Lecture 5. The CAPM

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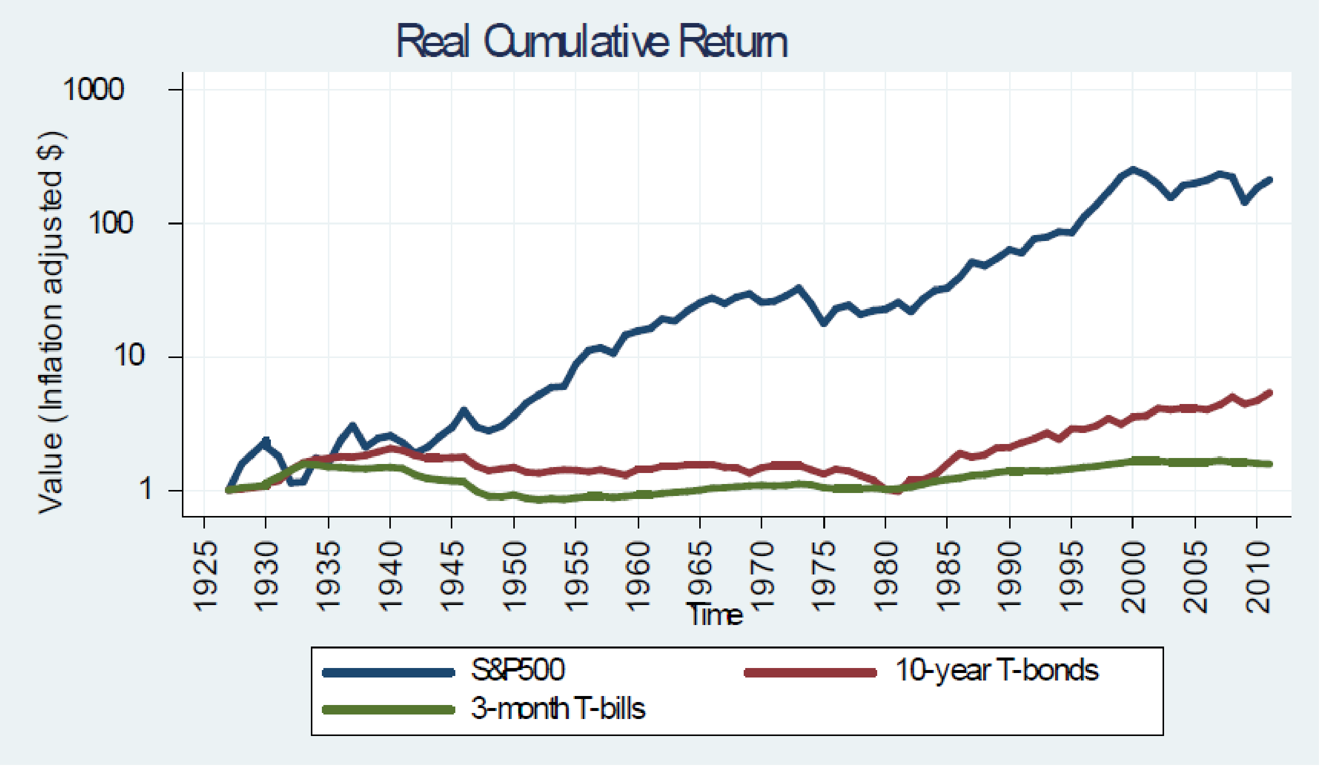
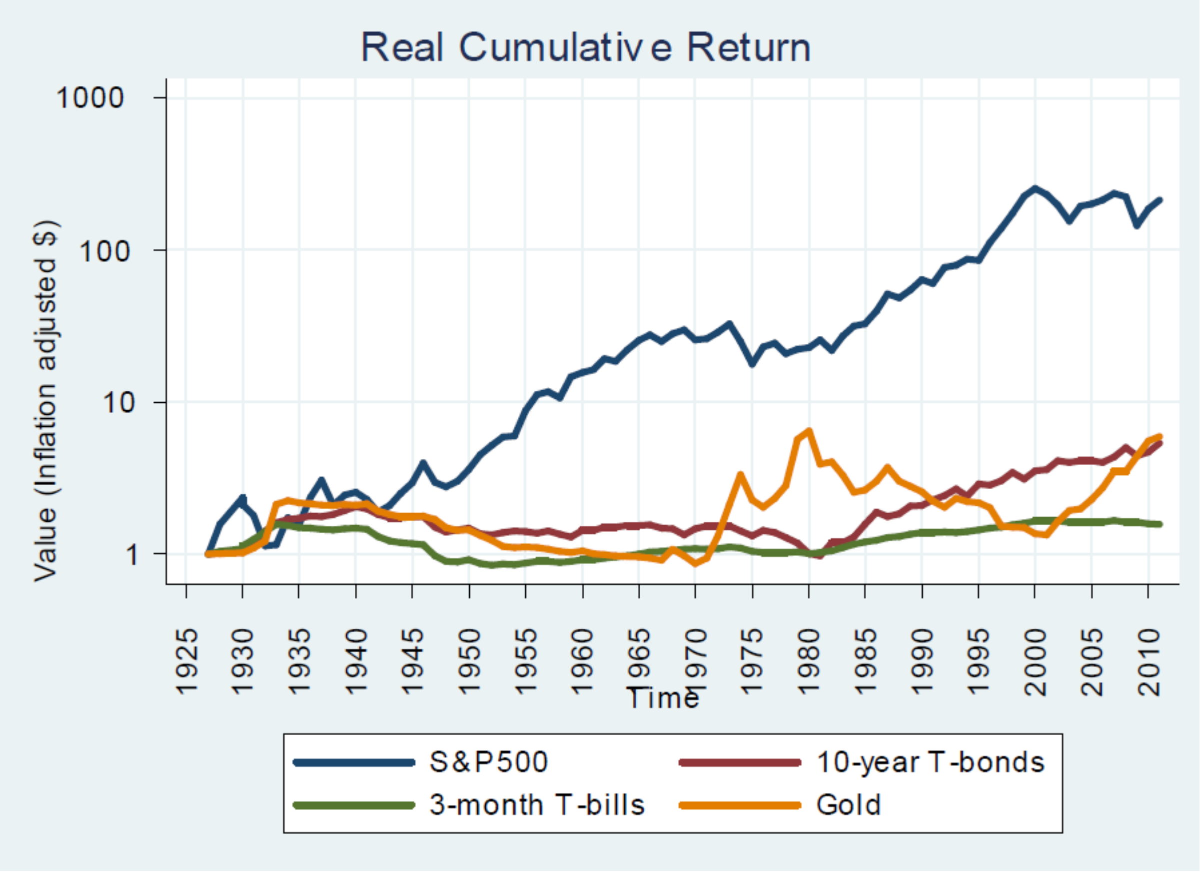
Motivation



- Among the three assets, the larger the return volatility, the larger the mean return.

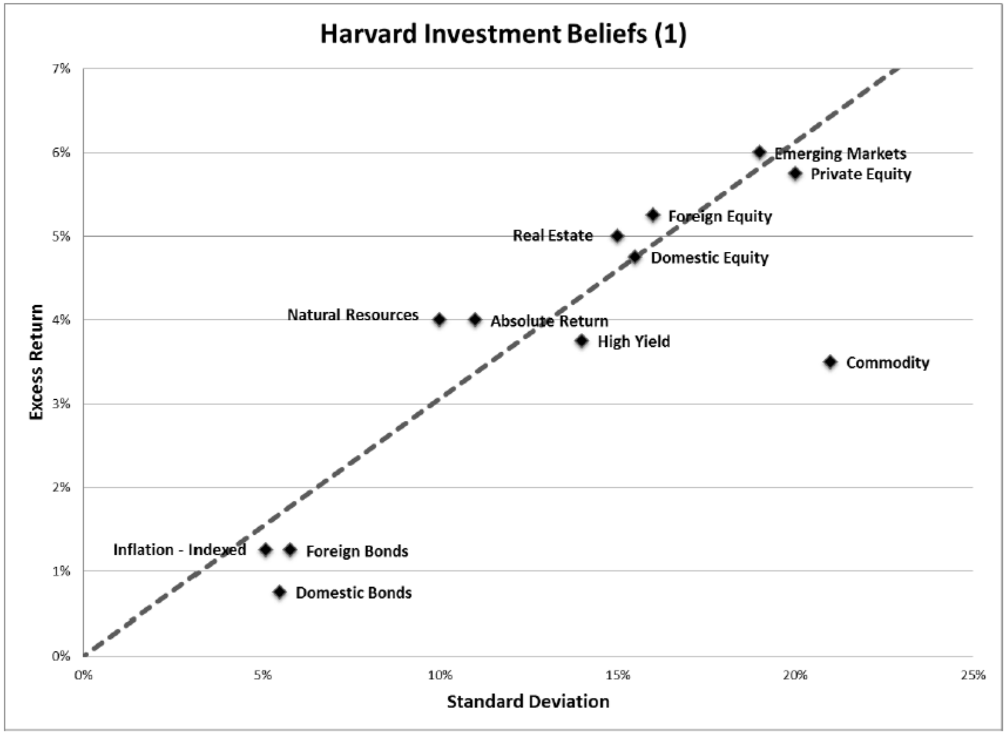
- This all seems sensible until we add another asset...

