Solutions for Real Options Pset

Question 1.

a) NPV = $-30+\dfrac{(0.5)(66)+(0.5)(34)}{1.1}=15.45.$

b) Let's compute the value of the project assuming we have commissioned the study. We first solve for the optimal decision (checked) in t = 1 for both scenarios (high and low demand):

t = 0 t= 1

High

√ Go ahead $-30+\dfrac{66}{1.1}=30$

Don't: 0

Low

√ Go ahead $-30+\dfrac{34}{1.1}=0.91$

Don't: 0

The NPV of the project in t = 0 is: $\dfrac{(0.5)(30)+(0.5)(0.91)}{1.1}=14.05$

The NPV is lower than in part a). You should therefore invest today even if the study is free. By implication, you are not willing to pay anything for the study. The reason why the additional information does not add value here is that it does not affect your decision to invest. Moreover, because you have to wait for the information to arrive it delays all cash flows by one year and reduces the NPV of the project.

Question 2.

a) NPV= $-300+\dfrac{(1/3)(600)+(1/3)(0)+(1/3)(-600)}{.5}=-300$. You should not invest.

b) Let's compute the value of the project assuming we have commissioned the two-year study. We first solve for the optimal decision (checked) in t = 2 for the high-, medium- and low-demand scenarios.

t = 2

High

√ Go ahead $-300+\frac{600}{0.5}=900$

Don't: 0

Medium

Go ahead $-300+\frac{0}{0.5}=-300$

√ Don't: 0

Low

Go ahead $-300+\frac{(-600)}{0.5}=-1500$

√ Don't: 0

The NPV of the project in t = 0 is: $\frac{(1/3)(900)+(1/3)(0)+(1/3)(0)}{(1.5)^2}=133.33$.

a) Let's compute the value of the project assuming we have commissioned the one-year study. Note that with this type of study we do not know for sure whether demand is high, medium or low. We only know whether the result of the study is positive or negative.

t =1

Positive

√ Go ahead $-300+\frac{(2/3)(600)}{0.5}=500$

Don't: 0

Negative

Go ahead $-300+\frac{(2/3)(-600)}{0.5}=-1100$

√ Don't: 0

The NPV of the project in t = 0 is: $\frac{(1/2)(500)+(1/2)(0)}{1.5}=166.67$

b) You should commission the one-year study since 166.67 > 133.33.

<u>Comment:</u> In this example the choice is between getting some information earlier and receiving more precise information later. For the specific numbers in this example the former alternative is better.

Question 3.

a) The profit in year 2 is either (1-0.75) or (1-0.25) per widget with equal probability. Therefore the NPV of buying the machine is:

$$-40{,}000+\frac{0.5(100{,}000)(1-0.75)+0.5(100{,}000)(1-0.25)}{(1.1)^2}=1{,}322.3$$

b) Let's calculate the NPV of the machine with the option to switch gases. You invest 40,000 today and can choose between "Gas A" and "Gas B" at the beginning of year 2. As we have done in class we first figure out the optimal decision and its associated cash flows:

Use gas A: 100,000(1-0.75) = 25,000
Use gas B: 100,000(1-0.80) = 20,000

→ Optimal decision: use gas A and get 25,000 at the end of year 2

-40,000

Use gas A: 100,000(1-0.25) = 75,000
Use gas B: 100,000(1-0.20) = 80,000

→ Optimal decision: use gas B and get 80,000 at the end of year 2

The NPV of buying the machine with the option to switch gases is:

$$-40{,}000+\frac{0.5(25{,}000)+0.5(80{,}000)}{(1.1)^2}=3{,}388.4$$

The maximum you are willing to pay for the device is therefore 3,388.4 – 1,322.3 = 2,066.1.

c) Using the same solution method as in part b) we obtain:

Use gas A: 100,000(1-0.75) = 25,000
Use gas B: 100,000(1-0.20) = 80,000

→ Optimal decision: use gas B and get 80,000 at the end of year 2

-40,000

Use gas A: 100,000(1-0.25) = 75,000
Use gas B: 100,000(1-0.80) = 20,000

→ Optimal decision: use gas A and get 75,000 at the end of year 2

The NPV of buying the machine with the option to switch gases is:

$$-40{,}000 + \frac{0.5(80{,}000) + 0.5(75{,}000)}{(1.1)^2} = 24{,}049.6.$$

The maximum you are willing to pay for the device is now 24,049.6 – 1,322.3 = 22,727.3.

<u>Comment</u>: The purpose of this problem is to show that the value of the option to switch to a different input of production depends on the correlation between the prices of the inputs. In part b) the prices of the two gases are positively correlated: They are either both expensive or both cheap. In part c) the prices of the two gases are negatively correlated. When gas A is expensive gas B is cheap and vice versa; hence you always have the option to use a relatively cheap gas. When the prices are positively correlated (part b), even though the switching device is valuable, you do not gain that much because you are switching to a gas that is only slightly cheaper. When the prices are negatively correlated you gain a great deal as you can switch from an expensive gas to a considerably cheaper one.