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The common dandelion uses a bundle of drag-enhancing bristles (the pappus) that enables seed dispersal over formidable distances; however, the scaling laws of aerodynamic drag underpinning pappus-mediated flight remains unresolved. In this paper, we study the aerodynamic shape of dandelion, derive the scaling law of resistance, determine the Vogel exponent. In particular, we find that the total drag coefficient is proportional to the -2/3 power of the dandelion pappus Reynolds number, and obtain the terminal velocity of the dandelion seed under gravitation field.

Keywords: Dandelion, pappus, flexible filament, wind-dispersal, aerodynamic shape, drag, Reynolds number, scaling laws

I. INTRODUCTION

The seeds of plants on Earth, such as dandelion, leave the mother and fly with the wind during the ripening season, and it is a very interesting phenomenon that plants use natural wind to spread seeds. After hundreds of millions of years of natural evolution, different seeds have evolved various but unique structures and thus have their own drag-enhancing pneumatic/aerodynamic behaviors. In order for seeds to spread over long distances, their structure is generally a disc structure composed of flexible filaments. During the process of seed flight, due to the action of hydrodynamic pressure, the flexible filament will deform to form an aerodynamic shape, thereby reducing the flow drag [1–13].

The common dandelion uses a bundle of drag-enhancing bristles (the pappus) that helps to keep their seeds aloft (as shown in Fig.1). This passive flight mechanism is highly effective, enabling seed dispersal over formidable distances and decreasing its terminal velocity; however, the physics underpinning pappus mediated flight had not been understood until the discovery of the separated vortex ring attached to the pappus [1].

FIG. 1: Dandelion wind dispersal.

Inspired by the amazing aerodynamics features of the dandelion, very recently, Lyer et al.[9] demonstrated wind-dispersal of battery-free wireless sensing devices.

Whether it is the experimental study of Cummins, et al.[1], or the sensing devices development of Lyer et al.[9], all need to understand the pneumatic/aerodynamic shape and drag behavior of dandelion structures. Regarding the aerodynamic drag of dandelion, their study only gave experimental scatter plots of drag versus Reynolds numbers, and did not obtain a universal scale law; As for the pneumatic/aerodynamic shape of the dandelion, there have been no relevant theoretical and experimental research reports so far.

It is not difficult to understand that the drag of dandelion is closely related to its pneumatic/aerodynamic shape. During the movement of the dandelion, its filaments are pneumatically deformed, so that the dandelion is deformed as a whole. This least drag shape is the aerodynamic shape of a dandelion.

By bending and twisting under fluid loading, on the one hand, dandelion reduce their projected area perpendicular to the flow, and, on the other hand, they also become more streamlined [5, 6]. Through these two mechanisms of reconfiguration, the drag load that dandelions must support does not grow with the square of the velocity of the flow they are subjected to – as it would on a rigid bluff body–but rather more slowly. How much slower is described with the Vogel exponent $\mathcal{V}$ [5] such that

$$\text{Drag} \sim U^{2+\mathcal{V}}, \tag{1}$$

where U is the flow velocity. However, the exponent $\mathcal{V}$ for dandelion has not been determined yet [1, 5, 6, 9].

This article will examine the aerodynamic shape and drag of dandelion. According to the structural characteristics of dandelion, assuming that all filaments are the same, the elastic deformation of each filament is first studied. To simplify, the filament is seen as a free elastic

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cantilever beam with a fixed bipartisan in the middle, and hydrodynamic pressure acts on the filament. In this way, the aerodynamic shape of the elastic filament is obtained. We regard the overall aerodynamic resistance of the dandelion as the sum of the drag of each filament, and obtain the relationship between the aerodynamic drag coefficient and the dandelion Reynolds number.

II. AERODYNAMIC SHAPE OF THE DANDELION

Dandelions have evolved mechanisms to use wind for seed dispersal over a wide area [1–5] including creating lightweight diaspores with plumose or comose structures that act as drag-enhancing parachutes [7–9]. A bundle of flexible filaments bend in a wind in order to absorb deformation energy and reduce the drag force of the wind on the dandelion as shown in Fig.1. The flexible filaments immersed in a flowing medium will adjust its shape to counteract flow drag to minimizes drag by reducing, or delaying, the turbulence in boundary layer of the flow nearest the moving body.