Motivation



- Why would anyone buy gold? It seems as volatile as stocks (S&P500) but generates lower returns.

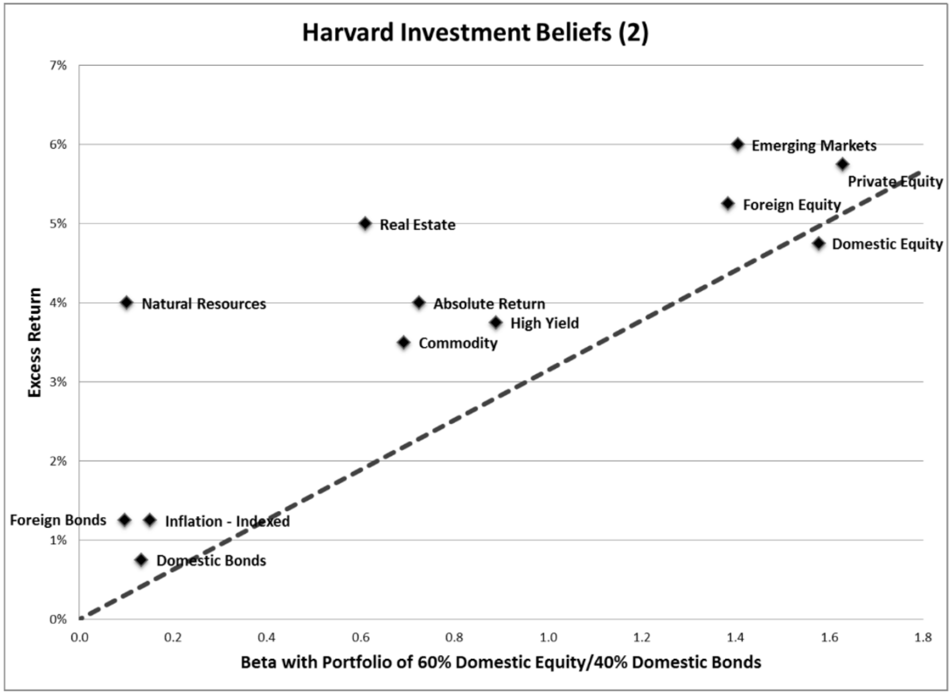
Topics Covered



Source: HMC Capital Market Assumptions, 2010.

- More generally, commodities seem to be an awful investment in terms of the return-volatility tradeoff.

Topics Covered



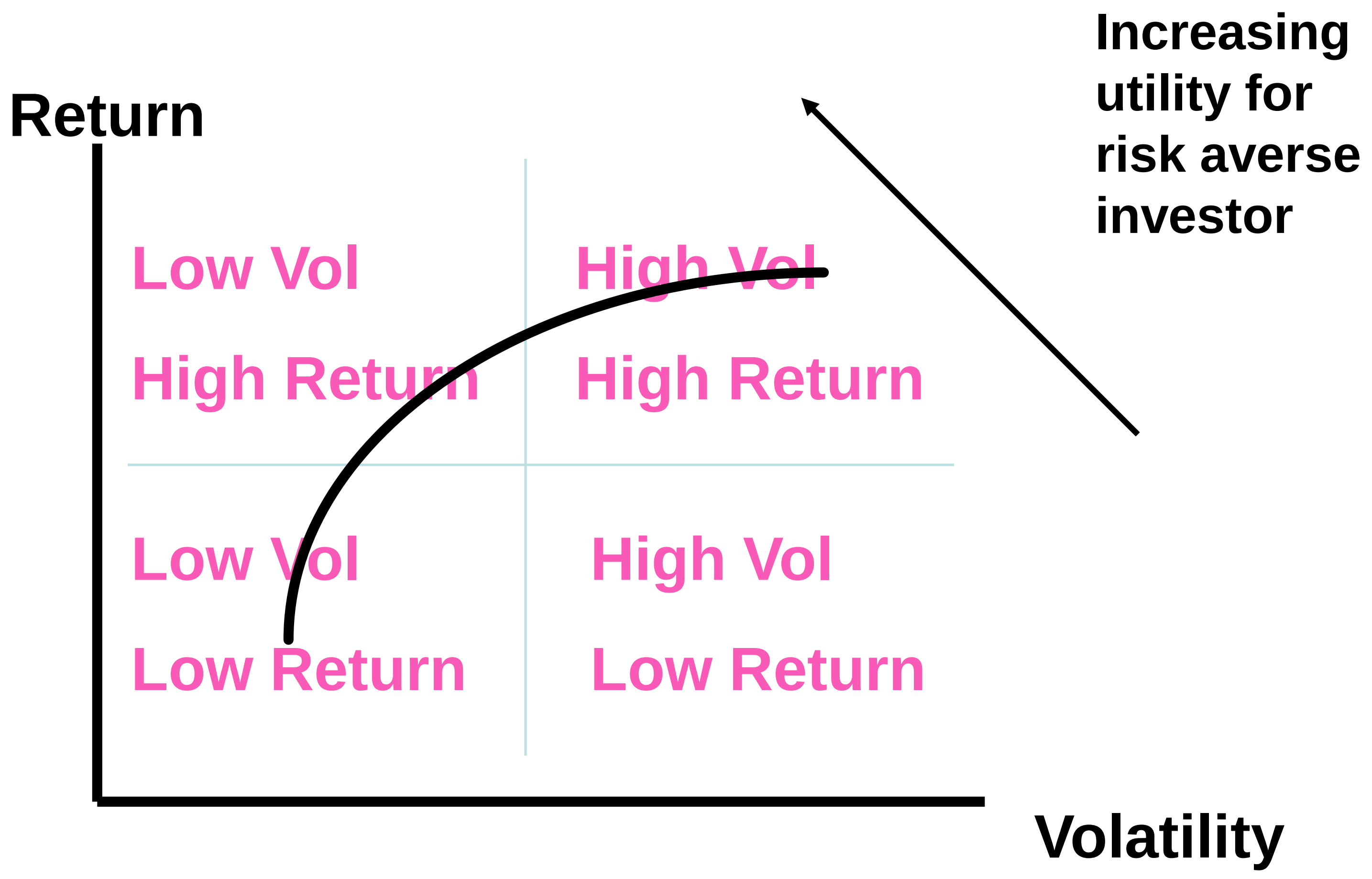
Source: HMC Capital Market Assumptions, 2010.

- The answer is that commodity is great for diversification. And risk that matters to investors is <u>NOT the asset's own volatility</u>, but how it <u>contributes to the overall portfolio volatility</u>. CAPM formalizes this idea.

