Example 1.1.1. Suppose Skippy records the outdoor temperature every two hours starting at 6 a.m. and ending at 6 p.m. and summarizes the data in the table below:

time (hours after 6 a.m.)	outdoor temperature in degrees Fahrenheit
0	64
2	67
4	75
6	80
8	83
10	83
12	82

1. Explain why the recorded outdoor temperature is a function of the corresponding time.

2. Is time a function of the outdoor temperature? Explain.

3. Let f be the function which matches time to the corresponding recorded outdoor temperature.

   (a) Find and interpret the following:

   - $f(2)$
   - $f(4)$
   - $f(2+4)$
   - $f(2)+f(4)$
   - $f(2)+4$

   (b) Solve and interpret $f(t) = 83$.

   (c) State the range of f. What is lowest recorded temperature of the day? The highest?

2. Let $h(t) = -t^2 + 3t + 4$.

(a) Find and simplify the following:

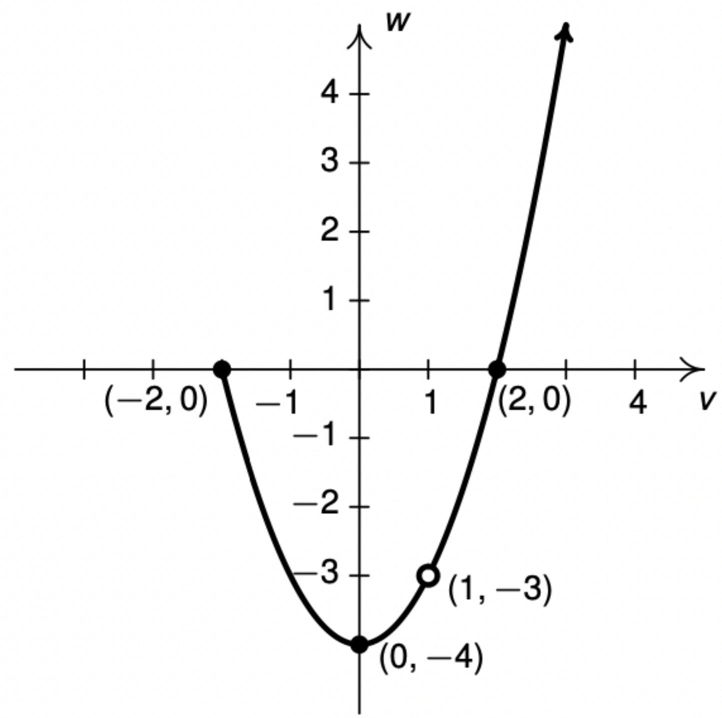
i. $h(-1)$, $h(0)$ and $h(2)$.

ii. $h(2x)$ and $2h(x)$.

iii. $h(t+2)$, $h(t) + 2$ and $h(t) + h(2)$.

(b) Solve $h(t) = 0$.

Example 1.1.4. Consider the graph below.



1. (a) Explain why this graph suggests that w is a function of v, $w = F(v)$.

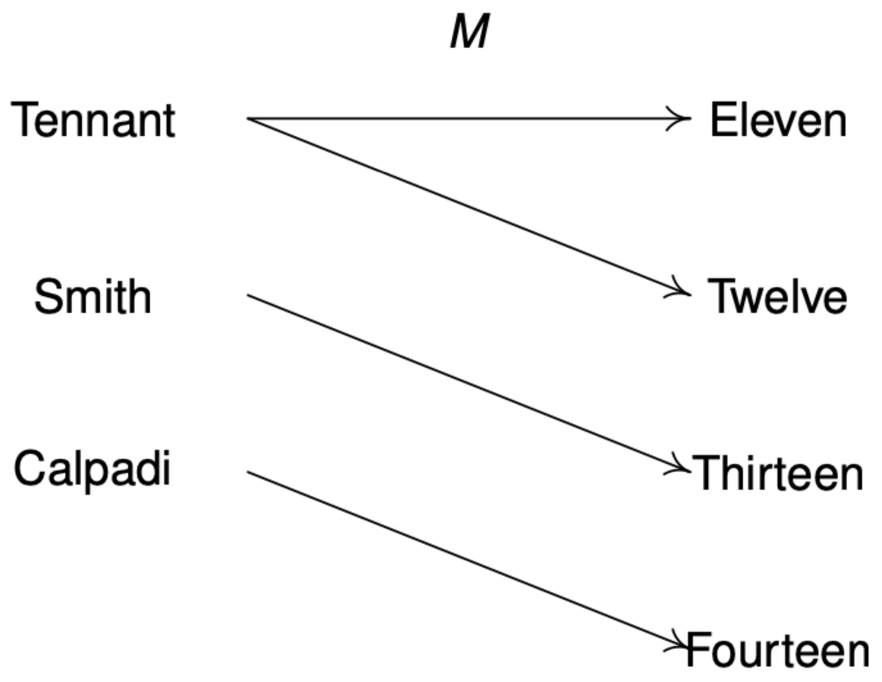
   (b) Find $F(0)$ and solve $F(v) = 0$.

