Ramsey number of P_{3n}^2 and C_{3n}^2

P. Allen, D. M., B. Roberts, J. Skokan

The Ramsey Problem

A first example

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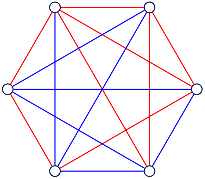

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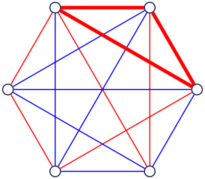

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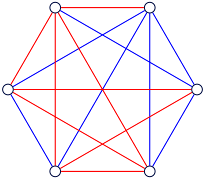

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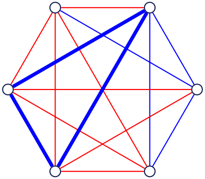

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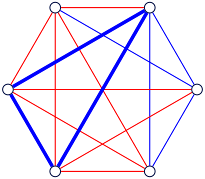

The 2-colours Ramsey problem

Let H be a graph, what is the smallest K_n for which any {red, blue}-edge-colouring of K_n allows a monochromatic copy of H?

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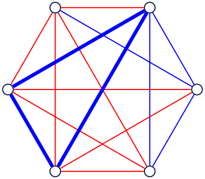

The 2-colours Ramsey problem

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Because we can colour K_5 as , we write $R(K_3, K_3) = 6$.

Ramsey for paths and stability

Ramsey number of the path (Gerencsér, Gyárfás, 1967)

$$\forall n \geq 2,\ R(P_{2n}, P_{2n}) = 3n - 1.$$

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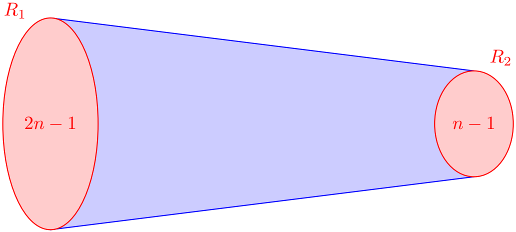

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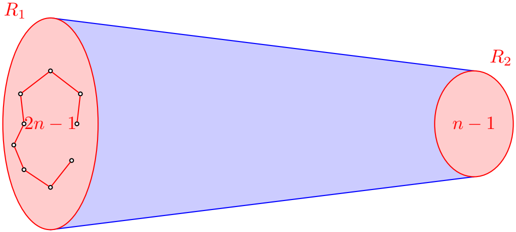

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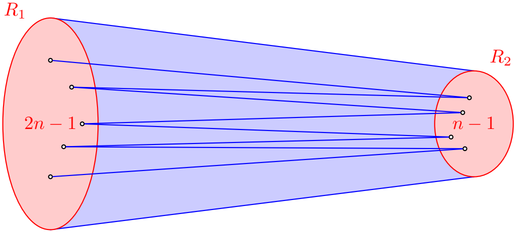

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Thm (Gyárfás, Sárközy, Szemerédi, 2009)

In any 2-colouring of the edges of $K_{(3-\epsilon)n}$ we can either find a monochromatic path substantially longer than $2n$, or the colouring is close to the extremal colouring.

Ramsey for square of paths

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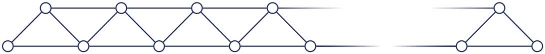

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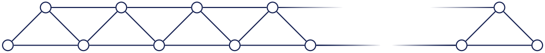

Thm (Allen, M., Roberts, Skokan, 2022+)

For n large enough, $R(P_{3n}^2, P_{3n}^2) = 9n - 3$.

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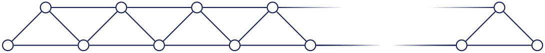

Thm (Allen, M., Roberts, Skokan, 2022+)

For n large, $R(P_{3n}^2) = R(P_{3n+1}^2) = R(P_{3n+2}^2) - 4 = R(C_{3n}^2) = 9n - 3$.