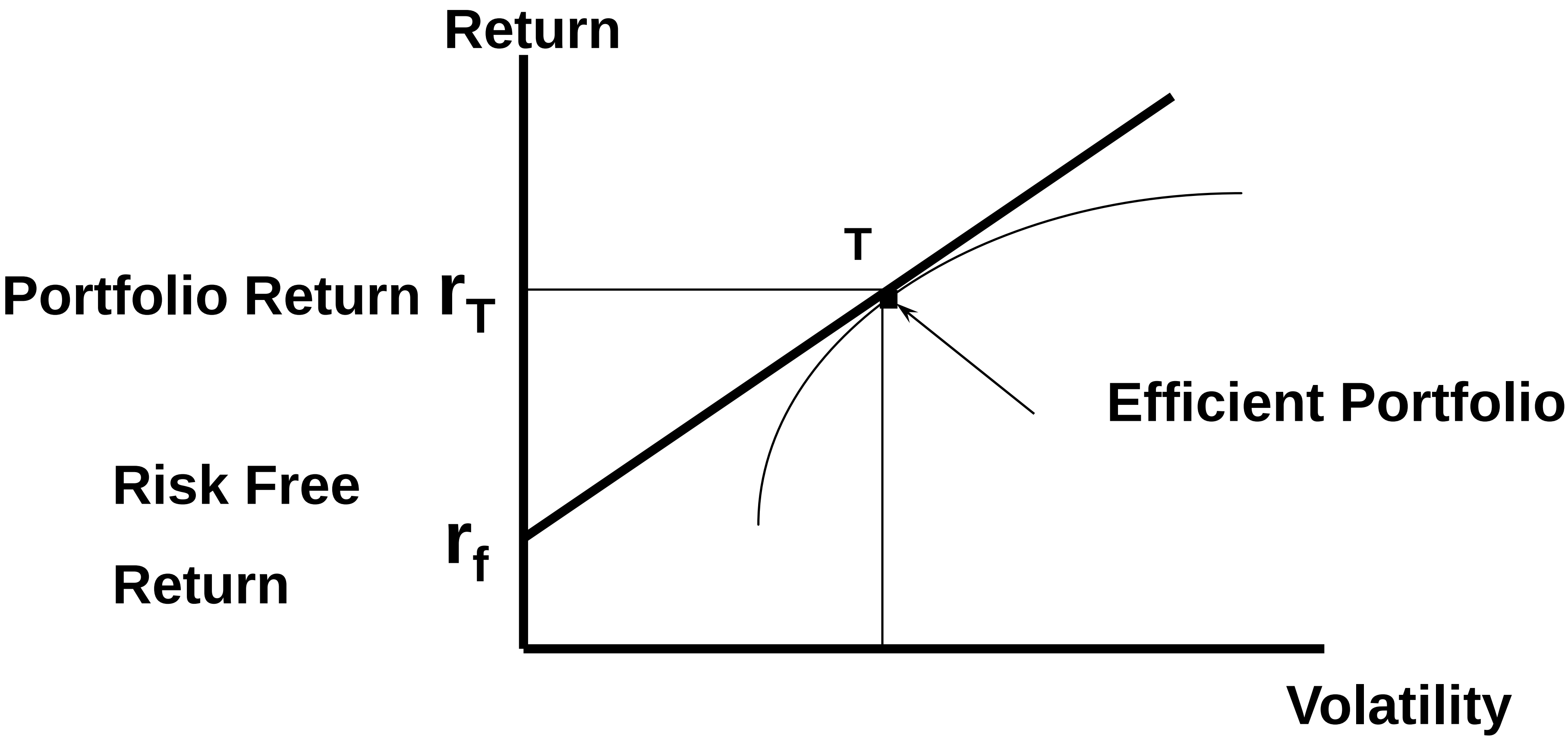
Topics Covered

- The CAPM
- Measuring betas
- Use of the CAPM
- Empirical evidence on the CAPM
- Multifactor models

How does the investor choose?



Efficient Frontier



The Capital Asset Pricing Model

- The CAPM accomplishes two goals:
  - To give an economic meaning/interpretation to the tangency portfolio.
  - To determine the expected returns of assets and portfolios of assets by using some nice features of the tangency portfolio.

The Capital Asset Pricing Model

- Assumptions:
  - There are N stocks and one riskless asset in the economy (i.e., lending and borrowing at a single riskless rate);
  - Each investor holds a mean-variance efficient portfolio, i.e., a portfolio with the highest expected return given volatility;
  - Investors have a one-period horizon;
  - Investors have the same beliefs;
  - Market clears, i.e., demand = supply.

The Capital Asset Pricing Model

- As we discussed, all investors hold a combination of the risk free asset and the tangency portfolio (with different weights).

- If we add up all investors, and consider them as a single group, this group also invests in the risk free asset and the tangency portfolio (with some average weights).

- Hence, the total demand for risky assets is represented by the tangency portfolio.

The Capital Asset Pricing Model

- We now characterize the supply of stocks.
- Suppose, there are N stocks in the economy and stock i has market capitalization (number of shares times share price) V_i.
- The proportion of wealth allocated to asset i is given by:

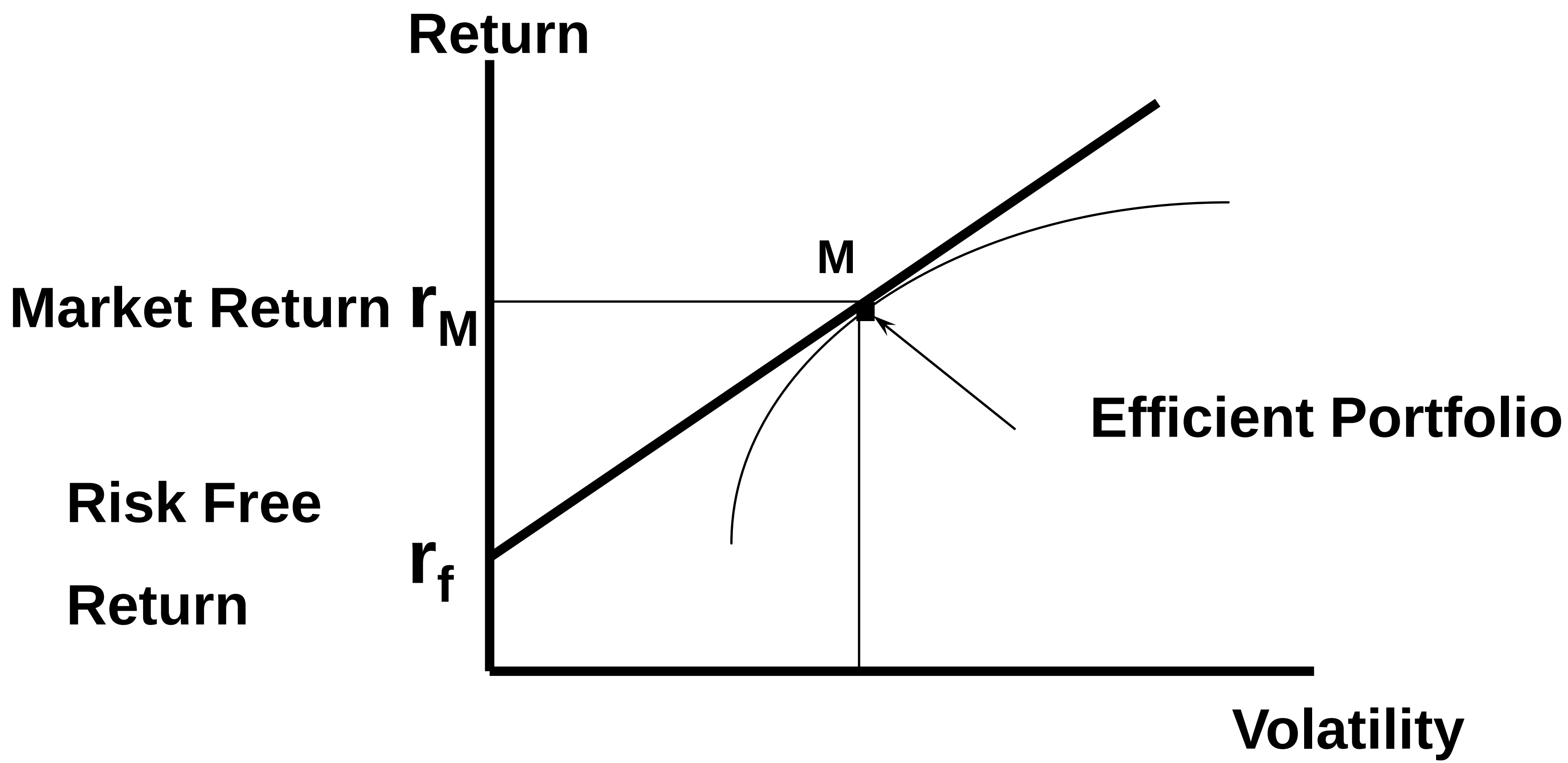
$$w_i^M = \frac{V_i}{V_1 + \ldots + V_N}.$$

- The market portfolio has portfolio weight w_i^M in stock i.
- We can approximate the market portfolio by a value-weighted index of all stocks traded in the market.

The Capital Asset Pricing Model

- The supply of all risky assets is the market portfolio.
- Matching demand and supply, we see that the market portfolio is the tangency portfolio!
- Said differently, market weights are determined by stocks’ expected returns, standard deviations, and return correlations.