   (c) Find the domain and range of F using interval notation.[14] Find the extrema of F, if any exist.

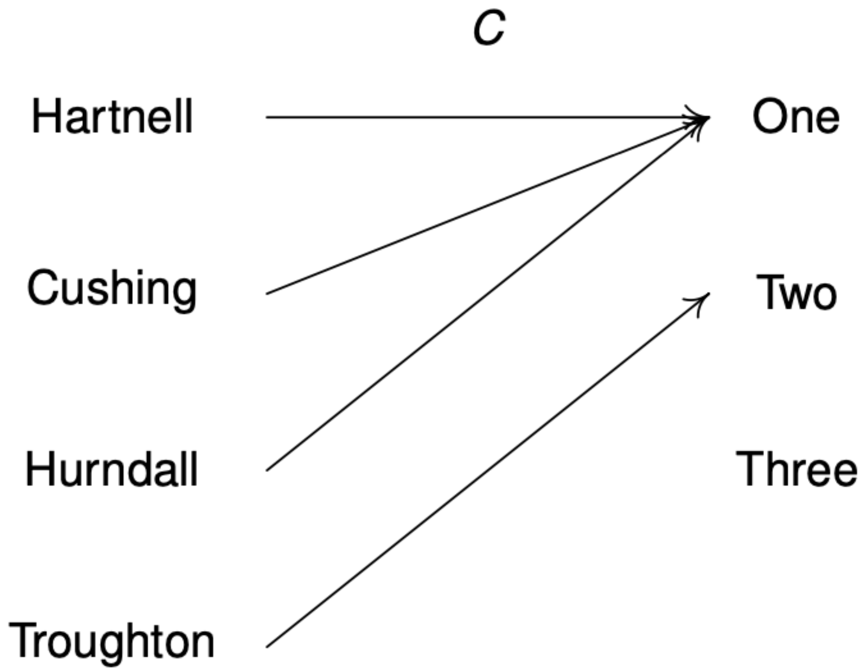
2. Does this graph suggest v is a function of w? Explain.

In Exercises 1 - 2, determine whether or not the mapping diagram represents a function. Explain your reasoning. If the mapping does represent a function, state the domain, range, and represent the function as a set of ordered pairs.

1.



2.



In Exercises 3 - 4, determine whether or not the data in the given table represents y as a function of x. Explain your reasoning. If the mapping does represent a function, state the domain, range, and represent the function as a set of ordered pairs.

3.

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

4.

x	y
0	0
1	1
1	-1
2	2
2	-2
3	3
3	-3

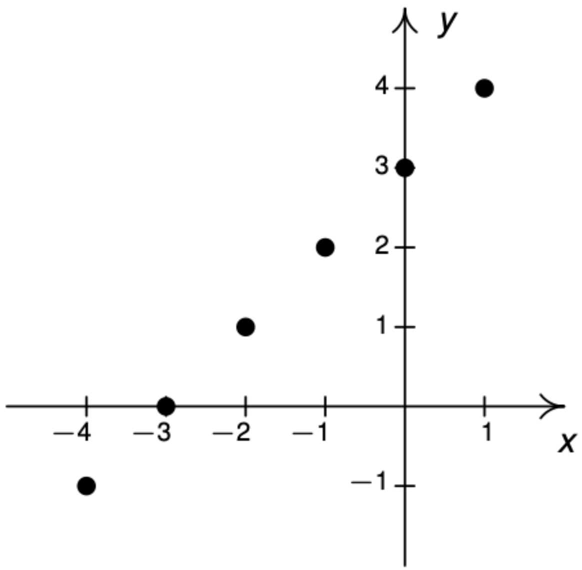
5. Suppose W is the set of words in the English language and we set up a mapping from W into the set of natural numbers $\mathbb{N}$ as follows: word $\rightarrow$ number of letters in the word. Explain why this mapping is a function. What would you need to know to determine the range of the function?

6. Suppose L is the set of last names of all the people who have served or are currently serving as the President of the United States. Consider the mapping from L into $\mathbb{N}$ as follows: last name $\rightarrow$ number of their presidency. For example, Washington $\rightarrow$ 1 and Obama $\rightarrow$ 44. Is this mapping a function? What if we use full names instead of just last names? (**HINT:** Research Grover Cleveland.)