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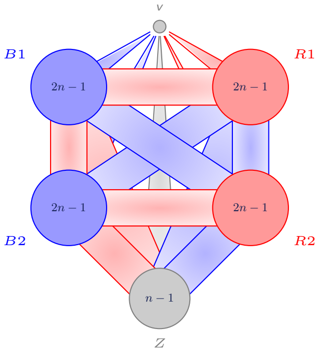

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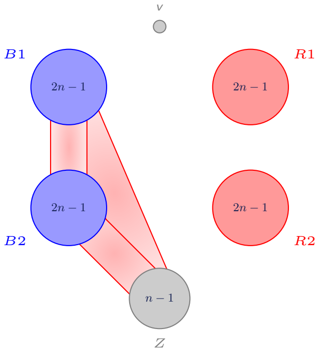

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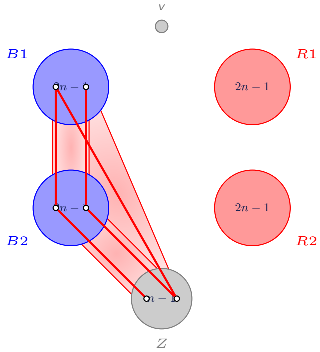

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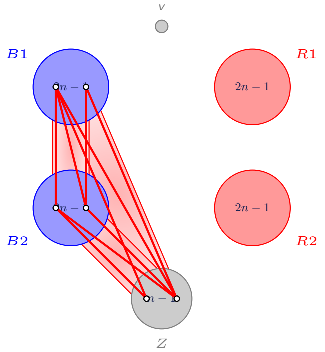

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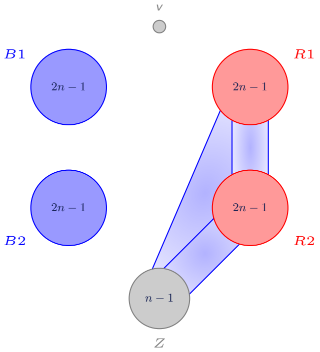

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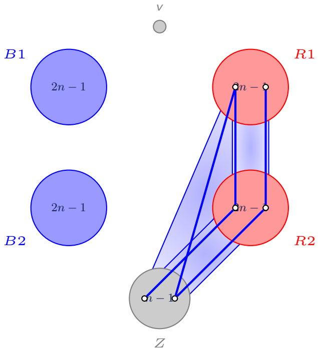

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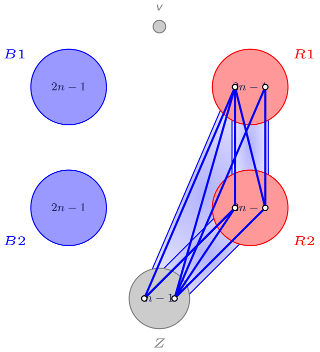

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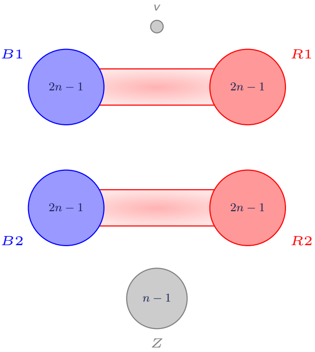

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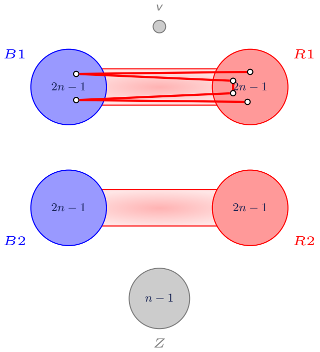

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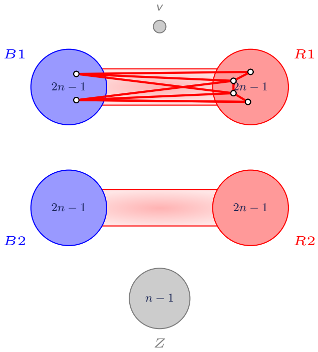

