Lecture 2. Bonds

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Today

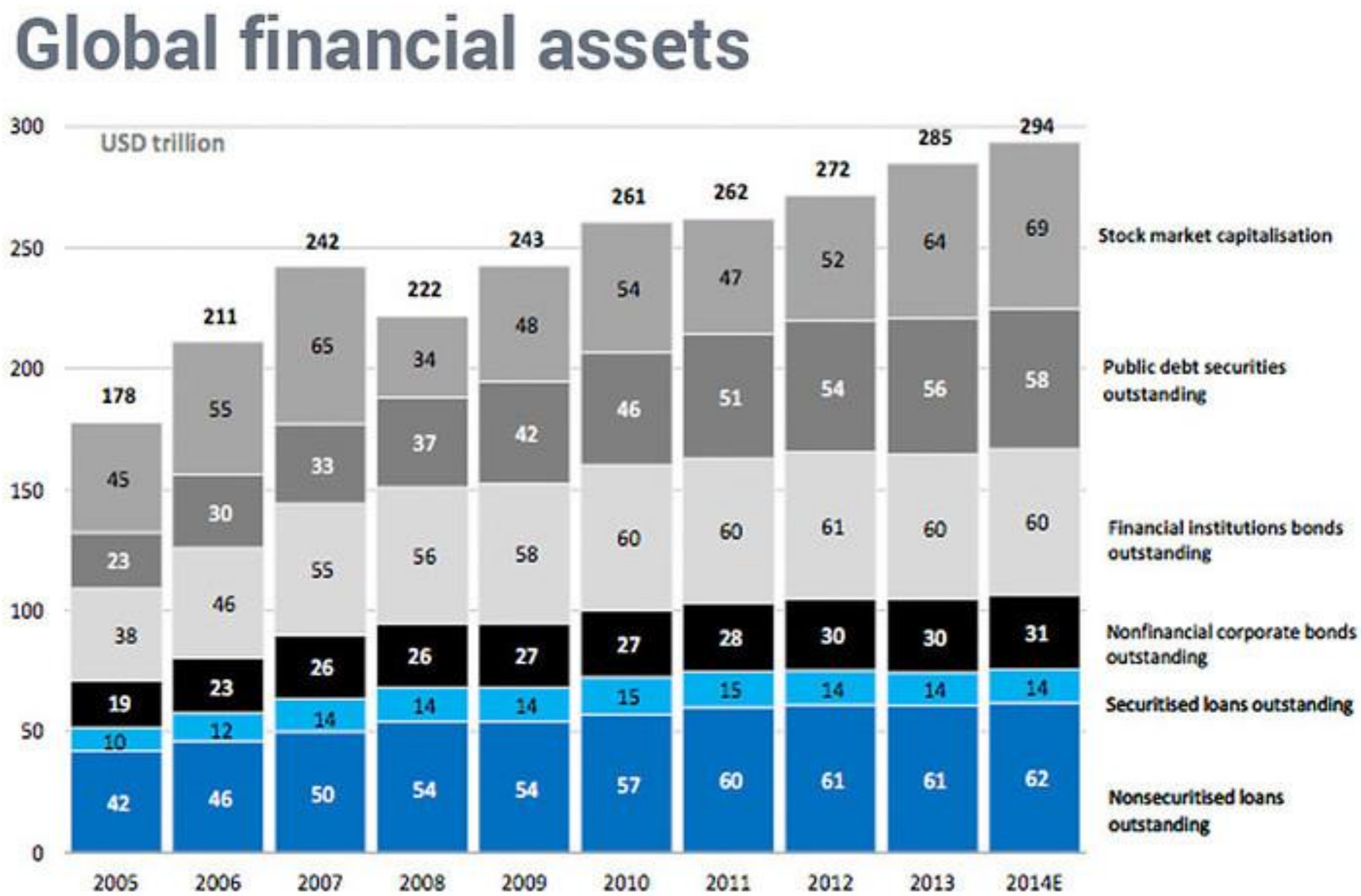
- What are bonds?
- Bond price and interest rates
- Term structure of interest rates
- Duration and immunization

Bonds

- Bond is a claim to **fixed periodical interests** and **principal repayment at maturity**
  - Entities issue bonds to borrow money. So if you buy a bond issued by the government, you’re lending money to the government.
  - Issued by firms, governments, cities
  - No ownership
  - Lower or no risk
  - Usually finite maturity (exceptions: UK)
  - The interest payments are called **coupons**.

- We will focus on government bonds with no default risk.