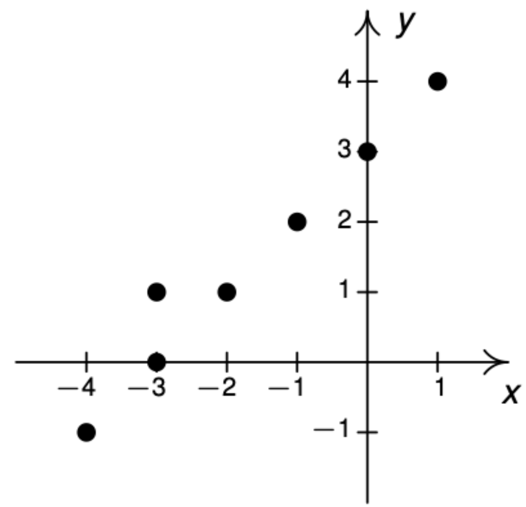
7. Under what conditions would the time of day be a function of the outdoor temperature?

In Exercises 58 - 61, determine whether or not the graph suggests y is a function of x. For the ones which do, state the domain and range.

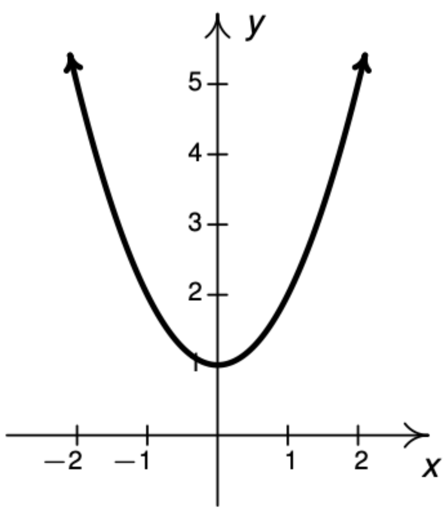
58.



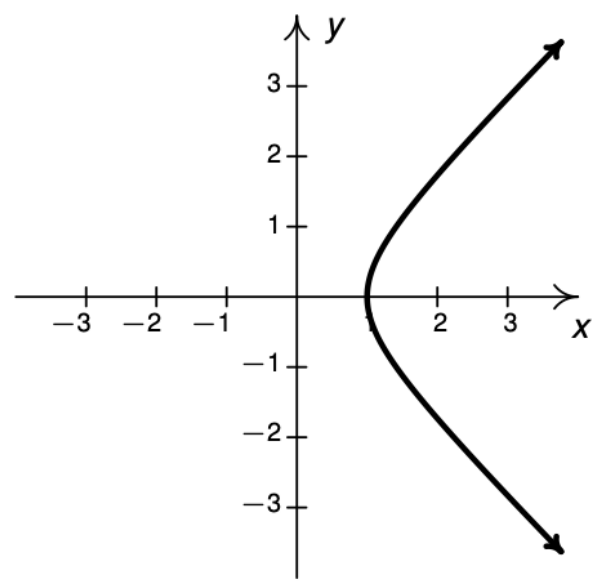
59.



60.

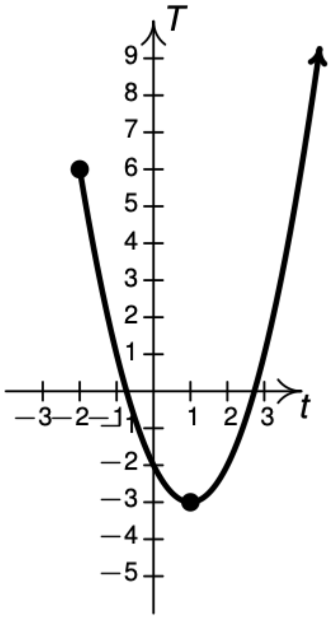


61.

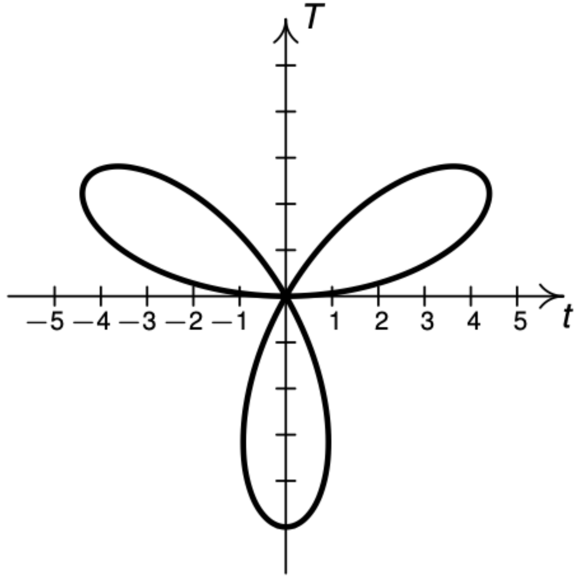


In Exercises 68 - 71, determine whether or not the graph suggests T is a function of t. For the ones which do, state the domain and range.