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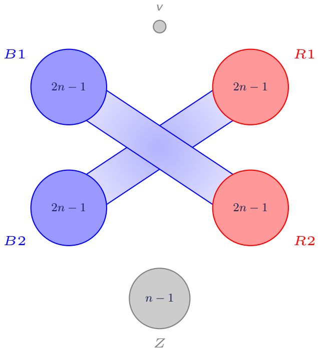

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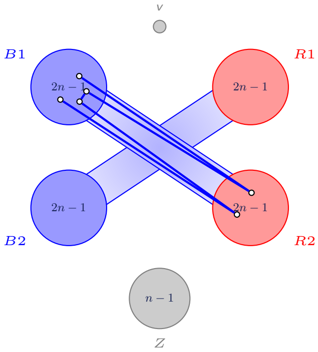

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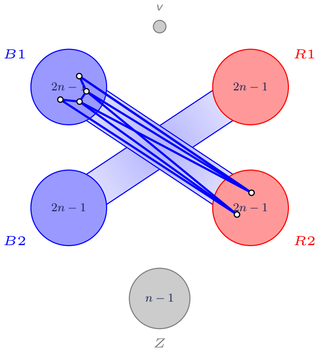

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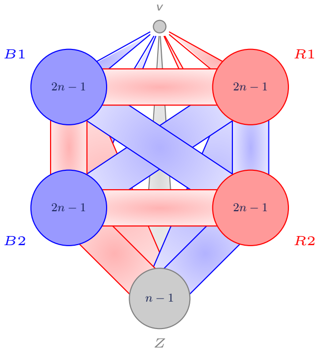
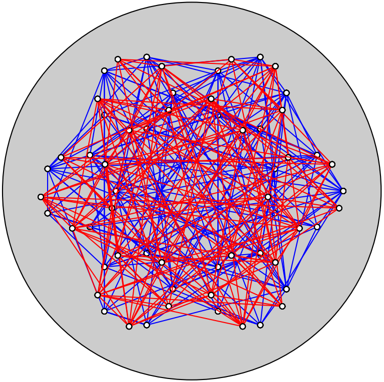


Regularity Method



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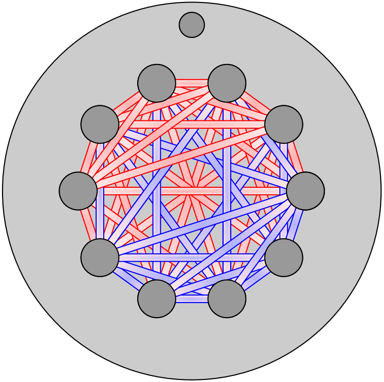
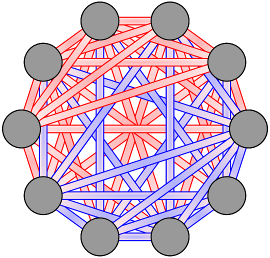


A large triangle-connected triangle factor

We want to find a large P^2.



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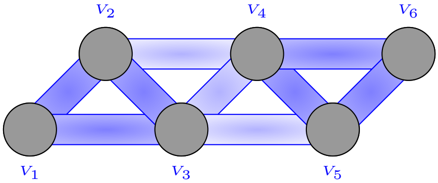

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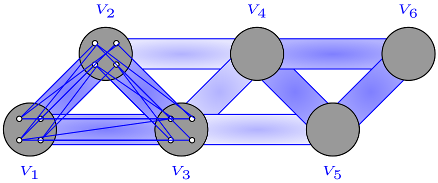

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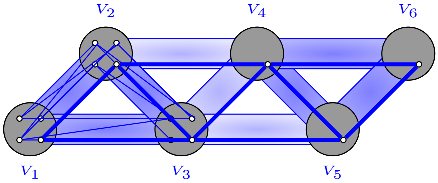

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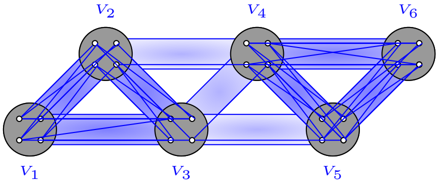