Capital Market Line



CAPM: derivation by intuition

- Recall that investors are maximizing the expected returns while minimizing the risk, in a mean-variance analysis framework.

- Now let’s derive CAPM by perturbation.

CAPM: derivation by intuition

- If you increase the weight of asset j in your portfolio (a combination of the market portfolio and the risk free asset) by Δ and borrow at the risk free rate:
  - Then expected returns increase by:

$$\Delta(E(r_j)-r_f)$$

  - Then the variance of the portfolio increases by:

$$2\Delta Cov\left(r_j, r_M\right)$$

  - Hence, the return/risk gain is:

$$\frac{\Delta(E(r_j)-r_f)}{2\Delta Cov\left(r_j, r_M\right)}=\frac{(E(r_j)-r_f)}{2Cov\left(r_j, r_M\right)}$$

  - This must be the same for all assets. Why?

CAPM: derivation by intuition

- Suppose that for two assets A and B:

$$\frac{E(r_A)-r_f}{Cov(r_A,r_M)} > \frac{E(r_B)-r_f}{Cov(r_B,r_M)}$$

  - Asset A offers a better return/risk ratio than asset B
    - Buy A, sell B
    - What if everybody does this?
  - Hence, in equilibrium, all return/risk ratios must be equal for all assets

$$\frac{E(r_A)-r_f}{Cov(r_A,r_M)} = \frac{E(r_B)-r_f}{Cov(r_B,r_M)}$$

CAPM: derivation by intuition

- If the risk-return trade-off is the same for all assets, then the equality also holds for the market:

$$\frac{E(r_A)-r_f}{Cov(r_A,r_M)}=\frac{E(r_B)-r_f}{Cov(r_B,r_M)}=\frac{E(r_M)-r_f}{Var(r_M)}$$

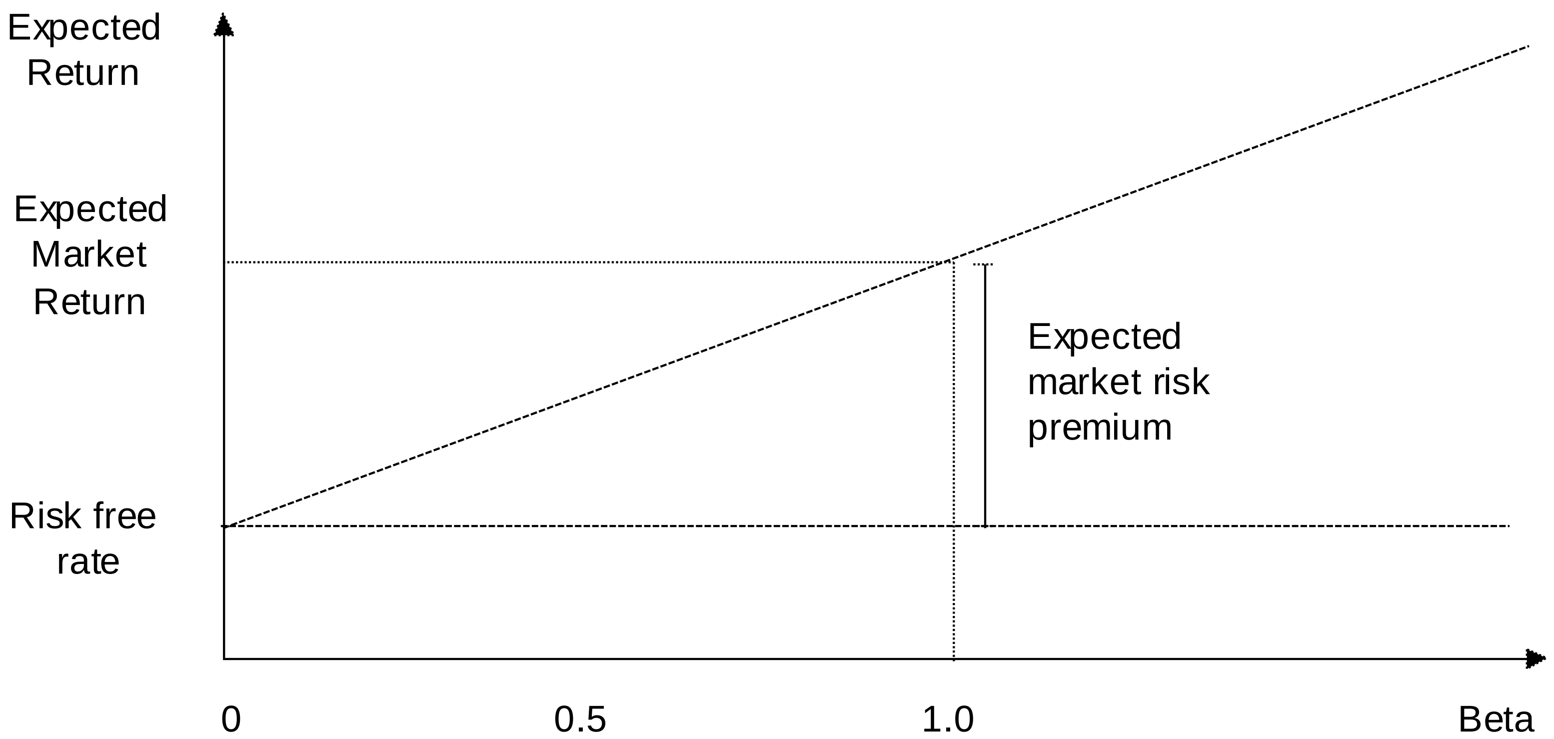
This gives the relationship between risk and expected return for individual stocks and portfolios.

$$E(r_A)=r_f+\frac{Cov(r_A,r_M)}{Var(r_M)}\left(E(r_M)-r_f\right)=r_f+\beta_A\left(E(r_M)-r_f\right)$$

$$\beta_A=\frac{Cov(r_A,r_M)}{Var(r_M)}$$

This is called the **Security Market Line**.

Security Market Line



Expected return = Risk free rate + Beta factor x Expected market risk premium

Implications of CAPM

- If β = 0 then $E[r_j] = r_f$ as the asset does not contribute to the riskiness of an efficient portfolio.
  - Risks associated with this asset can be diversified away, and hence investors do not require a risk premium for holding this asset.
- If β > 0 then $E[r_j] > r_f$ as the asset increases the riskiness of an efficient portfolio.
  - Investors require a risk premium for holding the asset.
- If β < 0 then $E[r_j] < r_f$ as the asset decreases the riskiness an efficient portfolio.
  - The asset is valuable, and investors are willing to buy it even if its expected return is lower than the risk free rate.