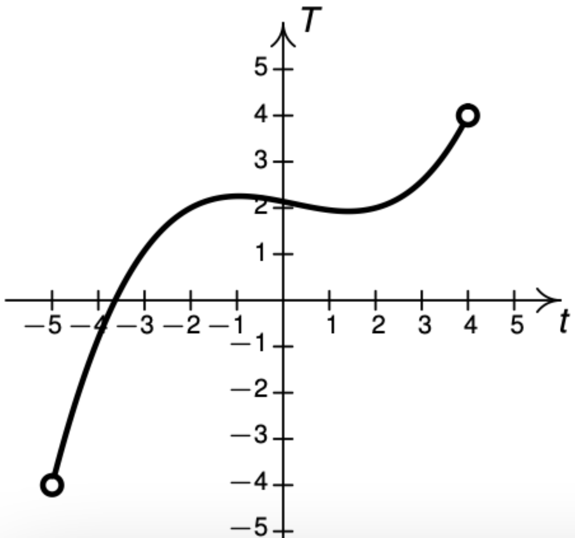
68.



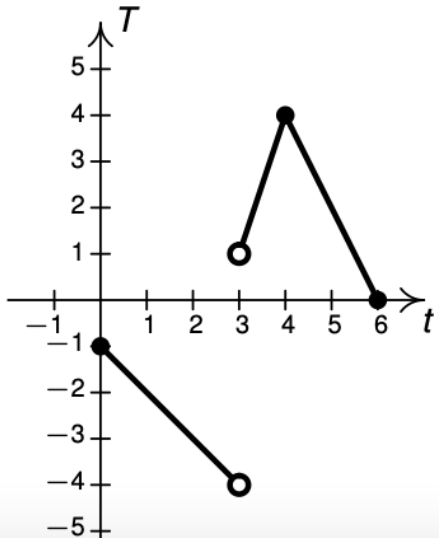
69.



70.



71.



Example 1.2.3. The cost, in dollars, to produce x PortaBoy[11] game systems for a local retailer is given by $C(x) = 80x + 150$ for $x \geq 0$.

1. Find and interpret $C(0)$ and $C(5)$ and use these to graph $y = C(x)$.

2. Explain the significance of the restriction on the domain, $x \geq 0$.

3. Interpret the slope of $y = C(x)$ geometrically and as a rate of change.

4. How many PortaBoys can be produced for \$15,000?

Example 1.2.4. The local retailer in Example 1.2.3 is trying to mathematically model the relationship between the number of PortaBoy systems sold and the price per system. Suppose 20 systems were sold when the price was \$220 per system but when the systems went on sale for \$190 each, sales doubled.

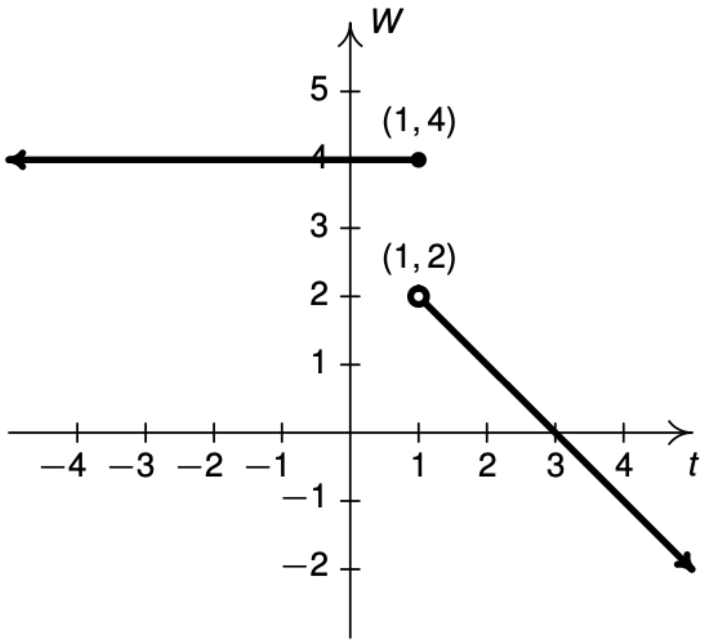
1. Find a formula for a linear function p which represents the price $p(x)$ as a function of the number of systems sold, x. Graph $y = p(x)$, find and interpret the intercepts, and determine a reasonable domain for p.

