passage is from Sir Henry Sumner Maine, *Ancient Law* (London, 1861), and will be found on p. 4 of the World’s Classics edition published in London by the Oxford University Press, 1931, 1959.

[87] Both phrases were added in the 1909 edition. Neither version makes very much sense in the first of these cases, since the wording implies that the maxims originated from adherence to the maxims. A wording which would probably convey Pareto’s meaning is:

"originated exclusively ... from people upon whom certain rules of action or abstention were imposed."

83

[88] This paper, "L'individuel et le social," appears in the *Congrès International de Philosophie, II^{me} Session Tenue à Genève du 4 au 8 Septembre 1904, Rapports et Comptes Rendus*, publié par le soin du Dr. Ed. Claparède, Geneva: Henry Kundig, Éditeur, 1905, pp. 125–139. It has been reprinted in V. Pareto, *Oeuvres Complètes*, Vol. VI, *Mythes et Idéologies* (edited by G. Busino), Geneva: Librairie Droz, 1966, pp. 259–265.

85

[89] 1906 version. The 1909 version reads: "most advanced".

[90] 1909 version. In the 1906 version the word is "error" in place of "envy".

86

[91] 1909 version. The 1906 version reads: "1860; although already by the end of that period it had made a fleeting appearance in the literature; but it looked like something that was bound to remain confined to romanesque fiction."

[92] 1906 edition. Omitted in the 1909 edition.

[93] 1906 version. The 1909 version is: "is certainly no longer the same."

[94] 1906 version. The 1909 version is: "then."

87

[95] Sentence added in the 1909 edition.

[96] 1906 edition; "are" in the 1909 edition.

[97] Added in the 1909 edition.

[98] 1906 version. Changed to "Some" in the 1909 edition.

[99] 1906 version. The 1909 version was "the humanitarians." This change could have been deliberate on Pareto's part, but it does seem to intrude on the original train of thought.

[100] Added in the 1909 edition.

[101] 1909 version. The 1906 version is "select part of the population."

89

[102] Here Pareto is using the symbols A, B, C to denote relations between things, which in §6 were denoted by $AB, A'B'$, etc. In the remainder of this section, however, he reverts to the previous notation in which A, B, C, \dots stand for "things" such as facts, sentiments, etc.

[103] Added in the 1909 edition.

90

[104] The past tense in this clause and in the following sentence were replaced by the present tense in the 1909 edition.

91

[105] 1906 version. The 1909 version was: “what”.

[106] 1909 version. The 1906 version reads: “fine for receiving, but never any good for giving.” The change may be assumed to be a deliberate one on Pareto’s part, since virtually the same sentence reappeared in the *Trattato di Sociologia Generale*, §1152 (p. 685 of *The Mind and Society*—cf. note 48 above).

[107] Here, as elsewhere, the 1909 edition substitutes “francs” for “lire.”

[108] 1906 version. The 1909 version reads: “those who earn twenty sous a day.”

[109] Omitted in the 1909 edition, as were a number of other ironic expressions.

[110] 1909 version. The 1906 version reads: “An Italian couple is turned back from their sacred soil because they are not joined in legitimate matrimony, whereas the inhabitants of the New World come to Europe to display much worse evidence of immorality.”

[111] 1909 version. The 1906 version was “1904.”

[112] 1909 version. The 1906 version reads: “but this applies only to war against foreigners, because”.

[113] 1909 version. The 1906 version reads as follows: “Meanwhile they injure gendarmes; at Brest, in July 1904, they drew a noose around the neck of one of them and tried to strangle him; they wounded thirteen of them with stones. At Armentières they set fire to factories and looted private homes and banks. In Marseilles, throughout 1904 there was no end to strikes with acts of violence of all kinds; in the Departments of the South, agricultural strikes turned into veritable riots. But all this did not disturb the dreams of the middle class, who are in raptures over the imminent arrival of a ‘new and better humanity.’”

92

[114] 1906 version. The 1909 version reads: “The proverb: he who breaks windows pays for them.”

[115] 1909 version. The 1906 version is “dearly beloved and very reverend.”

[116] Omitted in the 1909 edition.

[117] Omitted in the 1909 edition.

93

[118] 1909 version. The 1906 version was, of course, “just now.”

[119] 1906 version. The 1909 version was: “the men who belong to religious orders are not married, any more than the women.”

94

[120] 1909 version. The 1906 version was “observed”.

95

[121] Georges Sorel, *La ruine du monde antique*, Paris: G. Jacques, 1902. 2nd ed, 1922. Third (posthumous) edition, Paris: Marcel Rivière, Éditeur, 1933.

[122] The quoted passage is from Georges Sorel, *Introduction à l'économie moderne* (*supra*, Ch. I, note 36).

[123] Footnote 66 was placed here in the 1909 edition (in error). For the Grote reference see note 45 above.

97

[124] Added in the 1909 edition.

100

[125] 1909 version. The 1906 version was “carried out,” which conveys the same meaning but less sharply.

[126] 1906 version. The 1909 version reads: “Sociological phenomena are sometimes . . .”. As in the case of note 99 above, this may have been a deliberate change on Pareto’s part, but it seems like an afterthought, or an unnecessary intrusion into the earlier train of thought.

101

[127] Omitted in the 1909 edition.

[128] Omitted in the 1909 edition.

[129] Footnote 71 was added in the 1909 edition, where the French translation by de Rousiers, p. 130, was cited. The original will be found in Theodore Roosevelt, *American Ideals*, New York: The Review of Reviews Company, 1897, p. 265.

[130] “If you want peace . . .”: “para bellum” (prepare for war).

[131] The reference is evidently to Herbert Spencer, *Principles of Sociology*, in three volumes, New York: D. Appleton and Co., 1882–1896. According to Gabriele de Rosa, Pareto relied on the French translation of this work, which may have used the title cited by Pareto; cf. *Lettere a Pantaleoni* (*op. cit. note 132 below*, Preface, note 3), Vol. II, pp. 66n, 145n, 245n. It is interesting to note Pareto’s opinion of Spencer expressed in a letter dated

January 8th, 1898 (pp. 145–6): “Basically he is the only writer who has produced a truly scientific work on sociology.”

[132] The allusion is evidently to socialist theories espoused by the French chemist Pierre Berthelot (1827–1907) and the German economist and “socialist of the chair” Adolf Wagner (1835–1917), as well as to the writings of Schmoller (see note 133 below). Cf. Gabriele de Rosa (ed.), *Vilfredo Pareto, Lettere a Pantaleoni* (Rome: Banca Nazionale de Lavoro, 1960), Vol. II, p. 151; and V. Pareto, “Solidarité Sociale,” *Journal des Economistes*, 57^e Anné, 5^e Serie, Vol. 33 (February 1898), 161–171, reprinted in V. Pareto, *Oeuvres Complètes*, Vol. IV, *Libre-échangisme, protectionisme, et socialisme* (edited by Giovanni Busino), Geneva: Librairie Droz, 1965, pp. 266–276.

[133] The allusion is apparently to Gustav von Schmoller (1838–1917), leader of the (Younger) German Historical School. Cf. Joseph A. Schumpeter, *History of Economic Analysis*, New York: Oxford University Press, 1954, pp. 809–814. Pareto described his methodological differences with Schmoller in an interesting letter to Maffeo Pantaleoni dated April 2nd, 1907 (cf. G. de Rosa (ed.), *Lettere a Pantaleoni, op. cit.* (note 132 above), Vol. III, pp. 26–8):

“But mind you that, as in every other science, before arriving at a knowledge of the dependence of complex phenomena such as the ones you mention—happiness and unhappiness of a people, and a greater or smaller degree of economic prosperity—you must carry out the analysis (separation) into the various relatively simple phenomena of which the complex is composed.

“The usefulness of linking sociology and economics in an introduction [the reference is to Chapter II of the *Manuale*] consists in showing concretely how the second is only a part of the first, and therefore how, by itself, it cannot resolve any, or can resolve hardly any, practical problems.

“The difference between a theory of money fashioned—as you mention—after the statistics and one fashioned after pure economics may be decomposed into two very different kinds of differences: (1) Economic differences. Pure theory provides the broad *economic outlines*, and the facts present *economic differences*. (2) Sociological differences, i.e., economic phenomena interwoven with other ones. For example, a short time ago the French were reluctant (out of patriotism) to purchase German securities. It is necessary to perform an analysis of all those differences; if you put them *all* together you don’t get anything. In one pot I’ve planted grain with fertilized earth and in another just grain and sand. And yet the first has produced a smaller yield than the second. Oh! I forgot to mention that a sparrow came by and ate nearly all the seeds in the first pot!

“It is precisely in this analysis that the difference lies between the empirical and experimental methods. And similar considerations explain why the historical method of Schmoller and Co. is a bunch of nonsense, while the rational (experimental) historical method is the foundation of the social sciences.

“Today in France the discount rate of the Bank of France is 3 1/2%, and a few days ago it was 3%, while in London it was 5% and in Berlin, 6%. The explanation for this fact is only partly economic; in part it lies in the sentiments of the French people, who are timid in financial operations and afraid to get into trouble with the government, and who through the large banks are entirely dependent on the government.

"In Geneva you pay 50% more for clothing than you would pay ordering it directly from Paris (duty and freight included). How is it that there can be two prices in one market? Because, in this democratic country, people are afraid to get into trouble with the local merchant by ordering clothing from the outside.

"To my mind, the defect of economic theories up to now has consisted in wanting to explain concrete cases *all the while* failing to take account of facts similar to these. If you remain in the abstract, with pure science, you *don't* have to take account of this, but then you *must* give warning that you are dealing with an abstract and not a concrete case. When you deal with a concrete case you have to take account of all those facts—to the extent possible, of course.

"The novelty of my *Manual* consists precisely in my having insisted on this."

The following amusing anecdote is told by Pareto's biographer, C. H. Bousquet, in *Pareto, Le savant et l'homme*, Lausanne: Payot et C^{ie} S.A., 1960, p. 67:

"At a congress, Pareto had spoken about the 'natural laws of political economy.' Now, he was always dressed very unpretentiously, and even though he was very well off he looked like a poor wretch. Schmoller, in his frock coat, and at the peak of his glory, interrupted him with 'There's no such thing.' At the end of the session Pareto asked him humbly if he knew of any restaurant in the city where one could eat free of charge. And the member of the Prussian House of Lords replied, 'No; but I know of one where the price is reasonable.' At that, Pareto said, 'There you have the natural laws of political economy,' and took off."

[134] This probably refers to Maurice Block (1816–1901), a French financial and statistical analyst. Cf. Gabriele de Rosa (ed.), *Lettere a Pantaleoni*, Vol. I, pp. 246–9; and Pareto's series of letters in *Le Monde Économique*, 1892, reprinted under the title "Les nouvelles théories économiques," in Vilfredo Pareto, *Oeuvres Complètes*, Vol. IX, *Marxisme et économie pure* (edited by G. Busino), Geneva: Librairie Droz, 1966, pp. 1–32.

[135] 1906 version. The 1909 version reads: "or nearly so, to."

[136] Cf. Theodore Roosevelt, *American Ideals*, (*op. cit.*, note 129 above), p. 44. The complete passage, which comes from a chapter entitled "True Americanism", reads: "To bear the name of American is to bear the most honorable of titles; and whoever does not so believe has no business to bear the name at all, and, if he comes from Europe, the sooner he goes back there the better."

[137] Modern choice theorists would not accept this without the addition of an explicit assumption of asymmetry of the relation "is foremost with respect to." But this is a quibble in the present context.

[138] 1906 version. The 1909 version reads: "yesterday's universal consensus may well not be tomorrow's."

[139] "6" was erroneously printed as "5" in the 1909 edition.

[140] 1909 version. The 1906 version reads: "is like the Arabian phoenix."

[141] The reference is to Part II of Herbert Spencer's *Principles of Ethics* (see note 56 above), Vol. I, p. 346.

[142] 1906 version. The 1909 version reads: "many."

[¹⁴³] The complete quotation is as follows (*Principles of Ethics*, Vol. I, p. 346): "No reading is more popular than narratives of battles; and the epithet 'great,' as applied to Alexander, Karl, Peter, Frederick, Napoleon is applied notwithstanding all the atrocities they committed. Occasionally, indeed, we meet with overt expression of this sentiment. Lord Wolseley says of the soldier:—'He must believe that his duties are the noblest that fall to man's lot. He must be taught to despise all those of civil life': a sentiment which is not limited to the 'duties' of the soldier as a defender of his country, which in our day he never performs, but is extended to his 'duties' as an invader of other countries, and especially those of weak peoples: the appetite for aggression transforms baseness into nobility."

[¹⁴⁴] Cf. Herbert Spencer, *Principles of Sociology*, Vol. II, second edition, New York: D. Appleton and Company, 1909, Part V, Political Institutions, Ch. XIX, Political Retrospect and Prospect, §582, p. 664: "From war has been gained all that it had to give. The peopling of the Earth by the more powerful and intelligent races, is a benefit in great measure achieved; and what remains to be done, calls for no other agency than the quiet pressure of a spreading industrial civilization on a barbarism which slowly dwindle."

103

[¹⁴⁵] 1909 version. The 1906 version reads: "circulation of the aristocracy;" and this is followed by a footnote: "It would really be more accurate to use the longer expression: circulation of the select parts of the population." The Italian word corresponding to "select" is *elètto*, which can also mean "chosen," or "elect". The contemporary Italian usage is the same as that of the English, namely the French term *élite*; see, for example, Norberto Bobbio, *Pareto e il sistema sociale*, Florence: G. C. Sansoni S. p. A., 1973.

104

[¹⁴⁶] Italian *decaduti*, past participle plural of *decadere*, to decay. It has the general meaning of impoverished, or jaded, having gone into decline. It is to be distinguished from *decadenti* (decadent) and *scadenti* (inferior), although all three terms could be translated as "decadent" or "degenerate." The corresponding French term used in the 1909 edition—undoubtedly approved by Pareto, since it had been used by him previously in the *Systèmes socialistes*—is *dégenerés* (degenerated, degenerate). While this term tends to carry a derogatory and special connotation in English, its meaning as defined in the Oxford English Dictionary, "having lost the qualities proper to the kind, having declined to a lower type," seems to come closest to the technical meaning intended by Pareto.

106

[¹⁴⁷] 1909 version. The 1906 version reads: "also wins the support of the degenerate part of the former élite, which is willingly soothed by the sound of so sweet a song."

[¹⁴⁸] Footnote 77 was added in the 1909 edition. The reference was of course to Pareto's then forthcoming *Trattato di Sociologia Generale* (see note 48, above). Another, earlier, source is Vilfredo Pareto, "Un'applicazione di teorie sociologiche," *Rivista italiana di sociologia*, 4 (July 1900), 401–456, reprinted in V. Pareto, *Scritti sociologici* (edited by Giovanni Busino), Turin: Unione Tipografico-Editrice Torinese, 1966, pp. 232–293, and

translated into English under the title, *The Rise and Fall of the Elites* (edited by Hans L. Zetterberg), Totowa, N.J.: The Bedminster Press, 1968.

107 and 107 bis

[¹⁴⁹] The last paragraph of section 107, as well as the new section marked 107 bis, were added in the 1909 edition.

[¹⁵⁰] Georges Sorel, *Réflexions sur la violence*. Paris: Librairie de “Pages libres,” 1908.

109

[¹⁵¹] Footnote 80 was added in the 1909 edition.

110

[¹⁵²] 1906 version. The 1909 version is “utility.” In neither case has the concept been defined up to this point, but it may be that Pareto had in mind the concept of Pareto optimality (see Chapter V below). See also Ch. I, note 42, above.

III

[¹⁵³] 1906 version. The 1909 version reads: “fully”.

[¹⁵⁴] 1906 version. The 1909 version reads: “an upholder.”

114

[¹⁵⁵] Footnote 82 was added in the 1909 edition.

116

[¹⁵⁶] Cf. *The Roman Antiquities* of Dionysius of Halicarnassus, with an English translation by Earnest Cary, in seven volumes (Loeb Classical Library, London: William Heinemann Ltd., and Cambridge, Mass.: Harvard University Press, 1939), Vol. II (Books III and IV), p. 335. The translation of the complete passage given there reads as follows: “For they all thought that they had an equal share in the government because every man was asked his opinion, each in his own century; but they were deceived in this, that the whole century, whether it consisted of a small or a very large number of citizens, had but one vote; and also in that the centuries which voted first, consisting of men of the highest rating, though they were more in number than all the rest, yet contained fewer citizens; but above all, in that the poor, who were very numerous, had but one vote and were the last called.”

[¹⁵⁷] Omitted from the 1909 edition.

[¹⁵⁸] “Let the people have their ballots as a safeguard of their liberty, but with the provision that these ballots are to be shown and voluntarily exhibited to any of our best and most eminent citizens, so that the people may enjoy liberty also in this very privilege of honourably winning the favour of the aristocracy.” Cf. Cicero, *De re publica* and *De*

legibus, with an English translation by C. W. Keyes, Loeb Classical Library, London: William Heinemann, and New York: G. P. Putnam's Sons, 1928, p. 505.

[¹⁵⁹] 1906 version. The 1909 version was “best men.” The term used in the 1906 edition was *gli ottimati* (the patricians, or optimates); it could be that this was confused by the French translator Alfred Bonnet with *gli ottimi* (the best), which corresponds to the term *les meilleurs* appearing in the 1909 edition. On the other hand, the 1909 version also makes perfectly good sense, and applies furthermore to both Rome and the modern states.

118

[¹⁶⁰] Omitted in the 1909 edition.

[¹⁶¹] The Carthusian monks were expelled from France during the French Revolution, and after returning later they were expelled again in 1903 under a law forbidding all religious orders. France had enacted a law requiring them to renounce their vows in order to become “useful citizens”. Some of the monks moved to England and the rest to Spain where they started a second distillery of their well-known liqueur, Chartreuse. (This information comes from <http://www.bbc.co.uk/dna/h2g2/A759774>.)

[¹⁶²] 1909 version; “lire” in the 1906 edition.

[¹⁶³] 1906 version. The 1909 version was simply: “elections.”

120

[¹⁶⁴] See the comment on (and superb translation of) this passage by Philip H. Wicksteed in his review of the *Manuale* in the *Economic Journal*, 16 (December 1906), 553–557, reprinted in *The Common Sense of Political Economy*, Vol. II, London: Routledge and Kegan Paul Ltd., 1933, 1957, pp. 814–818.

121

[¹⁶⁵] 1909 version. The 1906 version reads: “at least if they could be made to express it.”

123

[¹⁶⁶] Friedrich Julius Stahl, *Die Philosophie des Rechts*, in three volumes (I, IIa, IIb), Heidelberg: im Verlag der akademischen Buchhandlung von J. C. B. Mohr, 1846, Vol. IIb (Zweiter Band, Zweite Auflage), p. 72: “Was aber könnte untereinander verwandter sehn, als dass meine Willkür das Princip der Rechtsordnung und mein Genuss das Princip der Vermögensordnung seh?”

CHAPTER III

I

[¹] 1906 edition. The corresponding passage in the 1909 edition reads: [not to set forth the theory, but to give a few examples].

3

[2] The word “mutual” was added in the 1909 edition.

4

[3] 1909 version. The 1906 version was “is”.

16

[4] 1906 version. The 1909 version is “in the presence of”.

19

[5] Added in the 1909 edition.

27

[6] 1909 version. The 1906 version reads: “We must therefore now try to find out which movements at the equilibrium point are not permitted, and which are made possible by the tastes; and similarly, which movements are not permitted and which are made possible by the obstacles.”

28

[7] This section was mislabelled 29 in the 1909 edition.

29

[8] This section was properly labelled 29 in the 1906 edition, but the section number 29 was missing in the 1909 edition.

30

[9] For an English translation of this article see *History of Economic Ideas*, 17 (No. 1, 2009), 158–179.

35

[10] 1906 version. The 1909 version reads “It is a mistake”.

[11] The reference to part 8 of the (1906) Appendix is contained only in the 1906 edition.

[12] The reference to part 5 of the (1906) Appendix is contained only in the 1906 edition.

36 bis

[13] Section 36bis (including the footnote) was introduced in the 1909 edition.

38

[14] Passages from the 1906 edition omitted from the 1909 edition.

45

[¹⁵] 1906 edition; replaced by “French” in the 1909 edition.

58

[¹⁶] Further references to the appendices are omitted from these annotations.

59

[¹⁷] 1906 version. The 1909 version is simply “the hill”. The “blissful mountain” is an allusion to the “dilettoso monte” in Dante’s *Inferno*, I, 77. The Longfellow translation of the relevant passage reads:

But thou, why goest thou back to such annoyance?
Why climst thou not the Mount Delectable,
Which is the source and cause of every Joy?

(Henry Wadsworth Longfellow, *The Divine Comedy*, Vol. I, p. 27.)

[¹⁸] 1906 version; “mountain” replaced by “hill” in the 1909 edition.

63

[¹⁹] 1909 edition. In the 1906 edition these were referred to as *b* and *mb* respectively.

72

[²⁰] The qualification was added in the 1909 edition.

76

[²¹] This reference was missing in the 1909 edition.

85

[²²] 1906 edition (*fabbriche*). The term used (presumably by the translator Alfred Bonnet) in the 1909 edition was “commodities” or “merchandise” (*marchandises*). The 1906 version makes more sense.

87

[²³] Here, Pareto’s notation is possibly ambiguous. He has defined A and B (both set in roman) as the *names* of the two commodities; but in Figure 11, *B* and *A* (as measured from the origin) both stand for *amounts* of commodity A, and *AB* stands for the *amount* $A - B$ of commodity A. Likewise, *BD* stands for the *amount* *D* of commodity B. Here, the letters A, B, C, etc. (set in roman) stand for *names* of commodities (following Pareto). The symbols A, B, C generally stand for *amounts* of commodities, not necessarily corresponding to the names.

[²⁴] The original “into” in the 1906 edition was translated to “or” in the 1909 edition.

97

[25] t'' is erroneously reproduced as t''' in the 1909 edition.

122

[26] The point l is misprinted as Xl in the 1909 edition.

125

[27] In the 1906 edition, this line is *muadbfγ am*; the 1909 edition has l in place of f .

132

[28] An exact translation would be the very awkward “ophelimitous”. Pareto’s term *ophelimity* has not made it to the 1971 edition of the Oxford English Dictionary, or to any edition of the Webster dictionary that has been consulted. It was introduced by Pareto in his *Cours d’économie politique*, p. 3, following from the Greek *ώφελυμος* “to designate the relationship that causes a thing to satisfy a need or a desire, whether legitimate or not.”

133

[29] 1909 edition; the symbols ac in place of acb appeared here in the 1906 edition.

[30] t' appeared here in the 1906 edition, but t in the 1909 edition.

145

[31] This was mn in the 1906 edition.

156

[32] 1906 version. The 1909 version reads:

“**156.** The VALUE IN EXCHANGE of the economists, when it is cleared of the haze with which it has been surrounded by the literary economists (§226), corresponds more or less to price as just defined; but the authors who use the term *value* have rarely had a clear idea of what it represents.

“Moreover, there have been economists who distinguished between the *value* which was an arbitrary fraction, i.e. $6/3$, and the price, which was a fraction whose denominator was unity, i.e., $2/1$. ”

168

[33] The 1909 edition has the erroneous “portion” here in place of “position”.

178

[34] These three paragraphs were added in the 1909 edition.

181

[35] 1906 version. The word “know” was changed to “do not know” in the 1909 edition.

196

[³⁶] Passage from 1906 edition omitted from the 1909 edition.

206

[³⁷] Passage omitted in the 1909 edition.

213

[³⁸] The name *mc* of this line was omitted in the 1909 edition.

217

[³⁹] Omitted in the 1909 edition.

219

[⁴⁰] “Le nuove teorie economiche: Appunti”, *Giornale degli Economisti* [2], 23 (September 1901), 235–252.

[⁴¹] This is from Phaedrus’s fable in verse, “The cockerel and the pearl”, where the cock says to the pearl:

“But my finding you—since I’m much more interested in food than in pearls—is of no possible use either to you or to me.”

Cf. *Babbius and Phaedrus*, edited and translated by Ben Edwin Perry, Cambridge, Mass.: Harvard University Press and London: William Heinemann Ltd., 1965, pp. 278–9.

219 and 222

[⁴²] The indicated paragraphs were added in the 1909 edition.

223

[⁴³] William Thomas Thornton, *On Labour. Its Wrongful Claims and Rightful Dues, Its Actual Present and Possible Future*. London: Macmillan and Co., 1869. The cited passage corresponds to the following one on p. 54: “How can we say that the equation of supply and demand determines price, if goods are almost always sold at prices at which supply and demand are unequal?”

224

[⁴⁴] The word “literary” was added in the 1909 edition.

226

[⁴⁵] Passage added in the 1909 edition.

[⁴⁶] Paragraph added in the 1909 edition.

[⁴⁷] These 11 paragraphs were added in the 1909 edition.

227

[48] Passage added to the footnote in the 1909 edition.

228

[49] This paragraph was added to the 1909 edition.

[50] These words from the 1906 edition were removed from the 1909 edition.

CHAPTER IV

5

[1] Omitted in the 1909 edition.

9

[2] 1906 version. The 1909 version reads “very inconvenient”. The paragraph ends here in the 1909 edition only.

[3] The original texts have “dependence of consumptions”, which of course is not idiomatic English.

II

[4] Here, by “ophelimity of a commodity” Pareto clearly means what he calls its “elementary ophelimity” (i.e., its marginal utility), hence he is assuming independent utilities.

12

[5] 1906 version. The 1909 version is “occur” or “take place”.

[6] It may be assumed that Pareto meant “of that of the cup”.

16, 18 and 62

[7] 1906 edition. In the 1909 edition, cornflour or cornmeal (*farine de maïs*).

Polenta is a popular Italian dish made from corn meal mixed with butter and grated cheese. Cf. Rose L. Sorce, *The Complete Italian Cookbook [La cucina]*. New York: Gosset and Dunlap, Publishers, 1953, pp. 83–84.

Dictionary listings:

The Shorter Oxford English Dictionary, 3rd edn. 1944, p. 1536: a. Pearl-barley. b. A kind of barley meal. c. Porridge made from steeped and parched barley, or, later, of meal of chestnuts, maize flour, etc.; much used in Italy.

Webster's 9th New Collegiate Dictionary, Springfield, Mass., 1987, p. 910: mush made of chestnut meal, cornmeal, semolina, or farina.

17, 19, 50 and 53

[8] In the 1909 edition “polenta” is here translated as “corn” (maize).

18

[⁹] 1906 version. The 1909 version reads: “if he has bread he will eat less corn [maize]; if he has meat, he will even reduce his consumption of bread.”

19

[¹⁰] 1906 edition. Replaced by “A, B, C, etc.” in the 1909 edition.

[¹¹] 1906 version. The 1909 version reads: “of A, B, C, etc.”

20

[¹²] This clause was omitted from the 1909 edition.

25

[¹³] 1906 version. The 1909 version is simply “narrow”.

26

[¹⁴] 1906 edition; This epigram was incorrectly cited in the 1909 edition as XII, 93. Cf. Martial (Marcus Valerius Martialis), *Epigrams* (edited and translated by D. R. Shackleton Bailey) (Cambridge, Mass.: Harvard University Press, 1993), Vol. III, p. 165: “You are wont to ask me, Priscus, what sort of person I should be if I were suddenly to become rich and powerful. Do you suppose that anybody can foretell his character? Tell me, if you were to become a lion, what would *you* be like?”

27

[¹⁵] 1906 edition. In the 1909 edition “economic statics” was changed to “economic statistics”, which of course makes no sense.

32

[¹⁶] This paragraph was added to the 1909 edition.

33

[¹⁷] 1906 version. The 1909 version was “its first characteristic”.

[¹⁸] The article “A” in the 1906 edition was replaced by “The” in the 1909 edition.

[¹⁹] Fechner’s law states, roughly, that individuals’ sensations vary as the logarithm of the stimulus, a property that has been compared with a logarithmic utility function in economics, of the kind that was proposed by K. Wicksell (*Finanztheoretische Untersuchungen* (Jena, 1896, p. 53) [see also G. Stigler, *Production and Distribution Theories* (New York, 1941, p. 295)], and C. W. Cobb and P. H. Douglas, *American Economic Review*, 18 (March 1928), 139–165], for production functions. Cf. H.-W. Sinn, *Journal*

of *Economic Psychology*, 6 (1985), 185–206. The idea of Fechner’s law goes back to E. H. Weber (1834). Wundt’s work was published in Leipzig, 1880, 1893, 1902–03.

[20] Literally, “consumptions”.

35

[21] 1906 edition. In the 1909 edition, “shirts” (*camicie*) was changed to “commodities” (*marchandises*).

39

[22] 1906 edition. In the 1909 edition, A was replaced (erroneously) by B.

47

[23] 1906 edition. The 1909 edition has (erroneously) γ here in place of α . The dependences α and β of the first kind were defined in §§8–12; there was no type γ .

54

[24] This curve was labelled *ogh* in the 1906 edition, but corrected to *bgh* in the 1909 edition.

[25] The distance *oa* in the 1906 edition was incorrectly changed to *oA* in the 1909 edition.

[26] This correct *gh* in the 1906 edition was replaced by the incorrect *bgh* in the 1909 edition. See pp. 196–7 of Montesano et al. (2006).

[27] The phrase “from *oB* to *G*” in the 1906 edition was replaced by the incorrect “from *oB* to *g*” in the 1909 edition. It seems further that Pareto must have intended to write “from *ob* to *G*”. The reference is obviously to the top of the shaded area in Fig. 30.

[28] This correct “from *b* to *f*” in the 1909 edition replaces the incorrect “from *o* to *f*” in the 1906 edition.

[29] The expressions “contract line” and “contract curve” (from Edgeworth) are used in both the 1906 and 1909 editions, but in accordance with note 37 below, Pareto probably meant to change these expressions to “exchange line” and “exchange curve” in the 1909 edition.

[30] This clause from the 1906 edition was struck from the 1909 edition, evidently by Pareto.

[31] The correct “at *a'*” from the 1906 edition was replaced by the incorrect “at *a*” in the 1909 edition.

[32] The correct “the point *a*” from the 1906 edition was erroneously replaced by “the points *a*” in the 1909 edition.

[33] The line *ba* is not explicitly shown in Figure 30, only the particular line *ba'*.

[³⁴] In the 1909 edition Figure 30 was erroneously referred to here as Figure 40.

55

[³⁵] The correct “in *G*” from the 1906 edition was incorrectly indicated as “in *g*” in the 1909 edition.

58

[³⁶] 1909 version. The 1906 version is: “Yet, such gentlemen are not a rare occurrence when discussing economic science.”

59

[³⁷] 1909 edition, correcting “contract line” in the 1906 edition.

[³⁸] This passage on p. 266 of the 1906 edition was erroneously typeset as c, c', e, \dots instead of as c, c_1, \dots . The error was pointed out by Pareto in the *errata corrigere* on p. 577 of the 1906 edition, except that it also had an error in replacing c, c', e, \dots by c, c, \dots instead of by c, c_1, \dots . The error was further compounded on p. 278 of the 1909 edition, which simply replaced c, c', e, \dots by c, c', c_1, \dots , instead of by c, c_1, \dots .

62

[³⁹] Here, the translator Alfred Bonnet uses the Italian word “polenta” in quotation marks.

[⁴⁰] Now, Bonnet uses the word “polenta” without quotation marks.

63

[⁴¹] The previous clause in both the 1906 edition (p. 268) and the 1909 edition (p. 280) stated that “4 units of A are always equivalent to 3 units of B”, hence the ratio of *oa* to *ob* must be as 4 to 3, as indicated in Figure 36, not as 3 to 4 as stated in both the 1906 and 1909 editions.

65

[⁴²] This was erroneously displayed as q, q', q'' in the 1909 edition.

[⁴³] This necessary statement, missing from the 1906 edition, was supplied in the 1909 edition.

[⁴⁴] This (erroneously) is printed as q' in both the 1906 and 1909 editions.

[⁴⁵] $q''m''$ in the 1909 edition corrects $q''m'$ in the 1906 edition.

[⁴⁶] 1906 edition; “makes use of” in the 1909 edition.

69

[⁴⁷] Two quotations from the *Purgatorio*. Longfellow’s translation of the first of these is

And he to me: “This mount is such, that ever
 At the beginning down below ’t is tiresome
 And aye the more one climbs, the less it hurts.

(p. 27, lines 88–90) and the second is

The summit was so high it vanquished sight,
 And the hillside precipitous far more
 Than line from middle quadrant to the centre.

(p. 26, lines 40–42). (Pareto omitted the first line.) These quotations from the *Purgatorio* were omitted from the 1909 edition.

70

[⁴⁸] In both the 1906 and 1909 editions, Figure 39 is erroneously referred to here as Figure 29.

CHAPTER V

2

[¹] By “elements” here, Pareto evidently means inputs or factors of production.

[²] The word “our” in the 1906 edition was omitted in the 1909 edition.

[³] 1909 edition. The 1906 version is simply “says”.

[⁴] The work cited here is a part of one of the many prefaces by Francesco Ferrara to translations of works by classical economists, contained in the *Biblioteca degli Economisti*, which were later reprinted. The relevant passage is found in Vol. II, Part 1, of the work *Esame storico-critico di Economisti e Dottrine economiche del Secolo XVIII e prima metà del XIX*, raccolta delle Prefazioni dettate dal Professore Francesco Ferrara alla 1^a e 2^a della *Biblioteca degli Economisti* (prefaces to Vol. I–VI), 2nd series (Roma, Torino, Napoli: Unione Tipografico-Editrice, 1890). [Volumes I and II are available in Google Books.] The passage quoted by Pareto is found on p. 22 of Vol. I. A more complete passage on pp. 21–22 had already been cited (in French) by Pareto in Vol. 2 of his *Cours d'économie politique* (Lausanne: F. Rouge, 1896–7), p. 52, note 654². The sentence cited here appears as the last two sentences of the quotation in the *Cours*.

[⁵] Added in the 1909 edition.

[⁶] 1906 edition; omitted in the 1909 edition. This word was also omitted in Pareto’s longer quotation of this passage in the *Cours* (see annotation 4 above).

3

[⁷] 1906 edition. Replaced by “best” in the 1909 edition.

4

[⁸] 1909 edition. The 1906 version was “substantive character”.

6

[⁹] In the 1906 edition, n^{'''} was misprinted as n''. In the 1909 edition, both m^{'''} and n^{'''} were omitted.

[¹⁰] Added in the 1909 edition.

[¹¹] 1906 edition (*soltamente*). This word was apparently mistranslated in the 1909 edition as “exclusively” (*exclusivement*), which would be a correct translation of *solamente*.

8

[¹²] 1906 edition. Omitted in the 1909 edition.

[¹³] This footnote, not contained in the 1906 edition, was added in the 1909 edition, but misplaced at the end of §15, surrounding the footnote that had already appeared there in the 1906 edition. See Montesano et al. (2006, p. 556, §8). See also note 7 below. Note that the reference to pursuit curves in this added footnote repeats what had already been said in §11 of the 1906 edition; see note 3c.

II

[¹⁴] Pursuit curves go back at least to Pierre Bouguer (1732). For a systematic analysis and historical references see Arthur Bernhart, “Curves of Pursuit”, *Scripta Mathematica*, 20 (1954), 125–141 and 23 (1957), 49–65; “Polygons of Pursuit”, *ibid*, 24 (1959), 23–50; and “Curves of General Pursuit”, 24 (1959), 189–206.

[¹⁵] The colon in the 1906 edition was replaced by a semicolon in the 1909 edition.

I2

[¹⁶] Here, Pareto does not make precise what can be meant by the maximum welfare of a group of persons.

I4

[¹⁷] The term “transatlantic liner” in the 1909 edition replaces “modern steamship” in the 1906 edition.

[¹⁸] Clause added in the 1909 edition.

[¹⁹] 1906 version. The 1909 version is “One could multiply such examples at will.”

I5

[²⁰] This footnote, correctly placed here in the 1906 edition, was surrounded in the 1909 edition by material that was intended for §8. That extra material is now included as footnote 2. Cf. Montesano et al. (2006), p. 209 and p. 556, §8.

17

[21] This sentence was added in the 1909 edition.

[22] The word “millstones” in the 1906 edition was mistakenly replaced by “machines” in the 1909 edition.

18

[23] 1906 edition. These two sentences were replaced in the 1909 edition by: “This construction necessarily rests on the surface of the soil.”

19

[24] The word is “consumed” in both editions.

20

[25] 1906 version. The 1909 version is “political economy”.

[26] 1906 version, removed in the 1909 version.

27

[27] A scudo was a coin worth five lire.

[28] A marengo was originally a gold Napoleon, namely a French gold twenty-franc coin, here used to indicate a coin worth 20 lire, hence worth four scudi.

[29] 1906 edition. The last two sentences of this paragraph read as follows in the 1909 edition (pp. 297–8):

When one says that an individual has had a five-franc dinner, one does not say that he has eaten a five-franc piece; when one says that to produce bread one needs something worth a thousand francs, one does not say that one must materially use two hundred crowns [five-franc pieces], or fifty louis [20-franc pieces] in order to produce bread. In one case as in the other, in order to spend a total of a thousand francs, it is enough to make material use of ten louis; and it is then these ten louis, i.e., 200 francs, that one can consider as capital.

28

[30] These last two paragraphs of §28 were added in the 1909 edition.

29

[31] The word used is “consumed” in both editions.

[32] The indicated passage from the 1906 edition (in which the bracketed passage has been added for clarity), was removed from the 1909 edition.

30

[³³] 1906 version. Replaced by “The insurance premium” in the 1909 edition.

[³⁴] The term is the same in both editions.

[³⁵] 1909 version. The 1906 version was “Frequently”.

[³⁶] 1909 version. The 1906 version was “considered”.

[³⁷] 1909 version. The 1906 version was “The modes of operation are diverse.”

31

[³⁸] 1906 version, The 1909 version is “always”.

32

[³⁹] 1906 version; removed from 1909 edition.

[⁴⁰] Paragraph added in the 1909 edition.

34

[⁴¹] 1906 version. The 1909 version is “will be dealt with later on.” This is apparently a reference to §§38–40 (cf. Montesano et al., 2006, p. 557, §34).

37

[⁴²] Section 37 was mislabelled section 38 in the 1909 edition.

38

[⁴³] 1906 version. Deleted in the 1909 edition.

[⁴⁴] 1906 version. Deleted in the 1909 edition.

[⁴⁵] 1906 edition; deleted in the 1909 edition.

43

[⁴⁶] This sentence was set in boldface in the 1909 edition.

45

[⁴⁷] 1906 edition. Deleted in the 1909 edition.

46

[⁴⁸] 1906 edition. A *marengo* was (in Italian) a coin equivalent to the Napoleon, a twenty-franc coin under Napoleon I. In the 1909 edition this is rendered as a “louis”.

47

[⁴⁹] 1906 edition. (B) was misssing from the 1909 edition.

48

[50] 1906 title. The 1909 title is “Income from capital”.

[51] 1906 edition. Deleted from the 1909 edition.

[52] Footnote 4 was added in the 1909 edition.

57

[53] 1906 edition. Deleted from the 1909 edition.

58

[54] 1909 version. The 1906 version is “some of these”.

59

[55] 1909 edition, p. 318. The decimal point was missing in the 1906 edition, p. 304.

61

[56] 1906 version. Removed from the 1909 edition.

[57] 1906 edition, p. 305. The 1909 edition omitted the word “apparent” (p. 319).

[58] Literally, “with his own firm” (1906, p. 306). This was missing in the 1909 edition, p. 319.

[59] Paragraph added to the 1909 edition.

63

[60] The diagram in the 1909 edition omitted the double-prime from h'' .

66

[61] 1906 edition. Passage removed from the 1909 edition.

67

[62] Passage added in the 1909 edition.

[63] Added in the 1909 edition.

[64] 1906 edition. The 1909 edition changed “subjective” to “objective”.

[65] 1906 version. The 1909 version is “a small advantage, or at least a small difference.”

69

[66] 1906 version: “profit”. The 1909 version is “income”.

[⁶⁷] 1906 version: “profits” (*utile*); 1909 version: “incomes”.

[⁶⁸] Landowners in both editions.

[⁶⁹] 1906 edition (“profit”) versus 1909 edition (“revenue”).

[⁷⁰] 1906 edition (“profit”) versus 1909 edition (“revenue”).

[⁷¹] 1906 edition, “fruits”, 1909 edition “revenues”.

70, 71 and 76

[⁷²] 1909 edition: literally “mobile capital”.

70

[⁷³] All but the last paragraph in §70, including footnote 7, were added in the 1909 edition.

[⁷⁴] Pareto does not specify who these mathematicians were.

[⁷⁵] Added in the 1909 edition.

78

[⁷⁶] In the 1909 edition, “quantities” was translated by “qualities”.

79

[⁷⁷] 1909 version. The 1906 version was “industry”.

81

[⁷⁸] This footnote was added in the 1909 edition.

[⁷⁹] 1909 edition. The 1906 edition read “for so much wheat”.

82

[⁸⁰] 1909 edition. The 1906 edition reads “in this way”.

83

[⁸¹] 1906 version. The 1909 edition has “must” in place of “may”.

85

[⁸²] This reference to the Appendix was in the 1906 edition only.

87

[⁸³] These passages were added in the 1909 edition.

[⁸⁴] 1906 version; the 1909 version was “in”.

88

[⁸⁵] 1906 edition. In the 1909 edition, “this interest” became “the interest”.

[⁸⁶] The word “if” was added in the 1909 edition.

[⁸⁷] These passages were added in the 1909 edition.

90

[⁸⁸] Passage added in the 1909 edition.

91

[⁸⁹] As pointed out in Montesano et al. (2006), p. 559, this concept had already been introduced by Pareto in the *Cours*, Vol. II, pp. 106–7.

95

[⁹⁰] 1906 version. The 1909 version is “for all the proprietors”.

96

[⁹¹] 1906 version. The 1909 version replaces *vst* by *ost* (incorrectly).

CHAPTER VI

I

[¹] 1909 edition. This appeared as *mp* in the 1906 edition.

2

[²] In both the 1906 and 1909 editions, these numbers are set as “one” and “minus one” respectively.

3

[³] In both editions, the word for “shifted” was “transported”.

4

[⁴] The placement of this footnote-mark follows the 1909 edition. In the 1906 edition, it was placed after “Fig. 46”.

[⁵] This is the same in the two editions, but Pareto must have meant *acd'*. Note that in both the 1906 and 1909 editions, the point *c* in Figure 36 was inadvertently placed just above the point *h'*.

[⁶] See note [b] to §4 of Chapter VI, in the Annotations from the 2006 Italian Edition, p. 499.

7

[⁷] The section number (33) of the 1906 appendix is cited in the 1906 edition; no corresponding section number is cited for the appendix to the 1909 edition.

9

[⁸] This was added in the 1909 edition.

12

[⁹] The originals have “one” and “two” respectively for the numerals “1” and “2”.

13

[¹⁰] This clause was added in the 1909 edition, where the next sentence starts a new paragraph.

18

[¹¹] This phrase was added in the 1909 edition.

23

[¹²] See note [a] to §23 of Chapter VI, in the Annotations from the 2006 Italian Edition, p. 500.

[¹³] 1906 edition. Changed to “increasing” in the 1909 edition.

24

[¹⁴] 1906 edition. This phrase was removed from the 1909 edition.

27

[¹⁵] 1906 edition, removed from the 1909 edition.

[¹⁶] The qualification was added in the 1909 edition.

31

[¹⁷] See note [a] to §31 of Chapter VI, in the Annotations from the 2006 Italian Edition, p. 501.

33

[¹⁸] This phrase is at best ambiguous. Clearly, if one moves along the equilibrium price line in the “Edgeworth box” away from the equilibrium point, both traders will lose.

[¹⁹] This is of course meant to be the statement of what is now called Pareto optimality. However, as the previous annotation explains, it is not strictly correct.

35

[20] The reference to §34 of the (1906) Appendix occurs only in the 1906 edition.

[21] 1909 edition. In the 1906 edition, “a general concept”.

37, 40, 41 and 42

[22] 1906 edition. No corresponding sections are provided for the Appendix to the 1909 edition.

43

[23] 1909 edition, a correction to *amc* in the 1906 edition.

[24] 1909 edition, correcting *all'c* in the 1906 edition.

[25] 1909 edition, correcting *c* in the 1906 edition.

44

[26] Pareto is not treating market prices as parameters, which is standard for competitive equilibrium, because he is dealing with production in the face of decreasing average cost.

45

[27] The meaning is presumably variable (nonconstant) per unit of output.

47

[28] The prices 6.20 and 6.00 in the 1906 edition appear as 620 and 600 in the 1909 edition.

48

[29] The reference is apparently to Georges Sorel, *Introduction à l'économie moderne*, 1903, 2nd edition (revised and augmented), Paris: Librairie des sciences politiques et sociales, Marcel Rivière, 1922, 229 pp. 3rd (posthumous) edition, Paris: Librairie G. Jacques, n.d., 385 pp. See especially Chapter I of Part 2, pp. 87–9 of the 2nd edition and pp. 133–7 of the 3rd edition. Electronic copies of the 2nd and 3rd editions are available from Google. The first edition (to which Pareto obviously referred) appears to be very scarce.

[30] This sentence was added in the 1909 edition.

49

[31] As before, §31 in the 1906 edition refers only to the 1906 appendix.

51

[32] 1909 edition. I need a good translation of the 1906 version.

52

[33] Correct 1906 version; “maximum” was changed to “minimum” in the 1909 edition!

61

[³⁴] 1906 edition; “or competition” did not appear in the 1909 edition.

65

[³⁵] 1906 edition. In the 1909 edition this was replaced by “This theory carries the name of the theory of international trade, and we shall keep to this name.”

[³⁶] This qualification was added in the 1909 edition.

69

[³⁷] The section number 69 was omitted from the 1909 edition.

79

[³⁸] 1906 edition. The 1909 version is “to establish the existing equilibrium”.

[³⁹] 1906 edition. The 1909 word is “external”.

86

[⁴⁰] 1906 edition. The 1909 edition replaced “benefited” by “encouraged” and “injured” by “discouraged”.

92

[⁴¹] 1906 edition. The 1909 edition replaced “consumption” by “circulation”.

95

[⁴²] 1906 edition. This clause was removed from the 1909 edition.

97, 99 and 100

[⁴³] Literally, “mobile capital”.

99

[⁴⁴] 1906 edition. Replaced by “abstract things” in the 1909 edition.

101

[⁴⁵] The singular “capitalist” in both editions.

C H A P T E R V I I

2

[¹] 1906 version. This passage was omitted from the 1909 edition.

3 and 4

[²] Italian *tipo medio* (1906) and French *type moyen* (1909). This term can be translated as either mean or median. In fact, as annotation 11 below indicates, it should perhaps in some places be interpreted as the mode.

3

[³] In the 1909 edition, this is erroneously set as *ab'' b'' a''*.

6

[⁴] 1906 edition; word omitted in the 1909 edition.

[⁵] This footnote, placed at the end of §5 in the 1906 edition, was moved to the new end of the revised §6 in the 1909 edition.

[⁶] Passage added in the 1909 edition (including the footnote).

7

[⁷] 1906 edition. The word used in the 1906 edition could also be translated as “simple”, as was done in the 1909 edition; cf. Montesano et al. (2006), p. 565, item 7.

9

[⁸] While Pareto’s figures always have the abscissa indicated vertically and the ordinate indicated horizontally, the latter is always to the right of the abscissa, whereas in the 1909 edition Figure 53 has the ordinate to its left.

[⁹] 1906 edition; removed in the 1909 edition.

II

[¹⁰] This refers to Pareto’s article “Aggiunta allo studio sulla curva delle entrate”, Vol. 14, pp. 15–26, recently translated as “Supplement to the Study of the Income Curve”, *Giornale degli Economisti e Annali di Economia*, 67 (December 2008), 441–551.

12

[¹¹] The 1906 edition contains the misprint *sk* for *s*, corrected in the 1909 edition. Pareto used the term *media* (mean) for the point *s*, but it is clear from Figure 54 that it is not the mean but the mode.

14

[¹²] British terminology. The U.S. terminology is “grades”.

[¹³] 1909 edition, correcting *abc* in the 1906 edition.

16

[¹⁴] This appears in both editions, but evidently it should be *bqc* in Figure 54.

17

[¹⁵] 1906 version. The 1909 version reads: “takes the form II”.

18

[¹⁶] 1906 version; the indicated passage was eliminated in the 1909 edition.

19

[¹⁷] 1906 edition. In the 1909 edition, Chapter IX of the *Systèmes* is erroneously listed as being in Vol. I.

20

[¹⁸] 1906 edition. The 1909 edition has the erroneous $a''bc$.

22

[¹⁹] 1906 edition. The 1909 edition has the erroneous $a'b''la''$.

24

[²⁰] See p. 320 of Vol. II of the *Cours*. Where this printing error was pointed out is not made clear.

[²¹] This long section (including the footnote) was added in the 1909 edition, partly replacing the first paragraph of §24 in the 1906 edition. The part that was replaced (including the original footnote) is translated as follows (translation by Vincenzo Savini):

24. Bringing our discussion back within the limits indicated in §23, we see that in the course of the 19th century there were cases in which the curve blc slightly changed its shape, its form remaining the same but the constants [parameters] varying; and this change has taken place in the direction of lower inequality of incomes. It is necessary to define what is meant by these words ‘lower inequality of incomes’. Incomes may tend to equality in two quite different ways, namely by higher incomes declining or by lower incomes increasing. We give the latter meaning to the decrease in the inequality of incomes; this increase will therefore take place when the number of individuals whose income is lower than a certain income x declines in comparison with the number of people whose income is higher than x .¹

[²²] This interesting methodological article appeared in Vol. I (1907), No. II, pp. 296–315. An Italian translation has been published in Vilfredo Pareto, *Oeuvres complètes*, Tome XXII, *Écrits sociologiques mineurs* (Geneva: Librairie Droz, 1980, pp. 324–343) under the title “L’economia e la sociologia dal punto di vista scientifico”.

¹ “Cours, §964. There is a misprint in the text, p. 320, 14th line from the bottom; it should read ‘revenu inférieur à x diminue par rapport’ instead of “revenu inférieur à x augmente par rapport” [‘income lower than x decreases relatively to’ instead of ‘income lower than x increases relatively to.’]”

25

[²³] Paul Leroy-Beaulieu, *Essai sur la répartition des richesses et sur la tendance à une moindre inégalité des conditions*, 4th edition (revised and augmented), Paris: Guillaumin et Cie, n.d. (approx. 1896).

[²⁴] 1909 edition. The corresponding opening passage of §25 in the 1906 edition may be translated as follows: “The fact that has now been brought rigorously to light by the study of the income curve had first been recognized as an induction derived from the study of many economic phenomena. Mr. Leroy-Beaulieu made it the object of a famous work.” [This translation has been influenced by one provided by Vincenzo Savini.]

27

[²⁵] Literally, “squashed”.

[²⁶] The term used in both the 1906 and 1909 editions is “real”, but the meaning is surely “actual”.

28

[²⁷] The term “relative” was added in the 1909 edition.

29

[²⁸] 1906 edition, removed in the 1909 edition.

31

[²⁹] Both the 1906 and 1909 editions refer to *fb'dc*.

32

[³⁰] 1906 edition; this passage was removed in the 1909 edition.

33

[³¹] Émile Levasseur (1828–1911), French geographer. This information may come from one of his scarce atlases. See note 73 below.

38

[³²] Italicized in the 1909 but not the 1906 edition.

40

[³³] Gustaaf Cauderlier, French demographer, known for his book *Les lois de la population en France* (Paris: Librairie Guillaumin & Cie, 1902). This thesis is expounded in Chapter III of that work.

44

[³⁴] Cf. Alfred Marshall, *Principles of Economics*, 2nd edn. (1891), Book IV, Ch. IV, §8 (§7 in the 3rd and later editions).

48

[³⁵] 1906 edition; omitted from the 1909 edition.

[³⁶] The meaning is presumably “time lag”. Cf. note [a] to §48 of Ch. VII, in the Annotations from the 2006 Italian Edition, p. 504.

51

[³⁷] The word “only” was added in the 1909 edition.

53

[³⁸] 1909 edition. In the 1906 edition this was “or”.

54

[³⁹] 1909 edition; the 1906 version is “in wealthy modern nations”.

[⁴⁰] 1909 edition; the 1906 version is “worse”.

[⁴¹] George Barr Baker, “Dollars vs. Pedigree. The Truth About International Marriages,” *Everybody’s Magazine* (New York: The Ridgway Company, etc.), Vol. XVI, No. 2 (February 1907), 167–176. The passage quoted by Pareto is found on p. 172, column 2.

[⁴²] Frances Trollope, *Domestic Manners of the Americans*, edited with a history of Mrs. Trollope’s adventures in America, New York: Alfred A. Knopp, 1949, pp. 156–7. The original reads: “In America, with the exception of dancing, which is almost wholly confined to the unmarried of both sexes, all the enjoyments of the men are found in the absence of the women. They dine, they play cards, they have musical meetings, they have suppers, all in large parties, but all without women. Were it not that such is the custom, it is impossible but that they would have ingenuity enough to find some expedient for sparing the wives and daughters of the opulent the sordid offices of household drudgery which they almost all perform in their families.”

[⁴³] The two extra paragraphs in this footnote were added in the 1909 edition.

[⁴⁴] 1909 edition. The corresponding passage on the 1906 edition reads: “for whom a moderate destruction of wealth is not too damaging.”

[⁴⁵] 1906 edition; removed in the 1909 edition.

[⁴⁶] 1906 edition. The 1909 edition reads (rather ungrammatically) “which is generally ascribed to”.

[⁴⁷] Cf. James E. Thorold Rogers, M.P., *Six Centuries of Work and Wages: A History of English Labor*, New York: The Humboldt Publishing Co., 1890. The original, found on p. 70, is somewhat different from that quoted by Pareto, as follows: “The peasants were

dispersed and defeated; their leaders were tried, sentenced, and hanged; but the solid fruits of victory rested with the insurgents of June, 1381.” For a full account of the Tyler rebellion see Vol. I, pp. 83–95 of Rogers’ *A History of Agriculture and Prices in England ...*, 8 vols., Oxford: Clarendon Press, 1866–1902.

[48] Cf. *Cronica di Matteo e Filippo Villani con le vite d'uomini illustri fiorentini di Filippo e la Cronica di Dino Compagni*, Milano: per Nicolò Bettoni e Comp., 1834, p. 3, Ch. IV. See also Matteo Villani, *Cronica, con la continuazione di Filippo Villani*, Parma: Ugo Guanda Editore, 1995, pp. 15–16 (the latter edition renumbers the chapters, and the passage appears in Chapter VI).

[49] 1906 edition, *beni terreni*; translated as “real estate” (*biens immobiliers*) in the 1909 edition.

[50] 1906 edition; these two passages were omitted from the 1909 edition.

[51] The Statute of Labourers was an act passed by the English parliament in 1351, in response to the high wages following the Black Death; it prohibited employers from paying wages above the level prevailing in 1346.

55

[52] 1909 edition; the 1906 version is “extensively destroys”.

[53] Pareto does not refer to the source of this criticism.

[54] 1906 edition; omitted in the 1909 edition.

[55] 1906 edition, omitted in the 1909 edition.

57

[56] The phrase “as the English say” from the 1906 edition was omitted in the 1909 edition. Of course, the English term is “standard of living”.

58

[57] The original source is the pamphlet *Herr Bastiat-Schülze von Delitzsch, der ökonomische Julian; oder Kapital und Arbeit* (Berlin: R. Schlingmann, 1864) by Ferdinand Lassalle (1825–1864). Lassalle’s term was *das eherne Lohngezet*, literally “the bronze law of wages”; “the bronze law” was rendered in Italian as “la leggi di bronzo” and in French as “la loi d’airain”.

60

[58] 1909 edition. The corresponding sentence in the 1906 edition reads (translation by Vincenzo Savini): “The fact that the increase in wealth proceeds in waves is a contributing factor to this.”

61

[⁵⁹] This conclusion is based on Pareto's article "La mortalità infantile ed il costo dell'uomo adulto", *Giornale degli Economisti* [2], 7 (November 1893), 451–456, translated into English as "Infant Mortality and the Cost of a Human Adult", *Giornale degli Economisti e Annali di Economia*, 67 (December 2008), 355–360.

65

[⁶⁰] 1906 edition. This appeared as "1805" in the 1909 edition.

[⁶¹] The word for "rates" is from "proportions" in the 1909 edition, correcting "propositions" in the 1906 edition.

68

[⁶²] 1906 edition. In the 1909 edition, "curve" (the Italian *curva*) was translated as *couche* ("couch" or "bed", or "stratum") instead of *courbe*. This error was undoubtedly caused by the fact that in the 1909 edition, in the previous sentence the Italian *strati* ("strata") had been translated (correctly) as *couches*.

69

[⁶³] J.-C.-L Simone de Sismondi, *Nouveaux principes d'économie politique, ou De la richesse dans ses rapports avec la population*, Paris, 1819, Book VII, Chapter III, esp. pp. 272–3. 2nd edition, 1827, p. 275. 3rd (posthumous) edition, Geneva and Paris: Edition Jeheber, 1953, Vol. II, pp. 184–5. English translation, *New Principles of Political Economy. Of Wealth in Its Relation to Population*, New Brunswick, N.J.: Transaction Publishers, 1991, p. 528.

[⁶⁴] 1906 edition; removed in the 1909 edition.

[⁶⁵] Literally, "wishing".

70

[⁶⁶] Passage added in the 1909 edition.

71

[⁶⁷] T. R. Malthus, *An Essay on the Principle of Population, as it affects the Future Improvement of Society, with remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers*, London, J. Johnson, 1798. Malthus distinguished in Chapter IV between "preventive" and "positive" checks on population growth (1798, pp. 62–70); he did not use the terminology "repressive" for the latter. See also the Everyman edition (based on the 7th edition of 1872), London 1973, Chapter II, p. 14.

72

[⁶⁸] 1906 edition. This sentence was removed in the 1909 edition.

[⁶⁹] 1906 version. The 1909 version reads: "Obstacle (α) may operate on legitimate births, obstacle (β) on illegitimate births."

74 and 96

[⁷⁰] Pareto used the English term, in italics. Malthus introduced this prescription in the later editions of his book, e.g., Chapter II of the Everyman edition, p. 14, note 1: “By moral restraint I would be understood to mean a restraint from marriage from prudential motives, with a conduct strictly moral during the period of this restraint; and I have never intentionally deviated from this sense.”

[⁷¹] 1909 edition; the 1906 edition reads “infertility”.

79

[⁷²] 1906 edition. Passage removed in the 1909 edition.

[⁷³] Émile Levasseur, *La population française* (3 vols.), Paris: Arthur Rousseau, 1889, 1891, 1892.

80

[⁷⁴] Added in the 1909 edition.

81

[⁷⁵] 1906 edition; word removed in the 1909 edition.

[⁷⁶] 1906 edition; word removed in the 1909 edition.

82

[⁷⁷] 1906 edition. Passage removed in the 1909 edition.

[⁷⁸] 1906 edition; replaced by “the population” in the 1909 edition.

[⁷⁹] 1906 edition; replaced by “increasing population” in the 1909 edition.

83

[⁸⁰] 1906 edition; “comfort” in the 1909 edition.

84

[⁸¹] William Godwin, *Enquiry concerning Political Justice and its Influence on Morals and Happiness*, 3rd edition, 2 vols., London: G. G. and J. Robinson, 1798. Critical edition, in 3 vols., Toronto: University of Toronto Press, 1946. Cf. Vol. II, Book VIII, Ch. IX, pp. 515–519. *Of Population*, London: Longman, etc., 1820. See also Malthus (1798), Chs. X–XV.

85

[⁸²] In the 1909 edition, section 85 was wrongly renumbered 84 (the number of the previous section).

[⁸³] Alexis de Tocqueville (1805–1859), author of *De la démocratie en Amérique* (Paris: Librairie de Charles Gosselin, 1836 and 1840), translated as *Democracy in America*.

86

[⁸⁴] 1909 edition. The 1906 edition reads: “Besides the economic motives, we find other ones”.

[⁸⁵] 1906 edition; replaced by “their theories” in the 1909 edition.

87

[⁸⁶] Charles Fourier (1772–1837), utopian socialist.

[⁸⁷] 1906 version; the 1909 version is “activity”.

[⁸⁸] 1906 edition; the sentence was removed in the 1909 edition.

90

[⁸⁹] 1906 edition; omitted in the 1909 edition.

91

[⁹⁰] In the 1909 edition, the next sentence becomes a new paragraph.

93

[⁹¹] Pareto uses the English terms (in italics).

96

[⁹²] 1906 edition. In the 1909 edition this is replaced by the sentence: “A part devoted to preaching certain rules of conduct.”

[⁹³] 1906 edition. In the 1909 edition this was replaced by “the new faith”.

[⁹⁴] 1906 edition; literally, “blows neither hot nor cold”. The 1909 edition reads: “really adds nothing new to our knowledge”.

[⁹⁵] Édouard Fournier, *L'esprit des autres*, 4th edition (Paris: E. Dentu, 1861), p. 174. One wonders whether he really meant “it is not the majority that governs.”

[⁹⁶] The quotations are from two articles published anonymously by Sir Henry Sumner Maine (1811–1888) in *The Quarterly Review*, the first being “The Prospects of Popular Government” in Vol. 155 (January and April 1883, 551–576; see p. 582), and the second being “The Nature of Democracy” in Vol. 158 (July and October 1884, 287–333; see pp. 314–15). These were reprinted (with some added footnotes) as Essay I and Essay II respectively in Maine’s subsequent book, *Popular Government: Four Essays*, 3rd edition, London: John Murray, 1886 (see pp. 42 and 89), reprinted, Indianapolis: Liberty Fund, 1976 (see pp. 63 and 105).

The other essay cited by Pareto was actually Essay III, "The Age of Progress", Vol. 159 (January and April 1885, 267–298), but his quotation was from Essay II. Essay IV was "The Constitution of the United States", first published in Vol. 157 (January and April 1884, 1–31). Maine (1886, p. 42n, 1976, p. 63n) also referred to the French philosopher Ernest Renan (1823–1892), who had in his drama *Caliban*—included in his *Drames philosophiques* (Paris: Calman Lévy, 1888, pp. 85, 99)—had his subjects proclaim that all civilization is of aristocratic origin.

[97] David Friedrich Strauss (1808–1874), controversial German Lutheran theologian, in his last work *Der alte und der neue Glaube. Ein Bekennntniß* (3rd edn., Leipzig: Verlag von S. Hirzel, 1872, §84, p. 285; 7th edn., Bonn: Verlag von Emil Strauss, 1874, §84, p. 287; 12th to 14th stereotyped edition, Bonn: Emil Strauss, 1895, §84, pp. 189–90. The relevant clause is "Nein, die Geschichte wird fortfahren, eine gute Aristokratin, obwohl mit volksfreundlichen Gesinnungen, zu sein".

The English translation of this passage by Mathilde Blind, from the 6th edition, *The Old Faith and the New. A Confession* (New York: Henry Holt and Company, 1873, Vol. II, §80, p. 104), reads: "No: history will continue a thorough aristocrat, although with convictions friendly to the people". Note that her translation omits the words "zu sein" ("to be") from Strauss's text. A possible alternative translation, kindly suggested by Gerhard Weiss, is: "No, history will continue to be a good aristocrat, though with convictions agreeable to the people." Undoubtedly Maine relied on Blind's translation. The earlier passages in Strauss's book are fully in accord with Maine's views.

[98] Maine (1883, p. 572; 1886, p. 42; 1976, p. 63).

[99] Maine (1884, p. 314; 1886, p. 89; 1976, p. 105).

[100] Ernest Renan (1823–1892), French philosopher and dramatist, with whom Strauss carried out an extensive correspondence (cf. *The Old Faith and the New*, Vol. I, §24, p. 73; *Der alte und der neue Glaube*, 1872, §25, p. 64; 1895, §25, p. 42). Maine (1886, p. 42n; 1976, p. 63n) pointed out the similarity between Strauss's ideas and Renan's. In Renan's drama *Caliban*—included in his *Drames philosophiques* (Paris: Calman Lévy, 1888)—he had Gonzalo proclaim that "all civilization is of aristocratic origin" (p. 85), and he had the prior of his Carthusian convent proclaim: "Yes: all civilization is the work of the aristocrats" (p. 99).

[101] The indicated paragraphs of this footnote, from the 1906 edition, were removed in the 1909 edition, while the remaining (first) paragraph of the footnote was moved to the end of §97 in the 1909 edition.

[102] 1906 edition, omitted in the 1909 edition.

[103] New location in the 1909 edition of the greatly abridged preceding footnote.

Not oftentimes upriseth through the branches
The probity of man; ...

His translation of the verse from the *Paradiso*, XVI, 76–78, is (p. 82):

To hear how races waste themselves away
Will seem to thee no novel thing nor hard,
Seeing that even cities have an end.

[¹⁰⁶] 1906 edition; replaced by “been” in the 1909 edition.

99

[¹⁰⁷] 1906 edition; replaced by “the” in the 1909 edition.

[¹⁰⁸] 1906 edition; “cannot be considered as such” in the 1909 edition.

100

[¹⁰⁹] 1906 edition; replaced by “a large number” in the 1909 edition.

[¹¹⁰] 1909 edition, replacing “it” in the 1906 edition.

[¹¹¹] 1909 edition, replacing “ratio” in the 1906 edition.

101

[¹¹²] 1906 edition; omitted in the 1909 edition.

103

[¹¹³] 1906 edition; replaced by “useful” in the 1909 edition.

[¹¹⁴] Passage added in the 1909 edition.

[¹¹⁵] Added in the 1909 edition.

104

[¹¹⁶] Added in the 1909 edition.

107

[¹¹⁷] 1906 edition, removed in the 1909 edition.

[¹¹⁸] 1906 edition. In the 1909 edition, o° was changed to 6° .

[¹¹⁹] The units here are of course centigrade.

109

[¹²⁰] 1906 edition; omitted in the 1909 edition.

[121] 1906 edition; replaced by “change” in the 1909 edition.

[122] 1906 edition; omitted in the 1909 edition.

[123] 1906 edition; replaced by “the *Accademia della Crusca* in Italy” in the 1909 edition.

[124] This clause was inserted by Pareto to take the place of Maine’s original passage “It would be wearisome to enter on a detailed comparison or contrast of English and Roman Equity, but it may be worth while to mention two features which they have in common. The first may be stated as follows: Each of them …”. See the Everyman edition of *Ancient Law* (London: Oxford University Press, 1931), p. 56.

[125] Passage added in the 1909 edition.

II10

[126] Added in the 1909 edition.

[127] 1909 edition, replacing “diverse” in the 1906 edition.

III

[128] 1906 edition, literally “the establishment of castes”; 1909 edition: “castes”.

III3

[129] 1906 edition; the singular “reform” in the 1909 edition.

III4

[130] 1909 edition, replacing “European” in the 1906 edition.

III5

[131] Added in the 1909 edition.

[132] 1909 edition, replacing “is opposed to the mutability of” in the 1906 edition.

[133] This may refer to a scandal in 1892–93 involving activities by Ferdinand de Lesseps in 1888 and the accusation against many members of the French parliament of receiving bribes from the Panama Canal Company.

III6

[134] The word “elsewhere” was added in the 1909 edition.

[135] 1909 edition, replacing “often” in the 1906 edition.

III7

[136] 1906 version. The 1909 version is “led people who had certain common interests”.

[137] 1906 edition; removed in the 1909 edition.

CHAPTER VIII

4

[¹] 1906 edition; removed in the 1909 edition.

6

[²] 1909 edition, replacing “in an absolute way” in the 1906 edition.

[³] The Italian *mezzadria* and French *métayage*. *The Concise Oxford English Dictionary* defines “métayage” as “land tenure in which a farmer pays part … of the produce as rent to the owner …”, and a “sharecropper” as “a tenant farmer who pays his rent with a part of his crop”. The word “metayage” was used in classical English economic writings. Adam Smith used the term in Book III, Chapter II of the *Wealth of Nations*: “To the slave cultivators of ancient times gradually succeeded a species of farmers known at present in France by the name of Métayers. They are called in Latin, *Coloni Partiarii*. They have so long been in disuse in England that at present I know no English name for them.” J. S. Mill included a chapter “Of Metayers” in his *Principles*. Alfred Marshall in Book VI, Chapter x, section 4 of his *Principles of Economics* considered the “fundamental distinction between the ‘English’ system of rental and that of holding ‘shares’, as it is called in the New World, or the ‘Metayer’ system as it is called in the Old.” The word “sharecropping” was introduced along with the practice of it following the U. S. Civil War.

For writings on sharecropping and its history, see Steven N. S. Cheung, *The Theory of Share Tenancy*, Chicago: The University of Chicago Press, 1969, and “Transaction Costs, Risk Aversion, and Choice of Contractual Arrangements”, *Journal of Law and Economics*, 12 (April 1969), 23–42; Joseph E. Stiglitz, “Incentives and Risk Sharing in Sharecropping”, *Review of Economic Studies*, 41 (April 1974), 219–255; Franklin Allen, “On the Fixed Nature of Sharecropping Contracts”, *Economic Journal*, 95 (March 1985), 30–48; and Jonathan J. Liebowitz, “Tenants, Sharecroppers, and the French Agricultural Depression of the Late Nineteenth Century”, *Journal of Interdisciplinary History* 19 (Winter 1989), 429–445.

8

[⁴] Literally, “mobile capital”.

[⁵] The meaning is presumably “inventories”.

[⁶] 1906 edition: *la moneta metallica*; 1909 edition: *la monnaie métallique*.

10

[⁷] Literally, “simple savings”, or “savings pure and simple”.

[⁸] The word used in both the 1906 and 1909 editions is *locomobile*, which was a portable steam-engine.

[⁹] Literally, “capitals” in the 1906 edition, but the singular “capital” in the 1909 edition.

II

[10] Added in the 1909 edition.

[11] This long footnote was added in the 1909 edition.

12 bis

[12] The meaning is presumably “homogeneous”.

[13] Section 12bis was added in the 1909 edition.

24

[14] 1906 version. The 1909 version reads: “his ways of feeling”.

[15] 1906 version; the 1909 version is “the theories are numerous”.

25

[16] 1906 version. The 1909 version reads: “understand how human concepts originate and develop”.

[17] Pareto referred in the *Systèmes*, II, p. 86, to the French translation of Menger’s work *Das Recht auf den vollen Arbeitsertrag in geschichtiger Darstellung*, 2nd edition, Stuttgart: J. G. Cotta, 1891 (1st edition 1886). Cf. the English translation, *The Right to the Whole Produce of Labour*, London: Macmillan and Co., Ltd., 1899.

26 and 28

[18] 1906 edition; removed in the 1909 edition.

[19] 1906 edition; this passage is found on pp. 164–5 of Vol. II of the second edition of Leroy-Beaulieu’s 4-volume work (Paris: Librairie Guillaumin et C^{ie}, 1896). In the 1909 edition of the *Manual* the page reference is given as p. 105, undoubtedly in error.

[20] In both editions, this footnote was apparently inadvertently interchanged with the succeeding one. Cf. Montesano et al. (2006), p. 569, note 26.

28

[21] 1906 edition, removed in the 1909 edition.

[22] This clause was added in the 1909 edition.

29

[23] 1906 edition; removed in the 1909 edition. The complete title and publisher of Martello’s work are supplied in Montesano et. al. (1906, p. 569, §29 [a]) as *La moneta e gli errori che corrono intorno ad essa*, with an Introduction by Francesco Ferrara, Florence: Le Monnier, 1883.

[²⁴] 1906 edition; these words were removed in the 1909 edition.

[²⁵] 1909 edition; the 1906 version reads: “Concrete money is put into operation materially in exchanges; ideal money is not.”

[²⁶] 1906 version; the marengo was a gold napoleon, a French twenty-franc gold coin. The 1909 version is “francs”.

[²⁷] 1906 version; the 1909 version is “crowns”.

[²⁸] 1906 edition; replaced by “In” in the 1909 edition.

31

[²⁹] Literally, “exchange” (*cambio* in the 1909 edition, *échange* in the 1909 edition).

32

[³⁰] Literally, “a certain money” (*una certa moneta*) in the 1906 edition; but “coined money” (*monnaie frappé*) in the 1909 edition.

33

[³¹] Literally “moneys” (plural): *monete* in the 1906 edition, *monnaies* in the 1909 edition. (In English, the word “money” is seldom used in the plural, nor does one usually speak of “a money”.) In the *Sansoni-Harrap Standard Italian and English Dictionary*, the word *moneta* is translated first as “coin” or “piece”, then as “currency”, then as “money”, and finally as “small change”. In what follows, “currency” always corresponds to Pareto’s *moneta*, but the latter is also translated as “money”, depending on the context.

[³²] 1909 edition; “given” in the 1906 edition.

37

[³³] 1906 edition; omitted from the 1909 edition.

40

[³⁴] The word used for “currency” is *moneta* in the 1906 edition and *monnaie* in the 1909 edition.

41

[³⁵] 1906 edition, reading “true moneys” (*vere monete*); the word “true” was removed in the 1909 edition.

42

[³⁶] In contemporary terminology, bank notes (e.g., dollar bills) would be considered as a form of “money”.

45

[37] 1906 edition; omitted in the 1909 edition.

46

[38] Section 46 was mislabelled 49 in the 1909 edition.

[39] Added in the 1909 edition.

47

[40] 1906 edition; replaced by “price in gold” in the 1909 edition.

49

[41] Literally “moneys” (*monete* and *monnaies*) in the 1906 and 1909 editions.

[42] Erroneously referred to as §49 in the 1909 edition.

CHAPTER IX

3

[1] 1909 edition “en gros”. The 1906 version was “per vendite ingenti” (“for huge sales”).

[2] 1906 version, omitted in the 1909 edition.

6

[3] The Italian is *prepotenze*, plural of *prepotenza*, translated in the 1909 edition as *préten-*
sions (pretensions or claims), clearly in error.

8

[4] 1906 version (*assai perfetto*). The 1909 version is “often good” (*souvent bonne*). The Italian word *assai* means “very”, and the French translator Alfred Bonnet evidently confused this with the French word *assez*, which means “enough” or “sufficiently”.

[5] 1906 version. The 1909 version is “have had little or no success.”

9

[6] 1906 version; the 1909 version is “deal with”.

[7] 1906 version (*braccianti*). The 1909 version is “diggers” (*terrassiers*) (earthwork contractors).

[8] This long passage comes from Maffeo Pantaleoni, “Alcune osservazioni sui sindacati e sulle leghe. A proposito di una memoria del prof. Menzel”, *Giornale degli Economisti*, [2], 26 (March 1903), 236–265, continued in Vol. 27 (April 1903), 346–378 and 27 (December 1903), 560–581. [The last installment is marked “to be continued”, but in Pantaleoni’s collected papers, *Scritti varii di economia*, Serie 2 (Milan: Remo Sandron,

Editore, 1909), pp. 145–260, no additional material was added.] It should be noted that the passage cited by Pareto is a paraphrase of Pantaleoni's passage, not an exact quotation, and continues to p. 241 of the March 1903 first installment; it is found on p. 451 of the 1909 reprint.

12

[⁹] 1906 edition (*aiuto*). In the 1909 edition this is *intervention* (“intervention”).

[¹⁰] 1906 edition; this sentence was omitted from the 1909 edition.

13

[¹¹] 1906 edition, omitted from the 1909 edition.

[¹²] 1906 edition, omitted in the 1909 edition.

[¹³] 1906 edition. The 1909 edition reads “few in number”.

[¹⁴] 1906 edition. The 1909 edition reads “success”.

[¹⁵] 1909 edition. In the 1906 edition this was “facts”.

14

[¹⁶] 1906 edition. The 1909 edition reads “nonchalance”.

[¹⁷] 1906 edition; these two paragraphs were removed in the 1909 edition.

15

[¹⁸] 1906 edition; omitted in the 1909 edition.

[¹⁹] 1906 edition; the 1909 edition reads “cause an increase in prices”.

16

[²⁰] This paragraph was added in the 1909 edition.

17

[²¹] 1909 edition. The 1906 edition reads: “regarding the artifices by which certain people cause commodities to become more expensive, in order to benefit from their higher prices.”

[²²] 1906 version. The 1909 version reads: “Among various nations, especially in ancient times, war allowed the strong to appropriate the goods of the weak; in the case of a single nation, it is by means of the laws and, from time to time, of revolutions, that the strong still strip the weak.”

18

[²³] 1909 edition. The 1906 edition reads: “dependent on”.

19

[²⁴] 1906 version. The 1909 version is: “affirmative on the subject of”.

[²⁵] The reference to §35 was added in the 1909 edition.

20

[²⁶] 1909 edition. The 1906 version is “learned”.

[²⁷] 1906 edition. The 1909 edition is “hoped”.

[²⁸] 1909 edition. The 1906 edition is “man’s”.

[²⁹] 1909 edition. The 1906 edition has “therefore have much greater power”.

21

[³⁰] 1906 version; the 1909 version is “for which Say has provided the theory”.

23

[³¹] Jehan Cherruyt, seigneur de Malestroict, conseiller du roy & Maistre ordinaire de ses comptes, *Les Paradoxes du seigneur de Malestroict sur le fait des Monnoyes, presentez à sa majesté au mois de mars MDLXVI* [Jehan Cherruyt, Lord Malestroict, King’s Council and Chief Monetary Accountant, *The Paradoxes of Lord Malestroict on Monetary Matters, presented to His Majesty in the month of March 1566*], (Paris, 1566). The 1909 edition uses the modern spelling “Malestroit”. This is the work that led to the famous Reply by Jean Bodin which is acknowledged to be the first formulation of the quantity theory of money: *Réponse aux paradoxes de M. de Malestroit touchant l’enrichissement de toutes les choses et des monnaies* (1568). Cf. Arthur Eli Monroe, *Monetary Theory Before Adam Smith* (Cambridge, Mass.: Harvard University Press, 1923. An English translation of Bodin’s tract is contained in Monroe’s *Early Economic Thought* (Cambridge, Mass.: Harvard University Press, 1924), pp. 121–141.

[³²] 1906 version. The 1909 version is “this”.

24

[³³] 1906 edition; omitted in the 1909 edition.

[³⁴] 1906 edition; omitted in the 1909 edition.

[³⁵] 1909 version. The 1906 version is “desire”.

[³⁶] These two paragraphs were added in the 1909 edition.

25

[³⁷] This is a summary rather than a direct quotation from “The Statute of Labourers; 1351”. The original wording will be found in Ernest F. Henderson, *Select Historical Documents of the Middle Ages*, London: George Bell and Sons, 1892, pp. 165–8.

[³⁸] 1906 edition, removed in the 1909 edition.