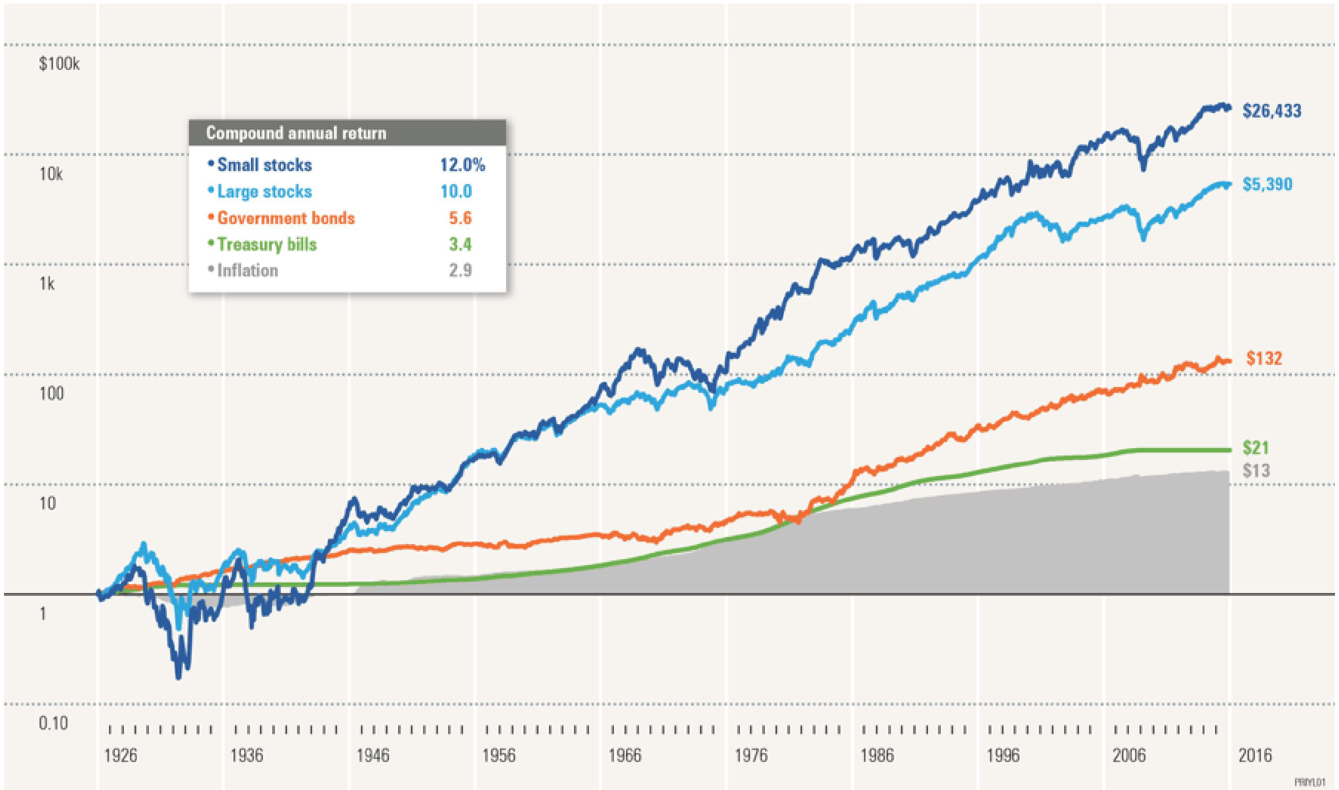
Size of the Bond vs. Stock Market



GFM Asset Management

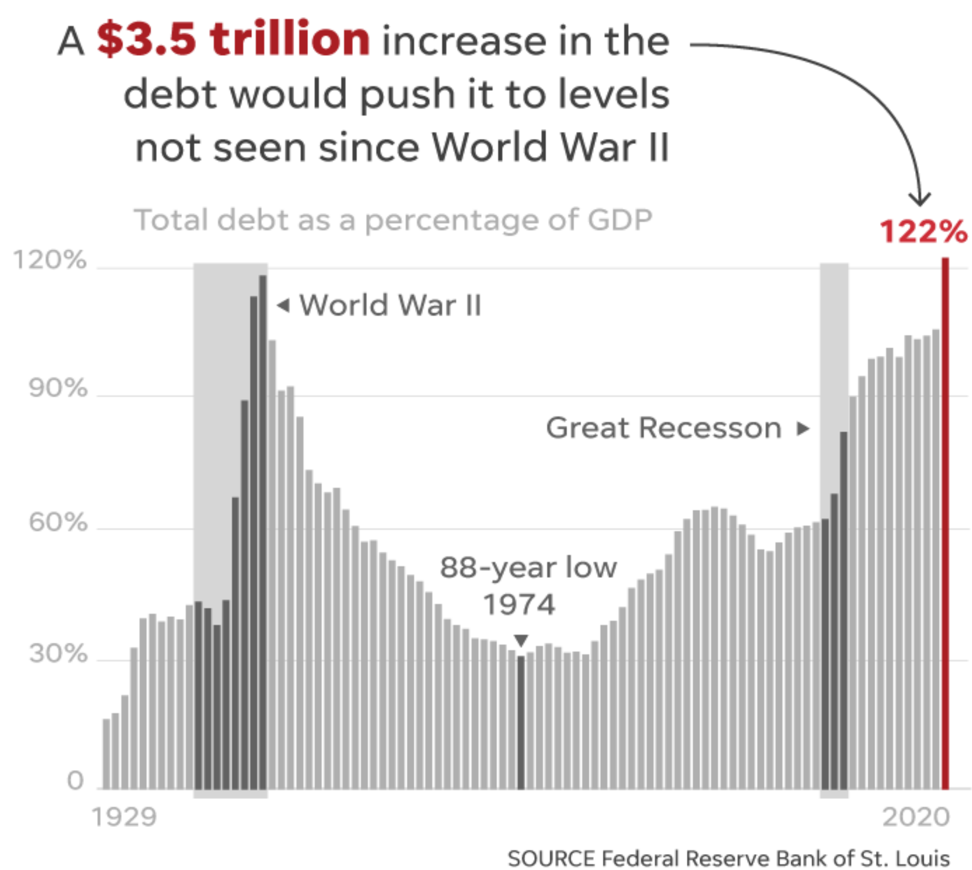
Bond vs. Stock Return

Ibbotson® SBBI®
Stocks, Bonds, Bills, and Inflation 1926–2015



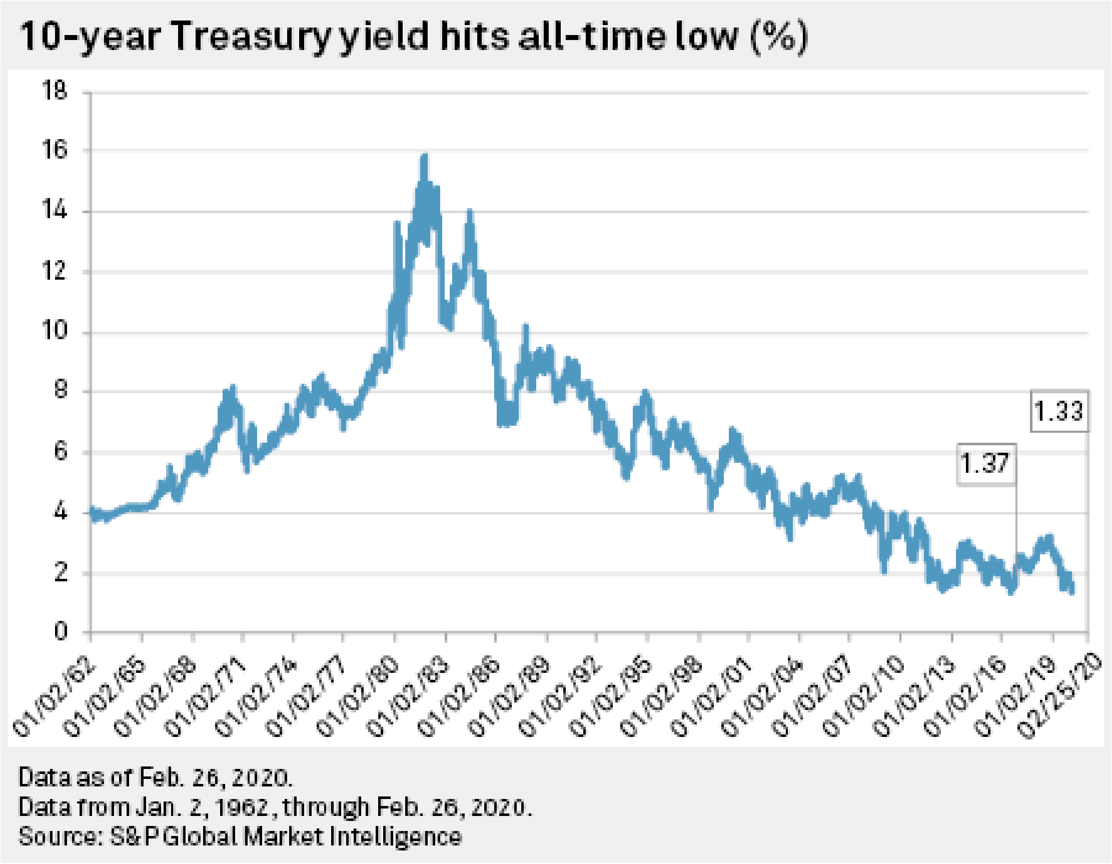
Past performance is no guarantee of future results. Hypothetical value of $1 invested at the beginning of 1926. Assumes reinvestment of income and no transaction costs or taxes. This is for illustrative purposes only and not indicative of any investment. An investment cannot be made directly in an index.

COVID and Public Debt



Economist

How Much Interest?



S&P Global

Bonds

- Cash flows to a bond:

		$\$C$	$\$C$	$\$C$	$\cdots$	$\$F + \$C$
years from now:	0	1	2	3		m

- Fixed **coupon** *C* every period except the last period m (**maturity**) when it also pays **face value** *F* (“**principal**”)
  - **Coupon rate** is C/F.

- But <u>**coupon rate isn’t the actual interest rate you get**</u>**!** The actual interest rate depends on the price of the bond and is called yield to maturity (YTM)

Bonds

The interest you earn on the bond is the discount rate that equates the present value of cash flows with price.

$$P = \frac{\mathrm{C}}{1+Y} + \frac{\mathrm{C}}{(1+Y)^2} + \frac{\mathrm{C}}{(1+Y)^3} + \frac{\mathrm{C}}{(1+Y)^4} + \frac{\mathrm{C}+\mathrm{F}}{(1+Y)^5}$$

- The discount rate on a bond, *Y*, is called "**yield to maturity**" (aka "yield", "YTM", or "interest rate").
- If *P* = *F*, you get *Y* = *C* (more on this soon).
- <u>Example</u>: *Price of a bond with face value £100, 4% coupon rate, 5 year maturity, and 5% yield required by investors*

$$P = \frac{4}{1+.05} + \frac{4}{(1+.05)^2} + \frac{4}{(1+.05)^3} + \frac{4}{(1+.05)^4} + \frac{104}{(1+.05)^5}$$

Today

- What are bonds?
- **Bond price and interest rates**
  1. How does the bond price depend on the interest rate?
  2. Why does the interest rate change?
- Term structure of interest rates
- Duration and immunization

