Lecture 10: Discount Rates

Outline

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1. Estimating Discount Rates using the Capital Asset Pricing Model

2. The Unlevered Cost of Capital

2. Weighted Average Cost of Capital (WACC)

0. Motivation

1. What's the Question?

2. Who cares?

3. Why don't we already know the answer?

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1. What's the Question?

If I use the NPV framework, how do I determine the appropriate discount rate? In other words, how do I figure out what is a reasonable rate of return on an investment?

Examples:

- Suppose you are investing your savings, and are choosing among government bonds of different countries. What rates are best?

- Suppose you are an I-Banker, and you need to figure out the right discount rate to price a merger transaction.

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2. Who cares?

Presumably we want to maximize our profits and make the best use of our capital…whether you're a CEO, middle manager, consultant, or banker, etc.; a key input for NPV calculations is the discount rate.

0. Motivation

1. What's the Question?

2. Who cares?

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3. Why don't we already know the answer?

Our first approach to answering this question is to use our INTUITION.

Examples:

0. Motivation

1. What's the Question?

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3. Why don't we already know the answer?

A SECOND approach is to use a statistical model: Look at historical data or comparable data, and take an average of observed rates of return.

What are the pros and cons of this approach?

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3. Why don't we already know the answer?

A THIRD approach is to use an economic model.

What's the difference between an economic model and a statistical model?

1. Estimating Discount Rates using the Capital Asset Pricing Model

The CAPM is an economic model that provides a way of numerically estimating a discount rate for a given asset.

The key ingredients in the CAPM are the following:

1. Assumptions, such as supply = demand (of market securities), investors have perfect information, there are no transaction costs, investors are risk averse, etc. etc.

2. Optimization problem: Investors choose portfolio weights over securities to maximize their portfolio's expected return subject to a risk constraint.

- When we solve for the equilibrium conditions implied by the model, we derive the following condition for any asset *i :*

$$r_i = r_f + \beta_i (r_m - r_f),$$

where

$$\beta_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$$

- Terminology: the "risk premium" of an asset is the extra compensation above and beyond the risk-free rate that contributes to the asset's expected return. In the CAPM, this risk premium is proportional to the asset's covariance with the market portfolio.

- In the CAPM, the expected return on an asset depends only on the asset's non-diversifiable market (a.k.a. "systematic") risk, which is captured by the asset's beta. This is because in the CAPM, we assume that investors can fully diversify and eliminate the contribution of assets' "idiosyncratic" risks to their portfolio.

- Example 4-1: Your firm is considering buying a license for oil exploration in Alaska. Immediately after paying for the license you will invest $50M in a drilling tower. There is a 10% chance that there will be 10M barrels of oil in the ground and a 90% chance that there will be nothing. The license specifies that you **must** sell all the oil you find to the government in one year from now for $95 per barrel. The risk free rate is 5%, the market risk premium is 6%, and the beta of oil is 0.5. What is the maximum amount you are willing to pay for the license?

- Example 4-2: The CEO of Blade Runner Inc. argues that she does not use the CAPM for capital budgeting because the firm is not well diversified. Is she right?

- Example 4-3: You are considering taking all your money out of the stock market and investing it in a single biotech project. Should you use the CAPM to evaluate this investment?

OK, so we have a model that provides a way of approximating a reasonable discount rate given several inputs. But how do we compute these inputs in practice?

1. Computing the Risk-Free Rate in practice:

- The risk-free rate is often proxied using the return on safe government securities. The yields on these securities can be easily obtained from Bloomberg or from newspapers.

- Government securities have different maturities with different rates of return. Which maturity should you use?
  - Ideally you want to match the maturities of the government securities to the timing of the project's cash flows.
  - As a result you may end up with different risk-free rates for different cash-flow maturities. In practice, though, people often use a constant risk-free rate for simplicity.

2. Computing the Market Risk Premium in practice:

- The size of the market risk premium is one of the most debated issues in finance.

- One popular way to compute the market risk premium is to compute the arithmetic average of the annual excess returns of stocks over government securities over a long time horizon. For example, the average annual difference between the return on the stocks and that on U.S. Treasury bonds between 1928 and 2016 is around 5-7%.

- The trouble with using historical data is that you are assuming that the future will look like the past. But perhaps the U.S. stock market has had a great century that will not repeat itself.

- An alternative way to estimate the market risk premium is to use consensus surveys asking “experts” what they think the market risk premium will be. The downside to this approach is that it is based on subjective opinions rather than market data. The upside is that it is forward looking.

3. Computing Betas in practice:

•Most of the time (in practice), CAPM is used to estimate discount rates on equity securities.