[³⁹] The word used is *krumiri* in the 1906 edition, and *krumirs* in the 1909 edition.

26

[⁴⁰] This paragraph was added in the 1909 edition.

[⁴¹] 1906 edition; removed in the 1909 edition.

[⁴²] *contribution mobilière*. This is the name of a tax imposed during the French revolution (18 February 1791) on movable or personal property, in contrast to the *contribution foncière*, a tax on real estate. The word *mobilier* can mean “personal” as well as “movable”. This tax was replaced by (i.e., was renamed as) a *taxe d’habitation* in 1974, which is an occupancy tax. (I thank Prof. Kathryn Reyerson of the Department of History at the University of Minnesota for suggesting the translation “occupancy tax”.)

In the words of J. R. McCulloch (*A Treatise on the Principles and Practical Influence of Taxation and the Funding System* (London: Longman, Brown, Green, and Longmans, 1845), p. 73: “Assuming that the annual value or rent of the houses occupied by different individuals afforded on the whole the best practical test that could be found of their fortune or of their ability to bear taxes, the Assembly selected it as the principal evidence on which to assess the *contribution mobilière*” [which] “has since undergone various changes and modifications.”

[⁴³] The 1909 edition has an erroneous end-quote here which was not in the 1906 edition.

[⁴⁴] These two paragraphs, including the two footnote references to G. Sorel and the one to Molinari, were added in the 1909 edition.

27

[⁴⁵] This word was added in the 1909 edition.

[⁴⁶] 1906 edition; replaced by “every economic phenomenon” in the 1909 edition.

[⁴⁷] “woe to the vanquished”.

[⁴⁸] This paragraph, including the two-paragraph footnote, was added in the 1909 edition.

28

[⁴⁹] 1906 version; the 1909 version is “reality”.

[⁵⁰] 1906 edition; omitted in the 1909 edition.

[⁵¹] 1906 edition, misprinted as 205 in the 1909 edition.

[⁵²] These three paragraphs were added in the 1909 edition.

29

[⁵³] Added in the 1909 edition.

[⁵⁴] 1909 edition. The passage in the 1906 edition reads: “Other facts contribute in an important way”.

30

[55] 1906 edition; removed in the 1909 edition.

[56] 1906 edition, removed in the 1909 edition.

[57] These two paragraphs were added in the 1909 edition.

[58] 1906 edition, removed in the 1909 edition.

[59] 1906 edition; removed in the 1909 edition.

[60] 1906 edition. Replaced by “one is content” in the 1909 edition.

[61] 1906 edition (“*torcendo*”). In the 1909 edition the word used is *forçant* which could be translated as “straining”.

[62] 1906 edition, removed in the 1909 edition.

[63] Parenthetical insertion in both editions.

[64] Word added in the 1909 edition.

[65] This work has gone through many editions; the 11th was published in Paris by Marcel Rivière et Cie., 1950. An English translation, *Reflections on Violence* (authorized translation of the 3rd edition by T. E. Hulme) was published in New York by B. W. Huebsch, c. 1912, and a later translation by T. E. Hulme and J. Roth was published by Collier-Macmillan in London, 1950.

This two-paragraph footnote was added in the 1909 edition.

[66] Antonio Pertile (1830–1895), *Storia del diritto italiano: dalla caduta dell’Impero romano alla codificazione* (*History of Italian Law: From the Fall of the Roman Empire to the Codification*), 6 vols., 1892–1902.

[67] 1906 edition; word omitted in the 1909 edition.

[68] 1906 edition; the 1909 edition has the plural “laws”.

[69] 1906 edition. The 1909 edition reads: “interfere with others’”.

[70] 1906 edition; omitted in the 1909 edition.

[71] 1906 edition. In the 1909 edition the word is “different”.

[72] Footnote added in the 1909 edition.

[73] Word omitted in the 1909 translation.

[74] 1909 edition, replacing “the latter” in the 1906 edition.

[75] 1909 edition (*appliquait*); the word in the 1906 edition was *volute* (“wanted” or “desired”).

31

[76] 1906 edition (*prepotenze*), replaced by *prétensions* (pretensions) in the 1909 edition (an obvious mistranslation).

[77] Footnote added in the 1909 edition.

[78] Word omitted in the 1909 edition.

32

[⁷⁹] 1909 edition (*l'usine de la maison Beaulieu*). The 1906 edition reads “the Beaulieu plant and firm” (“*l'officina e la casa Beaulieu*”). The 1909 version would be a correct translation of *l'officina della casa Beaulieu*), which may have been Pareto's intention in the 1906 edition.

[⁸⁰] Sarcastic word omitted in the 1909 edition.

[⁸¹] Words omitted in the 1909 edition.

[⁸²] 1906 edition. The 1909 edition reads “who lashed out at the bourgeois”.

[⁸³] 1906 edition (“ferrovieri”); this is replaced in the 1909 edition by “the employees of the railway”.

[⁸⁴] 1909 edition. In the 1906 edition, “support”.

[⁸⁵] This footnote was placed here in the 1909 edition; in the 1906 edition it had been placed at the end of the paragraph. And the 1906 “great merit” was replaced by “courage” in the 1909 edition.

[⁸⁶] 1906 edition; omitted in the 1909 edition.

[⁸⁷] This footnote was added in the 1909 edition.

[⁸⁸] 1906 edition, omitted in the 1909 edition.

[⁸⁹] 1906 edition, replaced by “lack of good sense and energy” in the 1909 edition.

[⁹⁰] This paragraph was added to the footnote in the 1909 edition.

[⁹¹] These paragraphs were added to section 32 in the 1909 edition.

32 bis

[⁹²] Section 32bis was added in the 1909 edition.

[⁹³] French *un assez bon*; the French *assez* (“quite” or “rather”) should not be confused with the Italian *assai* which means “very”. Cf. Montesano et al. (2006), p. 350n.

[⁹⁴] The Duma was the Russian parliament. The reference is to a famous speech, delivered in French (on 23 July 1906) in the Royal Gallery of the House of Lords, by Sir Henry Campbell-Bannerman, Prime Minister of England, welcoming members of the newly opened Russian parliament. Cf. William Jennings Bryan (ed.), *The World's Famous Orations*, Vol. V, Great Britain: III, pp. 239–246.

[⁹⁵] A fairly common phrase in early 20th-century French writing, probably based originally on a biblical source (Ezekiel 32:7–8). Cf. Édouard Tallichet, *La question de la paix et sa solution* (Paris: Félix Alcam Éditeur, 1907), p. 87: “How many times in the last nineteen centuries have we not wanted to extinguish the lights of heaven!”. [See also Paul Allard, a book review in *Revue des questions historiques* (Paris: Bureaux de la revue, 1907), pp. 347–8] and Georges Demanche in *Revue française de l'Étranger et des Colonies*, Vol. 33 (1908), p. 651.] And although Henri Poincaré's “Last Thoughts” were published four

years later (*Dernières pensées*, 1913, Ch. 8, p. 224), they discuss the popular view that science is incompatible with religion, and thus that “it will—as was said by I no longer know what famous author—extinguish the lights of heaven, or at least deprive them of their mystery and reduce them to vulgar gas-burners.”

The French word for “extinguishing” in Pareto (1909) is *éteindre*, not to be confused with *étendre*, which means “spreading”; cf. Montesano et al. (1906), p. 351, line 13.

[96] French *défendre*, which can mean either “to defend” or “to prohibit”, but in this context, “to prohibit”. Cf. Montesano et al. (2006), p. 352, line 1.

[97] French *assez*, translated in Montesano et al., p. 352, line 4, as “very” (*assai*).

[98] A reference to Léon Daudet’s 1894 novel *Les Morticoles*, (*The Sawbones*), which is a scathing indictment of the medical profession of his day.

[99] French *directeur*, or *directeur de conscience*, an ecclesiastic chosen by a person to direct his conduct (cf. *Le Nouveau petit Larousse illustré*, 1943).

[100] The subject of Molière’s satirical play *Le malade imaginaire* (*The imaginary invalid*).

[101] As translated by Grant Showerman in *Ovid: Heroides and Amores, with an English translation* (Cambridge, Mass.: Harvard University Press, 1958), pp. 336–7: “... and so may you be forever lightened of your long chain, nor have to drink for all time the waters of slavery!”

[102] End of section 32bis.

33

[103] These initial paragraphs to section 33 were added in the 1909 edition.

[104] 1906 edition; replaced by “resistance” in the 1909 edition.

[105] 1906 edition; replaced by “months and months” in the 1909 edition.

34

[106] This is the original section 34 of the 1906 edition, which was removed and replaced by the new section 34 in the 1909 edition, which follows it here.

[107] The word in the text is the French *on* (“one”) which is presumably a misprint for *ou* (“or”).

[108] The French *préfet*, chief administrator of the French department (*département*) in which the strike took place.

35

[109] The first paragraph of this section is section 35 of the 1906 edition, reproduced with additions in the 1909 edition.

[110] Added in the 1909 edition.

[¹¹¹] *êtres veules*, which also means “weak” or “powerless” beings.

[¹¹²] These five paragraphs were added in the 1909 edition.

35 bis

[¹¹³] This section, labelled a second section 35, was added in the 1909 edition.

[¹¹⁴] The word used in the 1909 edition was *liaisons*.

[¹¹⁵] This of course is the compensation principle, first stated by Pareto in “Il massimo di utilità dato dalla libera concorrenza,” *Giornale degli Economisti* [2], 9 (July 1892), 48–66. An English translation of this article appeared in the *Giornale degli Economisti e Annali di Economia*, 67 (December 2008), 387–402.

36

[¹¹⁶] The word used in the 1906 edition was *vincoli* (“links”), which could also be translated as *liaisons* or constraints. Curiously, this word was omitted in the 1909 edition.

37

[¹¹⁷] 1906 edition, omitted in the 1909 edition.

[¹¹⁸] 1909 edition. The 1906 edition reads “benefit”.

[¹¹⁹] In the original of both editions, “those people of little sense who go by the name of ‘anti-alcoholists’”.

[¹²⁰] These paragraphs were added to the footnote in the 1909 edition.

[¹²¹] 1906 edition; removed in the 1909 edition.

38

[¹²²] 1906 edition, omitted in the 1909 edition.

[¹²³] 1906 edition. The 1909 edition instead reads: “it follows that it would also be”.

39

[¹²⁴] 1906 edition, omitted in the 1909 edition.

41

[¹²⁵] 1906 edition; the 1909 edition repeats “the quantity of gold”.

43

[¹²⁶] Cf. David Ricardo, *On the Principles of Political Economy and Taxation* (London: John Murray, 1817), Chapter VI, p. 160n.

[¹²⁷] 1906 edition; the 1909 edition reads “diversified”.

[128] Cf. C. Francis Bastable, *The Theory of International Trade, with Some of Its Applications to Economic Policy* (Dublin: Hodges, Giggis, & Co., 1887), p. 15. Bastable quoted Ricardo's *Works* (edited by J. R. McCulloch), p. 34.

45

[129] 1909 edition, replacing "hypothesis" in the 1906 edition.

47

[130] The number (47) of this section was missing from the 1909 edition, p. 509.

49

[131] 1906 edition, Italian *protetta*, mistranslated in the 1909 edition as *produite* ("produced").

50

[132] 1906 edition; the word was omitted in the 1909 edition.

52

[133] 1906 edition; omitted in the 1909 edition.

[134] 1909 edition; the 1906 text is "the".

[135] 1906 edition, omitted in the 1909 edition.

53

[136] 1906 edition. In the 1909 edition this is replaced by "it may be to this community's advantage to replace the monopoly prices by the competitive prices".

54

[137] 1906 edition; word omitted in the 1909 edition.

[138] 1906 edition; word omitted in the 1909 edition.

[139] 1906 edition, replaced by (VI, 57) in the 1909 edition.

[140] 1906 edition, word omitted in the 1909 edition.

55

[141] 1909 edition; replaced by "the" in the 1909 edition.

[142] 1906 edition; "a number of" in the 1909 edition.

56

[143] 1906 edition; the section was erroneously labelled 59 instead of 56 in the 1909 edition.

[¹⁴⁴] Added in the 1909 edition.

[¹⁴⁵] 1906 edition; the word “entirely” was omitted in the 1909 edition.

57

[¹⁴⁶] 1906 edition: *nonche al.* Replaced by “and also” in the 1909 edition.

[¹⁴⁷] 1906 edition: *disfatta*; “destroyed” (*détruit*) in the 1909 edition.

[¹⁴⁸] 1909 edition (*décadente*). The term in the 1906 edition is *scadente* (“inferior” or “declining”).

58

[¹⁴⁹] 1906 edition; removed in the 1909 edition.

59

[¹⁵⁰] Added in the 1909 edition.

61

[¹⁵¹] Mistranslated as “Production” in the 1909 edition.

[¹⁵²] 1906 edition; replaced by “And yet” in the 1909 edition.

[¹⁵³] This article has been reprinted in Giovanni Busino (ed.), Vilfredo Pareto, *Écrits politiques*, in *Oeuvres complètes*, Vol. 17, pp. 218–234, with a supplements on pp. 235–240.

62

[¹⁵⁴] The word used by Pareto in the 1906 edition was “production”—evidently a slip. However, instead of this being corrected in the 1909 edition it was replaced by “of which we have just spoken”.

[¹⁵⁵] J[ean] Bourdeau, *Socialistes et sociologues*, Paris: Félix Alcan, Éditeur, 1905.

[¹⁵⁶] 1906 edition, omitted from the 1909 edition. The quotation is from the *Sententiae* (Sentences, or Maxims) of Publilius Syrus (a Syrian slave brought to Italy, and educated and later freed by his master), and arranged alphabetically by the first letter. One English rendering is: “Worthless are words, when gold’s the talisman.” Unfortunately this particular Sentence is not included in Gulielmus Meyer (ed.), *Publilius Syri mimi Sententiae* (Leipzig: G. Teubner, 1880), nor in R. A. H. Bickford-Smith (ed.), *Publilius Syri, Sententiae* (London: C. F. Clay and Sons, 1895).

63

[¹⁵⁷] 1906 edition; replaced by “or” in the 1909 edition.

[¹⁵⁸] 1906 edition; removed from the 1909 edition.

64

[159] 1906 edition; replaced by “current” in the 1909 edition.

[160] 1906 edition; removed in the 1909 edition.

65

[161] Pareto must have meant to refer to §63, not §62.

66

[162] 1906 edition; removed in the 1909 edition.

[163] 1906 edition; omitted in the 1909 edition.

[164] 1906 edition; “produced” in the 1909 edition.

[165] 1906 edition; not included in the 1909 edition.

[166] 1909 edition. The 1906 edition reads: “subtle shrewdness and timely opportunism”.

[167] 1906 edition; replaced by “energy” in the 1909 edition.

[168] 1906 edition; replaced by “him” in the 1909 edition.

[169] 1906 edition; removed in the 1909 edition.

[170] 1906 edition; omitted in the 1909 edition.

67

[171] 1906 edition; erroneously replaced by 64 in the 1909 edition.

[172] The reference to §35 was added in the 1909 edition.

68

[173] Qualification added in the 1909 edition.

[174] 1909 edition, replacing “changes” in the 1906 edition.

71

[175] 1906 edition; omitted in the 1909 edition.

[176] 1906 edition; removed in the 1909 edition.

[177] 1906 edition; removed in the 1909 edition.

72

[178] 1906 edition; replaced by “If protectionist policy triumphs in England” in the 1909 edition.

74

[¹⁷⁹] 1909 edition; “the ages of men” in the 1906 edition.

75

[¹⁸⁰] 1906 edition; omitted in the 1909 edition.

82

[¹⁸¹] Literally, “mobile capital”, as “fixed capital” is “immobile capital”.

85

[¹⁸²] This was the pen name of Joseph Clément, physician (1819–1905), known for his work on business cycles. His major work was *Des crises commerciales et de leur retour périodique en France, en Angleterre, et aux États-Unis* (Of Commercial Crises and their Periodic Return in France, England, and the United States), 2nd edition, Paris: Guillaumin, 1862. 5th edition, 1889. It was reprinted by Burt Franklin, New York, 1969. An English translation of the section on pp. 445–490 on business cycles in the United States was published in Clément Juglar, *A Brief History of Panics and their Periodical Occurrence in the United States*, edited by DeCourcy W. Thom, in New York by G. P. Putnam’s Sons, 1916, pp. 25–134, with supplementary translations on pp. 135–144, and an Introduction on pp. 1–23.

[¹⁸³] 1906 edition; replaced by “of” in the 1909 edition.

[¹⁸⁴] Paragraphs added in the 1909 edition.

86

[¹⁸⁵] 1906 edition: *incasso*; translated as *encaisses* (“reserves”) in the 1909 edition.

[¹⁸⁶] 1906 edition; omitted in the 1909 edition.

87

[¹⁸⁷] Pierre des Essars, “La vitesse de la circulation de la monnaie,” *Journal de la société statistique de Paris*, tome 37 (April 1895), 143–151. Cf. Irving Fisher “A Practical Method of Estimating the Velocity of Circulation of Money,” *Journal of the Royal Statistical Society*, 72 (September 1909), 604–618. Fisher’s opening sentence stated: “Hitherto no actual statistics for money-velocity have been attempted, while for deposit-velocity Pierre des Essars alone has given figures.” See also Thomas M. Humphrey, “The Origins of Velocity Functions,” *Federal Reserve Bank of Richmond Economic Quarterly*, 79 (Fall 1993), 1–17, esp. pp. 13–14, and Robert M. Dimand, “Irving Fisher and the Quantity Theory of Money: The Last Phase,” *Journal of the History of Economic Thought*, 22 (No. 3, 2000), 329–348, esp. p. 336.

88

[¹⁸⁸] 1906 edition; replaced by “facts” in the 1909 edition.

APPENDIX TO THE ITALIAN (1906) EDITION

2

[¹] Here, the symbols X and Y denote the *names* of two goods, and x and y their *quantities*. Later in this appendix, Pareto uses the symbols X and Y to denote their quantities also. Pareto's typesetters evidently did not possess symbols for upper-case italic letters. Here, the italic X and Y will always denote the *quantities* of X and Y.

7

[²] The numbering of formula (7) was omitted in the original.

[³] In the original, the second of these equations was written as $\varphi_y = \Psi_x F'$.

8

[⁴] The original has “the elementary ophelimity of X is a function only of X, and that of Y is a function only Y.”

9

[⁵] The original has Y instead of Y' .

13

[⁶] Here, Pareto refers to these inequalities as “equations”.

20 and 32

[⁷] Literally, barter.

21

[⁸] Literally, barters.

22

[⁹] Literally, line of barters.

23

[¹⁰] Literally, barter.

[¹¹] Literally, barter path.

[¹²] This must be Gaetano Scorza in his article “Osservazioni su alcune teorie di economia pura” (“Observations on Some Theories of Pure Economics”), *Giornale degli Economisti* [2], 25 (December 1902), 503–516. On p. 503 he quotes a passage from Pareto’s *Cours d'économie politique*, Vol. I, §46 (which he erroneously refers to as §48), p. 90, which states: “The trader submits to the market prices without deliberately trying to modify them. These prices are effectively modified by his supply and demand, but unwittingly.”

This is what is called *free competition*.” Scorza apparently did not understand the distinction between this definition of free competition and that of monopoly.

[¹³] This must again be Gaetano Scorza, in his first article “Osservazioni sulla teoria del baratto secondo il prof. Walras” (“Observations on the Theory of Exchange according to Professor Walras”), *Giornale degli Economisti* [2], 24 (April 1902), 282–302. He remarked (p. 301) that because of their literary preoccupations “several economists” failed to recognize the “extreme importance of strict rigor in proofs”—a sentiment that Pareto surely shared. He then (p. 302) held up as an example of the highest rigor in mathematics the work of the then “recently deceased Professor Weierstrass of the University of Berlin” (Karl Weierstrass’s dates were 1815–1897). It seems that Scorza was unaware (and Pareto too) of the close relationship of one of Weierstrass’s theorems, proved in the late 1860s, to the “Pareto optimality” of competitive equilibrium. This is the theorem—later discovered to have already been proved by Bernard Bolzano as a lemma in 1817, and now called the Bolzano–Weierstrass theorem—which states that every bounded sequence of points in n -dimensional Euclidean space has a convergent subsequence, hence at least one limit point. For an elementary proof and an exposition of its relationship to Pareto optimality see Truman F. Bewley, *General Equilibrium, Overlapping Generations Models, and Optimal Growth Theory* (Cambridge, Mass.: Harvard University Press, 2007), pp. 19–24.

24

[¹⁴] Pareto actually wrote formula (42) as $x - x_0 + p(y - y_0)$, but presumably intended it to be written as shown.

29

[¹⁵] The original has v_2 , evidently in error.

30

[¹⁶] The original in system (A) appears to have two misprints: p_z for p_z , and p_2 also for p_z .

41

[¹⁷] The original has p_x, d_x .

46

[¹⁸] There is no article by Pareto in the November 1903 issue of the *Giornale*. The intended reference must be to his article in the November 1902 issue, Vol. 26, pp. 401–422, “Di un nuovo errore nello interpretare le teorie dell’economia matematica.” An English translation will be found in the *Giornale degli Economisti e Annali di Economia*, “On a New Error in the Interpretation of the Theories of Mathematical Economics,” 67 (December 2008), 515–544.

47

[¹⁹] In the last term, the symbol C_0 replaces the symbol C in the original.

APPENDIX TO THE FRENCH (1909) EDITION

3

[¹] The word “by” has been added to the original.

12

[²] This refers to Volterra’s article “L’economia matematica ed il nuovo manuale del prof. Pareto,” found on pp. 296–301 of the April 1906 issue of that journal. An English translation of that article, “Mathematical Economics and Professor Pareto’s New Manual,” will be found in *Preferences, Utility, and Demand* by J. S. Chipman, L. Hurwicz, M. K. Richter, and H. F. Sonnenschein, eds. (New York: Harcourt Brace Jovanovich, Inc., 1971), pp. 365–369.

[³] This refers to Pareto’s article “L’ofelimità nei cicli non chiusi” in the *Giornale degli Economisti* [2], 33 (July 1906), 15–30. An English translation of this article, “Ophelimity in Nonclosed Cycles,” was included in Chipman et al. *op cit.*, pp. 370–385.

15

[⁴] The original has z, y, \dots, s, t .

18

[⁵] The term dy was omitted by Pareto.

19

[⁶] This refers once again to Pareto’s article responding to Volterra’s criticisms; see annotation 3 above.

24

[⁷] The original has “ $m + 1$ in number.”

35

[⁸] Evidently this refers to Gaetano Scorza, whose polemic with Pareto is discussed in J. S. Chipman, “The Paretian Heritage” (*Revue européenne des sciences sociales et Cahiers Vilfredo Pareto*, Tome XIV, No. 37, 1976, 65–173) and Michael McLure, “The Pareto-Scorza Polemic on Collective Economic Welfare” (*Australian Economic Papers*, 39, Issue 3, September 2000, 347–371). The particular alleged error in Scorza singled out by Pareto has not been identified, however. Pareto did not help his case with his terminology “maximum ophelimity” which did not specify what was being maximized.

[⁹] This evidently refers to the fourth of the five equations in (41), proceeding left to right by rows.

37, 52 and 135

[¹⁰] The word in French is *monnaie*.

42 and 43

[¹¹] The original has p_x here instead of p_z .

45

[¹²] The original has “and” here rather than “but”.

[¹³] This inequality was printed as

$$\delta_z \frac{dy}{dx} > 0.$$

[¹⁴] Presumably Pareto meant “ $\frac{dy}{dx}$ decreases in absolute value as x increases”.

[¹⁵] In the original this is X, which Pareto usually interprets as the name of the commodity whose amount is x .

48

[¹⁶] The original has “propositions” rather than “proportions”.

[¹⁷] The word “inequalities” replaces the word “equations” in the original.

49

[¹⁸] In the original, the plus sign in the parenthetic expression is a minus sign.

50

[¹⁹] This presumably refers to the 1906 appendix.

52

[²⁰] This appears to be the first and only use of the term “elementary index”.

[²¹] In the original there was a misprint p_y for p_z in this last term of (72).

[²²] This was part 3 of Pareto’s five-part article “Considerazione sui principi fondamentali dell’economia politica pura,” *Giornale degli Economisti*, 5 (August 1892), 119–157. An English translation is contained in Roberto Marchionatti and Fiorenzo Mornati, *Considerations on the Fundamental Principles of Pure Political Economy* by Vilfredo Pareto (London and New York: Routledge, 2007), pp. 45–74.

56

[²³] The first of these items is Pareto’s article “La teoria dei prezzi dei Signori Auspitz e Lieben e le osservazioni del Professore Walras,” Vol. 4 (March 1892), 201–239. An English translation will be found in the *Giornale degli Economisti e Annali di Economia*, 67 (December 2008), “The Theory of Prices of Messrs. Auspitz and Lieben and Professor Walras’s Observations,” 321–354. The second article cited by Pareto here is his “Teoria

matematica del commercio internazionale,” Vol. 10 (April 1895), 476–498. An English translation, “Mathematical Theory of International Trade,” will be found in the *Giornale degli Economisti e Annali di Economia*, 67 (December 2008), 405–424.

58

[²⁴] Literally, “false”.

59

[²⁵] This symbol had been printed as φ_x .

60

[²⁶] The original has p_{xz} .

69

[²⁷] This is Edgeworth’s three-part article “La teoria pura del monopolio,” *Giornale degli Economisti* [2], 15 (July, October, December 1897), 13–31, 307–320, 405–414, translated into Italian from the English original, which was lost. A subsequent English translation of the Italian, “The Pure Theory of Monopoly,” was included in Vol. I of Edgeworth’s *Papers Relating to Political Economy*, London: Macmillan and Co., Limited, 1925, pp. 111–142.

71

[²⁸] In the original, this second term appears as $\partial f_2 / \partial p_y$.

76 bis

[²⁹] In the text, the inequality (67) is referred to, but evidently Pareto meant to refer to equation (87).

77

[³⁰] The original French reads *on* instead of *ou*.

[³¹] The original reads b_b instead of p_b .

84 and 88

[³²] The word used here by Pareto was *numéraire*. However it appears to be used to denote money rather than as an arbitrary representative good in the Walrasian sense, in the same way as *monnaie* elsewhere in this appendix.

89

[³³] The French for “constraints” is *liaisons*.

[³⁴] The words “all decrease” are obviously incorrect.

92 bis

[³⁵] This is a second section 92, renamed “92 bis” here.

95

[³⁶] The original has *B'* here instead of *B''*, evidently in error.

98 bis

[³⁷] This is a second section 98 in the original, renamed “98 bis” here.

108

[³⁸] This author has not been definitely identified. This so-called law had been discussed by Pareto's friend Maffeo Pantaleoni (*Principii di economia pura*, Florence, Barbèra, 1899, 1894; *Pure Economics*, London, Macmillan, 1898, Ch. IV, §5, pp. 81–5), but not in the rigid form quoted here by Pareto.

[³⁹] “The case is still before the court.”

[⁴⁰] Cf. *The Fables of Aesop*, ed. Joseph Jacobs (New York: The Macmillan Company, 1834), pp. 154–5, or *Fables* by Jean de la Fontaine (New York: H. M. Caldwell Company, 1841), p. 148. The fox, having lost its tail in a trap, called a meeting of all the foxes, advising them to do away with their useless tails. Given his motive, the foxes were not persuaded: “Distrust interested advice.”

III

[⁴¹] The reference to §91 should be to §92.

115 and 119

[⁴²] There is no article by Pareto in the November 1903 issue of the *Giornale degli Economisti*. Possibly he meant to refer to his article “Il massimo di utilità dato dalla libera concorrenza,” *Giornale degli Economisti* [2], 9 (July 1894), 48–66. An English translation, “The Maximum of Utility given by Free Competition,” is contained in the *Giornale degli Economisti e Annali di Economia*, 67 (December 2008), 387–403.

[⁴³] Simpler ways have been shown by contemporary mathematicians and economists. Cf., e.g., Truman F. Bewley, *General Equilibrium, Overlapping Generations Models, and Optimal Growth Theory* (Cambridge: Harvard University Press, 2007), Chapter 3, pp. 19–24, based on the mathematical theorem due to Bernard Bolzano (1781–1848) and Karl Weierstrass (1815–1897).

115 bis

[⁴⁴] This is a second section 115, renamed “115 bis” here.

I39

[45] The original has $\partial R_i^0 / \partial x_i^0 = 0$, just as in the previous equality, whereas the partial derivative should be with respect to y_i .

I40

[46] It is not clear what this refers to. There is no formula (168) in this appendix. Note also that formulas (110) and (115) are missing from this appendix.

Annotations from the 2006 Italian Edition*

NOTE

[a] This section [*Avvertenze* in the Italian original] is missing from the French edition, 1909.

PREFACE

[a] The important *Preface* (*Manuale*, 1906, pp. V–XII) is missing from the French edition, 1909. A great deal has been written on the subject of the Preface relating to the dualism between free exchange and protectionism, but no one has synthesized Pareto's thoughts as effectively as Pareto himself in his *Trattato di sociologia*, §2208. See also E.N. 57.

CHAPTER I

I

[a] See E.N. 1

7

[a] See E.N. 2.

17

[a] For an opinion by Croce on Pareto, see E.N. 4.

CHAPTER II

17

[a] The *Manuale* (1906, p. 45) shows MB. The reprints of the *Manuale* (1909 and 1919, p. 45) and the *Manuel* (p. 51) correctly show MB'.

38

[a] In the *Manuale* (1906, p. 63) “per esempio, coi principii della classe di cui è tipo il principio del Kant”, which was translated in French “par exemple avec des principes dont

* These notes are to be read, and will be referred to, as in the following example: [§14, b] = see note [b] of paragraph 14. Where unspecified, the paragraph is implied to be in the same chapter.

celui de Kant nous fournit le type". It would appear that the French translator suppressed the word "classe" because of its uncertain meaning. It may be useful to mention that in Pareto's time, "class" was a term used in abstract logic to mean *set* or *category*. Readers will find confirmation of this in §42.

39

[a] The *Manuale* (1906, p. 64) includes a footnote with the original French paragraph. For obvious reasons, this footnote is missing from the French edition.

55

[a] In the *Manuale* (1906 and reprints, p. 80), this footnote is not indicated by a reference mark in the text. The *Manuel*, 1909, p. 85, positions it at this point.

78

[a] The classification of capital follows two general traditions in the ambit of neoclassical economics: the Marshallian classification commonly adopted in the English speaking world, which commenced with the *Economics of Industry* (1879) and was developed by Keynes with the distinction between 'fixed capital', 'working capital', 'liquid capital' (and 'loan capital' for an open economic system) employed in the *Treatise on Money* (1930) and, to a lesser extent, the *General Theory* (1936); and the Walrasian classification more commonly adopted in continental Europe, which was initiated in the *Éléments* (1874) and utilized the distinction between *capitaux fonciers*, *capitaux personnels* and *capitaux mobiliers*. Pareto followed Walras on this matter, albeit more closely in the *Cours* (1896–97) than the *Manual*. To maintain consistency with William Jaffe's translation of the *Éléments* (1954), the Italian phrase *capitali mobiliari* (the French *capitaux mobiliers*) can be translated as 'capital goods proper', indicating capital goods that are neither fixed natural endowment nor human capital. The types of goods that Pareto considered 'capital goods proper' are examined in Chapter VIII, §8. In a similar vein, *capitali fondiari* is translated as 'landed capital'. Of course, what is important is to know which is the capital included in each of the three classes. With regard the 'capital goods proper', see Chap. VIII, §8.

81

[a] In the French edition, 1909, p. 99, the footnote ends with the following specification: "Trad. Courcelle-Seneuil, p. 5".

86

[a] The *Manuale*, 1906 p.100 shows "Sand" instead of "George Sand".

91

[a] In the *Manuel* 1909, the episodes regarding Brest, Armentières, Marseille, etc. are not reported. Pareto had dwelled upon the various episodes recalled on this page (*Manuale*,

1906, p. 107) on a number of occasions in other works, including the *Trattato di sociologia*. As for the specific episodes recalled in the *Manuale* (Brest, etc.), Pareto had dwelled upon them in more detail in *L'ordre dans les grèves*, in V. P. *Oeuvres*, vol. VI, pp. 240–42.

101

[a] Pareto is referring to Theodore Roosevelt (1858–1919), 26th president of the United States.

[b] Pareto quotes the French translation by de Rousiers, p. 130.

103

[a] The French edition, 1909, p. 129, has: “that we shall call simply the *élite*”, instead of “that we shall call simply an *aristocracy*, or *élite*”.

[b] If one compares the theory of the *élites* in J.S. Mill and in Pareto, one is led to conclude that Pareto only added some important considerations on the *circulation* of the élites. Allow us a further remark. It is well known that Pareto moved from a liberal-radical position in youth to a liberal-conservative position, but his earlier position cannot be summarized by the aphorism “the greatest good for the greatest number of people”, but by “the greatest good for the greatest number of people and the most deserving”—even in this Pareto always remained Millian in his approach.

107 bis

[a] An extensive literature has developed with reference to Sorel and Pareto and their respective assessments of the political climate prevailing in Italy and in Europe before the establishment of authoritarian political systems. Sorel's historiography centred on revolutionary syndicalism and his concept of “violence”, with his book on violence translated into Italian in 1909 (with an introduction by Croce). Pareto's historiography, which drew on the ideas of Machiavelli and Marx, pointed to the great importance of interests in political system, with elite protagonists, and the mix of political and religious passions, featuring in Pareto's sociology.

While it is not appropriate here to deal with this subject in depth, it should be noted that the subject has been treated extensively in famous texts by R. Aron, I. Berlin and N. Bobbio. More recently, two books of note discuss Pareto's position on politics in general and early fascism: P. Bonetti, *Il pensiero politico di Pareto*, Bari, Laterza, 1994, G. Barbieri, *Pareto e il fascismo*, Milano, Angeli, 2007.

123

[a] For a consideration on the content of the first two chapters of the *Manual* and the corresponding topics of the *Cours*, see E.N.5.

CHAPTER III

4

[a] See E.N. 6.

8

[a] It has been suggested that Pareto confuses ‘statics’ and ‘dynamics’. For comment on this topic, see E.N. 7.

30

[a] The original (*Manuale*, 1906 and reprints, p. 154, as well as in the Authors’ names index, *Indice degli autori*, p. 574) wrongly shows “Fischer”.

[b] Both here, in the *Manuale*, 1906, p. 154, and in the *Manuel*, 1909, p. 157, note 1, Pareto should have pointed out that his encyclopedia entry *Economia matematica*, which had appeared in German in 1902, had been translated into Italian by G. Sensini in 1906. This encyclopedia entry by Pareto is important for a number of reasons. See E.N. 8.

35

[a] It appears from the Pareto–Pantaleoni *Corrispondence* that these questions present themselves to Pareto as soon as he started thinking about the “new school of political economy”, even before reading I. Fisher.

From a mathematical viewpoint, the secondary literature concerning this paragraph—especially when dealing with the problem of quantifying ophelimity—is perhaps more unsettled areas in the historical literature on Pareto. For instance, the following two essays do not always reach the same conclusions: 1) G. Ricci, *Commento alla memoria di G.B. Antonelli dell’anno 1886: “Sulla teoria matematica della Economia politica”* [Memorial comment on G.B. Antonelli’s 1889 booklet “On the mathematical theory of political economy”], in *Giornale degli Economisti*, n.s. 10, 1951, pp. 264–297 and 345–385; 2) A. Montesano, *The Paretian theory of ophelimity in closed and open cycles*, in *History of Economic Ideas*, XIV/2006/3, pp. 77–100.

Of course, the great merits of N. Georgescu-Roegen’s Paretian essays from the 1930s also come readily to mind, but even he reveals a certain lack of understanding and, at times, even a certain lack of fairness towards Pareto (see E.N. 25).

[b] Pareto refers to §8 of the *Appendix* of the Italian *Manuale*. In the *Manuel*, the cross-reference is simply to the *Appendix*, without any indication about the paragraph. In our case, the same topic can be found in §10 of the *Appendix* to the *Manuel*. From now on, we shall leave this kind of comparison to the mathematical reader, who, in the present edition, has the mathematical appendices from the 1906 Italian edition and the 1909 French edition at his disposal.

But by way of general observation, it is useful to note that Pareto simply made the case that the conditions required for the utility (ophelimity) function to be additively separable (as in Walras, Jevons, Marshall) are also the conditions that facilitate the recognition of the mathematical existence of cardinal utility. Following a remark by Volterra (1906) on the integrability conditions, Pareto went on to undertake analysis of the general case, where the utility function is not additively separable and the integrability conditions are not met (the line integral—of the vector field generated by the marginal rates of substitution—taken on a closed curve is not zero).

Finally, a terminological point. Since, by definition, a cycle is a closed curve, it seems to us the Paretian distinction between close and open cycles is somewhat misleading. It would have been preferable to write that the line integral of *closed* curve may be zero or different from zero.

38

[a] Criticisms of various kinds have been raised against Pareto's habit of establishing whether a problem has a determinate solution by comparing the number of unknowns with the number of independent equations. Readers will find an implicit response to such criticism in annotations 82 and 200 to the 1909 *Appendix*.

40

[a] On the reason for Pareto's strange terminology (type I and type II), see E.N. 9.

48

[a] It is evident, from Pareto's correspondence published in G. Busino, *Vilfredo Pareto e l'industria del ferro nel Valdarno* (Milan, Banca Commerciale Italiana, 1977), that the mind that lay behind these abstract hypotheses concerning exchange and pricing had also spent many years reflecting on the practice of price setting within the world of business.

52

[a] The individual postulated by Pareto, therefore, feels the psychological state of indifference.

56

[a] Of course, this *postulate of completeness* (given any basket of goods, the individual is able to recognize all the other infinite number of baskets that in the space of goods are indifferent to it, those he prefers and those he does not prefer) can be conveniently relaxed.

57

[a] For all commodities, obviously, that with regard to a given need, a certain level of opulence can be guaranteed by increasing the quantity of one commodity and decreasing the quantity of the other, and vice versa.

That of registering a subject's preference by means of an imaginary interview is not the only approach noted by Pareto. For the alternative consisting in registering the subject's demand corresponding to imaginary variations of the relative prices of goods, see the mentioned essays by G. Ricci (1951) and A. Montesano (2006).

58

[a] In the French edition, 1909, p.171, Pareto refers to §3 of the *Appendix*.

59

[a] From 1870 onwards Pareto “spoke a Tuscan Italian” (Manon Michels Einaudi) and he knew some works of Machiavelli and Dante (“blissful mountain”, Dante, *Inferno*, I, 77) by heart.

60

[a] On paths in Pareto, see E.N. 10.

62

[a] On terminal points, see *Editors' Introductory Note*, point 5, and E.N. 11.

65

[a] This is an aspect that Pareto considered fundamental from the moment he started thinking about the “new school of political economy”, after meeting Pantaleoni. See E.N. 24.

75

[a] On the line of complete transformations, see E.N. 12.

82

[a] In the French edition, 1909, p.179, §46 mistakenly becomes §16.

87

[a] In §§87, 99 and 106 of Chapter III of the *Manuale*, Pareto has clearly explained that the exchange line for consumers as a collective group is the aggregated (or average) exchange line; in other words, it indicates the sum (or the average) of the quantities of goods chosen by single individuals with respect to each price. But, in addition, Pareto also recognized that political rulers need to know the preferences of very similar but non-identical individuals, albeit in a very approximate manner. Indeed, some authors, such as Zawadzki (quoted in E.N. 15 bis, p. 157) and Fanno (*Beni succedanei*, 1926, p. 385) have cited Pareto when proposing the idea of collective preferences, or, if one likes, preferences for the representative, or average, individual. Now, even though Pareto comes insistently back to this average individual, in the above-mentioned paragraphs he speaks of it somewhat incidentally (“if one wishes” [*se si vuole*], §99) and with the premise (§87) that it is a “fictitious” consumer, that is—we add—imaginary, handy to use but non-existent. According to this authentic interpretation, it seems to us that the arithmetical operations proposed by Zawadzki in order to construct the preferences of the average individual go beyond Pareto’s discussion of the issue. At any rate, while an individual collective—on which we shall come back in E.N. 24—with its own preferences has a place in political, and therefore sociological, analysis, when dealing with economic theory it is necessary to recall the Debreu-Sonnenschein-Mantel theorem, which suggests that it is impossible to aggregate the full set of consumers into one single collective consumer who

is endowed with preferences that are represented by an ordinal utility function (at least not without resorting to very restrictive conditions, such as the Antonelli-Nataf-Gorman conditions).

88

[a] Pareto will deal with it in §§147–150 of this chapter.

94

[a] It has already been observed (E.N. 11) that Pareto includes the *terminal points* among the obstacles of the first kind.

97

[a] For a comment on Figure 12, see E.N. 13.

98

[a] What Pareto writes on this matter can also be illustrated graphically. See E.N. 16.

100

[a] The straight lines l, l', \dots drawn from the origin of the axes obviously regard alternative constant prices. It should be noted that when the indifference lines for the producer have the shape shown in figures 9 and 13, production takes place with increasing average costs, whereas if these lines have the shape shown in figure 14 (only with regard to their characteristic segment, namely the segment that goes up to and not beyond point T), production takes place with decreasing average costs.

Readers will see that Pareto associates marginal equilibrium and incomplete competition with the case of increasing average costs, while he associates complete competition and non-marginal equilibrium with the case of decreasing average costs. On this matter, see the table in E.N. 17. (The dashed line starting from point 0 has been inserted to find point T.)

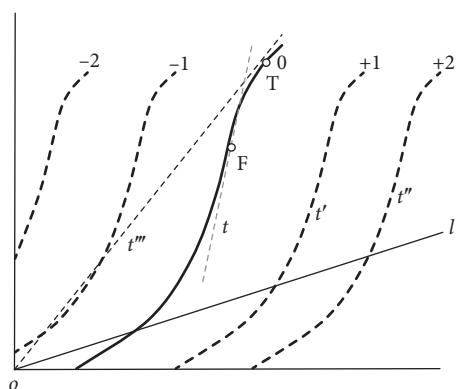


FIG 14

103

[a] Readers may place the letter F at the point of inflection on Pareto's production function (the line of complete transformations) to mark the shift from increasing to decreasing marginal productivity. On that premise, point F corresponds to the minimum marginal cost, whereas point T, which Pareto introduced to the original diagram, corresponds to the minimum *average total*

cost. When marginal productivity is always decreasing, the line of complete transformations does not present such an inflection and it has the shape shown in Fig. *a* in E.N. 12.

104

[a] Pareto's conclusion may be restated as follows: the production field delimited by point T, where unit average cost starts to be increasing, may be so extensive that it practically excludes production with increasing unitary cost.

It is perplexing that Pareto's realistic consideration of production was never referred to by Piero Sraffa in his first famous essay "Sulle relazioni fra costo e quantità prodotta", *Annali di Economia* II, 1, Bocconi editrice, 1925. Perhaps even more perplexing is the fact that neither Sraffa nor the extensive literature on Sraffa notes that Pareto's notion of complete competition is never a marginal equilibrium: when production is associated with persistently decreasing average total costs, equilibrium is always determined at a terminal point.

On the limits within which the average cost (inclusive of general expenses) can be decreasing, see Ch. VI, §11.

106

[a] Figure 15 shows a novelty about which see E.N. 18.

107

[a] Pareto did not realize he wrote "producers" instead of "consumers". Nor did the French translator, 1909, p. 188.

110

[a] Pareto would have avoided the interpretative misunderstandings that arose, even among the most astute of mathematical economists, if he had clearly specified, in §110, that two parties to exchange must be treated as isolated individuals in the case where one trader is a monopolist who offers product A in exchange for product B and the other trader is a monopolist who offers product B in exchange for product A. If this restriction is not explicitly stated, the former must be considered as representative of a variety of individuals competing to offer A for B, the latter must be considered as representative of a variety of individuals competing to offer of B for A.

116

[a] The *Manuale* indicates 'quality', but this was corrected to read 'quantity' (amount) in the French edition, p. 191.

[b] Figure 16 clearly shows what is called the *Edgeworth Box* diagram, which others argue should be called the *Pareto Box* diagram. On this matter, see E.N. 19.

118

[^a] By “insoluble” Pareto here means “contradictory”. In other words, the predetermined path introduces two conditions that are in conflict with each other. There was a polite debate between Pareto and Edgeworth over what is contradictory and what is indeterminate. See E.N. 20.

119

[^a] The last case considered in §118, namely when the path is predetermined but one of the two parties to exchange imposes his conditions on the other and compels the other to follow up to point k , must, therefore, be kept apart from the case in which one party to exchange may impose the path, but not the segment to cover. Readers will see that §128, which Pareto refers to when considering this latter case, anticipates what will subsequently be referred to as Stackelberg duopoly.

120

[^a] Also in the French edition text, 1909, p. 193, “for the second individual” Pareto indicates the coordinates $\omega\alpha$ and $\omega\beta$, that, in figure 17 (wrongly indicated as figure 19 in the *Manuel*), are instead marked with ωp and ωq .

[^b] Here Pareto is calling (1906 and repr., p. 189, but also 1909, p. 193) “contract line” the line he had just labelled “exchange line”. In a beautiful mathematical demonstration, (see E.N. 19) Amoroso systematically inverts these two terms.

121

[^a] This paragraph displeased Wicksell, who rejected the three graphs of figure 18 after declaring them to be a nonsense. It is possible that the difficulties that Wicksell encountered are attributable to: Pareto’s inclusion of terminal points in the most general definition of equilibrium; and his use of these graphs to distinguish between stable and unstable equilibrium.

122

[^a] If readers encounter some difficulty, they are advised to read §122 again after reading §§215 and 216 dealing with stable and unstable equilibrium: this is the same advice that Pareto gave to his own readers in his correspondence (see de Pietri-Tonelli, *Corrispondenza*, 1961, p. 194).

[^b] In a letter to A. de Pietri-Tonelli (*Corrispondenza*, 1961, p. 119), Pareto invites him to replace “A” with “B” in the French and in the Italian texts. After making this suggestion, we have decided to leave the original text as it was.

[^c] On the issues of dynamics that Pareto described in §122, and again in §138 and elsewhere, see E.N. 21.

125

[^a] The *Manuel*, 1909, p. 196, wrongly shows an *l* instead of the *f*.

I28

[a] It would have been advisable to include a cross-reference to §119 at the beginning of this paragraph.

I31

[a] In §134 Pareto explicitly states that equilibrium at point *s* in figure 20 does not indicate a maximum of *ophelimity*.

I33

[a] Pareto's comment (*Manuale*, 1906 and reprints, p. 195) to figure 21 requires a correction. The indifference line for tastes must be indicated by *tt'* rather than by *t'*. In the French edition, 1909, p.199, not all the symbols in figure 21 correspond to the symbols of the Italian version. Furthermore, possibly because of the oversight we have just pointed out, the French version (*ib.*) ends with the following nonsensical statement: "Let *acb* be a transformation line, and *c* the point at which it is tangent to an indifference line of tastes, *t* is the point at which equilibrium takes place".

I35

[a] It is line *cd* from figure 18, rather than *hk*, that typically indicates the maxima of ophelimity for the consumer or consumers. After what has already been specified under §107, it is perhaps unnecessary to point out that figure 22 refers to the case of increasing costs (incomplete competition), the only one for which the line of maximum profit exists.

I38

[a] It should be recalled that the producer transforms A (e.g., olives) into B (e.g., oil) and that, for this reason, if A is plotted on the abscissa and B on the ordinate, the increase in the slope of the rectilinear path starting from *m* is synonymous with a reduction in the price of B (i.e., of oil). It should also be noted that in the excerpt from §138 that is the subject of comment here, Pareto's reasons by contradiction and he assumes that a monopolist (type II) acts as if he were in competition (type I).

I41

[a] Both graphs in figure 23 refer to the case of decreasing average costs (complete competition). They differ only with regard to the different shape of the exchange line for the consumer. Readers will be able to find that in the two graphs in question, Pareto uses the two different segments of the exchange line as they turn out, for instance, from figures 17 and 20.

I47

[a] Pareto implies that among the various indifference lines for the producer *s*, *s'*, . . . , the line *ss* of figure 24—that is, the unbroken line—is the zero index line (line of complete

transformations). Obviously, the figure refers to the case of increasing average costs (incomplete competition), the only one for which the maximum profit line *hk* exists. Furthermore, in the figure an arrow has been added that only appears in the *Manuel*, 1909, p. 205.

148

[a] This lapidary conclusion by Pareto, which reminds us of economists that are so different, such as, for instance, A. Smith, Allyn Young, Georgescu-Roegen, is in lieu of a very important proposition: *ceteris paribus*, the entry of new producers (and the level of investment that is associated to it) depends on the size of the market, or, if one likes, on the level of the actual demand (on the position of the exchange line, in Pareto's terminology).

149

[a] This "case already dealt with (§141)" is the case in which there is no line of maximum profit. So, Pareto states that *i*) in the case of production at increasing average costs, in which the line of maximum profit exists; *ii*) when, on the other hand, the exchange line, due to the shifting of the curves, no longer intersects the line of maximum profit, equilibrium takes place as per the case in which that line does not exist (§141).

151

[a] The French translation, 1909, pp. 206–7, is faulty: the sentence where equilibrium "will be the point of tangency between the exchange line and the line of maximum profit" becomes, in the *Manuel* "will be the point of tangency between the exchanges and the lines of maximum profits". But this faulty translation is not the cause of a criticism by Triffin, for which see E.N. 22.

168

[a] In the French edition, 1909, p. 211, "several authors" of the Italian original becomes "some authors": it is a way, this, to tone down, or to somewhat distance himself from, a well-known controversy between Pareto and the worthy mathematician Gaetano Scorza. Readers who know about this controversy will have no difficulty in recognizing that all the graphs appearing in the following §169 constitute an answer by Pareto to Scorza. That in the French edition Pareto makes a point of making his thoughts clear by renouncing his highly argumentative tones with Scorza, we can especially infer from the fact that the following vitriolic passage, which can be read in the Italian *Appendix* §23, does not reappear in the French edition:

"In the case in which the exchange path is given by a constant price, when we differentiate in passing from one path to another, the price must be made to vary. This observation, too, is so elementary that it would be absolutely superfluous had it not been forgotten by a certain author who even imagined that in the *Cours* I made a mistake in differentiating along the exchange path, assuming *p* to be constant; and in order to lend credence to this and similar observations and discoveries, he dug up Weierstrass who, poor soul, has really not the slightest connection with all this business, and who never committed such a blunder in his whole life."

[b] For the whole of section (β), the French edition, p. 212, speaks of “kilograms” instead of “grams”. This forgivable mix up does not occur in the other sections (α) (β) (γ).

I69

[a] These cases will be briefly considered by Pareto with regard to collectivism. We shall see in Chapter VI, where Pareto (§7) introduces a subject who is both producer and consumer at the same time, a sort of Robinson Crusoe who acts as an introduction to collectivism. We shall see this issue again in Ch. VI, §§47–8, where collectivism is considered by Pareto applying pure economics—thanks to the twofold possibility of applying variable prices and discriminating prices according to the classes of individuals, collectivism may achieve a *first best* collective welfare that is unattainable in capitalist economies where constant prices prevail there.

I70

[a] That is, prices that are constant but changeable from one day to the next, as in the preceding examples, or changeable from early to late morning, as in the fish example: we are still in case (δ).

I71

[a] In §171 of the French edition, 1909, p. 215, there are two footnotes 1, whereas at the bottom of the page there is only one. On the next page, p. 216, at the bottom, there is a footnote 1 without any reference in the text. The latter footnote, which has obviously been wrongly positioned and wrongly numbered, is very important. In it, Pareto states: “Note 2 of §928 of the *Cours* is based on erroneous considerations and must be completely modified”.

As far as we know, the mistakes made by Pareto were first noticed by Wicksell in 1899, while reviewing Vol. II of the *Cours* (today, in K. Wicksell, *Selected papers on economic theory*, ed. by E. Lindhal, London, George Allen, 1958). When the present author (A.Z.) first read the *Manuale*, he verified together with his friend Piero Zattoni that Pareto was actually guilty of some mathematical oversights.

On the other hand, we do not share the criticism levelled at Pareto’s §171 by Kirman. On this matter, see E.N. 7.

I74

[a] In the *Manuel*, 1909, p. 217, the term “*total*” was not translated.

I76

[a] Here Pareto refers to §§75 and 76.

I77

[a] Since the numerator of the ratio includes “all the expenditures”, Pareto’s cost of production is obviously an average cost.

182

[a] In the Italian edition, 1906, p. 216, Figure 12 is replicated.

[b] The *Manuale* (1906 and reprints, p. 216) shows *c* instead of *c''*. The *Manuel*, 1909, p.220, corrects the mistake.

187

[a] In the Italian edition, 1906, p. 217, Figure 7 is replicated.

[b] The *Manuale* (1906 and reprints, p. 218) wrongly shows *va* instead of *ra*. The oversight has been rectified in the *Manuel*, 1909, p. 223.

194

[a] In the Italian edition, 1906, p. 219, Figure 15 is replicated.

[b] In §106 (1906 and reprints, p.184; 1909, p. 188), where fig. 15 had already been shown for the first time, it was clearly specified that the line *hk* is the line of complete transformations. From logical reconstruction, the cost that is equal to the price is inclusive of fixed costs.

195

[a] The French edition, 1909, p. 224, mistakenly has fig. 10 instead of fig. 18.

208

[a] In the Italian edition, 1906, p. 226, the theorem is in italics and ends with a cross-reference to “(Appendix 26, 28)”.

210

[a] See E.N. 23 on the marginal rate of substitution and the theory of choice.

218

[a] Note 1732² of Pareto's *Trattato di sociologia*, Vol. III, p. 61, contains a criticism to Walras of the kind Pareto had levelled at Walras during his lifetime:

“1732² *Manuale*, III, 217, 218, p. 229–230. A number of economists have made the mistake of assuming that the theories of Pure Economics could directly rule over the concrete phenomenon, and Walras thought he could, in that way, reform society. On this matter, see P. Boven, *Les applications mathématiques à l'Économie politique*.”

219

[a] The sentence quoted by Pareto (*Manuel*, 1909, p.235) is tangled up, but it can be inferred, also from what follows, that the French adjective *qualificatif* (“le degré de limitation qualificative”) is a misprint: qualificative instead of *quantitative*.

221

[a] In the Italian edition, 1906, p. 232, there is “our Ferrara”. The omission of “our” occurs, in the *Manuel*, every time the expression “our Ferrara” is used.

222

[a] In the *Manuale*, 1906, Pareto quotes this passage from p. 35 (but it is p. 33) of the Italian translation of Cairnes’ book. It is *Alcuni Principi Fondamentali di Economia Politica*, translated from English by Sidney Sonnino and Carlo Fontanelli, Florence, Tipografia di G. Barbèra, 1877. It is the issue N. 2 of the *Biblioteca di Scienze Sociali* published by Leopoldo Franchetti and Sidney Sonnino. The *Manuel*, 1909, p.240, shows a French translation of this passage without indicating the page. See also 223^[a].

223

[a] As per the preceding annotation, in the *Manuale*, 1906, Pareto quotes from (p. 70) of ‘the Italian edition’ of William Thomas Thornton, *Del Lavoro delle sue pretese e dei suoi diritti del suo presente e del suo futuro possibile*, translated from the second English edition by Sidney Sonnino and Carlo Fontanelli, Florence, Tipografia di G. Barbèra, 1875. It is the issue N. 1 of the *Biblioteca di Scienze Sociali* published by Leopoldo Franchetti and Sidney Sonnino.

This second edition of Thornton’s book, the economist who brought about the well-known “recantation” by J.S. Mill, is the same edition referenced in *The Economics of Industry* by Alfred and Mary Paley Marshall (1879¹) and in the *Mathematical Psychics* by Edgeworth (1881). Pareto read Thornton in Florence (the young translators Sonnino and Fontanelli were his friends) and, in the successive course of his work, made him the object of several considerations.

224

[a] The so-called input–output tables associated with W. Leontief allow us today to readily recognize the interdependencies that Pareto is hinting at in this paragraph.

225

[a] It is perhaps worthwhile noting here that the definitive solution to the problems of the labour-value theory is due to Piero Sraffa (1960), who, more than anyone else, worked to clarify its presuppositions for the purpose of infusing new life into the classical theory of the prices of production.

226

[a] 1909 version. This footnote indicates, in the Italian edition, p. 235, “Systèmes, II, cap. XIII.” This reference is placed, in the French edition, in the successive footnote 18, which, in the Italian edition, contains the references placed in footnote 17 in the French edition.

The recently published book to which Pareto refers is G. Valenti, *Principii di scienza economica. Introduzione allo studio dell'economia politica, i principii dell'economia individuale, i principii dell'economia sociale*, Firenze, G. Barbera, 1906.

CHAPTER IV

4

[^a] Pareto's solution to this problem did not immediately appear evident to him; this is clear from the Vol. I of the *Pareto-Pantaleoni* correspondence.

5

[^a] The italics have been suppressed in the French edition, 1909, p.250.

7

[^a] Pareto is hinting at the increased difficulties that must be confronted when consumers account for the "order of consumption" (to use Pareto's unfortunate expression). This problem, and the related issue of the order in which individuals acquire commodities, is dealt with exhaustively by Pareto in the 1909 *Appendix* to the *Manuel*, which is critically discussed in the notes to the French appendix (see notes 2, 9 and 20).

10

[^a] The two kinds of dependence (α) and (β) are *sub-classes* of the first kind of dependence.

27

[^a] In order to correctly represent what Pareto wrote in §27, all dashed lines (indifference lines t and the straight lines from a) have to be lowered until the dashed lines from a coincide with the solid lines.

32

[^a] In moving from *cardinalism* to *ordinalism* (theory of choice), on several occasions Pareto wrote that while Edgeworth starts from marginal utilities in order to arrive at the indifference curves, the theory of choice assumes an experiment—two baskets of n goods are put in front of a consumer, who is then asked whether he finds them indifferent or prefers one to the other—that allows one to start from the map of curves (varieties) of indifference. In the neoclassical cardinalism, utility measures the intensity of the need for goods; in other words, it is a psychological entity. In this paragraph, Pareto also introduces a cardinal function of utility in terms of preference, that is, without resorting to the logic chain: sensation, emotion, need, good, utility. The individual must be able to compare not only the combinations of goods (whether he prefers combination I to combination II), but also their variations (whether he prefers moving from I to II, rather than from II to III). This hypothesis has brought about lively discussions. We limit ourselves to recall G. Zaccherini, *Determinatezza della funzione dell'ofelimità*, G.d.E., March–April 1948, who tackles the topic with mathematical competence, but concludes that Pareto made a *fatal error of logic* ("salto logico fatale"), which, however, is due to his misinterpretation of what Pareto wrote.

[^b] For some comments on *no bridge* in Pareto, see E.N. 24.

33

[a] The *Manuale*, 1906, pp. 253 and 254, and the *Manuel*, 1909, pp. 265 and 266, wrongly show “§3” instead of “§4”. In our opinion, some misinterpretations of Pareto’s pure theory of consumption derive from the idea that this theory concerns the effective consumption of commodities. However, on the contrary, Pareto is clear in §4 that his theory refers to the commodities that are at the disposal of consumers. In other terms, the theory regards the purchase and the selling of commodities by consumers, not their consumption proper.

40

[a] According to Georgescu-Roegen, the transparent definition of complementary goods and competitive goods contained in paragraphs §39 and §40 provide “unsurpassed insight” and suggest that these notions must be ascribed to Pareto. See Georgescu-Roegen, “A diagrammatic Analysis of Complementarity”, *Southern Economic Journal* XIX (1952) and Georgescu-Roegen, “Vilfredo Pareto and his theory of ophelimity”, *Accademia dei Lincei*, 1973, cit.

41

[a] A cross-reference by Pareto to Chapter IV, §7 would have been helpful.

50

[a] In the French edition, 1909, p. 273, “*proposition*” was wrongly translated as “*proportion*”.

51

[a] It should be recalled that §19 deals with the hierarchy of goods that are able to satisfy a certain need.

54

[a] The French edition, 1909, p. 275 wrongly shows *bgh* instead of *gh*. The segment *gh* represents a high plateau and therefore must be parallel to the abscissa as in the original Italian text. On the other hand, the French text is defective because it is missing the symbol *G*, which indicates the high plateau. In other words, while the Italian text states that “the cylindrical surface leads into a high plateau *G*”, the *Manuel* states that “the cylindrical surface leads into a high plateau”.

[b] This sentence has to be corrected in “... explains the decrease in the slope of the hill from ∞ to 0, i.e., on the section, from the slope in *b* to that ones in *f* and *g* (§32)”.

[c] Here and in the *Manuel*, Pareto does not realize he is using the expression “contract line” to indicate the “exchange line”.

[d] The *Manuel*, 1909, p. 276, shows “at *a*” in place of “at *a'*”. However, the effect indicated by Pareto is produced if “the curve cuts *oA*”; it does not matter where.

57

[a] Both editions (1906, p. 265; 1909, p. 277) indicate a, a', \dots . They are instead the angles c, c' , that correspond to the points a, a' of the abscissa.

59

[a] Pareto uses again, this time by also including it in the *Errata-Corrigé* (p.577) of the Italian edition, “contract line” instead of “exchange line”. The French edition, 1909, p. 278, is correct.

[b] With reference to figure 31, the *Errata-Corrigé* (p.577) invites readers to replace $c, c', e \dots$ with $c, c' \dots$, which subsequent reprints correct in the main text.

62

[a] With regard to the “second kind of dependence” one must not fail to mention the thoughtful developments contained in M. Fanno, “Contributo alla teoria economica dei beni succedanei”, *Annali di Economia* II, 2, Milano, Bocconi Editrice, 1926.

65

[a] The text of the French edition, 1909, p. 281, wrongly shows q, q', q'' when in figure 37, as in the Italian edition, we have q', q, q'' .

66

[a] In the French edition, 1909, p. 281, the important and correct cross-reference to §19 regarding the hierarchy of goods is missing. For our commentary on the hierarchy of needs and goods, see E.N. 25.

70

[a] Figure 39 shows some imperfections both in the *Manuale* and in the *Manuel*: in the Italian edition (1906, p. 272), the point m has not been drawn and in the French edition, the same point has been wrongly positioned where the curve fhc'' intercepts the abscissa.

[b] Both the Italian (1906, p. 272) and the French (1909, p. 284) editions wrongly repeat “from above to below” when point c'' is considered.

CHAPTER V

II

[a] On the pursuit curve, which constitutes one of Pareto’s dynamic models, see E.N. 26.

[b] The *Manuale* (1906 and reprints, p. 277) wrongly shows bm . The *Manuel*, 1909, p. 289, is correct.

24

[a] Among the possible conventions, one should recall the one followed by J. von Neumann and P. Sraffa whereby durable capital goods are treated as joint products. The neo-classical tradition, following the lines of Marshall-Keynes and Walras-Pareto, maintain them as durable goods. In particular, Pareto and Keynes point out that the durability of capital goods is a cause of uncertainty. See E.N. 26.

30

[a] In the past, what Pareto calls “*economic deterioration*” was also called “*moral deterioration*”; nowadays it is usually called “*obsolescence*”.

[b] In both versions Pareto begins with direct self-insurance and reserves the term *insurance premium* only to the case of insurance underwritten by an insurance company. But it is anyway a matter of terminology.

36

[a] Pareto’s opinion according to which the representation through factors of production and the representation through production coefficients are equivalent dates back to the early 1890s, when Pareto read Walras again (production coefficients) and read Marshall (factors of production).

49

[a] Whilst the title of §48, “*Il frutto dei capitali*” (Yield on capital), has been translated in French with “*Le revenu des capitaux*” (Income from capital), here “*il frutto lordo di quei capitali*” (gross yield on this capital) has been translated with “*intérêt brut de ces capitaux*”, which retains explicit reference to the word ‘interest’. In analogous fashion, “*il frutto netto*” (net yield) in §52 has been translated in French with “*net interest*”.

66

[a] The word “firms” is always used to translate “*imprese*” (Italian) and “*enterprises*” (French).

68

[a] It would seem that the differences in the propensity for risk, with different consequent effects, as Pareto noted, is one basis for differentiating the behavior of Americans, who live in the ‘new’ world, from that of Europeans, who live in the ‘old’ world. This point had been noted previously by de Tocqueville and was noted subsequently by Keynes. It certainly did not escape Pareto the ‘sociologist’, although he does not mention it in the *Manual*.

70

[a] It is likely that once again Pareto had Gaetano Scorza in mind.

[b] On production coefficients in the *Cours*, see E.N. 27.

74

[a] What Pareto expounds in §§72–74 warrants comparison with various authors. As far as Keynes' *General Theory* is concerned, see E.N. 28.

75

[a] In many places Pareto has quite clearly explained that this point of equilibrium is of a virtual nature, that is, it disregards all the continuous changes that in actual fact make it impossible. In other words, at this point we are in the presence of an abstraction aimed at isolating a *tendency* that coexists with other tendencies.

[b] In a letter to Pantaleoni, Pareto comments on a graph that is missing in the *Cours*, where it is nonetheless possible to read an analogous comment. On this, see E.N. 29.

80

[a] With this paragraph, Pareto concludes a series of considerations on the efficiency of production in connection with *i*) the dispersion of production among a more or less large number of firms, and *ii*) the size of each single firm. Pareto had the merit of not falling into the kind of apologetic dogmatism that favors large firms, or into the opposite kind, which favors the small ones. It is as if he were telling us: here, I am offering you some tools to help you decide, from time to time, on which side the efficiency of production tends to be greatest. On the other hand, for a noteworthy case of inefficient production in connection with the numerousness of firms, see Ch. VI, §§10–11.

85

[a] Both the Italian and the French editions erroneously indicate (E) in place of (F).

91

[a] The topic “rent” prompts a new comparison between Cambridge and Lausanne (see note to Ch. II, §78), this time limited to Marshall and Pareto.

The former—Marshall—is a young professor who knows well J.S. Mill's *Logic* (1843) and *Principles* (1848), and the second edition of a book by W. Thornton where Mill's famous *Wages Fund* is questioned. The latter—Pareto—is a young manager who is passionately interested in social sciences and who, in Florence, reads the same book by Thornton, translated by two young friends of his, S. Sonnino and C. Fontenelli, who are followers of Are's and Mill's proportionalism, as Pareto himself is.

This is perhaps enough to recall the Millian root of the explanation of rents put forward by both Marshall and Pareto in terms of *reproducibility of capital goods* in the new, more profitable sectors (positive rents), and difficulty in withdrawing them from obsolete sectors (negative rents).

In the period 1870–90, both Marshall and Pareto, each unbeknown to the other, saw Ricardo's absolute rent as a particular case of their own theories. What essentially differentiates Marshall from Pareto, starting from the least important question, is language. In the *Cours*, Vol. II, §751, Pareto approves of Marshall's *quasi rent*, but prefers to cover all

cases of differential advantages (disadvantages), in the short and in the long term, with the term ‘rent’ alone.

Pareto contributes to this branch of economic knowledge by developing the concept of *acquired rent*. It is a concept that Pareto had in mind before meeting Pantaleoni (1890) and before reading (1891) Marshall’s *Principles*. But it is only with the *Cours*, Vol. II, that he sets this concept within the context of general equilibrium. It is also for this reason that in 1912, in reviewing G. Sensini’s “Teoria della rendita” in *The Economic Journal* (September), Pareto observes that: “Science, indeed, is never constant; it is a development continuous throughout. Ricardo’s theory of rent has been corrected and broadened—first by Marshall, then by Pareto, now by Sensini”. In short, what is here implicit, with regard to rent—and the same could be said with regard to the theory of consumption—is that Pareto’s *pure theory* moves the theory of rent forward by including it in the context of general interdependences for specific capital goods (rather than economic sectors).

Pareto’s pure theory of acquired rent passes unchanged from the *Cours* to the *Manuale* and to the *Manuel*. Without resorting to mathematics and without getting to the heart of the matter, its central core can be summarized as follows. If in an economic system represented by the general equilibrium equations the net return rate of capital goes, let us say, from 5% to 7%, the capital goods that in the renewed general equilibrium have a return rate higher (lower) than 7% will record a positive (negative) acquired rent.

Pareto’s understanding of ‘rent’ prior to reading Marshall’s *Principles* is evident from V. Pareto, “Il Signor Yves Guyot e il suo libro ‘La scienza economica’”, 1888, in V.P., *Scritti politici*, Vol. I, G. Busino, Torino, Utet, 1974, pp. 275–88. It may also be of interest to know that both Enrico Barone (1901) and Pareto (1902) reviewed L. Einaudi, *La rendita mineraria*, Torino, Utet, 1900, with admiration and friendship, although they also pointed out that Einaudi had not attained the Lausanne School’s notion of “acquired rent”.

96

[a] The French edition, 1909, p. 341, wrongly shows *ost* instead of *vst*. Readers will have no difficulty in noting that the straight line *ost* forms an angle of 45° with the axis *ox*. If we indicate the point at which *os* intersects *ah* with *r*, we have *rh* = *hs* = 20. Finally, we note that in the French edition, the segment *os* is a solid line, differently from the Italian edition. The present edition reproduces figure 43 from the *Manuale* (1906, p. 325).

97

[a] A cross-reference by Pareto to Ch. VI, §7 would have been appropriate.

CHAPTER VI

3

[a] When writing “cost of production” Pareto means “average cost of production”.

4

[a] Obviously, the segment $mh = 14$ litres of wine, as in fig. 46 and in Pareto's example, represents constant costs.

[b] It would have been preferable if Pareto had indicated that there will be further references to fig. 46: in §7 (with regard to products with decreasing cost, and with regard to two possible points of equilibrium); in §40 (to say that the equilibrium points c, c' do not give maximum ophelimity in the transformation); and in the mathematical *Appendix*: §36 of the *Manuale* (1906, p. 532), and §27 of the *Manuel* (1909, p. 560).

8

[a] In this context, the “bare cost” (*Manuale*, 1906, p.329) is simply the average variable cost, which does not include, therefore, the average fixed cost. This is confirmed further on, in §58 (1906, pp.346–47), where the same concept can be found without the use of the term “bare cost”.

9

[a] We shall see in this same Ch. VI that the collectivist production of Pareto's pure economy can provide “more pleasure”, which Pareto mentions in this §9.

10

[a] Cf. Ch. III, §103, footnote 1 and §104.

II

[a] For a comment on this, see **E.N.30**.

12

[a] Obviously, a pure mathematician could remark that Pareto could have reached the same result without any “rounding”. It is worth recalling that in Marshall and Pareto's day, the students of economics who knew differential calculus were few in number, and that Pareto explicitly adopted Marshall's method, which consisted in teaching theoretical economics by using geometrical representations alone, while relegating mathematics to separate notes. On the other hand, the following comment expressed by V. Volterra in his review of Pareto's *Manuale* (G.d.E., April 1906, pp. 300–301) is also worthy of a mention:

“But one should not think that the mathematical part of this work is only that in which the symbols of algebra and calculus are used. It is concepts and demonstrative and logical procedures that constitute the essence of the mathematical method, and it can therefore be said that this method pervades Pareto's work from as far back as Chapter III, in which the general concept of economic equilibrium is introduced.”

13

[a] “Even in this case”, that is, in the case *with increasing costs* that was examined in the preceding §12. “Competition” must be interpreted, here, as the entry of new producers,

entry competition, as it is also called. From what follows, it can be inferred that even in the case of increasing costs, entry competition can be inefficient and parasitic. For a reformulation, we refer readers to E.N. 30. We would also like to point out that in §13 there is an oversight: Pareto is dealing with parasitic competition in the case of increasing costs and shifts the line of maximum profit instead of shifting the line of complete transformations, as he had correctly done in the case of decreasing costs (§10). We also draw readers' attention to the following instance concerning the meanness of words. In §§10–11, which deal with parasitic competition in the case of decreasing costs, and more precisely at the end of §11, one reads: "within certain limits, of course". Well, it is impossible to understand and reformulate various aspects of Pareto's theory of production if this generic addition is not explained by recalling that the production function (the line of complete transformations) in fig. 14 of Ch. III, §103 shows two notable points, corresponding to notable points on the unit cost lines [Ch. III, §103, ^a]. Even if Pareto does not repeat it, these notable points must also be taken into account in the case of increasing costs found in §13. The interpretation of §13 is made even more complex by the puzzling disappearance of fixed costs. With regard to this topic we refer readers once again to E.N. 30.

16

[a] The point l'' here indicated by Pareto has not been included in fig. 47 of the Italian edition, nor in the equivalent figure in the French edition, p. 349.

17

[a] Summing up, we have seen that the hypothesis of Robinson Crusoe producing and consuming his own product has been found in Ch. V, §97, in Ch. VI, §7, and in Ch. VI, §17.

22

[a] As a matter of fact, the point c in fig. 48 corresponds to the point a in fig. 17 (2) of Ch. III, §120; and we have seen that at the point a the equilibrium is stable.

23

[a] It should be noted that the line of maximum profit crosses the line hk in the point l , where the average cost is at a minimum and the marginal cost is increasing. Indeed we know—even Pareto recalls it straight afterwards—that the line of maximum profit does not exist in the decreasing costs section.

27

[a] On pure economy and the State in Pareto's work, see E.N. 31.

30

[a] For a comment on this topic, see, once again, E.N. 31.

31

[a] It is obvious that by “individuals’ total income” Pareto means the income of all the individuals, and that the “salaries” shown in parentheses together with “public debt” are the salaries paid by public bodies. Obviously, by “firms’ expenditures” Pareto means the expenditure for the goods the firms produce.

33

[a] On the theorem of maximum utility in economics, readers are referred to the Notes to the *French mathematical Appendix*.

35

[a] Pareto correctly says that in going from o towards ω , the symbols t, t', t'', \dots , indicate increasingly high levels on the *hill of pleasure*. Fig. 50 has therefore been rectified since, even in the French edition, 1909, p. 355, t' wrongly precedes t .

36

[a] What Pareto implies is that type II (monopoly) is inefficient, that is, it destroys wealth.

37

[a] It is figure 50. In the French edition too, (1909, p. 356; as in 1906, p. 339), Pareto wrongly refers to figure 49.

40

[a] The epilogue of this paragraph, where Pareto affirms the profit of the producer can itself accompany and increase the welfare of the consumer, must be read together with the important conclusions of §46, where Pareto recalls efficient competition as well as the case of parasitic competition (§10) in which a high number of firms and zero profits combine with production inefficiency. Considering Pareto’s early readings, it is perhaps not inappropriate to add that the most distant and most authoritative forerunner of this hypothesis is perhaps J.S. Mill (*Principles*, 1848, Vol. I, Ch. “Competition and custom”), in which he contrasts the inefficiency of small shops with the efficient concentration of the department stores. It should be added that the hypothesis according to which monopoly affords a greater collective welfare than competition was established some years before the *Manuale* was first published. This hypothesis had always been ascribed to Marshall, and it would be taken up again by Pigou, Schumpeter and many others. But we would rather defer the discussion of this topic until §46, E.N. 32, where we shall mention Cournot.

43

[a] The *Manuale*, 1906, p. 341, wrongly has *all’c* in place of *all’e* and, few rows below, *c* in place of *e*. The French edition, 1909, p. 358, is correct, but the line *all’e* of fig. 51 does not appear to be tangent to the line of complete transformation and the indifference line *t*,

as it is instead in the Italian edition. In redrawing fig. 51 for the present edition we have followed the pattern of fig. 51 of the Italian edition.

46

[a] See E.N. 32, which deals with monopoly and collective welfare.

48

[a] It is useful to recall that Pareto's famous theorem on the maximum of ophelimity given by free competition—even if for the purposes of welfare it poses a problem of compensation—is still essentially a problem of efficient production.

53

[a] Pareto later deals with these issues in “Il massimo di utilità per una collettività in sociologia”, *G.d.E.*, April 1913, pp. 337–341, where he refers back to pp. 648–49 of the *Manuel*. The 1913 article anticipates some pages from the *Trattato di sociologia*, which at the time was being printed by Barbera in Florence. See E.N. 55.

59

[a] In the French edition, 1909, p. 364, the word *rent* is not in italics.

61

[a] For a reflection on the pure theory of collectivism in Pareto see E.N. 33.

67

[a] Pareto applies to the case of two countries (X and the rest of the world, Y) what is generally called “Walras’ law” but should more appropriately be called Cournot–Walras’ law.

70

[a] The title of §70, “Equilibrium and prices”, becomes “The equilibrium of prices” in the French edition.

71

[a] Similarly to other economists of his generation, Pareto uses the term “money surrogates” to indicate what in the Anglo-American literature going back to H. Thornton (1760–1815) was sometimes called “near money” or, by using a Latin word, “quasi money”, and in the Keynesian literature regarding *liquidity*, as well as in central banks’ statistics, is indicated by M_2 , M_3 ... to distinguish it from the category M_1 , which is constituted by liquid money par excellence. Readers should appreciate how modern the proposition by Pareto is.

73

[a] On the quantitative theory of money see E.N. 34.

90

[a] For a comment on what is expounded in §§87–90, see E.N. 35.

91

[a] For an observation in this regard, see E.N. 36.

92

[a] In this paragraph Pareto's term "consumption" includes both directly "ophelious" goods, and the goods that are sometimes called "means of production", or "industrial consumption goods".

93

[a] In both the Italian and the French editions Pareto wrongly refers to (IX, 94).

96

[a] In both editions (1906, p. 360; 1909, p. 377) Pareto wrongly refers to (IX, 92, 93).

CHAPTER VII

6

[a] At this point the French edition, p. 382, inserts a reference to footnote 1, which is the one that in the Italian edition refers to Bertrand. We believe the French edition to be correct.

II

[a] On Pareto's law see E.N. 37.

19

[a] Pareto was confused when he referred readers to the "*Systèmes*, II, Chapter IX", in the *Manuale* (1906, p. 368), and the "*Systèmes*, I, Chapter IX", in the *Manuel* (1909, p. 386). He was actually thinking of Volume II, Chapter X, which deals with the question of selection (and not Chapter IX). In Chapter IX, Pareto advanced the purist's need to separate the moral component of human action from general 'synthetic' action (also drawing on de Molinari's views, especially those concerning the importance of morals for the human life). This was intended to facilitate consideration of the moral component of human action in a more advanced scientific way, by departing from the first approximation under which the moral component of action is, typically, neglected. Pareto's scientific maxim must be: *recompose after separation*. Indeed, this maxim would have been all the more evident had the second edition of the *Cours* (see E.N. 3) seen the light of day.

