Partition universality

P. Allen, D.M.C., J. Böttcher

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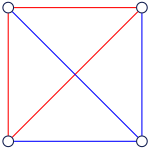

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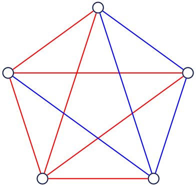

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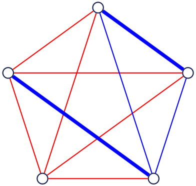

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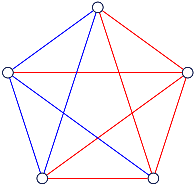

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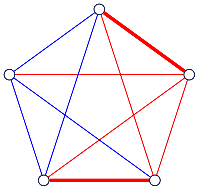

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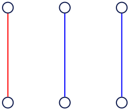

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$\hat{r}_3(2K_2)$

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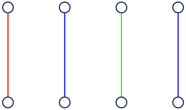
$\hat{r}_3(2K_2) = 4$



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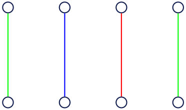

Bounds

Thm. (Chvátal, Rödel, Szemerédi and Trotter, 1983)

If $\Delta(F) \le \Delta$, then:

$$e(F) \le \hat{r}_r(F) \le O(n^2).$$

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Thm. (Beck, 1983)

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Thm. (Friedman and Pippinger, 1987)

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Thm. (Haxell, Kohayakawa and Łuczak, 1995)

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Thm. (Clemens, Miralaei, Reding, Schacht, Taraz, 2019)

- We have $\hat{r}_r(F) \sim e(F)$ when F is a path, or a tree, or a cycle, or a power of a path.

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Thm. (Berger, Kohayakawa, Maesaka, Martins, Mendonca, Mota, Parczyk; Kamčev, Liebenau, Wood, Yepremyan; 2021)

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Thm. (Rödl and Szemerédi, 2000)

- We have $\hat{r}_r(F) \sim e(F)$ when F is a path, or a tree, or a cycle, or a power of a path, or has bounded treewidth.
- Graphs F with $\Delta(F) = 3$ and $\hat{r_2}(F) = \Omega(n(\log(n))^{\frac{1}{60}})$.

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  - Improved for $\Delta = 3$ to $O(n^{\frac{8}{5}})$.

Bounds

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Any graph F in $\mathcal{G}(\Delta, n)$ has

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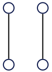

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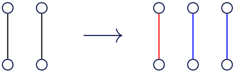

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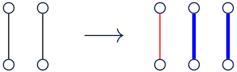

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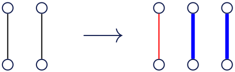

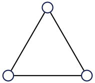

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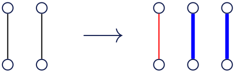

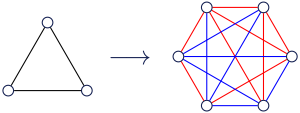

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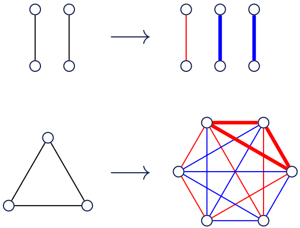

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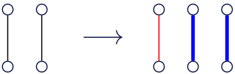

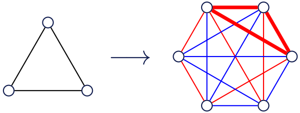

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Opt. 2 *(stronger)* : Find a graph that works for all.

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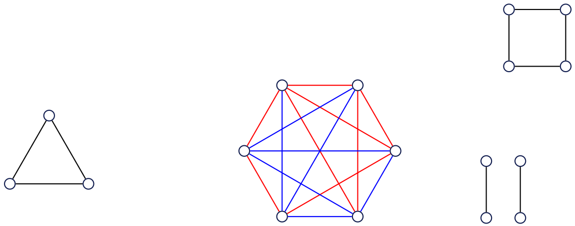

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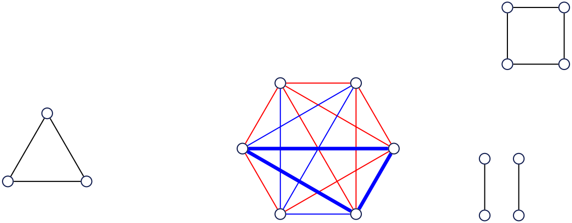