2. Interpret the slope of $p(x)$ in terms of price and game system sales.

3. If the retailer wants to sell 150 PortaBoys next week, what should the price be?

4. How many systems would sell if the price per system were set at \$150?

Example 1.2.5. Find a formula for the function L graphed below.



$W = L(t)$

1.2.5 Exercises

In Exercises 1 - 6, graph the function. Find the slope and axis intercepts, if any.

1. $f(x) = 2x - 1$
2. $g(t) = 3 - t$
3. $F(w) = 3$
4. $G(s) = 0$
5. $h(t) = \frac{2}{3}t + \frac{1}{3}$
6. $j(w) = \dfrac{1 - w}{2}$

In Exercises 7 - 10, graph the function. Find the domain, range, and axis intercepts, if any.

7. $f(x) = \begin{cases} 4 - x & \text{if} \quad x \leq 3 \\ 2 & \text{if} \quad x > 3 \end{cases}$
8. $g(x) = \begin{cases} 2 - x & \text{if} \quad x < 2 \\ x - 2 & \text{if} \quad x \geq 2 \end{cases}$
9. $F(t) = \begin{cases} -2t - 4 & \text{if} \quad t < 0 \\ 3t & \text{if} \quad t \geq 0 \end{cases}$
10. $G(t) = \begin{cases} -3 & \text{if} \quad t < 0 \\ 2t - 3 & \text{if} \quad 0 < t < 3 \\ 3 & \text{if} \quad t > 3 \end{cases}$

17. An on-line comic book retailer charges shipping costs according to the following formula

$$S(n) = \begin{cases} 1.5n + 2.5 & \text{if} \quad 1 \leq n \leq 14 \\ 0 & \text{if} \quad n \geq 15 \end{cases}$$

where n is the number of comic books purchased and $S(n)$ is the shipping cost in dollars.

(a) What is the cost to ship 10 comic books?

(b) What is the significance of the formula $S(n) = 0$ for $n \geq 15$?

18. The cost in dollars $C(m)$ to talk m minutes a month on a mobile phone plan is modeled by

$$C(m) = \begin{cases} 25 & \text{if} \quad 0 \leq m \leq 1000 \\ 25 + 0.1(m - 1000) & \text{if} \quad m > 1000 \end{cases}$$

(a) How much does it cost to talk 750 minutes per month with this plan?

(b) How much does it cost to talk 20 hours a month with this plan?

(c) Explain the terms of the plan verbally.

19. Jeff can walk comfortably at 3 miles per hour. Find an expression for a linear function $d(t)$ that represents the total distance Jeff can walk in t hours, assuming he doesn't take any breaks.

20. Carl can stuff 6 envelopes per *minute*. Find an expression for a linear function $E(t)$ that represents the total number of envelopes Carl can stuff after t *hours*, assuming he doesn't take any breaks.

21. A landscaping company charges \$45 per cubic yard of mulch plus a delivery charge of \$20. Find an expression for a linear function $C(x)$ which computes the total cost in dollars to deliver x cubic yards of mulch.

22. A plumber charges \$50 for a service call plus \$80 per hour. If she spends no longer than 8 hours a day at any one site, find an expression for a linear function $C(t)$ that computes her total daily charges in dollars as a function of the amount of time spent in hours, t at any one given location.

23. A salesperson is paid \$200 per week plus 5% commission on her weekly sales of x dollars. Find an expression for a linear function $W(x)$ which computes her total weekly pay in dollars as a function of x. What must her weekly sales be in order for her to earn \$475.00 for the week?

29. In response to the economic forces in Exercise 28 above, the local retailer sets the selling price of a PortaBoy at \$250. Remarkably, 30 units were sold each week. When the systems went on sale for \$220, 40 units per week were sold. Rework Example 1.2.4 with this new data.

30. A local pizza store offers medium two-topping pizzas delivered for \$6.00 per pizza plus a \$1.50 delivery charge per order. On weekends, the store runs a 'game day' special: if six or more medium two-topping pizzas are ordered, they are \$5.50 each with no delivery charge. Write a piecewise-defined linear function which calculates the cost in dollars $C(p)$ of p medium two-topping pizzas delivered during a weekend.

31. A restaurant offers a buffet which costs \$15 per person. For parties of 10 or more people, a group discount applies, and the cost is \$12.50 per person. Write a piecewise-defined linear function which calculates the total bill $T(n)$ of a party of n people who all choose the buffet.