24

[a] For Pareto's comments on C. Bresciani-Turroni's article, "Sull'interpretazione e comparazione di seriazioni di redditi o di patrimoni", *G.d.E.*, January 1907, see *Corrispondenza Pareto-Pantaleoni*, Vol. III, pp. 19–20, 22, and 26. When Pareto met Bresciani in 1907, he expressed himself in the following way in a letter to Sensini (*Corrispondenza*, 1948, pp. 31–32): "Yesterday I had the pleasure to see Bresciani here [...]. He is a man of great intellect and knowledge and I believe he will do well."

[b] This passage by Pareto is not at all clear if one does not specify that, as inferred from the *Cours*, h , with $h < x$, expresses minimum income, and that in the formula $u_x = N_x / N_h$, N_h expresses therefore the number of individuals whose income is not lower than that minimum.

31

[a] It should be $akb'f$ for $akbf$, which is found in both the *Manuale* and the *Manuel* (1906, p. 373; 1909, p. 393). In figure 58 we shifted the symbol b' in such a way that it appears at the end of the horizontal segment that starts from f .

44

[a] Here "volume of trade" stands for business trend, if not precisely for "business cycles", or for crisis. The latter is an old term used also by Pareto to indicate the economic cycles (see below, §46 and Ch. IX, §73).

45

[a] The volume of external trade is simply the total sum of imports and exports.

48

[a] In contrast with the synchronous facts he has previously referred to by the somewhat puzzling expression "facts that act on each other with a certain latitude", Pareto hints at facts that act on each other with time "lags", as they are now commonly referred to.

57

[a] The important theory of "human capital" can be considered a development that starts from classic economic theory. In something of a significant oversight, V. P.'s pioneering study on "La mortalità infantile e il costo di produzione dell'uomo adulto", *G.d.E.*, Nov. 1893, is rarely, if ever, recalled among the economic studies on human capital.

71

[a] In the *Manuale*, 1906, p. 392, the title of this paragraph is "Quality of the obstacles".

97

[a] In the *Manuel*, 1909, p. 423, the footnote concludes at this point. For a reflection on a possible reason for this, see E.N. 38

100

[a] With regard to this, see E.N. 39

112

[a] Readers should notice how Pareto the “social scientist” once again recalls, if only implicitly, the limits of “pure economy”.

116

[a] In the French edition, 1909, p. 432, the footnote has been suppressed.

[b] It is one of the constants in Pareto’s sociology. The starting point was the way in which, according to the young Pareto, Italian militarism in Africa and protectionism were identified with the interest of the whole nation by the individuals involved.

117

[a] A comparison with an opposing optimistic position by Keynes would be out of place here. It must be pointed out that the position expressed here is the last constant position by Pareto and we shall find it again in the *Trattato di sociologia*: ideas have a chance—which is always very slim—to affect real events only if they generate sentiments. After all, as Pareto used to say, *auro suadente nil potest oratio*.

CHAPTER VIII

8

[a] Terminology in regard to the various components of investment has not remained constant across epochs, with different authors placing diverse emphasis on the micro/macro and stock/flow dimensions to the concept of ‘stocks’, as well as using the word in the diverse contexts of pure theory and applied economics. Keynes, for example, after proposing subtle distinctions in vol. I of the *Treatise on money. The pure theory of money* (1930, cit., p. 116) ended up writing in the *General Theory* (1936, cit., p. 332) about “fluctuations in the stocks of finished and unfinished goods—‘inventories’—as it is becoming usual to call them”. In addition, “inventories” is found in Alvin Hansen’s famous book *Business cycles and national income*, New York, W.W. Norton, 1951, p. 345, also in connection with the acceleration principle. As for Pareto, during his life as an industrial manager he used to often refer to ‘stocks’ by the beautiful ancient expression “l’inessere”, which has now disappeared from the Italian language.

9

[a] Pareto defines savings, which are made up of the goods that people do not consume, and considers their transformations, which come to make up investment and capital goods proper. The resulting equality between savings and investment is subject to the same considerations—linked to the distinction between investments in desired inventories and investments in undesired inventories—that were discussed in relation to the *ex-post* equality $S = I$ determined in Keynes’ *General theory*. On the other hand,

Pareto was aware of this problem generated by business cycles, as is shown by his private correspondence during the “great crisis” that took place at the end of the 19th century. He also presumably knew G. Montemartini’s book on *Il risparmio nella economia pura* (with a Foreword by Carl Menger, Milano, Hoepli, 1896). The topic of savings had been analyzed in greater detail by Pareto in the *Cours*, §§105–134 and, in particular, §§417–473. The observations on savers made by Pareto in the *Trattato di Sociologia* are also interesting.

10

[a] On “simple savings” see E.N. 40.

II

[a] It is impossible for us to stop to compare Pareto’s position with regard to savings with those of other economists, including Italian ones (e.g. Einaudi, Fanno, and Del Vecchio). On the independence of savings from the interest rate, we point out to the reader the formalization contained in Pareto’s correspondence with L. Amoroso. See *Epistolario*, 1973, Vol. I, pp. 656–57. In this paragraph Pareto is giving examples of how the income effect and the substitution effect operate.

21

[a] As it has already been pointed out [VI, §13, b], Pareto knew J. S. Mill’s works well, but he was also an assiduous traveler to his native Paris, where the *department stores* actually gave a different social imprint compared with the towns that were dominated by a category—the *shopkeepers*—of which Pareto was never particularly fond. Readers must have realized by now that the shopkeepers constitute the constant example of Pareto’s *parasitic competition*.

25

[a] That in Italy, needless to recall, found its most famous supporter in Achille Loria (1857–1943).

27

[a] The *Manuel*, 1909, p. 449, shows the expression “in this division in three parts” instead of the more biting “in these trinities” of the original Italian.

29

[a] With regard to the depreciation of silver, see, in this chapter, comment [§41, ^a].

34

[a] Also in Pareto, as in the majority of Italian economists, the word “cambio” is an elliptical expression that can have three meanings. On this, see E.N. 42.

36

[^a] In general, “discount rate” has the meaning of interest rate (or set of rates) for short-term loans, which is also called, at times, the “money market interest rate”, as opposed to the interest rate (or set of rates) for long-term loans of the financial market. From §§49–51, it appears that Pareto used the phrase “discount rate” in reference to the particular rate that issuing banks (or what Pareto called the “bankers to the banks”) apply to other banks.

[^b] On governments, discount rate, and public debt, see **E.N. 43**.

37

[^a] To verify the exactness of Pareto’s example, one should keep in mind that Pareto starts from a price level equal to 1 and supposes that this level goes to 1.25, and then to 2.0. Under these hypotheses, the purchasing power in gold (e.g.) of 100 units of paper money, say dollars, goes from 100 to 80, and then to 50.

38

[^a] With his examples Pareto is here discussing what could be called a revision, or a realignment, of a currency’s gold parity. In Pareto’s time, these revisions of official parities were not governed by international regulations, as they will be, for instance, with the 1944 Bretton Woods Agreement.

39

[^a] Or to say it in another way: monetary stabilization (a new official settling of the exchange rate for a fluctuating currency) requires a previous policy of economic recovery; only in this case can a loan, whether it be in gold or in hard currencies, be a contributing factor towards the return to an official parity.

41

[^a] Pareto was writing when silver had greatly depreciated in terms of gold and the hopes of the bimetallists had by now faded. For a comparison of Pareto and Marshall on this topic, see **E.N. 44**.

42

[^a] On commodity-money in Pareto see **E.N. 45**.

48

[^a] This is a way of controlling the quantity of money through a kind of open market policy that is very similar to the operations that are usually called “swaps”. In the Italian economic literature, open market policy only starts to attract the attention of scholars in the 1930s.

CHAPTER IX

2

[a] On the habit and rationality of man's choices, see E.N. 46.

3

[a] The reader is referred to our Annotation [VIII, §21, a].

6

[a] Pareto's theory of inefficient—"parasitic"—competition aspires to generality, even though Pareto's examples are mostly drawn from the monopolistic agreements among retailers and from the numerousness range of commercial enterprises. In this paragraph, however, Pareto hints at the inefficiencies attributable to the larger category of the "bullies", among whom he also includes the trade unions. With regard to this, it is interesting to notice that around that same time, starting from the 5th edition of the *Principles* (1907), Marshall will condemn—more markedly than before—the inefficiencies connected with the wage levelling (equal wages for workers with different productivity) demanded by the *Trade Unions*. There is hardly the need to highlight the moral essence of the terminology—"parasites", "bullies"—used by an "experimentalist" such as Pareto, who never missed an opportunity to denounce its intrusion in economic theory.

8

[a] Inexplicably, in the French edition, 1909, p. 461, the expression "by shopkeepers' syndicates or by trade unions" has been translated as follows: "the shopkeepers or the trade unions". This means that the French version drops the regime of coalition—"syndicalization", which was the preferred term at the beginning of the 1900s—that characterizes Pareto's shopkeepers. On the contrary, in §14, footnote 2, the shopkeepers' syndicate will be correctly translated into French with "retailers' syndicate".

II

[a] For a further remark on the figure of the entrepreneur in Pareto, see E.N. 47.

20

[a] It is worthwhile recalling that among those who had put so much hope in the spreading of culture—especially economic culture—for the purpose of modifying human behavior, one must also include the "young Pareto". One need only read his beautiful correspondence with that nobleman (in both senses of the word) Francesco Papafava. See G. Busino, quoted in E.N. 1, where, on pp. 820–1, one reads: "You are telling me that you only became interested in political economy because of the desire you had to learn from it the way to improve the lot of the less well-to-do classes. It is the same for me."

[b] For a comparison between Keynes and Pareto on ideas and interests, see E.N. 48.

24

[a] Two comments: *i*) it is in these quick glimpses at history – which are devoid of any polemic topicality, but where, as Croce would put it (E.N. 5), one can discern the “sense of social antitheses” of someone like Marx—that in our opinion the best fusion between Pareto the sociologist and Pareto the economist can be found. In other words, it is here that Pareto the “social scientist” can be found in its mature entirety; *ii*) the second comment is a corollary of the first: some of the additions to the *Manuale* that are found in the *Manuel* follow the path of polemic topicality.

40

[a] Readers who are familiar with this subject will have no difficulty in recognizing that Pareto’s international transactions include all flow variables of the *balance of payments*. That is, they include the *current account* (goods and services) as well as the *international movements of capital*.

41

[a] In Pareto’s interpretation, Say’s law implies therefore the following two hypotheses: *i*) gold is a value reserve which also definitively regulates the various balances within the balances of payments; *ii*) if we consider an adequate lapse of time, the algebraic sum of the said balances can be assumed to be zero. This obviously does not mean that Pareto did not appreciate the economic and social importance of temporary imbalances: it was Pareto himself, in *Cours*, Vol. II, §632, who accused all those of Panglossianism—and we are afraid that they would include nearly all of Pareto’s followers—who deal with the *Paretian optimum* while neatly skipping over all the *social costs of adaptation*.

42

[a] Clearly, Pareto has in mind the mathematical expositions he proposes elsewhere, particularly in some paragraphs of the *Cours* (Vol. I, §294, n. 1, Vol. II, §§852–64), as well as in *Giornale degli Economisti*, Feb. 1894, April 1895.

43

[a] Paragraphs 43–53 of Ch. IX, regarding Ricardo’s theory of comparative costs are the most criticized of the whole *Manual*. On this matter, see E.N. 49.

46

[a] In the French edition, 1909, p. 509, the condition $\theta' > (t - \theta)x$ has been copied with a mistake: $\theta > (t - \theta)x$. In order to make it easier to read, this mathematical footnote by Pareto has been reformulated in E.N. 50.

48

[a] In the French edition, 1909, p. 510, the reference to §55 is missing.

54

[a] On the controversy about dumping see E.N. 51.

[b] On this mathematical footnote see E.N. 52.

57

[a] For a comparison with Einaudi's opinion on this matter, see E.N. 53.

61

[a] In the French edition, 1909, p. 519, the word "problem" is rightly followed by a colon instead of a full stop. On the other hand, "protection" is wrongly translated in French with "production".

66

[a] Among contemporary advanced societies, the interconnections between businessmen and politicians (even when they are not *petty politicians*, with Pareto's polemical meaning) are so numerous and so frequent—let us even say, normal—that it is a cause of great surprise to see the neglect in which they are kept in that part of today's *economics of industry* that arguably aspires at drawing general conclusions by comparing the "small business system" with the "large business" one. It is therefore worthwhile to note that Pareto puts forth "a remark that is of a general nature"—a remark, in other words, that goes beyond the specific topic he is discussing.

[b] In the French edition, 1909, p. 525, the question mark has been replaced by an exclamation mark.

[c] It is impossible to compare this Pareto on trade with the recent literature that draws on informational asymmetries and the theory of games, but, nevertheless, we would like to express our admiration for these pages (§§62–66), which meditate so cautiously on the topic of protectionism. The next paragraphs §§67–72 constitute some sort of vendetta against those who radically separate Pareto the economist from Pareto the sociologist instead of seeing in his sociology the inescapable last phase in his method of successive approximations. Scholars who make this mistake forget that: Pareto intended to replace his *Cours* with a *Trattato* that never appeared; that the intended book was primarily focused on sociology and inductive economics (see E.N. 3); and that that intended book was to be entitled *Trattato di economia*.

73

[a] It should be noted how Pareto arrives at macroeconomics starting from a non-aggregate point of view. In other words, it is as if Pareto approached the general movement of business by starting with a vector of economic activities, with some of the associated multipliers having discordant effects.

74

[^a] In addition to not being “very precise”, the two opposing phases characterisation of “crisis” being overviewed by Pareto is also misleading. The best literature on *business cycles*, such as that associated with the American Alvin Hansen, started to abandon that approach from the 1930s.

75

[^a] In his monumental work on economic cycles, where he distinguished among Kitchin cycles, Juglar cycles and Kondratieff cycles, Schumpeter repeatedly stated that innovative facts, which are at the origin both of economic evolution and of the various cycles, always occur in clusters, whether they be from a private source or from a public source. But with regard to Kondratieff cycles, we have added something in reference to Pareto in E.N. 56.

77

[^a] Pareto’s example recalls to our minds the cobweb theorem, an analysis tool that suits productions with rigid production periods.

79

[^a] The potential role that the memory of past “crises” can play in present and future “crises” was subsequently raised in the 1900s (by O. Morgenstern, for instance), but without reference to Pareto.

85

[^a] *Available liquid assets* in the banks of issue (for instance, the European banks’ deposits with the ECB), which can grow during periods of depression, are definitely in some relation to the *possible savings* of a country, but they are not, of course, identical with (*ex post*) “statistical” savings. However, this is matter of terminology. We believe that Pareto’s notion of “available savings” is synonymous with available liquid assets.

88

[^a] See E.N. 55.

Editors' Notes

E.N. 1. SOCIOLOGY AND SOCIAL SCIENCE IN COMTE, J. S. MILL, AND PARETO

Chap. I, §1. The current interpretation, under which Pareto is seen as having developed a serious interest in political economy after his first contact with Pantaleoni (1890) and from re-reading Walras (1891) at Pantaleoni's urging, is not entirely convincing. Prior to that, when he was still an industrial manager, Pareto was regarded by the young Francesco Papafava (1864–1912) as an engineer who was also a competent teacher of political economy. This very Papafava (1888) recalls Professor Pareto—an unpaid and *de facto* Professor, of course—advising him to carefully study:

“the part regarding the social sciences in Vol. II of Mill’s *Logic*”. (See G. Busino, *Vilfredo Pareto e l’industria del ferro nel Valdarno*, Milano, Banca Commerciale Italiana, 1977, p. 820).

Now, if we open this Volume II of Mill’s *Logic*, not only do we find there a term of which Pareto would later become quite fond—*The concrete deductive method*—but we also find that Mill had an aversion for the term “Sociology”, while Comte, in contrast, was rather fond of the term:

“The Social Science, therefore (which, by a convenient barbarism, has been termed Sociology) . . .” (J. S. Mill, *A system of logic, ratiocinative and inductive*, London, Park, Son, and Bourn, 5^a edit., Vol. II, 1862, p. 481).

On this matter, two points are pertinent: 1) in his ‘Tuscan period’ (1870–1893) Pareto was above all influenced by J. S. Mill until 1891, when he read Marshall and re-read Walras; and 2) Pareto always used the words *Scienza sociale* and *Sociologia* interchangeably, with both used to refer to all the disciplines of the social sciences, including political economy.

We regard Marshall and Pareto as the only two great economists to have further developed J. S. Mill’s sociological message. But while Pareto separates economic theory (pure economy more exactly) from sociology as much as he can, Marshall instead tends to blend them together. It is, therefore, rather ironic that it was Marshall, rather than Pareto, who proposed “Economics” as the term to replace “Political Economy” (see A.Z., “Economics or Political Economy? Marshall and Pareto as Mill’s heirs”, in *Studi Economici*, 2010/2).

SOME REFERENCES. Thanks also to two Florentine scholars, namely Giacomo Becattini and Marco Dardi, the literature on the relationship between Mill and Marshall is so vast that we shall avoid giving any bibliographical reference. As a methodological introduction to Pareto, we point to V. Tarascio, *Pareto’s methodological approach to economics*, University of North Carolina Press, 1968. As for the interpretation of Pareto’s sociology,

which is often made difficult by his uncommon, often very personal, conventions concerning the meaning of words, the works by G. H. Bousquet, R. Aron and N. Bobbio are still valuable texts. Since politics is a very dominant element in Pareto's sociology, it is unfortunate that there is no definitive essay that compares Pareto's sociology of politics (including the "massimo di utilità in sociologia") with John Rawls' (1921–2002) important philosophy of politics. For some recent contributions by M. McLure on Pareto's sociology in the field of public finance, see E.N. 31.

E.N. 2. PARETO AND MARSHALL ON MECHANICAL ANALOGY AND BIOLOGICAL ANALOGY

Chap. I, §7. In §7 Pareto clarifies why, in his assessment, the uniformities or scientific laws (see also §4) "have no objective existence" and they do not "suffer exceptions". In the eyes of some authors, these statements point to a contrast between Pareto's scientific method and that of Marshall, with Pareto often resorting to the mechanical analogy whereas Marshall was more inclined towards the biological analogy in reference to living organisms subject to growth and decline.

In the *Manual* Pareto does not dwell on the biological analogy, but this does not mean that he had abandoned the position expressed in Vol. II of the *Cours*, or that he would not have confirmed it in the *Trattato di economia*, the book that Pareto planned as his replacement for the *Cours*, but which never progressed beyond the initial planning stage (see E.N. 3). In relation to the evolution of social phenomena, Pareto wrote, in the *Cours*, that the mechanical analogy does not evoke anything real, whereas the analogy with living organisms may be useful. According to J. Whitaker, it is possible to discern a biological foundation to Marshall's reconciliation of competition with increasing returns through his "three-generation rule" for the life of a business: a firm is established by the grandparents, subsists with their children and perishes with their grandchildren. But, Pareto's famous reference to the "graveyard of aristocracies", which indicates that all economic and political elites fall at different intervals, is similarly suggestive of a biological analogy.

Pareto maintains that the substantive attributes of the biological analogy fall into difficulty when we decompose the various concurrent influences on social phenomena: it cannot survive the guillotine of *ceteris paribus*. On the contrary, just as it is possible in physics to abstract from the influence of centrifugal or centripetal forces when undertaking a first approximation investigation of some physical phenomena, so too is it possible, for instance, to abstract from the influence of transportation costs in economic science by assuming a punctiform market under which all contracts are carried out in a single place: but it is impossible to conceive of a human being as living without its head.

It seems to us that careful comparison between Pareto and Marshall reveals a paradox, which has escaped the attention of those who insist too forcefully on the contrast between general equilibrium and partial equilibrium, between economic interdependence and *ceteris paribus*. The paradox lies in the fact that Pareto implicitly resorted to the *ceteris paribus* as a simplifying assumption in the most drastic terms, namely, when theorizing about interdependences from the perspective of pure economics—which is confined to the field of logical action as he defined them: see E.N. 14—as a first approximation,

whilst temporarily setting aside the second and subsequent approximations of applied economics and sociology—which are not confined to the field of logical action. Pareto and Marshall both effectively employed the *ceteris paribus* simplifying assumption for analytical purposes, although they did apply it in different circumstances. But the fundamental difference between Marshall and Pareto lay not in the presence or absence of the *ceteris paribus* assumption, rather, it lay in their deeply contrasting positions on the scope for value judgments and ethics to be introduced within economics. In Marshall's opinion, Economics is a science of human motivations in which value judgments and ethics—and in particular the ethics of solidarity—have a rightful place; whereas Pareto—who in the 1880s wrote with a humanitarian spirit and admitting in his private correspondence that his interest in Political Economy was due to moral reasons—ended up by ridiculing his own earlier pacifist and humanitarian position and arguing against the intrusion of sentiments within scientific research.

So, contrary to the received view, and without considering any of Pareto's personal resentment towards Marshall, Pareto's profound scientific disagreement with the Master of Cambridge was not centered on the much-cited difference in their application of the mechanical and biological analogies to economics, rather, it was centered on their contradictory views on the appropriate methodology of science to be utilized in economics. In other words, the dualism between Marshall/Pareto has its roots mainly in two different ways of resolving what Mill, in the central point of his splendid *Autobiography* (London, Longmans, 1873, 1st edition, p. 163), calls the “many-sidedness” of human life and social phenomenology.

SOME REFERENCES: On the contrast between Pareto and Marshall see F. Vito, *La concezione biologica dell'economia. Considerazioni sul sistema di Marshall*, Milano, Vita e Pensiero, 1934. See also N. Georgescu-Roegen, “L'economia politica come estensione della biologia”, *Note Economiche*, 1974, n.2, even though he failed to study Pareto's *Cours* with the same passion he studied the *Manuale*. For a more general comparison of Marshall and Pareto, see M. Dardi and A. Zanni, “Pareto's ‘third way’ between Marshall and Walras”, in T. Raffaelli, G. Becattini, K. Caldari and M. Dardi (eds), *The impact of Alfred Marshall's ideas. The global diffusion of his work*, London, Elgar, 2010. One final consideration: Pareto, like J. S. Mill (*Logic*, cit. Book III, Chap. XX), saw analogies as performing an important evocative near-didactic function, but played no role in providing the proof of a proposition. That very position was subsequently affirmed by J. M. Keynes, in the *Treatise on probability*, and again in other famous studies, such as in recent works by M. Hesse and M. Bunge.

E.N. 3. THE COURS, THE MANUAL, AND THE 2ND EDITION OF THE COURS THAT NEVER SAW THE LIGHT OF DAY

(A Note to complement various other Editors' Notes)

When Pareto became professor of political economy at the University of Lausanne in 1893, he soon appreciated how difficult it was for him to live with Walras' ‘metafisiche’ [metaphysical nonsense]. But, he was not eager to enter into conflict with Walras over

the manner in which general equilibrium should be expounded. It was only two years since he had discovered that Walras' notion of interdependencies could be used in his battle against Italian protectionism and he was also personally grateful to Walras for backing his call to the Chair in political economy at Lausanne, which was significant because Pareto was no longer a young man and he had declared that he was running away from the country he loved because Italy was being ruled by petty politicians. Moreover, Walras had welcomed Pareto to Lausanne by giving him the outline of the lectures that Walras wanted Pareto to present. Indeed, Pareto's *Cours* was structured around Walras' famous trilogy of exchange, production, capitalization, although the *Cours* also incorporated many of the more original ideas that Pareto published in the *Giornale degli Economisti* between 1891–1894 (although his more sophisticated analysis, such as the 1892 article that inspired Slutsky's famous 1915 paper, had been excluded).

When compared to the *Cours*, Pareto's *Manual* marks a radical movement away from Walras' formulation of pure economics. In 1911, one year after the death of the great scholar that Pareto always acknowledged as the founder of general equilibrium, Pareto observed that:

“If I had wanted, I could have presented my theory of equilibrium in such a way that it would not seem to have anything in common with Walras's theory.” (See Sensini, *Corrispondenza*, 1948, cit., p. 61).

Aspects of this transformation include theoretical innovations related to: variable prices for successive quantities of product before reaching equilibrium (curvilinear paths as opposed to rectilinear ones); resorting to a *line of complete transformations* to separate profitable firms from unprofitable ones; the systematic use of terminal points (non-marginal equilibria) in the study of production with decreasing unitary costs; and consideration of the so-called Antonelli experiment instead of the so-called Pareto experiment alone in the theory of ophelimity. From a formal point of view, these innovations, as well as others like the investigation of unstable but lasting equilibria, suggest that the general equilibrium presented in the *Manuale* is almost unrecognizable when compared to Walras' *Éléments* and Pareto's *Cours*.

Nevertheless, a question remains: why did Pareto see something superior in the *Cours* relative to the *Manuale*? Even before the *Manuale* saw the light of day, Pareto was planning a second edition of the *Cours* to be produced in partial collaboration with Guido Sensini. The reason cannot lie in the fact that Pareto used to advise people to start reading the *Cours* from the end: Pareto in the *Manual* had already eliminated a cause of his discontent by having the sociological sections precede the pure economy (and not *vice versa* as in the *Cours*). The true reason is found in Pareto's correspondence with Sensini, which confirms Pantaleoni's view that Pareto had intentionally limited the number of pages in the *Cours* that are dedicated to pure economy, compared to applied economics and sociology. In the new edition of the *Cours* Pareto wanted realism—the observation of fact—to be the dominant moment compared to the unavoidable moment of hypothetical reasoning. Above all, he wanted to extend the search for new ‘Paretoian’ laws to the arduous field of the sociology, where instincts, passions and misleading rationalizations reign. It should not be forgotten that when Pareto first read Marshall's *Principles* in 1891, he admired it for its realism and theoretical sobriety. Above all, we should also recall

that some of the text that ended up in the *Trattato di Sociologia Generale* was originally intended for inclusion in the *Trattato di economia*, which was effectively regarded by Pareto as a second edition of the *Cours*, although it was planned as a five, not two, volume work: “1. Compendium of Sociology, 2. Pure Economy (without mathematical formulae), 3. Mathematical Economy, 4 and 5. Applied Economy”. In a letter (April 9, 1905) Pareto added that the *Trattato di economia* would be published by UTET in Turin (as part of the fifth series of the renowned *Biblioteca dell'Economista*: see *La Riforma Sociale*, 1905, p. 249).

In the literature, many mistakes have been made even after the *Corrispondenza Sensini* had appeared in 1948. Notable in that regard is the continuing contrast between Marshall's *Principles* and Pareto's *Manuale*, rather than with the new edition of the *Cours*. In view of this, it is helpful to extensively quote the details contained in a letter from Pareto to Sensini dated 8/12/1906:

“My publisher is putting me under great pressure for the 2nd edition of my *Cours*. It will take a long time for the first three volumes, namely *Sociologia*, *Economia pura*, *Economia matematica*, to be ready, but the last two volumes, namely *Economia applicata*, could be ready quite soon, were I to have your assistance; and the publisher is prepared to publish them *before* the other volumes. [...] To give you an idea of the work that would need to be done, I think it is better to provide some examples.

1. The statistical information in the *Cours* needs to be completed. For instance, Vol. I, p. 241, footnote 369⁴, *Change de Paris sur Londres* is given until 1888; it is necessary to give it until 1906. On p. 244, the Latin Union: it is necessary to quote the latest conventions. Monetary systems: Austria, Hungary, Russia, etc.: *idem*. In short, it is necessary to review *all* the statistical data and complete them.

2. Revision of the statements made at the time of the publication of the *Cours*. See which ones need changing according to later facts.

3. Additions due to recent facts. For instance: English municipal socialism, Italian and German municipal socialism. Trusts in the United States [Monopolized industries were called “trusts”, as reported by J.B. Clark and his son J.M. Clark. See also the good inquiry: Paul De Rousiers, *Les industries monopolisées (Trusts) aux Etats-Unis*, Paris, Arman Colin, 1898. This book was reviewed by Pareto in 1899. A. Z.] etc.

As for anything regarding classical Greek and Latin antiquity, I shall deal with it [...]. But as for the whole of the modern era, you will have to deal with it. Of course, your collaboration in producing these two volumes will be acknowledged and your name will appear in the title.

We shall remove all mathematical notes, because they will be placed in the volume on Mathematical Economics; [...].

We shall remove the whole sociological part, which will find its place in the volume on Sociology. For instance, the whole of Chapter 1, Book II (Vol. II) will go.

In this way, the two current volumes of the *Cours* would be reduced to one. But we would add *many—a great many*—pieces of information on economic history, and not only for the past, but also for the present. So we shall end up with two volumes again, even larger than the current ones.”

In short, not only did Pareto want sociology to precede pure economy and to act as a kind of *caveat* for it, as it does in the *Manuale*, he above all wanted the sections on sociology and applied economics, including economic history, to be the most predominant sections that provide the flavor of the entire multi-volume work. This proposed structure brings to

mind Marco Fanno's *Il mercato monetario* (1912), of which it is difficult to decide which to admire more: the first part, which is mainly historical, or the second one, which is theoretical. Similarly, it brings to mind the two well-known volumes of Keynes' *Treatise on money* (1930). Going back to a conviction of ours, it makes one think of the seed of Social Science planted by J. S. Mill, which found two different but equally fertile soils in the works of Marshall and Pareto. Indeed, it is amazing that Schumpeter (1949) wrote such an excellent essay on Pareto the economist and sociologist without even mentioning J. S. Mill.

In talks Pareto had with Sensini in February 1914, when the *Trattato di Sociologia Generale* had been largely finished (it would be published in 1916), Pareto indicated that Vol. III of the new *Cours* could simply be an expansion of his own 1911 encyclopedic entry on *Économie mathématique*. As for Applied Economics, Pareto outlined some considerations about the monetary economics, which would have helped Wicksell, who, in contrast to G. Del Vecchio in the past and P. Bridel today, never appreciated that Pareto's contribution to monetary economics mainly lies outside the scope of pure economy (see E.N. 34, 40 and 41). But it is worthwhile to report Pareto's words (Sensini, *op. cit.*, p. 24):

"For the volume on Mathematical Economics in the 2nd edition of the *Cours*, one only needs to repeat what is written in the article for the French Mathematical Encyclopaedia, apart from expanding some points, if possible. The two volumes on Applied Economics, instead, are a totally different matter. The two volumes of the first edition must be completely re-written. Applied economics must keep one foot in Pure Economy (which was done in the 1st edition) and the other in Sociology (which was not done—and could not be done—in the 1st edition). For instance, in the chapter on money the theory of monetary equilibrium remains unchanged; but the whole theory about the issue of false money by the Governments must be re-done. In this section, the author of the first edition of the *Cours* used ideas involving ethics, sentiments etc.; he condemned the issuing of false money without realizing that the problem was infinitely more complex than an ethical and economic problem. Indeed, those issues were actually able to help the nations. And here we are in the sociological field. Economics can certainly refer to sociology."

At the end of this note we will refrain from recalling all those who have reflected on whether the *Manuale* was more important than the *Cours* or *vice versa*—it is an ill-specified issue if one is not aware of Pareto's complete view, as evident from his correspondence with Sensini. Instead, we would like to note that Sensini, who worked hard on his unsuccessful attempt to realize Pareto's project, was of the opposite opinion (*op. cit.*, p. 24), that it was preferable to proceed from the simpler to the more complex, and not the other way around:

"I take the liberty to point out that this work would possibly have turned out better had it been subdivided in four volumes as follows: 1. *Pure Economy* (with the use of mathematical analysis, and nothing else); 2 and 3. *Applied Economy*; 4. *Sociology*. Among other things, in this way one would have proceeded from the simpler to the more complex."

However, the sequence of topics such as Pareto wished to apply, if he had lived long enough, was a reflection of the burning disappointment of a man who had placed too

much credence in the Enlightenment-based faith in the rational behavior of men, with the conviction—which was typical of the Enlightenment—according to which “educated” men can escape the misleading sirens, the myths (on the Enlightenment in the young Pareto, see E.N. 57). If we remove the dead leaves that are floating on the surface, at the bottom of Pareto’s failed project one can see an admission of guilt, an act of humility by a proud man, as well as an antidote against myths, religions, and fundamentalisms of any kind. In the *Manuale*, and even more so in the *Manuel* and in the *Trattato di sociologia*, Pareto pours scorn on all unarmed prophets, he tells us about wolves and lambs, lions and foxes. But there is a difference that escapes those who talk so freely about Pareto’s cynical Machiavellism: Machiavelli shows no sign of internal suffering, even when he makes use of the famous phrase, quoted by Pareto, according to which “men must either be pampered or destroyed”. Whereas in Pareto—who at times is even overcome by an affected type of sarcasm that lies between Voltaire and a form of spontaneous Tuscanism that can even be found in the *Liber Facetiarum* and in the *Contra hipocritas* of a humanist such as Poggio Bracciolini—in Pareto’s sarcasm, there is always the bitter aftertaste of a man who, especially with regard to the evolution of the history of Italy, cultivated some illusions—even about socialists—that ended up being all frustrated. Pareto’s last word seems to tell us: learn from me, who made a mistake. In our opinion, Pareto also made a mistake when he gave us the recipe of a *wertfrei* social science that is as bare as possible, exasperatingly bare, of its own subjective values: the very same Pareto who—as Giovanni Vailati once very perceptively pointed out—was fiery by temperament and irrepressibly exuberant.

SOME REFERENCES. In asking for all the benevolence we need, we remind the reader that Vol. XXXII (2005) of Pareto’s *Oeuvres complètes*, a volume planned by Giovanni Busino and brought to fruition by Fiorenzo Mornati, contains a seventy-page bibliography not of works *on* Pareto, but of works *by* Pareto! With things being as they are, this E.N. 3 of ours only aimed at filling a bibliographical gap, also in view of a better comparison between Marshall and Pareto. We had indeed noticed that the *Corrispondenza Sensini* (1948) was also missing in the bibliography appearing in John Chipman’s masterly 1976 essay, “The Paretian heritage”, *Revue Européenne des Sciences Sociales*, 1976, n.37. Unfortunately, in 2009, in commenting on a system of equations, John Chipman overlooks the *Corrispondenza Sensini* once again. See therefore A.Z., “A letter on professor Chipman’s translations of Pareto”, *History of Economic Ideas*, 2009/3.

E.N. 4. AN ASSESSMENT OF PARETO BY BENEDETTO CROCE

Chap. I, §17. There is a vast literature on Croce’s exchange of opinions with some leading Italian economists (notably Pareto and Einaudi). We shall only recall: *a)* a course of lessons that marks the passage from the *philosophy of politics* to the *science of politics* by the political scientist G. Sartori, *La filosofia pratica di Benedetto Croce*, Università degli Studi di Firenze, academic years 1953–55, C. a M., v. Castellaccio, 4r, Firenze; *b)* A. Montesano, “Croce e la scienza economica”, in *Economia Politica*, August 2003. These papers demonstrate that, while it is a useful that Croce’s *Filosofia dell’Economia*

and the economists' *Economia politica* coexist, they are quite distinct, which even Croce eventually realized.

As men and as scholars, Pareto and Croce were very different but they always held each other in high esteem. Evidence of this can also be found in the following most penetrating passage taken from the philosopher's forgotten review of Pareto's *Systèmes socialistes* (1902):

“What was good and vital in Marx has been preserved, and the rest has been rejected; and of those good things I find traces (unless I am mistaken) in almost every page of Pareto's new book, where there is a very keen sense of the social antitheses (class struggle) and of the lies or ideologies due to which the contrasting economic interests are often unwittingly covered: that is, precisely two of Marx's main directive concepts.”

See B. Croce, *Conversazioni critiche*, Bari, Laterza, 1918, p. 280. The fact that Pareto, who opposed Marx's economic theory, can be credited with a fine tribute to Marx's genius is something on which we cannot dwell, but which should not come as a surprise. The matter is anyway brilliantly exposed by G. La Ferla in a book (*Vilfredo Pareto filosofo volteriano*, Firenze, La Nuova Italia, 1954), whose only fault is not having placed the last 50 pages of the book at the beginning as the introduction.

A CONSIDERATION AND A BIBLIOGRAPHICAL ADDITION. On the ocean that separates the Croce continent from the Pareto continent we have referred the reader to a political scientist, an economist, and a sociologist. Let us be allowed a further consideration. If Croce is right in thinking that Marx enriched the canons of historical interpretation, then it must be recognized that one is much more enriched with interpretive canons by Pareto's *Sociologia*. One needs only think of the theory of the circulation of the *élites*, be they political, intellectual, or economic. On Pareto and Croce there exist thoughtful pages by many scholars, but we advise anyone wishing to venture in those two continents to start from G. Busino, “Pareto e Croce”, *Accademia dei Lincei*, 1973, cit.

E.N. 5. THE THREE LEVELS OF KNOWLEDGE IN PARETO

Chap. II, §123. A comparison between the topics of the first two chapters of the *Manual* (I. *General Principles*, II. *Introduction to Social Science*) and the corresponding topics of the *Cours* (1896–97) is beyond the scope of these *Notes*. However, it is worth pointing out that the three levels of knowledge that every social scientist should aspire to are never as clearly specified in the *Manuale*, as they are in the *Cours* (Vol. II, §580). In short, given the phenomena A, B, C, . . . , “our knowledge with regard to their reciprocal dependence can go through three degrees”: *i*) we can learn that the presence and the variations of A affect B, C, . . . ; *ii*) the level of knowledge is much greater when we know that B varies directly—let us say—whereas C varies inversely with the variation of A; *iii*) finally, we can “not only [know] the direction of these variations, but also calculate their size. Once we get to this point, our knowledge of the phenomena A, B, C, . . . , is complete and

perfect". In other words, perfect knowledge consists in an econometric representation of interdependences.

If one considers the importance that sentiments have upon human behavior in Pareto's works, and the difficulty in ascertaining them (in the *Manuale*, 1906, Ch. II, §100, p. 116, one reads: "Phenomena related to sentiment cannot be measured with precision; we therefore lack the statistics, which are so useful in political economy"), one will appreciate that, in Pareto's opinion, the social scientist's highest level of knowledge is actually unattainable.

The problem of the econometric decomposition of the many concurrent causes of a phenomenon started from Pareto's collaboration with Edgeworth's *The Economic Journal* (September 1892) and occupied Pareto for the rest of his life as a scientist. In 1908, anticipating Keynes' well-known position *vis-à-vis* Tinbergen, Pareto concluded that "some knowledge, even imperfect, of the nature of the phenomenon can be more useful than the best interpolation" and that it is necessary "to convince oneself that the future cannot be inferred from the empirical knowledge of the past, even for the phenomena that can be measured and can therefore be calculated. *A fortiori* this is not possible for sociological phenomena, which cannot be measured" (V.P., "L'interpolazione per la ricerca delle leggi economiche", G.d.E., June 1908, p. 446). In regard to the 'three' degrees of knowledge, see the considerations by A.Z. ("Demaria negli anni trenta attraverso un epistolario", *Storia del pensiero economico*, 1996, n.31–32, p. 37, and "Pareto in Tuscany. Discontinuity and premonitory signs of an economist and a sociologist", *Revue des Sciences Sociales*, 1999, n.116) about the econometric decomposition of the concurrent causes of a phenomenon and some points of contact between Pareto and Keynes.

E.N. 6. PARETO THE MANAGER WANTS TO WRITE A TREATISE ON RATIONAL ECONOMICS

We have already expressed dissatisfaction with the prevailing view that Pareto developed an interest in Political Economy following his first encounter with Pantaleoni (1890) and his re-reading of Walras (1891). As noted (E.N. 1), Pareto was still a manager when he started to reflect on activities that would culminate in his appointment as Professor of Political Economy at the University of Lausanne. Indeed, the textual evidence clearly confirms his earlier desire to write a treatise on rational economics, with the protectionist measures introduced in Italy in 1887 proving the catalyst in that matter. For example, Pareto wrote from Florence to his first disciple, Francesco Papafava, on 27 November 1888, in the following terms:

"It would be very useful for me to know the hurdles at which an independently natured person like you stops, so that I can avoid them in expounding the economic principles, when I happen to do so. Among the resolutions that I have made, but unfortunately not carried out, there is also that of writing a treatise on *rational* political economy. In other words, I would like to explain the general principles of economic science in their most general features, similarly to what is done for mechanics in those treatises that are precisely called treatises on *rational* mechanics. The problem lies in carrying out this explanation in the most concise and clear terms as possible, and perhaps this is too great

a task for me. Anyway, I am trying. And it is for this reason that I keep annoying my friends by expounding economic theories to them and asking for opinions”
 (G. Busino, *Vilfredo Pareto e l'industria del ferro*, cit. p. 817).

On 13 December 1888 he again wrote to discuss the possibility of writing a treatise on rational economics, this time while giving Papafava a stimulating lesson that falls somewhere between the humanitarian and pacifist radicalism of the young Pareto and neo-Ricardism, under which economic surplus varies with alternative investments. As regards the only section that is relevant here, Pareto wrote:

“At this point, it is necessary to leave behind the synthetic part and come to the analytic part. Unfortunately, in this case the basis of the latter is *value*—I am saying ‘unfortunately’ because it is one of the most difficult theories in political economy [...]. But I am facing a serious problem. It would be relatively easy to expound this theory by using mathematics and *assuming* that the reader is familiar with physics and mechanics [...]. Without this help it becomes extremely difficult. Last month, I spent some days in Turin to discuss this topic with a very dear friend of mine, [Galileo] Ferraris [1847–1897], a Professor at the university there and one of the first scientists in the field of electricity in Italy. He is of the opinion that the book should have a physical-mathematical introduction. [...] I am working at it, but I am afraid this is too much for me. When I have written that part of my work, I shall send it to you before printing it, so that you can frankly tell me your opinion.

Let us try to do without this general theory of value [...].” (*ib.*, pp. 849–50)

When he wrote these lines in 1888, there is no doubt that Pareto had not read Walras' *Éléments* in the edition that would appear in 1889 (with a mathematical introduction that unfortunately is also missing from the American edition edited by W. Jaffé). Similarly, he had not read Marshall's *Principles*, which appeared in 1890 and would suggest to him an alternative solution (that of forsaking the inclusion of mathematical exposition in the text of a book and confining mathematical developments to separate notes). Nor had he read Pantaleoni's *Principii*, which did not appear until 1889 and would become a valuable historiographical source for Pareto. Indeed, he remained bound to this work for an important aspect of the theory of consumption, namely, statics and the evolution of the hierarchies of needs and of the goods that satisfy them (see E.N. 25 and 32).

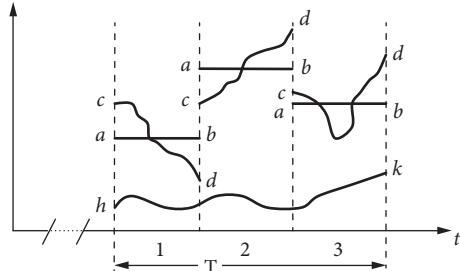
Having said that, and without trying to see the entire tree within the seed, it is evident from Pareto's correspondence that his 1888 project to write a treatise on rational economics was the forerunner of the *Cours*, which he subsequently wrote in Lausanne and in which some of the features that came to characterize Pareto as a social scientist are defined: the Millian idea (also embraced by Marshall) that anyone who does not cultivate Social Science cannot be a good economist; the idea that pure economy must be separated from the rest of Social Science in the same way rational mechanics is separated from the rest of mechanics (the latter idea was also developed by Marshall, although along lines that are more flexible than Pareto's and less preoccupied with methodological transgressions); the idea that the secret of an increase of welfare for the greatest number is fundamentally governed by production, and governed, in a lesser way, by consumption and distribution. As he states: “As the older economists already realized, the pursuit of the greatest advantage for society is mainly a problem of production” (*Manual*, Chap.VI, §48).

There is no need to go on. It was enough for us to document the limitations of a theory regarding the birth of Pareto the economist, even by means of a remote project for a treatise on Political Economy.

E.N. 7. STATICS AND DYNAMICS

Ch. III, §§7–8 and §171. The distinction between statics and dynamics that Pareto proposes for pure theory in §§7–8 of Ch. III must not be entirely clear, so much so that even a very careful scholar such as A.P. Kirman (“Pareto as an economist”, *New Palgrave*, Vol. 3, p. 806) has written, in reference to §171, that Pareto confuses statics with dynamics. Let us reflect further on the matter starting with the following diagram.

Pareto’s analysis is *static* when it is mono-periodical, that is, when it regards a single period considered as a unit. The period can be the period marked with number 1—that is, today, or the current semester—or, alternatively, it can be period 2—that is tomorrow, or next semester—and so on. The duration of the period is given by the context and it is, in general, the duration that is necessary for the causal nexuses established by the theoretical context to exercise their full effect within the final instant of the period.



Since statics makes it possible to compare alternative situations referring to one and the same period, Pareto believes that statics only lends itself to represent phenomena that vary slowly in time and whose changes are therefore minor for short periods (Ch. IV, §27). More importantly, in Pareto’s view, static analysis does not determine the effective value of a phenomenon as it continuously unravels during period 1 or, alternatively, in period 2 or 3. *Mono-periodical static analysis determines a normal average value*. In other words, whether it be period 1 or 2, 3, . . . , the value of the phenomenon is not the value *cd* that varies with the passing of time, but is the constant value that is determined by the ordinate of the rectilinear segment *ab*.

By the word *dynamics* Pareto means two forms of representation. The first is an ordered sequence of mono-periods. To distinguish it from usual comparative statics, in which only two static equilibria are compared, we could call it *pluriperiodical comparative statics*.

Whatever the duration of the period under consideration (it could be period 1, or period 2, or period T in our diagram), the second meaning Pareto gives the word *dynamics*, is the representation of a phenomenon as time passes. In other words, time is now considered as an argument in the functional representation of a given phenomenon. The *pursuit curve* found in the *Cours* and in the *Manuale*—which, in Rational Mechanics textbooks, is represented by differential equations—is an example of this second kind of representations. Two further dynamic representations in this sense are the following: *a)* the “toothed waning moon” shaped diagram, as we shall call it, drawn as Figure 38 in Vol. II of the *Cours*, §718; and *b)* the enigmatic formulae in the *Cours*, Vol. II, §698, footnote 2,

where we think we can glimpse a pioneering application of Fourier series (see E.N. 56). If our empirical knowledge had provided us with the necessary information about it, the variation law for a phenomenon, that is, its dynamic representation according to the second meaning, could be, for instance, that of line *hk* in our diagram. However, at times, Pareto knowingly went against his own method (see E.N. 30) and resorted to pluriperiodical comparative statics to represent that “half of an industrial revolution” (as we shall call it) that consists of an imitation process without innovations, which usually involves a long period.

We stated at the beginning of this note that the statics and the dynamics discussed in §§7–8 of Ch. III regard pure theory, which does not include the numerous and significant changes that modify the structure of an economic system (E.N. 32), which Pantaleoni used to call “dynamics of a second kind”.

SOME REFERENCES AND A CONSIDERATION. G. Borgatta remarked (*L'Economia Dinamica, studio critico sui problemi dinamici dell'economia pura*, Torino, Utet, 1915) that the shorter and the more numerous the Paretian periods considered in a dynamic sequence (in the first sense) are, the closer one gets to forms of representation where the passing of time is explicitly present (dynamics in the second sense).

Finally, it may be noted that there is no unique convention in economics that defines statics and dynamics. To confirm that, one need only read F. Machlup, *Statics and dynamics: kaleidoscopic words*, 1959, which can today be found in Machlup, *Essays in Economic Semantics*, Englewood Cliffs, 1963. As for the Italian literature, see A. Montesano, “La nozione di economia dinamica”, *G.d.E.*, March-April 1972.

E.N. 8. PARETO'S 1902 ENCYCLOPEDIA ENTRY ON MATHEMATICAL ECONOMICS

Ch. III, §30, footnote 1. There are various reasons to recall this encyclopedia entry, which appeared in German in 1902 and was translated into Italian by Guido Sensini (*G.d.E.*, November 1906). I) The first reason is pointed out by Pareto himself in the footnote in question: the 1902 entry contains a rich history of pure economy. This fact certainly escaped the attention of Baumol and Goldfeld: see E.N. 54, on *Pareto and quotations*. II) It is never emphasized that the influential, although perhaps debatable, advice by Volterra (*G.d.E.*, April 1906) was given in response to two works by Pareto, including his 1902 encyclopedia entry. III) This entry contains a valuable auto-biographical allusion: Pareto lets the reader understand that as soon as he arrived in Lausanne, he started searching for some statistical invariants regarding distribution, in order to fill, by induction, a major gap existing in Walras' system: the endowments with which Walras' individuals enter the market. IV) In this encyclopedia entry, finally, Pareto clearly explains what would ensue if consumer preferences were abandoned (see E.N. 15).

It is also worthwhile adding that, based on a letter sent to Sensini by Pareto, Sensini's translation corrects a system of equations that appeared in the original German version.

On the interpretative consequences of ignoring the *Corrispondenza Sensini* (1948), see comments by Riccardo Faucci and by Alberto Zanni in the *History of Economic Ideas* 2009/2 and 2009/3.

E.N. 9. MISUNDERSTANDINGS ON PARETO'S "TYPE I" AND "TYPE II"

Ch. III, §§39–48 and 160–61. The well-known dispute between Pareto and Scorza, which is so evident in §23 of the Appendix to the *Manuale* (1906), resembles a comedy of misunderstandings, not so much because of mathematical issues, but because of the preliminary understanding of what one wishes to represent with the aid of mathematics. At any rate, in the *Cours* (Vol. I §59, footnote 1) Pareto had quite clearly specified that, in the case of free competition,

"it is necessary to differentiate by assuming the prices as constant; I also repeat it many times. And here is a talented mathematician, namely Scorza, who thinks that it is by mistake that I forgot to make prices vary! And so, here I am again, repeating things I have already repeated many times; and in the *Manuale*, in order to try to be understood, I give this thing a new form. I discuss about type I and type II. No use! Do you want to bet that there will still be someone who does not understand?"

(*Corrispondenza Pareto-Pantaleoni*, Vol. III, p. 57)

The origin of Pareto's peculiar terminology—"type I" and "type II"—is therefore explained by the above letter. The two "types" have given rise to misunderstandings, which we shall try to highlight here.

Everybody clearly understands that even in the case of entrepreneurs, types I and II are based on the distinction between intentional behavior and the unintentional effects of some behavior. Whilst in type I an entrepreneur accepts the market price—or prices—and his production unintentionally causes a change in the product price, in type II the firm is able to and intends to affect the price. No one has ever doubted that Pareto's type II refers to monopoly.

What if after a monopolist-inventor, a second, a third firm appear, until after a tenth firm none of the existing firms shows any profits or losses? In a word, from the second firm until the ninth (included), is it type I that applies, or type II?

If one reads the French appendix, §92 (1909, p. 622), one realizes that, in Pareto's view, type I applies and he uses the expression *free competition* to encompass the entire competitive process, predicted on the free entry condition, which he conceives in terms of temporary static equilibria (E.N. 7) and assuming *ceteris paribus* (for instance, the demand schedule does not vary). It is a fact that Pareto does not always distinguish the eight temporary equilibria of our example (we could call them eight different states of *oligopolistic competition*), which are intermediate between monopoly and Cournot's unlimited competition (see E.N. 32), by using two different terms. In short, the fact that Pareto did not always use the expression "limit state of free competition" to distinguish it from the intermediate states of our example has given rise to considerable misunderstandings. Indeed, some believed that Pareto's type I referred only to the limit state, and never to the intermediate states where oligopolistic profit exists. This incorrect

interpretation led to criticisms about some mathematical propositions by Pareto, which, when translated into usual language become the following correct propositions: for as long as competition at entry does not reach its limit state, the equilibrium of every firm is given by the equality between price and marginal cost, and there is no equality between total receipts and total costs. In the final state, price evens out on the marginal cost and on the average cost, and there is equality between the total receipts and the total costs of every firm. However, Pareto's explanation is anything but clear because he resorts to incidental expressions to signify the failure in reaching final equilibrium (he abandons the *ceteris paribus* assumption and introduces a variation in the demand), and because, different to Cournot, he must consider changes in general equilibrium when firms adopt type I behavior but nevertheless obtain oligopolistic profits.

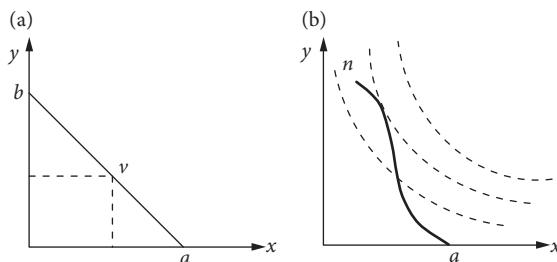
Confusion between statics and dynamics, confusion between average cost and marginal cost, failure to understand how equilibrium is determined in the case of increasing unit costs (and not all commentators realized that for Pareto, as well as for Amoroso after him, "unit cost" always means "average cost" and it never means "marginal cost") with all that follows: these are some of the criticisms levelled at Pareto by first rate mathematical economists. The fact is that Pareto did not point out clearly that type I also applies to oligopolistic competition; his generic "free competition" leads one to think that he goes straight from monopoly to unlimited competition, skipping the intermediate states. Furthermore, if one takes into account that, at times, Pareto talks about *monopoly with complete and incomplete competition*—perhaps in order not to be confused with other economists (E.N. 17)—one understands why in the varied world of historiography, he is never on the side of angels.

SOME REFERENCES. On the Pareto–Scorza polemic, see J. Chipman's seminal work, *The Pareian heritage*, 1976, as well as M. McLure, "The Pareto–Scorza polemic on collective economic welfare", *Australian Economic Papers*, 2000, pp. 347–371. On types I and II, see A.Z., "The adjustment mechanism of Cournot and Pareto in the monopolistic competition hypothesis", *Rivista Internazionale di Scienze Economiche e Commerciali*, June 1995. Our opinion is that in the *Manuale* (1906), while arguing with Scorza, Pareto clarified the difference between variations along a function and variations in a function, whereas it was only in the *Manuel* (1909) that he gave a hint that in the case of oligopolistic competition he was applying type I.

E.N. 10. PATHS

Ch. III, §60, §96, §172. Let us consider an individual, either a consumer or an entrepreneur, who can buy consumer goods or factors of production X, Y at the prices p_x , p_y , with a monetary income equal to M . The budget equation $M = xp_x + yp_y$ tells us that if the individual puts all of his income towards X, that is if $y = 0$, he will be able to buy a quantity $M/p_x = a$. Similarly, if $x = 0$, he will be able to buy the quantity $M/p_y = b$ of good Y. With M remaining unchanged, he will obviously be able to buy the infinite number of linear combinations, that is, the infinite intermediate pairs (x, y) between the

two extremes that we have just indicated. To use Pareto's language: the individual follows the path ab , the slope of which expresses the ratio between the two prices, that is, the relative price of the two goods.



However, Pareto does not start his analysis by assuming that the individual has an endowment of money M . He starts by assuming that the *initial endowment* is a quantity of good X alone—which the individual transforms in a quantity of the other good along the path ab —or a combination of the quantities of the two goods (for instance, the combination indicated by point v in fig. a). On the other hand, in Pareto's view rectilinear paths, that is the paths with a constant slope, are just one particular, albeit notable, case. In fact, by never ruling out relative price changes between any two points along the path an —a path that, together with the indifference lines, determines the optimum position of the individual (fig. b)—, Pareto effectively made an aspect of a well-known *barter controversy*, between Edgeworth and Marshall, his own.

According to N. Georgescu-Roegen ("Note on a proposition of Pareto", *The Quarterly Journal of Economics*, 1935), in the 1911 encyclopedia entry on *Économie Mathématique* Pareto was guilty of an oversight—he did not realize he was resorting to a "single equation" rather than to a system of equations.

Aldo Montesano, with whom the present writer (A.Z.) discussed this point, observes that

"Georgescu-Roegen does not refer to the *Manuel* (1909) because the oversight he criticises is only found in the entry on *Économie Mathématique* (1911). However, Georgescu-Roegen does not distinguish between the ideal experiment conducted by Pareto in the *Manuel* (1909) and that in *Économie Mathématique*, and he ends up (p. 713) by also pooling together in his criticism what is found in the *Manuel* (p. 542, that is, §5 of the mathematical Appendix). In fact, the experiment that Pareto establishes in §5 of the mathematical Appendix of the *Manuel* consists in directly identifying the marginal rates of substitution through an imaginary interview. Conversely, in *Économie Mathématique*, Pareto starts from the demand functions that have been obtained, in the more general case, on families of exchange paths (not necessarily rectilinear). The oversight that Georgescu-Roegen pointed out is limited to some considerations that Pareto makes about demand functions (equations (4) in *Économie Mathématique*). However, the oversight does not lead to error if the demand functions satisfy the integrability conditions (as pointed out by Georgescu-Roegen himself). On the contrary, if the integrability conditions are not satisfied, then the demand functions do not make it possible to obtain the marginal rates of substitution relating to each consumption vector, but only indifference

relations depending on the exchange path with regard to which that consumption vector is chosen.

Georgescu-Roegen indicates in a footnote that this problem is not connected to the path dependence that is found in the case of open cycles, probably because he believes that the order of consumption has to do with the pleasure of actual consumption and not with the path along which the consumer obtains possession of the goods. However, if one adopts the latter interpretation of order of consumption, Georgescu-Roegen's finding is strictly connected to open cycles where, as highlighted in note (13) by the editors of the French mathematical Appendix, the varieties of indifference depend on the path even when the marginal rates of substitution are independent of it."

The above comment draws on the reinterpretation of Pareto's theory of ophelimity outlined in the already mentioned (in the annotation to Ch.III, §35) essay by Montesano (2006), which avails itself of, but does not always coincide with, the noteworthy essay by the mathematician Giovanni Ricci (1951, cit.).

E.N. 11. TERMINAL POINTS

Ch. III, §62. In view of the importance of terminal points (or final points, or stop points) in Pareto's pure economy, particularly in the case of competition with production at decreasing unit costs, it is regrettable that he did not note that terminal points were first introduced to economic theory by Cournot for all types of markets, starting with the famous case of natural monopoly (*Recherches*, 1838, p.94). According to Cournot, a terminal point ("point d'arrêt") that prevents a firm from attaining a marginal optimum may depend on the conditions of supply—on the "forces productives" (and it is on these that Cournot's emphasis falls)—or on demand (see also E.N. 16. For further developments see A.Z. "Marshall and Sraffa on competition and returns in Cournot", *History of Economic Ideas*, 2012/1).

Ch. III, §94 shows that Pareto places the terminal points among the "obstacles of the first kind", which include "the obstacles that depend on the social system" (Ch. III, §69). In order to understand the generality and the importance of this latter class of terminal points, one need only think of laws that regulate working hours, such as those regulating shopping hours.

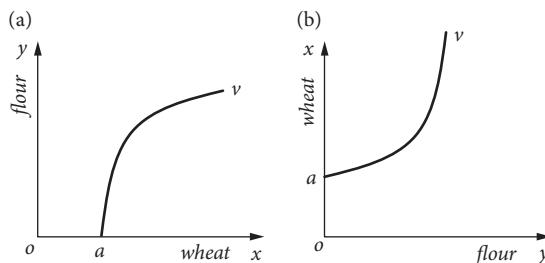
Scholars like Zawadzki (1914) who suggest that terminal points should be ignored do not take into account that Pareto (as well as Keynes subsequently) was not primarily interested in optimizing *a priori*, abstractly. Rather, he was mainly interested in the difficulties encountered in optimizing in the real world, in spite of all the accuracy that may have been used in forecasting. One can think, for instance, of an ultra-modern plant that has been planned on the basis of three work shifts but starts operating when, due to a range of institutional problems, barely two shifts are possible. Giovanni Demaria's well-known "entelechian facts" themselves are daily sources of terminal points (see, e.g., G. Demaria, *A New Economic Logic. Indeterminacy, Propagators and Entelechians*, Padova, Cedam, 1996). Even Pareto's own experiences as an industrial manager lead one to consider the optimization of production as it happens in the real world as a kind of Penelope's cloth that at times has to be thrown away.

E.N. 12. THE LINE OF COMPLETE TRANSFORMATIONS AND THE INDIFFERENCE LINES OF THE PRODUCER

Ch. III, §75 and Ch. VI, §4. Pareto refrains from introducing prices in terms of money at this stage of his analysis. It would, therefore, have been helpful to the reader if he had used a simple example to highlight that point, as he did in his correspondence of 1912 (see de Pietri-Tonelli, *Corrispondenza*, 1961, p. 119):

“when I was in Italy, the peasants would take a certain quantity of wheat to the small country mills and would get back a certain quantity of flour, no payment whatsoever took place, the miller would take wheat or flour as payment for himself”.

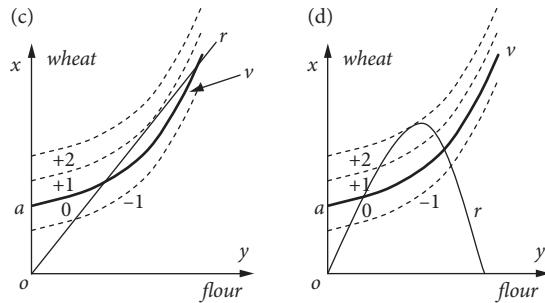
Let us, therefore, use the example of a mill that transforms X (wheat) in Y (flour) assuming, as Pareto does in fig. 9, that total production of flour incurs some fixed costs and grows less than proportionally to the quantity of wheat used in the transformation process. The curve *oav* in fig. (a) describes this situation. Pareto calls this curve “line of indifference of obstacles” or “line of complete transformations”. It does not differ from the usual representation of the production function. What is unusual, however, is the representation of the isoprofit curves, which Pareto calls “indifference lines of the producer”. In order to understand their meaning, it is advisable to represent the total cost function, fig (b), keeping in mind that Pareto measures costs in terms of units of good X.



In fig. (a) the curve *oav* represents the production function. In fig. (b) the curve *oav* represents the total cost function, with fixed costs represented by the segment *oa* and total variable costs represented by the section *av*.

The producer can choose the point that is most advantageous for him on the total cost line. If he wishes to maximize his profit, the producer plots a total revenue line and seeks the maximum difference between total revenue and total cost. The total revenue line is: linear if the producer is a price-taker, that is, if he operates according to type (I); or a curve, which typically increases initially and then decreases, if he has market power; that is, if he operates according to type (II). These two cases are represented by figures (c) and (d) below. The maximum difference is obtained by plotting curves that are parallel to the total cost line and identifying the one among them that is tangent to the total revenue line. Let us assign to each of these parallel curves, which Pareto termed the “indifference lines of the producer”, an index number equal to the distance between the particular indifference line and the total cost curve (measured in terms of units of X—the index is

positive if the parallel is higher than the total cost line, negative if it is lower). This index measures (in terms of units of X) the profit associated to each pair wheat-flour in the figures (c) and (d). Then Pareto's "indifference lines of the producer" can be thought of as isoprofit curves. On that basis, the greatest profit is attained at the point where the total revenue line reaches the highest index, that is where it is tangent to an isoprofit curve.



In figures (c) and (d), the line *or* represents two different total revenue functions. In fig. (c), the point of maximum profit is the point of tangency between the straight line *or* and the isoprofit curve (which is parallel to the total cost line) that has the index +1; in figure (d), the point of maximum profit is the point of tangency between the curve *or* and the isoprofit curve with the index +2.

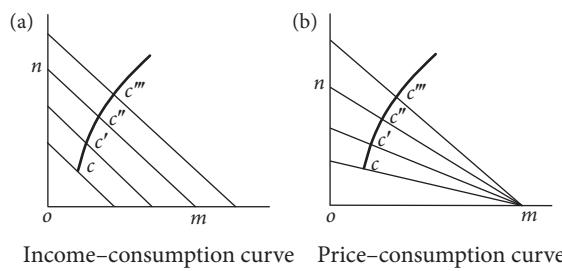
In short, the "indifference curves of the producer" are, therefore, parallel to the total cost line and the indices on them represent the profit obtained if the producer can choose one of their points. The choice possibilities, which Pareto calls "path", are represented, in the case of the producer, by the total revenue line.

SOME REFERENCES. As far as we know, L. Amoroso was the only scholar who dwelled on the indifference curves of the producer in Pareto. He did so in his *Lezioni di economia matematica* (Ch. III, §§34–35), Bologna, Zanichelli, 1921, by applying a geometrical method he had expounded in "La teoria matematica del monopolio trattata geometricamente", *G.d.E.* August 1911. He confined himself to the case of a monopolist who imagines that he is dealing with a total revenue line similar to that in our fig. (d). Amoroso (1921, p. 225) explained that the indifference curves of the producer are only plotted on the basis of the supposed knowledge of the production technology (the one established by the line of complete transformations), similar to the way that the indifference curves for the ordinalist consumer are only plotted on the basis of the supposed knowledge of his preferences.

E.N. 13. THE INCOME-CONSUMPTION CURVE AND THE PRICE-CONSUMPTION CURVE: ON A FIGURE BY PARETO

Ch. III, §97, fig. 12. Fig. 12 in the *Manuale* raises problems of interpretation that we shall try to solve here by degrees.

A first reading. The difficulties in interpreting this figure arise from the fact that: *i)* Pareto plots a decreasing path mn and an increasing curve that he calls the *exchange line*; *ii)* he specifies that c'' (point of tangency between the path mn and an indifference curve) is an optimum point, but he does not specify how to get to the other optimum points $c, c', c''' \dots$ that, together with c'' , form the exchange line. He limits himself to saying that the other optimum points are “situated on other paths that the individual may be assumed to follow”. Which other paths are they? Since the exchange line in fig. 12 is an increasing curve, they could indeed be the different paths—all issuing from point m —that are tangent to the indifference curves at the points c, c', c''' ; or, by the same token, they could all be paths that are parallel to path mn . In the latter case, the curve $cc'c''c'''$ could be called an *income-consumption curve*; in the former case, it could be called a *price-consumption curve*. The two curves—those in our figures (a) and (b)—only have in common the fact that when one moves from one point of equilibrium to another (from c to c' to $c'' \dots$), one has positive variations of the two goods.



If we had to decide on the basis of fig. 12 alone, we would have to opt for the income-consumption curve for the reason that we obtain points c, c', c'', c''' very near to those indicated in Pareto's fig. 12 if we use paths that are parallel to path mn and points significantly different from them if we use straight lines issuing from m .

A second reading. §97 of the *Manuale* (where fig. 12 is found) refers to §17 of the mathematical appendix. Pareto wishes to highlight how individual preferences could be represented in many ways: using indifference curves, exchange lines or preference lines.

“These lines are those with the maximum gradient on the ophelimity surface and are normal to the indifference lines”.

The preference curve is yet a third curve, different both from the income-consumption curve and from the price-consumption curve.

For instance, in the case of quasi-linear preferences represented by the utility function $u = x + \ln y$, one finds that at point $(1, 1)$ —which is the preferred point if income is equal to 2 and the price of good B in terms of A is equal to 1—there are the following three curves: the income-consumption curve, which is horizontal and has the equation $y = 1$; the price-consumption curve, which is vertical and has the equation $x = 1$; and the line of maximum slope, which has the equation $y = (2x - 1)^{1/2}$. Now, it is true that the preferences indicated by the indifference curves (that are supposed to be continuous and differentiable) are represented equally well by the preference lines (the latter are perpendicular to the former at every point). However, it is not true that preferences can

be represented by the price-consumption curves alone, or by the income-consumption curves alone, but both sets are needed. In other words, it is necessary to know the demand functions, that is, how choice depends on prices *and* income, and not just on the former or the latter alone. In this regard, in his *Considerazioni* of August 1892 (those quoted by Slutsky), Pareto starts by assuming that marginal utilities and prices are known, and from these data he derives the demands for the products at different prices. On the contrary, when he wants to go back from given prices to the unknown marginal ophe-limities (pp. 126–35), he introduces the marginal utility of the good “savings”. However, in his *Considerazioni* of October 1893 Pareto claims he has found a better solution: in 1893 he has the consumer's income (initial endowment of goods) vary, and for the first time proposes (p. 293) an *income-consumption* graph like the one shown in our fig. (a). More generally, in October 1893 Pareto proposes that all the laws of demand (supply) be studied by making both the point identifying the individual's income (initial endowment) and the angle (the price) of the path passing through it vary in commodity space.

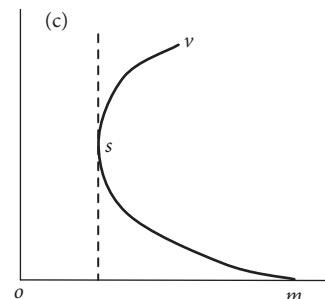
The doubt therefore arises that the ambiguity of fig. 12 stems from the fact that Pareto intended to use it to represent two related, but distinct, cases; the price-consumption curve and the income-consumption curve.

A third reading. Starting from the graphs in fig. 15 (1906, §106, p. 184), Pareto defines some curves shaped like curve *msv* in our fig. (c) as exchange lines.

There exist, therefore, two definitions, one explicit (§97) and one implicit (1906, §106, p. 184 ff.), that do not coincide. We believe that Pareto would have reached an agreement within himself if he had explicitly defined the *exchange line* as the curve *msv* in our fig. (c), that is the price-consumption curve.

Pareto's fig. 12 raises some doubts, but brings about a certainty. Starting from the already mentioned page 184 in the *Manuale*, and then various other instances, Pareto uses the expression *curva dei contratti* (“contract curve”) instead of *curva dei baratti*, (“exchange curve”). Pareto corrects himself only once, in the

Errata corrige. The reason why he is right in correcting himself will be explained in E.N. 19, which deals with the Edgeworth Box. There we shall see that Amoroso explains the difference between contract curve (Edgeworth) and an exchange line (Pareto) without realizing that he is consistently using *contratti* (“contracts”) instead of *baratti* (“exchanges”), and vice versa. Pareto's exchange lines were definitely born under a bad sign!



E.N. 14. PARETO FROM CARDINALISM TO ORDINALISM AND THE IDENTIFICATION OF PREFERENCES

The commonly held historiography on the measurement of utility, under which Pareto's ideas are presented as developing from his initial ‘cardinalist’ phase to his more mature ‘ordinalist’ phase, is not satisfactory. That view ignores various facets of Pareto the cardinalist. More importantly, it ignores the most enigmatic aspect of Pareto the ordinalist:

namely, the fact that he gives up cardinal measurement of ophelimity for pure economy, but continues to ask himself whether an index of ophelimity represents cardinal ophelimity.

As it is not a simple matter to interpret Pareto on this issue, we shall try to offer the reader something of a guide, aimed at reconnecting the earlier, cardinalist Pareto—the author of the 1892 and 1893 *Considerazioni*—with the later, ordinalist Pareto—the author of *Économie mathématique* (1911)—while also illustrating some peculiar aspects of Pareto's ordinalism.

In the 1892 and 1893 *Considerazioni* Pareto expounded his theory of demand, which he would never abandon, and which already contains all the relations that express the dependence of consumption choice on prices. These relations were later arranged and transmitted by E. Slutsky in his famous 1915 essay. It is true that here Pareto maximizes an ophelimity function that is numerically defined, but the result of this maximization does not depend on the cardinality of the objective function and therefore remains unchanged even when cardinal ophelimity is replaced by the ordinal index-function. Pareto is fully aware of this and, in the *Manuale*, he restricts himself to repeating the previous analysis on this topic, although he imposes properties on his specification of the ophelimity function that are incompatible with the ordinalist statute. Those properties were later corrected by J. R. Hicks, who introduced a reformulated version of Pareto's ordinalism to the Anglo-American world.

However, Pareto, who had always posed for himself the problem of the nature and of the measurement of utility and ophelimity, realized that measurement is not necessary for the construction of general equilibrium and for the analysis of demand. In his encyclopedia entry *Anwendungen der Mathematik*, 1902, (which Pareto refers readers to in the *Manual* §30, footnote 1, of Chapter III), Pareto wrote:

“Knowledge of these projections [the indifference lines] suffices to construct the fundamental equations of economic statics. As against this, we need not know whether or not pleasure (utility, ophelimity) is a measurable quantity in the mathematical sense of the word; still less do we need an exact measure of pleasure; the knowledge of the indifference lines suffices. The only measurable quantities on which the reasoning is based are the commodities themselves.” (p. 172)

In the encyclopedia entry *Économie mathématique* (1911, §15), Pareto revisits this topic (the following extract is from the English translation of that entry published by *International Economic Papers*, 5, 1955):

“There is something superfluous in this theory for our purposes, which is the determination of equilibrium; and that which is superfluous is precisely that which is dubious in the theory. As a matter of fact, in order to determine economic equilibrium we have no need to know the measure of satisfaction; an index of satisfaction is sufficient. From this consideration springs the theory which V. Pareto began to set forth in 1898, a theory subsequently developed and presented here in its general form”.

Though not necessary for the theory of equilibrium, a cardinal measurement of ophelimity may anyway be useful for other purposes. In that regard, Pareto, who still defined Bentham's considerations as “lucubrations”, did not appear to have totally abandoned the Benthamism that had been transmitted to him by J. S. Mill, and which is manifest in the *Considerazioni* (1892). In this line of thought, a measurement, albeit rough, of the

ophelimities of the various classes of the governed constitutes a guide in the political choice among the maxima of utility for the society (see E.N. 31). For Pareto, the maximum of utility in sociology represents the unavoidable absolute discretion of politics, whereas the maximum of ophelimity in pure economy represents, perhaps, what is left of the dream Pareto had entertained as a young pacifist radical and a passionate follower of the Enlightenment (see E.N. 57). Nevertheless, in his high idea of politics Pareto never completely gave up the Benthamite habit of his youth, nor did he consider it irreconcilable with the respect of the most deserving in all the social classes. In fact, the recognition of merits was one of the aspects of his productivistic vision of distribution: let it be the largest possible pie that is distributed. We believe that Pareto never completely turned his back to cardinal utility—which is not indispensable either for general equilibrium or for welfare economics—even because of that sentiment of benevolence with which in our mature years we tend to look back at our own youth. On the other hand, it is not difficult to understand that an economist as mathematically prepared as Pareto was interested in knowing all the formal conditions for a cardinal measurement of utility.

Ordinalism exacerbates, for Pareto, the problem of the experimental identification of preferences, the latter being the starting data in the analysis of individual choices. Pareto follows two paths. In the first path he establishes individual's preferences through an ideal experiment in which the consumer is asked to indicate his preference between two alternative consumption bundles, and for many pairs of possible alternatives; by using this path Pareto identifies the marginal rates of substitution and, possibly, the individual curves of indifference. The other path has the preferences deriving from choice, that is, from the functions of demand, as indicated, for instance, in the French Appendix, §42 (where the functions expressing the marginal rates of substitution are determined by means of the inverse functions of demand).

A theory that may explain preferences is quite a different problem from their simple identification and representation. Pareto is not hiding behind the old tenet *de gustibus non est disputandum*. Let us take our cue from this topic (see point (b) below) in order to briefly discuss the important distinction that Pareto makes between "logical actions" and "non-logical actions".

In the real world, every human action is "synthetic", that is, it contains a multiplicity of characteristics. Pareto believes that in analyzing them, it is worthwhile to separate these characteristics according to the purist way introduced by J.S. Mill. It is from this purism that Pareto's distinction between logical and non-logical actions originates. In Pareto's interpretation, the latter actions show a deviation between the *subjective* means-goal relationship, as it appears to the awareness of the acting individual, and the *objective* means-goal relationship, as it appears to *persons other than the acting individual*, to persons endowed with the best knowledge and driven by methodical spirit of observation—in short, to followers of J. S. Mill's concrete deductive experimental method (see E.N. 1). If, on the contrary, there is a coincidence between subjectivity and objectivity, then we are dealing with logical actions.

Pareto (*Sociologia*, 1923, §§149–151) distinguishes four categories of non-logical actions. Unfortunately, while both economists and sociologists observe that Pareto's non-logical actions are characterized by being dominated by sentiments and instincts, only economists point out that: a) Pareto's pure economy only explains actions that are repeated and experimented many times, and which therefore are logical in the

specified sense; b) although determined by preferences where sentiments and instincts prevail, even non-logical actions may be represented, in Pareto, by marginal rates of substitution.

It is in a short footnote of the *Sociologia* (§159, n.2) that Pareto includes the example of non-logical actions of the fourth category that should be of particular interest to economists, but that, on the contrary, as far as we know, economists ignore, as also do a number of Pareto scholars such as G.H. Bousquet, R. Aron and N. Bobbio. Pareto places the monopolist's actions in pure economy, among the logical actions, whereas he deems the actions of every single individual in a competition regime as non-logical. Pareto observes that in a competition regime, the acting individual ends up by obtaining a different result from the desired one, he ends up in the region of zero or negative profits instead of the region of positive profits. In Pareto's classification, the actions of an individual in a free competition regime belong to the kind of non-logical actions that an individual *ex post* would not repeat. In this case, of course, Pareto personifies the category of the individuals endowed with the best knowledge.

In Pareto's view, it is the activity of production, rather than the activity of consumption, that generates the strongest impulse towards rational choices; however, as it has just been shown, his *Sociology* identifies a significant non-logical component even in the production activity.

SOME REFERENCES. The encyclopedia entry V.P., *Anwendungen der Mathematik*, 1902, is mentioned above. In addition, three of Pareto's articles published in the *G.d.E* are important to developing an understanding of both the cardinalist and the ordinalist sides of Pareto's work. Two of them are the August 1892 *Considerazioni* and the October 1893 *Considerazioni*, with the following difference: whilst the October 1893 *Considerazioni* are mostly important to understand Pareto the analyst, the August 1892 *Considerazioni* help the reader better understand the politically significant reasons (see for instance pp. 136–37 and p. 144—respectively pp. 58–59 and 64–65 in the Engl. trans.) why he was interested in a numerical, albeit rough and indirect, measurement of ophelimity, and why he was therefore unhappy that Jevons, who had felt a similar need, had embraced the constant marginal utility of money. These are pages in which the Benthamism absorbed by Pareto in his youth show through. The third article, the June 1892 *Considerazioni*, contains Pareto's greatest scientific debt towards Pantaleoni. On this topic see E.N. 25 and E.N. 32.

In regard to Pareto's cardinalism (August 1892), we limit ourselves to one consideration: the fact that Pareto supposes that a numerical measurement of ophelimity exists in the same way the distance between the Earth and any star is believed to exist, can be questioned with the introspection (a tool that is undervalued in Pareto's *Sociologia*, even though an appreciation of it is clearly present throughout his private correspondence). In other words, whether or not ophelimity is cardinally measurable is a question each person has to ask themselves.

As for non-logical actions, we would like to add that in published university lectures from 1962, R. Aron (*Emile Durkheim, Vilfredo Pareto, Max Weber*, Paris, Centre De Documentation Universitaire, 1962, p. 103) proposed the following provocative example of Pareto's fourth category of non-logical actions, the category that in the present E.N. we have exemplified with competition as opposed to monopoly: those who backed the

Russian Bolshevik revolutionaries would not have acted as they did if they had imagined that from gaining power as they had wished (Pareto, *Sociologia*, 1923, §151, p. 65, writes Yes, Yes, respectively to the question of whether there is a subjective connection and an objective connection in an action based on a means-ends relationship), they would eventually obtain a heavy tyranny, instead of the maximum freedom they desired. For action to be logical, Pareto not only requires a 'Yes, Yes' response to questioning about the existence of subjective and objective connections, but he also requires that the actor reveal no regrets over the outcome of her action, as evidenced by an action being repeated by the actor when circumstances are unchanged.