Bond Price and Interest Rate

A bond **yield** *Y* differs from the **coupon rate** C/F when the bond **price** *P* differs from the **face value** *F*

If ..	*Y* = C/F,	Price = face value	➔ trading at	*par*
	Y > C/F,	Price < face value	➔ trading at	*discount*
	Y < C/F	Price > face value	➔ trading at	*premium*

Example: *Again, a bond with face value £100, coupon rate 4%, and 5 year maturity. Find the bond price when its yield to maturity is 4%, 5%, or 3%.*

$$P = \frac{4}{1+Y} + \frac{4}{(1+Y)^2} + \frac{4}{(1+Y)^3} + \frac{4}{(1+Y)^4} + \frac{104}{(1+Y)^5}$$

Bond Price and Interest Rate

Example: *Bond with face value £100, coupon rate 4%, and 5 year maturity. Find bond price when investors require yield to maturity of 4%, 5%, or 3%.*

$$P = \frac{4}{1.04} + \frac{4}{(1.04)^2} + \frac{4}{(1.04)^3} + \frac{4}{(1.04)^4} + \frac{104}{(1.04)^5} = 100$$

$$P = \frac{4}{1.05} + \frac{4}{(1.05)^2} + \frac{4}{(1.05)^3} + \frac{4}{(1.05)^4} + \frac{104}{(1.05)^5} = 95.7$$

$$P = \frac{4}{1.03} + \frac{4}{(1.03)^2} + \frac{4}{(1.03)^3} + \frac{4}{(1.03)^4} + \frac{104}{(1.03)^5} = 104.6$$

- If..

$Y = 4\%$,	Price = 100	trading at	*par*
$Y > 4\%$,	Price < 100	trading at	*discount*
$Y < 4\%$,	Price > 100	trading at	*premium*

Bond Price and Interest Rate

Example: *A bond with face value $100 is trading at a discount for $97. If its coupon rate is 2% and has 2 years of maturity left, what is the yield investors are requiring on the bond? (round to 0.1) Given the answer, is the bond a good buy?*

Bond Price and Interest Rate

<u>Example</u>: *A bond with face value \$100 is trading at a discount for \$97. If its coupon rate is 2% and has 2 years of maturity left, what is the yield investors are requiring on the bond? (round to 0.1) Given the answer, is the bond a good buy?*

$$97 = \frac{2}{1+Y} + \frac{102}{(1+Y)^2}$$

- Multiplying both sides by $(1+Y)^2$ and solving for *Y*,

$$y = 3.5\%$$

- Trading at a discount relative to the face value doesn't mean it's a good buy or a bad buy.

Bond Price and Interest Rate

Example:

A bond with face value \$100, coupon rate 2%, and 2 years to maturity is trading at a par (hence, investors are requiring a 2% yield).

A year later, the yield investors require on this bond rises from 2% to 3%. What is the bond price now? What if the yield stayed the same?

Bond Price and Interest Rate

<u>Example</u>: *A bond with face value $100, coupon rate 2%, and 2 years to maturity is trading at a par (hence, investors are requiring a 2% yield). A year later, the yield investors require on this bond rises from 2% to 3%. What is the bond price now? What if the yield stayed the same?*

Case 1:
$$P = \frac{102}{1.03} = \$99$$

Case 2:
$$P = \frac{102}{1.02} = \$100$$

- Hence bond price **decreases** if interest rate (yield) **increases**!

Bond Price and Interest Rate

Bond price changes in the **opposite direction** to interest rate (yield) changes.

- If the yield rises, the fixed coupons become relatively less valuable.
- So bonds are subject to interest risk.

In fact, interest rate (yield) change is the only factor that can move bond prices:

$$PV = \frac{\mathrm{C}}{1+Y} + \frac{\mathrm{C}}{(1+Y)^2} + \frac{\mathrm{C}}{(1+Y)^3} + \frac{\mathrm{C}}{(1+Y)^4} + \frac{\mathrm{C}+\mathrm{F}}{(1+Y)^5}$$

- Interest rates move around, making bond prices volatile.
- Unlike stocks, where the dividends move around, the coupons and the principal are fixed.

Bond Price and Interest Rate

Bond pricing:

$$PV = \frac{C}{1+Y} + \frac{C}{(1+Y)^2} + \frac{C}{(1+Y)^3} + \frac{C}{(1+Y)^4} + \frac{C+F}{(1+Y)^5}$$