•The equity beta of a publicly traded firm can be estimated from historical stock returns. The estimate is the coefficient in a regression in which you regress the firm's stock return on the market return (e.g., the return on the S&P 500):

$$r_i = a + br_m + \varepsilon$$

•Analysts typically use a two- to five-year time horizon and weekly or monthly stock returns to estimate equity betas.

•These betas are often computed in real-time by companies like Bloomberg and Yahoo finance.

•What are some examples of high vs. low beta stocks?

Betas are often times not computed for debt securities and simply assumed to be 0. Why?

For investment grade debt (above BBB)

1. Investment grade debt has a low default likelihood and thus little correlation with the market.
2. If a firm has investment grade debt, usually means its debt ratio is not too high. Hence, even if you underestimate the true debt beta, the effect of this measurement error on the numerical value of the asset beta in the unlevering formula is likely small.

For non-investment grade debt it is arguably less reasonable to assume a debt beta of zero, however, practically speaking:

1. Historical returns on debt securities are often difficult to observe. Furthermore, even if you are able to obtain such historical returns you will only capture the beta of the firm's traded debt (e.g., bonds), which tends to be junior to and thus riskier than the firm's non-traded debt (e.g., bank loans).
2. Even for non-investment grade debt the beta is likely to be small (studies estimate a range of debt betas from 0.1 to 0.4).

2. The Unlevered Cost of Capital

The unlevered cost of capital is the discount rate associated with the "unlevered" assets of the firm, i.e. the discount rate for the assets of a firm that is entirely financed by equity. This rate is often denoted R_a.

R_a is important because it reflects the fundamental assets of the business, devoid of any financing considerations such as tax shields from leverage.

Based on accounting identities, there is a mechanical relationship between the unlevered cost of capital and the equity and debt cost of capital.

This relationship is characterized by the following equation:

$$R_a = \frac{E}{E + (1 - t)D} R_e + \frac{(1 - t)D}{E + (1 - t)D} R_d$$

Unlevered Cost of Capital Special Case

There is one special case in which the general equation for the unlevered cost of capital becomes simplified: firms maintain a constant leverage ratio.

In this case, the equation becomes:

$$R_a = \frac{E}{E+D} R_e + \frac{D}{E+D} R_d$$

Unlevered Asset Betas

Similar relationships can be described when relating the unlevered asset beta of a firm with the equity and debt betas of the firm.

$$\beta_a = \frac{E}{E + (1 - t)D}\beta_e + \frac{(1 - t)D}{E + (1 - t)D}\beta_d$$

Unlevered Asset Beta Special Cases

There are two special cases in which the general equation for the unlevered asset beta becomes simplified:

1) Firms maintain a constant leverage ratio. In this case, the equation becomes:

$$\beta_a = \frac{E}{E+D}\beta_e + \frac{D}{E+D}\beta_d$$

1) Firms do not maintain a constant leverage ratio, but they maintain risk-free debt. In this case, the equation becomes:

$$\beta_a = \frac{E}{E+(1-t)D}\beta_e$$

Example 4-4: General Conglomerates is evaluating an oil exploration project proposed by the manager of its aviation division. What is the project's asset beta? Assume all firms maintain constant leverage ratios.

Company	E/(D+E)	Equity Beta	Debt Beta
General American Oil	0.85	1.81	0.00
Flight Airlines	0.50	2.00	0.25
Everything Conglomerates	0.95	1.10	0.00
Louisiana Oil Exploration	0.63	2.36	0.12
Mesa Petroleum	0.77	1.90	0.05

Portfolio Special Case

The return (beta) of a portfolio is a weighted sum of the returns (betas) of the individual assets in the portfolio:

$$r_A = \sum_{i=1}^{N} \frac{A_i}{A_N} r_i \qquad b_A = \sum_{i=1}^{N} \frac{A_i}{A_N} b_i$$

- Example 4-5: Your firm is considering building a plant to produce cars. You have the following information:

Company	E/(E+D)	Equity Beta	Debt Beta
Car & Food Conglomerate	0.60	2.375	0.00
Food Industries	0.80	2.50	0.00

Car and Food Conglomerates has two divisions (Cars and Food) which each account for half of the firm's market value. Food Industries is only involved in the food business. Assume all firms maintain constant leverage ratios.

What is the beta of your project?

3. Weighted Average Cost of Capital (WACC)

Once we have estimates for a firm's expected return on debt and equity securities, we can estimate the firm's weighted average cost of capital using the following formula (assuming the firm maintains a constant debt-to-equity ratio):

$$r_{WACC} = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D (1-T)$$

Using this discount rate, we can compute the NPV of a firm's cash flows:

$$\text{NPV}_{\text{levered}} = \sum_{t=0}^{\infty} \frac{C_t}{\left(1+r_{WACC}\right)^t},$$