32. A mobile plan charges a base monthly rate of \$10 for the first 500 minutes of air time plus a charge of 15¢ for each additional minute. Write a piecewise-defined linear function which calculates the monthly cost in dollars $C(m)$ for using m minutes of air time.

    HINT: You may wish to refer to number 18 for inspiration.

33. The local pet shop charges 12¢ per cricket up to 100 crickets, and 10¢ per cricket thereafter. Write a piecewise-defined linear function which calculates the price in dollars $P(c)$ of purchasing c crickets.

1. Graph the following functions using Theorem 1.3. Find the vertex, zeros and axis-intercepts (if any exist). Find the extrema and then list the intervals over which the function is increasing, decreasing or constant.

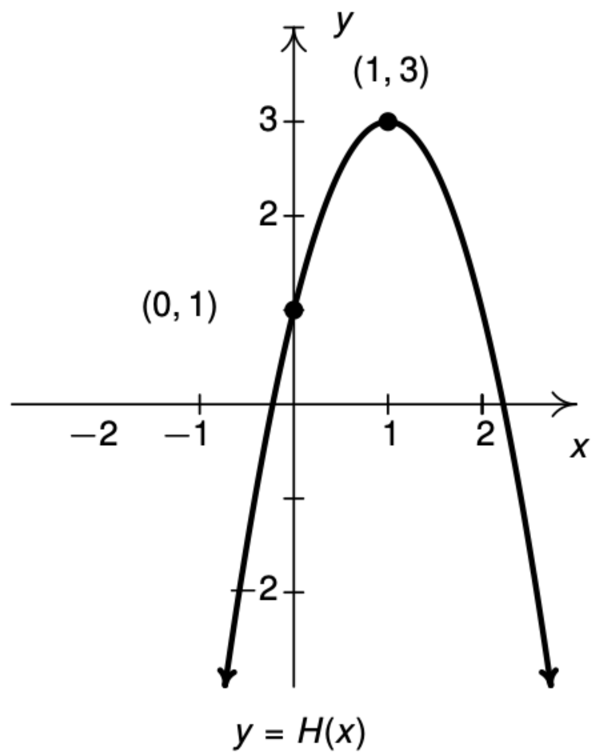
   (a) $f(x) = \dfrac{(x-3)^2}{2}$

   (b) $g(x) = (x+2)^2 - 3$

   (c) $h(t) = -2(t-3)^2 + 1$

   (d) $i(t) = \dfrac{(3-2t)^2 + 1}{2}$

2. Use Theorem 1.3 to write a possible formula for $H(x)$ whose graph is given below:



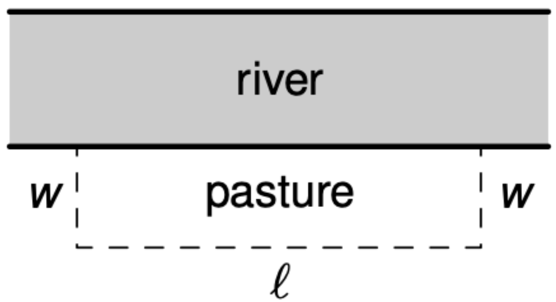
Example 1.4.2. Graph the following functions. Find the vertex, zeros and axis-intercepts, if any exist. Find the extrema and then list the intervals over which the function is increasing, decreasing or constant.

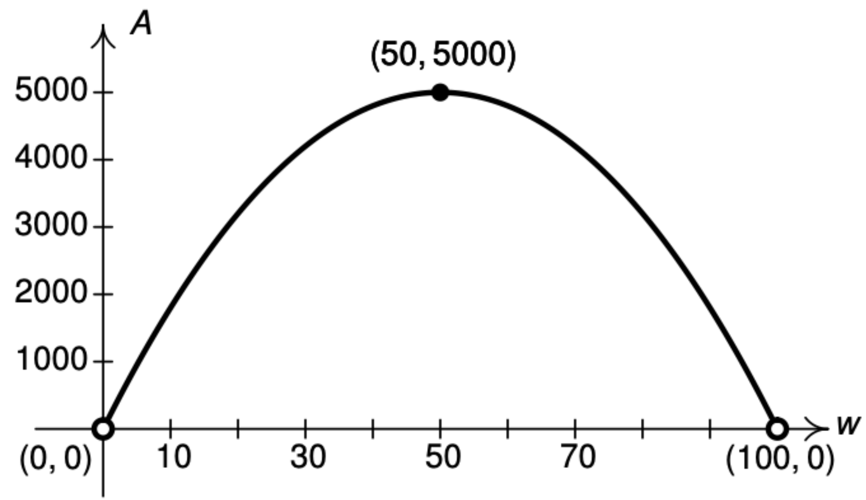
1. $f(x) = x^2 - 4x + 3.$
2. $g(t) = 6 - 4t - 2t^2$