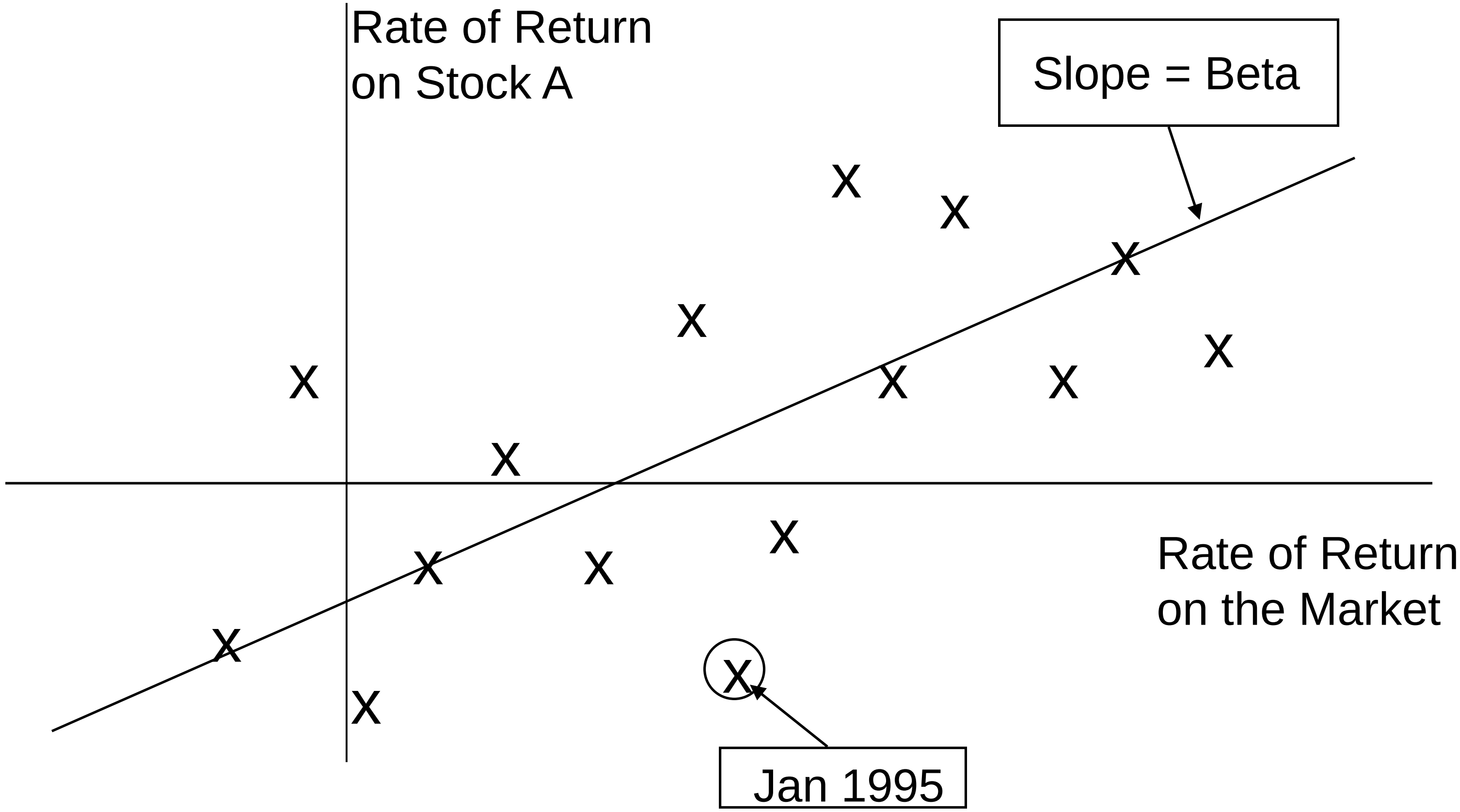
Measuring Betas

Market Portfolio – the portfolio of all assets in the economy. In practice a broad stock market index, such as the S&P Composite, is used to represent the market.

Beta – How do we estimate beta? A linear regression.

Beta measures the sensitivity of a stock's return to the return on the market portfolio.

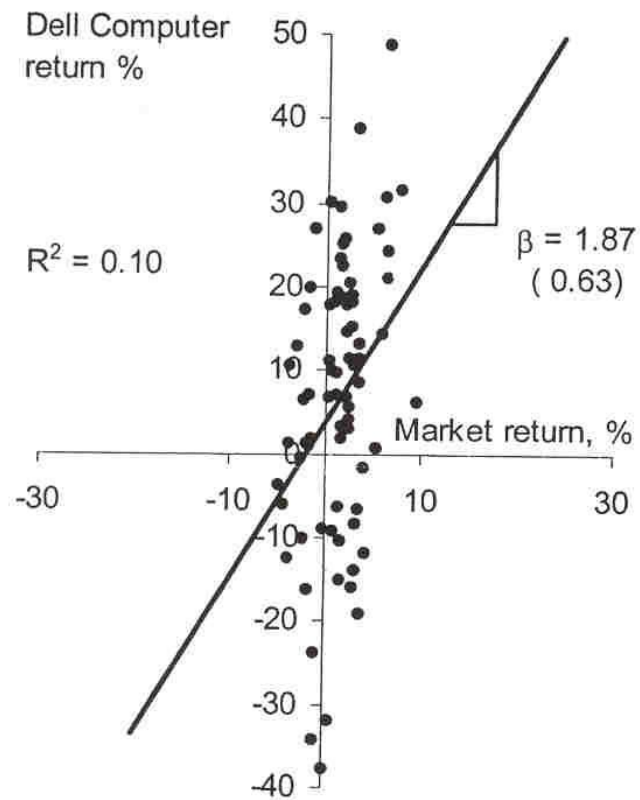
Measuring Betas



Measuring Betas

Dell Computer

Price data: May 91- Nov 97



$R^2 = .10$

B = 1.87

Slope determined from plotting the line of best fit.

Market return (%)

Dell return (%)

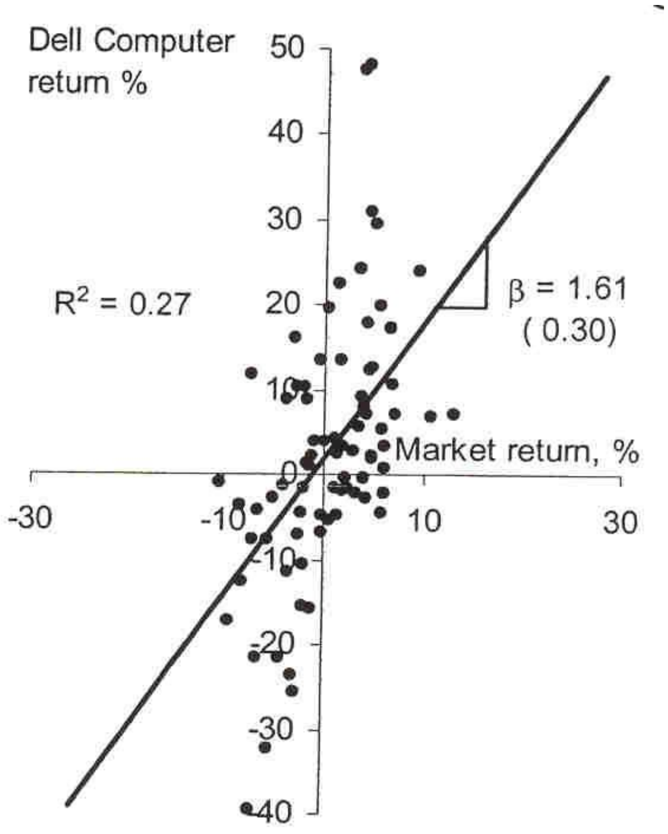
Measuring Betas

Dell Computer

Price data: Dec 97 - Apr 04

$R^2 = .27$

B = 1.61



Dell return (%)

Market return (%)

Slope determined from plotting the line of best fit.

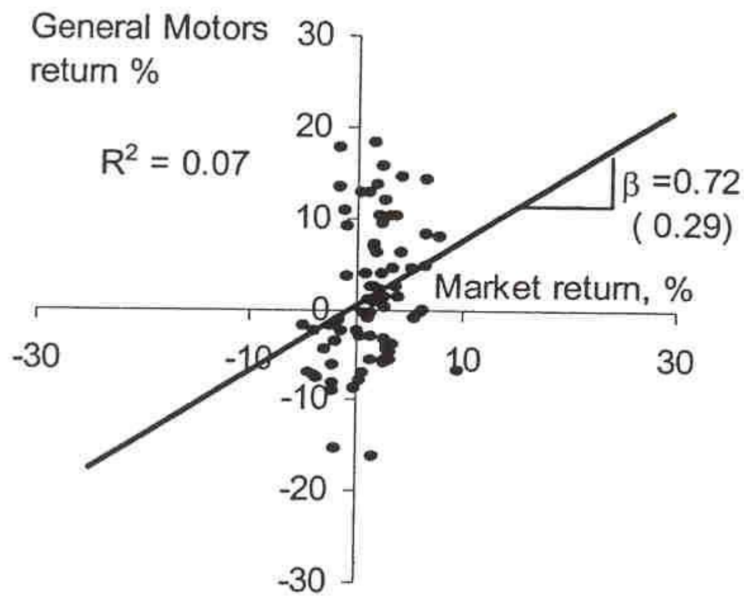
Measuring Betas

General Motors

Price data: May 91- Nov 97

$R^2 = .07$

B = 0.72



GM return (%)

Market return (%)

Slope determined from plotting the line of best fit.