E.N. 15. PARETO AND THE ALTERNATIVE BETWEEN PREFERENCES AND DEMAND

Pareto was not fond of superfluous notions for the theory of equilibrium, but could not but discuss Cassel's proposal to introduce the functions of supply and demand directly, without any mediation, in a manner that does away with the notion of utility: not just cardinal utility, but utility in general including ordinal utility.

Cassel's name appears in the Pareto–Scorza polemic (1902), but by the time Pareto had finished *Les Systèmes Socialistes*, he already knew that Scorza would review an 1899 essay by Cassel. Pareto saw Cassel's position as one which advocates "a return to Cournot" in the sense that he refuses to incorporate the preferences within the analytics that underlie representation of supply and demands. Pareto explicitly discusses this issue in his 1902 German encyclopedia entry (see E.N. 8), where he shows the equivalence of the two starting points (preferences or demand functions) in determining equilibrium. This equivalence was probably interpreted by Barone as a *crescite et multiplicamini*. It is indeed in Barone's famed *Ministro della produzione* (1908) that one finds a general equilibrium and the disappearance of ordinalist indifference curves. The equivalence is also argued by H. von Stackelberg ("Zwei kritische Bemerkungen zur Preistheorie Gustav Cassells", in *Zeitschrift für Nationalökonomie*, 4, 1933, pp. 456–472), who refers extensively to the *Manuel* and to Pareto's 1911 *Économie mathématique* entry, but does not quote the above mentioned German encyclopedia entry.

Well, Pareto saw preferences and demand functions as compatible and never changed his opinion in this regard. But he was by no means prepared to do away with preferences. Without preferences it would not be possible to introduce the principle that individual choice is intentional (in the sense that it maximizes ophelimity) and the very definition of Paretian optimality would disappear, and the notion of allocative efficiency would have no solid foundations. As Pareto (1902) puts it:

"These equations represent the laws of *supply and demand*. One can put them in place of systems (3) and (7) [which respectively indicate the equality between weighted marginal ophelimities and the individual budget constraints, as pointed out by Sensini, in his 1906 Italian translation of this paper, when correcting an oversight by Pareto which was suggested by Pareto himself]. But, if the [supply and demand] functions are taken as given, we do not know whether they are consistent with system (3) i.e., whether or not they are conducive to the maximum of pleasure, and we then miss the most important aim of economic research." (Pareto, 1902, English translation, 2009, p. 178)

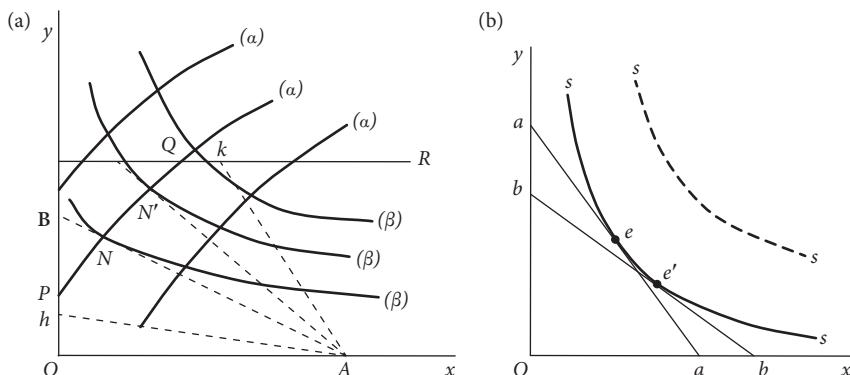
The full equivalence between preferences and demand implies the possibility of determining the latter based on the former, and vice versa. The determination of demand (and supply) starting from preferences is commonplace in economic theory. It is in this way, for instance, that Walras and Marshall proceed when starting from cardinal utility functions. And so does Pareto, who starts from indifference curves or, more generally, from the functions representing the marginal rates of substitution. The opposite path, from functions of demand to preferences, was introduced by G.B. Antonelli, who showed that a utility function that rationalizes the functions of demand does not always exist. It exists if the integrability conditions are satisfied. As is well known, Pareto tackled questions that should have led him to the integrability conditions on a number of occasions, but he never succeeded in writing them down correctly for the problem under consideration. On the other hand, Pareto does not necessarily take the index-function of ophelimity as the starting data in the most general case (for instance in the French appendix), rather, he commences from the functions that express the marginal rates of substitution. In other words, Pareto does not require, for the individual, global rationality (that is, complete and transitive preferences), but only local rationality (that is, for every set of three goods A, B, and C, the marginal rate of substitution between A and C is equal to the product of the marginal rates of substitution between A and B and between B and C). All that Pareto required as starting data with regard to preferences were the marginal rates of substitution, so the path from demand to preferences was immediately covered by taking into account the inverse functions of demand. In other words, the integrability conditions are necessary if it is required that preferences be represented by an ordinal utility function; they are not necessary if it is required that preferences be represented by marginal rates of substitution.

SOME REFERENCES. With regard to G. Scorzà's review of Cassel (1899), see *G.d.E.*, August 1902. As for the alternative between preferences and demand, one should always read Pareto's (1902) encyclopedia entry. On Pareto's continuity about the compatibility between preferences and functions of demand, see *Sociologia*, §2408 (footnote 1). While the fact that this was the path followed by Barone (1908) is well known, we mention again the position contained in Pareto's (1902) encyclopedia entry. As for the relationship between Pareto and G. B. Antonelli, perhaps Pareto did reach his positions on his own. Anyway, all that we know for certain is: that Pantaleoni lent his copy of Antonelli's book to Pareto; that Pareto read it and returned it to Pantaleoni before moving to Lausanne; and that there is now no trace of the book at the University of Lausanne (either among the books that belonged to Pareto, or among the other books held at the University's *Centre Walras-Pareto*, where we were kindly assisted by Prof. Pascal Bridel and by Prof. Roberto Baranzini).

E.N. 15. BIS. PARETO–HICKS AND ZAWADSKI–SLUTSKY (a continuation of the previous three notes)

Our doubts about the interpretation of Pareto's fig. 12 (E.N. 13), together with a conviction that in October 1893 (*G.d.E.*, p. 293) Pareto had introduced income-consumption

curves, led us to reread a number of works. Our attention was finally attracted by the curves (β) of the graph in Pareto's *Economie Mathématique* (1911) that accompanies some equations, which specify the prices of the goods and the income of any one individual (the value of his initial endowment of goods) as parameters. In Pareto's figure—which is our (a)—besides the curves (a) (preference curves), we also find some curves (β) . Pareto supposes that, in the space of goods A and B, the individual has an income OA, and that the optimum is at point N, if the relative price between A and B is expressed by the slope of the constraint AB. So far there is nothing new compared with what was attributed to Pareto by Hicks in *Value and capital* (1939). The novelty arrived when reading Pareto's very economical definition of a curve (β) . Pareto (1911, p. 323) writes: "Starting from point N, let us trace curve (β) , which is the envelope of the tangents such as ANB" (p. 62 in the English transl. quoted in E.N. 14). Now, if out of this infinite number of tangents we only take two, as in the following fig. (b), we realize that Pareto had in mind the reasoning that led Hicks to introduce the *compensated demand curve*. We were aware of Hicks' scrupulousness, we also checked *A revision of demand theory* (1956). However, in this work Hicks states that, while starting from Pareto, he arrived at the compensated demand curve on his own, from which we infer that Hicks had not noticed Pareto's characterization of curves (β) .



Was it worthwhile to call attention to the fact that Pareto preceded Hicks? Yes, but only to point out that in 1918 Pareto had publicly expressed his desire for the history of economic theory to be written. As everyone knows, Walras had hoped for it to be written by Barone. As for Pareto, we know that for the bibliographical part of *Économie Matématique* (1911) he asked Pantaleoni to help him, together with L. Amoroso and Umberto Ricci (see E.N. 54). We also know that with regard to Johnson's (1913) failure to quote Pareto, it was Amoroso (1916) who pointed out the existence of a previous work by Pareto. However, our impression is that Pareto was hoping for U. Ricci to write this history.

Let us now come to the second question in this note. Why was it necessary for V. Dominedò (1933) to write on demand for Slutsky to be discovered in Italy? Or rather, why did Amoroso—who was mathematically equipped and an assiduous reader of the literature on Pareto—not write something on the decomposition of the effect of a price

variation into a substitution effect and an income effect? The following is a possible explanation.

When W. Zawadzki, *Les mathématiques appliquées à l'économie politique*, Paris, M. Rivière, 1914, appeared, Amoroso (1914, p. 418) wrote a brief review of it. In short, Amoroso thought that Zawadzki had well understood and re-expounded Pareto's theories, and he had only omitted some recent studies about economic dynamics. Amoroso could have added something else. Specifically, Zawadzki shared Pareto's (1911) criticism of Cournot, and we have the impression that L. Amoroso and F. Vinci had discussions with Pareto on how to interpret Cournot in a different way. But Amoroso's remark on dynamics in Pareto was correct, whether one looks through Zawadzki's summary index, or whether one considers that Amoroso was working with Pareto to delve into a problem that was increasingly crucial to Pareto: statistical interpolations (see E.N. 5). Anyway, when one opens Zawadzki's book on pages 142–3, 180, 240, 245, 275–92, one realizes that he had thoroughly scoured Pareto's works, including Pareto (1911). The only missing work was Pareto (1913), *Il massimo di utilità per una collettività in sociologia*. More particularly, on pp. 181–6 one finds the same expressions with which in the *Manuel* Pareto went once again through his own August 1892 analysis, which also regarded substitution effect and income effect. Even though Zawadzki does not specify it, his equations on p. 181 are equations (74), (75), (76) of the *Manuel* (1909, p. 581) and the determinants on p. 182 are those that appear in the *Manuel* (1909, pp. 579–80). In Zawadzki one also finds a consideration that would become common sense among "indeterminists": dynamic reality contains more than just the effects of the variations in the relative prices of goods on their demands, just as reality contains more verses than Horace's poems. How much could Amoroso have added to the substance that was already present in Pareto and Zawadzki on this topic? Was it not preferable, from his point of view, to devote himself to other topics?

We do not know if there were any connections between Zawadzki (1914) and Slutsky (1915). We can only add that Zawadzki often does not appear in the best bibliographies, including the outstanding bibliography in Chipman (1976), even though as far as the theory of production and welfare is concerned, one can learn more from Zawadzki than from other general equilibrium theoreticians. It does not seem to us, for instance, that Zawadzki thought that Pareto confused average costs with marginal costs and vice versa, or that he confused statics with dynamics and vice versa, as it has sometimes been claimed.

AN EXPLANATION AND AN ACKNOWLEDGMENT. The present E.N. 15 bis touches upon topics that for once we did not deem should be compressed in a bibliographical note. Our debt towards Zawadzki is implied in what we have stated. On the contrary, we wish to express our gratitude for Aldo Montesano's help in interpreting the more strictly analytical aspects of the theory of consumption that are contained in the E.N. 14–15.

The following works were mentioned: Pareto, *Économie Mathématique*, 1911, in V.P., *Oeuvres*, Vol. VIII, 1966; W.E. Johnson, "The pure theory of utility curves", *Economic Journal*, December 1913; L. Amoroso's review of Zawadzki, *G.d.E.*, May 1914, p. 418; L. Amoroso, "Sulla teoria delle curve di utilità", *G.d.E.*, May 1916; V. Dominedò, "Considerazioni intorno alla teoria della domanda", *G.d.E.*, January and November 1933; and J. Hicks, *Value and capital*, Oxford, Clarendon Press, 1939.

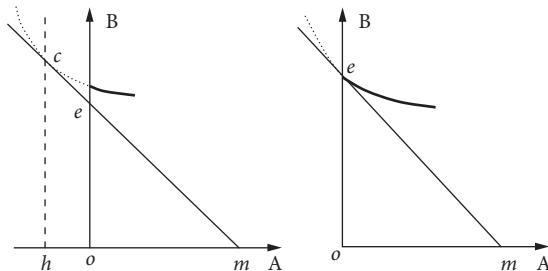
E.N. 16. NON-MARGINAL EQUILIBRIA IN PARETO

Ch. III, §98. With regard to entrepreneurs and consumers, we shall define *non-marginal equilibrium* as the equilibrium that Pareto determined by a terminal point rather than by the equality at the margin between different entities (*marginal equilibrium*).

Let us translate what Pareto states in §98 into graphical form. One would have an instance of *non-marginal* equilibrium if the indifference curves were, as illustrated in our graph on the left, where e is a point of non-marginal equilibrium. On the contrary, one would have an instance of marginal equilibrium at point c if the individual under consideration had at his disposal hm of A, rather than om .

In Ch. III, §98, which introduces the general possibility of non-marginal (or boundary) equilibria, Pareto does not refer the reader to Ch. IV, §54 (where he refers the reader back to Ch. III, 98). In Ch. IV, §54, Pareto analyses the particular case of an agent who uses money, which is not directly ophelious for him, for purchasing a directly ophelious commodity.

Our graph on the right indicates a borderline case of marginal equilibria in Pareto.



Obviously, a curve of indifference passes through point e in the graph on the left—it is the curve of indifference that is actually reached.

E.N. 17. INCREASING AND DECREASING COSTS, MARGINAL AND NON-MARGINAL EQUILIBRIA IN PARETO

Ch. III, §§100–101. If one compares §§100–101 with §151 of Ch. III, it is evident that one must distinguish clearly between Pareto's conception of complete and incomplete competition and Pareto's conception of free competition. In §151 Pareto uses the expressions complete and incomplete competition to mean, unequivocally, the presence of decreasing unit costs and increasing unit costs. Indeed, in this paragraph he talks about a *monopolist* (type II) in the cases of *complete competition* and of *incomplete competition*. How can one explain why Pareto's resorted to these enigmatic expressions?

Let us go back and start from what it seemed clear to us (E.N. 9): Pareto relates type I to free competition. The latter always implies freedom of entry and also involves oligopolistic competition that precedes the limit state of free competition, besides the limit state of free competition itself (E.N. 21 and E.N. 32). It also seems certain to us that Pareto could not have shared Walras' position, according to which, thanks to the

presence of fixed costs, decreasing unit costs are enough for there to be a monopoly (as Walras wrote in a letter to Bortkiewicz contained in Vol. II of the *Correspondence of Leon Walras*, brilliantly edited by W. Jaffé. North-Holland, Amsterdam, 1965). In Pareto's opinion there are various difficulties (terminal points) that in real life prevent firms from moving rapidly down the curves of decreasing unit costs. But by no means did Pareto intend to exclude the emergence of an industry that remains in a state of monopoly due to decreasing unit costs, even in the presence of complete freedom of entry, or the emergence of monopoly due to a process of competitive selection on costs. Whatever explanation one may offer for this terminology, the following table can help to remind us that Pareto associated complete free competition with the absence of differential profits (that is, profits that can be generated through the marginal increase in production) and incomplete free competition with the presence of differential profits.

	INCREASING UNIT COSTS	DECREASING UNIT COSTS
Competition	Incomplete (presence of differential profits)	Complete (absence of differential profits)
Equilibrium	Marginal	Non-marginal

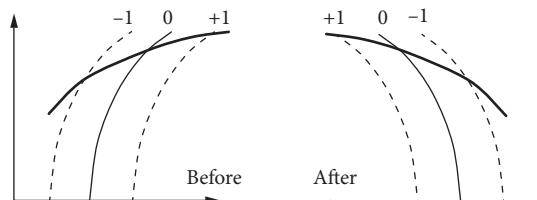
Here and in the rest of these notes we shall avoid using the expressions “decreasing returns to scale” and “increasing returns to scale”. In their place, we will respectively use increasing unit cost productions and decreasing unit cost productions. The reason, presumably, is the same as Pareto's for avoiding these expressions. The fact is that Marshall, in his *Principles*, resorted to these expressions, especially in reference to the long term, as he defined it (plants and technology change), to mean the dominance of human ingenuity over the nature's avarice, and vice versa. Now, in our interpretation (A.Z., “The Pareto's monologue with Marshall”, *Quaderni di Storia dell'Economia*, 1991, n.2/3, pp. 399–423), Pareto does not distinguish between increasing cost and decreasing cost productions with respect to Marshall's notion of the long period. Pareto's static, monoperiodical analysis always relates to a period considered as a unit (see E.N. 7), which comes after a “past” and before a “future”. But it is only from the context that it is possible to infer how much time has gone by—whatever be its length—to arrive at the period under exam. It is *only under the hypothesis of unlimited competition* that Pareto's monoperiod comes after a long period of time, even though this length varies according to the type of production and organization considered.

We also take the opportunity to add the following remark. It is often stated that Pareto's equilibrium is a *temporary equilibrium*. This is a legitimate expression, as are, too, the expressions “decreasing returns [to scale]” and “increasing returns [to scale]”. However, it must be recognized that: *i*) Marshall talks about *market equilibrium* or *temporary equilibrium* with some reluctance and with the warning that it is not even an equilibrium, given that in market equilibrium, or temporary equilibrium, as he defines it (similarly to what Walras assumes in the *theory of exchange*, that is before introducing production and accumulation), he studies the consequences of variations in demand by assuming production as given; *ii*) Pareto's temporary equilibrium is by no means bound to a given production; in other words, it can take into account production and accumulation.

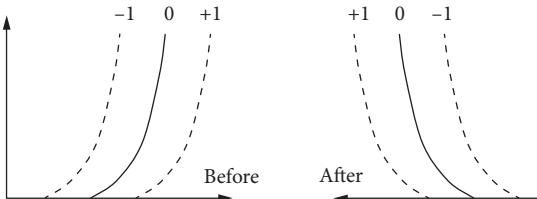
E.N. 18. COMPLETE AND INCOMPLETE COMPETITION AND RELEVANT GRAPHS

Ch. III, §106. Figure 15 shows the following novelty: it has an abscissa *om* delimited by two coordinates, one having origin at *o*, the other at *m*. What follows from it is implicitly obtained, but it would have been helpful if Pareto had alerted readers that the origin of the axes is *o* for the consumer; whereas, the origin is *m* for the producer. Pareto limited himself to the case of increasing unit costs (§100, fig. 13) and overlapped fig. 12 (which is the one containing the exchange line for the consumer) and fig. 13, relating to the producer with increasing unit costs, after rotating it by 180° around the ordinate axis. He proceeded similarly for the case of the producer with decreasing unit costs (no line of maximum profits).

In order to understand Pareto's graphs, it is useful to keep in mind that, as the origin of the graph shifts from the left to the right hand side, rotation of the producer's indifference curves demonstrates the following geometric configuration:



Incomplete competition (increasing costs) before and after rotation



Complete competition (decreasing costs) before and after rotation

E.N. 19. EDGEWORTH BOX AND PARETO BOX

Ch. III, §116. It is not easy to understand how the “Edgeworth box” derives from a graph in his famous *Mathematical Psychics* (1881). From Edgeworth's path breaking article (“The theory of international values”, *The Economic Journal*, Sept. 1894, pp. 425–26) it would appear that he drew two maps of opposing indifference curves on a graph together with Marshall's *reciprocal demands* (*The pure theory of foreign trade*, 1879).

It is impossible that Edgeworth's original brainwave—for it must be considered as such, especially when we consider that Marshall never used indifference curves—had escaped Pareto's attention, for two main reasons. First, a copy of Marshall's (1879) work had passed from Pantaleoni's library into Pareto's own library in Florence as soon as the two met, and Pareto at once wrote admiringly about it in his own “Considerazioni” (G.d.E.,

May 1892, p. 398, Engl. trans. p. 7). Second, Edgeworth's 1894 work was available to Pareto: apart from the fact that in it Edgeworth wrote at some depth on Pareto, this work can still be consulted to this day at the *Centre Walras-Pareto* in Lausanne, along with some other books that belonged to Pareto. Among the latter there is also the famous edition of Marshall's *Principles* (1891), where Marshall deals with Edgeworth and his graph, although by using the stick more than the carrot. This being the case, it is not even imaginable that Pareto could claim as his own the additions, in the *Manuale*, that allowed him to complete and make the Edgeworth box wholly understandable. In fact, Pareto continued to credit Edgeworth with the indifference curves even when he passed from "cardinalism" to "ordinalism".

The incomplete way in which Edgeworth had constructed the box was pointed out by W. Jaffé ("A review article", *Journal of Economic Literature* X (1972), by N. Georgescu-Roegen ("Vilfredo Pareto and his theory of ophelimity", in *Accademia dei Lincei*, 1973, cit., p. 130), and by M. Allais ("The general theory of surplus and Pareto's fundamental contribution", in *Accademia dei Lincei*, 1973, cit. p. 150). They went as far as to talk about "intellectual banditism" and renamed the Edgeworth box using Pareto's name. Our opinion is that Pareto's name could be added to Edgeworth's when referring to the box diagram, even though Pareto himself would be surprised by this, but his name could not replace that of Edgeworth. But it would be advisable to leave the 'box' aside for the moment and consider its contents, and, after that, try to distinguish clearly between Edgeworth's and Pareto's conceptions of the 'box' based on that criterion alone.

First of all, it is worthwhile to point out that Marshall (1891), who was the most qualified person to understand how much of his own theories flowed into Edgeworth's *Mathematical Psychics*, dealt with it as a mathematician and gave no importance to the fact that in Edgeworth's graph there is a single origin of the coordinates xy , instead of two opposing origins. In a course of lessons (1940-1) at the *Reale Istituto Nazionale di Alta Matematica*, L. Amoroso, too, ignored the box, but he also did something else. In the case of two goods and two traders, Amoroso distinguished between Pareto's *exchange curves* and Edgeworth's *contract curve*, and between an equilibrium obtained through Pareto's curves and an equilibrium obtained through Edgeworth's curve. We copy the demonstration and a graph by Amoroso here below, with the warning that the square brackets inserted in the text mean that we have substituted "exchange curves" for Amoroso's "contract curves", and "contract curve" for Amoroso's "exchange curve"—there was already enough terminological confusion between exchanges and contracts in Pareto (E.N. 13).

For two individuals and two goods, Amoroso starts from the following system by Pareto:

$$(6) \quad \begin{cases} \partial\Phi/\partial x_1 = 1/p \partial\Phi/\partial y_1 & \partial\Psi/\partial x_2 = 1/p \partial\Psi/\partial y_2 \\ x_1 - a_1 = p(b_1 - y_1) & x_2 - a_2 = p(b_2 - y_2) \\ x_1 + x_2 = a_1 + a_2 & y_1 + y_2 = b_1 + b_2 \end{cases}$$

where x_1y_1, x_2y_2 are the equilibrium coordinates for the first and the second individual, respectively; a_1b_1, a_2b_2 are their initial endowments; $\Phi(x_1, y_1), \Psi(x_2, y_2)$ are the two

index functions of ophelimity. Having supposed, in order to fix the ideas, that $a_1 < a_2$, Amoroso assumes, as Edgeworth does, that:

$$x = x_1 - a_1, \quad y = b_1 - y_1$$

and therefore

$$x = a_2 - x_2, \quad y = y_2 - b_2.$$

"We shall have

$$(7) \quad \begin{cases} \partial\Phi/\partial x_1 = +\partial\Phi/\partial x & \partial\Psi/\partial x_2 = -\partial\Psi/\partial x \\ \partial\Phi/\partial y_1 = -\partial\Phi/\partial y & \partial\Psi/\partial y_2 = +\partial\Psi/\partial y \end{cases}$$

and Pareto's equations [Amoroso is referring to system (6)] then become:

$$(8) \quad \begin{aligned} \partial\Phi/\partial x + 1/p\partial\Phi/\partial y &= 0, & \partial\Psi/\partial x + 1/p\partial\Psi/\partial y &= 0 \\ x &= py \end{aligned}$$

By eliminating p between the first and the third of equations [8], one obtains a curve that represents how the point of equilibrium of the first individual, 'Primus', varies as price changes. We shall call it Primus' [*exchange*] curve. Its equation is the following:

$$(9) \quad x\partial\Phi/\partial x + y\partial\Phi/\partial y = 0$$

where it is $\Phi = \Phi(a_1 + x, b_1 - y)$. Similarly, the [*exchange*] curve for the second individual, 'Secundus', is:

$$(10) \quad x\partial\Psi/\partial x + y\partial\Psi/\partial y = 0$$

where it is $\Psi = \Psi(a_2 - x, b_2 + y)$.

Finally, by eliminating p between the first and the second of Edgeworth's equations [Amoroso is referring to system (8)] one obtains a curve that represents the locus of the points of contact of the indifference curves of the two traders. This is called the [*contract*] curve and its equation is the Jacobian of the two functions Φ, Ψ :

$$(11) \quad \partial\Phi/\partial x - \partial\Psi/\partial x - \partial\Phi/\partial y - \partial\Psi/\partial y = 0.$$

The indifference curves form two bundles, the equations of which are, respectively:

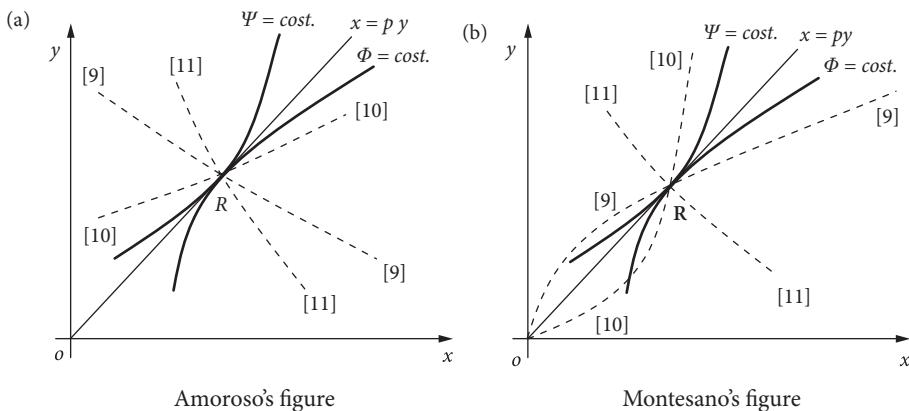
$$(12) \quad \begin{cases} \Phi(a_1 + x, b_1 - y) = \text{constant} \\ \Psi(a_2 - x, b_2 + y) = \text{constant} \end{cases}$$

and equations (8) tell us that the straight line $x = py$ is tangent to the two curves, one from the first and one from the second bundle, that pass through the point of equilibrium.

Thus, the point of equilibrium R can be identified: (a) as the intersection between the [exchange] curves (9) and (10); (b) as that point on the [contract] curve at which the tangent that is *common* to the indifference curves (12) passes through the origin of the coordinates." (Amoroso, pp. 50–51).

At this point Amoroso shows the following graph (a) where, as in Edgeworth, the box is missing and where the numbers in parentheses are those from Amoroso's equations.

Reading this E.N., in order to remove some imperfections in fig. (a), Aldo Montesano has revised it also in order to show its relationship "with the representation in the box". We also add this version: fig. (b).



Amoroso therefore clarified that Pareto's exchange curves are different from Edgeworth's contract curve, even though they have the equilibrium point in common. We can therefore distinguish between an Edgeworth diagram and a Pareto diagram (based on the analysis of the *Manuale*): in the Edgeworth diagram the origin of the axes represents the situation before the exchange and two opposite maps of indifference curves are drawn; in the Pareto diagram the origins of axes of both traders are represented in the opposite vertices of the box and the two maps of indifference curves are correspondingly drawn. But notwithstanding his ingenious graphical re-orientation, we are of the view that Pareto's name should not replace Edgeworth's when considering this 'box' diagram, although we believe that it would be entirely legitimate to refer to the 'Edgeworth-Pareto Box'. In that regard, it is useful to recall that in §144 of the *Cours*, in footnote 1 where the Pareto diagram is implied for the first time, Pareto himself indicates that he is developing "la voie ouverte par Mr. Edgeworth".

Among the economists of the post-Millian generation, Walras and Edgeworth mostly contributed to the theory of exchange, whereas Marshall and Pareto mostly contributed to the theory of production. Pareto was so aware of this that he was happy with the review of the *Manuale* (1906) by young G. Jona, who reproduced a graph regarding production. It is therefore somewhat disappointing to see that the valiant people who are arguing about the paternity of a box have failed to realize that the more difficult—but also more imaginative—Pareto graphical representations are those where the indifference curves of a consumer and the indifference curves of a producer are contrasted with each other.

A FURTHER REMARK ON PARETO AND SOME REFERENCES. The fact that Pareto was aware that Edgeworth's indifference curves were a substitute tool of analysis for Marshall's reciprocal demand curves can be inferred from an unequivocal remark contained in Pareto's encyclopedia entry *Anwendungen Der Mathematik* that appeared in German in 1902 and was translated into Italian by Sensini in 1906 (see p. 438 of Sensini's translation, quoted in E.N. 8, and p. 169 in Engl. trans.)

In §144 of the *Cours*, n.1, cit., Pareto considered two traders who could engage in exchange. In fig. 10 he presented a map of the indifference curves for one of the traders, in fig. 11 he presented a map of the indifference curves for the other. Pareto then commented on the two figures as if they had been superimposed over each other into a single graphic. Something similar can be read in the famous polemic between Pareto and Scorza (*G.d.E.*, 1902, pp. 422–23), but neither there, nor in the *Cours*, can one find the "box diagram" as presented in the *Manuale* and the *Manuel*; and as it is still taught today.

On the analysis by L. Amoroso, which was forgotten even by the Italian economists, see *Meccanica economica*, Città di Castello, Macrì, 1942.

J. Creedy's beautiful monograph, *Edgeworth and the development of neoclassical economics*, Oxford and New York, Basil Blackwell, 1986, clearly explains how indebted Edgeworth was to Marshall. We find some of Creedy's remarks on Pareto somewhat questionable, but we attribute them to difficulties associated with the language barrier. See, for instance, E.N. 20.

E.N. 20. PARETO AND EDGEWORTH ON CONTRADICTORINESS AND INDETERMINATENESS, AND ABOUT AMOROSO AND STACKELBERG

1. Ch. III, §118. It went down in history as *The great barter controversy*—an exchange of opinions on mathematical indeterminateness between the two greatest English economists of the end of the 19th century. But the reproach that Pareto directed at Edgeworth on two different occasions for not having distinguished between contradictoriness and indeterminateness has gone unmentioned. When Pareto could no longer answer to him (having died in 1923), Edgeworth (1925, p. 313) wittily replied in a way that Pareto could not but have endorsed: there is a certain indeterminateness in the word determinateness, but let us concern ourselves with the truth of our propositions. Presumably, Edgeworth had in mind both the Pareto–Scorza polemic (*G.d.E.*, Nov. 1902, p. 423; Engl. trans., p. 534), where Pareto refers to an essay by Edgeworth that focuses on duopoly, and §69 of the appendix of the *Manuel* (1909), where Pareto refers again to duopoly.

2. With regard to Pareto and the duopoly case, Amoroso expressed himself as follows in an important article of his (1930, p. 18):

"Against Cournot, who had stated that the problem of the two monopolists is determinate, against Bertrand and Edgeworth, who had considered it as indeterminate, Pareto maintains that the problem is *overly determinate*, that is, it is impossible".

Obviously, Pareto's *overdeterminate* duopoly relates to the case in which both duopolists intend to act as leader. Three years later, in 1933, in a journal edited by A. De Stefani, by Amoroso himself, and by another Pareto scholar such as F. Vinci, a lucid article appeared where Stackelberg, in relation to duopoly, ascribes the hypothesis of contradictory duopoly to Pareto. The present writer has found, and still finds it strange that Stackelberg too, as well as Amoroso, drew attention on Pareto's overdeterminate duopoly without mentioning the asymmetric duopoly that is missing in the *Cours* and in the Italian edition of the *Manuale* (1906), but is present in the French *Manuel* (1909). In the *Manuel* (1909, pp. 601–2) Pareto actually proposes the asymmetric duopoly in the following terms. There exists a firm that controls 80% of the market, is price leader and therefore fixes the price for the whole industry; there exist other firms that accept that price as in free competition: is this not analogous to the relationship *leader-follower* of Stackelberg's duopoly?

Amoroso—who in 1930 had already shown an inclination to follow a path that was similar, albeit more general, to the path that Stackelberg would follow—would give his own definitive mathematical formulation of market structures in 1938 (see E.N. 32) and, finally, in 1942.

SOME REFERENCES. See Edgeworth, *Papers relating to political economy*, Vol. II, London, Macmillan, 1925, p. 313. Overdeterminate duopoly in Pareto was again considered by L. Amoroso in "La teoria matematica del monopolio trattata geometricamente", *G.d.E.*, August 1911. He discussed it again in "La curva statica di offerta", *G.d.E.*, January 1930. I have mentioned H. Stackelberg, "Sulla teoria del duopolio e del polipopolio", *Rivista Italiana di Statistica Economia e Finanza*, 1933, pp. 275–89. F. Vinci, "Monopoli e concorrenze nel pensiero di Pareto", *G.d.E.*, Nov.–Dec. 1948, dwelled on the debt that Stackelberg felt towards Pareto and on the relationship he kept with some Italian "Paretiens". Amoroso's and Stackelberg's formulae were reproduced by O. H. Ehrlich, "Other countries economists", *Economia Internazionale*, Febr. 1956, where on p. 100, in relation to Amoroso (1930), one reads: "Stackelberg gives credit to Luigi Amoroso for the formulation of the functional relationship between marginal revenue and what we would call today point elasticity on the average revenue curve". The asymmetric duopoly scheme of the *Manuel* (1909, pp. 601–2) that could have inspired Stackelberg is specified in A.Z., *Rendimenti, concorrenza e monopolio nella teoria della produzione di Pareto*, Firenze, Università di Firenze, 1992, p. 139. On asymmetric duopoly and international economy in Pareto see E.N. 49.

J. Creedy, in an otherwise very beautiful essay ("Marshall and Edgeworth", *Scottish Journal of Political Economy*, Feb. 1990), attributes the sentence, on p. 32, "there is a certain indeterminateness...", mentioned in the text of the present E.N., to Marshall and Edgeworth dialogue rather than to Marshall and Pareto dialogue. We believe that this is due to a linguistic barrier that prevented J. Creedy himself from perusing the Italian literature. In what follows I try to reformulate Edgeworth's words—from the 1925 introduction to the reprint of the famous reply (1901) by Edgeworth to Marshall—to make them more understandable: "In the course of my life, economists have witnessed an Edgeworth-Marshall dialogue on determinateness and indeterminateness, and a monologue by Pareto with Edgeworth on indeterminateness and overdeterminateness".

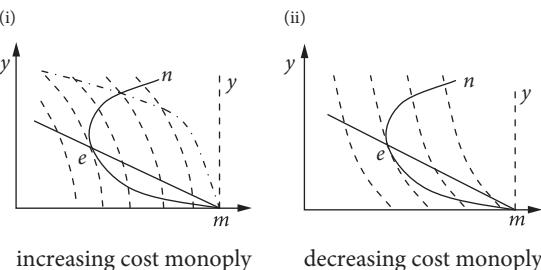
E.N. 21. STRATEGIC BEHAVIOR IN THE COURSE OF IMITATIVE COMPETITION

Ch. III, § 138 and *passim*. By rotating the radius whose slope represents the price of a product, Pareto introduces, in his own theoretical framework, the strategic behavior of entrepreneurs in the course of imitative competition. If one reads further in the *Manuale*, one realizes that Pareto uses Type I (a modest reduction in price, compared with the existing given price, which gives a transient economic advantage to those who apply it) as part of an adjustment mechanism that is also applicable to all the intermediate degrees of oligopoly between "artificial" monopoly (Cournot's *inventeur* monopolist, Marshall's *conditional* monopoly) and unlimited competition. In other words, in the case of unlimited competition, Pareto's Type I expresses the *price taker* behavior of each individual firm, whereas within an adjustment mechanism it expresses a strategic behavior (a price slightly modified by a firm, in comparison with the given price). Hence the misunderstandings caused by Type I (see E.N. 9).

The delay with which the competitors, however many, or few, they may be, react to the reduction in price strategically adopted by an entrepreneur implies that Pareto relaxes one of the postulates on which the construction of pure perfect competition is based: the unlimited knowledge (perfect transparency) postulate. In A.Z., *The adjustment mechanism of Cournot and Pareto*, 1995, a diagram is used to highlight a conclusion that is transparent in Pareto: different prices for one and the same good always exist in the process of imitative competition. This same conclusion, without any reference to Pareto, is found in Maurice Clark, both in his famous 1923 book on *overhead costs*, and in his last, little known book, *Competition as a dynamic process*, Washington, The Brookings Institution, 1961. In the latter work Clark reaches the same conclusion as Georgescu-Roegen: it is impossible to express the substance of any historical form of competition without representing it as a "process".

E.N. 22. ON TRIFFIN'S CRITICISM OF PARETO'S INCREASING COST MONOPOLIST

Ch. III, § 151. On the topic of the monopolist (type II) with decreasing unit costs (complete competition), Pareto correctly states that the equilibrium is given by the point of tangency between the exchange curve and the indifference curve of the producer in order to obtain the maximum achievable profit, as illustrated in fig. (ii). In the case of the monopolist with increasing unit costs (incomplete competition), the same principle



obviously applies, as shown by our fig. (i), and, as seen in Ch. VI, §16, where Pareto deals with the monopolist's (type II) optimum in the particular case of increasing costs (see E.N. 30, point 5).

In Ch. III, §151, however, Pareto slips up: instead of referring to the point of tangency between the exchange curve and the indifference curve of the producer, he discusses the point of tangency between the exchange curve and the maximum profit line. Triffin was, therefore, correct when pointing out this mistake (but it is an oversight, as we have just shown). The fact, Triffin added, that the tangency between the exchange curve and the maximum profit line

“makes no sense [...] does not deter the faithful Pietri-Tonelli [*Traité d'économie rationnelle*, Paris 1927, p. 160] from reproducing the same formulation of the conditions of equilibrium; a diagram is even supplied where curves are drawn in a perfectly fantastic manner in order to exhibit the impossible tangency”.

SOME REFERENCES. See R. Triffin, *Monopolistic competition and general equilibrium theory* (1940), Cambridge, Harvard University Press, 1960, p. 59. The diagram criticised by Triffin, is also found in A. de Pietri-Tonelli, *Lezioni di scienza economica razionale e sperimentale*, 2a ediz., Rovigo, Industrie Grafiche Italiane, 1921, p. 171, in addition to the already mentioned French edition.

E.N. 23. MARGINAL RATE OF SUBSTITUTION AND CHOICE THEORY

Ch. III, §210 and Ch. IV, §62 [a]. If the slope of the tangent to a point on the indifference curve is, as J. R. Hicks suggested, termed the *marginal rate of substitution* (an expression that in Italy was first used by Marco Fanno, “Contributo alla teoria economica dei beni succedanei”, *Annali di Economia* 2, Milano, Bocconi Editrice, 1936, pp. 369–70), Pareto's analysis gives the theorem that was (re)discovered by J. R. Hicks when avoiding reference to the words “utility”, “marginal utility”, “decrease in marginal utility”: one has equilibrium at the point of tangency between an indifference curve of the consumer and his budget line, that is, at the point where the marginal rate of substitution between two goods is equal to the ratio between their prices.

G. Demaria (in his article-review of Hicks, in *G.d.E.*, Sept.–Oct. 1939) and N. Georgescu-Roegen (“Choice, Expectations and Measurability”, *Quarterly Journal of Economics* 68, 1954, pp. 503–534) asked themselves whether Hicks' formally impeccable solution was akin to an ostrich burying its head in the sand—how was it possible to explain the movement of the marginal rate of substitution without referring to the corresponding movement of marginal utility? As Georgescu-Roegen wrote, in the abovementioned work, in reference to that solution

“This is simply an illusion because even though the postulates of the theory of choice do not use the terms “utility” or “satisfaction”, their discussion and acceptance require that they should be translated into the other vocabulary. Otherwise, one is forced to admit that the postulates have neither a rational explanation nor an experimental justification”

(footnote 9, p. 512).

Since we are dealing with two influential scholars such as Demaria and Georgescu-Roegen, we thought it appropriate to report their point of view. However, for our interpretation of Pareto on this matter, see E.N. 14 and 15.

E.N. 24. PARETO ON 'NO BRIDGE', 'REPRESENTATIVE INDIVIDUALS', AND 'HAPPINESS'

Ch. IV, §32. Contrary to what Georgescu-Roegen (1973, p. 230) believed, the drastic statement that there is *no bridge* in order to sum up the ophelimities of different individuals, which was added to the *Manuel*, is far from helping to clarify Pareto's position in relation to comparisons of ophelimities between individuals. This is a position that Pareto had already mentioned to Pantaleoni as far back as 1892 (*Corrispondenza Pareto-Pantaleoni*, vol. I, p. 275), and which he would develop in the *Cours* (vol. II, §§642–53).

First of all, it must be recalled that from Pareto's point of view, Political Economy in its first approximation deals with average, repeated phenomena "in such a way to eliminate most accidental variations" (Ch. IV, §30). In other words: Pareto believes that pure Economy deals with normal, representative individuals. This does not mean that in second or third approximation an economist may not also deal with blind and color blind people; rather, it means that to start with, the economist will deal with sighted and non-color blind people. This is the sense of the warning that Pareto expressed in 1892, when he met Pantaleoni and got closer to the new school of economics: "You must realize that this is a matter of life or death for the new theories. The economic science is a science of averages. If the final degrees [of utility] escape the use of averages, they cannot find a place in the science" (*Corrispondenza Pareto-Pantaleoni*, vol. I, p. 275). In the same letter a few lines earlier he had written: "The *no bridge* part, which you love so much, does not have the same effect on me" (*ibid.*). Why?

Because there is no contradiction between the statement according to which "Between two distinct individuals no comparison with regard to ophelimity is, *strictly speaking*, possible [...], it cannot make any sense" and the fact that on the other hand "the sensations of men from one and the same society and, at times, from different societies are compared every day" (*Cours*, Vol. II, §645–46). The fact is that the first statement relates to two 'well defined' individuals, of whom one could be—let us say—color blind, whereas the second statement refers to two individuals from one and the same environment who do not deviate very much from the normal or representative individual.

Of course, since Pareto was a tenacious advocate of the heterogeneity of the individuals, in his view aggregation is less arbitrary when more representative agents are introduced, i.e. when the number of sub-aggregated groups is greater. If a hierarchy of needs exists that is applicable to all individuals, comparisons between the welfare of two individuals would always be possible. But as individuals are heterogeneous, social hierarchies may, at best, be considered in terms of averages (*Cours*, § 650). In the *Manual*, Ch. III, §87 and §99, Pareto gives an authentic interpretation of his position: besides the consideration of the collective demand for a commodity, Pareto assumes that one can, if one wishes, introduce a fictitious representative consumer whose preferences generate this demand; where Pareto's fiction stands as a convenient but arbitrary theoretical assumption. Perhaps Pareto was starting from the concept of *comparability* between sets

of heterogeneous entities in order to pursue the historians' concept of *recognizability*: the middle class is recognizable compared to the high bourgeoisie, or with the proletariat, in the same way as the Vendée region was recognizable during the French Revolution, even though not everyone in Vendée was wielding a pitchfork.

Unfortunately, neither the Marxist literature nor the works of Georgescu-Roegen (who, notoriously, contrasted the "aritmomorphic propositions" of an analytical type to the "dialectical dimness" of common language) consider Pareto's claims on the existence of social classes in response to those who assert that "none can speak of a class of 'capitalists' or oppose the 'bourgeois' to workers": "It is certain that we pass by unnoticeable degrees from the class of the poor to that of the rich, but that does not prevent the existence of these two classes. It is a general observation: in current language, for real differences, which are quantitative (§97), we must substitute qualitative differences" (*Cours*, § 1052).

Someone may, for example, make the proposition that happiness consists in satisfying the same needs of certain—mainly pessimistic—writers. But if someone were to then put that proposition to derelict people, it would be to pose an absurd and incomprehensible problem; something like asking how we would feel if we were born lions (*Cours*, Vol. II, §651, *Manuale*, Ch. IV, §26). It almost seems that Pareto is anticipating and overturning Scitovsky's appealing theory (*The joyless economy, An inquiry into human satisfaction and consumer dissatisfaction*, Oxford, 1976), according to which the minorities conscious of a "good life", who feel stimulating needs, are unfulfilled because they are overcome by a consumerist environment in which mass production satisfies un-conscious people, who, in contrast, are fulfilled. In 1904, Pareto revisited comparative happiness in a famous work, arguing implicitly against his own humanitarianism of his younger years, and explicitly against solidarity needs—which are the *relational needs* of today's literature under a different name. This is how Pareto expresses himself: all the attempts that have always been done to negate or to attenuate the opposition of the interests in the social aggregate:

"rest on a circular argument. The problem to be solved is assumed to have been solved by stating that *true happiness* for an individual consists in doing what benefits "society", and starting from here it is stated that each individual who acts differently is only pursuing a *false happiness* and that he must be prevented from harming the others and himself. After Plato, arguments of this kind have been dished up to us in the most varied forms. A modern doctrine, called of *solidarity*, is only renewing it [...]. Some remark that the moral, intellectual, religious *unity* of society is something very desirable; but these people regularly mean that this uniformity must be brought about through the adoption of their own ideas; it so happens that the proposition that these ideas enunciate is nothing but a euphemism meaning that it is necessary to compel the whole world to think like them."

(V.P., *L'individuel et le social*, 1904, publ. 1905,
see V.P., *Oeuvres complètes*, VI, 1984, p. 261)

In Vol. I of the *Cours*, §§10–17, Pareto adopts an evolutionary concept of *utility*: when the people of "civilized nations" talk about utility, they generally identify it with the "prosperity" that has as characterizing features "moral and material welfare" and a certain progress in scientific knowledge. In Vol. II, §653, he deems comparisons between individuals in terms of *utility* to be easier than the comparisons in terms of ophelimity. This latter conclusion ignores the fact that comparisons in terms of utility, as he defines it here, concern entities that are as heterogeneous and vague as the many different assessments

of "moral and intellectual development". Yet, this vague nature of the ethical-religious aspects of utility emerges quite clearly from a work (1900) that only follows the *Cours* by a few years:

"A truth already known for many centuries has just been discovered again: the fact that man is driven by sentiment more than by reason. From this it can be inferred that religious sentiment plays a considerable role in preserving the organization of society, but from this alone it cannot be ascertained precisely how big that role must be in order to obtain maximum social utility [...]" (V.P., *Scritti sociologici minori*, G. Busino ed., Torino, Utet, p. 198 of 2nd edition, 1980).

It is also worthwhile to read §653 of the *Cours*, as that is where Pareto equalizes a community and an organism (biological analogy), implicitly confirming what was a constant element in his thought: in terms of development, the mechanical analogy is not applicable:

"653. *Comparison of utilities.* Such a comparison is much easier than the comparison of ophelimities, at least if there is agreement on what meaning should be given to the word *utility*. If one accepts, as is quite generally done, that for a people *utility* overlaps with material prosperity and moral and intellectual development, one has a criterion to draw comparisons between different peoples. A difficulty remains, stemming from the fact that society must be considered as a complex whole, as a system, as an organism. It may therefore happen that one of these organisms is superior to others for certain points; inferior for other points. In any case, if it is not possible to draw a comparison for the whole, it is at least possible to do so for some of the parts. For instance, it can be said that with regard to military power it was useful for the Japanese to adopt the organization of European armies and navies."

On the topic of comparisons between individuals in sociology see also E.N. 21 and E.N. 55.

A BIBLIOGRAPHICAL ADDITION. In the above text we allude to the famous essay on the philosophy of science (*Some orientation issues in economics*), which opens N. Georgescu-Roegen's volume *Analytical economics—Issues and problems*, Harvard University Press, Cambridge, 1966. A French translation of that work (overseen by H. Guitton, and prepared by a translator, M—Rostand, fully cognizant of the culture and language of mathematics) includes a *Preface* by P. Samuelson, a *Preface to the French edition* by H. Guitton and, at the end, an *Avant-Propos* by Georgescu-Roegen for the French edition, Paris, Dunod, 1970.

E.N. 25. NEEDS AND PREFERENCES: PARETO AND THE MOST IMPORTANT LAW OF POLITICAL ECONOMY

Ch. IV, §§ 9, 10, 11, 66. Unfortunately, there does not exist a wholly hierarchical structure of needs that can be applied to all people and to all times, as it would happen if need A were always satisfied before need B, need B before need C, and so on. This was one of Pareto's reflections after reading Pantaleoni's *Principii* in 1891. Of course, Pareto was reflecting on the simplifications that would stem from such a structure in the relationship between individuals who 'govern' and those who are 'governed', as well as in economic

theory. Pareto's meeting with Pantaleoni reinforced the former's conviction that, admittedly, there exist some essential needs, virtually imposed by the fact that people belong to the animal kingdom; needs, in other words, that are felt and satisfied before the other needs. But there also exists an evolution of needs and goods that is connected with the development of the communities in which people live. That is to say, as income increases, needs also expand and the preferences, the tastes of the individuals get the upper hand over the component virtually imposed by "nature". This variety and proliferation of needs seemed so important to Pareto—as they did to Banfield and Jevons—that they suggested to him a diagram—the following fig. 1—which tells us that in spite of the increase in income, the marginal utility of any one individual remains always positive (in fig. 1, Pareto puts it at the level m of the dotted line) because of the great number of the unfulfilled desires.

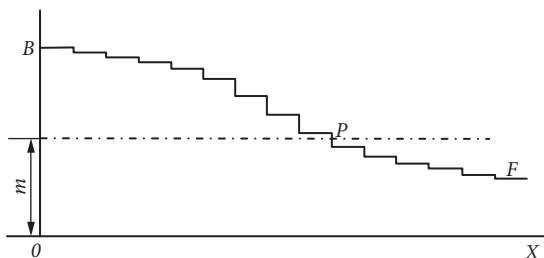


FIG 1

In Pareto's view, this is the most important law of Political Economy (see E.N. 32); so much so, that he would later imply it as a surrogate of Say's law. Since this allusion to the vastness of unfulfilled human desires was not meant to offend common sense, or the unemployed, Pareto's implication seems to us to be the same as the one found in Schumpeter's work: even when the economic trend keeps turning for the worst, inventors and innovators explore the inexhaustible vastness of human needs with the intention of successfully specifying them in a new, or renewed, way.

In this evolutionary view of humanity and their needs there is no room for the postulate that supports the usual static theory of consumption: needs and tastes are given, and individuals always remain identical to themselves. But nothing prevents one from focusing on a given preference structure, postulating unchanging individuals, and from analyzing the possible complementarities and substitutions between goods. It seems to us that no-one better than Georgescu-Roegen (1952 and 1953) has restated, even with diagrams, the well-known dependences "of the first kind" and "of the second kind" of Pareto's *Manual*. In fact, it seems to us that even the representation included in Georgescu-Roegen ("Choice, expectations and measurability", *Quarterly Journal of Economics* 58, 1954, fig. 1.1) owes much to the diagram in Ch. IV §66 of Pareto's *Manual*; a diagram by which Pareto intends to demonstrate that when a consumer's income increases, he tends to satisfy his needs with increasingly fulfilling goods.

Let us go back to Pareto at the time when he had just read Pantaleoni's *Principii* and let us ask ourselves: why, in his 1892 *Considerazioni* (June, p. 488 and p. 501, Engl. trans. pp. 26 and 34–35), does he contrast the consumer in fig. 3—who goes from an

income equal to 1 square to an income equal to 13-squares, but does not at all end up by equalizing the intensity of all his needs—to the Walrasian consumer in fig. 2—who also goes from income 1 to income 13, but equalizes the intensity of all his needs?

Figure 2 does not pose any problem of interpretation. Pareto had started by saying that it is necessary to be aware of what mathematical formulae include, as well as of what they exclude, and he had implied that fig. 2 establishes the Walrasian principle of the equalization of marginal utilities. It is hardly necessary to add that in Walras' work, goods are "independent" of each other, the utility of a good does not also depend on the utility of other goods (in Walras, the function of utility is additively separable). It is more difficult to understand figure 3, on which Pareto makes the following comment (p. 502, Engl. trans. pp. 34–35):

"if the law of human needs were instead illustrated by diagram [3], Political Economy would be totally different from the science that goes by this name. We shall therefore limit our comments to the case shown in diagram [2]",

that is, Pareto confines his comments to Walras' science.

In order to explain figure 3 one can follow two paths, which eventually run into one. The starting point is Pareto as he has just read Pantaleoni and discovered, besides Jevons, the most important law of Political Economy. *First explanation.* Like all Austrian marginalists, Wieser, after whom Pantaleoni (1889, p. 122) named an economic law, applies a method—later named *descriptive genetic method* by Rosenstein-Rodan (1933)—which consists in explaining how *in the course of time* economic phenomena take place. Figure 3 therefore tells us that an individual satisfies his needs starting with the most intense ones (for instance, thirst and hunger) and then, as his income increases, by moving on to those he feels with less intensity. Hans Mayer, who led the *Neue Phase der Wiener Schule* and "brought to trial", so to speak, the whole of the non-Viennese neo-classical theory, was convinced—and on this point we agree—that everyone leaves the dinner table having reduced to zero the marginal ophelimity of water in the presence of positive ophelimities for the other goods. In such a "trial" Pareto was acquitted, almost "for not having committed the crime", thanks to a diagram contained in the *Considerazioni* from January 1893 (p. 14, Engl. trans. p. 85), about which he made the following remarks: there are some needs and some goods, such as thirst and water, hunger and bread, "and other similarly essential commodities", the marginal utility of which is "very close to the abscissa axis", that is, very close to zero. Mayer (1937, p. 738) found an analogy between this way of reasoning and that of Wieser. *Second Explanation.* The most important law of Political Economy should be named after Banfield-Jevons, because, as even the Austrian marginalists later acknowledged, it was Banfield who in 1845 wrote:

"the first proposition of the theory of consumption is, that *the satisfaction of every lower want in the scale creates a desire of a higher character* [...]. The removal of a primary want commonly awakens the sense of more than one secondary privation: thus a full

5					
4					
3	3				
2	2	2	2		
1	1	1	1	1	
B	C	D	E	F	

FIG 2

6					
5	5				
4	4	4	4	4	
3	3	3	3		
2	2				
1					

FIG 3

supply of ordinary food not only excites to delicacy in eating, but awakens attention to clothing." (Cited by W.S. Jevons, *The theory of political economy*, 1871, Penguin Books, 1970, p. 104. Pareto read this passage in W.S. Jevons, *La teorica dell'economia politica*, vol. II of the third series of the *Biblioteca dell'Economista*, Torino, Utet, 1878, p. 202.)

Subsequently, Jevons and Menger would say that the satisfaction of an inferior need allows the superior need to manifest itself. Pareto's fig. 3 could therefore be thought of as a static cross-section of the evolution expressed in fig. 1. If we had such a cross-section at our disposal, we could come across so poor an individual, for instance, that the thought that the enjoyment of a meal may also depend on—let us say—Strauss' music, would never, not even remotely, cross his mind. Whereas another individual has even forgotten that water is also used to quench thirst—he quenches his thirst with champagne, and the opulence of his dinners depends on Strauss' music and crystal ware from Bohemia.

If through these examples we have succeeded in going back to the Austrian origins of Pareto's fig. 3, then we can definitely go and see what happens, in the *Manual*, to basic needs, that is, the needs that are most intensely felt, the needs that since Ancient Greece were opening the trilogy: *survive, live, live well*. The reader should note, first of all, that §11 has an autobiographical undertone that could be translated as follows: I, too, Pareto, had believed in Walras' independent goods; today, in the *Manual*, I am removing them from the picture, reserving the right to recall them in exceptional cases (thirst and hunger due to a siege, for instance) and for income levels that belong to far-off times. In short, Pareto follows the following path. He assembles a category of needs and goods that in §§9–10 he indicates by (a). In essence, he sets aside the thousands-of-years-old category of survival, that is, he considers income levels that are enough to bring about a small, even minimal, dependence of goods in the satisfaction of needs. In §10 Pareto writes (our italics):

"It is certain that anyone suffering for extreme cold cannot enjoy a delicately flavoured dish; a starving man does *not* derive *great* pleasure from looking at a fine painting or from listening to a well-told story, and if he were given some food, it would *hardly* matter to him whether it were served in coarse earthenware or in fine china."

Since, apart from this category (a)—in which, we reiterate, the dependence of goods is not zero, but is modest—in the other cases the dependence of goods in the satisfaction of the needs is always substantial, Pareto ensures he has a *continuum* that allows him always to be able to avail himself of his famous index function. This was the cause of two equally famous criticisms against him. The first of these criticisms was leveled at Pareto, starting from the 1920s, by the *New Vienna School*, which operated in a logical-mathematical environment in ferment (Karl Menger, too, the son of the economist, contributed to this ferment; even some sociologists were resorting to *set theory*) which was looking for a lexicographical representation of needs—for an ordering, in short, that cast aside Pareto's index function. The second criticism was leveled at Pareto in the years after the Second World War by Georgescu-Roegen. He had been the best equipped among the economists, and was certainly more aware than others about what he owed Pareto and what he owed to that *Neue Phase der Wiener Schule*, the memory of which was swept away by the Second World War.

SOME REFERENCES. In relation to fig. (1), Pareto writes (*Considerazioni*, 1892, cit., p. 507, English transl. p. 41): “The law of the increasing variety of human needs is indicated [...] by the shape of line BF , which extends indefinitely in the direction OX ”. The section PF represents the unfulfilled needs. On H. Mayer, see “Il concetto di equilibrio nella teoria economica”. It appeared in Vol. IV of the *N.C.E.*, Torino, Utet, 1937. The present writer dwelled on Pareto’s diagrams 2 e 3 in A.Z., “Un’opinabile interpretazione di Pareto da parte di Bresciani”, *Quaderni di storia dell’economia politica*, 1987, 1–2. Since Bresciani fell into a misunderstanding, the reader should note that in fig. 2, the marginal ophelimity of good B (first column) decreases from 5 to 4, 3, 2, 1, and that Pareto’s comparison between the two diagrams does not change if this step-shaped function is replaced by a function that decreases from 5 to 1 with monotonic continuity. The same applies to the other columns. We have mentioned P. Rosenstein-Rodan, “La complementarietà prima delle tre tappe del progresso della teoria economica pura”, *La Riforma Sociale*, May–June 1933. Of this essay (it should be noted that for the Viennese, psychic complementariness also included the needs satisfied by competing goods) only the part that shows the search for a lexicographical representation of needs is relevant to us. In the text we did not dwell on Pareto’s distinction, §§15–16, between ‘equivalence in tastes’ and ‘equivalence in needs’. On this topic, besides M. Fanno, “Contributo alla teoria economica dei beni succedanei”, *Annali di Economia*, vol. II, Milano, n. 2, Università Bocconi, 1926, we refer the reader to the forgotten translator of Rosenstein-Rodan, Vincenzo Porri, *Principii di scienza economica*, especially Vol. I, Torino, Giappichelli, 1932, particularly pp. 237–38, where Porri finds a connection between Pantaleoni, Böhm-Bawerk, Wieser, . . . , with regard to the irregularity in the decline of the degrees of intensity of needs. Georgescu-Roegen’s (1954, cit.) footnote quotations are witness to the fact that the consumer behavior represented by fig 1.1 of this essay owes to a diagram by Pareto (*Manuale*, 1906, p. 269, *Manuel*, 1909, p. 282) more than Georgescu-Roegen seems to imply in the text. In the present E.N., where Pantaleoni plays a very important role, ignored even by his admirers, we wish to reformulate a question that was posed by Pantaleoni and by Fanno, which in the last twenty years or so has been the topic of much literature: what happens in the satisfaction of needs and tastes when income decreases, or even just the suspicion arises that it may decrease?

E.N. 26. PURSUIT CURVES IN THE COURS AND IN THE MANUAL

Ch. V, §11. The pages that the *Cours* devotes to pure theory are so few in comparison with the rest of the book (see E.N. 3), that we wonder why they include a topic that, in the context of Walrasian equilibrium, is so impure, such as the topic represented by the diagrams 3 and 4 of §11. Indeed, the core of Pareto’s *pursuit curves* is made of the delusions into which entrepreneurs fall and the mistakes they make in their fixed capital investments. One would have to wait for the long-term expectations of Keynes’ *General Theory* (1936) to see equal importance be given to the uncertainty and the mistakes associated with the delays between the moment when the fixed capital investments are planned and carried out, and the moment when they start bearing fruit.

Why does Pareto include the economic cycles—which, too, are a source of uncertainty—in the applied economics section of the *Cours*, but not pursuit curves? We

can perhaps find the answer in the overt skepticism that Pareto showed quite early, even in the *Giornale degli Economisti*, with regard to the advent of an era of reliable predictions such as the one fondly imagined by Walras:

“This is the point which separates Professor Walras and me. He already foresees the day when we can have incontrovertible data and be able to forecast economic phenomena with certainty; and even now discusses possible measures to be taken at that stage. I believe that that day is still far off, so far off indeed that it is useless to discuss it.” (“Teoria matematica dei cambi forestieri”, *G.d.E.*, February 1894, p. 162; English transl. “Mathematical theory of the foreign exchanges”, *G.d.E* 67, 2008, p. 377)

Pareto would later explain quite clearly the reason why entrepreneurs are not able to make reliable forecasts, also in the *Manual* (Ch. IX, §76):

To produce commodities takes time, and often considerable time prior to consumption. For production to be perfectly adjusted to consumption, one should: (1) Be able to forecast consumption. (2) Be able to forecast the outcome of the productive process exactly. Neither the one nor the other can be done with any certainty.

Whatever one may think about the inclusion of pursuit curves in the pure theory section of the *Cours*, what is certain is that this topic is far better developed in the *Cours* than in the *Manual*, where, moreover, there is only one diagram instead of the two found in the *Cours*. In essence: (i) in the *Manual* (Ch. V, §11) Pareto confines himself to telling us that under a regime of free competition, entrepreneurs make decisions attracted by profit expectations, whereas in the real world they end up being pushed where they would not wish to go (that is, in the region of losses); (ii) in the *Cours*, Pareto takes us right to the heart of uncertain decisions by distinguishing between the mistakes in adaptation made in the field of consumption—which can be easily rectified (“If a man has bought too much wine for his own use, he will buy less next month”)—and the mistakes made in fixed capital investments:

“If [a man] has instead bought machinery to double his production while the consumption in that production does not increase, he will not resign himself to reduce production immediately, and the resulting disruption in the economic equilibrium will be deeper and longer lasting. It will be even more so in the case of transformations in landed capital or personal capital [...]; these mistakes in capital transformation cause important disruptions in the economic equilibrium”. (*Cours*, §42)

Finally, why is Pareto not fully satisfied with Fig. 3 alone (which uses the analogy of the dog that follows its master thinking it will follow a straight line that at the beginning goes from c to M , and which instead follows a curve $cc_1c_2c_3\dots$ because in the meantime the master moves from M to M_1 along the straight line that goes from M to X)? Why, in short, is Pareto resorting to Fig. 4? The fact is that in economic life the misleading effects of changes are much more marked than in Fig. 3; entrepreneurs formulate forecasts as if current trends were to continue, and throw themselves on them to fulfill future needs:

“In the case of the economic phenomenon the problem is more complex. If, for instance, consumption increases from M to M_1 , production does not turn so as to head towards M_1 but rather towards a point m further away. Producers tend to believe that the increase in consumption will continue indefinitely; not content with providing for current needs, they take into account the future needs: this is one of the causes of production crises”.

(*Cours*, §41)

Two comments. i) The expectations of Pareto's entrepreneurs are different from those (extrapolative, adaptive, rational expectations) found in contemporary literature. Pareto's entrepreneur is characterized more by his desire to get and occupy additional market slices as soon as they are detected, than by his ability to reason about ongoing changes. ii) Pareto gave such importance to uncertain entrepreneurial decisions, to the actions characterized by the "instinct of combinations", by "neophilia" as opposed to "neophobia", that he restricted pure economic theory to repeated, normal actions, and left applied economics above all to sociology, to deal with uncertain, "non-logical" actions—the actions, in other words, that are characterized by some incongruousness between the end that is subjectively pursued and the end that is actually achieved. One hardly needs add that this incongruousness is linked to the changes and uncertainty brought about by the passing of time. This split between a world of what is normal and certain, and a world of what is new and uncertain—which in Pareto's mind were not on the opposite sides of the Moon—ended up by backfiring on him. Few economists are interested in the "instinct of combinations" and "neophilia" in Pareto, and the path that in Pareto's intentions put those two worlds in close relation with each other has thus been lost. Unfortunately, the second edition of the *Cours* (the *Trattato di economia* discussed in E.N. 3), which was meant to give us Pareto's thought in its entirety, never saw the light of day. Not even Schumpeter (the *Corrispondenza Sensini* dates from 1948, Schumpeter died in 1950) got to know of Pareto's unfulfilled intention.

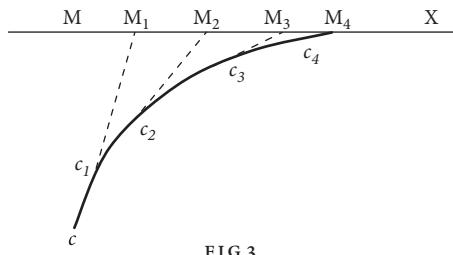


FIG 3

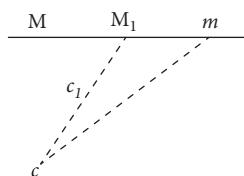


FIG 4

E.N. 27. PRODUCTION COEFFICIENTS IN PARETO'S COURS

Compared with the *Cours*, the theory of production in the *Manual* is expounded in a very different way. Suffice it to think of the various forms of that production function that Pareto calls *line of complete transformations* (see E.N. 12). Pareto excludes the possibility of production being represented, in general, by a single continuous and differentiable production function. Drawing on his former experience as an ironworks manager, Pareto proposes an example that, following a book by E. Schneider (1942), would later give rise, in the Italian literature, to a now familiar terminology: substitutional factors as opposed to limitational factors, with the latter "varying like shadows" as the other factors and the product vary. Here is the classic passage from the *Cours*, §714, with its relevant mathematical footnote:

"It should be pointed out that if production coefficients cannot all be assumed to be constant, neither can they all be assumed to be variable. From a certain quantity of iron ore, for instance, it is impossible to obtain a quantity of metallic iron greater than the quantity contained in this ore. Given a certain degree of technical knowledge, the quantity of metallic iron that can be obtained per ton of a certain ore is a fixed quantity. In other words, the quantity of ore that must be used is proportional to the quantity of iron that one wants to produce. This is therefore, in general, what we should envisage the nature of production coefficients to be like. Some of them are constant or almost constant; others are linked by relations such that a decrease in one of them can be offset by an increase in others.¹

"(714)¹ If $a_s \dots$ are constants, it will generally be possible to represent the production coefficients with the equations

$$(1) \quad \begin{cases} a_s = a_s \\ f_1(a_t, a_v \dots) = 0 \\ f_2(a_v, a_z \dots) = 0 \\ \dots \dots \dots \end{cases}$$

Almost all the authors who make use of the concept of *factors of production* reduce these equations to a single one. It is a mistake. Let us suppose that the quantities ... S_a, T_a, \dots of the services of the capital goods ... S, T, \dots are used to produce the quantity Q_a of A. The authors just mentioned write:

$$(2) \quad Q_a = F(S_a, T_a, \dots).$$

By the very definition of production coefficient one has,

$$(3) \quad S_a = a_s Q_a, T_a = a_t Q_a \dots$$

If equations [1] are reduced to a single one,

$$(4) \quad f(a_s, a_t, \dots) = 0,$$

by substituting in it the previous values one obtains

$$(5) \quad f(S_a/Q_a, T_a/Q_a, \dots) = 0$$

from which one can find the value of Q_a , given by equation [2]. If in equation [4] Q_a does not appear explicitly, equation [5] defines a function Q_a that is homogeneous of the first degree in S_a, T_a, \dots . This is the assumption that is very often made when factors of production are considered. It is evident that it cannot be generally accepted."

Thus, among other things, Pareto does not assume constant returns to scale. Admittedly, his equations [1] seem to imply them, but this does not exclude that Q_a may also appear in them, as Pareto indicates for equation [4]. Finally, it should be noted that in the *Cours* Pareto uses average production coefficients, whereas in the appendices of both the

Manuale and the *Manuel* he uses marginal production coefficients, that is, ratios between variations of factors and product, that is to say, $a_s = \delta S_a / \delta Q_a$, etc.

An additional comment. The writer (A.Z.) is indebted, on the topics of budgets, to the German literature that was progressively translated by the journal *Note di Economia Aziendale* edited by the *Associazione fra le Società Italiane per Azioni*. In that literature, the work of Eric Schneider represents the highest expression in the field of the economic theory. He is the author of a book, *Theorie Der Produktion*, Wien, Springer, 1934, and then the revised Italian edition (*Teoria della produzione*, Milano, Editrice Ambrosiana, 1942), which was enriched by F. Di Fenizio's Introduction. Schneider knew the Italian literature well, in particular that of Pareto, but neither Schneider nor Di Fenizio recalled that Pareto, in Chapter VII of the *Systèmes socialistes*, asserts that: one cannot completely separate, within the product, the part that is attributable to every factor of the production. In summary, Pareto dissented from those economists who were 100% certain how to extricate inextricable complementarities. We are sorry not to have recalled the *Systèmes socialistes* in A.Z., "Pareto's monologue with Marshall", *Quaderni di Storia dell'Economia Politica*, 1991/2-3, and not to have added that Pareto was generous to Francesco Ferrara (1810-1900), the famous Italian theorist of 'immaterial products' or services, when implicitly trying to explain that if he, Pareto, had been asked to specify his personal contribution (a service, or an 'immaterial' factor) to the Ferriera di Valdarno, he, Pareto, the director of that ironworks company, would not have been capable of specifying that contribution.

SOME REFERENCES. The literature on production coefficients in Pareto is vast. One needs only consult the beautiful *Corrispondence Walras* edited by W. Jaffé, J.S. Chipman's (1976) bibliography, and E. Schneider, *Teoria della produzione*, cit. The production coefficients topic offers an interesting aspect in Pareto's review (1902) of Aupetit (in V.P., *Oeuvres*, Vol. IX). Like Aupetit, Pareto optimizes by resorting to a Lagrange multiplicator in a static context. This makes Pareto's incomprehension of Laurent—as it appears from J.S. Chipman, "An episode in the early development of ordinal utility theory: Pareto's letters to Hermann Laurent", *Revue européenne des sciences sociales*, 1976, n.37—more enigmatic. According to Chipman (to whom we are in debt for identifying a Lagrange multiplicator in a "variational" context in Pareto), it appears that Pareto was incapable of distinguishing between an integration factor and a Lagrange multiplicator.

We refer the reader to Note [116], *French Mathematical Appendix*, where it is specified that in the *Cours*, Pareto considers average production coefficients, while in the *Manual* he considers marginal coefficients, which—as clarified by Zawadzki (1914, p. 209)—he defines with partial derivatives.

E.N. 28. PARETO AND KEYNES ON THE DISTRIBUTION OF THE BENEFITS OF TECHNICAL PROGRESS

§§72-74 of Ch. V are of paramount importance and they are also very beautiful. If someone sought to belittle them by saying that there is an old-fashioned Smithian flavor to them, he would be forgetting what happened, even recently, in areas such as: chicken farming on an industrial scale; the television screen industry; and, most

recently, the still ongoing process of price reduction in the field of cellular phones. And yet, the mechanism for the distribution of benefits from technical progress that Pareto embraced is still the classical Smithian one. This mechanism was so prevalent in Walras too that E. Barone tried to reproduce it using a diagram that was so beautiful that U. Ricci decided not to discard it even after realizing the correctness of a remark by Sraffa (1925) based on Marshall's famous distinction between *offer curves* and *particular expenses curves*.

If we turn over a new leaf and look at Keynes, at his *Treatise on money* and the *long period* mechanism of his *General theory* (p. 271), we find ourselves thrown in a different world: one which Pareto deals outside of the first approximation given by pure theory. Indeed, Keynes projects us into a monetary economy where the distribution of the benefits of technical progress may even take place through an increase in monetary remunerations alone, without any reduction in the monetary price of commodities.

"In the long period [...], we are still left with the choice between a policy of allowing prices to fall slowly with the progress of technique and equipment whilst keeping wages stable, or of allowing wages to rise slowly whilst keeping prices stable." (G.T., p. 271)

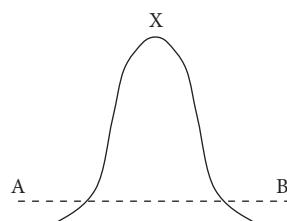
We referred to U. Ricci, *Éléments d'économie politique pure. Théorie de la valeur*, Milano, Malfasi, 1951, pp. 200–201, where the author alludes to a remark by Sraffa without quoting him.

E.N. 29. THE ENTREPRENEURS' HILL IN PARETO'S SOCIOLOGY

(from a letter to Pantaleoni)

Ch. V, §75. This topic is better developed in the following letter by Pareto to Pantaleoni. In it, Pareto comments on a diagram that is missing from the *Cours*, Vol. II, §718, where a similar comment can still be read:

"One should imagine entrepreneurs to be like this. They are on a hill. The average AB yields no profit, above AB there are those who make a profit, below there are those who make a loss. At X there is a lucky mortal who has maximum profit. Those below try to climb up. *In this there lies the stimulus of competition.* It is not enough for someone to try to obtain maximum profit, it is necessary for him to be pressured by those who sell at a loss. With the losses of the entrepreneurs who are below AB society buys the profit coming to the society itself from that stimulus. A socialist State might perhaps eliminate this stimulus, but it will also lose the profit. It will be a savings like that of a man who forsakes his spurs in order to climb on a horse. I said that *perhaps* there would be a savings, because those who try to climb up also have the duty to make experiments. And these experiments will have to be made anyway by the socialist State as well." (*Corrispondenza Pareto-Pantaleoni*, vol. I, pp. 426–27)



E.N. 30. EFFECTIVE COMPETITION AND PARASITIC COMPETITION IN PARETO THE SOCIOLOGIST-ECONOMIST

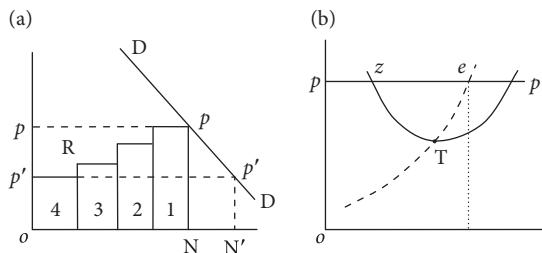
1. Ch. VI, §§10–13. The hypotheses that Pareto introduces to describe parasitic competition are the following: the dimensions of the market are given by the position of the exchange line, that is, they do not depend on the number of producers (it should be pointed out that this first approximation is of a kind for which Marshall is usually reproached); within the limits allowed by the exchange line, already existing firms operate in a monopolistic syndicate regime and therefore, whether they all produce with decreasing average costs (§§10–11) or with increasing average costs (§§12–13), they fix a price that maximizes their monopolistic profit. The entry of new firms does not modify the pre-existing monopolistic price whilst it causes the sum total of the industry's fixed costs to increase (§10). Since competition on entry cannot operate in an effective way (cut in profit by reducing prices), it operates in an ineffective, parasitic way—it cuts profits by increasing costs:

“the number of producers therefore increases; and as each one of them must earn his own living out of production, the cost of production necessarily rises. In other words, the line [...] of complete transformations shifts and will finally pass through the point where the producers had stopped. Such a phenomenon has become widespread in some countries where, owing to the syndicates, a large number of people live off productions like parasites.” [§10]

2. A reformulation aimed at opposing *efficient competition (on entry)* to *parasitic competition (on entry)* must start from fig. 14 in Ch. III, §§103–104. In this figure a production function appears—the line of complete transformations—which shows a point of inflection, which is labeled as point F in our Note [III, §103, a], and a point that Pareto indicates with T. Now, when point F exists, that point has the minimum of marginal cost, whereas at T one has the minimum of average cost. We already know that for Pareto point T may be virtual, in the sense that it may exist at such a high level of production that it is as if it did not exist at all: in actual fact, firms produce with a decreasing average cost. As for the inflection point F, in the remainder of the *Manual* Pareto ignores it because he analyzes a firm that operates—alternatively—in the segment where marginal cost is always constant or increasing, or decreasing. However, the point of inflection F will again be important in the case we shall examine under point 5 below.

That being stated, in order to reformulate the opposition between effective competition and parasitic competition we should start from a total cost function that shows a fixed cost and then increases more than proportionally with the quantity produced. In other words, let us assume that (total) average cost has a minimum at point T as in Fig. b, and that marginal cost increases with increasing increments, intersecting average cost at point T. Let us also assume that the surplus R of total proceeds over total costs of a firm is comprised of two parts. One is the surplus of price p over marginal cost, mg , multiplied by the quantity produced q , that is to say: $R_1 = (p - mg)q$. The other is the difference between marginal cost and average cost, md , multiplied by the quantity produced, that is to say: $R_2 = (mg - md)q$. Of course, the symbols R_1 and R_2 indicate two types of rents due to Paretian *irreproducibilities* of various duration. Now the

opposition between the two forms of competition may be represented by the following *Fig. a* and *Fig. b*.



3. EFFECTIVE COMPETITION. Let us suppose (Fig. a) that price is at level p and that some new firms enter the industry in question, which already includes 4 firms; of the new firms, some tend to replicate the most effective technology (that of firm n° 4, which was the last to enter and has the largest surplus), whereas a few others aim at achieving even greater effectiveness. The constraint of the demand DD is important to check *a posteriori* their investment decisions, but for Pareto their imagined demands also count. On this topic Pareto provided the following considerations. Contrary to what happens in the activity of consumption, in the production activity human *reasoning* is applied with the utmost intensity, even though competition is the realm of *non-logical* actions (see E.N. 14). Entrepreneurs, who embody *speculative spirit* and *instinct of combinations*, imagine they can take away some share of demand satisfied by other firms in the same industry, or they dream about appropriating the increase in income that will be generated by other industries; they fear failure at least as much as they yearn for success, and both these stimuli drive them to reflect, but also to decide quickly, in order to secure market shares.

