LSE Summer School
FM250 – Finance

Classwork 4: Portfolio Theory
Answer key

Question 1

$x_I = 0.60$ $\sigma_I = 0.10$
$x_J = 0.40$ $\sigma_J = 0.20$

$$\sigma_p^2 = [x_I^2\sigma_I^2 + x_J^2\sigma_J^2 + 2(x_I x_J \rho_{IJ}\sigma_I\sigma_J)]$$

(a) $\rho_{IJ} = 1$:

$$\sigma_P^2 = [(0.60)^2(0.10)^2 + (0.40)^2(0.20)^2 + 2(0.60)(0.40)(1)(0.10)(0.20)] = 0.0196$$

(b) $\rho_{IJ} = 0.5$:

$$\sigma_P^2 = [(0.60)^2(0.10)^2 + (0.40)^2(0.20)^2 + 2(0.60)(0.40)(0.50)(0.10)(0.20)] = 0.0148$$

(c) $\rho_{IJ} = 0$:

$$\sigma_P^2 = [(0.60)^2(0.10)^2 + (0.40)^2(0.20)^2 + 2(0.60)(0.40)(0)(0.10)(0.20)] = 0.0100$$

Question 2

For a two-security portfolio, the formula for portfolio risk is:

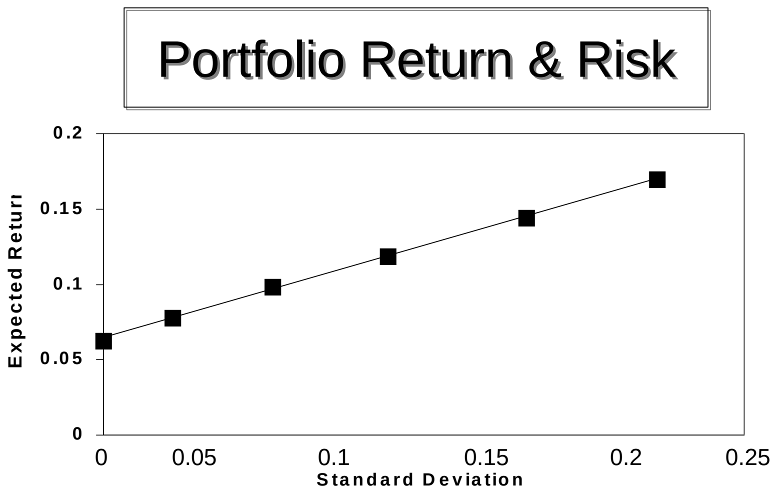
Portfolio variance = $x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2$

If security one is Treasury bills and security two is the market portfolio, then σ_1 is zero, σ_2 is 20 percent. Therefore:

Portfolio variance = $x_2^2\sigma_2^2 = x_2^2(0.20)^2$; Standard deviation = $0.20x_2$

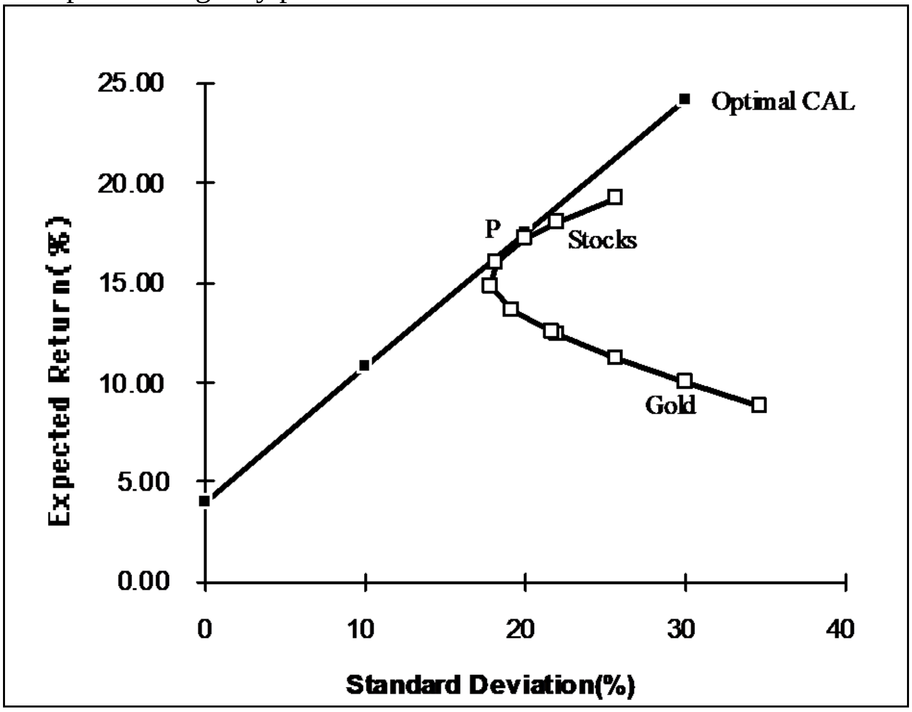
Portfolio expected return = $x_1(0.06) + x_2(0.06 + 0.85) = 0.06x_1 + 0.145x_2$

Portfolio	X_1	X_2	Expected Return	Standard Deviation
1	1.0	0.0	0.060	0.000
2	0.8	0.2	0.077	0.040
3	0.6	0.4	0.094	0.080
4	0.4	0.6	0.111	0.120
5	0.2	0.8	0.128	0.160
6	0.0	1.0	0.145	0.200



Question 3

Even though it seems that gold is dominated by stocks, gold might still be an attractive asset to hold as a *part* of a portfolio. If the correlation between gold and stocks is sufficiently low, gold will be held as a component in a portfolio, specifically, the optimal tangency portfolio.



Question 4

(a) False. It is the covariance of returns with the portfolio returns.
(b) False. Diversification can reduce total risk.
(c) True. Given that the stocks have the same return and risk, we look for the lowest return correlation.