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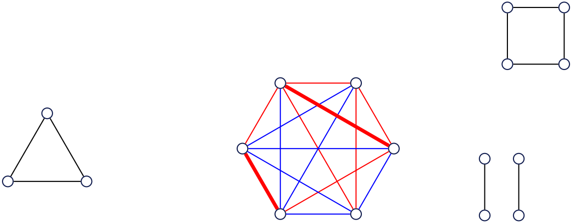

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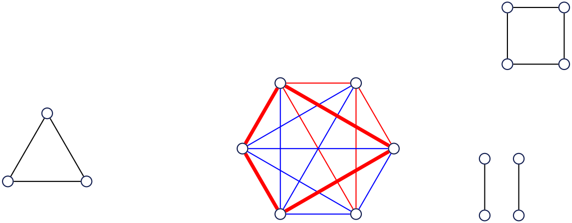

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There is Γ $(e(\Gamma) = \dots)$ s.t. $\Gamma \to_r F$ for each $F \in \mathcal{G}(\Delta, n)$.

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There is a graph Γ ($e(\Gamma) = \dots$) s.t. any r-colouring has a colour class χ containing $\mathcal{G}(\Delta, n)$.

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We say that Γ is r-partition universal for $\mathcal{G}$.

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Thm. (Kohayakawa, Rödel, Schacht and Szemerédi, 2011)

For the appropriate random graph $G(N, p)$, a.a.s. any r-colouring has a colour class χ containing $\mathcal{G}(\Delta, n)$.

Partition universality

We say that Γ is r-partition universal for $\mathcal{G}$.

Summary

Summary

While studying upper bounds for $\hat{r}_r(\mathcal{G}(\Delta, n))$ one realises that the p for which we can prove $G(N, p)$ is r size-Ramsey for $\mathcal{G}(\Delta, n)$ are the same for which $G(N, p)$ is r-partition universal for $\mathcal{G}(\Delta, n)$.

Our results

GOAL: Study partition universality properties of random graphs.

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Thm. (Allen, Böttcher, 2022)

A.a.s. $G(N, N^{\mu - \frac{1}{\Delta-1}})$ is r-partition universal for $\mathcal{G}(\Delta, cN)$.

Our results

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Rem: Better bounds for $\Delta = 3$.

Our results

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A.a.s. $G(N, N^{\mu - \frac{1}{\Delta-1}})$ is r-partition universal for $\mathcal{G}(\Delta, cN)$.

Coro. (Allen, Böttcher, 2022)

For any $F \in \mathcal{G}(\Delta, n)$,

$$\hat{r}_r(F) = O(n^{2+\mu-\frac{1}{\Delta-1}}).$$

Our results

GOAL: Study partition universality properties of random graphs.

Lemma/Thm. (Allen, Böttcher, 2022)

A.a.s. $G(N, N^{\mu-\frac{1}{D}})$ is r-partition universal for $\mathcal{G}(D,\Delta,cN)$.

Our results

GOAL: Study partition universality properties of random graphs.

Lemma/Thm. (Allen, Böttcher, 2022)

A.a.s. $G(N, N^{\mu - \frac{1}{D}})$ is r-partition universal for $\mathcal{G}(D, \Delta, cN)$.

Rem: By first moment method we cannot take $p = o(N^{-\frac{1}{D}})$.

Our results

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Thm. (Allen, Böttcher, M.C., 2023+)

A.a.s. $G^{(k)}(N, N^{\mu-\frac{1}{D}})$ is r-partition universal for $\mathcal{G}^{(k)}(D, \Delta, cN)$.