A posteriori, and by resorting to *ceteris paribus* (the demand DD is not altered by the number of firms; firms experimenting with better technologies than the existing ones are not taken into account), the fragmentation of production due to the entry of new firms that imitate the most effective technology tends to cause the price to fall from p to p' and therefore to cut the rents R_1 (the price tends to fall to the level of the marginal cost corresponding to point T) and the rents R_2 (the reduction in price causes the point e of equilibrium to drop down along the marginal cost, and the deviation between mg and md is reduced until it becomes zero at point T). As it can be seen, competition on entry *tends* to make the industry's total proceeds coincide with the industry's total cost. More precisely, according to Pareto, *when the imitative-competitive process has not yet reached its limit state*, equilibrium is determined by the equality between price and marginal cost, and the equality between total proceeds and total cost is not satisfied; the two equalities are reconciled in the limit state of free competition, when the firms cannot move either left or right of the quantity corresponding to the average cost minimum (*Manuel*, 1909, especially App., §92, p. 623). This is tantamount to saying that during the process that leads to the limit state of free competition, the distributable product increases until it reaches a maximum in the final moment when the product increase becomes zero. Before going ahead, let us carefully weigh what Pareto is saying, on the basis of this interpretation.

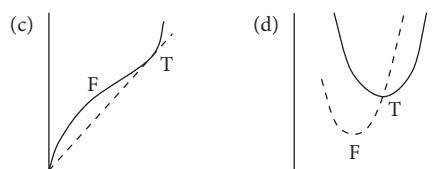
There is something great and courageous in Pareto's idea according to which the welfare of a community increases until it reaches a maximum, thanks to the diffusion of the most effective production technology. Its courage lies in freezing the innovative process so that producers confront a *given* set of technical production possibilities. If scholars like Marx and Schumpeter were to object, and rightly so, that capitalism cannot survive if innovation is frozen—that is, if only imitation is considered—Pareto would have the right to recall what he wrote twice in the official records of the Valdarno Ironworks [la Ferriera del Valdarno] (an ironworks makes a profit if it anticipates a potential demand with new iron products), and his agreeing-disagreeing statement towards Marx and Lassalle (who had seen the “perpetual movement of the entrepreneur”: *Cours*, Vol. II, §718). At any rate, whether, on the consumption side, one resorts to Pareto's notion of *equivalent surplus* or to Allais' notion of *distributable surplus*, the core of Pareto's idea is that maximum welfare for a community is reached when the best technologies are totally exploited on the production side. As it can be seen, it is a scientific project that bases welfare increase on what Pareto trusted the most—production. But it is also a project for which Pareto pays a price, because it exceeds the time duration that makes the *ceteris paribus* acceptable. Pareto arrives at the equilibrium corresponding to the maximum distributable product—and therefore the maximum welfare for the community—after loading on the *ceteris paribus* that half industrial revolution consisting of the imitation process without innovations. Pareto realizes he has frozen the continuous flowing of innovations, this is the reason why every now and then he drops the *ceteris paribus* assumption. That is, to remind us that the analysis of a tendential force (imitation) leaves out the existence of a disruptive force (innovation).

However, Pareto intended to fight the theory—which he almost considered as a moral dogma—according to which production effectiveness and community welfare depend on the numerosness of the firms. Competition on entry may indeed operate in an ineffective way. Let us see.

4. PARASITIC COMPETITION. In the case of parasitic competition it is not point p , in Fig. *a*, that tends to drop down towards p' , “draining” the rents, but it is p' that tends to climb up towards p , because of an increase in costs. More in particular, in Pareto's example (§10) the cut in R is due to an increase in fixed costs. However, it should be noted that in Pareto, the cut in R does not take place because of an increase in fixed costs in every firm, but because of the increase in fixed costs caused in the industry by the entry of new firms. From Pareto's point of view, the already existing firms are responsible for having created a monopoly, thus preventing p from falling as in the case of effective competition. But the cut in the rent R is the consequence of the entry of new firms: the quantity sold by the existing firms is reduced without any variation in price, up to the point where surplus R is zero, that is, up to point z in Fig. *b*. If for the sake of simplicity we assume that each of the 4 firms in Fig. *a* has the same general total costs and the same total fixed costs equal to K , the cut in R by parasitic competition consists in the total fixed costs in the industry going from $4K$ to $5K$, to $6K$, . . . to nK , against a total industry revenue that is constrained by a demand DD , which remains unchanged, and by the monopoly price imposed by the 4 firms in our example. It can therefore be concluded that: *i*) in the case of parasitic competition, the entry of new firms “inflates” total fixed costs for the industry whilst

the individual firms are forced to climb up, instead of down, the hyperbole of their own *unit fixed costs*; ii) for a sufficiently high value of n , the increasing fragmentation of the industry reduces sales to a point where the *average unit cost* curve is decreasing, even for the firms that were initially operating on the right hand side of Pareto's point T—that is, in the segment where average unit cost is increasing. This reformulation applies to parasitic competition both with firms that were initially operating in the decreasing costs segment, and with firms that were operating in the increasing costs segment. But in the latter case (§13) Pareto makes his analysis so complex as to suggest further clarifications, which are also made worthwhile by the oversight that has already been pointed out [III, §103, a].

5. PARASITIC COMPETITION IN THE CASE OF INCREASING COSTS. In §§12–13 Pareto discusses parasitic competition in the case of firms that have no fixed costs and operate in the segment where average unit costs are increasing. We could limit ourselves to observe that in the absence of fixed costs (and assuming that variable costs remain unchanged), the point T of minimum average cost in Fig. b is shifted to the left. In other words, point T corresponds to a smaller production. However, the silence that has always shrouded parasitic competition in Pareto suggests some further remarks. Let the reader go back to fig. 14 in Ch. III and suppose that the line of complete transformations starts from the origin of the coordinates (absence of fixed costs), rather than from a positive abscissa value. Under this hypothesis, the total cost function in Fig. c corresponds to the line of complete transformations. The marginal cost curve shows a point of minimum F and intersects the average cost curve at point T, as in Fig. d. This is tantamount to saying that the increasing average cost segment is *only* attributable to the course in variable costs, that is, to the fact that marginal unit costs show an inversion point that separates the decreasing marginal costs segment from the increasing marginal costs one. If Pareto is not interpreted in this way, his §13 must be blamed not only for the oversight that has already been mentioned, but also for a mistake. Indeed, if a point of minimum for average costs that is associated with the aforesaid course in variable costs did not exist, then the entry of new firms and the resulting fragmentation of production would lead to an increase in production effectiveness, rather than to a decrease (the firms would follow their average costs curve down, instead of following it up), whereas it has been seen that parasitic competition requires production fragmentation to reach such a level that every firm is forced to reduce its own production and climb up along its average cost curve.



The fact that in §§12–13 firms produce with zero fixed costs can be inferred from an example (§12) and from a diagram (§13). Why are there no fixed costs, considering that Pareto assumes them even when he does not explicitly state so? Moreover, we have seen from the passage quoted under point 1 that Pareto even includes the salary of small firm managers among the fixed costs. However, if we consider that from as far back as when

he was living in Florence Pareto wrote to Walras (*Correspondence Walras*, Vol. II, p. 483) that his theoretical system was compatible with fixed costs, their absence in the *Manuel* appendix, §92, must be seen as an extension of a dialogue between Pareto and his old master. An understandably satisfied Pareto concludes as follows:

“Ordinary economics had some inkling of the difference between the cases we have examined, but it never succeeded in having a precise idea of this difference, and it could not even explain the different ways in which competition operates.” (Ch. VI, §15)

Two considerations. *i*) Ricardo and Walras do not stand out for their theories of production and of market structures. Walras went well beyond Cournot in the field of interdependences, but he set the theory back when he started from the *competition in the structure*. In doing so, he overturned Cournot, who had instead started from monopoly and had arrived at the competition in the structure through the *competition on entry*. It is not one of Pareto's lesser merits, that of bringing us back into the heart of economic life by reintroducing monopoly in his own dynamic perspectives (see E.N. 32). *ii*) Of course, if we had assumed firms with decreasing unit costs for any quantity produced, *in the case of effective competition* we would have had to resort to successive equilibria determined by terminal points.

SOME HISTORIOGRAPHICAL HINTS. In the tradition of Italian economists after 1918, our surplus R was always divided into profits and rents. It was preferred to have $R=R_1+R_2$ in consideration of the importance that Pareto gave to incomes due to irreproducibilities (see E.N. 32). Without dwelling on details, we point out that our reformulation of effective competition refers to a necessary and sufficient condition for a maximum that Amoroso proposed in 1921 (p. 170, quoted in E.N. 12) with reference to Pareto, after introducing the concept of *virtual unit cost* (the greater between average unit cost and marginal unit cost). As far as we know, not even Barone or Amoroso used mathematics or diagrams to give an idea of what the present writer has called, here and elsewhere, *parasitic competition* in Pareto (see A.Z., *Rendimenti, concorrenza e monopolio nella teoria della produzione di Pareto*, Firenze, Università di Firenze, 1992, Part III, in particular §21, “La numerosità delle imprese non garantisce l'efficienza produttiva”).

We recall L. Amoroso's “La curva statica di offerta” (G.d.E., Jan. 1930), not because in it he named the point of minimum of average costs, “point of escape” (in that same year a friend of his, the mathematician G. Evans, named it “the critical point”), but because in it he reviewed his previous position and shared P. Sraffa's criticism to the opposition between increasing-cost industries and decreasing-cost industries. However, with an implicit reference to Pareto, Amoroso added that it is instead possible to talk about firms with a point of escape variously situated on the Cartesian plane according to market structures and other circumstances.

Fig. *a* takes its inspiration from Barone, Fig. *b* from Pareto. Fig. *a* contains a type of dynamics that calls upon the reader's imaginative intuition. Pareto would possibly have had some reservations about it because of the excessive weight given to the *ceteris paribus*, but a similar consideration can be proposed for the representation adopted by Pareto for the imitative productive process.

As for M. Allais' distinction between equivalent surplus and distributable surplus, see Allais, *The general theory of surplus*, quoted in E.N. 19, and A. Montesano, “Il Massimo

di ofelimità per la collettività: definizioni, analisi, interpretazioni di Pareto e loro generalizzazione" (in G. Busino ed., *Pareto oggi*, Bologna, Il Mulino, 1991), where Pareto's theorem is generalized taking into account Allais' distinction between the two concepts of surplus. As for a reconsideration of Hicks' objection to Pareto, mentioned in *N. Fr. App.* [135] of the present edition, we leave that matter for another occasion.

While we were reading the English translation of this E.N. 30, we received the outstanding essay by Aldo Montesano, "Price collusion with free entry: the parasitic competition", *International Review of Economics* 59, 2012, pp. 41–65.

E.N. 31. ECONOMICS AND THE "STATE" IN PARETO

1. PURE THEORY. Ch. VI, §27 and §30. It is helpful to start from *pure theory*, where Pareto distinguishes between *a*) a maximum of utility (ophelimity) for a collectivity in economics (*G.d.E.*, July 1894) and *b*) a maximum of utility for a collectivity in sociology (*G.d.E.*, April 1913).

In the optimum *a*), Pareto assumes a community governed by unlimited competition where interpersonal comparisons are excluded; a community where, in particular (*Pareto, G.d.E.*, Nov. 1902, p. 432), every person satisfies his own preferences *without envying* those who are better off than him. Whether one adopts Pareto's distinction between governed and governing classes, or Hegel's distinction between civil society and political society, in the case *a*), the State (the governing class *considered as a single unit*) is implicitly treated as a mere shadow; at best it exists like someone who watches a show without participating in it. In this case, Pareto means to suggest that the more efficient production is—and therefore the bigger the pie is—the greater the scope for a ruler to facilitate change through the application of the compensation principle (which, in the 1930s, would be attributed to Kaldor and Hicks, although that principle had already been explicitly used by Pareto in his *G.d.E.* paper of July 1894).

In short, the *a*) maximum implies that if the State really existed, it could make some individuals better off without making others worse off (given the lack of envy). In the maximum *a*), Pareto does not add up heterogeneous ophelimities (as Pantaleoni and Barone feared), but quantities of one and the same good, when he introduces the first order condition for the maximum of ophelimity for a collectivity by considering the equivalent surplus [see *N.Fr.App.*, 190].

A maximum under case *b*) is very different because (i) it does consider the political phenomena associated with the existence of the State; and (ii) it applies an approach to collective welfare that explicitly provides for interpersonal comparisons of utility. That approach is predicated on the view that every individual has a view about the relative benefits to society from their own consumption and from consumption by each and every other member of society. To estimate the maximum 'social utility', Pareto outlined a two-step process. First, every individual subjectively assigns coefficients to the welfare of themselves and to the welfare of every other individual in the community to establish each individual's idea of 'social utility'. Second, the governing elite assigns coefficients to each person's assessment of social utility. This transforms each person's view of 'social utility' into homogenous units that may be aggregated to derive a single cardinal measure of 'social utility'. Consequently, unlike Pareto's economic approach in case *a*), the socio-logical approach to maximization under case *b*) is made possible through interpersonal

comparisons of utility, and both individuals and the governing elite are accommodated. The outcome is a single measure of 'social utility' that is a cardinal quantity, although it is conditional on the prevailing social equilibrium. When social equilibrium changes, so too do the units by which 'social utility' is measured by individuals as well as by the governing elite.

The coefficients by which the governing class standardize the social utility of the diverse range of the governed individuals leave the door open to all possible interests and sentiments, including the envy, or the love for one's neighbor, the contempt or the admiration for the "humanitarians". Consequently, Einaudi's reference (*Morale et Économie*, 1936) to Pareto shifting from cardinalism to ordinalism in order to get rid of some superfluous hedonistic conception cannot be accepted. What Pareto considered as superfluous (for general equilibrium) was not the hedonistic conception of human behavior, but rather the *cardinal measurement* of ophelimity (see E.N. 14 and 15), but not the cardinal measurement of utility. (I will focus on the misunderstandings that have arisen on the optimum *a*) in the References, point 2.)

2. IN THE SUCCESSIVE APPROXIMATIONS. Pareto's program in passing from pure theory to the successive approximations is a very ambitious one. Indeed, as we today know from the *Corrispondenza Sensini* (see E.N. 3 on the unpublished *Trattato di Economia* in several volumes), Pareto intended to enrich pure economy by re-using the *Économie mathématique* (1911) and, above all, he wished to seek, by statistical induction, new *laws* in that sociology that, in the meantime, he had developed in theoretical terms.

The disappointment that the *Manuale* generated among those who had so much admired the author of the *Cours*, which was more inclined to use statistical induction even in sociological matters, is worthy of a passing mention. The only one who fully grasped Pareto's new direction was perhaps Einaudi, who was the editor in chief of *La Riforma Sociale* (where the Turin publisher had already announced, in 1905, the forthcoming publication of a *Trattato di Economia* by Pareto), when he read "Alcune relazioni tra lo stato sociale e le variazioni della prosperità economica" in 1913 (see E.N. 56).

An indication of the complexity involved in developing sociology in an inductive direction can be seen from the fact that Pareto wanted to disassemble and reassemble a reality where both neophilia and neophobia exist, where there are interests and political passions (*residuals*) that are shrouded in false rationalizations (*derivations*); a reality where the State itself, far from being a unitary entity, is also "plural" (because the political élites in power are undermined by those without it; because central State and local States coexist, with the possibility of having the "reds" dominating in the former and the "blues" in the latter, with all the resulting problems for the coordination of a fiscal policy).

We are leaving out other elements of Pareto's sociology, as well as the points on which he focused his attention for the advancement of econometric induction (in particular, his distinction between "random errors" in the survey of long-term trends and those in the survey of fluctuations along trends: E.N. 32 and 56). As for money and sociology in Pareto, see E.N. 34 and 40. However, it is worthwhile to mention the Italian school of the so-called "politological" approach to the study of public finance (as opposed to the so-called "hedonistic" finance).

SOME BIBLIOGRAPHICAL REFERENCE 1) ABOUT POLITOLOGICAL FINANCE AND ITS FUTURE AND 2) ABOUT SOME MISUNDERSTANDINGS ON MAXIMUM a).

1) The *Teoria delle illusioni finanziarie* (1903) by A. Puviani, mostly inspired by Marx, reminds us that in Italy not all politological public finance could be traced back to Pareto. However, there is a connection between Pareto and Puviani: Pareto's insistence on interests and false representations of reality ("derivations")—for which he draws from Machiavelli and Marx.

R. Murray was first to draw attention on the "Massimo di utilità per una collettività in sociologia" (1913) and politological finance in Pareto, in his *Principi fondamentali di scienza pura delle finanze, saggio di un'organica sistemazione teorica delle dottrine finanziarie nel loro duplice aspetto politico-economico*, Firenze, La Voce, 1914. However, it is M. Fasiani (G.d.E., March–April 1949) who must be credited with what we believe is the best essay on Pareto and public finance. As for the 35 years separating these two essays, many Italian scholars would be worthy of mention (one need only think of G. Sensini and G. Borgatta), both for extending general equilibrium to public finance and for "politological" finance.

Furthermore, in regard to the Italian scholars in the field of public finance at the end of the 19th century, the renowned works by J.M. Buchanan and the recent pages by I. Magnani, *Dibattiti fra economisti italiani di fine Ottocento*, Milano, Angeli, 2007, are invaluable. In his course on public finance (*Elementi di teoria generale della finanza pubblica*, Firenze, Università degli Studi, 1985), A. Petretto has always given space to the Italian socio-political approach, comparing it with the most recent contributions in the English-language literature, including Rawls' theory of justice. I shall now try to have a look at Pareto's politological approach with regard to the future.

Born in Paris in 1848 because he was the son of an exiled follower of Mazzini, Pareto always retained republican sentiments, even though he considered Cavour, who had embraced the cause of the Savoy house, as the great architect of the unification of Italy. From his deep knowledge of the *Storia delle Repubbliche Italiane* (Sismondi), Pareto was more inclined towards the republican federalism of Carlo Cattaneo (1801–1869), one of the great losers among the patriots of the Italian *Risorgimento*. Now, if Pareto's statistical studies on trends and cycles are still topical (as everybody knows, Pareto did not want to be confused with Vico's "corsi e ricorsi"), then the problem of the plural State, the problem of the coexistence of the central State with the local States and its related financial problems will perhaps accompany our future until the advent of a global government—the kind of government that today is once again being talked about, also in the wake of Keynes, even if we live in a vastly different world. The possibility that the unifying direction followed by the European Community after World War II may be included among the most interesting pages of the trend that is making the world smaller at an ever-increasing speed should not be discounted.

Even if M. McLure's reconstruction of the Italian socio-political approach to public finance stops at A. Scotto, his recent pages ("Pareto and contemporary economics: social equilibrium, fiscal decentralization and economic growth", in L. Bruni and A. Montesano eds., *New essays on Pareto's economic theory*, London-New York, Routledge, 2009; but see also M. McLure, *The Paretian school and Italian fiscal sociology*, Basingstoke, Palgrave-Macmillan, 2007) can be included in this perspective and are at any rate

characterized by their originality in the way they discover a new side to Pareto's financial thought.

2) The distinction between a collective *optimum* in economics (*case a*) and a collective *optimum* in sociology (*case b*) is made complicated, in Pareto, by the fact that in his abovementioned article from 1894 he deals with *case a*, which does not involve any interpersonal comparison, but already has in mind the *case b*, which does involve such comparisons, but to which he will dedicate an article only in 1913. This complication is made worse by Pareto's terminology: in 1894, he has not yet come into the habit of using the term "ophelimity" in *case a*, and the term "utility" in *case b*). He uses the term "utility" in both cases, even though in his mind he keeps them separate. It does not come as a surprise, therefore, that both Pantaleoni (who was less gifted than Pareto in mathematics but as gifted as him in terms of scientific inventiveness, and was at any rate the author, with A. Bertolini, of "Cenni sul concetto di massimi edonistici individuali e collettivi", *G.d.E.*, April 1892), and E. Barone (who was exceptionally swift at making the ideas of other scholars his own, re-presenting them with admirable geometric ingenuity) fell prey to the suspicion that Pareto had made the mistake of adding up the ophelimities of different individuals. The fact that Pareto, in 1894, had expressed himself in such a way as to give rise to misunderstandings (in the *Cours*, §721, n.2, Pareto himself would admit to expressing himself "in an elliptical fashion") can also be inferred from a letter—never sent—by Pareto to G. Vailati. This letter refers to the aforesaid §721 of the *Cours*, where Pareto tried to make the meaning of his equations transparent (see *Corrispondenza Pareto-Pantaleoni*, Vol. III, pp. 426–28).

Even though the *maximum a*) shows a mathematical inventiveness that was immediately noticed by Walras and would later be commented upon by M. Allais in 1973 and, in a more general form, by A. Montesano in 1989, there is a debatable side to it. This is not, let it be clear, from a logical-formal point of view (nothing prevents one from researching, as Pareto did in 1894, the *maximum* distributable production regardless of any distributive justice issues), but from the point of view of realism. In short, *maximum a*) does not take into consideration that those who see the wealth of the others increase, while their own does not change or increases at a slower rate, might be driven by their own dissatisfaction to fight the advent of an *optimum* they deem inequitable. The fact is that after he talked, in his polemic with Scorzà (*G.d.E.*, November 1902, p. 452), about the absence of "envy" as a prerequisite for *optimum a*), Pareto expanded on it in "L'individuel et le social", a paper presented at an International Philosophical Meeting in 1904 and published in 1905 (today found in V.P., *Oeuvres complètes*, Vol. VI, pp. 259–65).

As for the present writer, whilst I favor *optimum b*) over *optimum a*) in this E.N., in the following E.N. 32 I try to shift attention to what Pareto calls "the most important law of political economy", and on the fragility of partial equilibrium and general equilibrium when confronted with epochal changes, which Pantaleoni calls dynamic of the "second-kind".

We have mentioned: M. Allais, "The general theory of surplus and Pareto's fundamental contribution", International Meeting on *Vilfredo Pareto*, 1973, appeared in 1975, cit; A. Montesano, "Il massimo di ofelimità per la collettività: definizioni, analisi, interpretazioni di Pareto e loro generalizzazioni", in *Pareto oggi*, collective book edited by G. Busino, Bologna, Il Mulino, 1991, pp. 115–38.

E.N. 32. COURNOT, WALRAS, AND PARETO ON COMPETITION, MONOPOLY, AND COLLECTIVE WELFARE, ENDING UP WITH A DISCUSSION OF CAPITALIST DYNAMICS IN THE WORK OF PARETO AND PANTALEONI

Ch. VI, §46. This polemical dig by Pareto—clearly aimed at Walras’ “metaphysical nonsense”—prompts us to start from Walras and Pareto.

1. Walras and Pareto on competition and monopoly. From the very first edition of his *Éléments*, Walras is pleased with himself for having reversed the path followed by Cournot, who had set out from the monopolist “inventeur”, and had arrived at unlimited competition by going through duopoly and all the gradations of homogeneous oligopoly. Also for ethical reasons related to his idea of distributive justice, Walras elects to start from an ideal entrepreneur who makes neither profit nor loss after having remunerated all productive services. As we shall see under point 6, in the *Manuel* (1909) Pareto introduces into general equilibrium all the types and degrees of monopolistic power. If Pareto has been forgotten in the historiography regarding monopoly in general equilibrium, this is perhaps due to the fact that at times he expresses himself in a difficult and confrontational way—as when he talks about *monopoly* (type II) *with complete competition*: E.N. 17—and, at any rate, he never reaches the heights of Cournot’s rich and precise language. Let us, therefore, have a preliminary look at Cournot.

2. Competition and monopoly in Cournot’s language and in Pareto’s. Cournot (1838) distinguishes among natural monopoly, legal monopoly, and, most importantly, artificial monopoly, which is due to inventiveness. In a word, it is the monopolist “inventeur” who triggers that *imitative process* that Cournot calls *competition*.

Cournot gives a representation of this process through successive static equilibria. He indicates them with the expression “producer competition” (*concurrence des producteurs*), which, in today’s language, includes duopoly and homogeneous oligopoly with an increasing number of firms. As imitating firms enter the market, production increases and the price of the product falls without there being perfect transparency. Even though Cournot does not explicitly state it, his firms lack the fast transmission of data by telephone presupposed by Edgeworth (1881, p. 18). The Cournot duopolist or oligopolist will lower the price of the product in order to steal customers from their competitors because, in the absence of total transparency, they rely on a *transitory profit* (*bénéfice momentané*, §44, p. 92, italics in the original). In short, there are time lags between the moves of various competitors.

This imitative process ends with *unlimited competition*, at which stage the industry has become so fragmented that the price of the product does not vary when the production of a single firm varies. In other words, price is no longer a variable upon which the single firm would base its strategy, relying on a transitory profit: price has become an objective datum, impervious to any kind of decision by the firm.

Cournot’s unlimited competition is a *virtual limit state* because it is based on an initial innovation *una tantum*. Since in the course of the imitative process no other innovation takes place, unlimited competition constitutes a limit stationary state. Stationariness, too, implies savings and investments, as perhaps Pareto would make the young Schumpeter

understand, after the latter sent him *Das Wesen und der Hauptinhalt der theoretischen Nationalökonomie* (1908), for an account of which Pareto asked Pantaleoni: investments, of course, are those that are sufficient to prevent stationariness from degenerating into economic regression.

Something just as important, as it is ignored, still remains to be said. In Cournot's view, the monopolist inventor (*inventeur*) can be so cost efficient that imitative competition can stop at duopoly or even never go beyond monopoly itself. As for Marshall's famous accusation—later taken up by Sraffa (1925)—according to which Cournot did not recognize the incompatibility between competition and increasing returns, we have always regarded it as groundless. We have seen [VI, §40, a] that the hypothesis according to which a monopolist can produce and sell at increasingly lower costs and prices, and still achieve profits had always been ascribed to Marshall and was therefore old by the time of Pareto's *Manuale*. Einaudi had even converted it into a diagram in his *Rendita mineraria* (Torino, Utet, 1900, p. 723). We mention Einaudi because not even he, who, in later years would become such an admirer of Cournot, could dig out the jewel of logic that allowed Cournot to start from the manufacturing industry, with progressively decreasing unit costs, and conclude with a remark on monopoly and collective welfare that perhaps escaped attention because it was included in a seldom read book:

“The result of this is that some powerful capitalists or some large companies may kill competition, as they say, and artificially create a true monopoly for themselves, which involves a profit that is higher than the usual level of profits, that is, in actual terms, a rent or farm rent [fermage], the explanation of which, as one can see, has nothing in common with Ricardo's theory. The establishment of monopolies of this kind can be perfectly well associated with a decline in prices that is favourable to consumers, although the influence of the monopoly is always there, in the sense that the price does not drop as much as it would if the competition were compatible with the new conditions”. (Cournot, *Revue sommaire des doctrines économiques*, Paris, Hachette, 1877, pp. 178–79. We drew attention to this passage in our 1989 seminar notes published in 1992: A.Z., *Rendimenti, concorrenza e monopolio nella teoria della produzione di Pareto*, Università di Firenze, 1992, p. 42.)

In the field of industrial structures and collective welfare, we think we can make out two main, often overlapping, threads. Both threads have Cournot and Dupuit among their forefathers, and Marshall and Pareto—the two great theorists of production after Marx—among their most prominent successors.

Natural, legal or artificial monopoly, producer competition, unlimited competition: these are the rich terminologies that help the reader when Cournot tackles industrial structures and collective welfare. In terms of richness and clarity of language, Pareto is below Cournot's level. Pareto is quite clear only when he discusses the *limit state of free competition*, in which every firm makes (on average) neither profit nor loss: this is an abstraction he considers as only tendentially true (in the real world the elision of profit coexists with the tendency to recreate it). On the other hand, not all have understood that when Pareto is simply speaking about *free competition*, he limits himself to the supposition that firms are totally free to enter and exit the productive process in a given industry.

3. The reappraisal of Cournot in game theory ignores what Pareto read in Pantaleoni.
In order to compare Pantaleoni and Pareto (see points 7 and 8), it is advisable to start

from Pareto's lack of generosity towards Cournot, and from the great light that, on the contrary, Pantaleoni shed on the founder of mathematical economics.

Pareto's lack of generosity towards Cournot is only equalled by the silence that shrouds Cournot in K. Arrow's commendable *Opera omnia* (six volumes appeared in 1983–84–85, Harvard, Massachussets, Belknap Press). Today, at a time when economic culture speaks of a Cournot-Nash equilibrium, we can only add that when Pareto read Pantaleoni's *Principii* (1889) in 1891, he would certainly have noticed the ample acknowledgement given in that work to Cournot. Indeed, not only does Pantaleoni speak about the optimum of the Cournot monopolist, but he also speaks of a principle by Cournot, which states that a monopolist may opt for a price policy or a quantity policy but he does not have the coercive power to adopt them both. In addition, he also speaks of a "law of economic equivalents, attributable to Augustin Cournot [...], which is fundamental in order to explain a large category of *correlated prices*, that is, of prices of goods that cannot be varied without a correlated variation in the prices of other goods, with such variation going sometimes in the same direction, some other times in the opposite direction as the original variation." It is a "law of Cournot" that Pareto took up when dealing with price elasticity for competing and complementary goods. It is the same law that, between the two World Wars, would give rise in Italy to a vast literature rooted in Pantaleoni's thought, to which Schumpeter, who abandoned Europe in 1932, could not do justice in his famous *History*.

By now, the reader will have understood that we shall come to compare Pantaleoni with Pareto by going through the latter's scientific debts to the former. While we deem it misleading to consider Pareto the economist as being born from his encounter with Pantaleoni (E.N. 1), we do not want in the least to ignore these debts, even though for the sake of conciseness we shall only dwell on the main one: the endless evolution and expansion of human needs, which is what Pareto calls "the main law of political economy" (E.N. 25). Pantaleoni's potential influence on Pareto's distinction between a maximum of ophelimity in economics and a maximum of utility in sociology can be inferred from a noteworthy study by Marco Dardi ('Neither Lausanne nor Cambridge: Pantaleoni and the missing boundary between economics and sociology', forthcoming).

4. The main law of political economy: Pareto's debt to Pantaleoni. Pareto makes it very clear that in his own static equilibrium the given data that determine unknowns (prices and quantities of exchanged goods) are technology, which concerns production, and preferences (or tastes), which concern consumption. On this point, what an economist could say, and what Pareto did say in his *Considerazioni* just after he had met Pantaleoni, is that this static representation can only be acceptable for a short period of time. Indeed: a) production techniques and preferences change over time; b) in their evolution, techniques and preferences can no longer be considered as independent of each other; c) the question therefore arises whether in the dynamics of interdependencies, where "everything depends on everything", it is nevertheless production that dominates over preferences, or, on the contrary, it is the latter that guide production. It is in relation to this question—which evokes Marx, Marshall, Pantaleoni, Schumpeter, and Galbraith—that we place Pareto's main law of political economy. We detect Pareto's greatest debt to Pantaleoni in the passage from static equilibrium, where preferences are

given, to the dynamics of preferences where every subject sees his *ego* shattered, just as in Pirandello's famous play's *Sei Personaggi in Cerca d'Autore* (*Six Characters in Search of an Author*).

For the reader's ease of reference, we copy here the passage in E.N. 25 that led us to name the main law of political economy after Banfield-Jevons. Indeed, perhaps about ten years before he read Pantaleoni, Pareto also read, in Jevons, this passage by Banfield:

“The first proposition of the theory of consumption is, that the satisfaction of every lower want in the scale creates a desire of a higher character . . . thus a full supply of ordinary food not only excites to delicacy in eating, but awakens attention to clothing” (Banfield cited in W.S. Jevons, 1871. *The Theory of Political Economy*, London: Macmillan, §11, chapter 3).

Now, in Pantaleoni's *Principii* not only can one read §3 (pp. 61–69), where Pantaleoni rejects, as Pareto would later do (E.N. 25), the existence of a rigid hierarchy of needs that applies to all times and all individuals; in it one can also read the following passage (§4, “On the Variety and Progression of Needs”, pp. 59–70), which probably served as inspiration for Pareto's illustration with the marginal ophelimity of money being always positive (E.N. 25): “the satisfaction of more basic needs sharpens our sensitivity and gives rise to more refined needs. After taking care of present needs, we take care of the more remote ones. The progression of needs is therefore unlimited; and all the more so, as needs are not only directed towards the means of direct satisfaction of the needs themselves, but also towards the attainment of *tools for the making*—whether it be more abundant, or faster, or more perfect, for the same cost—*of the means of direct satisfaction*; and the only limits for this kind of needs lie in the inventive power of the human mind” (p. 70, italics in the original).

However, we do not think that Pareto arrived at the most important law of political economy just on the basis of his reading of Jevons—who quotes Banfield—and his successive reading of Pantaleoni—who quotes many economists including Spencer, with regard to the joy felt by some when working on the invention of new production tools. Behind these readings there is the industrial manager who, as soon as he reads Marshall, praises the latter's restrain in resorting to theoretical abstraction; there is the man in whom Walras sees a scholar inclined to look for the applicative political moment of theory; the man in whom Bobbio sees *panpoliticism* (E.N. 55) as the dominant moment of his sociology. After all, Pareto made his debut as an engineer by designing train funnels to reduce fires, and later, as an ironworks manager, he designed some alterations for a plant he considered badly planned, and, to that end, he had no disdain for going down to the workshop. Furthermore, on two occasions during his tenure as an industrial manager, he wrote that profit will reward the ironworks that anticipate other people's need for new iron tools, and not those who wait for the actual demand to fall on them as manna from heaven. If we add that Pareto's sociology contrasts *neophiles*, who love uncertain revenue (he calls them “speculators”), with *neophobes*, who are inclined to conservatism and low risk and that, arguably, this is still today the dominant philosophy in the business world, it is impossible not to think of Pareto as the economist who embraced the interdependences where “everything depends on everything”, but—like Marx, Marshall, Pantaleoni, Schumpeter, and Galbraith—he considered production to be the dominant moment in the dynamics of needs.

5. Pareto on monopoly in partial equilibrium and in general equilibrium. Various aspects of Pareto's work regarding competition and monopoly require some comment before we can compare Pantaleoni—who knows partial equilibrium and general equilibrium, but goes on a personal road—with Pareto—who, on the contrary, embraces and develops general equilibrium.

First of all, it should be made clear that Pareto did not go so far as to always consider the complete system of interdependencies. For example, in the present Ch.VI, §46, where Pareto implicitly criticises Walras by allowing for the possibility of concentrated production by a monopolist being more efficient than the atomistic production of the competition. This is especially evident in the case of parasitic atomistic production, where the elision of profit is not associated with a reduction in product price when new firms enter the industry, rather, it is associated with an increase in the cost of production if firms are linked by some monopolistic agreement. In particular, it has been seen that Pareto resorted to successive static equilibria without including parasitic industry among the general interdependencies. However, it would not have been difficult for Pareto to overcome this hurdle. Unfortunately, the mathematical technique by which Pareto included firms with some kind of monopolistic power, be it parasitic or not, in the system of general equilibrium has been ignored in the literature.

Contrary to the partial equilibrium of Cournot and Marshall, who start from given supply and demand functions for a product, in Pareto's theory of production and exchange the given fundamental data concern production techniques and preferences. In the work of Pareto, the quantities demanded and supplied of each commodity are therefore unknowns determined by a system of equations that can be divided into various subsystems. If one adopts given prices for each individual, the first subsystem includes, for all the individuals and all the products, the equations that express the equality of weighted marginal ophelimities and the equations that express the budget constraints. Let us now assume, for the sake of example, that bread is produced by a monopolist and that Pareto has observed that the monopolist uses his power on the price of bread to obtain maximum monetary profit. In relation to the monopolist, and the monopolist alone, Pareto eliminates in the first subsystem the equation that expresses the equality between the weighted marginal ophelimity of the numeraire commodity (the price of which is equal to 1) and the weighted marginal ophelimity of bread, and, in its place, adds the equation that expresses maximum monetary profit, in which the price of bread plays the role of an independent variable. For further details we refer to Pareto's famous polemic with G. Scorsa (V. P., "Di un nuovo errore nell'interpretare le teorie dell'economia matematica" *G.d.E.*, November 1902), where Pareto explains with greater clarity than in the *Cours* how to include monopolistic activity, which was originally conceived in the context of partial equilibrium, within general equilibrium.

6. For Pareto, rents (profits and losses) accompany the change that is inherent in economic evolution. It is more complex to find a central thread for all the types of profit introduced by Pareto in the *Manuel* (1909). Since we have learnt to distinguish between Ricardian rents, due to the avarice of nature, Cournot's rents, due to human inventiveness, and Pareto's parasitic rents due to the combined effect of production fragmentation and some monopolistic agreement, we could find the central thread in the fact that it takes time for the positive rents due to any monopolistic advantage to dissolve (where time is endless

in the case of Ricardian rents on land: total irreproducibility), in the same way as it takes time, at best, to recover the capital invested in productions with persistent negative rents. And yet, even if we recall that for Pareto: a) the limit state of free competition does not exist (it is an abstraction to isolate the virtual equilibrium that would be created if imitative competition alone were active when starting from a once-only innovation); and, therefore, b) while imitative competition is at play there is always an oligopolistic profit (a Cournot rent), not all market structures can be easily captured using the concept of rents from irreproducibility. We are thinking of all the cases of duopoly and bilateral monopoly considered by Pareto, starting from the case of indeterminateness he commented upon in 1891, the year of the famous “barter controversy” between Edgeworth and Marshall. However, if we consider that Pareto regarded the fundamental law of economics as synonymous with perpetual change, then we are led to represent Pareto's systematic rents by using an index that starts from negative values and becomes positive after passing through a notable value: zero; an index where each existing firm finds its image; an index that, in a changing economy, indicates the strength of each firm in their complex relations with regard to inputs and outputs. Since in terms of production, an economy may change in a “progressive” or in a “regressive” direction, we copy here a diagram to which Pareto often had resort. His vision of perpetual change must be projected on a society that progresses in terms of production, even though through recurring “crises”, as in Fig. 1: the abscissa represents time, the ordinate represents a generic production, the curve increasing in waves represents a historical course, whereas the broken curve, which never increases linearly, is an interpolated curve that represents an underlying trend, a “normal” tendency. The gap between the historical course and the normal trend allows Pareto to circumscribe the phenomena that are more strictly “cyclical”. In this framework, there is room for modest changes on the side of production that—when “the second order quantities” are ignored (Barone 1894 *a* and *b*, see References)—justify the hypothesis of modest variations in the relative prices of the products, and therefore, on the side of consumption, the Marshallian hypothesis of a constant marginal utility of money (see Pareto, *G.d.E.* June 1892, p. 496). But there is as much room for changes that Pantaleoni would later call a “second-kind dynamics”, where all the caution exercised in order to justify the analysis in the vicinity of a point of equilibrium, be it partial or be it general, is thrown to the wind (see **Point 8** below).

Pareto looks for some order in the changeable disorder of economic life especially in Vol. II of the *Cours*, where he deals with acquired rent. Well, here Pareto does not associate profit (a positive rent) with successive temporary equilibria that lead to a final state of virtual equilibrium. Here Pareto assumes that in any given period, something new happens, and therefore the scale on which he mentally sorts the index of the strength and vitality of each firm refers to the forces of change that give rise to the continuous dissolution of profit and its continuous regeneration through systemic cycles. This systemic heterogeneity is found again both when, in the *Cours*, Pareto

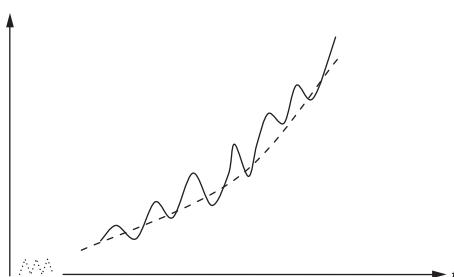


FIG 1

proposes periodical budgets for existing firms, and when he proposes closing balances that encompass the whole life of every ceased firm. Unfortunately, figure 38 of the *Cours*, §718, where the abscissa shows the time, and the ordinate shows the monetary values of a firm that is fighting for profit with the weapon of technical progress and which on average, considering highs and lows, because of competition, either does not make any profit or loss, or it records losses; that figure 38, we were saying, captures only the distributive mechanism for the fruits of the technical progress which was classical and then Walrasian-Paretian, but not also the distributive mechanism that takes explicitly into account the strength of the Trade Unions. If we consider the Trade Unions, we have a *long-term* distributive mechanism that we could name after Keynes (E.N. 28) and Sylos Labini. Indeed, the latter made it one of his favorite topics: it was based on the dualism that influences both the different market structures (competition/oligopoly) and the rate of increase in technical progress.

7. Pantaleoni and Pareto: two responses to a shared dissatisfaction. We have been playing on what Pareto considered the fundamental law of political economy and on an index of rent to try to overturn a repetitive interpretation of Pareto that is sometimes based on an alleged stationarity, and sometimes on an alleged lack of dynamics in his theoretical system. Considering that Pareto has been shown to be indebted to Pantaleoni, it comes naturally to us to also comment on a common comparison between Pantaleoni and Pareto that points to Pantaleoni as the originator of a “second-kind dynamics”, a sort of anticipation of the ideas of present disequilibrium theoreticians. We are referring to the Pantaleoni who, in “Di alcuni fenomeni di dinamica economica” (*G.d.E.*, Sept. 1909), contrasts the changes “of a first kind”, which lead from a previous to a new equilibrium (as in Pareto, Barone and B. Clark, Pantaleoni says), with Pantaleoni’s changes “of a second kind”, which affect the structures of a society and change its history. Since change and uncertainty are two major points that lead us to count Pantaleoni and Pareto among the few economists who at the turn of the century made it easier to understand the contemporary world, allow us, first of all, to share the opinion expressed by Pareto when he read the press proof of the 1909 article by his friend Pantaleoni. We must again mention Cournot.

We have stressed (point 3) Pantaleoni’s generous appreciation of Cournot’s monopoly theory. However, Pantaleoni forgets what even Walras recalls at the end of his *Éléments* (Jaffé’s translation, p. 440), precisely in the passage where he recognizes that he has overturned Cournot by using unlimited competition, rather than monopoly, as the starting point for his theoretical system (point 2). That is, Pantaleoni fails to recall that Cournot arrived at unlimited competition through an oligopolistic imitative process. It is, therefore, understandable that Pantaleoni objected to Pareto—who, after reading the press proof of the 1909 article, pointed out to his friend that he, Pareto, had included monopoly in general equilibrium—that whereas “free competition” is a state of rest that tends to perpetuate itself, monopoly is, on the contrary, “full of dynamism”. As if Cournot and Pareto had not known that an imitative process originates from an innovative monopoly full of dynamism! It is apparent that Pantaleoni confuses free competition with the limit state of free competition, or, if one prefers, with unlimited competition: whilst the latter presupposes the former, the reverse is not true. Of course, one must try to interpret an author according to the meaning of his words. If one follows this rule, one

realizes that Pantaleoni uses the words: 1) “free competition” to mean “the limit state of free competition”; 2) “Ricardian rents” to mean the rents due to human inventiveness, which we have preferred to call Cournot rents in honor of the economist who knowingly distinguished between the avarice of nature (Ricardo) and human inventiveness (Cournot) (see point 3), and what Pareto simply calls rents on the basis of a general theory in which Ricardian land rents are a particular case; 3) “artificial components of social phenomenology” to mean formal laws and traditions as opposed to human inventiveness. In conclusion, when one looks at the extent to which Pantaleoni’s and Pareto’s words are interchangeable, one realizes that the former’s criticisms of the latter are reduced to a very small domain, especially when one excludes differences due to Pantaleoni’s excess of rationalism and his corresponding underestimation of the importance of human sentiments (Pareto, for instance, disagreed, as did also R. Benini, with Pantaleoni’s denial of the existence of differences between cooperative firms and other firms); and differences in the two scholars’ scientific temperament. We prefer, therefore, to dwell on some distinctive features of Pantaleoni’s work. Since the latter’s importance in the field of public finance is generally well known, we shall dwell on other, more frequently overlooked features of his profile.

8. Schumpeter: a follower of the “second-kind dynamics.” Pantaleoni and Pareto anticipate Keynes on uncertainty and “risk premium” and also on the distinction between the liquidity and solidity of the firms. According to Pantaleoni, if the changes are in large number, but insubstantial, or substantial, but in small number, we have an instance of Pareto’s, Barone’s and J.B. Clark’s first-kind dynamics, in which a disturbed economic system reverts to a previous equilibrium, or moves to a new one. In order to define second-kind dynamics, Pantaleoni gives us a passage that involves biological analogy, as favored by Marshall, and Pantaleoni’s main agreement–disagreement with Pareto:

“The fact that there are factors of social dynamism that alter the economic structure and do not lead back to an economic system of equilibrium is something that is more or less clearly felt by many. Those, for instance, who feel inclined to immerse the Economic Science in a sociology, the content of which is still being sought, are led, we think, by the desire to answer the justified need to arrive at following the effects on economic equilibrium of that dynamism we called of the second kind, and struggle against the sterility of the means they are resorting to [Pareto’s *Sociology* would only appear in 1916, but Pantaleoni was already aware of his friend’s dissatisfaction and intentions]. A similar spirit, it seems to us, drives those who are looking for a biological cloak for Economics [here Pantaleoni is obviously thinking of Marshall in particular], and, in essence, even the concept of historical relativism—which for the last 40 years has been being exploited in every way, giving us more bran than flour—was and is aimed at solving the problems shown by economic dynamism when it is the economic structure that undergoes changes” (Pantaleoni, “Di alcuni fenomeni . . .”, 1909, *cit.*, p. 221).

Pantaleoni’s substantial and numerous changes remind us of Schumpeter’s famous innovations in clusters. It was indeed Schumpeter who wrote, in his monumental *History of Economic Analysis*, that between the 19th and the 20th centuries there also were “suggestions that point toward the dynamics of our time [. . .]. I can only refer to the (relatively) clearest and most important of them, which are all due to Pantaleoni” (Schumpeter, 1954, Routledge edition, 1994, p. 933). The Austrian economist had not forgotten a

Pantaleonian horse race when he wrote that chasing profit is akin to hitting a moving target. Today, when not even R. Swedberg (*Joseph A. Schumpeter: his life and work*, Cambridge, Polity Press, 1991) mentions Pantaleoni, it is difficult to explain the degree of imaginative creativity with which his discussion of second-kind dynamics is interwoven. It is, nonetheless, worthwhile to try to explain his position and to also compare him with Keynes, who, in his *Treatise on money* (Vol. I, p. 154, Vol. II, pp. 85–6), showed high regard for Schumpeter's treatment of innovations.

Pantaleoni does not repeat what is already known: analogy—Pantaleoni also calls it “allegory”—has an evocative rather than demonstrative function. We know (E.N. 2) that with regard to changes, the mechanical analogy is given no voice in Pareto as well, and that for Pareto, current language, even though inaccurate, still has a cognitive force (he also reiterates this point in §1052 of the *Cours* when rejecting the views of those who deny the existence of social classes). Finally, we have seen that in regard to second-kind dynamics, Pantaleoni also dispenses with Marshall's biological analogy. What else to do, other than invent an imaginary horse race, to then explain to the reader what it captures and what it misses of the more complex social-economic life? Since the essay by Pantaleoni quoted by Schumpeter (“Nota sui caratteri delle posizioni iniziali e influenza che le posizioni iniziali esercitano sulle terminali”, *G.d.E.*, Oct. 1901), also hinges on the changes that result in the profit target shifting (Pantaleoni remarks elsewhere that sometimes innovative firms fail because some modest changes take place: capital in monetary form, once “specified”, that is, invested in capital goods, cannot easily be transformed back into money, it becomes illiquid), it can be seen why Pantaleoni, as well as Pareto, believed that the high profits of some firms include a premium for uncertainty and why some interest rates on loans for productive activities (including the State, producer of public goods) show some “spreads” compared with the interest rates for loans that are considered less risky. Of course, the problem arises of the duplication of the premium for uncertainty recalled by Keynes in his *General Theory* (1936, pp. 144–45); the same problem that Einaudi had, I assume, already read in Pareto, as well as reading Pareto's observation that the auditors appointed by the same assembly that also appoints the board of directors for a corporation are unlikely to carry out their duties. Finally, for “conjuncturalists” such as Pareto and Pantaleoni, the conviction—which we also find in Keynes—of a general underestimation of the risks in the upward phases of economic cycles, precisely the best phases for a wished preventive anti-cyclical stabilization, was deep-rooted.

Neither Pareto nor Pantaleoni had at their disposal tools like those available to someone like Nash or the worthy Harsanyi and Selten. But this was perhaps their good fortune, because it allowed their superior knowledge of the business world and their humanistic culture to be tested, by tackling—Pantaleoni with some essays, Pareto with a *Trattato di sociologia*—the powerlessness of economic analysis when faced with “epoch-making” changes.

Before Einaudi's generation, Pareto and Pantaleoni were the only two Italian economists who were able to critically contribute to theoretical and practical questions in the most diverse international forums. In this, they are reminiscent of the best of Keynes, the Keynes who, after the pound sterling had returned, in 1925, to pre-war dollar parity, agreed to discuss the structural crisis in the cotton industry with the cotton industrialists

from Lancashire. According to Keynes, the crisis of an industry that showed an evident “problem of excess of productive capacity” (C.W.J.M.K., Vol. XIX, 1981, p. 591) could not be solved—in fact it would be deepened—by a reduction of working hours. It could be solved through a reorganization of the industry: 1) by focusing on that part of it that was still thriving; 2) by abandoning the idea that all forms of association among firms (which Pantaleoni, in 1903, had called *complessi industriali*, that is, “industrial conglomerates”: “Alcune osservazioni sui sindacati e sulle leghe”, *G.d.E.*, March, April, and December 1903) must necessarily mean ‘harmful monopolistic agreements’ (this had been Pantaleoni’s position in 1903, already implied in Pareto’s 1899 review of a book by P. De Rousiers, as already specified elsewhere).

On this latter point luck did not smile on Keynes, it smiled, *post mortem*, on Pantaleoni. We are not referring here to his two articles, from 1915 and 1916, on “The shareholding State”, which, together with a Pantaleonian article by Einaudi that also appeared in 1916, opened debate on what a decade and a half later would become known as the Italian “entrepreneur State”. Rather, we are referring here to Pantaleoni’s essay “La caduta della Società Generale di Credito Mobiliare Italiano” (*G.d.E.*, April, May, and November 1895). The principles of this essay, which was focused on the fundamental distinction between the liquidity and solidity of a firm, and on the refusal to consider as losses—not just the credits of uncertain collectability (what Pareto called *sofferenze*, the “non performing loans” of the English speaking world), but even the *immobilizzazioni*, that is the immobilization of capital (the slow-recovery investments, or, if one prefers, the “fixed investments” of the Cambridge school of thought, or “the capital goods proper” of the Lausanne school of thought)—had the great merit of starting a tradition. Indeed, that essay from 1895 drew the attention of men from various political leanings on German and British bank systems, and, above all, on the “crises” and on the “rescues” of firms, starting with the banks. Even Mussolini had to resort to the heirs of that tradition. After World War II, on a number of times, the heirs of those heirs would close ranks around the independence of the Bank of Italy, where independence is intended as a dialectic confrontation between banks of issue and a Government in power. Two of these heirs even became Governors of the Bank of Italy and Presidents of the Italian Republic. This tradition, which is built both on technical and humanistic knowledge, can also claim Pantaleoni and Pareto as among its spiritual forefathers, but Pantaleoni far more so than Pareto, thanks to his essay from 1895, with an essay that reveals an awareness of the Carl Menger’s notion of the “liquidity of money”.

SOME FURTHER CONSIDERATIONS AND SOME SUPPLEMENTARY REFERENCES.

Point 1. First of all, in this E.N., too, we have used the expression “Walras’ metaphysical nonsense”, which expresses a politico-social vision, in order to understand Pareto’s point of view. This does not mean that in order to understand Walras it is not necessary to identify with his point of view. This has been done in an excellent way by a scholar who has devoted many years to Walras’ writings: see therefore P. Dockès, *La société n'est pas un pique-nique*, Paris, Economica, 1996.

With regard to Cournot’s *Recherches*, we have kept at hand the original 1838 edition. It is hardly necessary to add, also because it was reviewed by Pareto, that there exists an American translation.

Point 2. On the occasion of the triple centenary of Marx (1818–1883), M. Keynes (1883–1946) and Schumpeter (1883–1950), the journal founded by Piero Barucci, *Storia del Pensiero Economico—Bollettino di Infomazioni* (see s. n. 11, 1983) asked us to review the Italian translation of Schumpeter's first book (*Das Wesen etc.*, 1908). We took the opportunity (A.Z., “Il ‘primo’ Schumpeter e Pantaleoni. Sull'avanzamento dell'analisi economica nella Mitteleuropa”) to recall that the young Schumpeter sent that book to Pareto, who, wishing to thank the young author, asked his friend Pantaleoni for an account of the book itself (as discussed in the *Corrispondenza Pareto-Pantaleoni*, Vol. III, pp. 358–60). As for the existence of that letter, it should be kept in mind that Pareto always replied to his correspondents, and that on that occasion he had two additional reasons to do so: Schumpeter had given him a book; and Pantaleoni had prepared for him a stimulating account of that book. As for my conjecture (that perhaps in that letter Pareto touched upon Schumpeter's theory of interest), I shall endeavor to shed some light on some Italian literature (Pantaleoni, Pareto, Barone) that was totally ignored in the best collection of essays published on Schumpeter, namely: *Schumpeter social scientist* (Cambridge, Massachusetts, Harvard University Press, 1951), edited by S. Harris, with contributions from R. Frisch, A. Smithies, G. Haberler, P. Samuelson, E. Schneider, J. Tinbergen, A. Marget, A. Hansen, E. Chamberlin, E. Mason, F. Machlup, W. Stolper, H. von Beckerath, P. Sweezy, A. Usher, D. McCord Wright. Indeed, in that book, in which no one shares Schumpeter's theory of interest, even Schneider (“Schumpeter's early works, 1906–1917”) and Haberler (“Schumpeter's theory of interest”) fail to mention Pantaleoni, Pareto or Barone.

I shall start from E. Barone (“Sopra un libro di Wicksell”, *G.d.E.*, Nov. 1894), who examines the following theses by Wicksell: 1) Walras presupposes a stationary economic system; 2) in stationarity, the interest rate is not equal to zero 3) in Walras' production process the workers maintain themselves; in other words, it is not the entrepreneurs who advance salary payments. Our aim is to show that Barone's objections to Wicksell revolve around: a) the supposed stationarity of Walras' theoretical system and its related interest rate; b) the interest paid by the entrepreneurs on the credit obtained in order to make advance salary payment. With regard to Pareto, first of all it is common knowledge that the lessons contained in the two volumes of the *Cours*, 1896, 1897, were read in advance by Pantaleoni, who in turn gave them to Barone to read. Whatever happened, the present writer has noticed that i) Wicksell's name never appears in the *Cours*; but ii) there is a passage in footnote 1 of §704 of the *Cours* that clearly refers to Wicksell: “Other authors have imagined that by following Mr Walras' theories one would be forced to admit that the entrepreneur was not making advance payments for the production; that the workers must have had themselves something with which to feed themselves until the production was completed. These authors probably knew Mr Walras' *Éléments* only by hearsay”.

It appears, therefore, that the best Italian literature at the end of the 1800s was cognizant of the interest rate in stationarity. Consequently, it does not come as a surprise that, immediately, it was Gustavo Del Vecchio—who followed Pantaleoni as co-editor of the *Giornale degli Economisti*—and, in the 1920s, it was Umberto Ricci—who followed Pantaloni in the chair at Rome University—who critically drew attention to Schumpeter's thesis on interest and stationarity.

There are still some scholars to whom I would like to acknowledge my debt: 1) At the end of the 1950s, Cesare Alfieri's students in Florence had to study a difficult book by Prof. A. Franchini-Stappo (*Teoria macroeconomica della congiuntura*, Firenze, Editrice

Universitaria, 1955, see pp. 52–54) and prepare themselves to answer the question whether or not the interest rate exists in stationarity. Thanks to the bibliography in that book, the present writer acquired the abovementioned wonderful collection of essays on Schumpeter. 2) In the early 1970s, Prof. G. Becattini introduced me to Prof. N. Georgescu-Roegen, whom I asked if he knew about the existence of a letter written by Pareto to Schumpeter. Georgescu-Roegen spoke to me about Schumpeter's grief during World War II, mentioned some papers of Schumpeter's that had ended up in Japan and suggested that I should turn to Prof. W. Leontief, although he pointed out that he had not concerned himself very much with Schumpeter's papers. 3) Once again I have had proof that no one ever comes out empty handed from an exchange of opinions or advices with Prof. M. Dardi.

A final consideration: even though Italy had some worthy followers of Pantaleonian and Schumpeterian dynamics, first and foremost among them being Gustavo Del Vecchio, and his pupil Giovanni Demaria, no one in the past seems to have clarified that the “epoch-making” innovations were at the origin of the main disagreement (see points 6–7) between Pareto and Pantaleoni (we regret that we are unable to dwell on an exception identified by a young historian of game theory, N. Giocoli, who, in 1998 wrote a book with N. Bellanca on Pantaleoni, which would give rise to a debate in 2000 that also involved Demaria).

As regards Demaria, we harbour the suspicion that he has not correctly interpreted the tripartition of *pure* economy readable in Pareto (*Manuel*, 1909, p. 147). On the “dynamic part that studies the movement of the economic phenomenon” (III, §7), Pareto says, on p. 148, that “except for a special theory, namely the theory of economic crises, nothing is known” (III, §8). About this dynamics, Demaria (“Saggio sugli studi di dinamica economica”, *Rivista Internazionale di Scienze Sociali*, March 1930, p. 117), speaks *tout court* of Pantaleoni's second-kind dynamics. We believe, on the contrary, that Pareto and Pantaleoni agreed in thinking that second-kind dynamics cannot find a solution in *pure* economics. In more explicit terms, Demaria fails to notice that the deviations between a historical curve, which highlights the business cycles, and an interpolated curve, which expresses some underlying or “normal” trends, are placed by Pareto in *pure* economy, that is, in first-kind dynamics.

Point 6. We have recalled professor Paolo Sylos Labini also because his *Oligopolio e progresso tecnico* (Milano, Giuffré, 1957¹) never mentions Luigi Amoroso, although it includes him in its criticism. Indeed, we believe that some of Sylos Labini's criticisms concerned the developments that Pareto's idea of competition and monopoly had L. Amoroso in mind (L. Amoroso's *Principii di economia corporativa*, Bologna, Zanichelli, 1938). In this work, besides Cournot's two instruments, elasticity and its inverse, price flexibility as the quantity produced varies (Cournot uses the inverse of elasticity when moving from duopoly to unlimited competition), Amoroso introduces the magnitude σ . More precisely, by developing a work of his from 1930, which Stackelberg had appreciated and includes what from 1934 will be curiously called the Lerner index of monopolistic power, Amoroso proposed the following formula

$$(1) \quad \frac{p - m}{p\eta} = \frac{x}{x + y(1 + \sigma)}$$

where p is the price of a product; x is the quantity produced by the leading firm (enjoying monopolistic power) in that product industry; y is the quantity produced by the competition (the other firms in the industry; obviously $x + y = X = \text{total production}$); m , function of x , is the marginal cost of the leading firm; η is price flexibility, an expression of the reaction by the market; finally, σ expresses the responsiveness of competition (the firms other than the leading one). More precisely, σ is "the ratio between the percentage rate by which the supply of the competition varies as a consequence of a variation in price, and that by which total supply varies—in the opposite way—as a consequence of the same variation in price" (1938, p. 171), with $\sigma = 0$ in the limit case of a totally rigid supply by the competition. If we solve equation [1] with respect to x , it expresses the quantity produced by the leading firm as a function of market price, the production by competition, price flexibility, and competition responsiveness. In Amoroso the number of firms is no longer given exogenously, as in Cournot and Pareto (monopoly, duopoly, oligopoly with an increasing number of firms up to unlimited competition); it is determined by assuming an increase in p . Between unlimited competition ($x = 0$ and $p = m$) and absolute monopoly ($y = 0$, and consequently, as in Cournot, $p - m = p\eta$), Amoroso was interested in the middle ground of monopolistic power associated with industrial concentration, which he also calls "partial monopoly". In order not to overload this E.N., I shall not dwell any further on Amoroso, or on any other development of the concept of competition and monopoly that stemmed from Pareto's pure theory and precedes Sylos Labini. We shall only point out that neither Sraffa, nor Amoroso, nor Sylos Labini realized that in Pareto's view, firms generally produce a quantity that precedes the increasing unit costs; nor did they realize that the terminal points (from Cournot) were included by Pareto in the more general definition of equilibrium of the *Manuale*.

Going back to Sylos Labini and his implied criticism of Amoroso, the fact is that he felt he had something new to tell us on the relationship between market structures (microeconomics) and the course of business in the short and long term (macroeconomics); he also had a temperament that was reminiscent of Pantaleoni, who, when he found it difficult to jump over a ditch, would jump in it, as P. Jannaccone affectionately observed. Game theorists might find it interesting to know that Sylos Labini overcame the academic barrier that Amoroso had put in front of him also because Giovanni Demaria—who had questioned Amoroso's theory of "dynamics" to the extent of considering his "escape point" unbearable—found that Sylos Labini was the victim of an injustice, and he helped in solving a prisoner's dilemma. We must add that in the past, we interviewed Prof. Demaria on two occasions, the second time in the presence of Prof. Aldo Montesano, and that, on the first anniversary of Demaria's death, in the hotel where we were staying in Turin we met, by chance, professor Paolo Sylos Labini, an unforgettable scholar and citizen.

Points 6–7. We have recalled E. Barone: 1894 (a), "Sulla 'Consumer rent'", *G.d.E.*, September; 1894 (b), "Sul trattamento di quistioni dinamiche", *G.d.E.*, November. It seems to us that Barone's distinction between the changes that do or do not justify ignoring "the second order quantities"—a distinction that was readable also in Cournot and Marshall—is the forerunner of Pantaleoni's distinction between first- and second-kind dynamics and explains why Pareto, who started off from general interdependencies, resorted to the example of changes that on the side of production affect the price of nutmegs, and changes that affect the price of iron, in order to show to what extent he

was in agreement with Marshall on the constant marginal utility of money on the side of consumption.

For the most important law of political economy in Pareto, see A.Z. "Say's law of market as interpreted by Jenkin, Pareto, Einaudi and the Italian economists of the past", *History of Economic Ideas*, n.3, 2013.

Point 8. The quoted two works by Pantaleoni, "Lo Stato azionista" (15. 6. 1915) and "Lo Stato azionista e il monopolio dell'emigrazione" (February 1916) can be read in M. Pantaleoni, *Note in margine alla guerra*, Bari, Laterza, 1917. The Pantaleonian article by Einaudi that was mentioned by us earlier, "L'industria degli armamenti", appeared on G. Salvemini's *L'Unità* on 22 December 1916.

On the heirs of the Pantaleonian tradition, who were of vital importance even to Mussolini, there exists so vast a historiography. I shall only mention the volume *Banca e industria fra le due guerre*, Bologna, Il Mulino, 1981, where, in Part II (*Gli economisti italiani e il rapporto banca-industria*), there is a remarkable essay by P. Barucci: "Il contributo degli economisti italiani (1921–1936)".

E.N. 33. A CONSIDERATION ON THE PURE THEORY OF COLLECTIVISM

Ch. VI, §61. It would be worthwhile to stop and consider the closest forerunners of Pareto's paragraphs on collectivism. These paragraphs are rather beautiful, but what we admire the most in them is the modesty of their conclusion, where pure economy asks for the help of sociology. If we have to limit our reasons for being somewhat dissatisfied with Pareto's pure economics of collectivism to a single proposition, it would be the sharp contrast between the autonomy of the consumers' preferences and the subordination of production to the directives of a government (by the minister of production). In other words, it is difficult to understand how, in a political system where those in government control all the means of production, decisions may safeguard both production efficiency and individual preferences. Is it not the case that whoever has all the means at their disposal, also controls all the ends? Having said that, it is quite important to add the two following remarks.

First, together with Wieser (*Der natürliche Werth*, Wien, 1889) and Barone ("Il ministro della produzione nello stato collettivista", *G.d.E.*, September–October 1908), Pareto (*Cours*, §§ 720 ff.) was one of the founders of the analysis of collectivist economy.

Second, if one considers that the super-welfare (with respect to a private ownership competitive economy) that can be achieved in Pareto's pure collectivism is due to the possibility of recovering constant production costs through taxes and marginal costs through variable prices, it is easy to see Pareto's debt towards Marshall with regard to variable prices.

E.N. 34. PARETO AND THE QUANTITY THEORY OF MONEY

Ch. VI, §73. The merit of seeing a mirror of human motivations in the holding of money, and of replacing the Fisherian point of view, which focuses on the velocity of circulation of money (v_i , with $i = 1, 2, \dots$ as many as the types of liquidity are) with the Cantabrigian

point of view expressed by $k_i = 1/v_i$, is ascribed to Marshall and Keynes. But one did not have to wait for Marshall or Pareto or Keynes to conclude that the quantity theory of money is, as Pareto writes, "roughly true". We only need recall R. Giffen, *Stock exchange securities: an essay on the general causes of fluctuations in their prices* (London, 1877), as a text well known to Pareto's and Marshall's generation in which this remark about "roughness" can be found.

Thanks to the *Corrispondenza Sensini* (E.N. 3), we know today that Pareto intended to replace the *Cours* with a *Trattato di Economia*. In particular, we know that the evident regression, compared with the *Cours*, of the *Manual* on the topic of monetary matters was due to two reasons.

The first is that once he had finished the *Manuale*, Pareto intended to use a volume of the planned *Trattato* to update the historical aspects of the monetary phenomena considered in the *Cours*. I shall mention two problems that in the early 1900s and with the start of the First World War that would have fascinated scholars like Pareto. *a)* After bimetallism, *gold exchange standard* had come to the fore, especially in the dealings between Great Britain and India. *b)* As for the link between finance and money, suffice it to recall that J. M. Keynes asked Pareto himself—who, alas, did not accept!—to review for him a volume on *extraordinary* public finance by L. Einaudi, where one can still today read that inflation is a fast substitute for a property tax in the case of war.

The second reason is that Pareto felt that, although resorting to historiography and statistical induction, the *Cours* overlooked political science even with regard to money and inflation. In short, in Pareto's view pure economic theory is not enough when, with regard to the maximum of utility for a collectivity, (see E.N. 31) we have to weigh up complex problems such as, for instance, the *assignat* inflation during the French revolution. It is odd that this realization of the political dimension of the economic problems, derived from Mill, escapes the attention of some Pareto scholars even in relation to the free-exchange/protectionism dualism.

SOME REFERENCES (AND ON SOME WRITINGS BY MARGET). After the famous essays by A. Marget ("Léon Walras and the "Cash-Balance Approach" to the Problem of the Value of Money", *Journal of Political Economy*, Oct. 1931, 39(5); and "The Monetary Aspects of the Walrasian System", *Journal of Political Economy*, April 1935, 43(2)) and the relatively bitter disputes that followed, the literature on money in Walras has become so vast as to prompt us to not even mention Walras' name in this E.N. In confirmation of what we are saying, a new book has appeared: R. Baranzini, *Léon Walras e la moneta senza velo* (1860–1886), Torino, Utet, 2005.

A beautiful bibliography is included in P. Bridel, *Money and general equilibrium from Walras to Pareto (1870–1923)*, U.K., Elgar, 1997. In it, the reformulation of money in Walras by A. Montesano (1986) is pointed out (see also A. Montesano, "A restatement of Walras' theories of capitalisation and money", *History of Economics Review* 47, 2008, pp. 86–109), and one can also read an exception to a rule: Bridel points out (and shares) G. Del Vecchio's remark according to which one must look for Pareto's monetary theory also outside of pure economics.

In his first essays and in the *Cours*, Pareto's critique of the quantity theory of money was on the grounds of the interdependence between the real and the monetary sectors of the economy. But it is in the unpublished manuscript (1920–21), which finally appeared

in the quoted vol. XXXII of V.P. *Oeuvres complètes* (see E.N. 3), that Pareto's critique reaches a mature and sophisticated form. On this important manuscript of Pareto, we can now read M. McLure, "Pareto's manuscript on money and the real economy" (J.W. Femia and A. Marshall eds., *Vilfredo Pareto: beyond disciplinary boundaries*, Farnham: Ashgate, 2012).

I hope the reader will forgive me if I add that, as a homage to an economist such as Arthur Marget (1899–1962), who also wrote in Italian and loved to listen to music even at the "Maggio Fiorentino", I acquired in Florence all the journal excerpts that he had sent to Gustavo Del Vecchio (1883–1972) and the surviving correspondence between the two. The documents and writings regarding Marget that are in my possession, including the embittered ones written in self-defence and for private circulation (especially against N. Kaldor and P.B. Whale), are listed in the Appendix to A. Zanni's, "Walras' theory of money: a scientific debate between Marget and Del Vecchio", *Economists' Archives*, n.1 (Supplement to *Storia del Pensiero Economico—Bollettino di Informazione*, n.18, 1990).

E.N. 35. A CASE OF SOCIO-ECONOMIC DYNAMICS APPLIED TO A MONETARY PROBLEM

Ch. VI, §§87–90. Here is a case of socio-economic dynamics applied to a monetary problem: how do the conditions of the social classes—entrepreneurs, workers, and the fixed-income receivers—vary when monetary wages increase?

It should be noted that for Pareto the complete transparency postulated by pure competition theory does not exist for two reasons: people immediately feel only what touches their personal life, and not the personal repercussions of what happens in external environments; on the other hand, it is difficult for them to foresee the most distant effects of present events. At the beginning of the 1900s, the latter point of view, which reminds us of Cantabrigians such as G.E. Moore or Keynes, was widespread in Italy too: suffice it to recall Giovanni Papini, *Sul pragmatismo (saggi e ricerche)*—1903–1911, Milano, Libreria Editrice Milanese, 1913.

E.N. 36. EDUCATION, SOCIALISM, AND FREE EXCHANGE

Ch. VI, §91. In the latter sentence on Socialism and workers education there is something of a revival of the humanitarian optimism of Pareto's youthful years, which had morphed into dark pessimism at the turn of the century. However, it should be pointed out that while Pareto was a tireless opponent of Marxist economic theory, he always credited Socialism with contributing to give workers human dignity. Pareto could have added that the Italian Socialist Party itself had denounced protectionism. Suffice it to recall Marx's pamphlet, *Discorso sul libero scambio*, published in Milan by Filippo Turati's "Biblioteca della Critica Sociale". Now, in the opening *Nota* to the pamphlet by the *Critica Sociale* (p. 3), one reads:

"At this moment, when in the Italian Parliament the landowners are once again rearing their heads and have already obtained further to raise the cost of bread for the poor man by increasing custom duties [...]."

E.N. 37. PARETO'S POLEMIC ON PARETO'S LAW

With regard to *Pareto's law*, Pareto soon had to face two valiant antagonists: the engineer G. Sorel and the mathematical economist Edgeworth. We would have to allow ourselves a great deal of space if we wished to explain why Pareto answered the former very courteously and the latter in a very aggressive and sarcastic way. Anyway, Pareto was sincere when, in concluding his second-last reply to Edgeworth, he wrote:

“Professor Edgeworth has published some works of considerable value which I have often had occasion to praise and, if a very bad case could be won by sheer intelligence, this case would also have won. But even the most subtle sophistries cannot prevail against facts. If Professor Edgeworth will go back to studying these questions objectively, he will certainly contribute to the advance of science and afford us a fresh and welcome occasion to hail him as our master” (G.d.E., Nov. 1896, p. 448, English trans. G.d.E. 2008, p. 436)

Pareto was equally sincere when he was writing to Pantaleoni: “With Edgeworth we made peace; with the others [Flux and Sanger] there was never a war” (*Corrispondenza Pareto Pantaleoni*, Vol. II, p. 377). Since Edgeworth (1854–1926) outlived Pareto (1848–1923) and had the opportunity to write the entry *Pareto's law* for the 1926 edition of the *Palgrave's dictionary*, where he also highlighted Pareto's “short fuse”, it is interesting to notice that this work by Edgeworth does not even appear in the bibliography of the entry *Pareto's law* in the *New Palgrave dictionary*, whereas it is fully reported in the *Œuvres* edited by G. Busino, Vol. III, 1967, to whom we definitely refer the reader also for an essential bibliography on the topic.

Today, at a time when people are talking so much about unsustainable development for the very environment in which we live, about globalization inspired by a Darwinian view of social relations; today, when not one day goes by without a new essay appearing on the advent of new forms of distribution unworthy of democratic societies; today, it is worthwhile to go back and look at the cultural climate that drove Pareto to investigate income distribution in the advanced countries of the second half of the 1800s, and also to put forward a hypothesis about the most ancient societies. A strident contrast was undoubtedly rising between some socialist preaching on the “rising poverty” and what the statistics—especially in England—were revealing, after the advent and the strengthening of the Trade Unions. This contrast constitutes the leading theme of a book that Pareto liked: P. Leroy-Beaulieu, *Essai sur la répartition des richesses et sur la tendance à une moindre inégalité des conditions*, Paris, Guillaumin, 1881. In this book a number of criticisms are repeatedly leveled at Turgot, Ricardo, Malthus and Sismondi, Lassalle, Proudhon and Marx, all the way to the most contemporary authors. It is in this climate, thick with political values, that a sort of competition arose for the best indices to measure distribution, on which well-known works exist, even by C. Bresciani Turroni. It is in this climate that Pareto, freshly arrived in Lausanne, felt the compelling desire to stick out his claws.

However, this is not all. We must also take into account what Pareto himself specified—by saying and by not saying—in his 1902 encyclopedic entry (E.N. 8). He let it be understood that he wished to fill a great gap in Walras' theory (the distribution of the income with which individuals come to the market) by way of econometrics; but he did not disclose all the annoyance he felt for Walras' “metafisicherie moraleggianti”—“moralizing metaphysical nonsense”—that is, for a distribution theory based on the principle of weighted marginal productivities and perfect competition.

AN ADDITION. There is an aspect, in connection with *Pareto's Law*, on which Pareto failed to comment. Since the end of the 1800s, the theory of increasing poverty lay essentially in the following statements: a) the continuous evolution of needs in the developed countries is based on a capitalist production that generates a permanent disparity between the lifestyle of the "capitalist classes" and the lifestyle of the "working classes"; b) out of this there arises a permanent relative unhappiness in the "working classes" that escapes the attention of those who limit themselves to recording the rates of growth of the real income in the two classes. In Italy, the author who drew closest to this renewed Marxist theory of alienation was R. Michels in *Economia e felicità* (Milano, Vallardi, 1918) and in *La teoria di C. Marx della miseria crescente e le sue origini* (Torino, Bocca, 1922). Michels did not know that Pareto had thrown the doors wide open to all sentiments, including envy and frustration, in the maximum of utility in sociology (see E.N. 31).

E.N. 38. ON THE PARETO-GAETANO MOSCA POLEMIC

Ch. VII, §97. In the *Manuel* (1909) the content of footnote 3 of the Italian *Manuale* was purged of all the parts regarding the well-known polemic between Pareto and Gaetano Mosca. What is left of footnote 3—the part starting with "Eq. 62" and ending with "thick"—appears in the French edition as footnote 1 at the end of §97. In essence, Pareto preferred not to continue his polemic with Mosca in the French edition of the *Manuale*.

Since Mosca complained that Pareto had not quoted him, we shall point out a constant: Pareto was interested in historiography of all kinds and once he complained that a history of the economic theories between the 19th and the 20th century had not yet been written. But when he was tackling theoretical topics, he always preferred to draw from all authors, sometimes referring to his own more systematic works, where quotations were more numerous. At times, reasoned bibliographies were added, but they had been written "separately". For such bibliographies Pareto had resort to assistance from helpers, most often his friend Pantaleoni, especially when German language text was concerned. The most serious accusation for Pareto's missing quotations and self-quotations was leveled at him—possibly after Stigler—in one of the most beautiful anthologies on the evolution of mathematical Economics. See therefore E.N. 54.

E.N. 39. PARETO, THE PERFECT UNIT OF MEASURE, AND PRICES OF PRODUCTION

Ch. VII, §100. Pareto was right. In effect, when approaching political economy for the first time, one wondered why a perfect unit of measure has not been found in that subject. When the gold monetary system was still in force during the second half of the 1800s, there was a conviction that the yellow metal carried out three monetary functions at best (unit of measure, medium of exchange, reserves of value), Jevons questioned this point of view in a beautiful essay, the content of which is evident from its title: *A Serious Fall in the Value of Gold* (1863). During the time of Pareto, however, scholars (Walras, Marshall, and the same Pareto) returned to debate the possibility, evident from Cournot, of considering gold as an analogue of the "average sun" of the astronomers.

Within the ambit of the labor theory of value and the associated approach to the prices of production, the lack of a perfect unit of measure already had been denounced

by Ricardo. He observed that when one modifies the distribution of national product between “capital” and “labor” the relative prices between products differ from industry to industry (because the proportions of capital and labor used differ from industry to industry). It follows that we are not able to say if the variation in the price of a good depends on causes that lay behind the measured good, or if it instead depends on causes that lay behind the good used as a unit of measure. To manage that problem Ricardo and Marx theorized on the presumption that the proportions of “capital” and “labor” used in production are equal for all goods.

In 1960, Piero Sraffa set aside the labor theory of value and reopened the problem within the ambit of the theory of the prices of production. His perfect unit of measure (cit., 1960, Ch. IV, §§24–26) is an abstract ‘composite commodity’, a mixture of goods that are in identical proportion to each other on the side of inputs and on the side of the outputs. This ‘composite commodity’ is a construction that, within the theory of the prices of production, exceeded the objections of the constructive critic only in the restrictive case of ‘non-joint production’ (Schefold): all summed, it would be a beautiful step forward if it were not circumscribed to the production prices.

It seems to us, in fact, that the return to production prices would represent a backward step relative to the work of Pareto and Keynes. In the *Cours*, in order to stress the limited range of Ricardo’s theory of rent (analogous to what subsequently happened in the *Manuale* for the theory of the comparative costs: see E.N. 49), Pareto made Ricardian rent a particular case, albeit a remarkable one, of his own general theory of *acquired rent* (a general theory of the reproducibility of goods), which was set in the frame of general economic equilibrium. It is in that context that Pareto questioned the mathematical formula that determined the value of the capital goods, dividing expected *normal profits* (for those who sell their assets) by the current rate of interest. What Pareto implied here is clear: the entrepreneur who acquires real assets nurses a hope of obtaining some monopolistic privilege. For example: one who is acquiring land is disposed to pay more for it than an extrapolation of past profits would suggest, provided he believes that others find it difficult to reproduce his skills as a winemaker; similarly, one who is convinced in his own capacity to produce a new kind of machine may pay more when acquiring a mechanical garage or workshop, provided he thinks that reproducibility of that new machine by others will not be easy. It is also worth remembering that, as Director of the Ferriera del Valdarno, Pareto left two written records indicating that one who produces iron products that are easily reproducible does not earn much. In sum, when Pareto sustained that the main point in the formula for capital values are the expected uncertain profits, he was anticipating what Keynes thought lay at the heart of his *General Theory* (1936); a *Theory* that commenced from macroeconomic aggregates rather than the microeconomic heterogeneities of Pareto, but was, as Pareto’s theory was, light years away from the prices of production of Ricardo and Sraffa.

