Class 4

Recap - PORTFOLIO THEORY

- Probability for FINANCE:

  - Expected return : $\bar{R} = E[R] = \sum \underbrace{p_s}_{\text{prob. of state}} \underbrace{R_s}_{\text{RETURN in state}}$ s: POSSIBLE STATES

  - Variance : $\sigma^2 = Var(R) = E[(R-\bar{R})^2]$

    - VOLATILITY (std. dev.) : $\sigma = \sqrt{Var}$

  - Covariance : $\sigma_{i,j} = Cov(R_i, R_j) = E[(R_i - \bar{R}_i)(R_j - \bar{R}_j)]$

    - CORRELATION : $\rho_{i,j} = \dfrac{Cov(R_i, R_j)}{\sigma_i \; \sigma_j}$

- WHAT is a PORTFOLIO?

  Determined by the weight of each asset: $w_1, w_2, \ldots$ (w_2: percentage of wealth invested in asset 2)

  - COMPUTATIONS/REMARK: Given a portfolio $p = [w_1, w_2]$ over assets with (uncertain) returns R_1 and R_2,

    $$\bar{R}_p = E[w_1 R_1 + w_2 R_2] = w_1 \bar{R}_1 + w_2 \bar{R}_2$$

    $$\sigma_p^2 = \ldots = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 Cov(R_1, R_2)$$

    ⇝ Can reduce Variance by combining identical (but non-too-correlated) assets.

    ⇝ DIVERSIFICATION

    - One can reduce variance by combining uncorrelated assets.

      Diversifiable (or IDEOSYNCRATIC) risk

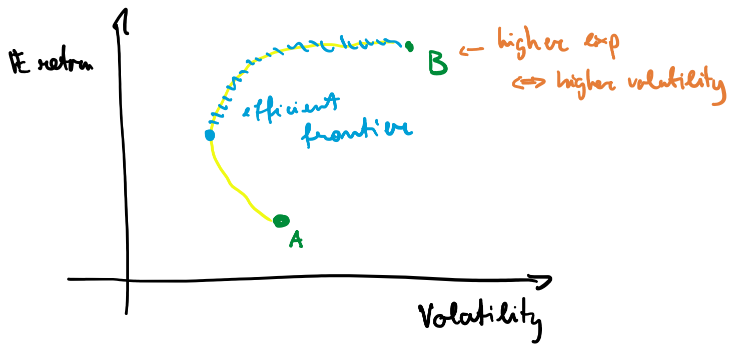
      VS Market (SYSTEMATIC) risk

- RISK AVERSION: level of preference of different types/levels of risk.

WANT: Highest exp. return for a level of risk
OR Lowest risk for a given exp. return.

⇝ If everyone had the same idea of $\mathbb{E}$, Var and Cov, they should all have the same portfolio (for a given risk level).

– REM Even if stock A is DOMINATED by other portfolios, you want to combine it to reduce RISK.



This is for POSITIVE portfolio weights. You can also do a NEGATIVE investment and assume a SHORT position (SHORT-selling : selling a stock you do NOT own)

REM ① A linear comb. of efficient portfolios is efficient
② N stocks are not worse than N-1 stocks.
③ Combine the optimal one with risk-free assets

Question 1

A fund manager invests 60% of her funds in stock I and the rest in stock J. the standard deviation of returns on I is 10%, and on J it is 20%. Calculate the variance of portfolio returns, assuming:

(a) the correlation between the returns is 1.0
(b) the correlation is 0.5
(c) the correlation is 0.

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2 \, \overbrace{\mathrm{Cov}}^{\rho_{1,2}\,\sigma_1\sigma_2}$$

```
def Two_stocks_easy(Weight_A_1, ExpR_1, SD_1, ExpR_2, SD_2, Corr_1_2):
    Exp_Comb = Weight_A_1*ExpR_1 + (1-Weight_A_1)*ExpR_2
    Var_Comb = Weight_A_1**2*SD_1**2 + (1-Weight_A_1)**2*SD_2**2 + 2 * (Weight_A_1) * (1-Weight_A_1) * Corr_1_2 * SD_1 * SD_2
    return round(Exp_Comb,3), round(Var_Comb, 5)
```

```
for corr in [1, 0.5, 0]:
    print(Two_stocks_easy(0.6, 10, 0.1, 10, 0.2, corr))
```

```
(10.0, 0.0196)
(10.0, 0.0148)
(10.0, 0.01)
```