Example 1.4.5. Much to Donnie's surprise and delight, he inherits a large parcel of land in Ashtabula County from one of his (e)strange(d) relatives so the time is right for him to pursue his dream of raising alpaca. He wishes to build a rectangular pasture and estimates that he has enough money for 200 linear feet of fencing material. If he makes the pasture adjacent to a river (so that no fencing is required on that side), what are the dimensions of the pasture which maximize the area? What is the maximum area? If an average alpaca needs 25 square feet of grazing area, how many alpaca can Donnie keep in his pasture?

Solution. We are asked to find the dimensions of the pasture which would give a maximum area, so we begin by sketching the diagram seen below on the left. We let w denote the width of the pasture and we let ℓ denote the length of the pasture. The units given to us in the statement of the problem are feet, so we assume that w and ℓ are measured in feet. The area of the pasture, which we'll call A, is related to w and ℓ by the equation $A = w\ell$. Since w and ℓ are both measured in feet, A has units of feet^2, or square feet.

We are also told that the total amount of fencing available is 200 feet, which means $w + \ell + w = 200$, or, $\ell + 2w = 200$. We now have two equations, $A = w\ell$ and $\ell + 2w = 200$. In order to use the tools given to us in this section to *maximize* A, we need to use the information given to write A as a function of just *one* variable, either w or ℓ. This is where we use the equation $\ell + 2w = 200$. Solving for ℓ, we find $\ell = 200 - 2w$, and we substitute this into our equation for A. We get $A = w\ell = w(200 - 2w) = 200w - 2w^2$. We now have A as a function of w, $A = f(w) = 200w - 2w^2 = -2w^2 + 200w$.





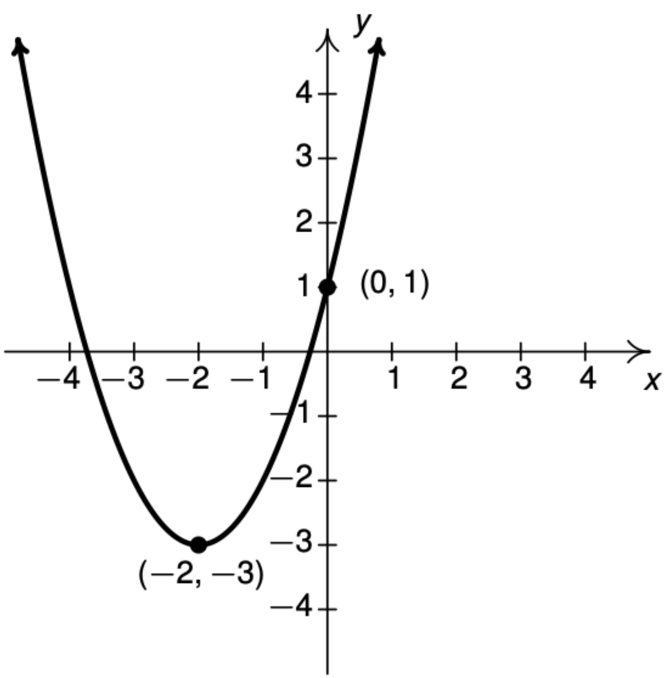
$A = f(w)$

In Exercises 1 - 9, graph the quadratic function. Find the vertex and axis intercepts of each graph, if they exist. State the domain and range, identify the maximum or minimum, and list the intervals over which the function is increasing or decreasing. If the function is given in general form, convert it into standard form; if it is given in standard form, convert it into general form.

1. $f(x) = x^2 + 2$
2. $f(x) = -(x+2)^2$
3. $f(x) = x^2 - 2x - 8$
4. $g(t) = -2(t+1)^2 + 4$
5. $g(t) = 2t^2 - 4t - 1$
6. $g(t) = -3t^2 + 4t - 7$
7. $h(s) = s^2 + s + 1$
8. $h(s) = -3s^2 + 5s + 4$
9. $h(s) = s^2 - \frac{1}{100}s - 1$