Pareto was a Keynesian *ante litteram* when writing: one who acquires land or a factory “does not have a privilege: he acquires a privilege, which is different” (*Cours*, vol. II, § 781). Also the parents who make their sons study at university hope that they will secure a privileged future on the labor market. It is curious that, in regard to these issues, Pareto is never recalled and yet he, more than others, highlighted forecast errors made on investments, including investments in human capital, when there is a large temporal lag between a decision and the moment of verification of the goodness of that decision.

SOME REFERENCES. I have recorded S. Jevons, *A serious fall on the value of gold, ascertained and its social effects set forth*, 1863, then in W. S. J., *Investigations in currency and finance*, London, Macmillan, 1884. The literature following from P. Sraffa, *Production of commodities by means of commodities, Prelude to a critique of economic theory*, Cambridge University Press, 1960 is so vast, especially in Italy, that I renounce the bibliographical reference. On that issue I recall only B. Schefold, "Joint production, triumph of economic over mathematical logic?", International Meeting on *Piero Sraffa*, Roma, Accademia Nazionale dei Lincei, 2003. In regard to Keynes, above all I record the beautiful book by L. Pasinetti, *Keynes and the Cambridge Keynesians, A 'Revolution in Economics' to be Accomplished*, Cambridge University Press, 2008, because in reading it I returned to ask myself if Sraffa, who was certainly part of Keynes' conspicuous circle, as Pasinetti wrote, did not end by theorizing in a way that had nothing more to do with Keynes. On the other hand, Pasinetti (p. 262) enlists Pareto as among the lovers of the theory of the exchange, when reading *Manuale* alone is enough to show that Pareto was, above all, a theorist of production. Pareto, above all, overturned Walras and placed monopolistic power at the centre of the production theory.

E.N. 40. "SIMPLE SAVINGS", "CAPITAL SAVINGS", AND SOME REMARKS ON MONEY IN PARETO

Ch. VIII, §10. It is not easy to define precisely the distinction between *simple savings* and *capital savings* because Pareto simply limits himself to referring the reader to §90 of the *Cours*, whereas, he should have also recalled §§ 425, 477, and 539. Nevertheless, we shall attempt to define these terms and then make a few general remarks on money in Pareto.

1. From §§ 425, 477, and 539 of the *Cours* it can be inferred that: *i)* *simple savings* [*semplice risparmio*] are stocks or deposits that have a latent productivity which becomes effective when they are transformed into capital. Simple savings are, for instance, the stocks of wheat stored for sowing and the deposits collected by banks for lending to firms; *ii)* *capital savings* [*risparmio capitale*] are instead the directly productive savings used by firms as a rotating fund tied to a purpose. For Pareto that purpose is threefold: *a)* capital savings are used by firms for anticipated payments of wages and raw materials, and *b)* they are used as a supply of valuable goods that enables traders to pick the best time to buy and sell. Similar to Keynes' quasi money M_2 , Pareto's purpose *b)* implies uncertainty because one can discern, at times, capital savings being accumulated in a speculative spirit, and at other times, they may be accumulated in a cautious risk-reducing spirit directed towards avoiding losses. For Pareto, the third purpose *c)* of capital savings is to generate a flywheel effect that makes production more stable. The reader who remembers the classification of *real capital* in Keynes' *Treatise on money* and *General theory* (*fixed capital, working capital, liquid capital*, see above, Ch. II, Note to the text 78^[a]) will have no difficulty in realizing that Keynes' notion of *working capital* would fall within Pareto's third purpose, *c)*, for capital savings.

We now have enough materials to make some remarks. 1) First of all, the existence of savings held by firms and families implies that forecasts have been made about an uncertain future. In the case of firms, suffice it to think of the uncertainty related to plant obsolescence. 2) For our second remark, let us consider the savings of families in the

advanced countries of Pareto's time. Well then, at the time of Queen Victoria, besides the well-off who bought long-term public securities, which in London also met the liquidity requirement, there were also modest savings of working families, whose motivation lay in the uncertainty of their future and their love for their children. Now, given that love of one's children belongs to the realm of sentiments and ethics, it can be inferred that Pareto could not entirely ignore sentiments and ethics in his first approximation of pure economics. Indeed, the *homo economicus* of the pure economics of the *Cours* (Vol. I, §106) saves up not only on the basis of reasoning about the interest rate (for Pareto there can be savings even with negative remuneration) but also on the basis of "prévoyance", a psychological state imbued with sentiments and moral values. The great distance from Marshall claimed by Pareto, whose aspiration was to commence theorizing from a type of rationality that is totally impervious to ethics, is therefore somewhat reduced. 3) In Pareto, the creation of savings and their transformation into investment goods are separate processes, even if the variations in stocks (whether desired or not, as post *General theory* Keynesian economists would add) make them coincide with each other in final national accounts. 4) A final remark is connected to the interpretation given in E.N. 32: Pareto classes the epochal savings and investments in the dynamics of a second kind, which imply uncertainty and structural modification of the trend of an economic system ascending in waves. On the other hand, he places normal savings and investments, which also imply uncertainty but which do not alter the existing trend in waves, with pure economy. That is equivalent to saying that stationarity does not exist even in the first approximation of Pareto's theoretical system, or, if one prefers, in that system some normal inventiveness also exists in pure economy. Of course, we are not dealing, here, with the savings and inventiveness that, during Pareto's life, were associated with the advent of railways and electricity, which changed the face of the world.

2. It is not easy to put forward some general remarks on money in Pareto. If we are starting from the quantity theory of money (E.N. 34), it is not in order to point out that money is only one of the components of Pareto's interdependencies between real phenomena and monetary phenomena; and neither is it to point out that Pareto's main objection to the quantity theory originated from the controversial issue of the hoarding, i.e. the *hoards of money* being maintained in unused bank deposits or 'money as treasure', as such deposits have been called in an allusion to outdated monetary systems. In fact, Pareto—who was familiar with Hume's work and, consequently, with the relationship between quantity of money on the one side, and prices and national production in the short as well as in the long term, on the other side—made his main objection to the quantity theory precisely on the basis of the component of money that in periods of crisis disappears from circulation (or circulates at zero speed, as others would say). Since with regard to this point one's mind turns to Keynes' M_3 component, we recall that in both *The economics of industry*, by M. and A. Marshall (1879, cit., p. 154), and *Il risparmio nella economia pura*, by G. Montemartini (Milano, Hoepli, 1896, p. 19, p. 117 and *passim*), one can read what used to be a very widespread diagnosis: namely, that in periods of economic depression, many people have the resources available to spend money but do not do so, and that, in particular, the monetary savings are left unused because of entrepreneurs' dreadful profit expectations. In Pareto's terms, one could say that the *hoards* are the degenerate children of "simple savings" when the latter

go from transient to persistent “potential spending”. Keynes’ great talent in this field lies in having made it clearer, compared to others, that an economic system may fall into a “liquidity trap”.

However, we have set off from the quantity theory of money not for a comparison between Pareto and Keynes—for which we refer to point 3—but in order to add that this is a topic in Pareto’s thought in which historical-institutional complications and theoretical forms of representation are particularly inextricable. What actually matters, for Pareto, in the relationship between quantity of money on the one side, and the level of the prices and national production on the other, is the potential for the various social classes, and their political agents, to influence the quantity of money and bank credit. The main difference, in Pareto’s work between an ideal efficient gold system, where only fiduciary money circulates (E.N. 34), and a ‘false’ money system is that the relationship between politics and the quantity of money goes through the indirect way of controlling the gold mines in the first ideal case; whereas, the influence of social classes and politics on money and bank credit is more substantial and direct in the case of ‘false’ money. Marshall’s delusion that *Political Economy* can be replaced with *Economics* because somewhere or other a political class existed, which embodied the general interest of a country, is never even remotely touched upon in Pareto’s “experimentalism”. In contrast with Keynes, who in regard to “class struggle” claimed to belong to the “educated bourgeoisie”, and was optimistic about the potential influence of this kind of bourgeoisie, we have Pareto, who thought that the reasoning of economists only has a practical influence if it strongly affects sentiments, and also thought that a utilitarian devoted to active politics will only survive for some time if he accepts the compromises of some political party.

3. Since “money is the functions it performs”, looking for an essential comparison between money in Pareto and in Keynes, I find that there are so many different institutional complications between countries and also so many theoretical points of view that this comparison is like an uphill hurdle race. Let us commence by a practical and theoretical example. Since Pareto knew Walras, he had at his disposal one term to define one function of money and a different term to indicate two other functions: “numeraire” and “monnaie”. But in practice, what “numeraire” is used in negotiations apart from the “monnaie” with which payments are made? Let us complicate the example. A few decades back—I allude to the years when the Euro did not exist, the Chinese currency had not yet come to the fore and the US dollar was unstable—for some big contracts between Italy and China payments were made in US dollars, but the negotiations were conducted in ECU, which was only an average of “numeraires” and not a “numeraire” and a “monnaie” like the Euro. Now let us consider a complication that is only theoretical. In our opinion it is preferable to discuss money by starting, like Marshall, from the motivations that lead to keep it—by starting, that is, from $k_i = 1/v_i$ —instead of from the point of view of the velocity of circulation v_i . However, we also agree with Marshall when he states that both these perspectives lead to the same mathematical results. We agree with him provided one keeps in mind: a) that $i = 1, 2, \dots, n$, includes money as legally defined and all the activities that have good levels of liquidity; and b) that money is used for the circulation of both the periodical flow of production and the stock of accumulated wealth, recklessly passing from one function to the other without any warning: not even

Pope's famous father, who retired from business and went to live in the country taking a "strong box containing nearly £20,000 out of which he took whatever was required for household expenses", had this kind of warning (for this famous example of hoarding see References).

Finally, since we had to opt for something essential for a Pareto–Keynes comparison, we have been attracted by the respective temporal durations of their balance sheets. Well then, in the *Cours*, Pareto resorts to two types of balance sheets: the periodical ones, that is the budgets of the firms that are still in operation, and the closing balances of the firms that have ceased operations. Now, periodical balance sheets may be annual, six-monthly or, today, virtually instantaneous. As for closing balances, their duration is indeterminate *a priori*, since the Pantaleonian case could arise of a firm that is illiquid because it has just prepared itself for a very innovative production but fails because of the sudden appearance of a competing firm with a very modest innovation. In terms of temporal duration, therefore, Pareto's two types of balance sheets show great theoretical flexibility, they are good "à tout faire". The same cannot be said of the macroeconomic balance sheets of Keynes' *General Theory*, if one considers that they imply some weighted averages of the production periods of the different production activities. Keynes' average production period obviously varies from era to era. But whether it be six months or a year, it is always too long compared with the periods implied by the component of Keynes' M₃ that regards speculation in its strict sense. What we are expressing here is not the same dissatisfaction felt by Richard Kahn, who, in 1929—having in mind Keynes as an industrial consultant in Manchester and the time needed for an industrial restructuring (E.N. 32)—would have perhaps contented himself with adding only a third period to the short period and to the long period that dominate Marshall's theoretical landscape. It is our opinion that in those cases of instability that, though associated with world situations that generate long-term expectations, nevertheless have their beginning and their end within a few hours without it being possible to predict the algebraic sign of the conclusion—in those cases, the preferable method is the method of the examples and of the historical frescoes that combine logical deduction with the complexity of history and its institutional complexities.

Finally, we shall mention the role played by money in the books that were promised by Marshall, Pareto, and Keynes; but were never published.

4. Though with its limits, perhaps the most complete Economics course to appear between J. S. Mill's *Principles* (1848) and A. Marshall's *Principles* (1890) is *The economics of industry* by Alfred and Mary Paley Marshall (1879). It is a complete course because it tackles the problems of production and monetary problems. The latter are presented in an industrial relations framework that, even though not yet the British framework of 1926 (first general strike in the United Kingdom), was light-years away from Ricardo's world. Marshall's *Principles*, which appeared in 1890 with an unfulfilled promise—"Vol. I"—is found wanting precisely on the monetary side: whatever Marshall had to say in this field, he consigned it to oral tradition. It would only be in 1923, one year before his death, that some old monetary writings of his would reappear, collected in *Money, credit and commerce* (London, Macmillan).

In Pareto's body of work, the *Manuale* (1906) is to the *Cours* (1896–97), what the *Principles* are to the *Economics of industry* in Marshall's body of work. The *Manuale* is

more innovative than the *Cours*, but it is inferior to the *Cours* on the topic of money. Those who were familiar with the *Corrispondenza Sensini* (1948) knew that when the *Manuale* was near to appear, Pareto wanted to replace the *Cours* with a *Trattato di economia*. Pareto was not satisfied with the monetary topic in the *Cours* because he had not clarified that it is not the role of pure economy to decide whether false money is a good or a bad thing, and also because he thought that the institutional part of money concerned largely obsolete arrangements (E.N. 3). For example, what was the point, at the turn of the 20th century, of giving space to bimetallism, as Pareto did in the old *Cours*, when various new studies explained how the expectations of a depreciation of gold from the inflow of newly extracted gold had been offset once Germany, in 1873, and many other countries, adopted gold monometallism and increased the demand for that metal? What was the point of posing the problem of the international movements of infra-European capital solely in terms of expectations of profit rates and interest rates, when France—contrary to Britain, which lent also to enemy countries—submitted its own international loans to the authorization of the foreign minister? Those who at the beginning of the 1900s were looking at monetary problems from an old London-centric perspective only were ignoring the fact that, one way or another, politics must weigh up not only the various interests, but also the different ideas of justice, and also the ideals of economists such as Pareto. When reviewing, in 1905, a book on the idea of a French-Italian Zollverein as a prelude to the United States of Europe, Pareto concluded as follows: “The day when these ideas will turn from theory to practice has perhaps not yet arrived; but they are in any case interesting to notice, because they reveal a certain current of opinion that may have its importance”.

As for the publications that never went past the planning stages, Keynes obviously offers a more topical picture. Though a passionate follower of the ever-changing events of the world and therefore closer to the 18th century genre of pamphlet literature than to general economics treatises, Keynes came close to writing one such work with his *General theory* (1936), in which he thought that the succession and the sum of shorter periods may also deal with long run problems. He did feel the need to rewrite that 1936 book, but limited himself to writing some articles in self-defense. In 1936, he had ignored monetary-international problems, so that if we were compelled to choose whether to reread the Bretton Woods Treaty (1944) in the light of the *General theory* or in the light of the two volumes of the *Treatise on money* (1930), where international problems are plentiful, we would regretfully opt for the *Treatise on money*. In fact, while in the 1900s other economists have been able to compete with Keynes with regard to the 1st Volume of the *Treatise*, which is devoted to monetary theory, the same cannot be said with regard to the 2nd Volume, which is unrivalled for what concerns the knowledge of international monetary complexities. In the 2nd Volume there are also some ideas that bridge the gap between the *Treatise* and the *General theory*.

The first idea that Keynes put forward as far back as in the 1920s with regard to German war reparation payments is that production and employment do not have the same lightning speed of adaptation as do monetary phenomena. This leads us immediately to Keynes' attempt at Bretton Woods (1944) to distinguish between international movements of capital that are desirable for economic development, and the movements of “hot money”, which are destabilizing; or, if one prefers, to the attempt to distinguish, for the purpose of declared, but reviewable, exchange parities (adjustable peg), between

transitory imbalances in the balance of payments and fundamental imbalances. But it is best to go back to 1942 to find two main points around which Keynes' project for a post-war monetary agreement (fourth draft) revolved: a) without any international monetary rules, the chaos of customs wars and depressions that degenerate into stagnation may arise again. b) In order to curb destabilizing international movements of capital ("the flight of funds for political reasons or to evade taxation or . . .") there must be checks "at both ends", especially by the two "founder-States", the USA and the UK. Such a political arrangement would lead one to reconsider the Anglo-Keynesian hope for an Anglo-American post-war *pax*, and to compare it with the words with which Pareto concluded his 1905 considerations on the United States of Europe. They were considerations on a possible method to arrive at a government of Europe in a non-belligerent way, and today, going beyond Pareto, towards a government of the whole world. But I have mentioned the total multilateralism required by Keynes for a check on hot money because I have strong doubts about those who hope to win a battle through the introduction of a "Tobin tax" on a unilateral or bilateral basis. The fact is that since the 1980s, the Earth on which we live is becoming increasingly small and interdependent and has moved progressively away from the control of general politics and monetary politics in particular. We live in a world that is very different from the world that Pareto and Keynes experienced, but clearly legible telephone numbers for the government of Europe and for the government of the world still do not exist.

SOME REFERENCES. For the gradual transformation of the "hoards" in the various monetary systems until they become the savings in monetary form of Pareto's *Cours*, we consider as invaluable, even for the recollection of the famous father of the poet Pope, the entry "Hoarding" in the old editions and reprints of the *Palgrave's dictionary of political economy*, H. Higgs (ed.), London, Macmillan.

Pareto does not specify it, but it is easy to verify that Hume's passages quoted in his *Cours* are taken from essay 4, *Of interest*, by D. Hume, *Political discourses*, appeared in Edinburgh in 1752.

The sentence "[. . .] the *class war* will find me on the side of the educated *bourgeoisie*", is found in M. Keynes, *Am I a liberal?* (1925), later in M. Keynes, *Essays in persuasion*, 1931. See C.W.J.M.K., Vol. IX, 1972, p. 297.

When referring to an average period of production in Keynes' *General theory*, we considered it obvious that it does not have anything to do with Böhm-Bawerk's average period of production.

We have mentioned R. Kahn, *The economics of the short period* (1929), which, thanks to an initiative by M. Dardi we have read in the Italian translation by L. Cecioni, *L'economia del breve periodo*, Torino, Boringhieri, 1983, before it reappeared in a well-known English edition.

Pareto's review, *Le Zollverein Italian-Français et les ?tats-Unis d'Europe*, 1905, can be read in V.P., *Oeuvres complètes*, Vol. VI, 1984, pp. 254-55.

The bibliography on international economics is vast. For having at times devoted pages to this topic that look like glossaries of words, our gratitude goes to the editions and reprints of the old *Palgrave's Dictionary*, cit., to the *Memorandum* of the League of Nations, to the publications of the I.M.F., to the beautiful collections of *Essays, Studies, Special Papers in International Economics and Reprints in International Finance* of

Princeton University. However, the topic of “hot money” (**point 4**) calls for two references. 1) For a comparison between the evolution of the real events and the original Article VI, *Capital Transfers*, of the Bretton Woods agreements, see *The International Monetary Fund 1945–1965, twenty years of international monetary cooperation*, K. Horsefield ed., Vol. III: *Documents*, Washington, I.M.F., 1969. 2) L. Einaudi asked M. Fanno to write *I trasferimenti anormali dei capitali e le crisi*, Torino, Einaudi (Giulio, son of Luigi's), 1935; a book that was translated into Spanish and reviewed for the American edition, M. Fanno, *Normal and abnormal international capital transfers*, Minneapolis, The University of Minnesota, 1939.

E.N. 41. ON THE WORD “MONETA” AND ON PARETO’S REFUSAL TO BE HIS OWN TRANSLATOR

Let us try a moment to imagine the difficulties that Pareto would have encountered if he had had to translate the word “moneta” (money), which he uses A) in §32 and B) in the following paragraphs of Ch. VIII, into English.

This distinction between paragraphs is quite important. Indeed, in B), in the paragraphs that follow §32, by “moneta” Pareto refers to any kind of circulating currency. In the *Cours*, vol. I, §276, Pareto uses the expression “monnaie de circulation” as inclusive of all the means of exchange in circulation (gold, warrants, cheques, etc.). This meaning of “monnaie de circulation” is confirmed in §409 of the *Cours* where, by using statistics, Pareto shows that in a monetarily developed country such as Great Britain, gold had for the most part been displaced, in domestic transactions, by more effective fiduciary instruments. It was primarily J. M. Keynes (*Indian currency and finance*, 1913) who in later years showed that in the “gold exchange standard”, which had been proposed at the end of 1800s by a follower of Ricardo, was reducing the use of gold in international transactions as well. In short, we believe that the “monnaie de circulation”—a synonym for any means of circulation—finds its conceptually corresponding expression in the English term “currencies”.

It is for this reason that Pareto might have found the word “currencies” to be an unsuitable translation of the word “moneta” as he used it in A), in §32, to mean gold minted in coins, as opposed to gold in ingots, in a system in which there exists a mint where raw gold may be freely minted, and already minted gold may be demonetized. In comparison to his contemporaries, who pessimistically distinguished between money and its surrogates, Pareto stood out, in turn, because he talked about maximum collective opulence for monetary systems where gold was the *true* numeraire and the *true* value reserve, while other instruments of circulation served as effective fiduciary means of exchange. Pareto talked instead about destruction of wealth, *false* numeraire and *false* means of exchange—for which he blamed the immorality of public authority, whether it be monarchic or republican and democratic, more than private arrogance—when the possibility to mint or demonetize gold on the basis of free choice ceased to exist; when, in other words, gold surrogates were imposed by “politicians”, that is, petty politicians. All this is quite well known and perhaps might have been enough to dissuade Pareto from having the word “moneta” as he used it in §32, while thinking of gold as *true* money, translated as “currencies”.

What is less known, however, is the importance that Pareto gave to the gold stocks that the governments of modern States let their ‘banks of issue’ hold. We are referring to the main monetary function that Pareto saw in the gold stocks of the banks of issue when the gold standard in force is not tempered by petty politicians. In essence, Pareto had in mind the British gold system that had worked more or less between the end of the Napoleonic wars and the First World War. Pareto thought that in monetarily developed countries such as Great Britain, where saving took on a monetary form (the “monnaie d'épargne” of the *Cours*, vol. I, §280), the greatest collective advantage would be provided by a fiduciary-only monetary circulation without gold, but with the right to ask for the conversion in gold from the banks of issue, which alone were the holders of the gold reserves. Only in this way, according to Pareto, those who save in a monetary form transforming present goods into future goods, would have avoided the risk—coming from the petty politicians—of not getting back the “consumption” they had forgone in the past. The fact that all this implies favorable historical circumstances and an “enlightened” political class, was already included in the *Cours*, vol. I, §539, and can be found again in the *Manuale*, Ch. VIII, §46, where Pareto reiterates that the banks of issue holding a non-interest-bearing metallic value reserve perform a *public function* because they allow an effective fiduciary-only circulation (for instance, banknotes and cheques), and because, by holding a gold reserve as guarantee of that trust, they prevent the destruction of wealth associated with false money. This certainty in the public function of gold reserve also involves the maximum of collective utility in sociology, as well as the maximum of collective ophelimity (see E.N. 31), but in this E.N. we will not go any further into the matter.

Naturally, we don’t know anything about the English translation, by Pareto, of the word “moneta” in the mentioned paragraphs A) and B). We know that Pareto was able to think and write both in Italian and French, but declared himself incapable to be his own translator. He knew that “translator, traitor” and preferred to have his *Manuale* translated by his friend Bonnet (see *Editors’ Introductory Note*, point 2) and regretted his choice a thousand times, rather than becoming his own “traitor”.

A REFERENCE ON PARETO’S LANGUAGE. In Manon Michels Einaudi, “Pareto as I knew him”, *Atlantic Monthly*, September 1935, p. 337, we read: “He spoke a Tuscan Italian and a French colored with argot equally well, passing with absolute ease from one language to the other”.

E.N. 42. PARETO AND THE THREE MEANINGS OF THE WORD “CAMBIO” (“EXCHANGE”)

Ch. VIII, §34. In works written by Pareto (a great fan of terms with univocal meaning), as in the writings of any other economist, the word “cambio” (literally, “exchange”) is an elliptic expression that can assume three meanings.

1) it can mean “official exchange rate”, “currency equivalent”, “currency parity”. When the *gold standard* was in force (including its classic *gold-exchange standard* variant), every currency was defined as the equivalent of a certain weight in gold. The “exchanges” between two currencies, intended as official exchange rates, were therefore given by the ratio between their metallic weights;

2) it can also mean “market exchange rate”, that is, the ratio at which two currencies are exchanged on the “exchange market”;

3) finally, it can mean negotiable credit instruments written with reference to a foreign currency (cheques, bills of exchange, etc. discussed by Pareto; cheques that have a variable price on their market).

Under the gold systems such as those that Pareto had in front of him until the First World War, exchanges as in 2)—that is, market exchange rates—fluctuated around the official currency parities within a fluctuation band called “gold points”, which was only determined by private convenience (transportation costs plus insurance on the gold). Such a band was, therefore, of quite a different nature to that of the fluctuation bands provided for by international agreements, which became prevalent later with the Bretton Wood agreement and, in Europe, with use of the ECU (European Currency Unit) before the introduction of the Euro.

A FURTHER REMARK. While Pareto was still alive, fluctuation in the exchange rate for the Italian lira was at the center of a scientific dispute between some of the most famous Italian economists. We know from Pareto's correspondence that he was preparing a new scientific essay on exchange rates. Thanks to Fiorenzo Mornati and the last volume of Pareto's *Oeuvres Complètes*, published under the direction of Giovanni Busino (volume XXXII, cit.), many pieces of that work have been found. Once again, we point out to the reader that volume XXXII of the *Oeuvres Complètes* includes a seventy-page chronologically ordered bibliography of Vilfredo Pareto (*Bibliografia Cronologica di Vilfredo Pareto*).

E.N. 43. GOVERNMENTS, THE REDISCOUNT RATE, AND PUBLIC DEBT

Ch. VIII, §36. Pareto's logical relations appear to be the following: *a)* all other conditions being equal, a worsening of the balance of payments would lead to a tightening of the money market and, as a consequence, increase discount rates and the rediscount rate; *b)* governments, however, apply some pressure for credit to be rationed, in order not to depress—so “they say”—production activity; *c)* as an alternative, they can prevent the tendential increase in interest rates mentioned under point *a)* by increasing the quantity of money; *d)* a policy of containing interest rates keeps the entrepreneurs' investment activity alive, as well as their competitiveness, which is one of the forces for redressing the balance of payments.

If we have some doubts about this logical sequence (which reminds us of the mixed price-quantity policy, of the “moral suasion” and of the “fringe of unsatisfied borrowers” of Keynes' *Treatise on money*), it is because of those words, “they say”, that Pareto puts into the mouth of the governments: one could almost conclude that Pareto's governments resort to a form of rationalization that hides the true purpose of their policy, which is first of all to watch the quotation of public debt securities. It should not be forgotten that for Pareto the holding of public securities and the fraudulent practice of insider trading could be traced back to a criminal conspiracy between the bourgeoisie and its political representation.

A CONSIDERATION AND MARSHALL'S FOURTH REMEDY. Since Pareto's time, the relationship between industry and finance—or, if one wishes, between the real world and the financial world—has changed a number of times in Italy, though it has always been characterized by a high level of illegality. All in all, Pareto's ideas belong to that tradition of criticism towards the financial world that dates back to Marshall and Keynes and stretches all the way to the valiant J. Stiglitz of today and his notion of "cash for trash". However, it is a pity that not even Stiglitz mentioned the fourth of the nine remedies against cyclical unemployment, which A. Marshall proposed in 1885. As Marshall was one of the first scholars (1879) who studied the various aspects of business cycles, it is worthwhile to go over this fourth remedy once again: "In one sense indeed I am a socialist, for I believe that almost every existing institution must be changed [...]. My fourth remedy is to reverse the presumption that if a dishonest bankrupt fails for a large sum of money he should therefore be left off with a small punishment on the ground that he is likely to feel any punishment deeply. In dealing with an ordinary criminal, recklessness as to the extent of the harm he does is a ground for a heavy sentence. This principle should be applied consistently. Given two acts of commercial dishonesty, similar in other respects, but of which one causes injury only to a few, while the other, like the Glasgow Bank failure, spreads desolation through thousands of homes, the latter ought to be far the more heavily punished. If judges could be induced to treat more severely fraud whenever it is found in the high rank of business, particularly among promoters of companies, the industry of the country would become steadier". See, various authors, *Industrial Remuneration Conference 1885, The remuneration of capital and labour*, Proceedings and Papers, London, Paris, N. York, Melbourne, Cassell, 1885, p. 173 and p. 177.

E.N. 44. PARETO AND MARSHALL ON BIMETALLISM

Ch. VIII, §41. The gold-silver monetary system was notoriously unstable because of the recurring deviations between the *legal* relative price between the two metals and the relative price determined by the *private* metals market. Marshall and Pareto highlighted this instability, as others had done before them. But whilst Pareto considered the advent of gold monometallism as obvious (England had been on a gold standard for a number of decades and, after Germany had adopted the gold system in 1873, gold had become the standard for an ever-increasing number of countries), Marshall formulated a project based on the immense quantity of silver and gold that had been accumulated—for monetary and non-monetary uses—by the "pearl of the British Empire", India. This project of Marshall's contains a vein of unrealism that was certainly not the main characteristic of this great English economist. But in the work by Marshall we are referring to (*Remedies for fluctuations of general prices*, 1887), precisely where Marshall acknowledges his debt to Ricardo, there is an allusion to some control of the quantity of money and of the interest rate (the sale of public securities known today as *open market operations policy*) which we do not believe should be ascribed to Ricardo. Perhaps Marshall was the first of the great economists to incorporate the open market policy in a monetary project.

MARGINAL CONSIDERATIONS. The plans to reform bimetallism—both when they resulted in proposals for a multi-good standard, and when they followed the path of a

combined gold-silver product with daily declarations of the price ratio between the two metals—saw the active participation of the most prestigious names of political economy: it suffices to mention Jevons, Walras, Marshall, and Pareto. Their discussions concerned delicate problems of pure theory and substantial interests. If at the onset of our new millennium, in India, recriminations about those plans have arisen once again, perhaps it is also because J.M. Keynes, who tended to talk about ideals more than interests, never made it explicit that Marshall's plan for the abovementioned combined gold-silver product was conceived with the Empire, essentially India, in mind. Allow us to refer the reader to A. Zanni, "Price stabilisation policies", *The Elgar Companion to Alfred Marshall*, T. Raffaelli, G. Becattini, M. Dardi editors, Cheltenham, United Kingdom—Northampton, MA, USA, E. Elgar, 2006.

E.N. 45. PARETO AND THE DISAPPEARANCE OF THE MONEY-COMMODITY

Ch. VIII, §42. Here and elsewhere, when Pareto talks about the tendency towards the disappearance of money in exchanges, he is clearly referring to the disappearance of the money-commodity *par excellence*, namely gold. Barely four years after the appearance of Pareto's *Manuel*, in *Indian currency and finance* (London, Macmillan, 1913), a young Keynes would give an historical-theoretical description of the tendency to replace gold with various negotiable credit instruments that is as succinct as it is clear. In short, even the *Manuale*, which, in comparison with the *Cours*, marks a step backward in the field of monetary theory (from E.N. 3 we know that Pareto wanted to revisit this topic in a *Trattato di Economia*), shows Pareto as someone who is rather mindful of an essential and historic juncture for international monetary economics.

AN ADDITION. Thanks to the efforts of Fiorenzo Mornati (see the unpublished works finely annotated by him in Vols. XXXI (2001) and XXXII (2005) of V.P., *Oeuvres complètes*, cit.), we know and prove today that Pareto intended to write some articles on monetary theory in 1920–21, but only various unpublished chunks have been found. See E.N. 34 and 42.

E.N. 46. HABIT AND RATIONALITY IN CHOOSING, IN PARETO

Ch. IX, §2. *Habit* (or, if one wishes, one's usual mental customs) played a major role even in Pareto's personal consumption model. Pareto himself revealed this in a 1891 letter to Pantaleoni (V.P., *Corrispondenza Pareto-Pantaleoni*, vol. I, pp. 43–49). This letter is very important because Pareto discloses, once again, the political genesis of what he would end up by calling "non-logical actions". Pareto would later maintain that the latter cannot be explained by using the pure economy, but by using sociology (this does not mean that they cannot be ordered with an index function of utility; in other words, while it is impossible to economically explain "non-logical" tastes, it is, however, possible to represent them and to explain demand and prices in reference to them). We do not rule out the possibility that Pareto's position with regard to *habit*, though actually based on introspection, may have contributed to the tendency of Pareto to overrate sociology

relative to pure economy, which characterized the writing of the mature Pareto. On the other hand, Pareto had learned about the importance that “customs and habits” assume in human behavior from J. S. Mill, according to whom the most important thing a generation inherits from the previous one are its traditions, its customs and its institutions. At any rate, we would like to add the following remark.

Shortly after publication of the *Cours*, in an important essay from 1900, “Un'applicazione di teorie sociologiche” (see V.P., *Scritti sociologici minori*, G. Busino ed., Torino, Utet, p. 179), Pareto restricts the attribute of logical rationality to production activity alone, especially when not dealing with small entrepreneurship:

“The majority of human actions do not stem from logical reasoning, but from sentiment; and this is mainly true for the actions that have a non-economic purpose. The opposite happens for economic actions, especially those involved in trade and production on a large scale”.

Considering *a)* the little faith that Pareto had in logical consumer behavior, and taking into account *b)* that the Paretian theorem on the maximum of ophelimity for a collectivity (1894) essentially belongs to the theory of production, the reading of Pareto has always left us wondering about the great importance given by later literature to Pareto's pure theory of consumption over Pareto's theory of production, as outlined in the *Manual*. This conviction of ours was strengthened by the injustice perpetrated by Georgescu-Roegen (“Vilfredo Pareto and his Theory of Ophelimity”, in *International Meeting on Vilfredo Pareto*, cit.) when he included Pareto among the pure equilibrium theorists of consumption, as opposed to the theorists of “change”, when Pareto had in fact always proclaimed his admiration for those authors, such as Gossen, Jevons, etc., who had shed light on the endless evolution of needs and on the disruptive changes that take place within the very hierarchies of needs in the course of history.

To conclude this Note, let the reader allow us a further remark: the fact that Pareto “Marshallized” Walras' general equilibrium with a variety of aspects pertaining to the theory of production and market structures does not yet appear to have been absorbed into the economic culture.

E.N. 47. THE ASPIRATION TO RISE AND THE FEAR OF FAILURE IN PARETO'S ENTREPRENEUR

Ch. IX, §11. This push towards enterprise, which stands between the aspiration to rise and the terror of failure, constitutes a considerable improvement compared to the sociology—of which Keynes represents the most mature expression—that restricts itself to recalling investor's “animal spirits”. However, it is fair to add that, particularly in the masterly essay of 1923 (*Some aspects of commodity markets*, in C.W.J.M.K., Vol. XII, 1983), Keynes provided an insightful sociology of businessmen (entrepreneurs, bankers and undertakers of speculative risks, that is, insurers) who differ from one another for a number of aspects. We shall recall two: their different contextual knowledge and therefore their different ability to predict the future; their different degree of confidence in the reliability of their own predictions, which will become “the state of confidence” of the *General Theory*—possibly the most important parameter of Keynes' function of the marginal efficiency of capital. However, these are aspects that, rightly or wrongly, Pareto

preferred to include with the second approximations of applied economics or sociology, that is, outside of the first approximations of pure economy. It is precisely because of this inclination of his that we were surprised to see the inclusion of the *pursuit curve* in the *Cours* (see E.N. 26).

E.N. 48. IDEAS AND INTERESTS IN PARETO AND IN KEYNES

Ch. IX, §20. It is worthwhile to recall a considerable difference and an analogy between this Pareto and Keynes. As for the difference between Pareto and Keynes, one only needs to read again the passage that concludes the *General theory* (p. 384):

“But sooner or later, it is the ideas, not vested interests, which are dangerous for good or evil”.

As for the analogy, it is suffice to recall that for Keynes even probability theory is, at times, a form of rationalization that is devoid of any objective grounds: it is a “derivation”, to use an expression from Pareto’s sociology that was also taken up by L. Amoroso in a polite argument with probability theorists, such as the mathematician Guido Castelnovo.

Since it is impossible for us to dwell on a comparison between the optimistic-humanitarian tradition, to which Marshall and Keynes belonged, and Pareto’s realistic tradition, we refer the reader to a suggestive work by a scholar who bestows a tribute of esteem to Pareto’s realism and who wishes that Economics, after having acknowledged Coase, would let Machiavelli finally be heard. We are referring to Jack Hirshleifer’s “The dark side of the force”, *Economic Inquiry*, Vol. XXXII, January 1994, where on p. 3 one reads:

“Human history is a record of the tension between the way of Niccolò Machiavelli and what might be called the way of Ronald Coase. According to Coase’s Theorem the people will never pass up an opportunity to cooperate by means of mutually advantageous exchange. What may be called Machiavelli’s Theorem says that no one will ever pass up an opportunity to gain a one-sided advantage by exploiting another party [...]. Our textbooks need to deal with both modes of economic activity”.

A CONSIDERATION AND AN ACKNOWLEDGMENT. Thanks also to Pareto, conflict today between economic theory and sociology is becoming increasingly less topical. The number of the economists who are venturing in the traditional domains of sociology is on the increase, as is the number of economist-sociologists. Nicolò Bellanca, who knows many border areas, has kindly directed us to Hirshleifer’s suggestive text, in which Marshall and Pareto are also compared.

E.N. 49. PARETO ON RICARDO’S AND BASTABLE’S COMPARATIVE COSTS

1. Criticisms of Pareto’s pure theory. From their very first appearance, Pareto’s pages on Ricardo’s comparative costs were the most criticized pages of the *Manuale* (1906). The appearance of the *Manuel*, where those pages were kept unchanged (1909, pp. 506–14), triggered further criticisms; so much so that in 1943 Gambino mentioned “Pareto’s unfortunate criticism” of Ricardo in one of his titles. Since then, the criticisms—

which started with Loria, Graziani and Wicksell—have kept piling up. However, we have discerned an interpretative key that allows us to exonerate Pareto from these criticisms.

First of all, it appears that the difficulties in interpreting Pareto depend: a) on the particular case considered by Bastable and criticized by Pareto; b) on Pareto's reticence. Indeed, Pareto does table some arithmetical examples but, as we shall see, he hides the main hypothesis in support of his criticism from the readers' view. Let us look at a) and b) more in detail.

a) Bastable believed that Ricardo's reciprocal advantage in the trade between two countries always exists, even in the following case: I and II, two countries closed to trade, produce good A and good B; after opening to trade and the division of labour, the two countries produce a world quantity of A greater than the quantity produced with closed markets, whilst the world production of B is smaller.

b) In the *Manuale* Pareto explains, as he does in the *Cours*, that I and II have some preferences (some tastes) with regard to products A and B. Having assumed this, Pareto maintains that the Ricardian *reciprocal* advantage of two countries in trading is not generally valid, but is only a *possibility*. Pareto shows this with an arithmetical example according to which the opening of the markets would result in a reduction of welfare for one of the two countries in Bastable's case under point (a). In short, Pareto states that if the world production of corals increased and the world production of bread decreased, it would not be possible to tell the hungry: "Eat coral!"

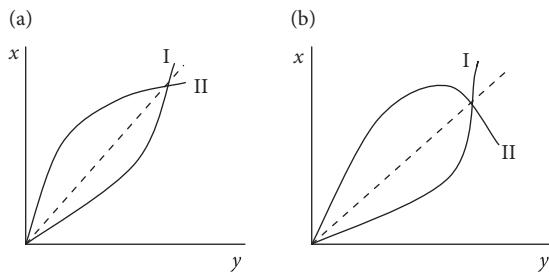
Let us then preliminarily establish two general points that can be inferred from the whole body of Pareto's works: i) when passing from pure economy to politology (which is the core of Pareto's sociology), protectionism may be preferable to free trade because a governing *élite* may discern lasting distant advantages that prevail over transient disadvantages (Pareto's "destruction of wealth"). ii) However, even within the field of pure economy it can be demonstrated that reciprocal advantages in trading may not exist, by taking into account I's and II's preferences.

We wish for some heirs of J. Viner's (1937) *Studies in the Theory of International Trade* to rediscover both of Pareto's points. But we hasten to add that the uproar triggered by Pareto's point (ii) has not yet settled. Even Gandolfo, in his excellent *Economia internazionale*, (vol. I, Torino, Utet, 1986, p. 20), speaks of a blunder by Pareto on comparative costs. However, in order to demonstrate this mistake of Pareto's—who resorts to various arithmetical examples to find the consequences of alternative preferences on welfare—Gandolfo *deliberately ignores preferences*: it is possible "to demonstrate the incorrectness of this thesis [of Pareto's] without having to resort to a specific utility hypothesis" (on this criticism by Gandolfo, who makes use of Dorfman Samuelson Solow *world transformation curve*, see A.Z., 2007, pp. 123–24).

As far as we are concerned, our interpretation went along the following path.

2. A reinterpretation. In the *Cours*, Pareto had reshaped the theory of comparative costs by taking into account the preferences (tastes) of individuals—I and II, for the sake of simplicity. In it, he had used two previous articles of his (1894 and 1895), in one of which (1895, p. 486) he showed Edgeworth that it was preferable to extend Walras' equilibrium to international economics rather than tackle 2^8 (that is, 256) possible cases with Marshall's demand supply curves. Pareto added that he was using Marshall's curves

in teaching. Indeed, in Volume II of the *Cours* (§876 but see also §§877–78) we did discover the following two diagrams.



According to Pareto, Fig. (a), where the demands and supplies of I and II are monotonically increasing, represents the case of competition, when neither of the two protagonists has any significant monopolistic clout because of the high *price elasticity* of the demand curve in which each of them operates. Fig (b), where the curve for II shows a noticeable hump (it decreases after reaching a maximum), represents instead the case where one of the traders has monopolistic power in terms of ratio of exchange. In other words, the curve for I meets the curve for II in the decreasing section, the section where price elasticity is very low. Pareto exemplifies Fig (b) with the by now crumbling international monopoly held by Sicilian sulphur. For the sake of completeness we point out that Pareto, who kept track of the economic studies and legislation on American industrial structures, did not think that international monopolies existed in large numbers, and that he distinguished between concentrated industries in a competitive regime and trusts (concentrated industries with monopolistic power): see Pareto's 1899 review-article, entitled as the book "Les industries monopolisées (trusts) aux États Unis" (author Paul de Rousiers, Paris, Armand Colin, 1898), in *Oeuvres Complétes* (Vol. VI, cit., pp. 152–53). While Pareto does not say so in as many words, we believe that when he puts forward the case in which it is not in the interest of one of the two countries to open up to trade, he is thinking of a country whose domestic production is governed by a pervasive industrial and commercial competition of an "atomistic" kind; he is thinking of a country that would therefore suffer a reduction in welfare, were it to open up to trade with another country where a large monopolistic firm produces a quantity of a good that vastly exceeds the quantity of the same good produced in the first country, and more in general in the world. If this is Pareto, then he is putting forward *his* hypothesis of *asymmetric duopoly* with one "active" and many "passive" individuals (see E.N. 20) at international level. For some time after World War II this hypothesis was still being taught in some Italian universities, with regard to domestic production alone and without mentioning Pareto (all references were to Stackelberg). Sometimes it was taught under the name of "partial monopoly" (for instance, G. Demaria in Milan), at other times it was called "imperfect competition" (for instance, R. Galli in Florence), with the warning that Joan Robinson's imperfect competition is something else. International monopoly is therefore the hypothesis that Pareto hides from the readers, the cause of the scandal. It was indeed considered a scandal (by Wicksell among others) that Pareto would challenge the general validity of Ricardo's theory with an example (*Manuale*, 1906, p. 474, *Manuel*,

1909, p. 513) in which the ratio of international trade falls outside the comparative costs interval. However, we do not believe there is any scandal. Indeed, there is no rule set in stone that prevents one from taking into account the preferences of I and II, as well as monopoly, to investigate what consequences the opening of markets would have on welfare. We should rather ask ourselves why Pareto conceals his debt to Marshall: indeed, in 1891 Pareto had read, and had expressed immediate admiration for, *The Pure Theory of Foreign Trade* (1879) by Marshall and the (1891) edition of Marshall's *Principles*, which contained point elasticity.

3. Why Pareto does not quote Marshall with regard to international monopolies? One could maintain that when writing the *Manuale*, Pareto was feeling so resentful towards Marshall that even Pantaleoni, who saw in Marshall another Ricardo, wrote: "Marshall, furthermore, is factious, he behaves in a mafia-style. He systematically passes your things under silence" (*Corrispondenza Pareto Pantaleoni*, Vol. III, p. 37). But leaving aside that potential explanation (which is legitimate, especially considering that Pareto knew about the famous failed publication of an article by Barone in *The Economic Journal*), an alternative—and more likely—explanation is evident from a report that Pareto wrote in 1875 for the "Società Adamo Smith di Firenze", which was republished in 2005. On reading Pareto (1875), we decided to test the basis of the criticism frequently leveled at Pareto for his failure to quote other authors (see A.Z., 2010a and 2010b, and E.N. 54), and found: i) that he had been following the discussions on the international monopoly held by Sicilian sulphur since the 1870s, that is, when he did not know that Marshall existed; these discussions were already involving the concept of "elasticity", albeit in terms of *percentage* variations in the prices and quantities of a product; ii) that Pareto had so well commented on the proportional relationship arguably existing between "offer and value" as contained in Mill's *Principles*, to lead one to believe that he knew Cournot's theory of monopoly and *point* elasticity before Cournot's *Recherches* (1838) appeared in Italian (1878). On the other hand, the fact that Pareto knew the *Recherches* before reading Marshall and was therefore not bound, in the *Manuale*, to quote Marshall with regard to monopoly and elasticity can also be explained by the following considerations. We know (E.N. 6) that Pareto intended to write a book on Political Economy before he met Pantaleoni (1890) and before he read Marshall (1891). The *political* motive was protectionism, which Italy had embraced in 1887; the *scientific* motive was given by the misleading conclusions he had found in the *Recherches*, where Cournot tackled international economics without taking into account economic interdependences. This can be inferred from the fact that, before starting his famous *Considerazioni* on the "new school of economics", Pareto was asked by Pantaleoni in 1891 to write an article on an error in Cournot's mathematical treatment of political economy ("Su un errore del Cournot nel trattare l'economia politica colla matematica", *G.d.E.*, January 1892). This is an article that from its very title announces one of the constants of Pareto's thought (mathematics exposes one to the risk of accurate conclusions based on inaccurate assumptions), but whose interest lies for us in the fact that in it (p. 1) Pareto specified that he was correcting the mistakes that had appeared in the Italian translation of Cournot's work (1878). This is a detail that leads us to infer that Pareto had acquired and read the *Recherches* (1838) a long time beforehand. Shortly later (*G.d.E.*, May 1892, p. 399), Pareto said that in his previous article from January he had not mentioned

Bertrand's criticism of Cournot, which had appeared in 1883; and from this, too, one can infer that Pareto had known Cournot for a long time. Consequently, given that Pareto had known about the discussions on the international monopoly held by Sicilian sulphur since as far back as the 1870s, and given that all evidence appears to point to the fact that he had known Cournot's theory of monopoly and elasticity since then, and at any rate before reading Marshall, we conclude that when in the *Manuale* Pareto re-examined Ricardo's comparative costs, he was not bound to quote Marshall with regard to monopoly and elasticity. On the other hand, if there is one thing that Pareto and Marshall have in common in this regard, it is the unacknowledged debts that, independently of each other, both of them contracted with the French mathematical economist (see E.N. 54).

As for Pareto's praise (*G.d.E.*, May 1892, p. 398) of *The Pure Theory of International Trade* by Marshall (1879), it is not any less important even considering that in 1892 Pareto did not yet have his own theory of international trade (1895). In that praise Pareto essentially says: here is an economist, Marshall, who makes "sparing use" of mathematics "while always keeping his mind trained on concrete facts" (*G.d.E.*, June 1892, p. 510); an economist who brings to fruition, with elegance, the theory of international trade, and puts an end to discussions—this implication of Pareto's is added by us—that had been dragging on for a number of decades.

ACKNOWLEDGMENTS, SOME REFERENCES (AND A "SNAPSHOT". TAKEN AT THE "SOCIETÀ ADAMO SMITH" IN FLORENCE, IN PARETO'S YOUNG YEARS). Our points 1 and 2 are based on A.Z., "Pareto on Ricardo and Bastable's comparative costs", *History of Economic Ideas*, 2007/2, where the author thanks Piero Zattoni, Piero Tani, Aldo Montesano for their generous pieces of advice offered at different times. We have referred to: Gambino A., "Il paradosso ricardiano e l'infelice critica del Pareto", *Rivista di Storia Economica*, March-June 1943; Dorfman, Samuelson, Solow, *Linear Programming and Economic Analysis*, New York etc., McGraw-Hill, 1958; Marshall A., *The Pure Theory of International Trade. The Pure Theory of Domestic values*, 1879, privately printed (Pantaleoni lent it to Pareto). As for Wicksell, see Ch. III, *Notes* [§171, a]. There was a reference to two works by A.Z., "Economics or Political Economy? Marshall and Pareto as Mill's Heirs"; *Studi Economici*, 2010/2; "Marshall and Pareto on Cournot's Elasticity and on W. Thornton", *Studi Economici*, 2010/3. For Pareto, see "La teoria matematica dei cambi forestieri", *G.d.E.*, Feb. 1894, and "La teoria matematica del commercio internazionale", *G.d.E.*, April 1895. As for Pareto, *Intorno ai trattati di commercio* (a report presented in 1875 at the "Società Adamo Smith di Firenze", in the presence of F. Ferrara (1810–1900), the economist, and P. Bastogi (1808–1899), a high-ranking representative of the Italian financial world), see V.P. *Oeuvres Complètes*, Vol. XXXII, 2005, cit..

A miscellaneous book (*Firenze Oggi*, Ariani, Firenze, 1896) contains a passage, a "snapshot" (p. 268), that can be of some interest to politologists: "One day Ubaldino Peruzzi [1822–1891: head of the provisional government of Tuscany in 1859, Minister of the Italian government in 1860–64, Mayor of Florence when it was the capital of Italy] had heard three or four young men in discussion at the Società Adamo Smith and had been impressed by their cleverness, by the elegance of their delivery, by the originality of their arguments, by the political and social importance of their discussions; and—

with no hesitation—on that evening he took them home, introduced them to his wife Emilia [...]. These young men's names were Genala, Sonnino, Franchetti and Pareto. If we remember correctly, the question at the centre of the lively debate in the first few days was that of proportional representation". This is a topic that in addition to Sonnino, Pareto and the Swiss National E. Naville, brings to mind above all the political theorists T. Hare and J. S. Mill, and the current Constitution of the Federal Republic of Germany.

E.N. 50. A FIRST MATHEMATICAL NOTE BY PARETO

By resorting to symbols that are different from Pareto's—using $(T - t)$ instead of $(t - \theta)$ —his mathematical condition may be reformulated in the following way. Let us assume that I and II have T hours of work at their disposal and that while II produces 1 of A, I produces x ; while II produces 1 of B, I produces y . In other words: x is the ratio between the productivities of countries I and II in terms of commodity A; y is the ratio between the productivities of countries I and II in terms of commodity B. Then, let (μ) be a combination where, with closed markets, I produces A for the period $(T - t)$ and B for the remaining time t , whereas II produces A for the period $(T - t')$ and B the remaining time t' . And let (π) be a different combination where, with open markets, for the whole time T , I only produces B and II only produces A. As it can be seen, Pareto's combination (π) refers to binary complete specialization. Now, the condition for the productions of A and B to be greater in combination (π) with open markets, than in combination (μ) with closed markets, is that

$$T > (T - t)x + (T - t'); Ty > ty + t'.$$

If we develop the two inequalities with respect to t' we obtain

$$(T - t)x < t' < (T - t)y.$$

In order for this condition to be possible it is necessary that $y > x$: this latter inequality of Pareto's is legitimate since x and y are two pure numbers.

E.N. 51. PARETO AND THE ITALIAN CONTROVERSY ON DUMPING: WHERE THERE IS ALSO A REFERENCE TO DUPUIT

Ch. VI, §47 and **Ch. IX, §54**. By now, readers must have noticed that Pareto—himself the very inventor of the term "ophelimity"—uses the expression "variable prices" to mean *i*) prices that vary in the course of trading for subsequently negotiated doses, and was accepted by Pareto even in the case of competition; and *ii*) prices differentiated according to the classes of demanding parties by a supplying party that has a monopolistic power (for instance, a trust). The hypotheses in Ch. VI, §47, and Ch. IX, §54, gave rise in Italy to a scientific dispute, in which the champions were P. Jannaccone ("Il 'dumping' e la discriminazione dei prezzi") and A. Cabiati ("Prime linee di una teoria del 'dumping'", both in *La Riforma Sociale*, March 1914). In our opinion, this dispute was possible

because Pareto allowed the two (or three) meanings of the expression “variable prices” to remain implicit. Indeed, a third meaning of the same expression (prices that vary with time) would take us back to the Pareto–Scorza polemic, which goes beyond the scope of the present E.N. We would like instead to add a consideration.

It has already been pointed out (E.N. 32) that it is unfair, with regard to production cost, monopoly, and collective welfare, to start from Marshall while forgetting Cournot. But it would be equally unfair to neglect Dupuit for his work on price discrimination and collective welfare. It was indeed Dupuit who observed that a theatre company does not make a profit and spoils the fun for part of the potential spectators if it offers one-price ticket shows. It was also Dupuit who formulated the paradox according to which a single-price fare destroys canals, bridges and railroads, whereas a differentiated-price fare does “some kind of miracles”: it creates bridges and canals “without any kind of work”.

SOME REFERENCES. The quotations are from J. Dupuit, *De l'utilité et de sa mesure*, ed. by M. De Bernardi, Torino, Einaudi, 1934, p. 142 and p. 138. The fact that Pareto knew some of Dupuit's works is shown by the *Corrispondenza Pareto–Pantaleoni*, Vol. I, pp. 116–17 and 356. Dupuit's works and, more in general, the *Ponts et Chaussées* tradition were held in high regard in L. Einaudi's first university courses and in a now forgotten work by the engineer F. Tajani (*Tariffe ferroviarie*, Torino, Utet, 1910). Dupuit does not appear in the index of names of the *Cours*, but the fact that his works were known to Pareto is shown by an obvious misprint: besides Cournot, “Doupont [Dupuit] a aussi traité des monopoles, mais il confond la courbe de prix avec celle d'ophelimité” [“Doupont [Dupuit] has also dealt with monopolies, but he confuses the curve of price with that of ophelimity”] (*Cours*, vol. I, §140, **Monopoles**, p. 63).

E.N. 52. A SECOND MATHEMATICAL NOTE BY PARETO

Ch. IX, §54, footnote. The proof by contradiction contained in Pareto's mathematical footnote is based on two hypotheses: a protective customs duty in favor of a product makes it possible to: 1) sell abroad at a price lower than that which is established in the absence of a duty, and 2) increase the domestic price, thus reducing domestic consumption. In other words, if we indicate with a the quantity of a good produced and consumed in a country before the introduction of a protective duty; with p its corresponding unit cost and selling price; with p' and b the domestic price and the corresponding quantity consumed after the introduction of the protective duty, and with p'' the selling price abroad, still after the introduction of the protective duty, the first hypothesis requires $p'' < p$, the second $p' > p$ and, consequently, $a > b$.

Starting from these hypotheses—and indicating with c the quantity of the good sold abroad at the price p'' after the introduction of the duty, and with q the cost of production of the same good when, after the duty, $b + c$ is produced—Pareto introduces the following three conditions asking that all three be true at the same time:

- [1] $p'b + p''c > (b + c)q$
- [2] $pa + (b + c - a)p'' < (b + c)q$
- [3] $p'b + p''c - (b + c)q > (p' - p)b$

which, after simplification, may also be written

$$[3'] pb + p''c > (b + c)q.$$

Since Pareto wants to demonstrate that customs protectionism causes losses for consumers, which exceed the profits of the protected entrepreneurs, it is clear that condition [3], which states the opposite, is introduced in order to generate a proof by contradiction.

Now, if we replace the right side of [2] with the first side of [3'], we obtain:

$$pa + (b + c - a)p'' < pb + p''c$$

which, after simplification, becomes:

$$(p - p'')(a - b) < 0.$$

Since, because of the first hypothesis, it is $p - p'' > 0$, this implies

$$a < b,$$

and this contradicts the second hypotheses, which requires $a > b$. It is therefore demonstrated (on the basis of hypotheses whose evaluation we leave to the reader) that the entrepreneurs' profits cannot exceed the losses suffered by consumers as a result of a protective duty.

E.N. 53. PARETO AND EINAUDI ON PROTECTIONISM AND LOGIC

Ch. IX, §57. In a polemical article against protectionism, L. Einaudi ("La logica protetzionista", *La Riforma Sociale*, Dec. 1913) took the opportunity to state:

"None of us has ever refused to acknowledge the truth of a protectionist line of reasoning, when the reasoning was actually carried out and found to be correct. Throwing heaps of statistics at each other is time wasted; following sensible lines of reasoning and producing beautiful and good statistics to corroborate them is time usefully employed. In fact, if we look closely, all the protectionist theories that have managed, within their logical limits, to withstand the fire of criticism have been expounded not by protectionist pseudoscientists, but by most pure economists" (p. 830).

In confirmation of this apparent paradox, Einaudi dwelled on John Stuart Mill, Pantaleoni, and Pareto. Wishing to know whether he had interpreted Pareto's thought correctly, Einaudi sent his article to him. Pareto replied with a letter that we only know of today (see *Vilfredo Pareto (1848–1923) – L'uomo e lo scienziato*, a miscellaneous volume edited by Gavino Manca, with 63 unpublished letters from the "Fondo Pareto della Banca Popolare di Sondrio", Milano, Libri Scheiwiller, 2002, pp. 356–58). In his reply to Einaudi, Pareto recalled the importance of §68, Ch. IX of the *Manuale* and once again mentioned the difference between the *Cours* and the *Manuale* with regard to protectionism. He then explained to Einaudi "the only point on which we disagree [...]" (which is an historical point that does not need to be discussed here).

For what concerns Einaudi on Pantaleoni, we plan to write an article on this subject where Einaudi will be compared with Marshall, Pantaleoni and Pigou as scholars in public finance.

E.N. 54. ON PARETO'S QUOTATIONS AND HIS PERCEIVED LACK OF GENEROSITY

Ch. IX, §72. By now readers must have worked out Pareto's way of quoting. We may, therefore, resume what we had put to the side in E.N. 8, 38, and 49.

In 1968, Baumol and Goldfeld, the editors of an excellent anthology in which Pareto's encyclopedia entry *Économie Mathématique* (1911) appeared in English, criticized Pareto on p. 50 of their work because he only quotes himself. A similar, more serious criticism of Pareto on this matter had already been raised by Stigler (1950, p. 146): "Cassel was not the equal of Pareto in this respect (see especially the latter's *Économie Mathématique*)".

If it were not because of the difficulties raised by the language barrier, we would say that a comparison between Pareto (1911) and Pareto's previous encyclopedia entry from 1902 (see E.N. 8) should have warned Stigler, too, that the 1911 entry saw the light of day in an incomplete form: the First World War prevented the appearance of the bibliographical-historiographical section, for which Pareto had sought the help of none other than Pantaleoni, with the collaboration of L. Amoroso and U. Ricci.

At any rate, we must ask ourselves to what extent economists who are endowed with theoretical instinct and driven by a moral and political sacred fire should recall their predecessors. In this regard it is worthwhile to compare Pareto with Keynes the writer of the *Treatise on probability* (1921). On the one hand, Keynes (1921) devotes a chapter to a *Historical retrospect*, but on the other hand he states that if an author intends to persuade (and Keynes would always try to persuade, on a scientific, moral, and political level), he must make sure that his positions have some hope of prevailing ("He must give his argument a chance, so to speak", 1921, then 1973, p. 467), and not undermine their strength with doubts and—we add—with quotations. In his *General Theory* (1936), for instance, Keynes tends to draw benefit from Marshall, but obscures him rather than quotes him (one only needs think of the self-fulfilling expectations). Pareto's position is different. At least in words, Pareto does not embrace persuasion, he pours scorn on it. And even if at times he seems to be excusing himself because he knows that his fiery temper will prevent him from resisting all temptations, he assures us that he has embraced the faith in objectivity, the one and only thing he actually wants is to be *wertfrei*. And the fact that this, with some goodwill on the part of readers, essentially provides a good self-portrait of Pareto, can be seen from his position on Ricardo's comparative costs (E.N. 49). As a matter of fact, it is precisely Pareto, the world citizen, who is so inclined to see the long run advantages of free exchange, the very same Pareto who denies the general validity to Ricardo's theory. It is in the political debate that Pareto and Keynes show some affinities: as Keynes wrote highly critical pages against Lloyd George, but then also dedicated pages of acknowledgments to him, so did Pareto, who commented so harshly on Giolitti, also acknowledge the credit that the latter deserved.

As regards generosity in quotations, we sum up here the conclusions we have drawn from our readings. Pareto always acknowledged what he owed to Walras, Marshall and

Edgeworth. As for J. S. Mill, Pareto (*Trattata di sociologia*, §§295–99) paid off his debt by actually resorting to a diagram to specify the extent to which he agreed with him and where he started to disagree. Basically, there is only one author, Cournot, towards whom Pareto was definitely not generous. Indeed, Pareto mentioned Cournot almost exclusively to criticize him, although it is from him that he derived the notion of *terminal points* (see E.N. 11), which he used in his own definition of economic equilibrium (see Ch. III, *Notes 121 a*) and to reply to Wicksell's remark that he had not taken into account Marshall's criticism of Cournot with regard to the incompatibility between competition and increasing returns (a criticism that we consider unfounded, even if it was embraced and made famous by Piero Sraffa).

We have also found that as far as Cournot is concerned, Marshall's lack of generosity exceeds even Pareto's. On the roof of the Hotel Oliva in Palermo Marshall cried *Eureka!*: he had discovered point elasticity. However, a note reveals that Marshall had a copy of Cournot's *Recherches* in his suitcase!

SOME REFERENCES. We have mentioned G. Stigler's famous essay, "The development of utility theory", 1950, today in G.S., *History of Economics*, University of Chicago Press, 1965. The anthology edited by W. Baumol and G. Goldfeld is: *Precursors in mathematical economics. An anthology*, London, Series of reprints of scarce works on political economy, 1968. In regard to *A Treatise on Probability* by M. Keynes (1921), we perused the C.W.J.M.K., Vol. VIII, MacMillan for the Royal Economic Society, 1973. For an essential bibliography, as well as for the already mentioned works (2010a and 2010b), we refer the reader to the following work by A.Z., "Marshall and Sraffa on competition and returns in Cournot", *History of Economic Ideas* (XX/2012/1).

E.N. 55. PARETO'S PANPOLITICISM AND THE MAXIMUM OF UTILITY IN SOCIOLOGY

Ch. IX, §88. The present note is a continuation of E.N. 24 and a reinforcement of E.N. 31. It also intends to be a bridge to a work by Pareto that is extremely important, but, as it was published in 1913, it is strictly speaking beyond the scope of the *Manual*.

If we follow Pareto's writings, starting from his Tuscan period, in parallel with events of history, and of Italian history in particular, we appreciate the expression "panpoliticism" used by N. Bobbio to qualify Pareto's thought. And yet, there are few like Pareto who confirm the thesis, that was dear to G. Vailati's and L. Einaudi's heart, according to which value judgments, sentiments, and even interests, though being something different from the scientific core of knowledge, can live within it and even fertilize it.

Considering Pareto's panpoliticism, one would have expected that he would formulate the *maximum of utility in sociology*, which involves a governing-governed dualism and interpersonal comparisons (as formulated by Pareto in 1913) much earlier than the *maximum of ophelimity*, which does not involve such comparisons (as formulated by Pareto in 1894). But, as these dates suggest, the opposite is actually true. It should also be added that the Paretian maximum in sociology was beset by misfortune. Indeed, it was ignored to such an extent by the greatest scholars of welfare economics in Italy (and abroad), that, even if one accepts a famous dismissive judgment by P. Rosenstein Rodan,

Italy's corporativist economists of the 1930s must be largely excused for demanding to be released from Pareto's *no bridge* constraint!

As we know (E.N. 31), Pareto's maximum of utility in sociology implies the governing-governed relationship. Now, it is indeed impossible that the governing class apply their coefficients to the evaluations of every governed individual. Zawadzki (1914) thought that Pareto presupposed the *preferences* of average individuals, representative of groups composed by *identical individuals*. However, since for Pareto the individuals are different even within each single social class, he must be interpreted in an "Einaudian" way: the interpersonal comparisons of the governing class always imply some residue of arbitrariness. In our opinion (E.N. 15), this would always keep alive the Benthamism absorbed by Pareto in his youth years, which would express itself in the dream to find out, even if only approximately, the numerical measurement of ophelimities before any arbitrary political evaluation. Hence, too, the high meaning of Politics in Pareto, his attachment to "preferences", in spite of any potential "return to Cournot" (that is, in spite of directly introducing demand functions) for the construction of general economic equilibrium.

SOME REFERENCES. Besides V.P., "Il massimo di utilità dato dalla libera concorrenza", *G.d.E.*, July 1894 (Engl. trans. "The maximum of utility given by free competition", *G.d.E.*, December 2008, pp. 387–403), where Pareto uses the term 'utility' for indicating that subjective utility which he will call 'ophelimity' later on, we have referred to V.P., "Il massimo di utilità per una collettività in sociologia", *G.d.E.*, April 1913 (Engl. trans. "The community's utility maximum in sociology", in *Italian Economic Papers I*, il Mulino and Oxford University Press, 1992, pp. 39–43). One had to wait until J.S. Chipman's beautiful essay, "The Paretian heritage", 1976, cit. E.N. 3 for economists such as P. Samuelson and A. Bergson to find out about Pareto's maximum of utility. We have mentioned P.N. Rosenstein-Rodan, "La complementarietà: prima delle tre tappe del progresso della teoria economica", *La Riforma Sociale*, May–June 1933, in particular, p. 272, footnote.

E.N. 56. JUGLAR CYCLES AND PARETO-KONDRATIEFF LONG WAVES

Ch. IX, §§73–88. On the topic of "crises" (or rather, economic cycles) one could repeat what Wicksell stated with regard to money in the journey from the *Cours* to the *Manuel*: Pareto had lost it along the way. Even the treatments of economic cycles in the *Manuale* and in the *Manuel* are well below that of the *Cours*. Once again, this can be explained by the fact, unknown to Wicksell, that in the years during which Pareto was writing his *Manuale*, he was hoping to draft a second edition of the *Cours* in the form of the *Trattato di economia* (see E.N. 3). But let us go back to Wicksell.

Pareto's mathematical slip-ups in dealing with economic cycles (*Cours*, §928, footnote 2) were detected by Wicksell. As we have seen (Ch. III, §171, a), Pareto pointed them out himself in the *Manuel*, possibly without being aware of Wicksell's comments on the matter. The latter however had not noticed a banal oversight by Pareto. We are referring to the enigmatic mathematical expressions in the *Cours*, Vol. II, §698, footnote 2, where a mathematician that we consulted thought he could detect two Fourier series, and, immediately after that, we had the impression that the "known term" in these two

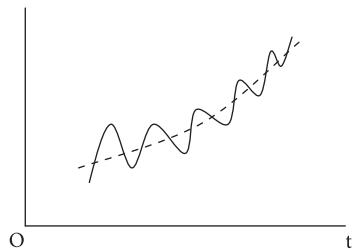
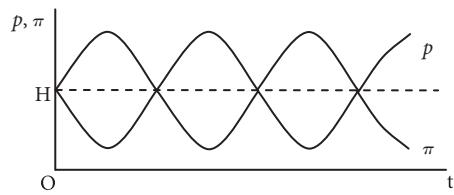
series—the average value around which two periodic functions fluctuate—is missing. Let us resort to a diagram. Let us suppose that we are illustrating the cyclical movement of profit for a Walrasian firm that, on average, does not make any profit or loss. Time is plotted on the abscissa axis, whereas the periodic function of price (p) and the periodic function of average cost (π) are plotted, perfectly out of phase, on the ordinate axis. The diagrammatic representation of Pareto's oversight consists in setting point H so that it coincides with point O, the origin of the axes; it follows that in the course of their oscillations, price and cost also take negative values. Had they noticed this error, it would surely have brought a smile to the faces of both Walras and Pareto.

Banal oversights of this kind can obviously be rectified mentally, without making a note of them. At any rate, in our perusal of a number of books on the forms of representation of economic cycles (including a famous work by W. Baumol), we have never found Pareto mentioned among the pioneers who availed themselves of Fourier functions. Yet, Pareto himself points out: *i*) that he is expounding a dynamic model; *ii*) that this is not in contradiction with general equilibrium; *iii*) that he is not developing his system any further “because of a lack of statistical data, and not because of theoretical flaws” (*Cours*, § 698, footnote 2).

To get to the core of this note issue—which is uncertainty, or the uninsurable risk in the language of F. Knight—we recall that on a number of occasions, both in the *Cours* and in Pareto's correspondence, we come across a diagram where cyclical fluctuations are not cleansed by the trend. The diagram in question—see below—was close to Pareto's heart because it evidently highlighted his experience as manager of an ironworks over a number of years, when he had been counter-speculating (that is, speculating with an insurance oriented mind) to try to shield himself from the fluctuations of the international price of a metal. In commenting on this diagram Pareto stressed that the art of forecasting becomes extremely difficult in the proximity of the high and low turning points of economic cycles. This is a truth that in the last decade of the 1900s—the years of the so-called *new economy*—virtually became a common experience.

The core of our discussion on the difficulties in forecasting, as we were saying, is Knight's notion of uninsurable risk, or uncertainty: in other words, uncertainty is the cause of the first disagreement between Pareto and Walras (E.N. 26). We have already pointed out, in E.N. 26, that—after distinguishing between short-term forecasts and easily rectifiable errors on the one hand, and long-term forecasts and virtually un-rectifiable errors on the other—this core brings Pareto and Keynes together.

Now we would like to draw attention to a more specific aspect that is shared by Pareto and Marshall-Keynes: we are referring here to the phenomenon of self-fulfilling forecasts, which fulfill themselves when the few end up prevailing—gradually at first, and then precipitously—over the many, who, in turn, end up becoming the few (for instance, the



bears who end up overcoming the bulls, who also turn into bears). It is precisely this subjective factor, involving the psychology of individuals as well as "crowd" psychology (it should be recalled here that, as a sociologist, Pareto also dealt with the psychology of the crowd), that for Pareto is the main cause of the intensity of crises, if not of the "crises" themselves.

If we extend our comparison in a deeper and more detailed fashion, we find that in Pareto, as in Keynes, the first and the only cause of uncertainty in long-term entrepreneurial decisions lies in the time between the planning of investment and the moment when the investment starts to yield fruits in terms of a return valued in money. One should read again all the passages regarding pursuit curves (E.N. 26) and the passage that in §76 of the present chapter contains the following lapidary conclusion: "To produce commodities takes time, and often considerable time prior to consumption. For production to be perfectly adjusted to consumption, one should: (1) Be able to forecast consumption. (2) Be able to forecast the outcome of the productive process exactly. Neither the one nor the other can be done with any certainty".

Many are the aspects of Pareto's reflection on "crises" that deserve some comment. We cannot abstain from making two considerations. 1) If Marshall gave so much importance in 1879 (in *The economics of industry*) to "business cycles", it is because, like Pareto, who had to live through those rough years as a firm manager, Marshall too found himself having to live through a "great crisis" as a professor of economics. That is, both Pareto and Marshall lived through, what Schumpeter would later call the ebb phase of a "long wave", or of a "Kondratieff cycle". 2) Whoever reads Pareto's essay "Alcune relazioni tra lo stato sociale e le variazioni della prosperità economica" (*Rivista Italiana di Sociologia*, Sept.–Dec., 1913) will find in it all the arguments of N. Kondratieff's 1926 famous essay ("Die langen wellen"), as well as an impressive combination of economic, financial, and sociological topics. The adjective *fondamentale* used by L. Einaudi in *La Riforma Sociale* (Dec. 1913, p. 852, footnote) as soon as he received Pareto's essay was not an overstatement. As far as we are concerned, while sensing a strong lack of generosity in Pareto regarding his intellectual debts to Cournot, we nevertheless find it unfair that Pareto's name is not inserted before Kondratieff's when long waves are considered.

E.N. 57. ON PARETO THE (DELUDED) ENLIGHTENMENT FOLLOWER AND ON PARETO THE (DISILLUSIONED) ANTI-ENLIGHTENMENT THINKER

When discussing Pareto and the Enlightenment, it is customary to recall the memories of his early belief, which he had confessed to A. Antonucci in 1907. I was 20 years old—says Pareto—and able to reason. But my inclinations led me to a simplistic view:

"In politics, the sovereignty of the people was an axiom, freedom a universal panacea. History showed us on the one side the people—good, honest, intelligent—oppressed by the higher classes, which were characterized by superstition. Militarism and religion were the worst scourges of mankind. Cesar among the ancient, Napoleon I and Napoleon III among the modern, were for me kinds of criminals. I used to deny, or at least excuse, the evils of democracy. The Terror was a slight mark in the glowing picture of the French revolution." (in V.P., *Epistolario*, Vol. I, p. 613)

With subtle self-irony, Pareto goes on:

"In Italy, increasing taxes were only due to the evil works of a corrupt and self-centred clique. If democracy had won, if we could have had the republic, the taxes would have been greatly reduced and would have almost entirely disappeared, because democracy is synonymous of freedom, and freedom does not require expenses by the State." (ib., pp. 613-14)

However, Pareto then adds that this early ardor soon cooled: the naive liberalism of his youth, which we could call radical-libertarian, gave way to a kind of liberalism that we find difficult to define, whether conservative or aristocratic elitist. At any rate, it was a liberalism that was closer to the British model, because it was based on the concept of individual responsibility and on the belief in the good that can be derived from joining "the greatest welfare for the greatest number" with the protection of the deserving ones in the various social classes.

An assessment of how much of this early naive enlightenment survived in what we could call *the younger Pareto*—roughly including the last 15 years of the 1800s—to distinguish it from *the older Pareto*, the disillusioned one, the pessimist, the anti-Enlightenment thinker of the 1900s, goes beyond the scope of this E.N.

What kind of enlightenment remained with the *younger Pareto*? It seems to us that there is a certain degree of vagueness in the terms *Enlightenment follower* and *anti-Enlightenment thinker* that are so often respectively related to the earlier and the later Pareto as intended here. It is indeed likely that the meanings of "Enlightenment follower" are more numerous than the 20 interpretations of Machiavelli to which Berlin decided to add a 21st. Putting our trust in an historian of ideas as admirable as Berlin, for our part we think that the Enlightenment of the earlier Pareto fits a definition that we read in "The Decline of Utopian Ideas in the West" (1978), today in I. Berlin, *The Crooked Timber of Humanity: Chapters in the History of Ideas*, (edited by H. Hardy, London, John Murray Ltd, 1990), p. 34:

"Failure, unavoidable or deliberate, to observe what there is in the world has robbed man of the knowledge needed to improve his life. Scientific knowledge alone can save us. This is the fundamental doctrine of the French Enlightenment, a great liberating movement which in its day eliminated a great deal of cruelty, superstition, injustice and obscurantism."

In the *older Pareto* even this residue of Enlightenment appears to be obscured. Suffice it to give one example. For the later Pareto anything experimentally scientific that can be said about protectionism and anti-protectionism has very little relevance in the decisions by the political classes in power. Essentially, the deciding forces in this contest are the dominant interests and their chances of persuasion through the press.

Born in Paris in the momentous year that was 1848 from a self-exiled Italian patriot, who presumably absorbed and instilled in his son the shrewd concretism of Cavour and the religious-like sense of duty of Mazzini, Vilfredo Pareto, at the turn of the 20th century, developed a dark pessimism of which a testimony was left to us by Manon Michels, the "small girl who has become his friend". It is worthwhile to copy here the words that Pareto, who by then was close to death, left in legacy to Manon. They are words that evoke a Millian vein of lay utilitarianism and are, in particular, reminiscent of Lucian (*Timor in*

orbe deos fecit) and Lucretius' protest against those who believe in the gods: "May you, when nearing the end of your life, consider false the words in Ecclesiastes which say that all is vanity and a striving after wind. It is the best wish that could be made for one who is just upon life" (M. Michels Einaudi, 1935, cit., p. 346; as is well known, Manon Michels, who was Robert Michels' daughter, would later marry one of Luigi Einaudi's sons).

AN EXPLANATION AND SOME REFERENCES. We were undecided whether to place the present *Note* at the beginning or at the end of the E.N.s. We opted for the end, but it is difficult to explain, for example, the self-criticism Pareto expresses in the *Preface* of the *Manuale*, which was eliminated in the *Manuel*, if one does not start from his early libertarian belief.

On the number of interpretations of Machiavelli, see "The Originality of Machiavelli", in I. Berlin *Against the Current: Essays in the History of Ideas* (edited by H. Hardy, London, Hogarth Press and Princeton University Press, 1979). As for Pareto's letter to Antonucci, S.B. Galli also recently dwelled upon it in a paper delivered at a conference (*Pareto e il Valdarno*).

E.N. 58. LAUSANNE AND CAMBRIDGE: SECOND-GENERATION LEADERS

Ch. 1, §27, Ch. 2 §§3–19 and §§97–112; Ch. III, §§98–101; Ch. VII, §§11–31; and Fr. App. §127. As Pareto was the second generation leader of the Lausanne School, it is perhaps useful to reflect on some similarities and differences between his work and that of A. C. Pigou, the second generation leader of the Cambridge School after Alfred Marshall. Such a contrast is pertinent for many reasons, but the most prominent is the fact that both scholars made seminal contributions to welfare economics. Indeed, the phrase 'welfare economics' is often prefaced by the adjectives 'Paretian' or 'Pigouvian', depending on whether one is respectively emphasizing preference ordering or market failure.

1. Ch. 1, §27; Ch. 2 §§3–19 and §§97–112. *Pigou, ophelimity and economic welfare.* When Pigou succeeded Marshall to the chair in political economy at Cambridge University in 1908, he was only 30 years old. In that year, Pareto turned 60 years old, his *Manuale* had already been published in Italian and the *Manuel* was shortly to be published in French. The scholarly influences between the second generation leaders of the Lausanne and Cambridge schools were, however, to flow in one direction only: from Pareto to Pigou. This was so for two reasons. First, Pareto's interest in following the development of Cambridge economics published subsequent to the second edition of Marshall's *Principles* waned considerably, perhaps because his work had been ignored by Marshall. Second, Pareto's major works were not only written in languages that Pigou read, French and Italian, they were also published during the formative years of Pigou's university studies. By way of illustration, Pigou was admitted to Cambridge University as an undergraduate student in the very year that the first volume of Pareto's *Cours* was published. As Marshall was to become Pigou's mentor, it is perhaps not surprising that Pareto exerted little or no influence on Pigou in regard to the equilibrium framing of

economic studies. He did, however, come to exert some influence on Pigou in matters concerning the relationship between ophelimity, utility and welfare.