Similar to a flexible fibre modelling in [1, 9, 13], the dandelion pappus (a bundle of filaments) structure is regarded as a porous disk composed of many flexible filaments as shown in Fig.2.

Consider a flexible filament in the flow medium as shown in Fig.3. The filament is modelled as a thin, inextensible elastic beam loaded by the difference in fluid pressure p between its upstream and downstream sides. The flow characteristic velocity is U and no wake is considered, thus the flexible filament acted fluid pressure $[p] \approx \frac{1}{2}\rho U^2$ uniformly.

The Euler-Bernoulli beam under fluid dynamic pressure was formulated by Alben et al. [10, 11], Sun and Guo [13], and can be used to dandelion flexible filament. According Sun and Guo [13], leads to a single ordinary differential equation (ODE): $J_c\frac{d^2\kappa}{ds^2}+\frac{1}{2}J_c\kappa^3=\frac{1}{2}\rho U^2 d_c\cos\theta$. If introducing $\bar{s}=s/L_c$ and $\bar{\kappa}=d\theta/d\bar{s}=L_c\kappa$, The ODE and corresponding boundary conditions can be nondimensionalized as to the form:

$$\frac{d^2\bar{\kappa}}{d\bar{s}^2}+\frac{1}{2}\bar{\kappa}^3=\eta^2\cos\theta, \quad \bar{\kappa}_{\bar{s}=\frac{1}{2}}=\left(\frac{d\bar{\kappa}}{d\bar{s}}\right)_{\bar{s}=\frac{1}{2}}=0, \qquad (2)$$

where $\eta=\left[\frac{\frac{1}{2}\rho U^2 L_c^2 d_c}{(J_c/L_c)}\right]^{1/2}$ is called Alben-Shelly-Zhang number [10, 13], d is the diameter of the filament, $J_c = EI_c$ is the bending rigidity, the Young modulus is E, area moment of inertia is I_c, $[p]=\frac{1}{2}\rho U^2$ is the fluid pressure jump across the filament.

The aerodynamic shape of the fibre can be reconstructed by relations: $\frac{d\bar{x}}{d\bar{s}}=\cos\theta(s)$ and $\frac{d\bar{y}}{d\bar{s}}=\sin\theta(s)$, where $\bar{x}=x/L_c$, $\bar{y}=y/L_c$, and shown in Fig.4.

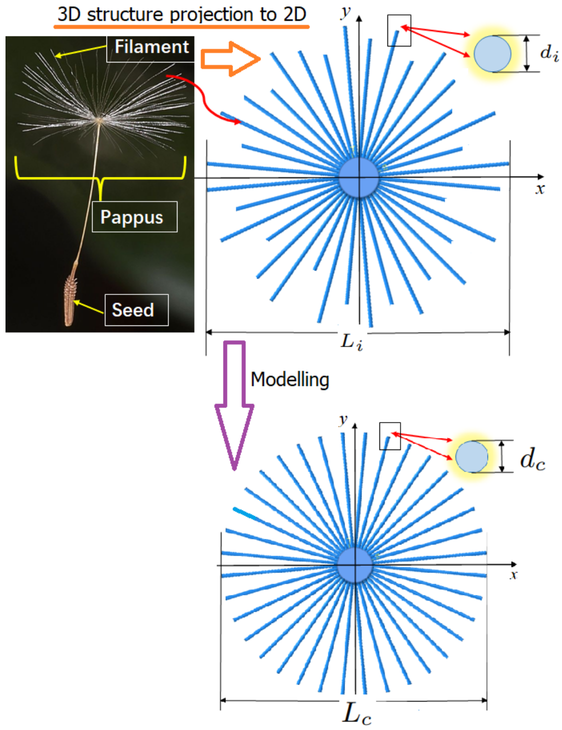

FIG. 2: The 3D structure pappus is modelled as a planner disk that is consist of filaments with different lengths l_i and diameters d_i. Transforming the disk to a model with a characteristic length $L_c=\frac{\sum_{i=1}^N L_i}{N}$ and a characteristic diameter $d_c=\frac{\sum_{i=1}^N d_i}{N}$, where N is total number of filaments.