Idea of the proof

Want

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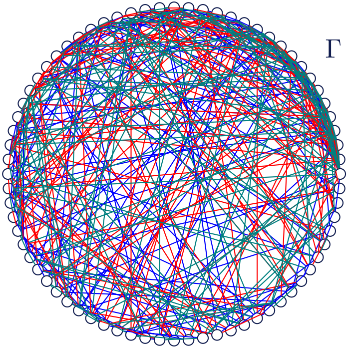

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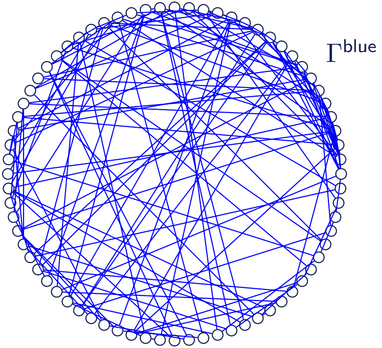

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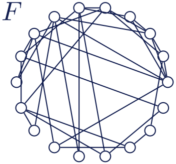

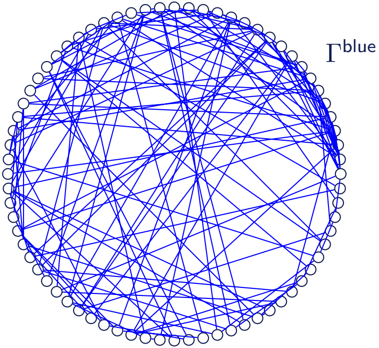

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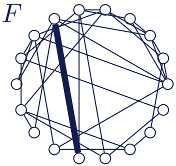

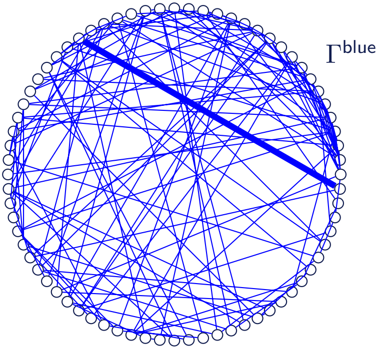

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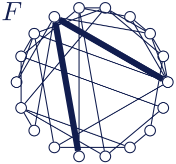

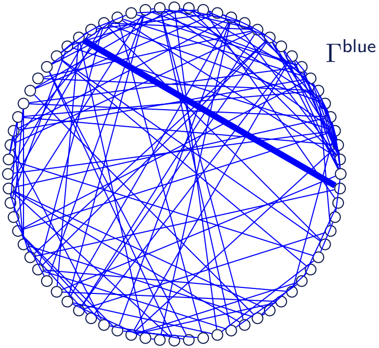

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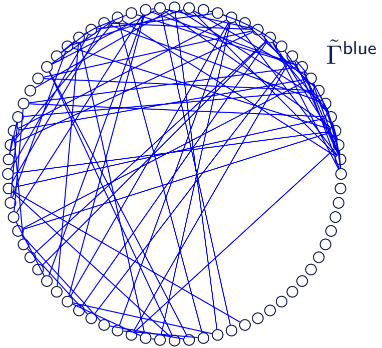

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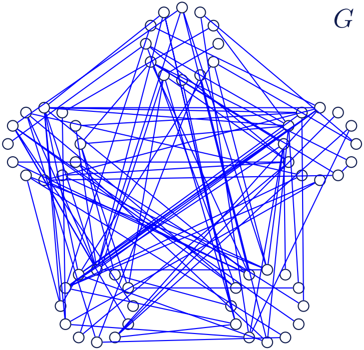

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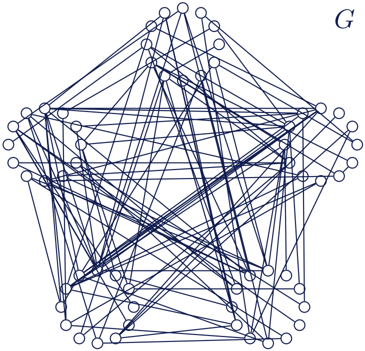

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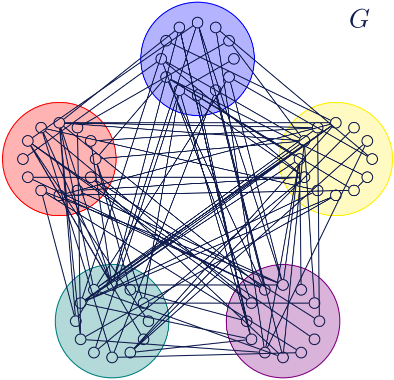

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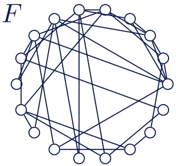