Ch. 1, §27 of the *Manuale* makes reference to *homo economicus*, *homo ethicus* and *homo religiousus*. These labels are logical extensions of the discussion in the *Cours* concerning the distinction between ophelimity and utility and the various categories of utility and ophelimity:

“One can distinguish different types of utility according to the diverse aspects of human nature which assure development and progress. Economic utility would be that which assures material well-being, moral utility that which would produce the development of more perfect morals etc. Similar divisions can be adopted for ophelimity; as it may satisfy material, moral or religious, needs and desires etc, will be called economic, moral or religious ophelimity etc.” (Pareto 1896–97, *Cours*, §14)

This implies that ethical and religious action could be studied using the same approach that economists adopt when theorizing about logical economic action. Just as economists had developed pure theory by considering the reaction of *homo economicus* to the force of *economic ophelimity*, so too could philosophers study the reaction of *homo ethicus* to the force of *moral ophelimity*, and theologians could study the reaction of *homo religiousus* to the force of *religious ophelimity*. However, Pareto’s concern with the non-logical aspect of human behavior—as discussed in Ch. II §§3–19 and §§97–112 of the *Manuale*, and elsewhere—would eventually lead him to abandon hope of developing rational actor models of ethical and religious behavior based on *homo ethicus* and *homo religiosus* as such models were unlikely to contribute to experimental social theory. In the *Trattato di Sociologia Generale*, Pareto focused on ‘residues’ (observable indications of sentiments that may motivate non-logical action) and ‘derivations’ (quasi-logical rationalizations, that is, false representations of the reality, that may be used to justify past actions or encourage particular future actions) when reflecting on whether, in sociology,

“we might not relegate the non-logical element to the residues . . . and proceed to examine the logical conduct that originates in the residues. That, indeed, would yield a science similar to pure, or even to applied, economics. But unfortunately the similarity ceases when we come to the question of correspondences with reality... Residues are not, like tastes, merely sources of conduct; they function throughout the whole course of the conduct developing from the source, a fact which becomes apparent in the substitution of derivations for logical reasonings.” (Pareto 1916 [1935] §2079)

In general, then, Pareto came to use the term ophelimity without any adjectives; becoming simply ‘ophelimity’ and being largely applied to the study of logical economic action. However, he did not limit non-logical action to the interdependence between residues and derivations and, indeed, his notion of non-logical action provides useful context to Pareto’s economic theory. For example, non-logical actions by individuals can be said to play a significant role in illustrating the equilibration process under conditions of free competition (see E.N. 14).

After commencing his studies of sociology, the diverse range of adjectives that preceded utility were also replaced, in Pareto’s major 1913 sociological study of collective welfare (see E.N. 31), with the single adjective ‘social’. This was appropriate because Pareto’s notion of ‘social utility’ reflects the fact that an individual’s preferences for ethical and moral outcomes are not limited to one’s own actions, but extend to the actions

of others. When human behavior is revealed to reflect non-logical influences, ethical and religious influences, as well as the economic influence of actions undertaken with regard to the welfare of others, were considered in Pareto's mature works with reference to 'social utility'. Consequently, the analytical dichotomy between ophelimity and utility (or social utility in Pareto's more mature studies), and the associated dichotomy between logical action and non-logical action, is of fundamental importance when considering the relationship between Pareto's pure economics and his sociological theory.

Aspects of the above distinctions, and associated dichotomies, resonated with the young Pigou. As a student of history and the moral sciences at Cambridge, he devoted much time to the study of ethics. As he turned his attention to the study of economics, he did so under the influence of mid to late Victorian era philosophers who had taken an interest in economic matters, such as Henry Sidgwick and T. H. Green. Drawing on the ideas of those philosophers, Pigou's first remarks on utility published in 1903 stressed that psychological hedonism was an untenable doctrine because people desire more than just pleasure. Shortly afterwards Pigou read the *Cours* and *Manuale*. In his 1910 article *Producers' and Consumers' Surplus* (*Economic Journal*, vol. 20(79), pp. 258–370), Pigou reported that Pareto's notion of ophelimity "is free from certain ambiguities involved in the common English term utility" (Pigou 1910, p. 359) and he outlined, for the first time, the formal basis for his analysis of externalities related to supply and demand—all of which was written in reference to 'ophelimity', not utility.

In subsequent publications Pigou reverted to the more traditional term 'utility'. He did, however, retain the distinction between 'welfare' and 'economic welfare' that he first introduced in *Wealth and Welfare* (1912 London: Macmillan), which is a distinction that has some similarity to Pareto's contrast between ophelimity and social utility; and to the consequent differentiation between Pareto's analysis of collective maximization in economics (based on ophelimity) and his analysis of collective maximization in sociology (based on social utility). Nevertheless, it should also be acknowledged that Pigou's dichotomy between 'welfare' and 'economic welfare' does not precisely match Pareto's differentiation between the economic approach to collective welfare, undertaken in reference to ophelimity, and the sociological approach to collective welfare, undertaken with reference to social utility. Specifically, Pigou provided for a broader range of economic activities to remain within the scope of his economic analysis of welfare than Pareto did. The contrast between these two theorists on collective welfare is considered further in "Pareto, Pigou and Third-party Consumption" (M. McLure, 2010. *European Journal of the History of Economic Thought*, 17(4), pp. 635–657).

2. Ch. VII, §§11–31. Pareto's law, income inequality, and economic welfare. While Pareto's distinction between ophelimity and utility was a positive influence on Pigou, Pareto's work on income distribution constituted a significant obstacle to Pigou developing welfare economics as a subject that grew out of the moral sciences tradition. The scope of Pigou's economics of welfare presented in *Wealth and Welfare* provided for the possibility of economic welfare being enhanced by both an increase in national income and an increase in the absolute share of national income accruing to the poor. As such, redistribution of income to the poor was a subject that fell within the scope of Pigou's welfare economics. In contrast, the scope of Pareto's economics of welfare was largely limited to economic efficiency. Of course, Pareto did consider redistribution in his eco-

nomic works, but he limited the scope of his enquiry to the issue of how governments can achieve their redistributive goals in an efficient manner. He did not consider the extent of income redistribution to be a question for pure economics, rather, he considered it a question for sociology. Consequently, matters that concern the ‘ministry of production’ in a collectivist economy could fall within the scope of Pareto’s economic approach to collective welfare; whereas distributive matters that concern the ‘ministry of justice’ were considered as part of his sociological approach to collective welfare. As the ministries of ‘production’ and ‘justice’ are two parts of the one government, Pareto effectively presented the economics and the sociology of redistribution as complementary perspectives on collective welfare.

But the main obstacle in Pareto’s work to Pigou did not concern the exclusion of the extent of redistribution from economics. Pigou simply ignored that aspect of Pareto’s work. Rather, the main obstacle was ‘Pareto’s law’. In its reduced double logarithmic form, the equation for the Pareto distribution of income is given by $\log N = \log A - a \log x$, where: x is an arbitrary level of income; N_x is the number of people with an income of at least x ; and A and a are estimated parameters. Pareto’s index of equality/inequality, as presented in the *Cours* and reproduced in the footnote to §24, Ch. VII of the *Manuel*, is defined as the ratio of N_x (the number of people accruing an income of at least x) to N_h (the number of people accruing the minimum income h). When the index rises, a reduction in the inequality in income distribution is indicated. The equation for the Pareto distribution provides the means for estimating both the numerator, N_x , and the denominator of that ratio, N_h , for inclusion in the index of equality/inequality.

Pareto investigated the properties of this index in the *Cours* and concluded that real per capita economic growth is necessary for an increase in the minimum income; and/or a decline in inequality of income distribution. Chapter VII §29 of the *Manuale* reproduces that conclusion, although Pareto did not reproduce the workings detailed in the *Cours* from which that conclusion was derived. Pigou, and others, subsequently referred to that finding as ‘Pareto’s law’.

One of Pigou’s goals in welfare economics was to consider when his efficiency goal, considered with reference to changes in national income, was in ‘harmony’, and when it was in ‘disharmony’, with his goal of distributive fairness, considered with reference to changes in the absolute share of income accruing to the poor. Pigou’s framing of welfare economics was predicated on his two goals often being in harmony, but not always. When efficiency and distributive fairness were not in harmony, he did not wish to exclude the possibility of the welfare gain from redistribution exceeding the welfare loss from a reduction in national income. This would result when the poor’s economic welfare gain from redistribution exceeds the reduction in the rich’s economic welfare from the decline in their income. The problem for Pigou was that Pareto’s law implied that an ‘absolute increase’ in the share of income accruing to the poor was only possible when *per capita* national income is increasing. To demonstrate the legitimacy of his multi-objective trade-off framing of the theory of economic welfare, Pigou dedicated one chapter in *Wealth and Welfare* (1912 London: Macmillan) and one chapter in each of the editions of *The Economics of Welfare* (1920, 1924, 1929, 1932 London: Macmillan), to rejecting Pareto’s law.

Pigou's analysis, however, was confused and seriously flawed. For example, Pigou maintained that Pareto derived his law of income distribution on the presumption that the coefficient α , which measures the slope of the double log Pareto distribution, is fixed across the range of all incomes in all circumstance. Pareto, however, explicitly considered two cases: the case when α is constant; and the case when α is variable when deriving his law. Pigou also incorrectly associated the Pareto distribution with standard deviation (Allyn Young, 1913, 'Wealth and Welfare by A. C. Pigou', *Quarterly Journal of Economics*, 27(4), pp. 672–86). These issues are dealt with more fully in "A.C. Pigou's rejection of Pareto's law" (M. McLure, 2013, *Cambridge Journal of Economics*, 37(4), pp. 775–78). For examples of the general reaction to Pareto's law, see E.N. 37.

3. Ch. III, §§98–101. The Marginal Theory of Distribution. Having rejected Pareto's law, Pigou immediately raised the possibility of using distribution theory to consider questions of economic efficiency. The final paragraph of the chapter entitled 'Pareto's law' restates the possibility of 'disharmony' between efficiency and distributive fairness, but then diminishes that possibility by shifting his attention back to harmony and stating that "we find ourselves confronted with the broad problem of distribution" (Pigou, 1912, *Wealth and Welfare*, p. 77). Pigou then went on to consider the 'laws of distribution' in an entirely marginalist manner, with the 'law of diminishing returns to individual factors of production' given a prominent role in efficiency considerations. This marks another difference with Pareto, whose concerns with the marginal productivity theory of distribution had been clearly outlined in the *Cours* and the *Manuale*. In Pareto's assessment, using the marginal theory of production to derive the coefficients of production assumes that all inputs are variable and independent, but he regarded them as neither necessarily variable, as production involves a combination of fixed and variable factors, nor necessarily independent. See also E.N. 27.

John Chipman has suggested that Pareto's position on this matter reflects his view that the theory of production should not be limited to the case of constant returns to scale (2002 'Pareto: Manuel di d'Économie Politique' in *Dictionnaire des grandes œuvres d'économie*, X. Greffe, J. Lallement and M. De Vroey (eds), Paris: Dalloz, pp. 424–433). In Ch. III, §§98–101 of the *Manuale*, Pareto accommodated non-constant returns by introducing marginal and non-marginal equilibria (see E.N. 16) and, in a related analysis, distinguished between 'complete competition', associated with decreasing unit costs, and 'incomplete competition', associated with increasing unit costs (see E.N. 17).

Pigou too did not wish to limit his treatment of economics to constant returns. To accommodate non-constant returns in *Wealth and Welfare* he calculated the supply price with reference to changes in the total supply of the industry, which, as Allyn Young pointed out, in his 1913 review of that book, has the effect of attributing rents that firms realize from increasing returns to the marginal net product of resource. The important point here is that Pareto imposed constraints on marginal product theory and provided some indication of how cases of non-constant returns to scale could be dealt with, but that work was largely ignored by Pigou. Instead, Pigou exploited the marginal theory of distribution beyond that which was considered legitimate by Pareto, although, in the process, he succeeded in formally presenting the concept of externalities, which is, of course, one of the most important concepts in welfare economics.

4. Fr. App. §127. It is perhaps appropriate to end this note by highlighting a common and fundamental position shared by both Pareto and Pigou on the question of economic welfare. In regard to the efficiency dimension of economic welfare, both Pareto and Pigou accommodated a 'material' interpretation of welfare. In §127 of the mathematical appendix to the *Manuel* (1909) Pareto pointed to the equivalence between his theoretical representation of the first law of welfare economics and the economic meaning of that law, which he expressed as the surplus that may be generated for the collective as a whole when moving from a non-optimal state to the point that would have prevailed under free competition. The objective indicator of that surplus is represented by a numeraire, which provides a material basis for indicating changes in aggregate economic output, although it should also be stressed that Pareto's equations highlight the economic wellbeing of individuals by transforming the objective indicator of the part of the surplus accruing to each individual member of the collective into individual subjective assessments of changes in economic welfare. Maurice Allais characterized Pareto's approach as the 'equivalent surplus' approach to collective economic welfare (see also E.N. 19 'A further remark on Pareto and some references').

Pigou, while initially adopting the non-Paretian presumption that ophelimity can be directly represented by the measuring rode of ophelimity, similary considered the efficiency goal for economic welfare with reference to changes in the national income, or the 'national dividend' in Pigou's terminology. Robert Cooter and Peter Rappoport ('Were the Ordinalists Wrong About Welfare Economics', *Journal of Economic literature*, 1984, vol. 22, pp. 507–530) have made clear that Pigou equated economic wellbeing with the 'material' notion of welfare, although there has been some debate on the issue (Ian M. D. Little *Journal of Economic literature*, 1985, vol. 23, pp. 1186–1188; Cooter and Rappoport, *Journal of Economic literature*, 1985, vol. 23, pp. 1189–1191).

M. McLure

Notes to the French Appendix

[¹] This is the translation of the mathematical *Appendix* that in the *Manuel* goes from p. 539 to p. 671. As specified in the *Editors' Introductory Note* (§2.2), the editors' notes to the French mathematical *Appendix* show a separate numbering sequence compared to the other editors' notes and, wherever necessary, they will be referred to by *F. Ap. N.* The drafter of these notes would like to thank E. Castagnoli, M. McLure, I. Moscati and A. Zanni for their valuable suggestions.

[²] It is not clear from this paragraph whether Pareto considered direct empirical data could reveal: the indifference curves (or indifference varieties) represented by equation (5); or the marginal rates of substitution, as explicitly stated in §5; or both. Until it also becomes necessary to introduce indifference varieties from note [13] onwards, it is assumed in these notes that the direct empirical data is represented by the marginal rates of substitution. That is, by the functions indicated by symbols b_y, c_z, \dots, n_t in equation (12). In this way Pareto implicitly assumes local rationality, because he assumes that the ratio between two marginal rates of substitution, both with regard to the same good, is equal to the marginal rate of substitution of the other two goods involved—that is, b_y/c_z is the marginal rate of substitution between goods Y and Z, where b_y and c_z are the marginal rates of substitution between goods Y and X and between goods Z and X. (This condition is quite similar to the no arbitrage condition adopted in the case of exchange ratios.)

In order to follow Pareto's line of reasoning, it is useful to recall the theory of fields because the analogy between Pareto's description of consumption and the description of a force field is self-evident. There exists a scalar field if a number I is associated with each point (x, y, \dots, t) of the space under consideration. There exists a vector field if a vector (A_x, B_y, \dots, N_t) is associated with each point of the space under consideration, as indicated in §§6 and 13. The empirical data is represented by the ratios $b_y = \frac{B_y}{A_x}, c_z = \frac{C_z}{A_x}, \dots, n_t = \frac{N_t}{A_x}$, which are the marginal rates of substitution of the goods Y, Z, ..., T with respect to good X. Therefore, the vector field (A_x, B_y, \dots, N_t) is defined except for an arbitrary factor, that is, it is empirically indistinguishable from the vector field $(HA_x, HB_y, \dots, HN_t)$, where H is an arbitrary positive function (it must be positive if it is to have an economic meaning). The vector field is conservative if there is a scalar field of which the vector field is the gradient: in the case under consideration, if there is a function $\Psi(x, y, \dots, t)$ such that $A_x = \frac{\partial \Psi}{\partial x}, B_y = \frac{\partial \Psi}{\partial y}, \dots, N_t = \frac{\partial \Psi}{\partial t}$. Since the vector field (A_x, B_y, \dots, N_t) is defined except for an arbitrary factor, if it is conservative it admits a multiplicity of scalar fields, that is, every strictly increasing transformation $F(\Psi(x, y, \dots, t))$ defines, through the ratios between its partial derivatives, the same marginal rates of substitution defined by the function $\Psi(x, y, \dots, t)$. A vector field is conservative if and only if it satisfies the so-called integrability conditions. In the analysis

under consideration, since the vector field is defined except for an arbitrary factor, the integrability conditions only exist when the number of goods is greater than two.

Pareto calls the function $F(\Psi)$, which is defined at least for a strictly increasing transformation, the ophelimity index function. If the field is conservative, the line integrals of the vector field only depend on the starting and ending points of the path followed for the integration, and therefore any integral on a closed curve is equal to zero. If the field is not conservative, the line integrals also depend on the path followed for the integration. Pareto calls the integration path “order of consumption”. When, in §13, he writes “the order of consumption does not affect the consumption choice”, he means that the line integral does not depend on the path, but only on the starting and ending points; the field is therefore conservative and the differential equation (12) can be integrated and its integral is the scalar field represented by the ophelimity index function. When Pareto writes “the order of consumption affects the consumption choice”, he means that the equation (12) cannot be integrated and there is the “non-closed cycle”. (The same interpretation is given by G. J. Stigler, 1950, “The Development of Utility Theory”, II, *Journal of Political Economy*, 44, pp. 373–396, on pp. 380–381, and by G. Ricci, 1951, “Commento alla memoria di G. B. Antonelli dell’anno 1886: ‘Sulla teoria matematica della Economia Politica’”, *G.d.E.*, N.S. 10, pp. 264–297 and 345–385, on p. 375.) However, Pareto never wrote out the integrability conditions. These conditions—which must be satisfied by the functions expressing the marginal rates of substitution in order for the vector field they define to be conservative—had already been introduced to economics by G. B. Antonelli, *Sulla teoria matematica della economia politica*, Pisa, Tipografia del Folchetto, 1886. (Engl. Trans. “On the Mathematical Theory of Political Economy”, in J. S. Chipman, L. Hurwicz, M. K. Richter and H. F. Sonnenschein, eds., *Preferences, Utility, and Demand*, New York, Harcourt Brace Jovanovich, 1971, pp. 333–364.) With the symbols that are being used here, they require $\frac{\partial b_y}{\partial z} - c_z \frac{\partial b_y}{\partial x} = \frac{\partial c_z}{\partial y} - b_y \frac{\partial c_z}{\partial x}$, etc. The presence of the “non closed [open] cycle” illustrates the case of a consumer that is locally rational (that is, with marginal rates of substitution that are consistent with each other, as already indicated) but globally irrational. Indeed, if one were to associate the integral with utility, the open cycle implies that utility may increase with a sequence of changes in the quantities of the goods, at the end of which the consumer finds himself with the same quantities he initially had. (This irrationality is of the same kind as the one that is currently indicated by the money pump argument, which can be introduced if preferences are not globally transitive.)

^[3] Provided it is injective, that is, such that $F(\Psi) \neq F(\Phi)$ if $\Psi \neq \Phi$. Immediately afterwards, Pareto also requires it to be derivable (by introducing the derivative F') and strictly increasing (by requiring for it to be $F' > 0$).

^[4] In this §4 Pareto introduces the notion of ordinal ophelimity by indicating that ophelimity can be represented through a multiplicity of index functions, connected with each other by strictly increasing functions.

^[5] Due to an oversight or a misprint, the original text reads “equation (6)” instead of “equation (5)”.

^[6] Here Pareto introduces the notion known today as “marginal rate of substitution”.

^[7] In these last two equations Pareto uses symbols that are not consistent with those he adopts later. These relations should be written in the same way as equations (12) and (12 bis).

[8] Pareto writes that the integration factor exists because he assumes that the equation (9) is similar to the equation (7) or to the (8), which have been obtained by differentiating the equation (5). Otherwise, what Pareto indicates in §12 should apply.

[9] The purpose of the digression is to illustrate the connection between the facts of experience (that is, the functions that express the marginal rates of substitution and the indifference varieties) on the one hand, and elementary ophelimities (marginal utilities), the ophelimity index function, and total ophelimity itself, on the other. Pareto's purpose is the measuring of the non-observable elementary ophelimities, measuring that he pursues both in the case of 'closed' cycles under which the integrability conditions are satisfied, and in the opposite case of 'open' cycles. In the latter case the order of consumption, that is, the integration path, is assigned. On this matter, see J. S. Chipman, "Introduction to Part II", in J. S. Chipman, L. Hurwicz, M. K. Richter and H. F. Sonnenschein, eds., *Pref-ferences, Utility, and Demand*, New York, Harcourt Brace Jovanovich, 1971, pp. 321–331.

[10] Pareto introduces an identification condition for elementary ophelimities for the case in which the vector field, as defined by the marginal rates of substitution, is conservative. He demonstrates that if each elementary ophelimitiy is only a function of the good it refers to (and therefore total ophelimity is an additively separable function), it is then possible to measure the elementary ophelimities except for a constant (that is, except for the unit of measurement). Pareto's line of reasoning, which he also employs in §18 of the appendix) is the following. The function of ophelimity is one of the index functions that can be obtained by integrating the vector field. Therefore, there exists an integral Ψ that is an additively separable function, so that every partial derivative of Ψ is a function of only one variable, specifically $\Psi_x(x)$, $\Psi_y(y)$, . . . The ophelimity function is an increasing monotonic transformation of this, that is, there exists an increasing monotonic function $F(\cdot)$ such that the ophelimity function is $F(\Psi)$. The elementary ophelimities are, then, $P_x = F'(\Psi)\Psi_x(x)$, $Q_y = F'(\Psi)\Psi_y(y)$, . . . Since the elementary ophelimity of the first good P_x is a function of x alone, that of the second good Q_y , a function of y alone, . . . and since $F(\Psi)$ is a function of the quantities of all the goods, then $F'(\Psi)$ is necessarily equal to a constant A . Therefore, the functions $\Psi_x(x)$, $\Psi_y(y)$, . . .—which can be inferred from experience by taking an additively separable integral function—are equal to the elementary ophelimities multiplied by a constant factor A . The identification condition outlined by Pareto is equivalent to the condition of independence, which means that the preference relationship between two baskets of goods does not depend on the goods that are contained in the same quantity in both baskets; its representation with an additively separable utility function has been formalized by G. Debreu in "Topological methods in cardinal utility theory" (in K. J. Arrow, S. Karlin and P. Suppes, eds., *Mathematical Methods in the Social Sciences*, Stanford University Press, 1960, pp. 16–26).

[11] The passage reported by Pareto is taken from Volterra's article and is also found in Pareto's reply, "Lofelimità nei cicli non chiusi", *G.d.E.*, July 1906, p. 15 (p.370 in the Engl. Trans. "Ophelimity in Nonclosed Cycles", in J. S. Chipman et al., *op.cit.*, pp. 370–385).

[12] What was indicated in note [7] applies here, too.

[13] Pareto analyses the case in which the vector field defined by the marginal rates of substitution is not conservative. For each point (x, y, \dots, t) of the space, he considers the line integral of the vector field along a certain path that starts from a given point (the point that represents zero consumption for all goods) and reaches the point under

consideration. In this way for each (x, y, \dots, t) he obtains a scalar, and researches the relations that exist between the value of this integral, the marginal rates of substitution, and the indifference varieties which are related to the integration path. Pareto's example of a consumption path starts from the point $(0, 0, \dots, 0)$, modifies the consumption of the various goods in sequence, that is, it goes from $(0, 0, \dots, 0)$ to $(x, 0, \dots, 0)$, then from $(x, 0, 0, \dots, 0)$ to $(x, y, 0, \dots, 0)$, and so on, until it reaches the point (x, y, \dots, t) . In the §15, Pareto calls this order of consumption "path (a)". He observes that the functions A_x, B_y, \dots, N_t are known except for an arbitrary function (since the functions A_x, B_y, \dots, N_t and the functions HA_x, HB_y, \dots, HN_t , where H is an arbitrary positive function, represent the same marginal rates of substitution); he does not explicitly note that this function affects the relation of indifference, though he appears to take it into account. He will determine the function H that makes the integral of the vector field compatible with the indifference varieties related to the path. In Pareto's judgment, the indifference varieties expressed by the function (5) are, like the marginal rates of substitution, also given by experience. It should be noted that the marginal rates of substitution (that is, the functions $\frac{B_y}{A_x}, \dots, \frac{N_t}{A_x}$) are independent of the path, whereas, if the field defined by the marginal rates of substitution is not conservative, the indifference varieties depend on the integration path and require specific information in addition to the marginal rates of substitution. Indeed, the arbitrary function H affects the indifference relation, in the sense that with the functions A_x, B_y, \dots, N_t , the value of the integral is the same in two points, that will not necessarily be the case when using HA_x, HB_y, \dots, HN_t , where H is the arbitrary function.

The following example just shows how the indifference varieties cannot be univocally inferred from the marginal rates of substitution when the field they define is not conservative, even if the path is given. Let $A_x = 2 + 2x + \beta y$, $B_y = 1 + x$ and $C_z = \frac{(1+x)(1+x+y)}{1+z}$, where β is a constant parameter. The marginal rates of substitution $\frac{B_y}{A_x}$ and $\frac{C_z}{A_x}$ will define a conservative vector field if and only if $\beta = 1$. The same marginal rates of substitution are obtained from the ratios of the functions $A'_x = (2 + 2x + \beta y)(1 + z)$, $B'_y = (1 + x)(1 + z)$ and $C'_z = (1 + x)(1 + x + y)$. In the former case, the integral along the path which has been described above as path (a) is

$$\begin{aligned} I(x, y, z) &= \int_0^x A_x(x, 0, 0) dx + \int_0^y B_y(x, y, 0) dy + \int_0^z C_z(x, y, z) dz \\ &= \int_0^x (2 + 2x) dx + \int_0^y (1 + x) dy + \int_0^z \frac{(1+x)(1+x+y)}{1+z} dz \\ &= -1 + (1+x)(1+x+y)(1+\ln(1+z)); \end{aligned}$$

in the latter case, it is

$$\begin{aligned} I'(x, y, z) &= \int_0^x A'_x(x, 0, 0) dx + \int_0^y B'_y(x, y, 0) dy + \int_0^z C'_z(x, y, z) dz \\ &= \int_0^x (2 + 2x) dx + \int_0^y (1 + x) dy + \int_0^z (1 + x)(1 + x + y) dz \\ &= -1 + (1+x)(1+x+y)(1+z). \end{aligned}$$

The two points $(1, 1, 0)$ and $(0, 2, e - 1)$ should be situated on the same indifference surface according to the integral I , because $I(1, 1, 0) = 5 = I(0, 2, e - 1)$, but not according to the integral I' , because $I'(1, 1, 0) = 5 \neq 3e - 1 = I'(0, 2, e - 1)$. On the contrary, if $\beta = 1$ (in which case the vector field is conservative), the indifference surface is univocally determined by the integration, by choosing the arbitrary function H in such a way that the functions $A_x H, B_y H, C_z H$ satisfy the conditions $\frac{\partial H A_x}{\partial y} = \frac{\partial H B_y}{\partial x}, \frac{\partial H A_x}{\partial z} = \frac{\partial H C_z}{\partial x}$ and $\frac{\partial H B_y}{\partial z} = \frac{\partial H C_z}{\partial y}$ (which cannot be satisfied if $\beta \neq 1$). A function H that satisfies these equalities when $\beta = 1$ is $H = 1 + z$. In that case, we obtain the functions $H A_x = A'_x = (2 + 2x + y)(1 + z)$, $H B_y = B'_y = (1 + x)(1 + z)$, $H C_z = C'_z = (1 + x)(1 + x + y)$, and, therefore, the function $\Psi = -1 + (1 + x)(1 + x + y)(1 + z)$. When $\beta \neq 1$ the vector field is not conservative and the two functions given by experience, namely the marginal rates of substitution defined by the ratios of the functions A_x, B_y, C_z , and the indifference surfaces defined by the function $\Psi(x, y, z)$, are not independent of each other even though the indifference surfaces are not inferable from the marginal rates of substitution. The functions $A_x(x, y, z), B_y(x, y, z), C_z(x, y, z), \Psi(x, y, z), H(x, y, z)$ and $F(\cdot)$ are connected by the relation

$$\begin{aligned} F(\Psi(x, y, z)) &= I(x, y, z) = \\ &= \int_0^x H(x, 0, 0) A_x(x, 0, 0) dx + \int_0^y H(x, y, 0) B_y(x, y, 0) dy + \int_0^z H(x, y, z) C_z(x, y, z) dz. \end{aligned}$$

Putting $H = 1 + z$ and $F(\cdot)$ as the identity function, this condition is satisfied by the functions

$$\Psi(x, y, z) = -1 + (1 + x)(1 + x + y)(1 + z),$$

and

$$A_x = 2 + 2x + \beta y, \quad B_y = 1 + x, \quad C_z = \frac{(1 + x)(1 + x + y)}{1 + z}.$$

It should be noted that the partial derivatives of the integral, that is

$$\frac{\partial I}{\partial x} = (2 + 2x + y)(1 + z), \quad \frac{\partial I}{\partial y} = (1 + x)(1 + z), \quad \frac{\partial I}{\partial z} = (1 + x)(1 + x + y),$$

differ from the functions

$$H A_x = (2 + 2x + \beta y)(1 + z), \quad H B_y = (1 + x)(1 + z), \quad H C_z = (1 + x)(1 + x + y),$$

since it is $\beta \neq 1$. However, they coincide with the latter along the integration path, since $\left. \frac{\partial I}{\partial x} \right|_{y=z=0} = 2(1 + x) = H(x, 0, 0) A_x(x, 0, 0)$, $\left. \frac{\partial I}{\partial y} \right|_{z=0} = 1 + x = H(x, y, 0) B_y(x, y, 0)$ and $\left. \frac{\partial I}{\partial z} \right|_{y=0} = (1 + x)(1 + x + y) = H(x, y, z) C_z(x, y, z)$.

[14] In line with what has been pointed out in note [13], it is not quite true that, "if the order of consumption influences the choice", then the differential equation (13) is equivalent to an equation of the same type as (5). Indeed, without additional information,

it is not possible to determine an equation of the same type as (5) from the differential equation (13)

[15] The equations (17) are missing the factor $H(x, y, \dots, t)$. It is true, as Pareto writes, that “the functions G may be understood to include this factor”, but this does not imply that this factor can be excluded from the relations given by equations (17). Equations (17) can be arrived at (including in them the factor H) by considering the line integral

$$\begin{aligned} I(x, y, \dots, t) = & \int_0^x H(x, 0, \dots, 0) A_x(x, 0, \dots, 0) dx + \\ & + \int_0^y H(x, y, 0, \dots, 0) B_y(x, y, 0, \dots, 0) dy + \dots + \int_0^t H(x, y, \dots, t) N_t(x, y, \dots, t) dt. \end{aligned}$$

The partial derivatives of the function $I(x, y, \dots, t)$ are

$$\begin{aligned} \frac{\partial I}{\partial x} &= H(x, 0, \dots, 0) A_x(x, 0, \dots, 0) + \int_0^y \frac{\partial H(x, y, \dots, 0)}{\partial x} B_y(x, y, \dots, 0) dy + \dots \\ \frac{\partial I}{\partial y} &= H(x, y, 0, \dots, 0) B_y(x, y, 0, \dots, 0) + \int_0^z \frac{\partial H(x, y, z, \dots, 0)}{\partial y} C_z(x, y, z, \dots, 0) dz + \dots \\ &\dots\dots\dots \\ \frac{\partial I}{\partial t} &= H(x, y, \dots, t) N_t(x, y, \dots, t). \end{aligned}$$

Thus, since along the integration path $I = F(\Psi(x, y, \dots, t))$, we have

$$\begin{aligned} \left. \frac{\partial I(x, y, \dots, t)}{\partial x} \right|_{y=\dots=t=0} &= \Psi_x(x, 0, \dots, 0) F'(\Psi(x, 0, \dots, 0)) = H(x, 0, \dots, 0) A_x(x, 0, \dots, 0) \\ \left. \frac{\partial I(x, y, \dots, t)}{\partial y} \right|_{z=\dots=t=0} &= \Psi_y(x, y, \dots, 0) F'(\Psi(x, y, \dots, 0)) = H(x, y, \dots, 0) B_y(x, y, \dots, 0) \\ &\dots\dots\dots \\ \left. \frac{\partial I(x, y, \dots, t)}{\partial t} \right|_{\dots} &= \Psi_t(x, y, \dots, t) F'(\Psi(x, y, \dots, t)) = H(x, y, \dots, t) N_t(x, y, \dots, t). \end{aligned}$$

[16] To Pareto, the measurability of pleasure corresponds to the existence of elementary ophelimities $P_x, Q_y, \dots, S_s, T_t$. The integral of these functions along a path is the ophelimity on that path, as indicated by the symbol G (this symbol, by the way, is also used in equations (15), but with a different meaning). The functions A_x, B_y, \dots, N_t represent, by means of their ratios, the marginal rates of substitution, which are also equal to the corresponding ratios between the functions P_x, Q_y, \dots, T_t .

[17] As indicated in note [13], $\Psi_x F', \Psi_y F', \dots, \Psi_t F'$ are the partial derivatives of the function that determines the indifference varieties only with regard to path (a).

[18] Due to an oversight or a misprint, there is $T_s(x, \dots, s, 0)$ instead of $T_x(x, \dots, s, 0)$.

[19] Pareto finds that it is $H = 1$ not because “ F' , being arbitrary, may always be supposed to include H ”, but because he did not take into account the factor H in the equations (17) (as observed in note [15]).

[20] The condition “the terms $\chi_s, \chi_u, \dots, \chi_x$ are all zero” implies that the vector field defined by the elementary ophelimities P_x, Q_y, \dots, T_t is conservative: indeed, if $T_t = \Psi_t F'$ and $S_s = \Psi_s F'$, then $\frac{\partial T_t}{\partial s} = \Psi_{s,t} F' + \Psi_s \Psi_t F'' = \frac{\partial S_s}{\partial t}$, etc. Therefore, “the quantities P_x, Q_y, \dots, T_t happen to be the partial derivatives of one and the same function”. Moreover, since the vector field defined by the elementary ophelimities P_x, Q_y, \dots, T_t , is conservative, the vector field defined by the marginal rates of substitution is also conservative—taking into account the relations of equation (19) and the function H that is represented there and which is precisely the integration factor of the vector field defined by the functions A_x, B_y, \dots, N_t . Thus, if “the terms $\chi_s, \chi_u, \dots, \chi_x$ are all zero”, consumption provides pleasure that is independent of the order of consumption, in the sense that the elementary ophelimities and total ophelimity at the point x, y, \dots, t do not depend on the path that led to this point. Therefore, in the “first category” there are no “commodities X, Y, \dots, T , whose consumption yields a pleasure that depends on the order of consumption [...]. Pareto examines a situation that is not that of the “first category” and compares it to the latter. The sense of Pareto’s argument can be interpreted in the light of what has been pointed out in note [13]. In other words, when the vector field defined by the functions (elementary ophelimities) P_x, Q_y, \dots, T_t is not conservative and the path is given, the integral of the vector field along this path is a function the partial derivatives of which differ from P_x, Q_y, \dots, T_t , even though they coincide with the latter along the integration path. The partial derivatives of the integral are “fictitious pleasures”, whereas P_x, Q_y, \dots, T_t are “real pleasures”.

Recalling the example introduced in note [13], let the marginal rates of substitution (given by experience) be represented by the ratios between the functions $A_x = 2 + 2x + \beta y, B_y = 1 + x$ and $C_z = \frac{(1+x)(1+x+y)}{1+z}$, and let the indifference surfaces (also given by experience) be represented, in the case of path (a), by the function $\Psi = -1 + (1+x)(1+x+y)(1+z)$. Let the pleasures (which are not given by experience) be represented by the elementary ophelimities $P_x = \frac{2+2x+\beta y}{(1+x)(1+x+y)}, Q_y = \frac{1}{1+x+y}$ and $R_z = \frac{1}{1+z}$ (with $\frac{Q_y}{P_x} = \frac{B_y}{A_x}$ and $\frac{R_z}{P_x} = \frac{C_z}{A_x}$ as required by the relations given by equation (19)). If $\beta = 1$, the vector field defined by these elementary ophelimities is conservative and the scalar field of which the vector field P_x, Q_y, R_z is the gradient is represented by the function $I = \ln(1+x) + \ln(1+x+y) + \ln(1+z)$. If $\beta \neq 1$, the vector field is not conservative and “consumption yields a pleasure that depends on the order of consumption”. In this case, the relations of equation (21) are $R_z = \Psi_z F', Q_y = \Psi_y F' + \chi_y$ and $P_x = \Psi_x F' + \chi_x$. The first of these relations requires $\frac{1}{1+z} = (1+x)(1+x+y)F'$, that is, $F'(\Psi) = \frac{1}{(1+x)(1+x+y)(1+z)} = \frac{1}{1+\Psi}$. Thus, $\chi_y = 0, \chi_x = \frac{(\beta-1)y}{(1+x)(1+x+y)}$. By integrating the vector field along the usual path, we obtain

$$\begin{aligned} I &= \int_0^x P_x(x, 0, 0) dx + \int_0^y Q_y(x, y, 0) dy + \int_0^z R_z(x, y, z) dz \\ &= \ln(1+x) + \ln(1+x+y) + \ln(1+z), \end{aligned}$$

the partial derivative of which are $I_x = \frac{2+2x+y}{(1+x)(1+x+y)} \neq P_x, I_y = \frac{1}{1+x+y} = Q_y, I_z = \frac{1}{1+z} = R_z$. It should be noted that $I_x(x, 0, 0) = P_x(x, 0, 0)$, which means that fictitious pleasures I_x, I_y, I_z coincide with the real pleasures P_x, Q_y, R_z on the integration path.

[21] The condition that some terms $\chi_s, \chi_u, \dots, \chi_x$ are not equal to zero is a necessary and sufficient condition (in line with what has been pointed out in note [20]) in order for “the pleasures [to depend on] the order of consumption”. This dependence means that the vector field defined by “the pleasures” is not conservative.

[22] Pareto introduces a multiplicity of paths that connect the point $(0, 0, \dots, 0)$ with the point (x, y, \dots, t) . Each path is also characterized by the vector (h, k, \dots, n) . Thus, the values I of the line integrals calculated on the vector field $(HA_x, HB_y, \dots, HN_t)$, with $H(x, y, \dots, n, h, k, \dots, n)$, namely,

$$\begin{aligned} I = & \int_0^h H(x, 0, \dots, 0, h, \dots, n) A_x(x, 0, \dots, 0) dx + \int_0^k H(h, y, \dots, 0, h, \dots, n) B_y(h, y, \dots, 0) dy + \dots + \\ & + \int_0^n H(h, \dots, m, t, h, \dots, t) N_t(h, \dots, m, t) dt + \int_h^x H(x, k, \dots, n, h, \dots, n) A_x(x, k, \dots, n) dx + \\ & + \int_k^y H(x, y, \dots, n, h, \dots, n) B_y(x, y, \dots, n) dy + \dots + \int_n^t H(x, y, \dots, t, h, \dots, n) N_t(x, y, \dots, t) dt, \end{aligned}$$

depend, in general, on the vector (h, k, \dots, n) , as well as on (x, y, \dots, t) , since they depend on the path. However, the ratio $\frac{1}{H} \frac{\partial I}{\partial t}$, which is equal to $N_t(x, y, \dots, t)$, does not depend on the vector (h, k, \dots, n) . Furthermore, any non-linear transformation $U(I)$ of I does not satisfy this property, since $\frac{1}{H} \frac{\partial U(I)}{\partial t} = U'(I) \frac{1}{H} \frac{\partial I}{\partial t}$, where $\frac{1}{H} \frac{\partial I}{\partial t}$ is independent of (h, k, \dots, n) , whereas $U'(I)$ depends on this vector because I and, therefore, $U(I)$ depends on it. In this case, the condition that requires that $\frac{1}{H} \frac{\partial I}{\partial t}$ be independent of (h, k, \dots, n) implies that the function I is determined except for a linear transformation. The purpose of this analysis is to investigate the possibility of measuring elementary ophelimities on the basis of the empirical data, consisting of the marginal rates of substitution (which are independent of the vector (h, k, \dots, n)) and of the indifference varieties (which are dependent on the vector (h, k, \dots, n) and are represented by a function I which is defined except for a linear transformation).

[23] Differentials (23) and (25) are carried out by only varying the vector (x, y, \dots, t) while maintaining the vector (h, k, \dots, n) constant.

[24] Pareto explores whether it is possible to measure the elementary ophelimities P_x, Q_y, \dots, T_t (which we are supposing exist, are not observable and do not constitute a conservative vector field) starting from the empirical data A_x, B_y, \dots, N_t (or rather, b_y, c_z, \dots, n_t , that is, starting from the marginal rates of substitution) and from the indifference varieties represented by the function $F(\varphi(x, y, \dots, t, h, k, \dots, n))$. As a consequence of what has been pointed out in note [13], the functions $F(\cdot)$ and $H(x, y, \dots, t)$ must satisfy the equality $I = F(\varphi(x, y, \dots, t, h, k, \dots, n))$, where I is the line integral of the vector field $(HA_x, HB_y, \dots, HN_t)$.

The identification condition for the measurement of elementary ophelimities, which Pareto introduced implicitly here but explicitly in his earlier “Lofelimità nei cicli non chiusi”, *G.d.E.*, July 1906, p. 30, Engl. Trans. p. 385, is the requirement that elementary ophelimities be independent of the vector (h, k, \dots, n) . By calculating the line integrals G (which are dependent on the path, and therefore also on the vector (h, k, \dots, n)) of the vector field defined by the vector $(P_x(x, y, \dots, t), Q_y(x, y, \dots, t), \dots, T_t(x, y, \dots, t))$, Pareto finds, under this assumption, that the last partial derivative of G is independent of the vector (h, k, \dots, n) ; in particular, it is $\frac{\partial G(x, y, \dots, t, h, k, \dots, n)}{\partial t} = T_t(x, y, \dots, t)$. The integral

G represents the indifference varieties, as does the integral $I = F(\varphi)$. Since the latter is defined except for a linear transformation (as we have already seen in note [22]), then $\frac{\partial G}{\partial t} = a \frac{\partial F(\varphi)}{\partial t}$, where a is an arbitrary constant. We obtain, therefore, being $\frac{\partial G}{\partial t} = T_t$ and $\frac{\partial F(\varphi)}{\partial t} = HN_t$, that $T_t = aHN_t$. If T_t is independent of (h, k, \dots, n) , as Pareto hypothesized, since N_t is independent of (h, k, \dots, n) , we have, on the one hand, that H is also independent of (h, k, \dots, n) , and, on the other hand, that the elementary ophelimity T_t is inferable, except for the factor a , from the empirical data (marginal rates of substitution and indifference varieties, which determine HN_t except for an arbitrary constant). The other elementary ophelimities can then be inferred from the relations $b_y = \frac{Q_y}{P_x}, \dots, n_t = \frac{T_t}{P_x}$, and consequently $P_x = \frac{1}{n_t} T_t, Q_y = \frac{b_y}{n_t} T_t, \dots$, as indicated by Pareto. (It should be noted that Pareto's line of reasoning is similar to the one he follows in the case of independent goods, which has been commented on in note [10].)

In order to illustrate Pareto's result let us consider again the example that was introduced in note [13] and used in note [20]. Let the marginal rates of substitution (given by experience) be $A_x = 2 + 2x + \beta y, B_y = 1 + x$ and $C_z = \frac{(1+x)(1+x+y)}{1+z}$ and let the indifference surfaces (they, too, given by experience) represented, along the path (γ) , by the function

$$\varphi = -1 + \left(\frac{1+h+k}{(1+h)(1+x+k)} \right)^{\beta-1} (1+x)^\beta (1+x+y)(1+z).$$

This function is compatible with the functions A_x, B_y, C_z because, with

$$H(x, y, z) = \frac{1}{(1+x)(1+x+y)} \text{ and } F(\varphi) = \ln(1+\varphi),$$

we have

$$\begin{aligned} \ln(1+\varphi) &= (\beta-1) \ln \frac{1+h+k}{(1+h)(1+x+k)} + \beta \ln(1+x) + \ln(1+x+y) + \ln(1+z) = \\ &= \int_0^h H(x, 0, 0) A_x(x, 0, 0) dx + \int_0^k H(h, y, 0) B_y(h, y, 0) dy + \int_0^j H(h, k, z) C_z(h, k, z) dz + \\ &\quad + \int_h^x H(x, k, j) A_x(x, k, j) dx + \int_k^y H(x, y, j) B_y(x, y, j) dy + \int_j^z H(x, y, z) C_z(x, y, z) dz = \\ &= \int_0^h \frac{2}{1+x} dx + \int_0^k \frac{1}{1+h+y} dy + \int_0^j \frac{1}{1+z} dz + \int_h^x \frac{2+2x+\beta k}{(1+x)(1+x+k)} dx \\ &\quad + \int_k^y \frac{1}{1+x+y} dy + \int_j^z \frac{1}{1+z} dz. \end{aligned}$$

If we assume Pareto's identification condition, that is, that the elementary ophelimity of good Z is independent from (h, k, j) , we obtain $R_z = \frac{a}{1+z}$, because $\frac{\partial F(\varphi)}{\partial z} = \frac{\partial \ln(1+\varphi)}{\partial z} = \frac{1}{1+z} = HC_z$. The other two elementary ophelimities are, then, $P_x = A_x \frac{R_z}{C_z} = a \frac{2+2x+\beta y}{(1+x)(1+x+y)}$ and $Q_y = B_y \frac{R_z}{C_z} = \frac{a}{1+x+y}$.

[25] The reasons for some sharp quarrels between Pareto and the French economist Charles Gide, and the subsequent and rather positive developments in their personal relationship, emerge clearly in the *Corrispondenza Pareto-Pantaleoni*, particularly Vol. III.

[26] In other words, Pareto demonstrates that the empirical facts expressed by the vector field of the marginal rates of substitution and by the indifference varieties relating to the given path make it possible to identify the elementary ophelimities, except for the constant needed to determine the unit of measurement of ophelimity, in the two following cases: if the vector field defined by the marginal rates of substitution is conservative and the cardinal function of ophelimity (which is the scalar field of which the elementary ophelimities are the gradient) is additively separable; and if the vector field is not conservative and the path is of a kind, like the path (γ) , that makes it possible to apply the identification condition suggested by Pareto (which requires that the elementary ophelimities be independent of the vector (h, k, \dots, t) that characterizes the path (γ)). More generally, elementary ophelimities are measurable if some characteristic of them are known and these characteristics are sufficient for identifying the elementary ophelimities, as in the two cases considered by Pareto. (For more on the identification of the elementary ophelimities in Pareto's work, see A. Montesano, "The Paretian theory of ophelimity in closed and open cycles", *History of Economic Ideas*, XIV/3, 2006, pp. 77–100.)

[27] Following Pareto's suggestion, the first of the two 'Additions' that he included on p. 684 of the *Manuel* (1909) has been inserted with this note.

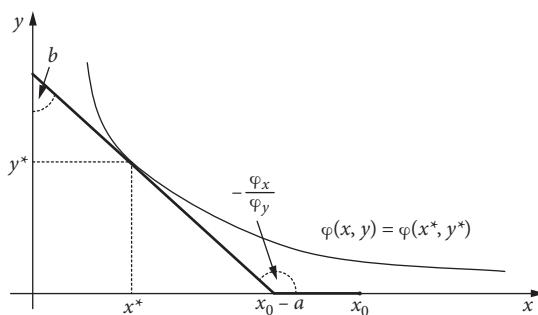
[28] Pareto should have written (following what has been pointed out in §3): "where the curve (27) is tangential to an indifference line" or "where the curve (27) is tangential to the projection of a level curve on plane xy ".

[29] Pareto only makes explicit the conditions of the first order and, furthermore, assumes that the solution is internal (that is, with $x > 0$ and $y > 0$).

[30] Pareto describes a material point that can move in three-dimensional space on a curve obtained from the intersection of the two surfaces $f(x, y) = 0$ and $z = -G(x, y)$, where z indicates the vertical direction and $G(\cdot)$ is a concave function.

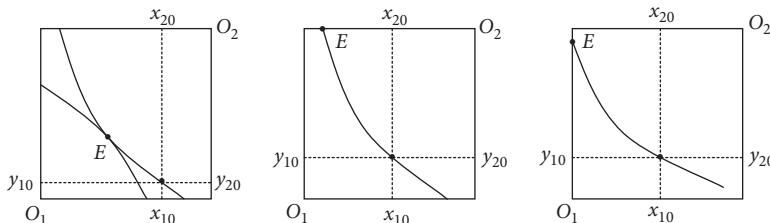
[31] Notice that Pareto is considering production with increasing returns to scale.

[32] The problem and the solution under consideration are represented in the following figure



[33] This equilibrium corresponds to the case that in the current literature is indicated as monopoly with price discrimination of the first kind. Here individual 2 (the monopolist) maximizes his utility (in the standard current analysis, the monopolist maximizes a monetary profit).

[34] This case differs from the preceding one because individual 2 maximizes the quantity of good X, and coincides with the preceding one if the utility of individual 2 is an increasing function of X alone. Pareto's intention here is to represent the case in which the monopolist's objective is to maximise profit, measured in terms of units of good X, and there is price discrimination of the first kind. The intersection of the indifference curve of individual 1 with the x-axis (of individual 2) represents a possible equilibrium. Another case of possible equilibrium is represented by the point of intersection of the indifference curve of individual 1 with the y-axis (of individual 1). What is being pointed out is immediately clear when observing the following three figures (which use the Edgeworth-Pareto box diagram), with the first box representing the equilibrium of §30 and the next two representing the equilibrium of §31.



[35] In what follows, this function is indicated as $f_1(x_1, y_1, \mu)$.

[36] After examining the case of the monopoly with price discrimination of the first kind (in which individual 2 can choose the path that is most convenient to him, which is the path lying on the indifference curve of individual 1 that passes through the latter's endowment), Pareto continues with his analysis of monopoly in an unconventional way. He considers the choice of a path within a family of paths that are not necessarily rectilinear (the choice of a path in the bundle of straight lines passing through the endowment of individual 1 corresponds to the choice of price by individual 2, who is the monopolist). This is a generalization of single-price monopoly that can introduce possible interesting applications such as the two-part tariff monopoly, where the monopolist individual 2 chooses a path from within a family of curves that is characterized by two parameters, x_a and p_y (rather than a single one, as in the case examined here by Pareto), where x_a represents the quantity of good X that individual 1 must give to individual 2 if he wants to enter into exchange and p_y is the price of good Y in terms of good X according to which the exchange takes place. This type of tariff is introduced by Pareto in Ch. VI, §8.

[37] In this equilibrium, individual 1's choice is represented by the solution of the problem $\max_{x_1, y_1} \varphi_1(x_1, y_1)$ subject to the constraint $f_1(x_1, y_1, \mu) = 0$. If the utility function is monotonic and strictly quasi-concave and the choice is internal (that is, with $x_1 > 0$ and $y_1 > 0$), then the first order conditions (38) determine the choice and define the demand function $x_1 = x_1(\mu)$, $y_1 = y_1(\mu)$. Individual 2's choice is represented by the solution of the problem $\max_{x_2, y_2, \mu} \varphi_2(x_2, y_2)$ subject to the constraints $x_1(\mu) + x_2 = x_{10} + x_{20}$ and

$y_1(\mu) + y_2 = y_{10} + y_{20}$, that is, $\max_{\mu} \varphi_2(x_{10} + x_{20} - x_1(\mu), y_{10} + y_{20} - y_1(\mu))$, of which (39 bis) is the first order condition.

[³⁸] Compared to the previous case, the only difference is that now individual 2's choice is represented by the solution of the problem $\max_{\mu} y_{10} + y_{20} - y_1(\mu)$, of which the equation $\frac{dy_1}{d\mu} = 0$ is the first order condition.

[³⁹] In the French text, p. 563, 16th line from the bottom, there is a misprint: "terminer" instead of "déterminer".

[⁴⁰] Pareto relates competition to the situation in which the individuals choose on a path that for each of them is given (and which is determined, within a given family of paths, by the condition of inter-individual equilibrium). Therefore, individuals are parameter-takers, and not necessarily price-takers.

[⁴¹] They are instead §§30 and 31.

[⁴²] They are instead §§32 and 33.

[⁴³] Pareto is obviously arguing with mathematician G. Scorza. On this matter see E.N. 9 regarding "type I".

[⁴⁴] With the symbols previously used, it should be written $f_{1x}\varphi_{1y} - f_{1y}\varphi_{1x} = 0$.

[⁴⁵] The supply and demand by individual 1 are determined by the third equation and the fifth equation of (41).

[⁴⁶] The prices p_y, p_z, \dots defined in this way are, in general, marginal prices, that is, prices relating to infinitesimal exchanges of goods. Marginal prices represent for exchanges on the market what marginal rates of substitution represent for individual preferences.

[⁴⁷] Pareto considers the case in which marginal prices are assigned, which do not originate from the differentiation of a surface $f(x, y, z, \dots) = 0$, and repeats what he pointed out (in §14) for the analogous case of the marginal rates of substitution.

[⁴⁸] Due to an oversight or a misprint the original text indicates x_2 instead of x^2 . There is also a misprint in the preceding equation, which indicates $\frac{az}{x}$ instead of $\frac{bz}{x}$.

[⁴⁹] In this case, the individual first gives Y in exchange for X until he reaches quantity y , and then gives Z in exchange for X until he reaches quantity z . Therefore, the quantity x_1 of X reached by the individual in the first phase is obtained following the relation $-\frac{\partial x}{\partial y} = \frac{ay+cz_0}{x}$. Then, we have

$$-\int_{x_0}^{x_1} x dx = \int_{y_0}^y (ay + cz_0) dy$$

and therefore

$$-\frac{1}{2}(x_1^2 - x_0^2) = \frac{a}{2}(y^2 - y_0^2) + cz_0(y - y_0).$$

The final quantity x of X is then obtained starting from the quantity x_1 and following the relation $-\frac{\partial x}{\partial z} = \frac{bz}{x}$, and so it is

$$-\int_{x_1}^x x dx = \int_{z_0}^z bz dz,$$

and therefore

$$-\frac{1}{2}(x^2 - x_1^2) = \frac{b}{2}(z^2 - z_0^2)$$

By eliminating x_1 from the two relations, one obtains equation (48) indicated by Pareto. Similar applies to the relations defined by equation (49).

[50] This relation indicates the value of the marginal price of good Y (in terms of good X) that corresponds to the point (x', y', z', \dots) . This price is chosen by the individual under consideration. The marginal price varies, in general, along the exchange path followed by the individual to go from the starting point (x_0, y_0, \dots) to the ending point (x', y', \dots) . At the ending point (if it is an internal point and if the second order conditions are satisfied), the marginal price of good Y (in terms of X) is equal to the marginal rate of substitution of good Y (in terms of X). Pareto indicates here with $f(x', y', \dots)$ the function that expresses the marginal rate of substitution, and with $F(x', y', \dots)$ the function that expresses the marginal price, so that the equality $f(.) = F(.)$ is a condition of equilibrium for the individual choice.

[51] Pareto does not explicitly point out the equilibrium conditions for the case with variable prices (whereas, in the subsequent §, he points out the equilibrium conditions for the case with constant prices). The first order equilibrium conditions for the case with variable prices require

$$\begin{aligned} p_y &= F_y(x', y', z', \dots) = f_y(x', y', z', \dots) \\ p_z &= F_z(x', y', z', \dots) = f_z(x', y', z', \dots) \\ &\dots\dots\dots \\ F(x', y', z', \dots) &= 0, \end{aligned}$$

where the latter is the equation of type (30) obtained, if necessary, by integrating the differential system $p_y = F_y(x, y, z, \dots), p_z = F_z(x, y, z, \dots), \dots$ on the exchange path starting from the point (x_0, y_0, \dots) .

[52] Pareto defines the inverse demand functions (that is, p_y, p_z, \dots as functions of x, y, z, \dots) and shows how these functions, if taken as empirical data, since they are equal to the marginal rates of substitution, concern the analysis of the indices of ophelimity.

[53] Pareto does not consider here the determination of the index of ophelimity starting from inverse demand functions because he has already examined this problem at the beginning of the appendix, from §1 to §21, both in the case of closed cycles and in the case of open cycles, that is, both in the case in which the inverse demand functions satisfy the integrability conditions (found by G.B. Antonelli, 1886, and never described by Pareto), and in the case in which they do not satisfy them (but the order of consumption is given). Subsequently Pareto explicitly examined the case where the empirical data is given by the inverse demand functions in *Economie Mathématique* (1911), although he did so with some imprecision as indicated in E.N. 10. Notice that marginal rates of substitution and inverse demand functions are not always equivalent data for the purposes of determining the indices of ophelimity. For instance, in the case of closed cycles, while they are equivalent if preferences are convex, they are not equivalent if preferences are not convex since the inverse demand functions are never equal to the marginal rates of substitution for the non-convex points of the indifference varieties.

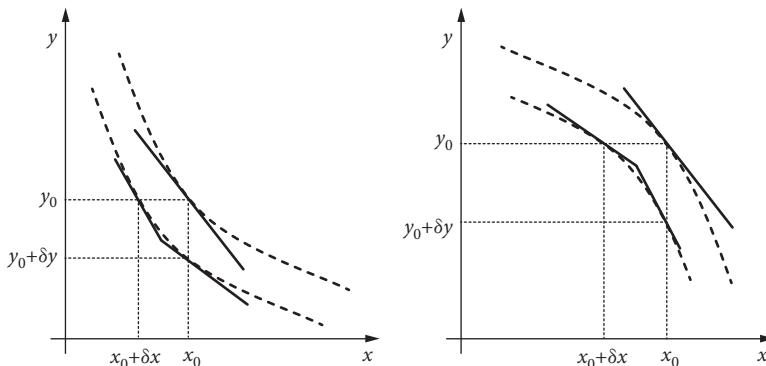
[54] In other words, the inverse demand functions and budget constraint determine individual equilibrium. Pareto adds that a theory of demand only based on the information provided by the inverse demand functions could not say anything before the latter are empirically known, whereas some pieces of information exist if one takes into account that they also are the marginal rates of substitution (and therefore must satisfy some properties, which he examines in the subsequent §§).

[55] Following Pareto's suggestion, the second of the 'Additions' that he included on p. 684 of the *Manuel* (1909) has been inserted with this note.

[56] However, the third property indicated by Pareto, according to which $\frac{d^3y}{dx^3} < 0$, even if it is reasonable for $\frac{dy}{dx}$ sufficiently close to zero, is in actual fact irrelevant for economic theory (it is only considered by Pareto in Ch. IV, §38, and, in the remainder of the appendix, at the end of §46). The dependence of the second kind (that is, an ophelimity function that shows a negative mixed second derivative, together with negative direct second derivatives) may invalidate the second property (being compatible with concave indifference curves, since it is $\frac{d^2y}{dx^2} = -\frac{1}{\varphi_y^3}(\varphi_y^2\varphi_{xx} - 2\varphi_x\varphi_y\varphi_{xy} + \varphi_x^2\varphi_{yy})$).

[57] If we indicate with (x_0, y_0) the starting point and with $-MRS(x, y) = -\frac{\varphi_x}{\varphi_y}$ (where φ_x and φ_y are marginal ophelimities) the function $\frac{dy}{dx}$ (that is, the function that expresses the marginal rate of substitution with the negative sign), the derivatives $-\frac{dMRS(x, y_0)}{dx}$ and $-\frac{dMRS(x_0, y)}{dy}$ correspond to what Pareto indicates with the symbols $\delta_x \frac{dy}{dx}$ and $\delta_y \frac{dy}{dx}$.

[58] The figure indicated by Pareto and the sense of his line of reasoning can be made clearer with the two following figures (where the indifference curves are also drawn and negative variations both of x , leaving y constant, and y , leaving x constant, are considered).



[59] What Pareto expounds follows from the relations

$$-\frac{dMRS(x, y_0)}{dx} = \frac{1}{\varphi_y^2} (-\varphi_y\varphi_{xx} + \varphi_x\varphi_{xy})$$

and

$$-\frac{dMRS(x_0, y)}{dy} = \frac{1}{\varphi_y^2} (-\varphi_y\varphi_{xy} + \varphi_x\varphi_{yy}).$$

Therefore, the relations (57) hold if $\varphi_{xy} \geq 0$, whereas they may or may not hold if $\varphi_{xy} < 0$. Moreover, since it is $-\varphi_y \frac{dMRS(x,y_0)}{dx} + \varphi_x \frac{dMRS(x_0,y)}{dy} = \varphi_y \frac{d^2y}{dx^2}$, at least one of the relations (57) holds if $\frac{d^2y}{dx^2} > 0$, whereas at least one of them does not hold (or rather, holds with the opposite sign) if $\frac{d^2y}{dx^2} < 0$. Pareto takes into account, here and elsewhere, some aspects of the ophelimity function—such as the sign of its second derivatives—which are not defined for an ordinal function (that is, they are not invariant with respect to increasing monotonic transformations).

[60] As shown in Ch. IV, §38.

[61] Due to an oversight or a misprint, the signs of the relations defined by equations (63) and (64) are indicated in the opposite way, that is the correct signs are

$$(63) \quad \varphi_{xy} > 0, \quad \varphi_{xz} > 0, \dots \quad \varphi_{yz} > 0, \dots$$

$$(64) \quad \varphi_{xy} < 0, \quad \varphi_{xz} < 0, \dots \quad \varphi_{yz} < 0, \dots$$

[62] With the inequalities (65) Pareto assumes that marginal ophelimity is decreasing for all goods. This property is not consistent with the ordinal notion of ophelimity (that is, it is not invariant with respect to increasing monotonic transformations of the ophelimity function). In the subsequent §48, Pareto also extends the property of decreasing marginal ophelimity to composite commodities and demonstrates (without explicitly indicating it) how this implies the concavity of the ophelimity function—in other words, that it is $d^2\varphi < 0$ for any (infinitesimal) variation of the bundle of goods.

[63] The numbering of the formulae repeats, by mistake, (64) and (65).

[64] To follow the line of reasoning expounded by Pareto immediately afterwards, it is better to write the inequality as

$$(\varphi_{xx}\varphi_{yy} - \varphi_{xy}^2)\varphi_x\varphi_y - ((\varphi_{xx}\varphi_y - \varphi_{xy}\varphi_x)\varphi_y + (\varphi_{yy}\varphi_x - \varphi_{xy}\varphi_y)\varphi_x) \varphi_{xy} > 0$$

[65] Pareto assumes that individual choice (that is, demand and supply) is represented by the relations (68). For this purpose, he implicitly assumes that the choice is internal (that is, the consumed quantity of each good is positive) and that the second order conditions are satisfied.

[66] Due to an oversight or a misprint, the original text shows φ^{xz} instead of φ_{xz} .

[67] Pareto meant to write “by deleting in R the i th row and the n th column”.

[68] The relations given by equations (75) and (76) indicated by Pareto are essentially identical of those that will later be found and commented on by Slutsky (who generously acknowledges Pareto’s antecedence on this matter, E. Slutsky, “Sulla teoria del bilancio del consumatore”, *G.d.E.*, July 1915, pp. 1–26, in a footnote on p. 12, where, however, he quotes Pareto’s article from August 1892 rather than the one from October 1893, which includes the analysis of the demand in the general case with non-additively separable utility function). In reference to relation (75) the substitution effect is represented by $m \left(\frac{MH_{2,2}}{RR_2} - \frac{M_{3,1}}{R} \right) \frac{R_2}{M}$ and the income effect by $-(y - y_0) \frac{R_2}{M}$. Taking into account Slutsky’s substitution matrix $S = (s_{i,j})$, we have, for its generic component, with Pareto’s symbols, $s_{i,j} = m \left(\frac{MH_{i,j}}{RR_i} - \frac{M_{j+1,1}}{R} \right) \frac{R_i}{M}$.

[69] The link between Pareto's and Slutsky's analyses is discussed by H. Schultz, 1935, "Interrelations of Demand, Price, and Income", *Journal of Political Economy*, 43, pp. 433–481 and P. C. Dooley, 1983, "Slutsky's Equation is Pareto's Solution", *History of Political Economy*, 15, pp. 513–517, among the others. English translations of the above quoted articles by Slutsky and Pareto are: E. Slutsky, "On the theory of the budget of the consumer", in G. Stigler and K. Boulding (eds), *Readings in Price Theory*, Irwin, 1952, pp. 27–56, and V. Pareto, *Considerations on the Fundamental Principles of Pure Political Economy*, edited by R. Marchionatti and F. Mornati, Routledge, 2007.

[70] Due to an oversight or a misprint, there is $\frac{MH_2}{RR_3}$ instead of $\frac{MH_{2,3}}{RR_3}$.

[71] In the case under consideration, these inequalities, which Pareto assumes as natural for an ophelimity index, are sufficient for the second order conditions of individual choice to be satisfied; but they are not necessary. The second order conditions could also be satisfied if one of the inequalities indicated by Pareto has the positive sign instead of the negative one. This possibility has been discussed by U. Ricci, "Curve crescenti di ophelimità e di domanda", *G.d.E.*, August 1904, pp. 112–138 and "The Psychological Foundation of the Law of Demand", *Journal of Political Economy*, 40, 1932, pp. 145–185, and by E. Slutsky, *op. cit.*, *G.d.E.*, 1915, p. 18 (on this matter, see A. Montesano, "Umberto Ricci, l'utilità marginale e la teoria della domanda", in P. Bini and A. M. Fusco eds., *Umberto Ricci (1879–1946)*, Firenze, Polistampa, 2004, pp. 99–116). In this case the proposition according to which "if commodity Y is demanded, then always $\frac{\partial y}{\partial p_y} < 0$ ", which Pareto puts forward shortly afterwards, is not always true.

[72] In the original text a symbol is missing, giving $T - \frac{p_y^2}{yy}$ instead of $T - \frac{p_y^2}{\varphi_{yy}}$.

[73] That is to say, the substitution effect is negative and the income effect is positive (because the good is in supply). Pareto then shows that, since an increase in the price of the good in supply is advantageous for the individual, it produces a decrease in the marginal ophelimity of money and an increase in the consumption of the other goods.

[74] The assumption that is currently used in the literature to justify the Marshallian hypothesis—according to which the marginal utility of money is constant and, as also assumed here by Pareto, the marginal utility of every good is a function of only the quantity of the corresponding good—requires that the utility function be additively separable and linear with respect to the good that represents money (that is, with respect to good X, in Pareto's example). This assumption is consistent with the case Pareto has just discussed: T is equal to infinity since $\varphi_{xx} = 0$, and then, by differentiating the condition $\frac{\varphi_y(y)}{p_y} = m$, we obtain $\frac{\partial y}{\partial p_y} = \frac{m}{\varphi_{yy}}$.

[75] Really, it is $\frac{\partial x}{\partial p_y} + \frac{\partial(p_y y)}{\partial p_y} = 0$. Indeed, as Pareto has just indicated, "if one assumes that $\frac{\partial m}{\partial p_y}$ can be disregarded because T is very large, it will follow that all the $\frac{\partial z}{\partial p_y}, \frac{\partial u}{\partial p_y}, \dots$ can be disregarded as well". This, however, does not imply that $\frac{\partial x}{\partial p_y}$ can be ignored. Furthermore, in the case under consideration, the relation (72) requires that $0 = \frac{\partial x}{\partial p_y} + y - y_0 + p_y \frac{\partial y}{\partial p_y}$, which coincides with $\frac{\partial x}{\partial p_y} + \frac{\partial(p_y y)}{\partial p_y} = 0$ as soon as we put $y_0 = 0$, as implicitly assumed in order for Pareto to write the relation $0 = \frac{\partial x}{\partial p_y} + \frac{\partial(p_y y)}{\partial p_y} + p_z \frac{\partial z}{\partial p_y} + \dots$, and explicitly stated in §59 (just before the relations (81)).

[76] Notice that the Cobb-Douglas utility function requires $\varphi_x = \frac{A}{x}$, which implies $u(x) = A \ln x$, while the form $\varphi_x = \frac{A}{x_0 - x}$ implies $u(x) = -A \ln(x_0 - x)$.

[77] If we put $\alpha = \beta = \gamma = \dots = 0$, the exercise carried out by Pareto in §§60–62 represents an individual's demand where preferences are of the Cobb-Douglas variety.

[78] Indeed it is $1 + \epsilon_y = (\frac{B}{A})^{\frac{\beta}{1+\beta}} x^{\frac{\alpha-\beta}{1+\beta}} p_y^{\frac{\beta}{1+\beta}}$, which has a magnitude that tends to 1 for α and β tending to zero.

[79] Due to an oversight or a misprint, there is $\frac{p_y^2 B x^2 (1+\epsilon_y)^2 y^\beta}{(1+\beta) A^2}$ instead of $\frac{B x^2 (1+\epsilon_y)^2 y^\beta}{(1+\beta) A^2}$.

[80] Pareto's line of reasoning can be made clearer by pointing out that $y_0 < y$ implies $h_0 - p_y y_0 > h_0 - p_y y = x + p_z z + \dots > 0$.

[81] Due to an oversight or a misprint, there is (III, 4) instead of (III, 40).

[82] This equality does not, of course, imply that a solution exists in every case, but it does suggest that the problem being tackled is not ill-posed, and that it is therefore possible to proceed with the analysis along the lines suggested. In Pareto's approach, the purpose of the theory is not to establish the sufficient conditions for equilibrium to exist, as in the case of the axiomatic approach to equilibrium, but to determine the properties of equilibrium on the presumption that equilibrium does exist. The importance of these properties, explicitly identified through formal theoretical investigation, is that they help to explain economic reality. The purpose of the theory in Pareto's system is discussed in §143 (and in note [200]).

[83] If “the holder of a commodity being supplied, say Y, does not use it to satisfy his tastes”, then $y_1 = 0$, and not $y_{10} = y_1$ as Pareto writes. Alternatively, Pareto is here no longer indicating with y_1 the quantity consumed, but the quantity supplied: then, those who do not consume good Y, sell all their endowment of it, that is, $y_1 = y_{10}$.

[84] As pointed out in the previous note, here it should read $x_1 = 0$, and not $x_1 - x_{10} = 0$, unless x_1 represents the quantity supplied rather than the quantity consumed.

[85] The assumption according to which individual 1 does not take into account the ophelimity index of Y only relates to what Pareto indicates immediately afterwards as case (a). Indeed, under this assumption there would be no difference between the two cases: case (β) would simply turn into case (a) because maximum ophelimity would be reached precisely with maximum income.

[86] The function $f(p_y)$ is the (excess) demand function for good Y by individuals $2, \dots, \theta$, so that the second of the equations (C) becomes the relation $y_{10} - y_1 = f(p_y)$ indicated by Pareto. The function $f(p_y)$ is obtained from the relations (A) and (B), excluding those that relate to individual 1 (that is, the first row of the relations (A) and the first relation (B)). For individual 2 and good Y, one obtains the demand function $y_2 - y_{20} = f_2(p_y, p_z, \dots)$; for individual 3, the demand function $y_3 - y_{30} = f_3(p_y, p_z, \dots)$, etc.; by adding them, we obtain the function

$$f(p_y, p_z, \dots) = f_2(p_y, p_z, \dots) + f_3(p_y, p_z, \dots) + \dots$$

It should be noted that Pareto does not explicitly indicate the prices p_z, \dots , but only the price p_y , among the arguments of the function $f(p_y, p_z, \dots)$. This fact (as well as other aspects of his subsequent analyses) leads to the belief that in Pareto's theory the monopolist of good Y, in choosing the sale price, does not consider the prices of the other

goods as independent of his choice, but also takes into account the interdependences, that is the fact the prices of all goods other than good Y are a function of p_y . However, under this assumption there arise some analytical problems, which will be discussed in the following note.

[87] Pareto analyses case (a) by replacing, in the system of equations (A), (B) and (C), the equation $\varphi_{1x} = \frac{1}{p_y} \varphi_{1y}$ with the equation $\frac{d(p_y f(p_y))}{dp_y} = 0$. The interpretation of this equilibrium deserves some discussion.

If individual 1 does not take into account the interdependences between the prices of goods—if, that is to say, he considers the demand function $f(p_y, p_z, \dots)$ —his choice is represented by the solution of the problem $\max_{p_y, x_1, y_1, z_1, \dots} \varphi_1(x_1, z_1, \dots)$ subject to the two constraints $y_1 - y_{10} + f(p_y, p_z, \dots) = 0$ and $x_1 - x_{10} + p_y(y_1 - y_{10}) + p_z(z_1 - z_{10}) + \dots = 0$. The first order conditions of this problem are represented by the two constraints and by the equations $\frac{\partial(p_y f(p_y, p_z, \dots))}{\partial p_y} = 0$, $\varphi_{1x} = \frac{1}{p_z} \varphi_{1z} = \dots$

If, on the contrary, we assume that individual 1, in choosing p_y , takes into account the interdependences (as pointed out in the previous note), then we have a different situation from the one discussed by Pareto. In particular, for individual 1 the relations $\varphi_{1x} = \frac{1}{p_z} \varphi_{1z} = \dots$ no longer apply. Indeed, his choice is the solution of the problem $\max_{p_y, x_1, y_1, z_1, \dots} \varphi_1(x_1, z_1, \dots)$ subject to the constraints

$$\begin{aligned} x_1 - x_{10} + p_y(y_1 - y_{10}) + p_z(z_1 - z_{10}) + \dots &= 0, \\ y_1 - y_{10} + f(p_y, p_z, \dots) &= 0, \quad z_1 - z_{10} + g(p_y, p_z, \dots) = 0, \quad \dots, \end{aligned}$$

where $g(p_y, p_z, \dots)$ is the demand function for good Z by the individuals $2, \dots, \theta$, obtained analogously to function $f(p_y, p_z, \dots)$ in note [86].

The analysis proposed by Pareto is logically inconsistent, but not unrealistic: it can be explained considering the approach often adopted by Pareto, who distinguishes between the behavior of the agent that operates on the basis of logical economic considerations and maximizes profit—represented here by the monopolist—and that of the consumers, whose rationality is the product of experience and habit, even in the extreme case in which monopolist and consumer are one and the same person. In that case the individual is price-taker for all goods other than good Y when choosing consumption, whereas he takes the interdependences into account when choosing price p_y . In other words, if we indicate with $s = p_y f(p_y, p_z, \dots)$ the monopoly revenue, individual 1's choice would be represented by two maximum problems. The first one, in which the monopolist is, as a consumer, price-taker, is the problem of the choice of consumption $\max_{x_1, z_1, \dots} \varphi_1(x_1, z_1, \dots)$ subject to the budget constraint $x_1 - x_{10} - s + p_z(z_1 - z_{10}) + \dots = 0$, which gives the first order conditions represented by the budget constraint and by the relations $\varphi_{1x} = \frac{1}{p_z} \varphi_{1z} = \dots$. These conditions determine the demand functions $z_1 - z_{10} = g_1(s, p_z, \dots)$, \dots for all goods other than X and Y . The equilibrium conditions for these goods (that is, the conditions $z_1 - z_{10} + z_2 - z_{20} + \dots = 0, \dots$), determine, through the demand functions for them by the monopolist and the other individuals, the prices of these goods as a function of p_y and s (that is, $p_z = p_z(p_y, s), \dots$). If we

insert these functions in the demand function $f(p_y, p_z, \dots)$ for good Y , we obtain the function $f(p_y, s)$. Since the maximum of ophelimity obtained from the first problem is an increasing function of revenue s (because the function $\varphi_1(x_1, z_1, \dots)$ is monotonically increasing), the second maximum problem concerns the choice of the monopolist (who seeks maximum profit, which in this case coincides with the revenue), that is, $\max_{p_y, s} s$ subject to the constraint $s = p_y f(p_y, s)$. In this problem the monopolist chooses the price of the good on which he holds the monopoly, taking into account that the prices of the other goods depend on his choice (and therefore he is not price-taker for them). As a consequence, the price p_y and the revenue s of the monopolist are represented by the solution of the first order conditions $s = p_y f(p_y, s)$ and $\frac{\partial(p_y f(p_y, s))}{\partial p_y} = 0$.

[88] In other words, Pareto points out that the condition $y_1 \geq 0$ must also be satisfied.

[89] Analogous to what has been indicated for case (a), if individual 1 does not take into account the interdependences and, therefore, he considers the demand function $f(p_y, p_z, \dots)$, his choice in case (β) is represented by the solution of the problem $\max_{p_y, x_1, y_1, z_1, \dots} \varphi_1(x_1, y_1, z_1, \dots)$ subject to the constraints $y_1 - y_{10} + f(p_y, p_z, \dots) = 0$ and

$x_1 - x_{10} + p_y(y_1 - y_{10}) + p_z(z_1 - z_{10}) + \dots = 0$, which yield the first order conditions comprised of the two constraints, of the relations $-\varphi_{1x}p_z + \varphi_{1z} = 0, \dots$ for all goods other than X and Y , and of the relation $\varphi_{1x} \frac{\partial(p_y f(p_y, p_z, \dots))}{\partial p_y} - \varphi_{1y} \frac{\partial f(p_y, p_z, \dots)}{\partial p_y} = 0$. The same equations can be arrived at by splitting individual 1's choice into two stages. In the first stage, individual 1 chooses the quantities of the goods on which he has no market power (that is, of all the goods except good Y) by solving the problem $\max_{x_1, z_1, \dots} \varphi_1(x_1, y_1, z_1, \dots)$ subject to the constraints $y_1 - y_{10} + f(p_y, p_z, \dots) = 0$ and $x_1 - x_{10} + p_y(y_1 - y_{10}) + p_z(z_1 - z_{10}) + \dots = 0$, so as to obtain the demand functions $x_1(p_y, p_z, \dots), z_1(p_y, p_z, \dots), \dots$, as well as $y_1 = y_{10} - f(p_y, p_z, \dots)$. In the second stage, he chooses the price of good Y by solving the problem $\max_{p_y} \varphi_1(x_1(p_y, p_z, \dots), y_1(p_y, p_z, \dots), z_1(p_y, p_z, \dots), \dots)$. This way leads to the same conditions indicated above. Indeed, by derivating the second constraint we obtain

$$\frac{\partial x_1}{\partial p_y} + y_1 - y_{10} + p_y \frac{\partial y_1}{\partial p_y} + p_z \frac{\partial z_1}{\partial p_y} + \dots = 0$$

and, therefore, taking into account the first order conditions $\varphi_{1x} = \frac{1}{p_z} \varphi_{1z} = \dots$, we find

$$\varphi_{1x} \frac{\partial x_1}{\partial p_y} + \varphi_{1x}(y_1 - y_{10}) + \varphi_{1x} p_y \frac{\partial y_1}{\partial p_y} + \varphi_{1z} \frac{\partial z_1}{\partial p_y} + \dots = 0,$$

and, therefore,

$$\varphi_{1x} \frac{\partial x_1}{\partial p_y} + \varphi_{1y} \frac{\partial y_1}{\partial p_y} + \varphi_{1z} \frac{\partial z_1}{\partial p_y} + \dots - \varphi_{1y} \frac{\partial y_1}{\partial p_y} + \varphi_{1x}(y_1 - y_{10}) + \varphi_{1x} p_y \frac{\partial y_1}{\partial p_y} = 0.$$

Finally, since it is, taking the first constraint into account,

$$-\varphi_{1y} \frac{\partial y_1}{\partial p_y} + \varphi_{1x}(y_1 - y_{10}) + \varphi_{1x}p_y \frac{\partial y_1}{\partial p_y} = \varphi_{1y} \frac{\partial f(p_y, p_z, \dots)}{\partial p_y} - \varphi_{1x} \frac{\partial (p_y f(p_y, p_z, \dots))}{\partial p_y},$$

we can conclude that relation (86) coincides with the condition identified in the first part of this note. If the monopolist takes interdependences into account, then similar considerations apply as those pointed out in note [87] for case (a).