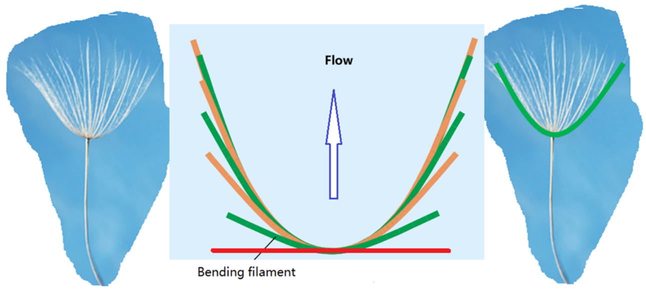

FIG. 3: The filament is supported by a thin stainless-steel rod, which is clamped at one end. Fluid drag force acting on the filament deflects this support slightly downwards.

III. SCALING LAW OF DRAG AND TERMINAL SPEED OF THE DANDELION

Similar to the analysis of a flexible fibre in a flowing medium [13], we can also deal with this problem by dimensional method. There are 6 quantities in this problem, namely D, ρ, J_c, ν, U, A_c, where ν is kinematic vis-

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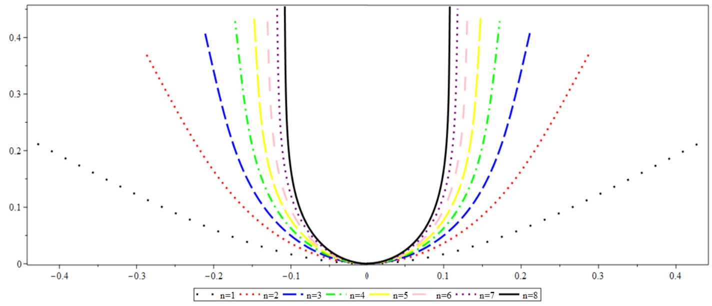

FIG. 4: The aerodynamics shape $(\bar{x}, \bar{y})$ for different parameter $\eta = 6n$, $n = 1, 2, 3..., 8$, the aerodynamic shape is symmetric to the y axis. The aerodynamic shape is a revolution of 2D curve in this figure.

cosity and $A = L_c d_c$. The quantity dimensions are listed in the following table I:

TABLE I: Dimensions of physical quantities

Variables	Notation	Dimensions
Filament's Drag	D_f	$\mathrm{M}\ell\mathrm{T}^{-2}$
Mass density	ρ	$\mathrm{M}\ell^{-3}$
Rigidity	J_c	$\mathrm{M}\ell^3\mathrm{T}^{-2}$
Kinematic viscosity	ν	$\ell^2 t^{-1}$
Flow velocity	U	$L\ell^{-1}$
Area($= L_c d_c$)	A_c	ℓ^2

The dimensional basis used is length (ℓ), mass (M) and time (T).

The drag D_f can be expressed as the function of quantities ρ, E, ν, U, A_c, namely,

$$D_f = f(\rho,\ E, \nu,\ U,\ A_c). \tag{3}$$

In the above relation, there are 6 quantities, two of them are dimensionless. Since only 3 dimensional basis L,M,T are used, so according to dimensional analysis of Buckingham [14], Eq. 3 produces 3 dimensionless quantities Π. The first one is: $\Pi_D = D\rho^a U^b A^c = \ell^0 M^0 T^0$,, we have $\Pi_{D_f} = \frac{D_f}{\frac{1}{2}\rho U^2 A_c}$, Similarly, for J_c, we have $\Pi_J = \frac{J_c}{\frac{1}{2}\rho U^2 A_c^2}$ and for ν, we have $\Pi_\nu = \frac{\nu}{\rho U \sqrt{A_c}}$.

From Buckingham Π theorem [14–17], the Eq.3 can be equivalency expressed as $\Pi_{D_f} = f(\Pi_J, \Pi_\nu)$, namely

$$D_f = \frac{1}{2}\rho U^2 A f(\frac{J_c}{\frac{1}{2}\rho U^2 A_c^2}, \Pi_\nu). \tag{4}$$

For a single characteristic filament, we can propose an approximate $f(\frac{J_c}{\frac{1}{2}\rho U^2 A_c^2}, \Pi_\nu) \approx C_0 \left(\frac{J_c}{\frac{1}{2}\rho U^2 A_c^2}\right)^\alpha \Pi_\nu^\beta$, therefore

$$D_f \approx C_0 \frac{1}{2}\rho U^2 A_c \left(\frac{J_c}{\frac{1}{2}\rho U^2 A_c^2}\right)^\alpha \Pi_\nu^\beta, \tag{5}$$

where C_0 is a constant, α and β are exponents to be determined by experiments.