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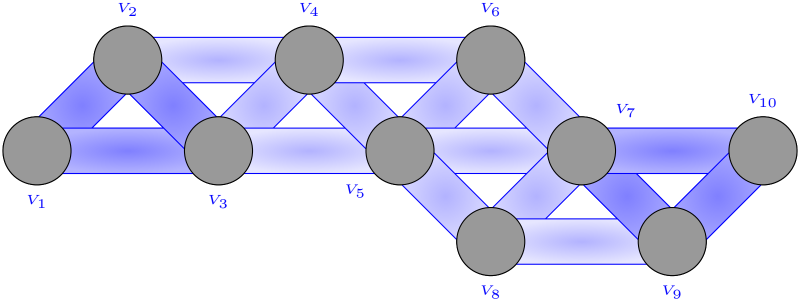
What if the situation is more complicated?



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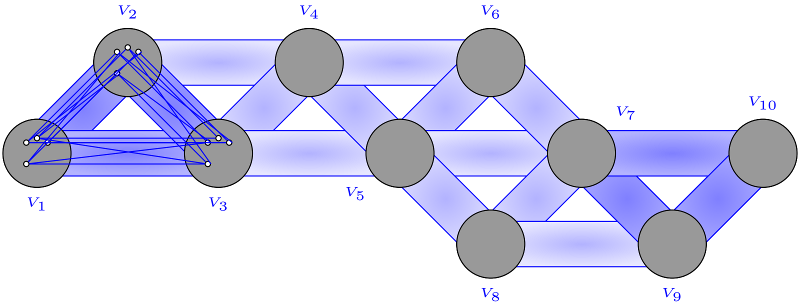

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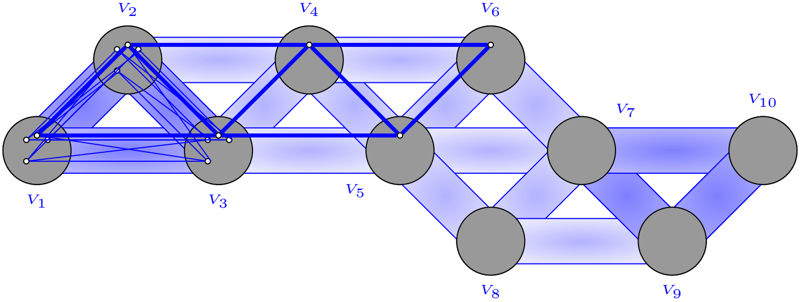

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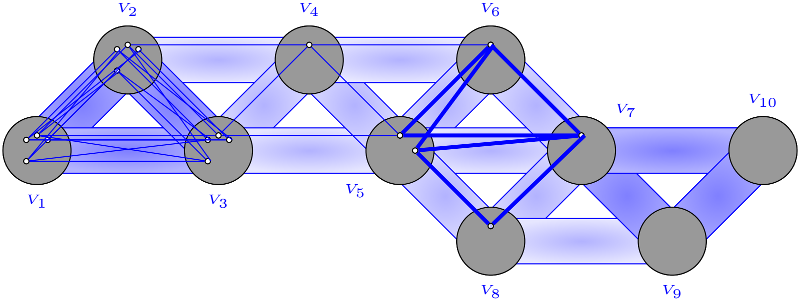

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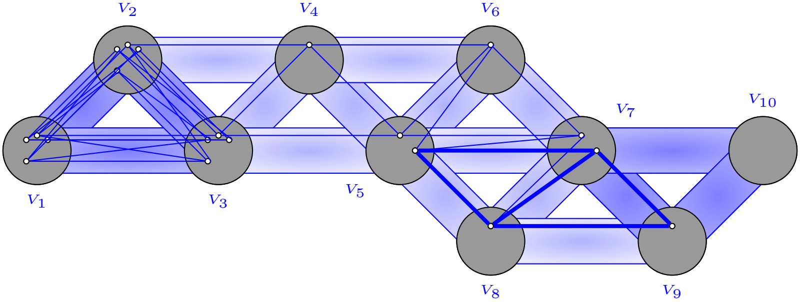