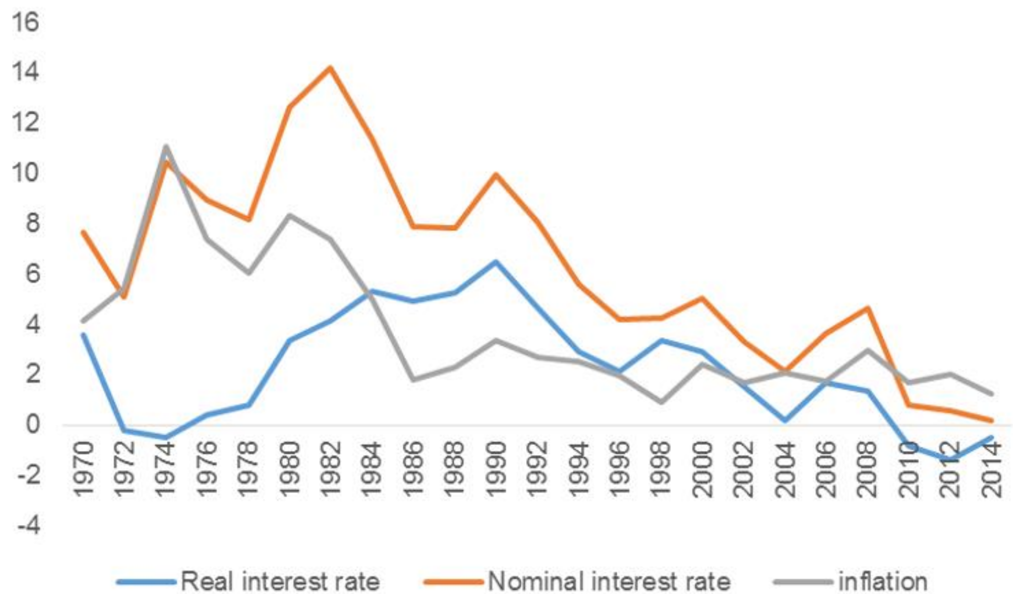
Two reasons why the (nominal) interest rate Y changes:

$$1+Y = (1+r_{real})(1+\pi)$$

1. Changes in **real interest rate** r_{real} (interest rate earned after adjusting for inflation)
   - Real interest rates can change due to the supply and demand for savings
2. Changes in the expected **inflation rate** π
   - Higher inflation rate means that the nominal interest rate should be higher to compensate for the faster decay in the value of money.

Bond Price and Interest Rate

- Nominal interest rate *Y*: $1+Y=(1+r_{real})(1+\pi)$
  1. r_{real} is the real interest rate
  2. π is the expected inflation rate



(Figure from LSE blog)

- The long-term trend in the nominal interest rate is due to the decline in inflation, but the recent short-term movements in the nominal interest rate is due to real interest rate movements.

Bond Price and Interest Rate

<u>Example</u>:

A bond with face value \$100, coupon rate 2%, and 2 years to maturity is trading at a par (hence, the current yield or nominal interest rate is 2%).

A year later, the <u>real interest rate</u> investors require on this bond rises from 1% to 2%, but expected inflation rate stays constant. What is the bond price now? (use approximations)

Bond Price and Interest Rate

- Example:

  Currently, inflation rate = $(1+Y)/(1+r_{real}) - 1 \approx 1\%$.

  Therefore, nominal interest rate (yield) next period

  $= (1+r_{real})(1.01) - 1 \approx 3\%$.

  So, $$P = \frac{102}{1.03} = \$99$$

Today

- What are bonds?
- Bond price and interest rates
- **Term structure of interest rates**
- Duration and immunization

Zero-coupon Bond

- Zero-coupon Bond:

		\$0	\$0	\$0	$\dots$	\$F
years from now:	0	1	2	3		m

- (Compare to coupon bond cash flows)

		$\$C$	$\$C$	$\$C$	$\dots$	$\$F + \$C$
years from now:	0	1	2	3		m

- Question: Why would anyone buy a zero-coupon bond?

Zero-coupon Bond

- Price of a zero-coupon bond with maturity *m* and face value, say, 1:

$$P = \frac{1}{\left(1+Y_m\right)^m}$$

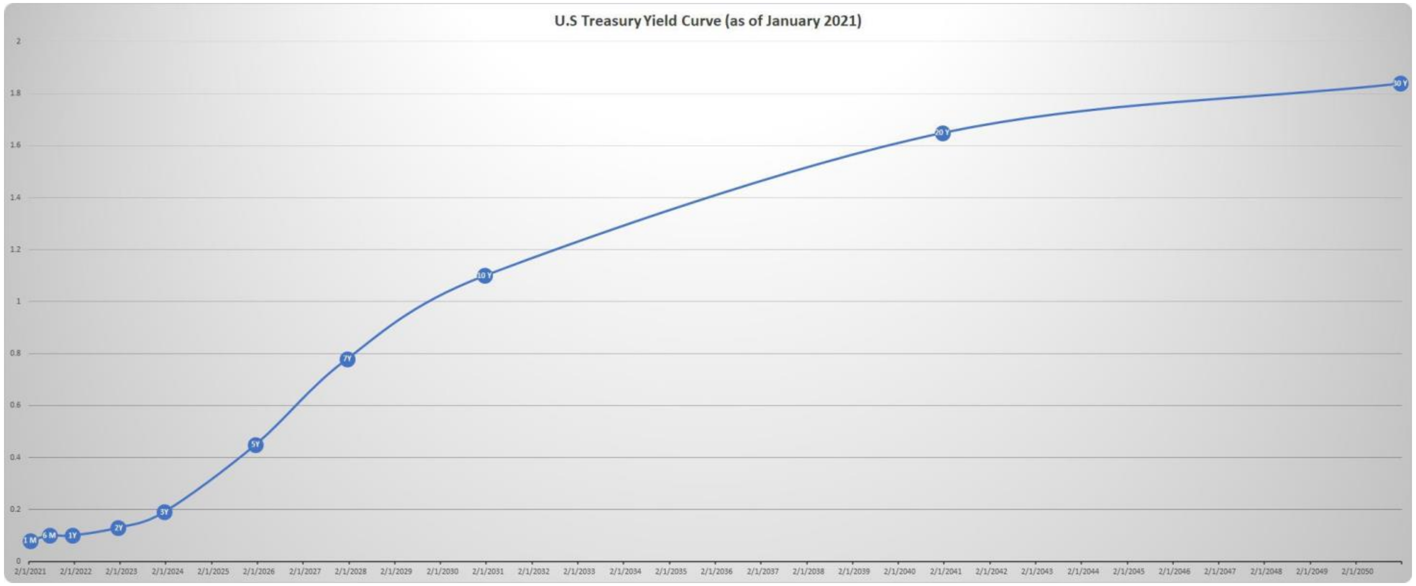
where we allow yield to differ by maturity. The yield on a zero-coupon bond is sometimes called the **spot rate**.