From Alben, Shelley and Zhang [10, 11], Sun and Guo [13], for $\eta \le 1$, we have $\alpha = 0$, $\beta = 0$, for $\eta \ge 1$, we have $\alpha = \frac{1}{3}$, $\beta = 0$, therefore the drag of single filament is given by

$$D_f = \begin{cases} C_1 \frac{1}{2}\rho L_c d_c U^2, & (\eta \le 1), \\ C\left[(\frac{1}{2}\rho)^2 J L_c d_c\right]^{1/3} U^{4/3}, & (\eta \ge 1). \end{cases} \tag{6}$$

where C_1, C are a constants.

Comparing with the Vogel law in Eq.1, we can determine the Vogel exponent as follows: $\mathcal{V} = -2/3$ in case of $\eta \ge 1$, and $\mathcal{V} = 0$ in case of $\eta \le 1$.

Since the pappus of the dandelion is consist of N filaments, the total drag of the dandelion is the summation of each filament, therefore we have the total drag of the dandelion

$$D = ND_f = \begin{cases} NC_1 \frac{1}{2}\rho L_c d_c U^2, & (\eta \le 1), \\ NC\left[(\frac{1}{2}\rho)^2 J_c L_c d_c\right]^{1/3} U^{4/3}, & (\eta \ge 1). \end{cases} \tag{7}$$

Thus we can get the total drag coefficients of the dandelion as follows

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 N L_c d_c} = \begin{cases} C_1, & (\eta \le 1), \\ C\left[\frac{J}{\frac{1}{2}\rho U^2 (L_c d_c)^2}\right]^{1/3}, & (\eta \ge 1). \end{cases} \tag{8}$$

in which, the case of $\eta \le 1$ is corresponding to the rigid filament. we will not going to use C_1.

With the total drag in Eq.7, if we assume the weight of total dandelion seed is mg, then we have balance relation $mg = D \approx ND_f$, which gives the terminal velocity U of the dandelion as follows

$$U = C_U (mg)^{3/4}, \tag{9}$$

where the coefficient $C_U = (NC)^{-3/4}(\frac{1}{4}\rho^2 J_c L_c d_c)^{-1/4}$. The analytical expression of the terminal velocity has not been seen in literature and whose log profile is depicted in Fig.5.

The Reynolds number is a non-dimensional parameter characterizing the relative importance of inertial to viscous forces in a fluid. The flow through and around the pappus involves two different Reynolds numbers: that of the entire pappus ($Re = \rho U L_c/\mu$, in which U is the velocity of the seed, L_c is the characteristic diameter of the pappus and μ the dynamics viscosity of the fluid) and that of an individual filament ($Re_f = \rho U d_c/\mu$). The study of Cummins et al.[1] revealed that the pappus of a dandelion benefits from a 'wall effect' at low $Re_f = \rho U d_c/\mu$. Neighbouring filaments interact strongly with one another because of the thick boundary layer around each filament, which causes a considerable reduction in air flow through the pappus.

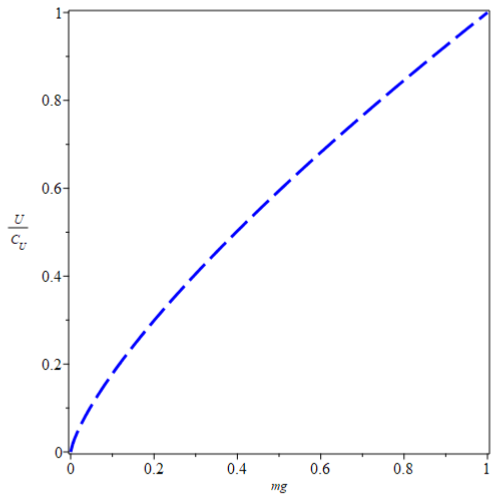

FIG. 5: Terminal velocity vs. mg.