[90] What Pareto means by “monopoly of two individuals and one commodity” is the situation in which each of the two monopolists would like to act as a pure monopolist. (This is also what transpires from the presentation of the problem in *Economie mathématique*, §39.) In von Stackelberg duopoly, this corresponds to the case in which both sellers would like to be leader (on this matter, see note [93] below). After ascertaining that this equilibrium is impossible, in §§73 and 74 Pareto then points out, when discussing the case of the duopolists who choose the price (known as Bertrand duopoly), that no monopolistic position can persist (on this indetermination see note [100] below).

[91] The relation written by Pareto is obtained from the equality between the total revenue of the two monopolists and the expenditure in good Y by the other individuals. With reference to note [86], if the two monopolists do not take the interdependences into account, by indicating with $f(p_y, p_z, \dots)$ the demand function for good Y of the other individuals, the equation (87) indicates that $s_1 + s_2 - p_y f(p_y, p_z, \dots) = 0$. If the monopolists take interdependences into account (as Pareto's analysis leads us to believe, and we consequently follow the analysis proposed at the end of note [87]), we have $s_1 + s_2 - p_y f(p_y, p_z(p_y, s_1, s_2), \dots) = 0$.

[92] The relation (88) can be interpreted as one of the first order conditions of the problem $\max_{s_1, p_y} s_1$ subject to the constraint $F(s_1, s_2, p_y) = 0$, or of the problem $\max_{s_2, p_y} s_2$ subject to the same constraint. Both these problems lead to the same first order conditions, which are the relations (87) and (88).

[93] Pareto assumes that each monopolist believes he can choose all the variables to be determined. Thus, the first monopolist maximizes his revenue by also choosing the other monopolist's revenue, and so does the second monopolist. In other words, the behavior of the first monopolist is represented by the problem $\max_{s_2, p_y} s_1$ subject to the constraint $F(s_1, s_2, p_y) = 0$, and the behavior of the second is represented by $\max_{s_1, p_y} s_2$ subject to the constraint $F(s_1, s_2, p_y) = 0$. These two problems, which Pareto solves in two stages (for the first monopolist, by maximizing s_1 with respect to p_y in the first stage, and with respect to s_2 in the second stage; and similarly for the second monopolist) are in general incompatible with each other. It is for this reason that Pareto maintains that the equilibrium is overdetermined. The problem examined by Pareto corresponds, in the subsequent literature, with reference to the von Stackelberg duopoly model, to the case in which both sellers try to behave as leader. This case is called “Pareto duopoly” by H. von Stackelberg himself, “Sulla teoria del duopolio e del polipolio”, *Rivista Italiana di Statistica, Economia e Finanza*, 11, 1933, pp. 275–289.

[94] Pareto is politely arguing with Edgeworth, who had asserted the thesis of indetermination (F. Y. Edgeworth, “La teoria pura del monopolio”, *G.d.E.*, 1897, pp. 13–31,

307–320 and 405–414). There is also a reply to Pareto's criticism by Edgeworth (F. Y. Edgeworth, *Papers Relating to Political Economy*, London, Macmillan, 1925, vol. II, p. 313), who, however, on this occasion does not deal with the case discussed by both himself and Pareto, of the two monopolists of the same commodity (or of two highly interchangeable commodities), but with the topic of bilateral monopoly.

[95] A possible interpretation of this equation is as follows. In the first stage (as indicated in note [89]), individual 1 chooses the quantities of the goods on which he has no market power (that is, all the goods except for good Y), by solving the problem $\max_{x_1, z_1, \dots} \varphi_1(x_1, y_1, z_1, \dots)$ subject to the constraint $x_1 - x_{10} + p_y(y_1 - y_{10}) + p_z(z_1 - z_{10}) + \dots = 0$, from which result the demand functions $x_1(y_1, p_y, p_z, \dots), z_1(y_1, p_y, p_z, \dots), \dots$ and the function

$$t_1 = \varphi_1(x_1(y_1, p_y, p_z, \dots), y_1, z_1(y_1, p_y, p_z, \dots), \dots).$$

By inverting this function with respect to y_1 , one obtains a function of the type $y_1 = g_1(t_1, p_y, p_z, \dots)$. If we proceed in a similar fashion for individual 2, we arrive at a function of the type $y_2 = g_2(t_2, p_y, p_z, \dots)$. If we take into account that the quantity of good Y sold by the two monopolists is equal to the quantity purchased by the other individuals, represented by the demand function $f(p_y, p_z, \dots)$, we arrive at the relation

$$y_{10} - g_1(t_1, p_y, p_z, \dots) + y_{20} - g_2(t_2, p_y, p_z, \dots) = f(p_y, p_z, \dots),$$

which can be synthetically represented as $F(t_1, t_2, p_y, p_z, \dots) = 0$.

[96] However, if individual 2 (monopolist of good Z) is a purchaser of good Y , the demand for the latter good also depends on revenue s_2 . In that case, monopolist 1's revenue is $s_1 = p_y f(p_y, p_z, \dots, s_2)$, that is, there is a relation $F_1(s_1, s_2, p_y, p_z) = 0$. Similarly, for the other monopolist we have $s_2 = p_z g(p_y, p_z, \dots, s_1)$, where $g(\cdot)$ is the demand function for good Z (in the same way as $f(\cdot)$ is the demand function for Y), and, therefore, $F_2(s_1, s_2, p_y, p_z) = 0$. Thus, a strategic context emerges, the Nash equilibrium of which is represented precisely by the relations $F_1(\cdot) = 0, F_2(\cdot) = 0, \frac{\partial F_1}{\partial p_y} = 0$ and $\frac{\partial F_2}{\partial p_z} = 0$.

[97] Due to an oversight or a misprint, there is (94) instead of (93).

[98] Pareto determines in this way the general equilibrium with many monopolists, when each monopolist controls the market of only one commodity of which he is the unique seller.

[99] Pareto re-examines the case of monopoly of two individuals and one commodity putting p_y and p_z in place of p_y and y_2 .

[100] In this paragraph Pareto very briefly introduces Bertrand duopoly (essentially, this is, after all, the case dealt with by Edgeworth in his 1897 article quoted in note [94]) and points out how, in it, it is to each seller's advantage to lower the price just below the price chosen by the other seller. There is indetermination, not because the sellers continue to lower the price ad infinitum, but because the price variations carried out by the sellers may not converge (as illustrated by Edgeworth in his 1897 article).

[101] The example is carried out according to the pattern introduced at the end of note [87]. In other words, the monopolists of good Y are price-takers when they choose their consumption and they take interdependences into account when they choose their behavior as monopolists.

[102] Due to an oversight or a misprint, in the original text there is p_y instead of p_z .

[103] The symbols y_1 and y_2 indicate the quantities sold by the monopolists.

[104] Pareto does not write the budget constraint of the consumer of good Y because it is implied by the budget constraints of the two monopolists and by the equality conditions between demand and supply (also including with them the relation $y = y_1 + y_2$ that Pareto points out further below).

[105] That is, s is the total revenue of the two monopolists.

[106] Taking into account that $s = s_1 + s_2$, and the relation that determines p_z as a function of p_y and s .

[107] By taking into account that $s = s_1 + s_2$.

[108] The numerical example put forward by Pareto is an unfortunate choice, because the revenue s is an increasing monotonic function of p_y in the preceding relation, and therefore the equation (88), which expresses revenue maximization, would require an infinitely high price for good Y and, consequently, the relative price of good X with respect to good Y would be zero. But good X is ophelious for all and thus a demand would arise for good X that would exceed availability. A situation would therefore ensue, in which equilibrium would not exist even if there were only one monopolist instead of two.

[109] What Pareto writes here is incorrect, even without considering the problem pointed out in the previous note. Indeed, let us assume that we have arrived at the relation (89 bis). The latter implies $s_1 + s_2 = k$ (where k is the solution of the equation $f(k) = 0$). In that case, the maximization of s_1 with respect to s_2 leads to the pair $s_1 = k$ and $s_2 = 0$ and the maximization of s_2 with respect to s_1 leads to the pair $s_1 = 0$ and $s_2 = k$: we find, in this way, the incompatibility described by Pareto. The inaccuracy consists in the fact that the relations (90) and (91) do not apply to the case in which the relation (89) is of the type (89 bis), because in this case the two maximum problems, of which (90) and (91) are the first order conditions, have boundary solutions.

[110] That is, the problem is overdetermined.

[111] Pareto describes the whole economy, indicating the consumption of every individual and the production of every good. However, he describes production as if there were one single firm, unless by the term *enterprise*, which he uses in defining the quantities $a, b, \dots, A, B, \dots, \dots$, he means the aggregate of all firms.

[112] Indices 1, 2, ... refer to individuals.

[113] The expression ‘intermediate position’ should be interpreted in the sense of a generic position (not necessarily of equilibrium), rather than in the sense of a position through which a firm goes during production.

[114] Due to an oversight or a misprint, in the original text there is y' instead of y'_2 .

[115] As correctly indicated further in the text (in §85), it is the difference between A'', B'', \dots and A''', B''', \dots (and not, as here indicated, the difference between A', B', \dots and A'', B''', \dots) that represents a profit.

[116] The production coefficients considered by Pareto are marginal, not average. A comparison with the standard analysis is put forward in note [157]. In the *Cours*, Pareto—like Walras and the great majority of economists—had used average production coefficients (ratios between quantities of input and output), instead of marginal coefficients (ratios between the variations of these quantities).

[117] In this way Pareto implicitly assumes that inputs cannot be substituted. This assumption will be abandoned in §§103 ff.

[118] This characteristic applies to the now standard analysis of production (as carried out, for instance, by G. Debreu, *Theory of value*, New York, Wiley, 1959, and based on the notion of production set). If, on the contrary, there were dependence on the path, one could apply the analysis of closed and open cycles (explicitly recalled by Pareto, in this regard, in *Economie mathématique*, §43).

[119] This assumption implies that there is no joint production.

[120] Due to an oversight or a misprint, in the original text there is Y' instead of Y'' .

[121] In §78 the possibility that the quantities of inputs depend on the path is considered unrealistic. Yet, the possibility still remains that prices depend on the path.

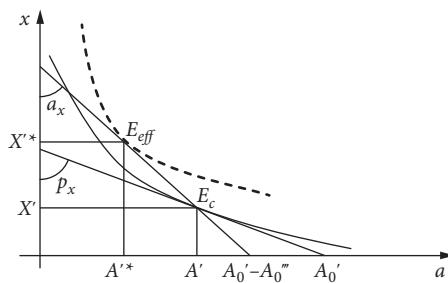
[122] Due to an oversight or a misprint, in the original text there is a_2 instead of a'_2 .

[123] In Pareto's opinion, there is free competition if there is free entry. This condition implies "the equality between the cost of production and the price of the commodities", which characterizes, in Pareto's analysis, free competition. (Pareto is not univocal when he defines competition in words. At times—as in this analysis and, previously, for instance, in the *Cours*, §705—he associates competition with free entry and the consequent zero-profit condition; at other times—as, for instance, in the *Manual*, Ch. V, §§8–9—he associates it with producers acting as price-takers on the market of their products). In the now standard analysis, Pareto's free competition (defined on the basis of free entry) corresponds to a type of Bertrand oligopoly (that is, Bertrand oligopoly with free entry), because firms compete with each other on the price. If price is constant (or linear, which means that all the units of the commodity are sold at the same price) and there are no fixed costs (general expenses), then there is also equality between price and marginal cost because competition requires the minimum price among those that generate zero profit, and this, under the conditions indicated, means a price equal to the minimum average cost (with a quantity of product, therefore, the marginal cost of which is equal to the average cost, since the latter is the minimum). On the contrary, in the standard analysis, for instance that of G. Debreu in the *Theory of value*, op. cit., free competition is characterized by the condition that firms are price-takers. In that case, if price is constant, the firm's choice leads to the equality between price and marginal cost. Profit can be positive if there is no free entry. It becomes zero in the long term, which is defined in this way because the assumption of free entry is added to the assumption that firms are price-takers. Pareto's position and the standard position coincide if firm production sets are convex and there is free entry. The typical case that is included in free competition in Pareto's analysis, but not in the standard one, is represented by productions with positive fixed cost and constant marginal cost (for an active firm). In the standard analysis, the outcome is a long-term supply function that indicates a quantity equal to zero for prices lower than, or equal to, the marginal cost, and a quantity equal to infinity for prices higher than the marginal cost. No competitive equilibrium is therefore possible if demand is positive and bounded. On the contrary, in Pareto's analysis, a competitive equilibrium (albeit one that is not a Pareto-optimum) is defined also for this case. In this case, however, only one firm ends up staying active for the production under consideration, and it is compelled by the potential competition of the inactive producers to choose the price that entails zero profit. Only one firm is active because in this way the fixed cost of the firm is spread over all the quantity sold, allowing

the firm to sell at the lowest possible price, thus beating the competition. In the case of a two-goods economy, one consumer and one producer with the technology represented by the relation $A''' = A_0''' + a_x X''$, Pareto's competitive equilibrium requires:

- (A) $\frac{1}{p_x} \varphi_x(X') = \varphi_a(A')$,
- (B) $A' - A'_0 + p_x X' = 0$,
- (M) $A'_0 - A' = A''$, $X' = X''$,
- (D) $p_x X' = A''' = A_0''' + a_x X''$,
- (E) $A'' = A'''$

In the following figure, the point E_c represents Pareto's competitive equilibrium, whereas the point E_{eff} represents the Pareto-optimal allocation.



However, Pareto does not exclude the possibility of a competitive equilibrium with variables prices (i.e., with free-entry and parameter-taker agents). In this case a competitive equilibrium with many active firms can emerge even if increasing returns to scale prevail and the equilibrium allocation may—or may not—be Pareto-optimal.

For instance, with reference to the example taken into account in the preceding figure, let $A_j = A_{0j}''' + a_x x_j$ for every firm j and let the variable price be represented, for every firm, by $p_x^f = a x_j^{-0.5}$. The profit of an active firm is $a_j - A_j$, where $a_j = p_x^f x_j$ is its revenue and $A_j = A_{0j}''' + a_x x_j$ is its cost (recalling that commodity A is the numeraire), so that the profit is equal to $a_j - A_{0j}''' - \frac{a_x}{a^2} a_j^2$. Profit maximization leads to $a_j = \frac{1}{2} \frac{a^2}{a_x}$, $x_j = \frac{1}{4} \frac{a^2}{a_x^2}$ and a profit of $\frac{1}{4} \frac{a^2}{a_x} - A_{0j}'''$. Thus, free entry requires that $a = 2a_x^{0.5} (A_{0j}''')^{0.5}$ (notice that the determination by free entry of parameter a corresponds, when price are constant, to the determination of price p_x), so that $a_j = 2A_{0j}'''$ and $x_j = \frac{1}{a_x} A_{0j}'''$. Consequently, total demand for input is $a = A = 2n_f A_{0j}'''$ and total supply of output is $x = \frac{1}{a_x} n_f A_{0j}'''$, where n_f is the number of active firms. Let us assume that consumers are all equal and that their ophelimity index function is $\Phi_i = \ln a_i + \ln x_i$, so that the marginal rate of substitution is represented by $\left(\frac{da_i}{dx_i} \right)_{MRS} = \frac{a_i}{x_i}$ for every consumer. If price is constant for the consumers

and is represented by p_x^c , then individual demands are $x_i = \frac{1}{2} \frac{a_{i0}}{p_x^c}$ and $a_i = \frac{1}{2} a_{i0}$ and the aggregate excess demand functions are $\theta x_i = \frac{1}{2} \frac{A'_0}{p_x^c}$ and $\theta a_i = \frac{1}{2} A'_0$, where $A'_0 = \theta a_{i0}$ is their total endowment. Equilibrium of demands and supplies by consumers and firms requires that $2n_f A''_0 = \frac{1}{2} A'_0$ and $\frac{1}{a_x} n_f A''_0 = \frac{1}{2} \frac{A'_0}{p_x^c}$, so that $p_x^c = 2a_x$ and $n_f = \frac{1}{4} \frac{A'_0}{A'''_0}$. We observe that the marginal rate of substitution for consumers, which is equal to $p_x^c = 2a_x$, differs from the corresponding marginal rate for firms, which is equal to a_x . Consequently, the allocation is not Pareto-optimal. If price is variable also for the consumers and is represented by $p_x^c = \beta(a_{i0} - a_i)^{-1}$, where $a_{i0} - a_i$ is the quantity sold by consumer i in order to buy the commodity produced by firms, then the budget constraint is $p_x^c x_i = a_{i0} - a_i$. The individual choice is the solution of the problem $\max_{a_i, x_i} \ln a_i + \ln x_i$ subject to the budget constraint $\beta x_i = (a_{i0} - a_i)^2$. Therefore: $x_i = \frac{4}{9\beta} a_{i0}^2$ and $a_i = \frac{1}{3} a_{i0}$, so that the aggregate excess demands are $\theta x_i = \frac{4\theta}{9\beta} a_{i0}^2$ and $\theta a_i = \frac{2}{3} A'_0$; and equilibrium requires $2n_f A''_0 = \frac{2}{3} A'_0$ and $\frac{1}{a_x} n_f A''_0 = \frac{4\theta}{9\beta} a_{i0}^2$, so that $n_f = \frac{1}{3} \frac{A'_0}{A'''_0}$ and $\beta = \frac{4}{3} a_{i0} a_x$. The marginal rate of substitution for every consumer results in equilibrium $\left(\frac{da_i}{dx_i} \right)_{MRS} = \frac{a_i}{x_i} = \frac{3\beta}{4a_{i0}} = a_x$, which is equal to the corresponding marginal rate for every firm. Consequently, in this case, the equilibrium allocation is Pareto optimal.

[124] In a letter to A. de Pietri-Tonelli (reported in P. de Pietri-Tonelli, ed., *Scritti paretiani*, Padova, Cedam, 1961, p. 141), Pareto highlights that the relation (109) is an identity and it is for this reason that one equation can be eliminated.

[125] Due to an oversight or a misprint, there are X and Y instead of X' and Y' .

[126] Therefore, the monopolist takes the interdependences into account, as indicated at the end of note [86].

[127] The monopolist is not included in the list of the θ consumers and is not endowed with any goods.

[128] Due to an oversight or a misprint, there is $\frac{1}{p_b} \varphi(b''')$ instead of $\frac{1}{p_b} \varphi_b(b''')$.

[129] In the paragraph that we have numbered 98 [bis]—see note [155].

[130] As can be seen, there are two paragraphs indicated with number 87. The second of them has been specified by us as 87 [bis].

[131] Pareto has the first order differential condition in mind when characterizing the maximum of ophelimity. This condition is satisfied also by the states which admit the possibility of a decrease in all ophelimities. However, he should like to exclude these states from those that determine a maximum of ophelimity. The definition of a maximum of ophelimity is indicated immediately above in the text: according to it, no shift from this position that is compatible with the constraints (of feasibility for the allocation) can increase the ophelimities of all the individuals of the community.

[132] The reference to §22 is wrong. Perhaps the correct reference is to the first of the two paragraphs indicated with number 92.

[133] Due to an oversight or a misprint, in the text of equation (113) there are $\delta\Phi_{1a}, \delta\Phi_{2a}, \delta\Phi_{3a}$ instead of $\delta\Phi_1, \delta\Phi_2, \delta\Phi_3$. From the very earliest works by Pareto on

this topic, the equation (113) is the first order condition that characterizes the maximum of ophelimity for the community (or Pareto-optimum allocation, or efficiency). In this paragraph Pareto highlights one of its implications—namely, the implication that excludes positive variations in ophelimity without there being any negative ones—but does not point out the meaning of the differential expressed by (112). This meaning will be pointed out further in the text, in §§127 ff.

[^{134]} Pareto is taking under consideration the case of variable prices (as pointed out in §83). Therefore p_x indicates the marginal price of good X . Then, π_x is its marginal cost and Π_x its total cost. The relation (114) indicates that there is a positive profit. However, contrary to what Pareto writes, the presence of positive profit in the production of commodity X is not a cause of inefficiency, since the profit pertains to the owners of the firms (who would suffer a loss if it were distributed to the other members of the community). The condition of production efficiency is that given in §92.

[^{135]} The zero-profit condition is not required by production efficiency because of what was pointed out in note [134]. On this matter, see J.R. Hicks, "Pareto and the Economic Optimum", in *Convegno Internazionale Vilfredo Pareto* (Roma, 25–27 October 1973), Roma, Accademia Nazionale dei Lincei, 1975, pp. 19–28, on p. 26.

[^{136]} Due to an oversight or a misprint, there is (109) instead of (105).

[^{137]} Due to an oversight or a misprint, the number of the equation (115) is missing.

[^{138]} This equation indicates the equality between marginal price and marginal cost at the equilibrium point. The equilibrium point gives the allocation of which the efficiency is tested (in other words, it is not necessarily the competitive equilibrium allocation, but a feasible allocation).

[^{139]} Due to an oversight or a misprint, there is φ_a instead of φ_{3a} .

[^{140]} Due to an oversight or a misprint, in the original text there is $\pi_x^0 \delta X''$ instead of $\pi_x^0 dX''$.

[^{141]} That is, this more general theory is connected to the analysis of economic efficiency.

[^{142]} As indicated in note [134] the first condition is not required in order to have a maximum of ophelimity.

[^{143]} Due to an oversight or a misprint, there is X instead of X'' .

[^{144]} This equality and the above equality (*sub 1st*), from which it derives, hold in equilibrium, they do not hold outside of equilibrium. In other words, they must contain directly X'' instead of x and, consequently, the further condition $X'' = x$ has not to be introduced.

[^{145]} Due to an oversight or a misprint, there is $p_x x = \pi_{0x} + \pi_x x$ instead of $p_x X'' = \pi_{0x} + \pi_x X''$. Indeed, this equality is required in the equilibrium position, and not in the intermediate positions.

[^{146]} Pareto excludes the possibility of efficiency if the producer is suffering a loss. In the now standard analysis, on the contrary, losses do not exclude efficiency (in the case pointed out by Pareto, efficiency requires price and marginal cost to be equal, that is, the condition $p_x = \pi_x$; a transfer equal to π_{0x} is needed in favor of the producers to balance their accounts).

[^{147]} Pareto is referring to free entry, which applies when, even though price and marginal cost are equal (which is the first order condition for a firm's profit to be

maximum), profit is not equal to zero. In that case, the number of firms increases if profit is positive, and decreases if profit is negative, so making profit tend to zero.

[148] These last two equations represent the zero-profit condition. Pareto requires this condition in competitive equilibrium, whereas he does not require marginal price and marginal cost to be equal (in the example under consideration the condition $p_x x = A''' + p_b B''' = 3 + 5p_b + (0.5 + p_b)x$ implies $p_x > 0.5 + p_b = \pi_x$). Contrary to standard analyses, as already indicated in note [122], Pareto defines competition as the situation in which there is free entry and associates the zero-profit condition to this. The inequality $p_x^0 > \pi_x^0$ (marginal price greater than marginal cost) does not determine an increase in the firm production, because this is not allowed by market conditions (that is, by the demand), as pointed out in §92 [bis].

[149] As in Pareto's time, the mantissa of the logarithm is positive, whereas the value of the characteristic is indicated with the minus sign above it, when it is negative: in other words, $\bar{1}.3$ indicates -0.7 .

[150] What is indicated by the tables is obtained from the relations

$$\frac{1}{p_x} \left(\frac{1}{\sqrt{x}} - \frac{1}{x+0.5} \right) = \frac{M}{(17 - A'')^{0.4}} = \frac{1}{p_b} \frac{N}{\sqrt{23-x}},$$

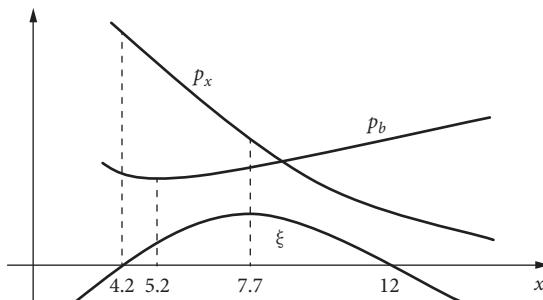
$$B'' = 5 + x, \quad A'' - A''' = (p_x - 0.5 - p_b)x - 3 - 5p_b, \quad A''' = 3 + 0.5x.$$

[151] In other words, the limit imposed by the syndicate of the suppliers of B prevents production from being increased. This effect does not eventuate when the increase in production (and therefore in the purchased quantity of B) determines an increase in the price of B .

[152] See note [154] for a representation of p_b as a function of x .

[153] Due to an oversight or a misprint, there is $\bar{1}$ instead of 0 in the characteristic of $\log p_x$.

[154] The following figure, which shows the values of p_x , p_b and ξ as functions of x , makes it easy to follow Pareto's line of reasoning on syndicate policy, as developed in this paragraph and in the subsequent ones.



[155] The figure in note [154] can be useful.

[156] The figure in note [123] can be of use in following Pareto's line of reasoning.

[157] The analysis carried out by Pareto in this paragraph and in the following ones is not the analysis that is currently followed, since Pareto uses marginal (instead of average) production coefficients and a transformation function on these coefficients. It is worthwhile, then, to compare his analysis with the standard currently followed, assuming, as Pareto himself also accepted in §78, that the production possibilities do not depend on the path and, for the sake of simplicity, that the prices of inputs are constant.

In the case of single production (which is the case examined by Pareto) the standard analysis describes the production possibilities with the production function $y = F(B_y, C_y, \dots, E_y)$. The first order conditions for minimum cost require the equalities

$$\frac{1}{p_b} \frac{\partial F}{\partial B_y} = \frac{1}{p_c} \frac{\partial F}{\partial C_y} = \dots = \frac{1}{p_e} \frac{\partial F}{\partial E_y},$$

which define, together with the production function, the conditional demand functions for inputs

$$B_y = B_y(y, p_b, p_c, \dots, p_e), \quad C_y = C_y(y, p_b, p_c, \dots, p_e), \dots, \quad E_y = E_y(y, p_b, p_c, \dots, p_e).$$

The derivatives of these functions with respect to the quantity produced (some of which could be negative)

$$b_y = \frac{\partial B_y(y, p_b, p_c, \dots)}{\partial y}, \quad c_y = \frac{\partial C_y(y, p_b, p_c, \dots)}{\partial y}, \dots, \quad e_y = \frac{\partial E_y(y, p_b, p_c, \dots)}{\partial y}$$

define the (marginal) production coefficients that minimize the production cost for y in the presence of prices p_b, p_c, \dots, p_e . These are the coefficients that in Pareto's analysis satisfy the relations (125) and the relation (121). The relation (121) can be obtained from the functions that express the marginal coefficients of minimum cost (namely, $b_y = b_y(y, p_b, p_c, \dots, p_e), c_y = c_y(y, p_b, p_c, \dots, p_e), \dots, e_y = e_y(y, p_b, p_c, \dots, p_e)$) by eliminating the prices p_b, p_c, \dots, p_e . We obtain a relation of the type $f(y, b_y, c_y, \dots, e_y) = 0$, which is the relation (121) introduced by Pareto. The relation (121) corresponds to the production function of standard analyses, in the sense that it is an alternative representation of production possibilities. The relation (121) only includes coefficients referring to inputs the conditional demand of which depends on the prices of the inputs, for which, therefore, substitution is possible. Of course, as Pareto points out further in the text, there may also be fixed coefficients (for example, of the type $a_y = 2$, if two units of good A are needed to produce one unit of good Y regardless of the quantity produced and of the prices of the inputs) and it may happen that the production possibilities of a certain good are represented by a multiplicity of relations of the type (121). The relation (121) does not highlight, among its arguments, the quantity y , probably because of what was pointed out at the end of §102: however, it is advisable to take into account that it is, in general, of the type $f(y, b_y, c_y, \dots, e_y) = 0$.

If we now examine Pareto's analysis, with its marginal production coefficients, the cost of production is $\Pi_y = \pi_{0y} + \int_0^{Y''} (a_y + p_b b_y + \dots) dy, \dots$, as pointed out in §102 and elsewhere. The problem $\min_{b_y, c_y, \dots, e_y} \Pi_y$ subject to the constraint $f(y, b_y, c_y, \dots, e_y) = 0$

requires as first order conditions the relations (125) indicated in §105, as well as the constraint. These relations lead to the same conditions required by the equations

$$\frac{1}{p_b} \frac{\partial F}{\partial B_y} = \frac{1}{p_c} \frac{\partial F}{\partial C_y} = \dots = \frac{1}{p_e} \frac{\partial F}{\partial E_y}$$

and by the standard production function, because they originate from the same problem. This result can be formally demonstrated by taking into account, on the one hand, that Shephard lemma requires $\frac{\partial \Pi_y^*}{\partial p_b} = B_y(\cdot), \frac{\partial \Pi_y^*}{\partial p_c} = C_y(\cdot), \dots$ (where Π_y^* is the minimum cost function, that is, $\Pi_y^* = \min_{B_y, C_y, \dots} p_b B_y + p_c C_y + \dots$ subject to the constraint $y = F(B_y, C_y, \dots)$). Moreover, taking into account the marginal coefficients of minimum cost $b_y = b_y(y, p_b, p_c, \dots, p_e), c_y = c_y(y, p_b, p_c, \dots, p_e), \dots$, since it is also $\Pi_y^* = \pi_0 y + \int_0^y (a_y + p_b b_y(\cdot) + p_c c_y(\cdot)) dy$, we have that $\frac{\partial \Pi_y^*}{\partial p_b} = \int_0^y b_y(\cdot) dy + \int_0^y (p_b \frac{\partial b_y(\cdot)}{\partial p_b} + p_c \frac{\partial c_y(\cdot)}{\partial p_b} + \dots) dy, \dots$, where $\int_0^y b_y(\cdot) dy = B_y(\cdot), \dots$, so that $p_b \frac{\partial b_y(\cdot)}{\partial p_b} + p_c \frac{\partial c_y(\cdot)}{\partial p_b} + \dots = 0, p_b \frac{\partial b_y(\cdot)}{\partial p_c} + p_c \frac{\partial c_y(\cdot)}{\partial p_c} + \dots = 0, \dots$

On the other hand, by deriving the relation $f(y, b_y(\cdot), c_y(\cdot), \dots, e_y(\cdot)) = 0$ with respect to prices, we obtain

$$\frac{\partial f}{\partial b_y} \frac{\partial b_y(\cdot)}{\partial p_b} + \frac{\partial f}{\partial c_y} \frac{\partial c_y(\cdot)}{\partial p_b} + \dots = 0, \quad \frac{\partial f}{\partial b_y} \frac{\partial b_y(\cdot)}{\partial p_c} + \frac{\partial f}{\partial c_y} \frac{\partial c_y(\cdot)}{\partial p_c} + \dots = 0, \dots$$

Therefore, from the relations

$$p_b \frac{\partial b_y(\cdot)}{\partial p_b} + p_c \frac{\partial c_y(\cdot)}{\partial p_b} + \dots = 0 \text{ and } \frac{\partial f}{\partial b_y} \frac{\partial b_y(\cdot)}{\partial p_b} + \frac{\partial f}{\partial c_y} \frac{\partial c_y(\cdot)}{\partial p_b} + \dots = 0, \dots,$$

we obtain the relations (125) $\frac{1}{p_b} \frac{\partial f}{\partial b_y} = \frac{1}{p_c} \frac{\partial f}{\partial c_y} = \dots = \frac{1}{p_e} \frac{\partial f}{\partial e_y}$, which make it possible, together with the relation (121), to determine, according to Pareto's procedure, the marginal coefficients of minimum cost.

For instance, let there be the production function $y = B_y^\beta C_y^\gamma E_y^\epsilon$. The corresponding conditional demand functions are $B_y = \beta \frac{k}{p_b} y^{\frac{1}{\beta+\gamma+\epsilon}}, C_y = \gamma \frac{k}{p_c} y^{\frac{1}{\beta+\gamma+\epsilon}}$ and $E_y = \epsilon \frac{k}{p_e} y^{\frac{1}{\beta+\gamma+\epsilon}}$, where $k = \left(\left(\frac{p_b}{\beta} \right)^\beta \left(\frac{p_c}{\gamma} \right)^\gamma \left(\frac{p_e}{\epsilon} \right)^\epsilon \right)^{\frac{1}{\beta+\gamma+\epsilon}}$. The marginal coefficients of minimum cost are $b_y = \frac{\beta}{\beta+\gamma+\epsilon} \frac{k}{p_b} y^{\frac{1}{\beta+\gamma+\epsilon}-1}, c_y = \frac{\gamma}{\beta+\gamma+\epsilon} \frac{k}{p_c} y^{\frac{1}{\beta+\gamma+\epsilon}-1}$ and $e_y = \frac{\epsilon}{\beta+\gamma+\epsilon} \frac{k}{p_e} y^{\frac{1}{\beta+\gamma+\epsilon}-1}$, from which one obtains $b_y^\beta c_y^\gamma e_y^\epsilon y^{\beta+\gamma+\epsilon-1} (\beta + \gamma + \epsilon)^{\beta+\gamma+\epsilon} - 1 = 0$, i.e. the relation (121). The latter and the relations (125) determine precisely the marginal coefficients of minimum cost that have been found by deriving the conditional demand functions with respect to y .

In conclusion, Pareto's analysis is correct and leads to the same relations as the current standard analysis. However, whilst the standard production function has an intuitive meaning (it indicates the maximum output that can be obtained from some inputs), for the corresponding function in Pareto's analysis, that is the (121), a similar intuitive meaning is missing.

[158] On the pursuit curve, even more than Ch. V, §11, Fig. 41 of the *Manuel* (1909), see *Cours*, Vol. I, §41, Fig. 3 and Fig. 4 and what Pareto states about forecast and adaptation errors in the following §42. See also E.N. 26.

[159] Due to an oversight or a misprint, there is (V, 82) instead of (V, 85).

[160] Due to an oversight or a misprint, in the original text there is Y instead of dY (as indicated by Pareto himself in the *Errata*).

[161] The production cost that must be minimized is the average cost. Pareto pointed this out in the *Cours* (§719 and in the last footnote of that same paragraph) and it is consistent with the current standard analysis. This is also required by the logic of Pareto's analysis, where firms achieve zero-profit in equilibrium under free competition because of free entry. Therefore, not only is average cost equal to price, but also price is equal to the minimum average cost. Here (and in the Appendix of the Italian edition of the *Manuale*, §39, relation (72)) Pareto writes the same expression he uses in the *Cours* to show how the average cost varies as the quantity produced varies. While in the *Cours* Pareto uses average production coefficients (and the condition he writes is correct), here he uses marginal coefficients and relation (126) no longer represents the condition required in order to minimize the average cost (but to minimize the marginal cost, which does not make sense in Pareto's analysis). The relation (126) should be replaced by the first order condition for the minimum average cost problem, which requires average cost and marginal cost to be equal, that is, $\frac{\Pi_z}{q_z} = a_z + p_b b_z + \dots$

[162] The main purpose of §§109–126 (partly anticipated in §§89–92 [bis]) is to demonstrate that competitive equilibrium implies that, when price and marginal cost are equal, the condition of maximum ophelimity, given by $\frac{1}{\varphi_{1a}}\delta\Phi_1 + \frac{1}{\varphi_{2a}}\delta\Phi_2 + \dots = 0$, is satisfied. It is, therefore, a discussion of what is now referred to as the first theorem of welfare economics.

[163] The relations (129), (130), and (131) are not as straightforward as Pareto believed. From §78 we obtain $\frac{da}{dX} = a_x + a_y \frac{dY}{dX} + \dots$, $\frac{db}{dX} = b_x + b_y \frac{dY}{dX} + \dots$, and therefore the relation (128) indicates $V = (p_x - a_x - p_b b_x - \dots) + (p_y - a_y - p_b b_y - \dots) \frac{dY}{dX} + \dots$. In that case, the integral $\int_0^{X'} V dX$ measures the profit of all productions and it is equal to zero because of perfect competition. (The integral is equal to zero even outside of perfect competition when one takes into account that the firms' profit pertains to the individuals). In the third of the relations (131) there is probably a wrong sign (which does not alter the subsequent analysis): it should be

$$\delta T = \left(-\frac{\partial V}{\partial Y} - \frac{dp_y}{dX} \right) \omega_y + \dots - \frac{\partial V}{\partial a} \omega_a + \left(-\frac{\partial V}{\partial b} + \frac{dp_b}{dX} \right) \omega_b + \dots$$

We obtain the relation (130) by differentiating the relation (129), that is, by putting $0 = \delta \int_0^{X'} V dX$, where V is, in general, a function of all the variables (including X'). We find

$$0 = \int_0^{X'} \delta X \left(\frac{\partial V}{\partial X} + \frac{\partial V}{\partial Y} \frac{dY}{dX} + \dots + \frac{\partial V}{\partial a} \frac{da}{dX} + \frac{\partial V}{\partial b} \frac{db}{dX} + \dots + \frac{\partial V}{\partial p_x} \frac{dp_x}{dX} + \frac{\partial V}{\partial p_y} \frac{dp_y}{dX} \right. \\ \left. + \dots + \frac{\partial V}{\partial p_b} \frac{dp_b}{dX} + \dots \right) dX + \dots - \int_0^{X'} \delta Y \frac{\partial V}{\partial Y} dX + \dots - \int_0^{X'} \delta a \frac{\partial V}{\partial a} dX \\ - \int_0^{X'} \delta b \frac{\partial V}{\partial b} dX + \dots + [V]_0^{X'} \delta X' + \delta X' \int_0^{X'} \frac{\partial V}{\partial X'} dX.$$

If we take into account, on the one hand, that the relation (128) requires

$$\frac{\partial V}{\partial p_x} = 1, \quad \frac{\partial V}{\partial p_y} = \frac{dY}{dX}, \quad \dots, \quad \frac{\partial V}{\partial p_b} = -\frac{db}{dX}, \quad \dots, \quad \frac{\partial V}{\partial X} = 0 \quad \text{and} \quad [V]_0^{X'} = 0,$$

and, on the other hand, the relations (131) (with the right sign, as indicated above), we find

$$0 = \delta R + \int_0^{X'} \delta T dX + \int_0^{X'} \delta X \frac{dp_x}{dX} dX + \int_0^{X'} \delta Y \frac{dp_y}{dX} dX + \dots - \int_0^{X'} \delta b \frac{dp_b}{dX} dX - \dots$$

Finally, since we have $\int_0^{X'} \delta X \frac{dp_x}{dX} dX + \int_0^{X'} \delta Y \frac{dp_y}{dX} dX + \dots - \int_0^{X'} \delta b \frac{dp_b}{dX} dX - \dots = [p_x \delta X + p_y \delta Y + \dots - \delta a - p_b \delta b - \dots]_0^{X'} \text{ and, because of the relation (128), } p_x \delta X + p_y \delta Y + \dots - \delta a - p_b \delta b - \dots = V \delta X + p_y \omega_y + \dots - \omega_a - p_b \omega_b - \dots = \delta U,$ we obtain the relation (130).

[164] Due to an oversight or a misprint, there is x' instead of X' .

[165] Due to an oversight or a misprint, there is dX instead of δX .

[166] Due to an oversight or a misprint, there is ω_b instead of ω_y .

[167] In the French text, p. 641, 8th line from the bottom, there is a misprint: "Quand" instead of "Quant".

[168] It is not immediately clear why it should be $\int_0^{X'} \delta T dX = 0$ "if the budget does not change no matter what path is followed to reach the point of equilibrium". An explanation can be obtained by differentiating the sum of consumers' budgets $p_x X + p_y Y + \dots - a - p_b db - \dots = 0$. If we take into account the relation (128), we obtain

$$V dX + X dp_x + Y dp_y + \dots - b dp_b - \dots = 0,$$

that is,

$$V + X \frac{dp_x}{dX} + Y \frac{dp_y}{dX} + \dots - b \frac{dp_b}{dX} - \dots = 0,$$

from which

$$\frac{\partial V}{\partial Y} + \frac{dp_y}{dX} = 0, \dots, \frac{\partial V}{\partial b} - \frac{dp_b}{dX} = 0, \dots$$

and, therefore, $\delta T = 0$.

[169] According to the condition (134), the variation $\delta X'$ serves to maintain firms' profit at zero. The profit differential is the sum of two terms: the first, δP , indicates the marginal profit obtained by determining the production quantities without taking into account their effect on prices—that is, the difference, leaving the prices unchanged, between the profit achieved with production quantities $X' + \delta X', Y' + \left(\frac{dY}{dX}\right)^0 \delta X', \dots$ and the profit achieved with the quantities X', Y', \dots ; the second, δQ , indicates the effect on the profit resulting from the variation in prices determined by the variation in the production quantities. On this matter, see also §§139–141.

[170] Due to an oversight or a misprint, there is dR instead of δR .

[171] As already pointed out in note [123], Pareto associates perfect competition with the zero-profit condition resulting from the free entry assumption, and not necessarily with the equality between price and marginal cost. Here Pareto points out that maximum ophelimity requires the latter condition.

[172] It is possible to arrive at this result in a simpler way by differentiating the sum of consumers' budgets at the point of equilibrium, according to the line of reasoning followed by Pareto in §115 [bis]. If—according to the relation (132)—we introduce the term $\delta U = p_x^0 \delta X' + p_y^0 \delta Y' + \dots - \delta A'' - p_b^0 \delta B''$, the differentiation of the sum of the budgets requires $\delta U + X' \delta p_x + Y' \delta p_y + \dots - B'' \delta p_b = 0$. The variations in the quantities for the consumers $\delta X', \delta Y', \dots, \delta A'', \delta B'', \dots$ and those in the quantities for the firms $\delta X'', \delta Y'', \dots, \delta A''', \delta B''', \dots$ are equal to each other (because of the competitive equilibrium condition). These variations and the variations in the prices $\delta p_x, \delta p_y, \dots, \delta p_b, \dots$ are constrained by the zero-profit condition, the differentiation of which requires (whether the coefficients be fixed or variable)

$$p_x^0 \delta X'' + X'' \delta p_x = \pi_x^0 \delta X'' + B_x''' \delta p_b + \dots, \quad p_y^0 \delta Y'' + Y'' \delta p_y = \pi_y^0 \delta Y'' + B_y''' \delta p_b + \dots, \quad \dots,$$

where π_x^0, π_y^0, \dots are the marginal costs and B_x''', B_y''', \dots are the quantities of good B used in the production of, respectively, X, Y, \dots , and therefore it is $B_x''' + B_y''' + \dots = B''', \dots$. If we add up these differentials, we obtain

$$(p_x^0 - \pi_x^0) \delta X'' + (p_y^0 - \pi_y^0) \delta Y'' + \dots + X'' \delta p_x + Y'' \delta p_y + \dots = B''' \delta p_b + \dots.$$

This relation, together with the differential of the sum of the budgets, requires

$$\delta U = (p_x^0 - \pi_x^0) \delta X'' + (p_y^0 - \pi_y^0) \delta Y'' + \dots,$$

and from this it can be inferred that $\delta U = 0$, whatever variations had been introduced, only if prices are equal to marginal costs.

[173] In §§112 and 113 Pareto considers the variation in two production coefficients of good Y , namely, $\delta' b_y$ and $\delta' c_y$, which are constrained to each other by the equation (121). He keeps the quantities X, Y, \dots unchanged, probably because their variations have already been examined in the previous paragraphs.

[174] In the original text, only the last two relations (136) are included in the brace.

[175] There should be $-\int_0^{Y'} \delta' K dY$ instead of $-\delta' K$ and, a bit further, δT instead of δR . Paragraph 112 must be seen as a complement to §109 that must be considered if there are

variable coefficients of production. In that case, in the second member of the (130) we also have the addends $-\int_0^{Y'} \delta' K dY - \int_0^{Y'} \delta' H dY$, and the fact that they are equal to zero—together with the other conditions pointed out in §109—leads to the result $\delta' U = 0$. At any rate, the relation $\delta' U = 0$ is demonstrated in a much easier way (still in the case of the quantities X, Y, \dots being fixed) in §113.

[176] Pareto is referring to what is now referred to as “the first theorem of welfare economics”.

[177] Due to an oversight or a misprint, in the original text there is Y''' instead of Y'' .

[178] Due to an oversight or a misprint, there is §91 instead of §92.

[179] This connection has been indicated in note [141].

[180] Pareto means that the consumers who operate according to type I maximize ophelimity on the budget constraint; in other words, $d\Phi_1 = 0, d\Phi_2 = 0, \dots$, are the first order conditions for these constrained maximum problems.

[181] The year cited in the footnoted reference appears to be wrong: instead of November 1903 it should probably read November 1902, a reference to Pareto's article “Di un nuovo errore nello interpretare le teorie dell'economia matematica”, *G.d.E.*, November 1902, pp. 401–433. (Engl. Trans. “On a New Error in the Interpretation of the Theories of Mathematical Economics”, *G.d.E.*, December 2008, pp. 515–544.) On this matter, see A. Montesano, “Il massimo di ofelimità per la collettività: definizioni, analisi, interpretazioni di Pareto e loro generalizzazioni”, in G. Busino, ed., *Pareto oggi*, Bologna, il Mulino, 1991, pp. 115–138.

[182] Due to an oversight or a misprint, there is p_x instead of p_y .

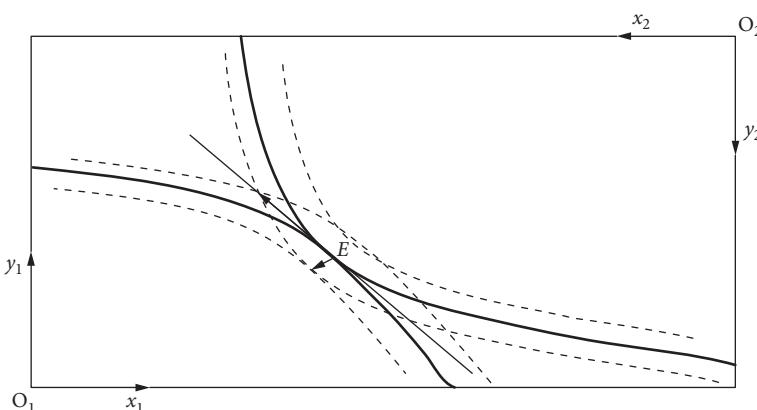
[183] Due to an oversight or a misprint, in the original text there is a_x instead of a'_x .

[184] Due to an oversight or a misprint, there is φ_{zy} instead of φ_{xy} .

[185] Due to an oversight or a misprint, there is φ_{zy} instead of φ_{xy} .

[186] Due to an oversight or a misprint, there is φ_{xx} instead of φ_{xz} .

[187] It is possible immediately to understand what Pareto is writing by looking at competitive equilibrium in the Edgeworth-Pareto box diagram. As shown by the following figure, any small displacement from the point of equilibrium (indicated by E) reduces the ophelimity of at least one consumer. They reduce the ophelimity of both consumers if the displacement is carried out along the budget constraint of the consumers.



[188] Pareto intends to establish sufficient conditions for the allocation of goods and services under competitive equilibrium to be Pareto optimal not only locally but also globally (that is, compared with all the other feasible allocations, be they equilibrium allocations or not). The conditions indicated by Pareto essentially require that the ophelimity functions be concave. Admittedly, the concavity of the ophelimity functions makes no sense if the ophelimity is ordinal, but in Pareto's line of reasoning all that is needed is their quasi-concavity (that is, the convexity of the preferences), which, however, will be introduced in economic analysis much later. On the other hand, the equilibrium allocation is Pareto optimal even if the preferences are not convex—in other words, the first theorem of welfare economics does not require that the preferences be convex (their convexity is among the sufficient conditions for the existence of equilibrium, but not among the conditions ensuring that equilibrium, which can exist even if the preferences are not convex, is Pareto optimal). On the influence that G. Scorza may have had on this analysis of the Pareto optimum in global terms, J. Chipman ("The Paretian Heritage", *Revue Européenne des Sciences Sociales, Cahiers Vilfredo Pareto*, 14, 1976, pp. 65–173) believes it to be relevant, whereas M. McLure ("The Pareto–Scorza Polemic on Collective Economic Welfare", *Australian Economic Papers*, 39, 2000, pp. 347–371) maintains the opposite view.

[189] In line with what has been previously pointed out in §124, it should be "in the case of a dependence of the first kind".

[190] In §§127–129 Pareto specifies the economic meaning of the condition

$$\frac{1}{\varphi_{1a}} \delta\Phi_1 + \frac{1}{\varphi_{2a}} \delta\Phi_2 + \dots = 0,$$

which is the first order condition for the Pareto optimum. Pareto's reasoning is carried out considering variations represented by first order infinitesimals, and it arrives at a correct result. However, by following Pareto's reasoning it would not be possible to obtain correct second order conditions or a definition also applicable to finite variations. Pareto examines an arbitrary feasible allocation to ascertain whether it generates a maximum of ophelimity for the community (that is, the allocation is efficient). Pareto modifies the allocation under consideration by introducing another feasible allocation, indicates with $\delta\Phi_1, \delta\Phi_2, \dots$ the variations of ophelimity generated by this modification, and measures them in terms of good A. Pareto says that the allocation under consideration is not optimal if the variations of ophelimity correspond to a positive total quantity of good A. In other words, the allocation under consideration is efficient if, with respect to any arbitrary feasible allocation, it is not necessary to increase the total quantity of good A in the allocation under examination to obtain the ophelimities of the other allocation. More precisely, if for each of the feasible allocations there exists an allocation which, on the one hand, is indifferent for all the individuals, and, on the other, coincides with the allocation under consideration apart from good A, of which it requires a smaller or equal total quantity, then the allocation under consideration is Pareto-optimal. If, on the contrary, one finds that the total quantity of good A is greater, then, according to Pareto, its distribution would make it possible to increase the ophelimity of all the individuals. In symbols, if we indicate with

$$\Phi_1(x_1, y_1, \dots, a_1, b_1, \dots), \quad \Phi_2(x_2, y_2, \dots, a_2, b_2, \dots), \quad \dots$$

the ophelimities obtained with the allocation under consideration, with

$$\Phi_1(x'_1, y'_1, \dots, a'_1, b'_1, \dots), \quad \Phi_2(x'_2, y'_2, \dots, a'_2, b'_2, \dots), \quad \dots,$$

the ophelimities obtained with any other feasible allocation (that is, with $X' = x'_1 - x_{10} + x'_2 - x_{20} + \dots, Y' = y'_1 - y_{10} + y'_2 - y_{20} + \dots, \dots, A' = a_{10} - a'_1 + a_{20} - a'_2 + \dots, \dots$ being compatible with the production possibilities), and with $\Delta s_1, \Delta s_2, \Delta s_3, \dots$ the variations in the quantity of good A such that

$$\Phi_1(x'_1, y'_1, \dots, a'_1, b'_1, \dots) = \Phi_1(x_1, y_1, \dots, a_1 + \Delta s_1, b_1, \dots), \quad \dots,$$

the condition for optimum requires, according to Pareto's line of reasoning, that the quantity $\Delta S = \Delta s_1 + \Delta s_2 + \Delta s_3 + \dots$ never be positive, that is, that its maximum value be equal to zero in the feasible allocations set (since it is equal to zero for the allocation under consideration). Since it is $d\Phi_1 = \varphi_{1a}ds_1, d\Phi_2 = \varphi_{2a}ds_2, \dots$, the first order condition, which is the one considered by Pareto, requires

$$dS = ds_1 + ds_2 + \dots = \frac{1}{\varphi_{1a}}d\Phi_1 + \frac{1}{\varphi_{2a}}d\Phi_2 + \dots = 0.$$

Pareto's line of reasoning, however, is not entirely correct, because the quantity dS derives from a relation of equivalence, it is not an available quantity of good A which can be distributed to consumers (both allocations under comparison are feasible and make use of the whole available quantity of good A). As shown by M. Allais (*La théorie générale des surplus*, Grenoble, Presses Universitaires, 1981, 1989), the correct concept, which he calls "maximum distributable surplus" is slightly different from the one introduced by Pareto, which he calls "maximum equivalent surplus". According to Allais, an allocation is Pareto-optimal if there is no other feasible allocation to which the allocation under consideration is indifferent for all the individuals, and which requires a smaller quantity of resources (in particular, equal quantities of the goods other than A and a smaller quantity of good A). Indeed, if it existed, it would be possible to distribute the excess quantity available of good A to the individuals and, because of the hypothesis of monotonic preferences, to arrive at an allocation that is preferred by all the individuals. In symbols, if we indicate with $\Delta \sigma_1, \Delta \sigma_2, \Delta \sigma_3, \dots$ the variations in the quantity of good A such that

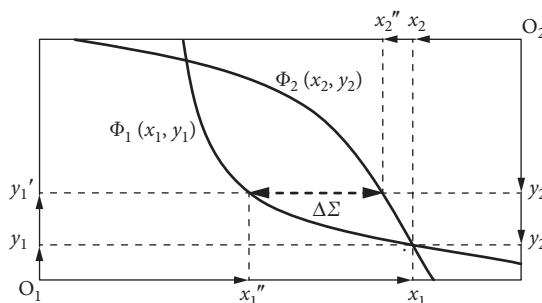
$$\Phi_1(x_1, y_1, \dots, a_1, b_1, \dots) = \Phi_1(x'_1, y'_1, \dots, a'_1 - \Delta \sigma_1, b'_1, \dots), \quad \dots,$$

the condition for optimum requires that the quantity $\Delta \Sigma = \Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3 + \dots$ never be positive, that is, that its maximum value be equal to zero in the feasible allocations set. Since $0 = d\Phi_1 - \varphi_{1a}d\sigma_1, 0 = d\Phi_2 - \varphi_{2a}d\sigma_2, \dots$, the first order condition requires

$$d\Sigma = d\sigma_1 + d\sigma_2 + \dots = \frac{1}{\varphi_{1a}}d\Phi_1 + \frac{1}{\varphi_{2a}}d\Phi_2 + \dots = 0,$$

which is identical to the condition pointed out by Pareto. The second order conditions, however, are different, as is also the related maximum problem. However, Pareto is very near to the notion of "distributable surplus" in Chap. IX §35bis, where he writes "Or, we can say that the persons suffering from a social order that departs from maximum ophelimity could, if they were allowed to reach this maximum position, pay a sum such that the new social order would be to everybody's advantage".

The definition of Pareto optimum—according to which an allocation is Pareto-optimal if there is no other allocation to which it is indifferent for all the individuals and which requires less resources—is different from the definition currently used, which we also owe to Pareto, according to which an allocation is Pareto-optimal if there is no other feasible allocation (that is, requiring the same quantity of resources) that proves to be preferred by all the individuals (as indicated in §89). The first order condition for the Pareto optimum according to the definition of maximum distributable surplus (or minimum resource use) leads to the Paretian condition $\frac{1}{\varphi_{1a}} d\Phi_1 + \frac{1}{\varphi_{2a}} d\Phi_2 + \dots = 0$; the first order condition for the Pareto optimum according to the definition of maximum ophelimity (examined by O. Lange, "The Foundations of Welfare Economics", *Econometrica*, 10, 1942, pp. 215–228) leads to the equality of the marginal rates of substitution for each pair of goods between all the agents. If the preferences of the individuals are monotonic, the two definitions of Pareto optimum are equivalent and between them there is a relation of duality (on this matter, see A. Montesano, "Equivalence and duality between the sets of Pareto-maximal allocations and the sets of maximal distributable surplus allocations, including externalities", in, P. Battigalli, A. Montesano and F. Panunzi, eds. *Decisions, games and markets*, Boston, Kluwer, 1997, pp. 239–263). In the Edgeworth-Pareto box diagram of the following figure an inefficient allocation is represented, which is such both because there is another allocation that is preferred by both consumers, and because there is another allocation to which the first one is indifferent and which requires a smaller quantity of good X (that is, we have $\Phi_1(x_1'', y_1') = \Phi_1(x_1, y_1)$ and $\Phi_2(x_2'', y_2') = \Phi_2(x_2, y_2)$, with $x_1'' + x_2'' < x_1 + x_2$ and $y_1' + y_2' = y_1 + y_2$).



^[191] As clarified in the subsequent paragraphs, the functions (152) are objective functions, that is, there are agents that operate in such a way as to maximize—or minimize—them.

^[192] Pareto will get back to this point in *Economie mathématique*, 1911.

[193] The meaning of this §139 is clarified by the subsequent §140. Pareto indicates with x_i, y_i, \dots the variables to be chosen by the agent under consideration. The system (153) corresponds to the case in which among the elements of R_i there are some quantities (which Pareto calls constants) that depend on the equilibrium values x_i^0, y_i^0, \dots but are not chosen by the agent under consideration (to him they are given quantities, or constants, or parameters). In other words, the agent determines, by choosing, some values of x_i, y_i, \dots , which are a function of the parameters of R_i . Some of these parameters are in turn determined by equilibrium conditions to which the values x_i^0, y_i^0, \dots chosen by the agent in question contribute. However, the agent performs his choice without taking into account its effects on the parameters of R_i .

[194] The system (154) corresponds to the case in which the agent takes these effects into account.

[195] Probably there should be (152) instead of (168).

[196] This note implicitly argues with G. Scorza. See E.N. 9 regarding “type 1”.

[197] The equations (156) and (157) can be exemplified in the following way. Let us consider the production choice by a firm. The profit is $px^0 - \Pi(x^0)$, where p is the price, x^0 the quantity produced and $\Pi(x^0)$ its production cost. Profit could also be indicated with the expression $\int_0^{x^0} (p - \pi(x))dx$, where $\pi(x)$ is the marginal cost function (that is, $p - \pi(x)$ specifies the function $f(x)$ used by Pareto). The price p depends on the quantity produced x^0 .

If the firm is price-taker, it does not take this dependence into account and therefore its choice is represented by the solution of the problem $\max_{x^0} \int_0^{x^0} (p - \pi(x))dx$, the first order condition of which is $p - \pi(x^0) = 0$ (relation (156)), which requires the usual equality between price and marginal cost.

If the firm is a monopolist, it takes into account the dependence of the price p on the quantity produced x^0 , which is expressed by the inverse demand function $p(x^0)$, and therefore its choice is represented by the solution to the problem $\max_{x^0} \int_0^{x^0} (p(x^0) - \pi(x))dx$, the first order condition of which is $p(x^0) - \pi(x^0) + \int_0^{x^0} \frac{dp(x^0)}{dx^0} dx = 0$ (relation (157)), that is, $p(x^0) + \frac{dp(x^0)}{dx^0} x^0 = \pi(x^0)$, which is the standard monopoly equilibrium condition.

[198] Due to an oversight or a misprint, there is §91 instead of §92.

[199] Pareto is probably referring to the second order conditions of the optimization problems the objective functions of which are index-functions. These conditions can be used to “distinguish the different kinds of equilibrium” if—as was usual in Pareto’s times—the first order conditions are indicated as equilibrium conditions, and the second order conditions as the stability ones (in that case, the second derivatives distinguish between stable and unstable equilibria). However, Pareto’s analysis of the second order conditions is flawed, both in regard to the individual choice problems (in which, for instance, Pareto considers some properties of the functions that are incompatible with the hypothesis that they are ordinal functions), and the definition of a maximum of opélimity for a community. These conditions have then been examined by the subsequent literature (among the first ones, for instance, Hicks and Allais).

[200] Here Pareto presents some types of problems that will be studied in more depth in the subsequent literature. However, it can be noted that Pareto separates the question

of the compatibility of the equilibrium conditions with each other (that is, the logical consistency of the theory or the existence of an equilibrium), from that of the determinateness of equilibrium (given by the equality between the number of equations and the number of unknowns), which is in connection with the problem of the regularity of equilibrium (and, therefore, with the problem of its local uniqueness). It is true that in Pareto's analysis the formal demonstration of the existence of equilibrium is missing. However, this demonstration requires the use of fixed point theorems, which were still unknown in Pareto's time (and one would have to wait until the 1950s to have the formal demonstration of the existence of a general competitive equilibrium). It is also true that in Pareto's approach the logical consistency of the theory (which is ensured by the existence of an equilibrium) is a topic of relatively little importance, subordinated to the goal that economic theory has—especially in Pareto's vision and in the epistemology of his times—of describing and explaining reality. (On this matter, see A. Montesano, "Approccio descrittivo-esplicativo e approccio assiomatico nella teoria dell'equilibrio economico generale", in G. Sabattini, ed. *Economia al bivio*, Milano, Franco Angeli, 1995, pp. 202–216). Furthermore, as for the regularity of equilibrium, the equality between the number of equations and that of unknowns is a condition that is still rudimentary (on this matter see, for instance, A. Mas-Colell, *The Theory of General Economic Equilibrium. A Differential Approach*, Cambridge, Cambridge University Press, 1985). Finally, the question whether "equilibrium in the conditions assumed is possible or impossible", which is not logically separated from the question of the compatibility of equilibrium conditions with each other, could have been highlighted by Pareto to show how theoretical analysis can be used to reject inconsistent proposals of social reform.

[201] The analysis of the stability of equilibrium was never undertaken by Pareto in a systematic way. Walras' analysis, based on the *tâtonnements* of the auctioneer, was later taken up again and completed by Hicks and Samuelson. Some alternative analyses without *tâtonnements* have also been introduced (for instance, F. M. Fisher, *Disequilibrium Foundations of Equilibrium Economics*, Cambridge, Cambridge University Press, 1983).

[202] Here Pareto is referring to what will later be called "comparative statics". Its first systematic application to general equilibrium was performed by J. R. Hicks, *Value and Capital*, London, Oxford University Press, 1939.

[203] Here the way is indicated which will later be followed by making use of statistical methods of hypothesis testing and econometrics.

[204] Due to an oversight or a misprint, in the original text there is y^2 instead of y'_2 .

[205] Pareto presents the case in which individuals can have the quantities they want of goods other than good A (there are no constraints on the variations $\delta x'_1, \delta y'_1, \dots, \delta x'_2, \dots$), whereas they cannot modify the quantities of good A they own.

[206] Due to an oversight or a misprint, in the original text there is Y'_1 instead of Y' .

[207] Keeping in mind that it is $\delta R = \delta X' \int_0^{X'} \frac{\partial V}{\partial X} dX$, as indicated in the relations (131), and that it is $\frac{\partial V}{\partial X'} = \frac{dp_x}{dX'} + \frac{dp_y}{dX'} \frac{dY}{dX} + \dots$ because of the relation (128).

[208] Due to an oversight or a misprint, in the original text there is dY' instead of dY'' and in the first relation (164) there is p_y instead of p_x .

[209] Due to an oversight or a misprint, there is §91 instead of §92.

[²¹⁰] Pareto is referring to an analysis that may bring to completion what he indicates in §§121–126; an analysis, that is, that may fully establish the first theorem of welfare economics in global terms. It will take time. The proposition of the first theorem of welfare economics in global terms can be found, for instance, in G. Debreu, *Theory of Value*, op. cit., 1959.

ALDO MONTESANO

Index of Authors Cited by the Editors

- Alfieri C. 580
Allais M. XXII, 542, 563, 565, 566, 569, 620, 655, 657
Allen F. 457
Amoroso L. XII, 487, 506, 525, 529, 531, 537, 538, 542–546, 565, 581, 582, 601, 609
Antonelli G.B. 422, 482, 485, 515, 536, 633
Antonucci A. 613, 615
Arena C. XX
Ariosto L. XVII, 407
Aron R. 481
Arrow K.J. 572, 623
Athenaeus 417
Aubert F. XVIII, XIX
Aupetit A. 559
- Baker G. B. 449
Banfield T. E. 552, 553, 573
Baranzini R. 536, 584
Barone E. XII, 498, 535, 536, 537, 560, 565, 566, 569, 575, 576, 577, 580, 582, 583, 604
Barucci P. 580, 583
Bastable C.F. 468, 601, 602, 605
Battigalli P. 656
Baumol W. 523, 609, 610, 612
Becattini G. XIX, 512, 514, 581, 599
Beckerath H. von 580
Bellanca N. 581, 601
Benini R. 577
Bergson A. 611
Berlin I. 481, 614, 615
Bernhart A. 437
Bertrand J. 503, 545, 605, 640, 641, 643
Bewley T. F. 473, 477
Bianco A. IX
Bini P. 636
- Block M. 424
Bobbio N. 425, 481, 513, 534, 573, 610
Bodin J. 462
Böhm-Bawerk E. von 555, 594
Bolzano B. 474, 477
Bonnet A. XII, 408, 415, 417, 427, 429, 435, 460, 596
Borgatta G. 523, 568
Bortkiewicz L. von 540
Bottai G. XX
Bouguer P. 437
Boulding K. 536
Bourdeau J. 469
Bousquet G. H. XXI, 424, 513, 534
Bracciolini P. 518
Bresciani-Turroni C. 504
Bridel P. 517, 536, 584
Bruni L. VII, VIII, IX, XIII, 568
Bryan W. J. 465
Bunge M. 514
Busino G. IX, XIII, XX, XXI, 406, 411, 420, 423, 424, 425, 469, 483, 498, 508, 512, 518, 519, 521, 551, 566, 569, 586, 597, 600, 653
- Cabiati A. 606
Cairnes J.E. 492
Caldari K. 514
Campbell-Bannerman H. 465
Cassel G. 535, 536, 609
Castagnoli E. 621
Castelnuovo G. 601
Cauderlier G. 448
Chamberlin E. 580
Cheung S. N. S. 457
Chipman J. S. VII, VIII, XIII, XIV, XXI, 474, 518, 525, 538, 559, 611, 619, 622, 623, 654

- Cicero. 411, 413, 419, 426
 Clark J. B. 516, 575, 577
 Clark J. M. 516, 547
 Cobb C. W. 433, 637
 Comte A. 512
 Cooter R. 620
 Courcelle-Seneuil J. 480
 Cournot A. XVII, XVIII, XXI, 501, 502, 524, 525, 527, 535, 538, 545, 547, 565, 570, 571, 572, 574, 575, 576, 577, 579, 581, 582, 587, 604, 605, 607, 610, 611, 613
 Creedy J. 545, 546
 Croce B. 407, 408, 479, 481, 509, 518, 519
 Dante. XIX, 410, 429, 484
 Dardi M. 512, 514, 572, 581, 594, 599
 Darwin C. XIX, 586
 Daudet L. XVIII, 466
 Davanzati B. XIX
 De Bernardi M. 607
 De Rousiers P. 422, 481, 516, 579, 603
 De Stefani A. 546
 Debreu G. 484, 623, 643, 659
 Dehem R. VII, XIII, XXI
 Del Vecchio G. 506, 517, 580, 581, 584, 585
 Demaria G. 520, 527, 548, 549, 581, 582, 603
 Di Fenizio F. 559
 Dimand R. M. 471
 Diogenes L. 410, 413, 417
 Dionysius 413, 426
 Dockès P. 579
 Dominedò V. 537, 538
 Dooley P.C. 636
 Douglas P. H. 433, 637
 Dupuit J. 571, 606, 607
 Eatwell J. XX
 Edgeworth F. Y. XVIII, XXII, 434, 443, 476, 487, 492, 493, 520, 526, 531, 541–546, 570, 575, 586, 602, 610, 631, 640, 641, 653, 656
 Einaudi L. 498, 506, 510, 518, 567, 571, 578, 579, 583, 584, 595, 607, 608, 609, 610, 611, 613, 615
 Einaudi M. M. 484, 596, 615
 des Essars P. 471
 Evans G. 565
 Fanno M. 484, 495, 506, 517, 548, 555, 595
 Fasiani M. 568
 Fechner, G. T. 433, 434
 Ferrara F. 436, 458, 492, 559, 605
 Fisher F.M. 658
 Fisher I. 471, 482, 483
 Flux A.W. 586
 Fourier C. 453
 Fournier E. 453
 Fontanelli C. 492
 Franchetti L. 492, 606
 Franchini-Stappo A. 580
 Frisch R. 580
 Furlan V. XII
 Fusco A.M. 636
 Galbraith J. K. 572, 573
 Galli S. B. 603, 615
 Gambino A. 601, 605
 Georges Demanche G. 466
 Georgescu-Roegen N. XIII, XXII, 482, 489, 494, 514, 526, 527, 542, 547, 548, 549, 550, 551, 552, 554, 555, 581, 600
 Gide C. 630
 Giocoli N. 581
 Godwin W. 451, 452
 Goldfeld G. 523, 609, 610
 Gorman W.M. 485
 Gossen H.H. 600
 Graziani A. XI, 602
 Grote G. 415, 422
 Guyot Y. 498
 Haberler G. 580
 Hansen A. 505, 511, 580
 Henderson E. F. 462
 Hesse M. 514

- Hicks R. D. 410, 417
 Hicks J. R. VII, XIII, 532, 536, 537, 548,
 566, 646, 657, 658
 Hirshleifer J. 601
 Horace 538
 Humphrey T. M. 471
 Hurwicz L. 474, 622, 623
- Jaffé W. XIII, XX, XXII, 480, 521, 540, 542,
 559, 576
 Jannaccone P. 582, 606
 Jaurès J. 412
 Jevons W.S. 482, 534, 552, 553, 554, 573,
 587, 589, 599, 600
 Johnson W.F. 537, 538
 Juglar C. 471, 511, 611
- Kant E. 479, 480
 Karlin S. 623
 Keynes J.M. XIX, XX, XXI, 480, 496, 497,
 502, 505, 508, 514, 517, 520, 527, 555,
 559, 560, 568, 576, 577, 578, 579, 580,
 584, 585, 588, 589, 590, 591, 592, 593,
 594, 595, 597, 598, 599, 600, 609, 610,
 612, 613
 Kirman A.P. 490, 522
 Kitchin J. 511
 Knight F.H. 612
 Kondratieff N.D. 511, 611, 613
- La Ferla G. 519
 La Fontaine J. 477
 Lange O. 656
 Lassalle F. 450, 463, 586
 Leontief W. 492, 581
 Leroy-Beaulieu P. 407, 448, 458, 586
 Levasseur E. 448, 452
 Liebowitz J. L. 457
 Little I. M. D. 620
 Lombardini S. XII
 Loria A. XI, 506, 602
 Lucian 614
- Machiavelli N. XIX, 481, 484, 518,
 568, 601, 615
 Machlup F. 523, 580
- Maine H. J. S. 419, 453, 454
 Malestroit J.C. de 462
 Malthus T.R. 451, 452, 586
 Manca G. 608
 Mantel R. 484
 Marchionatti R. XX, 475, 636
 Marget A.W. 580, 584, 585
 Marshall A. XIX, XXI, 449, 457, 480,
 482, 492, 496, 497, 498, 499, 501, 507,
 508, 512, 513, 514, 515, 516, 517, 518,
 521, 526, 527, 536, 540, 541, 542, 544,
 545, 546, 547, 559, 560, 561, 571, 572,
 573, 574, 575, 577, 578, 582, 583, 584,
 587, 590, 591, 592, 598, 599, 600, 601,
 602, 604, 605, 607, 609, 610, 612, 613,
 615, 636
 Martello T. 458
 Martial 433
 Marx K. 407, 411, 424, 481, 509, 519,
 550, 563, 568, 571, 572, 580, 585,
 586, 587, 588
 Mas-Colell A. 658
 Mason E. 580
 Mayer H. 553, 555
 McCord Wright D. 580
 McCulloch J. R. 463, 468
 McLure M. VII, VIII, XX, XXII, 474, 513,
 525, 568, 585, 617, 619, 621, 654
 Menger A. 585
 Menger C. 506, 554, 579
 Menger K. 554
 Milgate M. XX
 Mill J.S. 407, 412, 457, 481, 492, 497, 501,
 506, 512, 514, 517, 521, 532, 533, 584,
 592, 600, 604, 605, 606, 608, 610, 614
 Molinari G. de 463, 503
 Monroe A. E. 462
 Montesano A. VII-VIII, IX, XIX, 434,
 437, 438, 439, 442, 446, 458, 465, 466,
 482, 483, 518, 523, 526, 527, 538, 544,
 565, 566, 568, 569, 582, 584, 605, 630,
 636, 653, 656, 658
 Montesquieu 412
 Moore G.E. 585
 Morgenstern O. 511
 Mornati F. IX, XX, 475, 518, 597, 599, 636

- Morselli E. XII, XIX
 Mosca G. 587
 Moscati I. 621
 Murray R. 568
 Mussolini B. 579, 583
- Napoleoni C. XIX
 Nataf A. 485
 Neumann J. von 496
 Newman P. XX
- Page A.N. XIII
 Pantaleoni M. XIII, XVIII, XX, XI, 406,
 422, 423, 460, 461, 477, 482, 484, 493,
 497, 498, 504, 512, 515, 520, 521, 523,
 524, 534, 537, 541, 549, 551, 552, 553,
 555, 560, 566, 569–583, 592, 599, 604,
 605, 608, 609, 630
- Panunzi F. 656
 Papafava F. 508, 512, 520, 521
 Papini G. 585
 Perry Miller J. XIII
 Pertile A. 464
 Peruzzi E. XII
 Peruzzi U. 605
 Phaedrus 431
 Pietri-Tonelli A.de XX, 487, 528, 548, 626,
 645
 Pietri-Tonelli P.de 626
 Pigou A.C. 501, 609, 615–620
 Pirandello L. 573
 Plato 410, 550
 Pliny 408
 Plutarch 417, 418
 Poincaré H. 446
 Porri V. 555
 Proudhon P.J. 586
- Raffaelli T. 514, 599
 Rappoport P. 620
 Rawls J. 513, 568
 Renan E. 454
 Ricardo D. 467, 468, 497, 498, 509, 565,
 571, 577, 586, 588, 592, 595, 598, 601,
 602, 603, 604, 605, 609
- Ricci G. 482, 483, 527, 622
- Ricci U. 537, 560, 580, 609, 636
 Richter M.K. 474, 622, 623
 Robinson A. XIII, XIV
 Robinson J. 603
 Rogers J. E. T. 449, 450
 Roosevelt T. 422, 481
 Rosenstein-Rodan P. N. 553, 555, 611
- Sabattini G. 658
 Salvemini G. 583
 Samuelson P. 551, 580, 602, 605, 611, 658
 Sand G. 480
 Sanger C. P. 586
 Sartori G. 518
 Say J. B. 462, 509, 552, 583
 Schmoller G. 423, 424
 Schneider E. 557, 559, 580
 Schumpeter J. A. 502, 511, 517, 552,
 557, 563, 570, 572, 573, 577, 578, 580,
 581, 613
- Schwier S. A. XIII
 Scorza G. 472, 473, 474, 489, 496, 524,
 525, 535, 536, 545, 569, 574, 607, 632,
 654, 657
- Sensini G. XX, XXI, 482, 498, 504, 515,
 516, 517, 518, 523, 524, 535, 545, 557,
 567, 568, 584, 593
- Sinn H.-W. 433
 Sismondi J.C.L. Simone de 451, 568
 Slutsky E. 515, 531, 532, 536, 537, 538,
 635, 636
- Smith A. 407, 457, 462, 489, 559, 560,
 604, 605
- Smithies A. 580
 Sonnenschein H.F. 474, 484, 622, 623
 Sonnino S. 492, 497, 606,
 Sorce R. L. 432
- Sorel G. 409, 422, 426, 444, 463,
 481, 586
- Spencer H. 413, 414, 416, 422, 424,
 425, 573
- Sraffa P. XIII, XIV, XXI, 468, 492, 496,
 527, 560, 565, 571, 582, 588, 589, 610
- Stackelberg H. von 487, 535, 545, 546,
 581, 603, 640
- Stahl F. J. 427

- Steve S. XIX
Stigler G.J. XIII, 433, 587, 609, 610,
622, 636
Stiglitz J. E. 457, 598
Stolper W. 580
Strauss J. 454
Suppes P. 623
Swedberg R. 578
Sweezy P. 580
Sylos Labini P. XIX, 576, 581, 582
- Tallichet É. 465
Tani P. 605
Tarascio V. 512
Tertullian 411
Thornton H. 502
Thornton W. T. 431, 492, 497, 605
Tinbergen J. 520, 580
Tocqueville A. de. XIII, 453, 496
Triffin R. 489, 547, 548
Trollope F. 449
Turati F. 585
Turgot A. R. J. 586
- Usher A. 580
Vailati G. 518, 569, 610
Valenti G. 492
Villani M. 450
Vinci F. 538, 546
- Vito F. 514
Voltaire F.-M.A. de 518
Volterra V. 408, 474, 482, 499, 523, 623
- Walras L. XIII, XX, 473, 475, 476, 480,
482, 491, 496, 502, 512, 514, 515, 520,
521, 523, 536, 537, 539, 540, 544, 553,
554, 555, 556, 559, 560, 565, 569, 570,
573, 574, 576, 579, 580, 584, 585, 586,
587, 589, 591, 599, 600, 602, 609, 612,
642, 658
Weber E. H. 434
Weierstrass K. 473, 477, 489
Whitaker J. 513
Wicksell K. XIX, 433, 487, 490, 517, 580,
602, 603, 605, 610, 611
Wieser F. von. 553, 555, 583
Wundt W. 434
- Young A. 489
- Zaccherini G. 493
Zanni A. VIII, IX, XVIII, XX, 490,
512, 514, 516, 518, 520, 524, 525,
526, 527, 540, 546, 547, 555, 559,
565, 571, 580, 583, 585, 599, 602,
604, 605, 610, 621
Zattoni P. 490, 605
Zawadzki W. 484, 527, 538, 559, 611
Zeller E. 417