G&S3-11

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Background

Suppose that ordinary aspirin has been found effective against headaches 60 percent of the time, and that a drug company claims that its new aspirin with a special headache additive is more effective. We can test this claim as follows: $H_0 = 0.6$, $H_1 > 0.6$.
We give the aspirin to n people to take when they have a headache. We want to find a number m, called the critical value for our experiment, such that we reject H_0 if at least m people are cured, and otherwise we accept it. There are t types of error.

- Type 1 error:

$$P = \alpha(p) = \sum_{m \le k \le n} b(n, p, k) \quad = \textit{Type 1 error}$$

  We want to choose m so as to make $\alpha(p)$ small, to reduce the likelihood of a type 1 error.

- Type 2 error:

$$P = 1 - \alpha(p) = 1 - \sum_{m \le k \le n} b(n, p, k) \quad = \beta(p)$$

  We want to choose m so as to make $\beta(p)$ small, to reduce the likelihood of a type 2 error.

Figure 3.7

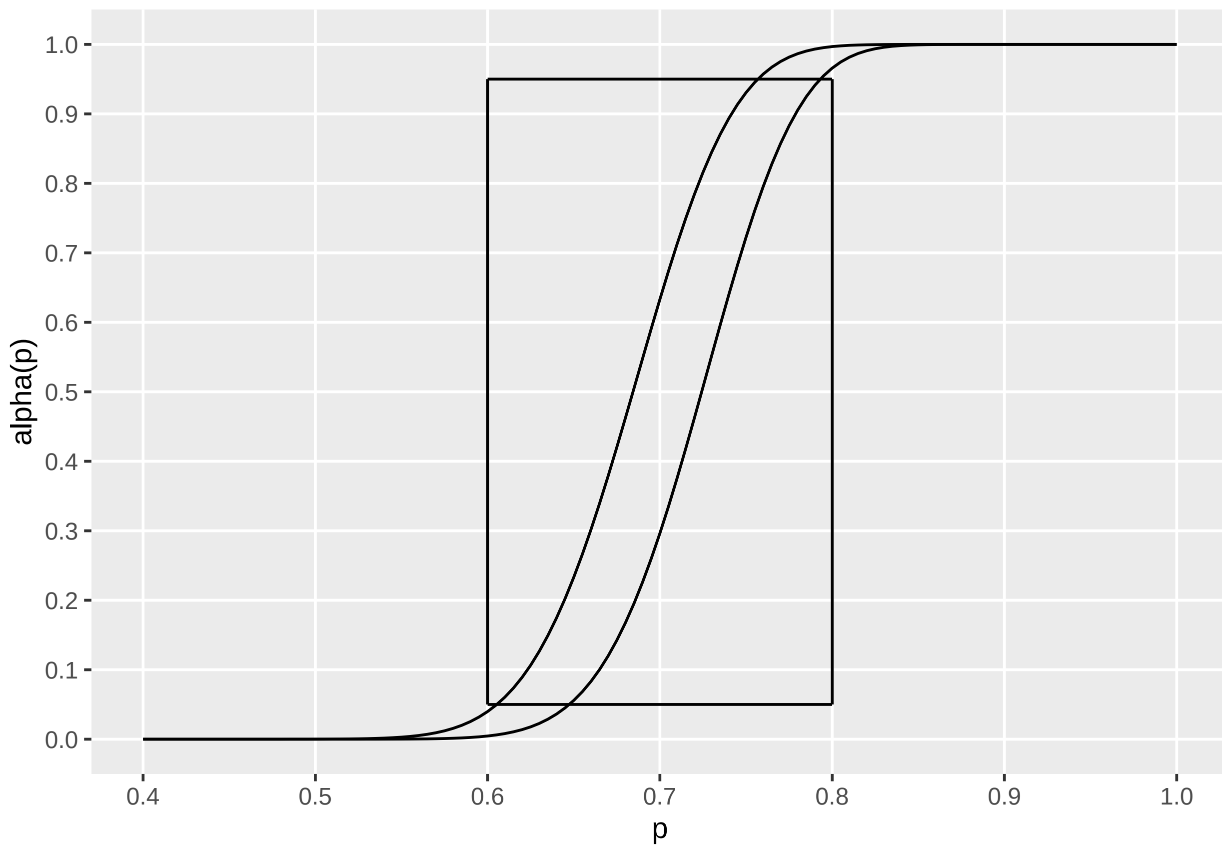
In Figure 3.7, the author set n = 100, plot the $\alpha(p)$ for p ranging from 0.4 to 1. The left line is when m = 69 and right line is when m = 73.

```r
PowerCurve <- function(p,m){
  sum <- 0
  for (i in m:100) sum = sum + dbinom(i, 100, p)
  return(sum)
}

x <- seq(.4, 1, .005)
y1 <- PowerCurve(x, 69)
y2 <- PowerCurve(x, 73)

ggplot() +
  geom_line(mapping = aes(x = x, y = y1)) +
  geom_line(mapping = aes(x = x, y = y2)) +
  scale_x_continuous(limits = c(.4, 1), breaks = seq(.4, 1, .1), minor_breaks = NULL) +
```

```r
  scale_y_continuous(limits = c(0, 1), breaks = seq(0, 1, .1), minor_breaks = NULL) +
  xlab("p") +
  ylab("alpha(p)") +
  geom_segment(aes(x = 0.6, y = 0.95, xend = 0.8, yend = 0.95)) +
  geom_segment(aes(x = 0.6, y = 0.05, xend = 0.8, yend = 0.05)) +
  geom_segment(aes(x = 0.6, y = 0.05, xend = 0.6, yend = 0.95)) +
  geom_segment(aes(x = 0.8, y = 0.05, xend = 0.8, yend = 0.95))
```



Critical value

We want $\alpha_{0.6}(m) < 0.05$ and $\beta_{0.8}(m) = 1 - \alpha_{0.8}(m) < 0.05$.

We now print the value of $\alpha_{0.6}(m)$.

```
##      m alpha0.6.m.
## 1   60 0.543294486
## 2   61 0.462075341
## 3   62 0.382187657
## 4   63 0.306809762
## 5   64 0.238610714
## 6   65 0.179469353
## 7   66 0.130336529
## 8   67 0.091253601
## 9   68 0.061503910
```

```
## 10 69 0.039847884
## 11 70 0.024782823
## 12 71 0.014775318
## 13 72 0.008432533
## 14 73 0.004600434
## 15 74 0.002395665
## 16 75 0.001189001
```

The cutpoint is 69. So $m >= 69$ satisfy $\alpha_{0.6}(m) < 0.05$.

- If we need $\alpha_{0.8}(m) > 0.95$, we now print the value of $\alpha_{0.8}(m)$.

```
##     m alpha0.8.m.
## 1  65   0.9998529
## 2  66   0.9996639
## 3  67   0.9992631
## 4  68   0.9984496
## 5  69   0.9968703
## 6  70   0.9939407
## 7  71   0.9887510
## 8  72   0.9799798
## 9  73   0.9658484
## 10 74   0.9441673
## 11 75   0.9125246
## 12 76   0.8686468
## 13 77   0.8109128
## 14 78   0.7389328
## 15 79   0.6540332
```

The cutpoint is 73. So, we need $m <= 73$ to satisfy $\alpha_{0.8}(m) > 0.95$.

So

$$69 <= m <= 73$$

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