- But, as posed before, if bonds are safe, should all maturity yields be the same ($Y_1 = Y_2 = \ldots = Y_M$)?

- Let's look at U.S. Treasury "**yield curve**."

Yield Curve

- The **yield curve** or **term structure of interest rates** is the set of yields, at a give time, on zero-coupon bonds of different maturities.



investopedia

- The yield curve is typically **positively sloped.** Why?

Explaining a Yield Curve

At least three different hypotheses:

1. Expectations hypothesis
   - The yield curve reflects expectations about future short-term (1-period) interest rates
2. Liquidity preference hypothesis
   - Long-term bonds are riskier since they have greater exposure to changes in interest rates or inflation rates.
3. Market segmentation hypothesis
   - Bonds with different maturities have different pools of investors, so they cannot be easily compared.

1. Expectations Hypothesis

Example: The one-year spot rate ($Y_{1,t}$) is 5%, and the two-year spot rate ($Y_{2,t}$) is 7%. Let $E[Y_{1,t+1}]$ be the expected one-year spot rate next year. If you wish to invest $100 for 2 years, you have 2 options.

- Option A (buy 1-year zero-coupon bonds twice consecutively)
  1. Buy $100 of 1-year zero-coupon bond today to get $105 next year.
  2. Buy $105 of 1-year zero-coupon bond next year to get $105 × (1+$E[Y_{1,t+1}]$) in year 2 in expectation.

- Option B (buy 2-year zero-coupon bond once)

  Buy 2-year zero-coupon bond today to get $100 × 1.07^2 = $114.5 in year 2.

If everyone is **risk neutral** and disregards the uncertainty involved in option A, the two options should have the same expected payoff:

$$(1+Y_{1,t}) \times (1+E[Y_{1,t+1}]) = (1+Y_{2,t})^2 \Rightarrow E[Y_{1,t+1}] = 9\%$$

1. Expectations Hypothesis

In fact, in everyone's risk neutral, you can infer all expected one-year interest rates in the future from the yield curve:

$$(1+Y_{1,t})\times\left(1+E\left[Y_{1,t+1}\right]\right)=(1+Y_{2,t})^2 \Rightarrow E\left[Y_{1,t+1}\right]=\frac{(1+Y_{2,t})^2}{1+Y_{1,t}}-1$$

$$\left(1+Y_{2,t}\right)^2\times\left(1+E\left[Y_{1,t+2}\right]\right)=(1+Y_{3,t})^3 \Rightarrow E\left[Y_{1,t+2}\right]=\frac{(1+Y_{3,t})^3}{(1+Y_{2,t})^2}-1$$

$$\left(1+Y_{3,t}\right)^3\times\left(1+E\left[Y_{1,t+3}\right]\right)=(1+Y_{4,t})^4 \Rightarrow E\left[Y_{1,t+3}\right]=\frac{(1+Y_{4,t})^4}{(1+Y_{3,t})^3}-1$$

$\ldots$

Expectations hypothesis: The shape of the yield curve is a perfect reflection of people's expectations about future 1-period interest rates

- So an upward sloping yield curve would mean people expect the one-year interest rates to go up and up.
- Again, its critical assumption is that everyone disregards risk, which is unlikely to be true.

2. Liquidity Preference Hypothesis

- If yield curves are normally positively sloped, the strict interpretation of the expectations hypothesis implies that short-term interest rate <u>must have been rising historically</u>.

- Not true. Instead, longer-maturity bonds generally have higher yields probably because they are generally riskier.

- To derive the expectations hypothesis, we assumed that all investors of 2-year bonds are sure about keeping their investment for two periods.

- In reality, there’s a risk of having to liquidate your investments because you suddenly need cash (“liquidity event”).

2. Liquidity Preference Hypothesis

- **Liquidity preference hypothesis**: Investors are faced with uncertain liquidity shocks when they have to sell a bond. And if you're selling a bond before maturity, the selling price is **uncertain**. This makes longer-maturity bonds riskier.

- m-year bond (face value 1) becomes a (m-1) year bond next year, and by then the required yield may change, so your holding period return may not be $Y_{m,t}$:

$$1+Y_{m,t} \neq \frac{P_{m-1,t+1}}{P_{m,t}} \text{ unless } P_{m-1,t+1} = \frac{1}{(1+Y_{m,t})^{m-1}}$$

2. Liquidity Preference Hypothesis

- And the longer the maturity, the more sensitively price responds to change in yield. Can you see this?

$$P_m = \frac{1}{(1+Y_m)^m}$$

- This explains the upward-sloping yield curve.

3. Market Segmentation Hypothesis

- **Market segmentation hypothesis**: Different investors hold different maturity bonds, so you can’t compare them easily.

Inferring from the yield curve

- What does an increase in the slope of the yield curve imply?