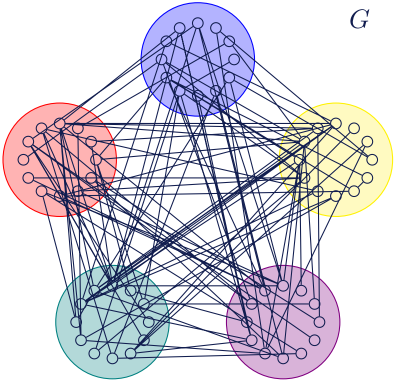

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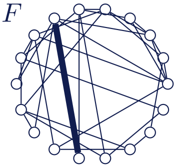

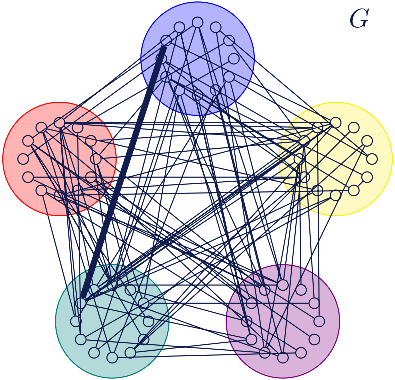

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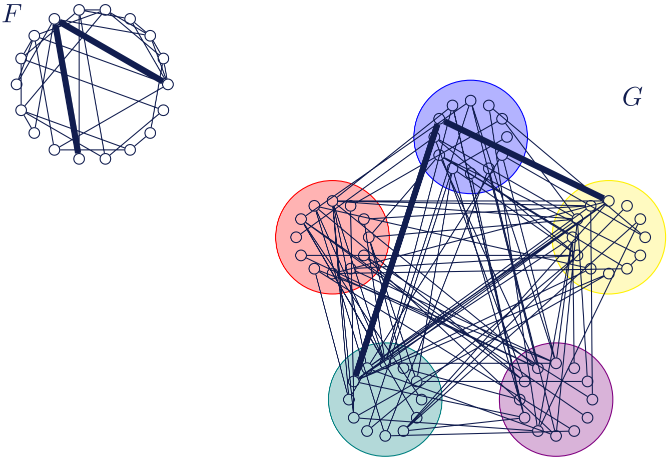

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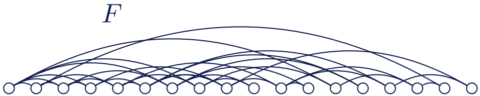

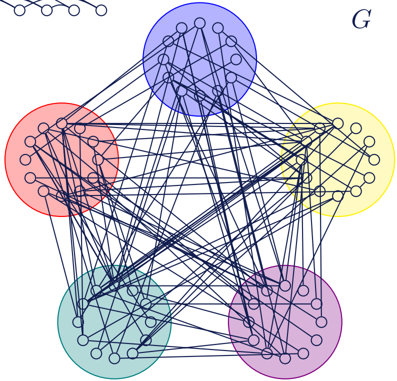

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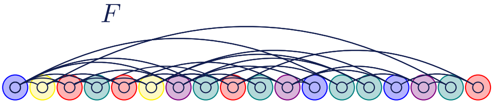
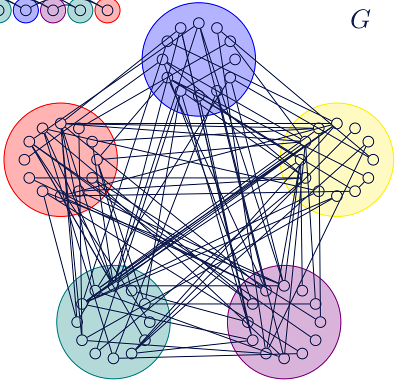


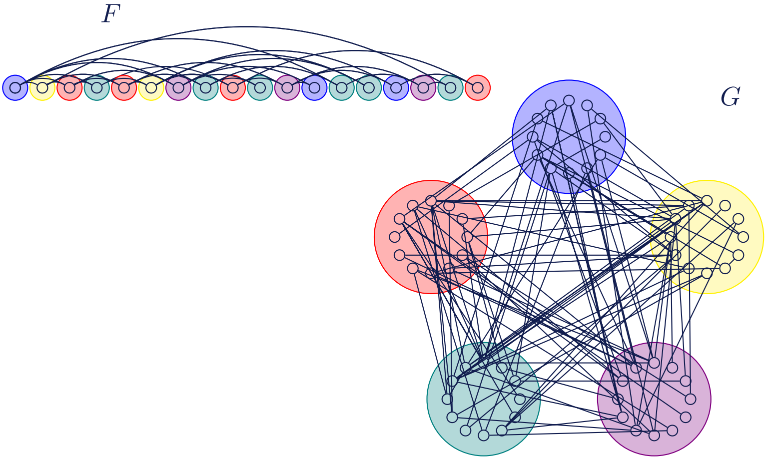
In F, vertices with the same colour are distant.