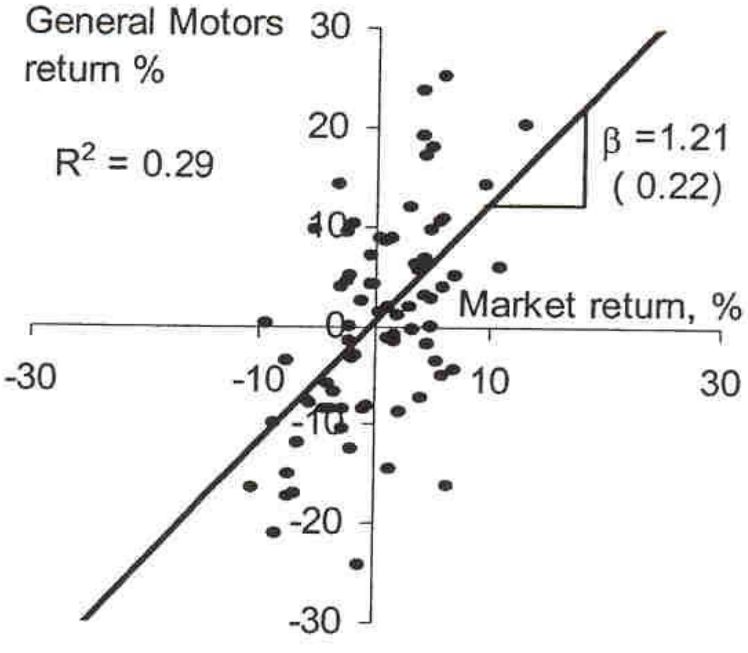
Measuring Betas

General Motors

Price data: Dec 97 - Apr 04

$R^2 = .29$

B = 1.21



GM return (%)

Slope determined from plotting the line of best fit.

Market return (%)

Betas of selected common stocks

Stock	Beta	Stock	Beta
AT&T	0.96	Ford Motor	1.03
Boston Ed.	0.49	Home Depot	1.34
BM Squibb	0.92	McDonalds	1.06
Delta Airlines	1.31	Microsoft	1.20
Digital Equip.	1.23	Nynex	0.77
Dow Chem.	1.05	Polaroid	0.96
Exxon	0.46	Tandem	1.73
Merck	1.11	UAL	1.84

Betas based on 5 years of monthly returns.

Uses of the CAPM

- Estimate expected returns for:
  - Project evaluation, project selection
  - Benchmark to evaluate investment strategies (risk adjustment)
    - It tells us what the required return on a stock should be, accounting for risk.
  - Performance evaluation and attribution of money managers.

Uses of the CAPM

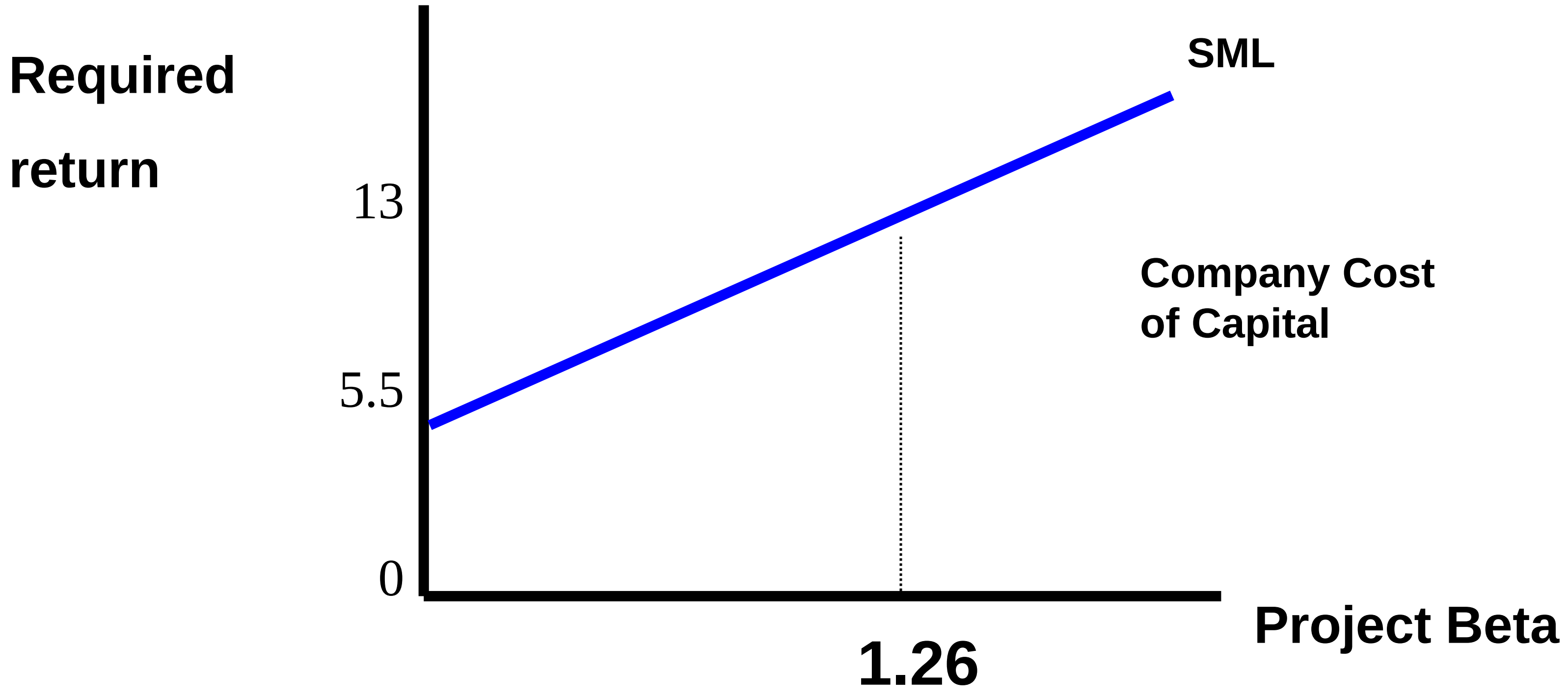
- The CAPM gives us a way to estimate the expected (or required) rate of return on equity.

$$E(r_j) = r_f + \beta_j\left[E(r_M) - r_f\right]$$

- We need estimates of three things:
  - Risk-free interest rate, r_f.
  - Market risk premium, $[E(r_M)-r_f]$.
  - Beta for the stock, β_j.

Uses of the CAPM

- A company's cost of capital can be estimated by the CAPM required return



Testing the CAPM

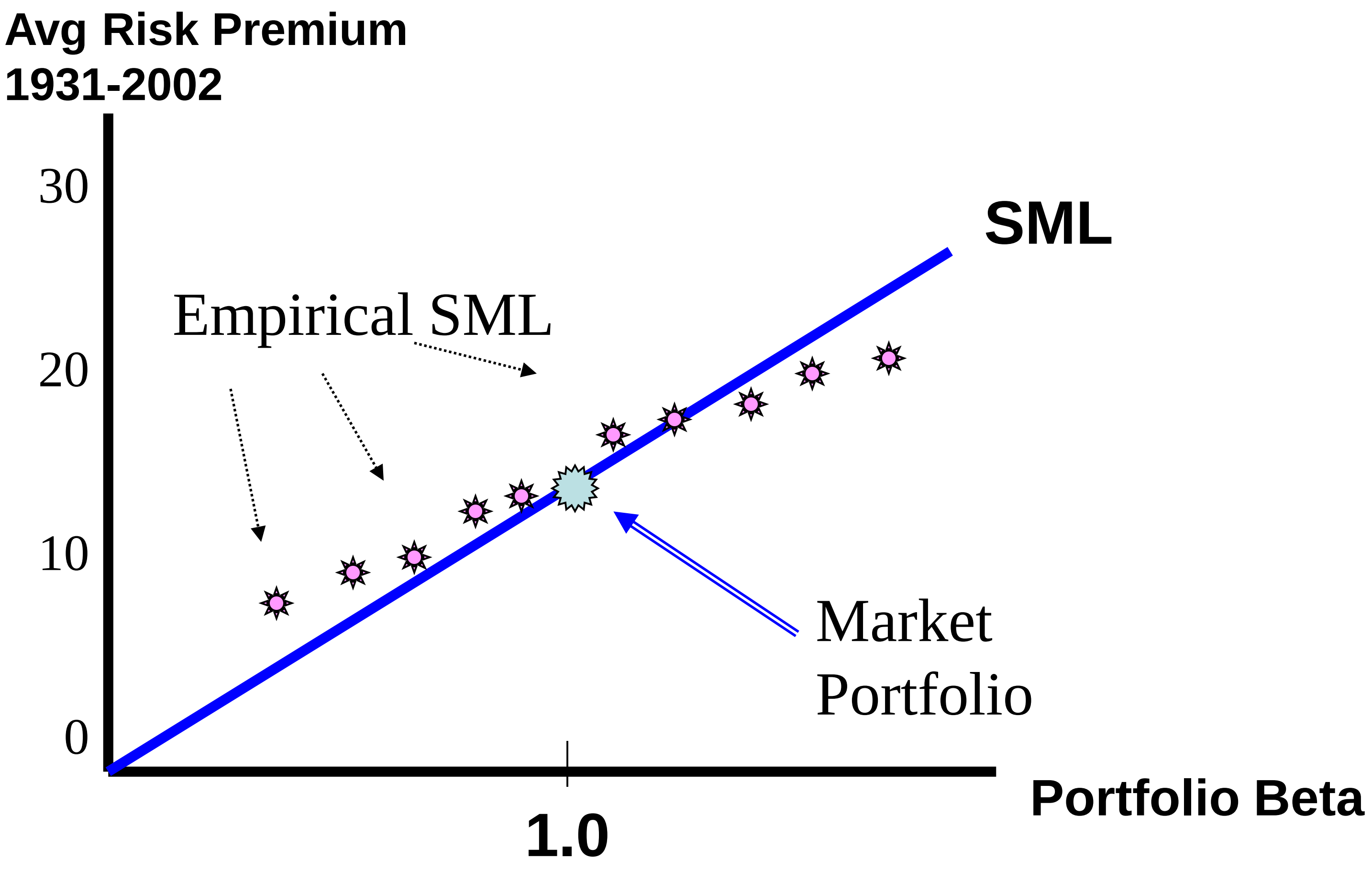
- Let's sort all stocks by beta into ten portfolios:
  - Portfolio 1 contains 10% of all stocks with the lowest beta;
  - Portfolio 2 contains 10% of stocks with the next-lowest beta ;…;
  - Portfolio 10 contains 10% of stocks with the highest beta.