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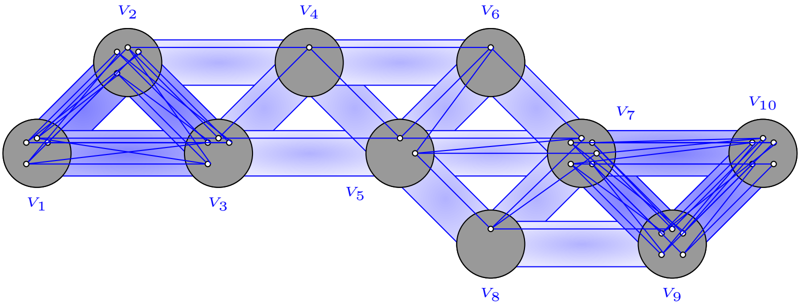

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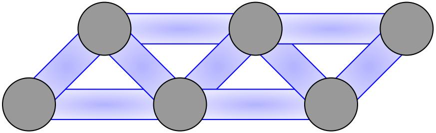
It suffices to find a triangle-connected triangle factor on $(3+\epsilon)n$ vertices.



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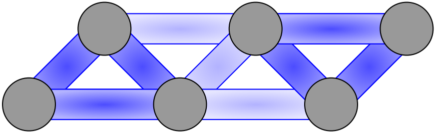

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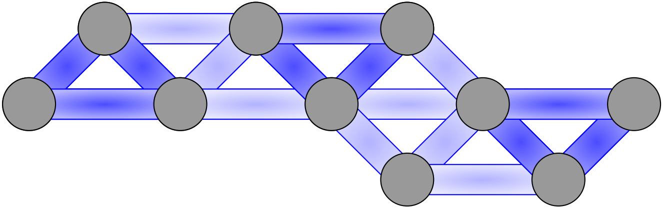

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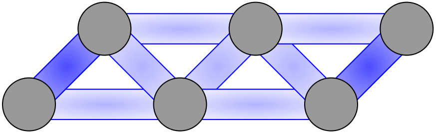

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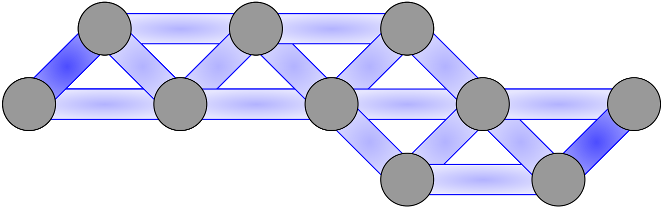

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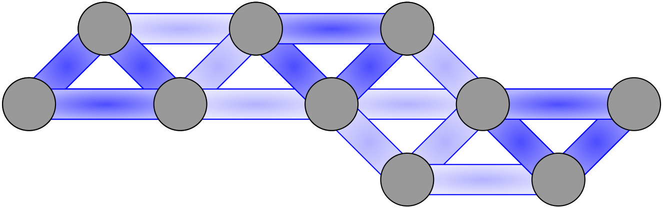
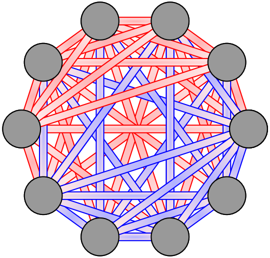


Recap

What if we cannot find a large TCTF?

Recap

What if we cannot find a large TCTF? Then we can deduce some properties of the reduced graph.