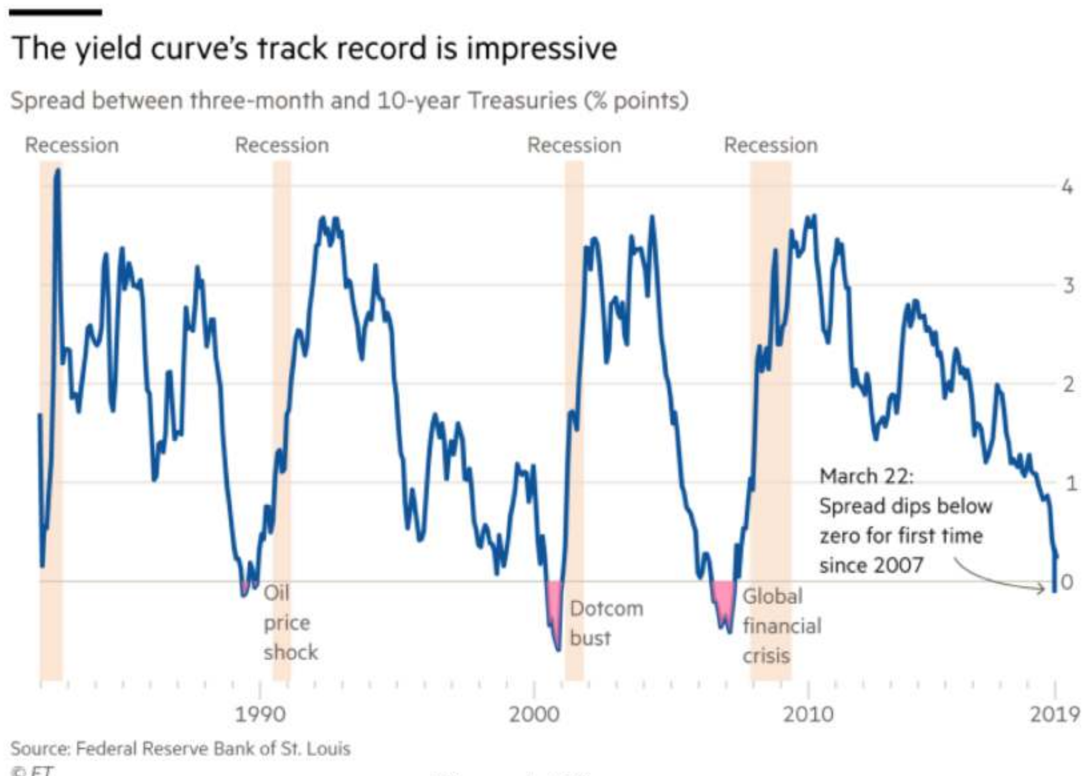
  1. Under EH, it implies that investors anticipate higher future interest rates

  2. Under LPH, it implies that risk premium associated with longer maturity increases (due to an increase in risk aversion or in the quantity of risk)

  3. Under MSH, we cannot infer much

US Yield Curve and Recessions

- An **inverted yield curve** usually predicts a recession. Why?



Financial Times

Today

- What are bonds?
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- **Duration and immunization**

Duration

- We saw previously that the longer the maturity of a zero-coupon bond, the more sensitive it is to yield changes.

- But for coupon bonds, price sensitivity to yield changes doesn't just depend on maturity but on the relative size of the coupon and face value (Do you see this?):

$$PV = \frac{\mathrm{C}}{1+Y} + \frac{\mathrm{C}}{(1+Y)^2} + \frac{\mathrm{C}}{(1+Y)^3} + \frac{\mathrm{C}}{(1+Y)^4} + \frac{\mathrm{C}+\mathrm{F}}{(1+Y)^5}$$

Duration

- So we came up with a measure of “effective maturity” called **duration**:

$$D = \sum_{t=1}^{m} \left(\frac{PV(C_t)}{PV} \right) t$$

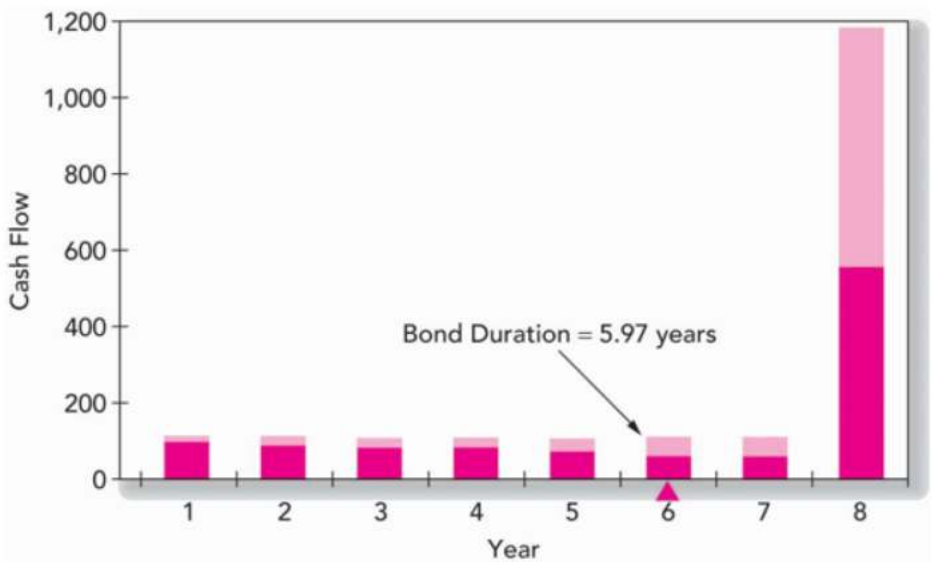
  – Duration is the the weighted-average period of time that the bond's cash flows are received.