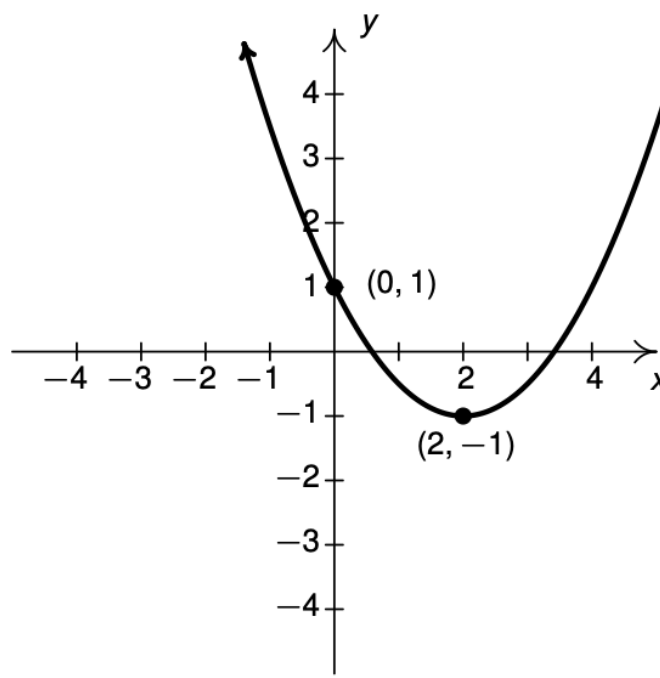
In Exercises 10 - 13, find a formula for each function below in the form $F(x) = a(x-h)^2 + k$.

10.



$y = F(x)$

11.



$y = F(x)$

In Exercises 30 - 34, cost and price-demand functions are given. For each scenario,

- Find the profit function $P(x)$.
- Find the number of items which need to be sold in order to maximize profit.
- Find the maximum profit.
- Find the price to charge per item in order to maximize profit.
- Find and interpret break-even points.

30. The cost, in dollars, to produce x "I'd rather be a Sasquatch" T-Shirts is $C(x) = 2x + 26$, $x \geq 0$ and the price-demand function, in dollars per shirt, is $p(x) = 30 - 2x$, for $0 \leq x \leq 15$.

31. The cost, in dollars, to produce x bottles of 100% All-Natural Certified Free-Trade Organic Sasquatch Tonic is $C(x) = 10x + 100$, $x \geq 0$ and the price-demand function, in dollars per bottle, is $p(x) = 35 - x$, for $0 \leq x \leq 35$.

32. The cost, in cents, to produce x cups of Mountain Thunder Lemonade at Junior's Lemonade Stand is $C(x) = 18x + 240$, $x \geq 0$ and the price-demand function, in cents per cup, is $p(x) = 90 - 3x$, for $0 \leq x \leq 30$.

33. The daily cost, in dollars, to produce x Sasquatch Berry Pies is $C(x) = 3x + 36$, $x \geq 0$ and the price-demand function, in dollars per pie, is $p(x) = 12 - 0.5x$, for $0 \leq x \leq 24$.

34. The monthly cost, in *hundreds* of dollars, to produce x custom built electric scooters is $C(x) = 20x + 1000$, $x \geq 0$ and the price-demand function, in *hundreds* of dollars per scooter, is $p(x) = 140 - 2x$, for $0 \leq x \leq 70$.

Example 1.4.3. In Example 1.2.3 the cost to produce x PortaBoy game systems for a local retailer was given by $C(x) = 80x + 150$ for $x \geq 0$ and in Example 1.2.4 the price-demand function was found to be $p(x) = -1.5x + 250$, for $0 \leq x \leq 166$.

1. Find formulas for the associated revenue and profit functions; include the domain of each.

2. Find and interpret $P(0)$.

3. Find and interpret the zeros of P.

4. Graph $y = P(x)$. Find the vertex and axis intercepts.

5. Interpret the vertex of the graph of $y = P(x)$.

6. What should the price per system be in order to maximize profit?

7. Find and interpret the average rate of change of P over the interval $[0, 57]$.

Solution.

1. The formula for the revenue function is $R(x) = x\,p(x) = x(-1.5x + 250) = -1.5x^2 + 250x$. Since the domain of p is restricted to $0 \leq x \leq 166$, so is the domain of R. To find the profit function $P(x)$, we subtract $P(x) = R(x) - C(x) = \left(-1.5x^2 + 250x\right) - (80x + 150) = -1.5x^2 + 170x - 150$. The cost function formula is valid for $x \geq 0$, but the revenue function is valid when $0 \leq x \leq 166$. Hence, the domain of P is likewise restricted to $[0, 166]$.

2. We find $P(0) = -1.5(0)^2 + 170(0) - 150 = -150$. This means that if we produce and sell 0 PortaBoy game systems, we have a profit of $-\$150$. Since profit = (revenue) − (cost), this means our costs exceed our revenue by \$150. This makes perfect sense, since if we don't sell any systems, our revenue is \$0 but our fixed costs (see Example 1.2.3) are \$150.