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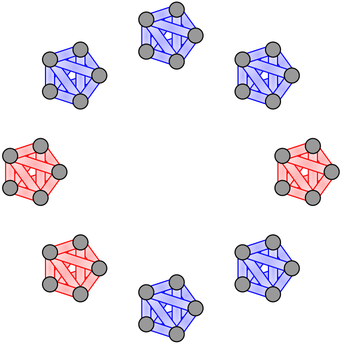
If we cannot find a large TCTF:

① We partition our graph into monochr. K_m using Ramsey's Thm,



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If we cannot find a large TCTF:

1. We partition our graph into monochr. K_m using Ramsey's Thm,
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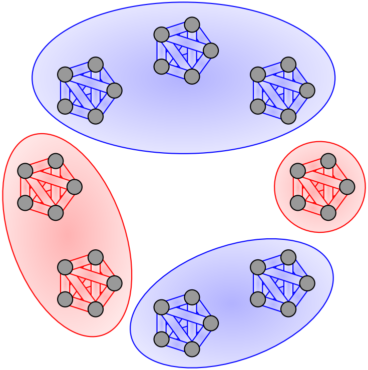

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If we cannot find a large TCTF:

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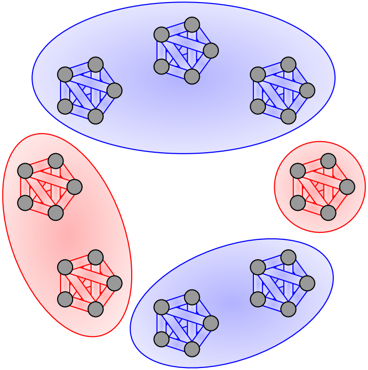
3. This last partition has some nice properties:



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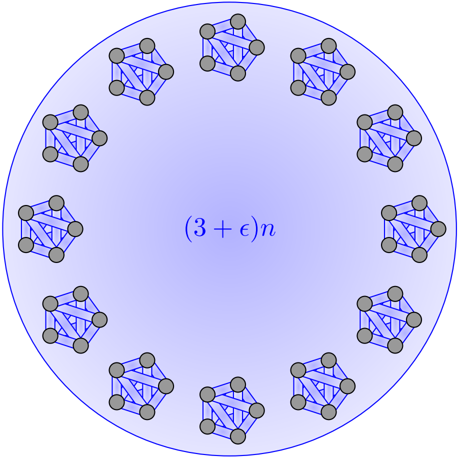
3. This last partition has some nice properties:
   1. no components is too large,



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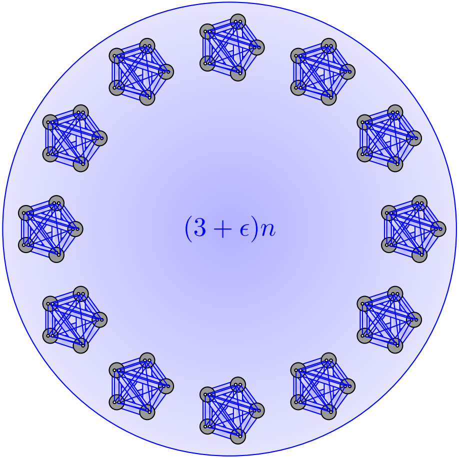

Recap

3. This last partition has some nice properties:
   1. no components is too large,
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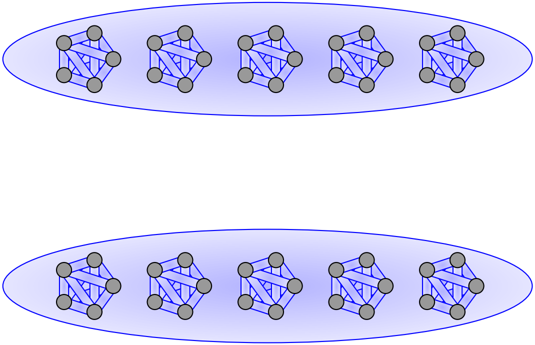