  – That weight is the present value of the cash flow divided by the price: $PV(C_t) = C_t/(1+Y)^t$

- Mathematically, duration is by what percentage price changes for a change in Y as a fraction of 1+Y:

$$D = -\frac{dP}{dY}\frac{1+Y}{P} \approx -\left(\frac{\Delta P}{P}\right) \div \left(\frac{\Delta Y}{1+Y}\right)$$

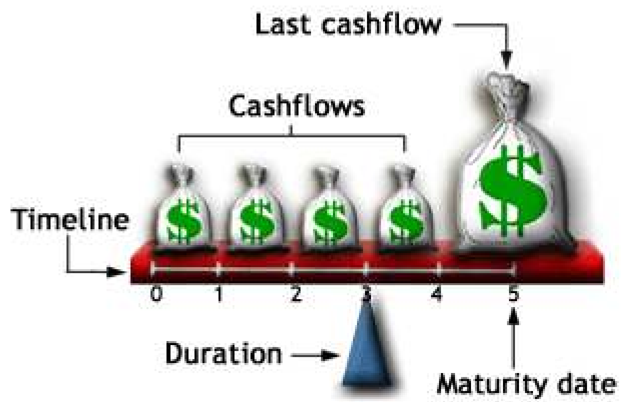
Duration: Example



Payments from a 9% (annual) coupon bond with 8-year maturity and 10% YTM. The height of each bar is the total of interest and principal. The lower portion of each bar is the present value of that total payment.

Duration

- Graphically, duration is the location of the fulcrum that balances the lever with discounted cash flows on top:

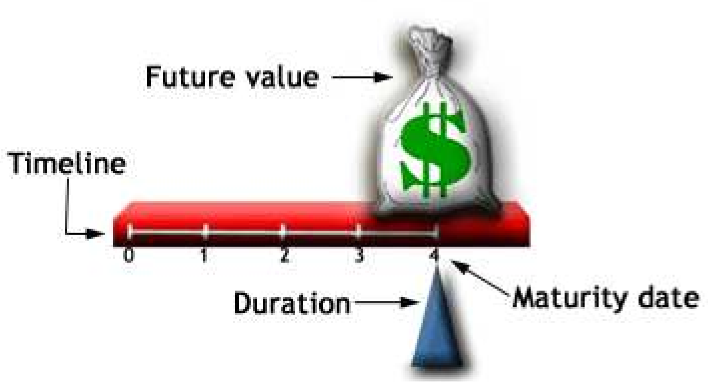


Source: Investopedia

$$D = \sum_{t=1}^{m} \left(\frac{PV(C_t)}{PV} \right) t$$

Duration: Examples

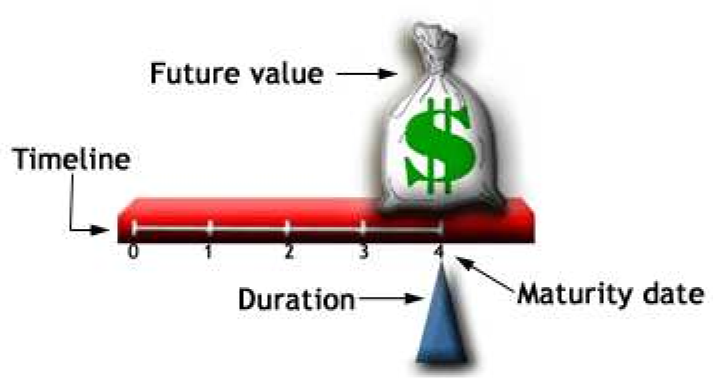
- <u>Example</u>: *What is the duration of a zero-coupon bond?*



Source: Investopedia

Duration: Examples

- <u>Example</u>: *What is the duration of a 2-year bond with face value $100 and coupon rate 5%? Can you also find by what percentage its price will change if its yield rises from 6% to 7%? (Hint: get the sign right, and don't use just the percentage point change in yield.)*



Source: Investopedia

Duration: Concept Check

- Duration decreases with the bond’s coupon rate
- Duration decreases with the bond’s yield to maturity
- Duration increases with the bond’s maturity
- All else equal, a higher duration makes the bond price more volatile

Duration: Concept Check

- Duration decreases with the bond's coupon rate

    Yes

- Duration decreases with the bond's yield to maturity

    Yes

- Duration increases with the bond's maturity

    Not necessarily.

- All else equal, a higher duration makes the bond price more volatile

    Yes. Between two bonds with the same YTM, the one with a higher duration will have higher price volatility.

Modified Duration and Immunization

- **Modified duration** D*:
  - $D^* = D \div (1+Y)$
  - Modified duration is by what percentage price changes for a change in Y (in percentage point)
  - Mathematically, $D^* = -\left(\frac{\Delta P}{P}\right) \div \Delta Y$

- **Immunization** is a strategy used by financial institutions to shield their financial wealth from exposures to interest rate fluctuations.

- Put differently, they want to set the modified duration *D** of their net wealth (assets minus liability) to 0.