Idea of the proof

Goal

Find $\psi_0, \ldots, \psi_{v(F)}$ growing seq. of partial homom. from F to G.



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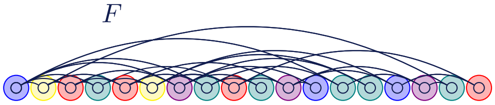

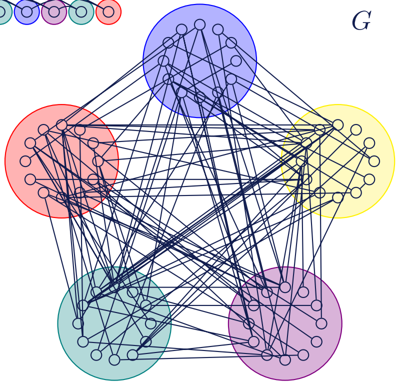

At each step, the number of viable embeddings of each of the following vertices should not decrease too much.

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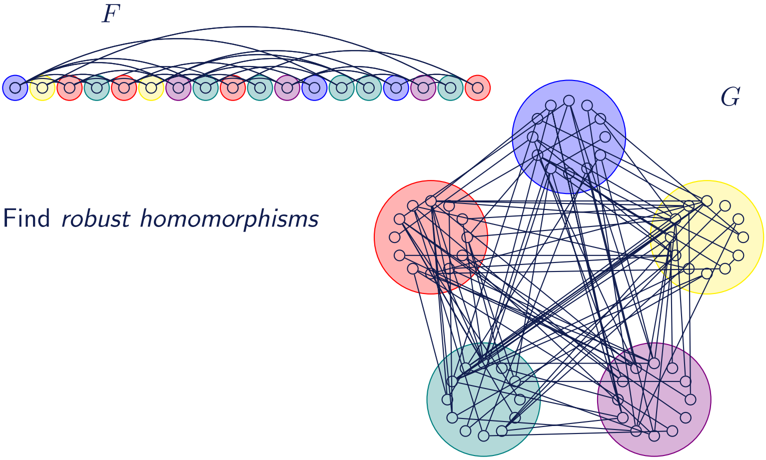

Find *robust homomorphisms*

Idea of the proof

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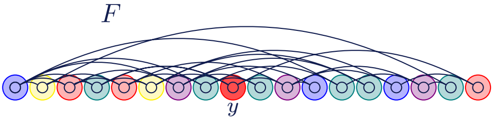
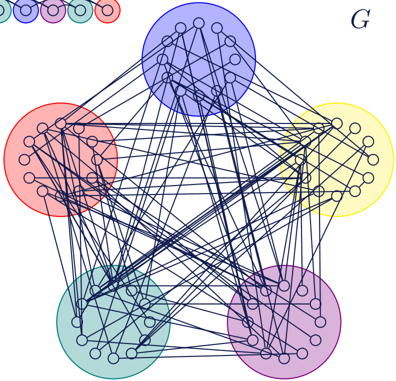


Find *robust homomorphisms:*
$\Omega(p^{d^-(y)}N)$ *choices for* y



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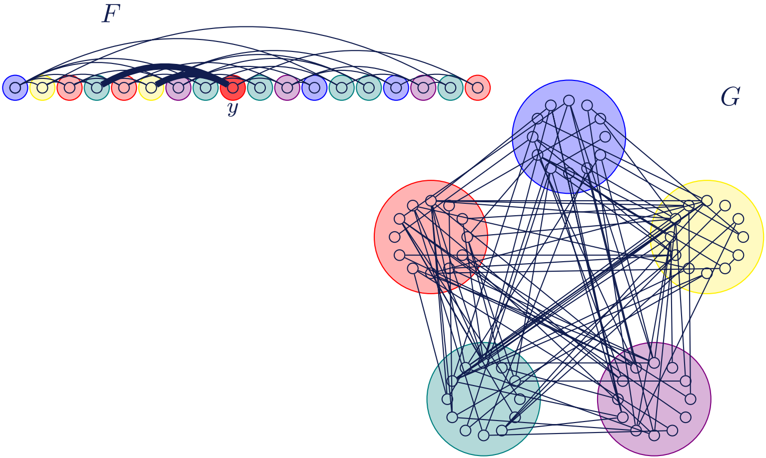

