MA103 - Class 2

Friday 15/10/2021

GENERAL REMARKS

→ PLAGIARISM VS COLLABORATION
→ SUBMIT ASSIGNMENTS ON TIME
→ SUBMIT FILES THAT ARE NOT TOO LARGE (< 5 Mb)

PROBLEM 1

1a) $A = \{ m \in \mathbb{N} \mid m \text{ is an odd integer} \}$
$B = \{ m \in \mathbb{N} \mid m^2 \text{ is an odd integer} \}$

1b) $A' = \{ m \in \mathbb{R} \mid m \text{ is an odd integer} \}$
$B' = \{ m \in \mathbb{R} \mid m^2 \text{ is an odd integer} \}$

⇝ ① Try to understand what A, A', B, B' are

$\in$	0	1	5	-11	$-\sqrt{7}$	π
A						
A'						
B						
B'						

② Decide which containment hold and what you need to prove

WANT: $A = B$ $\qquad$ $A' \neq B'$

NEED: $A \subseteq B \wedge B \subseteq A$ $\qquad \wedge \qquad$ $A' \subseteq B' \wedge B' \nsubseteq A'$

$\exists x \in B' \cup A'$ s.t. $x \in B', x \notin A'$

③ PROOF: We start with a claim. $\qquad$ (MISTAKES)

(BAD HABITS)

CLAIM $\forall m$ we have that m is an odd integer if and only if m^2 is an odd integer.

⇝ Let m be an odd integer. By definition, $m = 2k+1$ for some $k \in \mathbb{Z}$.
Then, we have $m^2 = (2k+1)^2 = \dots = 2(2k^2+2k)+1$.
Since $2k^2+2k$ is an integer, we are done.
Let us now show that if m^2 is odd, then m is odd. We can show the counterpositive, which is we can show that $\forall m \in \mathbb{N}$, m not odd implies m^2 not odd...

Can you find all the mistakes?

LESSONS → You NEED to justify your statements. *
→ "$\forall x$ we have" has NO meaning.
It is always "$\forall x \in X$"!

* E.g. "We have that $A \subseteq B$ since all the elements of A are also elements of B".
"We have that $A \subseteq B$ since the square of an odd integer is still an odd integer"

These are NOT proofs and will get you 0 in an exam.

PROBLEM 2 $X = \{ S \mid S \subseteq \{0,1\}\}$.
If we call $Z := \{0,1\}$ we have $X := P(Z)$. But $X \neq Z$.

Rem: 1) $X = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$

2) Q: • How many elements does $P(X)$ have?
• Can you name a few?
• what about $P(P(X))$?

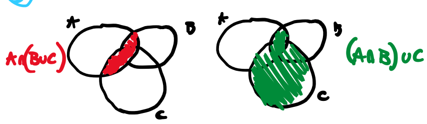
LESSON: → Sets are not always intuitive, but they are mechanical
(therefore, do try to get experience)

PROBLEM 3 For all sets A, B, C we have $A \cap (B \cup C) = (A \cap B) \cup C$

⇝ Do the following proofs both work? Which does/does not?

① For an element x in the universal set, consider the propositions
$a(x) := x \in A$
$b(x) := x \in B$ $\quad c(x) := x \in C$
We can show with a truth table that $a \wedge (b \vee c) \neq (a \wedge b) \vee c$
Indeed, we have
$a \; b \; c \;|\; a \wedge (b \vee c) \quad (a \wedge b) \vee c$

② Consider the sets $A = B = \emptyset$ and $C = \{1\}$.
We have
$A \cap (B \cup C) = \emptyset \cap (\emptyset \cup \{1\})$
$= \emptyset \neq \{1\} = (\emptyset \cap \emptyset) \cup \{1\}$
$= (A \cap B) \cup C$

③


It suffices to find A, B, C such that $A \cap (B \cup C) \neq (A \cap B) \cup C$.
This is true every time that

0 0 1 | 0 1 1

Therefore $a \wedge (b \vee c) \not\equiv (a \wedge b) \vee c$

1 $(C \cap B) \setminus A$ or $C \setminus (A \cup B)$ are non-empty.
We can take an example in which this happens.

WRONG Let $p(x) := 12 | x$,
$z(x) := 3 | x$, $q(x) := 6 | x$.
Using the same method we could prove
$\neg [(p \wedge q) \Rightarrow z] \Rightarrow [(p \vee q) \Rightarrow z]$ *
which is false.

By considering

p	q	z	$[(p \wedge q) \Rightarrow z] \Rightarrow [(p \vee q) \Rightarrow z]$
0	1	0	0

LESSON → An existential proof without an EXPLICIT example is NOT an existential proof.

PROBLEM 4 $s :=$ least among the largest elements in each row
$t :=$ largest among the least elements in each column.

Prove that $s \geq t$.

⇝ Consider the unique number k that is in the same row of s and in the same column of t.
Since s is in the same row of k, we have $s \geq k$,
Since t is in the same column of k, we have $t \leq k$.

LESSON → Not all problems with easy solutions are easy

EXTRA PROBLEM Consider $f(n) := n^2 - n + 41$. Is $f(n)$ prime for $n = 1, 2, 3$?
- Is it true that $\forall n \in \mathbb{N}$, $f(n)$ is prime?
- Are there inf. many n for which $f(n)$ is not prime?
- ——— " ——— is prime?

Q Can we find a polynomial g with constant term $c > 1$ such that $g(n)$ is always prime?
What if $c = 0$? What if $c = 1$?