- We now look at portfolios formed in 1931 and held for 34 years and portfolios formed in 1966 and held for 39 years.

- We see that the relationship for CAPM is weaker since the mid-1960s, especially for high-beta portfolios.

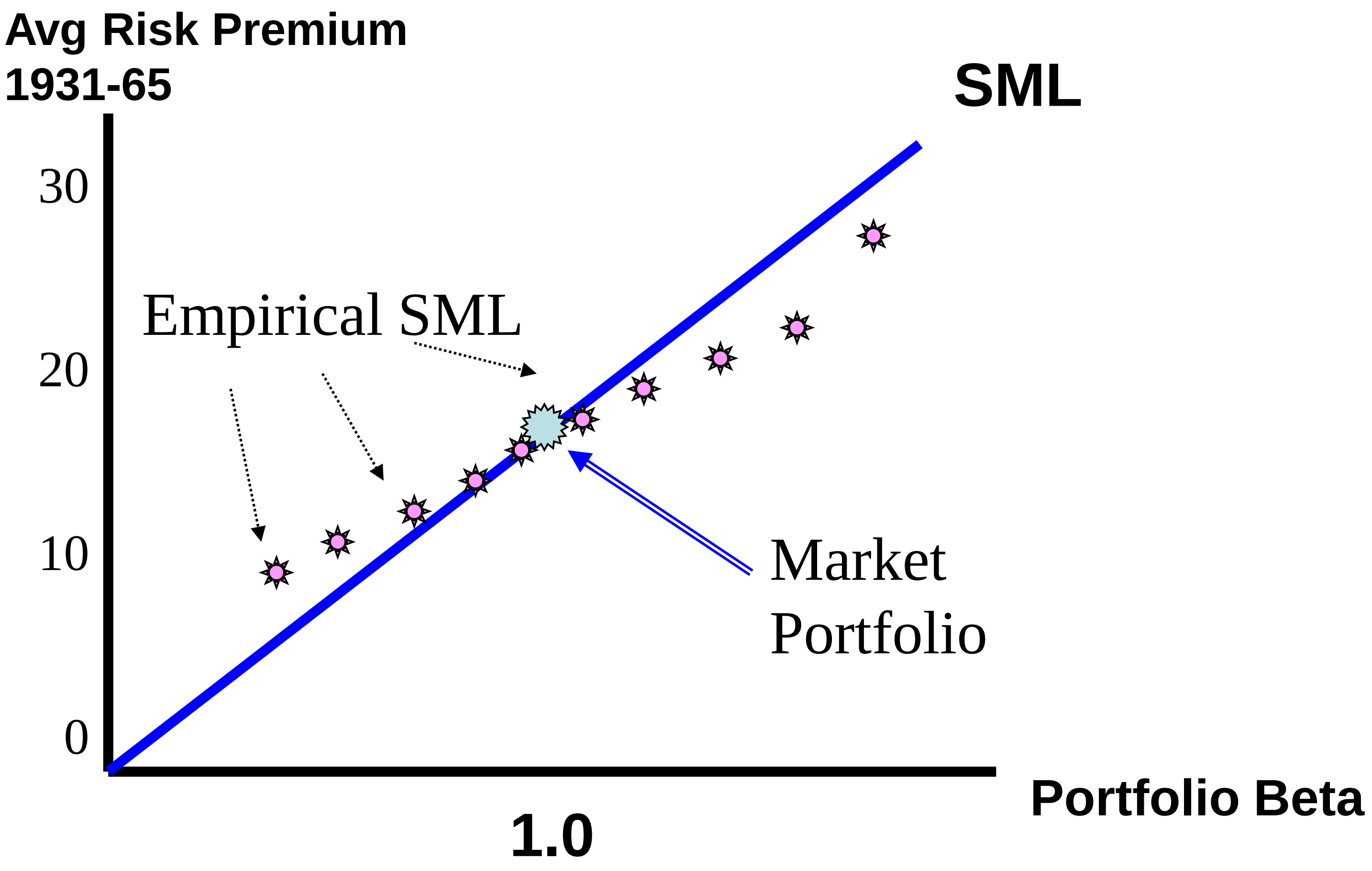
Testing the CAPM

Beta vs. Average Risk Premium



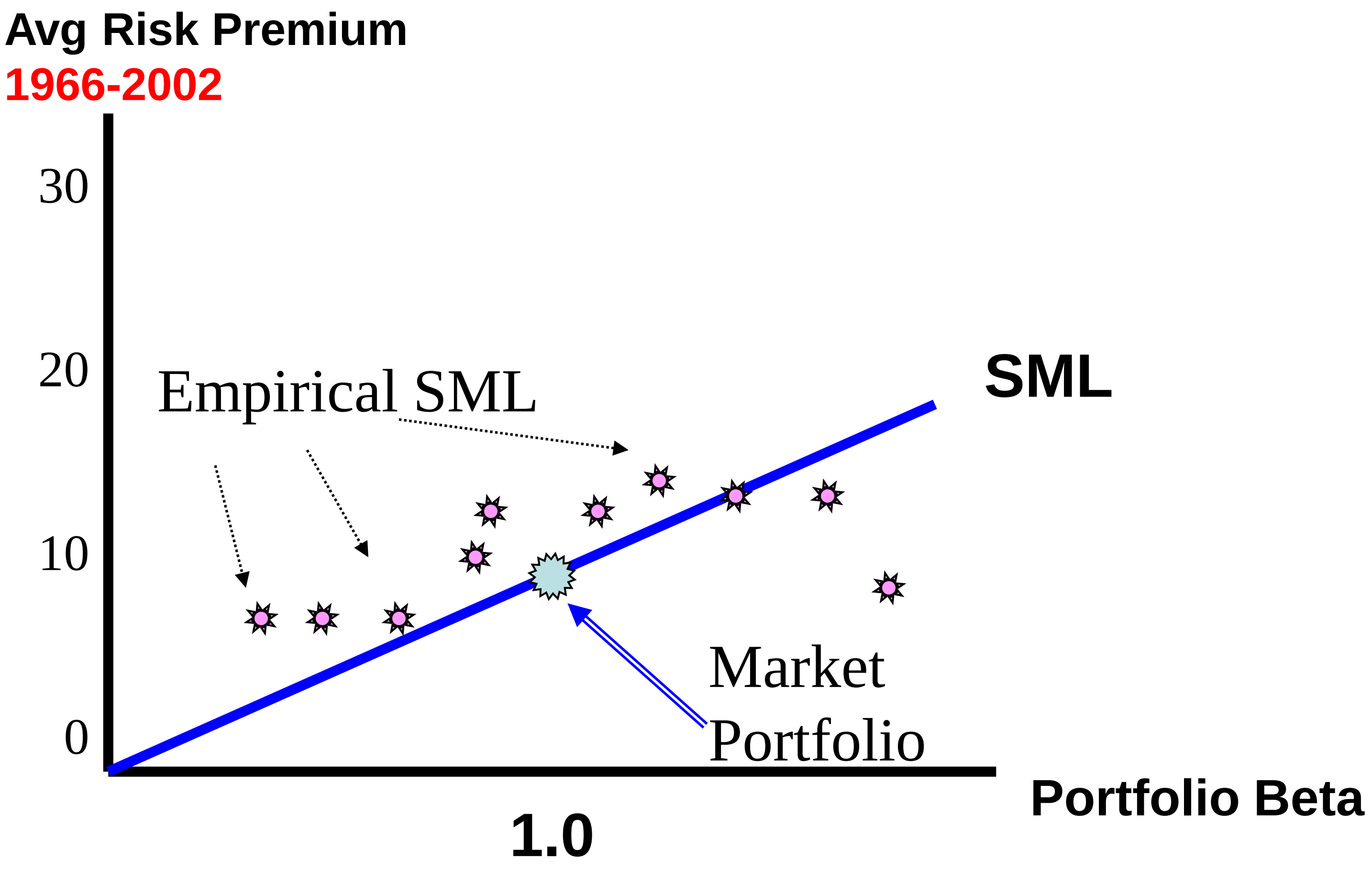
Testing the CAPM

Beta vs. Average Risk Premium



Testing the CAPM

Beta vs. Average Risk Premium



Empirical evidence on the CAPM

- Some evidence that expected returns increase with risk
- But at the same time:
  - high beta stocks have lower returns than predicted by the CAPM
  - low beta stocks have higher returns than predicted by the CAPM

Empirical evidence on the CAPM

- Is CAPM dead?
  - Theory says that *true* betas are related to average returns.
  - We regress average returns on *estimated* betas.
    - Estimation error in betas may explain part of this problem
  - We use a *proxy* for the market return (the “Roll critique”)
    - Market should include bonds, real estate, foreign assets, human capital, …
    - The real market portfolio is unobservable, thus the CAPM cannot be tested!

Empirical evidence on the CAPM

- CAPM is imperfect, but still useful in the estimation of the costs of capital.
- It has been found in the data that other sources of risk which are not captured by the market beta also matter.
- We can try to improve CAPM by:
  – Accounting for market frictions that limit borrowing, lending, and short selling;
  – Accounting for consumption (Consumption CAPM);
  – Accounting for multiple periods (Intertemporal CAPM);
  – Accounting for additional sources of risk (multifactor models; arbitrage pricing theory).

Multifactor Pricing Models

- Alternative to the CAPM.

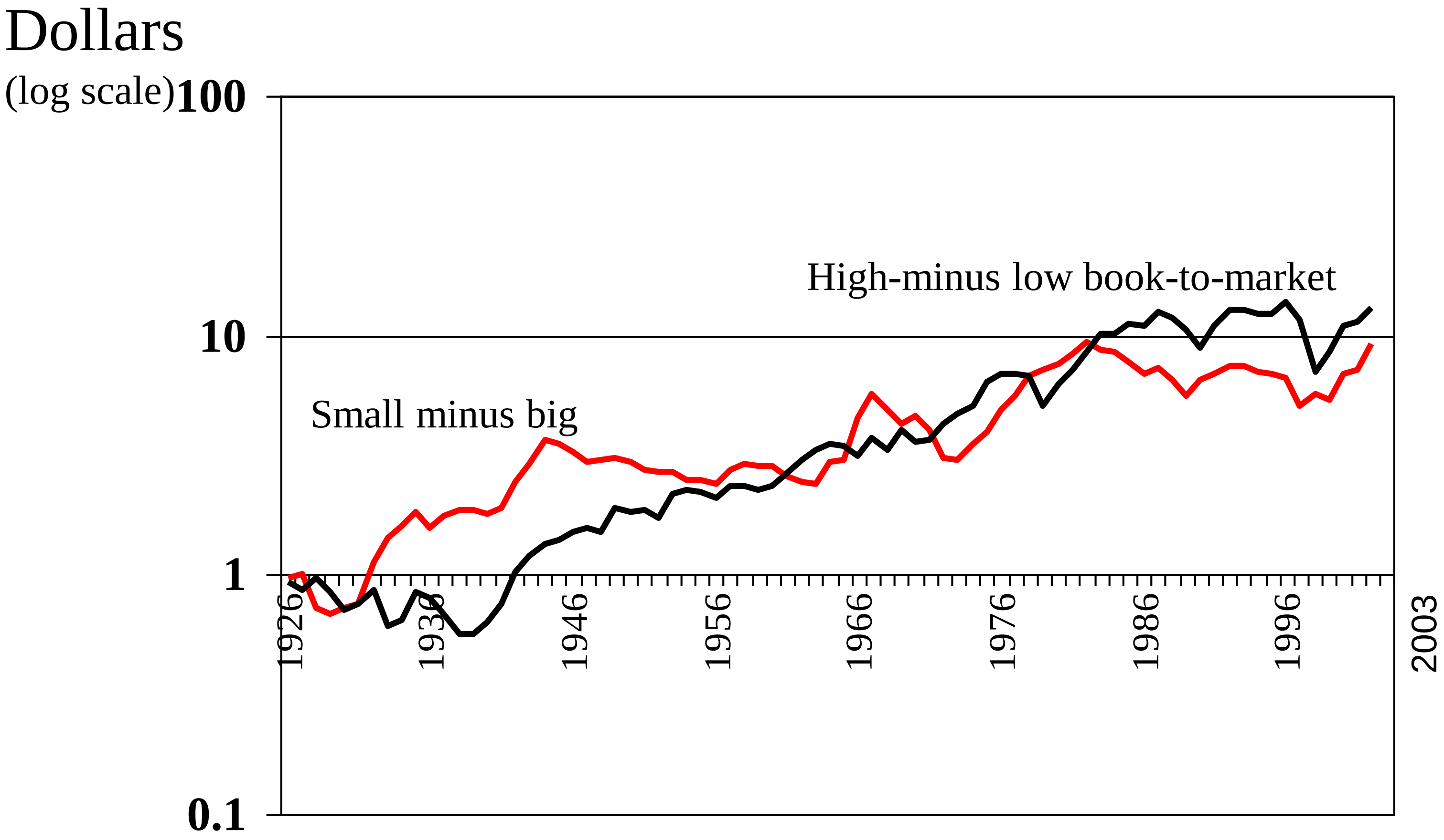