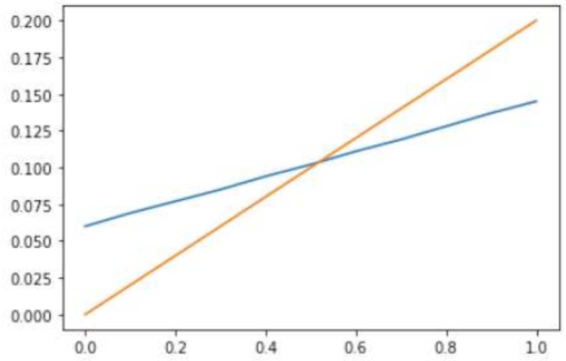
Question 2

Suppose that Treasury bills offer a return of 6% and the expected market risk premium is 8.5%. The standard deviation of Treasury bill returns is zero and the standard deviation of market returns is 20%. Use the formula for portfolio risk to calculate the standard deviation of portfolios with different proportions in Treasury bills and the market (what is the covariance of the two rates of returns?). Graph the expected returns and standard deviations.

```python
Wei = [round(0.1*i,2) for i in range(11)]
Exp = [0 for i in range(11)]
Std = [0 for i in range(11)]
for i in range(11):
    Exp[i], Std[i] = Two_stocks_easy(Wei[i], 0.145, 0.2, 0.06, 0, 0)
    Std[i]=Std[i]**0.5
print(tabulate([["Exp"]+Exp, ["Std"]+Std], headers=["Weights"]+Wei))
plt.plot(Wei,Exp)
plt.plot(Wei,Std)
```

Weights	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Exp	0.06	0.069	0.077	0.085	0.094	0.102	0.111	0.119	0.128	0.137	0.145
Std	0	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2

```
[<matplotlib.lines.Line2D at 0x7fce58224940>]
```



Question 3

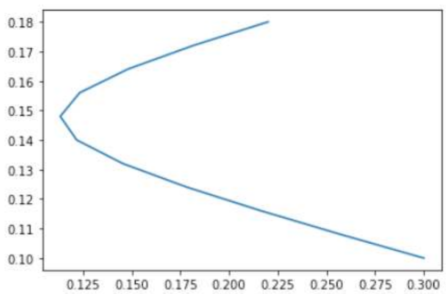
Stocks offer an expected rate of return of 18%, with a standard deviation of 22%. Gold offers an expected return of 10% with a standard deviation of 30%.

Given the apparent inferiority of gold with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why an investor would hold any gold.

```python
Wei = [round(0.1*i,2) for i in range(11)]
Exp = [0 for i in range(11)]
Std = [0 for i in range(11)]
for i in range(11):
    Exp[i], Std[i] = Two_stocks_easy(Wei[i], 0.18, 0.22, 0.1, 0.3, -0.6)
    Std[i] = Std[i]**0.5
print(tabulate([["Exp"]+Exp, ["Std"]+Std], headers=["Weights"]+Wei))
plt.plot(Std,Exp)
```

Weights	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Exp	0.1	0.108	0.116	0.124	0.132	0.14	0.148	0.156	0.164	0.172	0.18
Std	0.3	0.25741	0.216472	0.178382	0.145396	0.121655	0.113225	0.123207	0.147986	0.181604	0.22

```
[<matplotlib.lines.Line2D at 0x7fce30460130>]
```



Question 4

True or false?

(a) The measure of risk for a security held in a diversified portfolio is the standard deviation of returns. Covariance of returns.

(b) Proper diversification can reduce or eliminate systematic risk. No. But total risk yes!

(c) Stocks A, B, and C have the same expected return and standard deviation. The following table shows the correlations between the returns on these stocks:

	Stock A	Stock B	Stock C
Stock A	1.0		
Stock B	0.9	1.0	
Stock C	0.1	-0.4	1.0

< 0! Is good!

The portfolio having the lowest risk is a portfolio is invested in stocks B and C. Yes!