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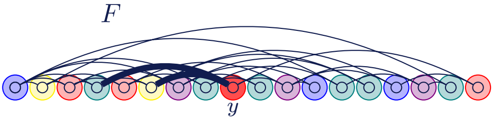

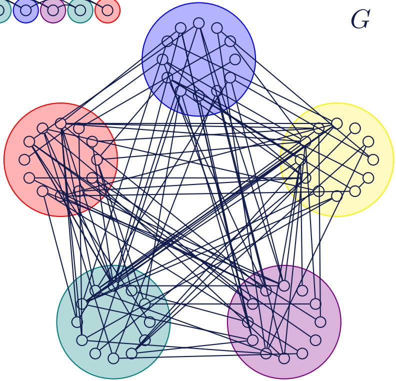

We must be careful when embedding $N^-(y)$

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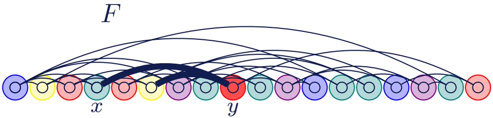
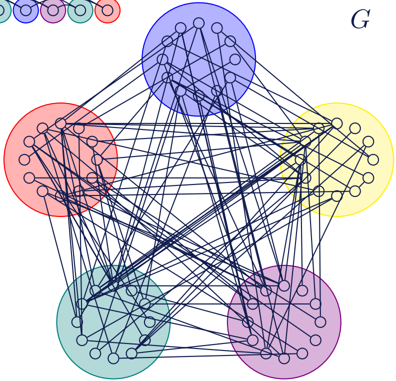


When embedding x we need to consider y.



Idea of the proof

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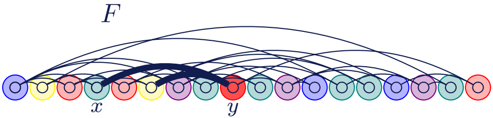
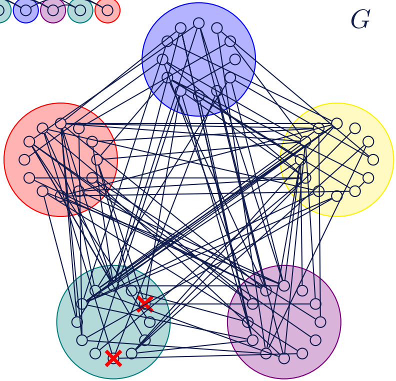


We won't embed x to *dangerous* vertices, but there are very few of them.



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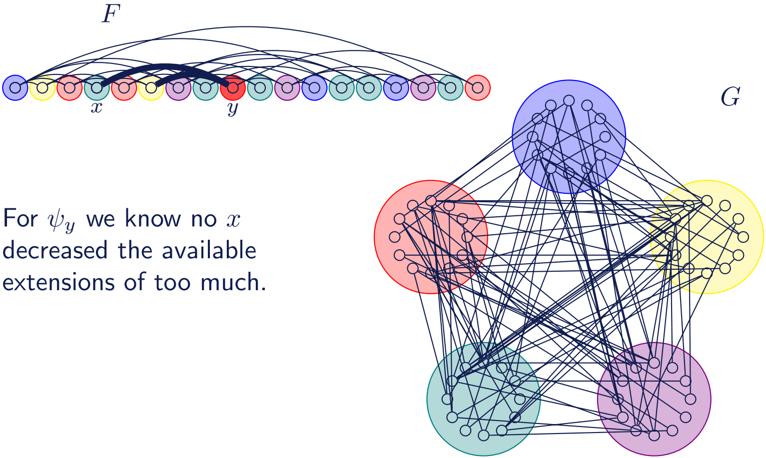

For ψ_y we know no x decreased the available extensions of too much.


Thank
You!!