Class 7

Question 1.

You are considering setting up a firm to produce widgets. The cost of the project is $30 today. The demand for widgets is uncertain. It can be either high or low with equal probability. When the demand is high cash flows in t = 1 are $66 and when the demand is low cash flows in t = 1 are $34. The discount rate is 10%.

a) What is the NPV of the project?

b) Suppose you can commission a study that tells you whether the demand for widgets will be high or low. The study takes one year to complete. That is, if you commission the study you must decide in t = 1 whether to invest. If you invest the cash flows will arrive in t = 2. What is the maximum amount you are willing to pay for the study today?

Setting: $P_0 = 30\$$; $P_1 = \begin{cases} 66\$ \\ 34\$ \end{cases}$; $r = 0.1$

a) $NPV = -30\$ + \frac{\bar{P}_1}{1.1} = \dots = 15.45\$$

b) Value of the project WITH THE KNOWLEDGE :

$$NPV^{1}_{\text{with knowledge}} = \begin{cases} u \begin{cases} -30 + \frac{66}{1.1} = 30 & \text{if invest} \\ 0 & \text{no invest} \end{cases} \\ d \begin{cases} -30 + \frac{34}{1.1} = 0.91 & \text{invest} \\ 0 & \text{no invest} \end{cases} \end{cases}$$

So $NPV_0 = \frac{0.5 \times 30 + 0.5 \times 0.91}{1.1} = 14.05$

But $14.05 < 15.45$, so there is no incentive in waiting.
You would NOT pay for the study!

Question 2.

You are considering setting up a firm to produce gadgets. The demand for gadgets can be high, medium, or low with equal probability. The corresponding cash flows are:

Demand	Annual Cash Flows
High	600
Medium	0
Low	-600

These cash flows will begin one year after the investment is made and continue forever. The cost of the project is $300 today. The discount rate is 50% (yes, that's not a typo).

a) What is the NPV of the project?

b) Suppose you can commission a study that tells you what the demand for gadgets will be. The study takes two years to complete. What is the NPV of the project if you commission the study?

c) Suppose that you can commission a different type of study that takes only one year to complete. The drawback of this type of study is that the information is less precise than in part b). The result of the study will be either "positive" or "negative" with equal probability. When the result is positive demand will be high with probability 2/3 and medium with probability 1/3 (and low with probability zero). When the result is negative demand will be low with probability 2/3 and medium with probability 1/3. What is the NPV of the project if you commission this type of study?

d) Which type of study, if any, should you commission?

Settings: $P_0 = 300\$$, $P_1 = \begin{cases} 600\$ & 1/3 \\ 0\$ & 1/3 \\ -600\$ & 1/3 \end{cases}$ $r = 0.5$

a) $NPV = -300 + \frac{\frac{1}{3}600 + \frac{1}{3}(-600)}{0.5} = -300$

b) $NPV^2_{\text{with knowledge}} = \begin{cases} u\ (1/3): & -300 + \frac{600}{0.5} = 300 \text{ invest} \\ & 0 \text{ not} \\ m\ (1/3): & -300 + 0 = -300 \text{ invest} \\ & 0 \text{ not} \\ l\ (1/3): & -300 - \frac{600}{0.5} = -1500 \text{ invest} \\ & 0 \text{ not} \end{cases}$

So $NPV = \frac{NPV^2_K}{(1.5)^2} = \frac{\frac{1}{3}\cdot 300 + \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot 0}{(1.5)^2} = 133.33\$$

c) $NPV^1_{\text{with part. knowledge}} = \begin{cases} + \ (1/2): & -300 + \left(\frac{2}{3}\cdot\frac{600}{0.5} + \frac{1}{3}\frac{0}{0.5}\right) = 300 \text{ invest} \\ & 0 \text{ not} \\ - \ (1/2): & -300 + \left(\frac{2}{3}\cdot\frac{-600}{0.5} + \frac{1}{3}\frac{0}{0.5}\right) = -1100 \text{ invest} \\ & 0 \text{ not} \end{cases}$

$NPV^0_{Ph} = \frac{NPV^1_{Ph}}{1.5} = \frac{\frac{1}{2}\cdot 300 + \frac{1}{2}0}{1.5} = 166.67\$$

d) $166.67\$ > 133.33\$$

Question 3.

You are considering buying a machine to produce widgets (again!). It takes you one year to fine-tune the machine so that it produces exactly the kind of widgets you want. The machine costs $40,000 today and produces 100,000 widgets in year 2. (You cannot produce in year 1 because you have to fine-tune the machine first.) Expected revenues are $1 per widget. The cost of producing widgets depends on the type of gas the machine uses. The machine uses one unit of gas per widget produced. In its current version the machine runs on a gas called "Gas A". The price of "Gas A" in year 2 is uncertain and will be known only at the beginning of year 2. This price will be either $0.75 or $0.25 per unit of gas with equal probability. All revenues and costs (except the cost of the machine) accrue at the end of year 2. The discount rate is 10%.

a) What is the NPV of the project?

b) The manufacturer of the machine offers you a device that can be attached to the machine. You will have to buy the device together with the machine, that is, today. The device allows the machine to run on either "Gas A" or "Gas B" (a different type of gas). If you buy the device you can choose between the two gases at the beginning of year 2 (after observing the prices). The prices of the two gases are:

- With probability 0.5 the price of "Gas A" will be \$0.75 per unit and the price of "Gas B" will be \$0.80 per unit.
- With probability 0.5 the price of "Gas A" will be \$0.25 per unit and the price of "Gas B" will be \$0.20 per unit.

How much would you be willing to pay for the device?

c) Suppose now instead that the prices of the two types of gas are:

- With probability 0.5 the price of "Gas A" will be \$0.75 per unit and the price of "Gas B" will be \$0.20 per unit.
- With probability 0.5 the price of "Gas A" will be \$0.25 per unit and the price of "Gas B" will be \$0.80 per unit.

How much would you be willing to pay for the device now?

Setting: $P_0 = 40$ $P_1 = 0$ $P_2^+ = 100.$ $P_2^- = \begin{cases} -75 & 0.5 \\ -25 & 0.5 \end{cases}$ $r = 0.1$

a) $NPV = -40 + \frac{100\left[(1-0.75)\,0.5 + (1-0.25)\,0.5\right]}{1.1^2} = 1322.3\,\$$

b) $P^{(2)}_{\text{with device}} = \begin{cases} 1 \text{ (1/2)} \begin{cases} 100\,(1-0.75) = 25 \\ 100\,(1-0.80) = 20 \end{cases} & \text{A} \\ & \text{B} \\ 2 \text{ (1/2)} \begin{cases} 100\,(1-0.25) = 75 \\ 100\,(1-0.2) = 80 \end{cases} & \text{A} \\ & \text{B} \end{cases}$

$NPV_{\text{with device}} = -40 + \frac{25 \times 0.5 + 80 \times 0.5}{1.1^2} = 3388.4\,\$$

So the value of the switch is $3388.4\,\$ - 1322.3\,\$ = 2066.1\,\$$

c) EXACTLY THE SAME PROCESS