From the Reynolds numbers of the entire pappus $Re = \rho U L_c/\mu$, we have $U = \frac{\mu Re}{\rho L_c}$. Replacing the velocity U in the 2nd expression of Eq.8, we have the total drag coefficient in terms of entire pappus Reynolds number as follows

$$C_D = C\left(\frac{2\rho J_c}{\mu^2 d_c^2}\right)^{1/3} Re^{-2/3},\ (\eta \geq 1). \tag{10}$$

The expression reveals that the total drag of the dandelion is proportional to $-2/3$ power law of the pappus Reynolds number, namely $C_D \sim Re^{-2/3}$.

If assume all filaments are circular solid cross-section, then the area moment of inertia I can be calculated as $I = \pi d_c^4/64$, inserting it to Eq.10, we have

$$C_D = C\left(\frac{\pi \rho E d_c^2}{32\mu^2}\right)^{1/3} Re^{-2/3},\ (\eta \geq 1). \tag{11}$$

To verify our scaling law in Eq.10, we depicted comparisons with Cummins et al. [1] and Lyer, et al. [9] in Fig. 6 and Fig.7, respectively.

With the help of our scaling law in Eq.10 and data fitting, we obtain $C_D = f(Re)$ approximate analytical relations for Cummins et al. [1]. The solid lines are from our formula Eq.10, for blue solid line $C_D = 350Re^{-2/3}$ and for red solid line $C_D = 250Re^{-2/3}$ as shown in Fig. 6.

In the same way, we obtain $C_D = f(Re)$ approximate analytical relations for Lyer et al. [9]. The solid lines are from our formula Eq.10, for green solid line $C_D = 380Re^{-2/3}$, for blue solid line $C_D = 295Re^{-2/3}$ and for red solid line $C_D = 250Re^{-2/3}$ as shown in Fig.7.

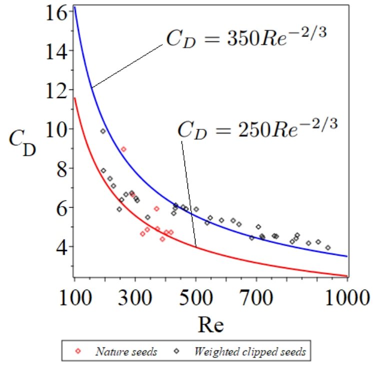

FIG. 6: Comparisons with experimental data provided Cummins et al. [1]. The drag coefficient C_D for natural (red filled circle) and artificially weighted/clipped (black filled circle) dandelion seeds as a function of Re.

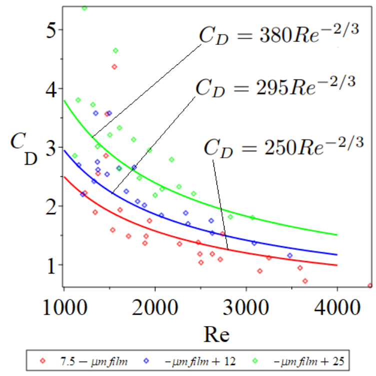

FIG. 7: Comparisons with experimental data provided Lyer et al. [9]. The drag coefficient C_D for disk with film thickness 7.5μm (red filled circle), for disk with film thickness 12μm (blue filled circle) and for disk with film thickness 25μm (green filled circle) dandelion seeds as a function of Re.

IV. CONCLUSIONS AND PERSPECTIVES

Our study obtained not only the aerodynamic/aerodynamic shape of the filament, but also the universal drag scaling laws of the dandelion, as well as the terminal velocity of the dandelion seed under gravitation field.

To the best of the authors' knowledge, this is the first detailed study of dandelion's drag in the context of the dimensional analysis. The investigation showns that the drag of the dandelion is proportional to $-2/3$ power law of the pappus Reynolds number, namely $C_D \sim Re^{-2/3}$, which reveals that the softer filament the smaller the drag, which is the secret of reducing drag by fine hairs [?].

For insight perspectives, due to the generality of the scale law we obtain using dimensional analysis, we can even bravely predict that the resistance coefficients of all structures with flexible filaments obey the same law as $C_D \sim Re^{-2/3}$.

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Data availability The data that support the findings of this study are available from the corresponding author upon reasonable request.

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