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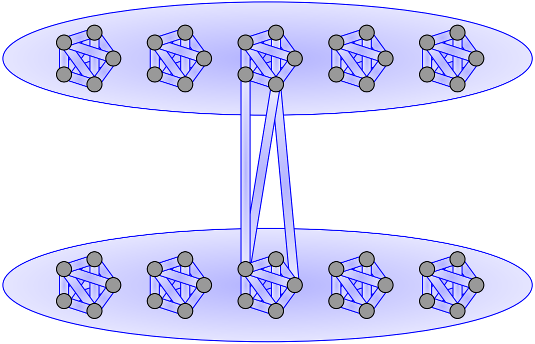

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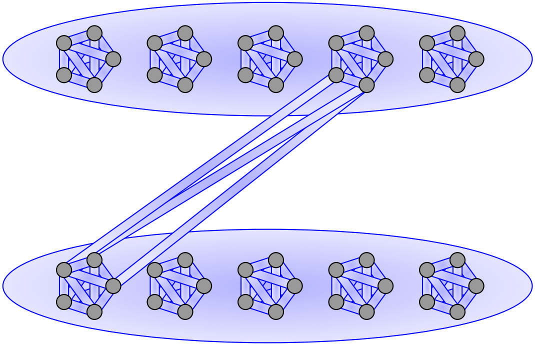

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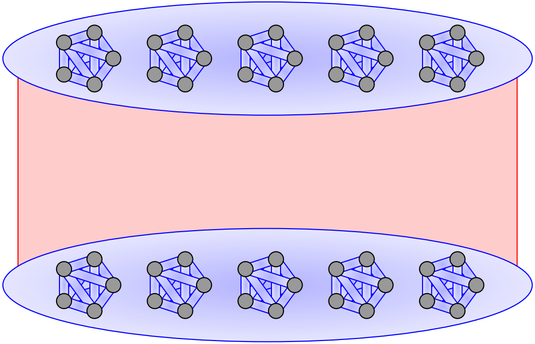
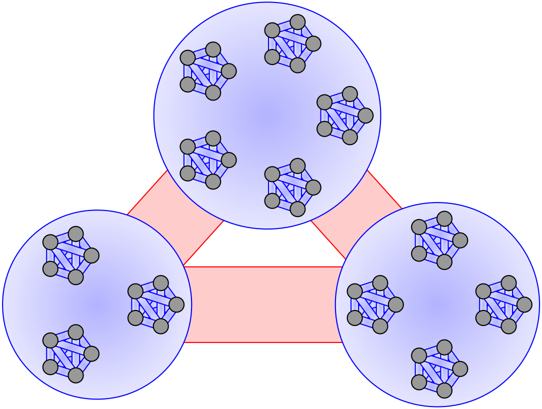


Recap

3. This last partition has some nice properties:
   1. no components is too large,
   2. between two blue components, most edges are red,
   3. the third largest blue component is not too large.



Recap

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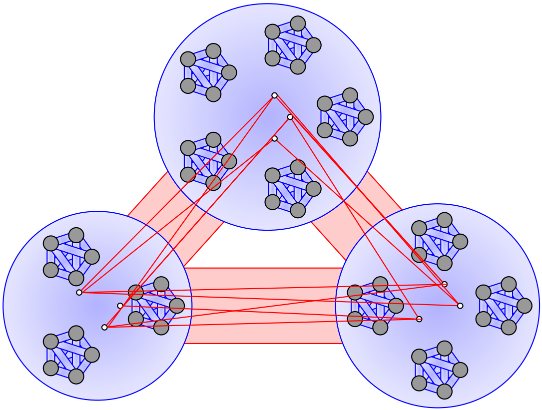

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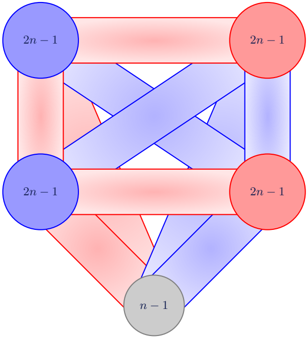

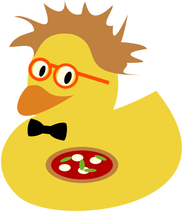


Thank
You!!