  Include more factors in the regression, beyond the market factor. For example:

$$F_{At} = R_{M,t} - r_f$$
$$F_{Bt} = R_{small,t} - R_{big,t.}$$
$$F_{Ct} = R_{high,t} - R_{low,t.}$$

- As in the CAPM, the betas (sensitivities) are the coefficients of the regression

$$R_{it} = \alpha_i + \beta_{Ai} F_{At} + \beta_{Bi} F_{Bt} + \beta_{Ci} F_{Ct} + \varepsilon_{it}$$

Multifactor Pricing Models



Multifactor Pricing Models

How to use a multi-factor model:

- Construct a series of factors F_{kt} through time
- Examples of factors:

  (1) Macroeconomic and financial variables: Chen, Roll, and Ross (1986)

  (2) Return spreads between index portfolios formed according to cross-sectional sorts based on size and book-to-market: Fama and French (1993)
- Estimate the sensitivities b_{ik} for each asset i by running a time-series regression of R_{it} on the F_{kt}.

Multifactor Pricing Models

Estimated risk premia for macroeconomic risk factors

Factor	Estimated Risk Premium
Term spread	5.10%
Market	6.36%
Exchange rate	-0.59%
Real GNP growth	0.49%
Inflation	-0.83%

Multifactor Pricing Models

- Example: you are considering making investment in the pharmaceutical industry. Your project is expected to have the same risk exposures as an average firm in this industry.
  - Fama-French betas for this industry are given by:
    $\beta_{market} = 0.68$, $\beta_{SMB} = -0.62$, $\beta_{HML} = -0.43$;
  - Factor premiums can be estimated from historical data: $\bar{r}_M - r_f = 7\%, \bar{r}_{SMB} = 3.7\%, \bar{r}_{HML} = 5.2\%;$
  - What is the expected return (cost of capital) of your project?

Multifactor Pricing Models

- Based on the multifactor pricing model:

$$
\begin{aligned}
r_{new} &= r_f + \beta_{market} \times 7\% + \beta_{SMB} \times 3.7\% + \beta_{HML} \times 5.2\% \\
&= 5\% + 0.68 \times 7\% - 0.62 \times 3.7\% - 0.43 \times 5.2\% \\
&= 5.23\%.
\end{aligned}
$$