Class 1 FM 250 Domenico Mergoni

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INTRODUCTIONS

- Name
- Country of origin
- Studies
- Interests / hobbies

RECAP

- When thinking of cash flow:
  - How much? (expectation)
  - When?
  - How certain?

- Q How much is it worth TODAY a cash flow of C at time t? Can we compute $\overbrace{PV}^{\text{present value}}(C_t)$

$$\rightsquigarrow \quad \underbrace{PV(C_t)}_{\text{(1) Not clear which one is the indeterminate}} = \frac{C}{(1+r)^t}$$

↳ discount rate (r)

- Remark! PV is ADDITIVE (assuming r is constant).

  I.e. $PV(A+B) = PV(A) + PV(B)$ ← SKETCHY, do not use lightly

- FORMULA Given a sequence $C_0, C_1, C_2, \dots$ of cash flows with discount rate r; we have $PV(C) = \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t}$

  Also called NET PRESENT VALUE (when including C_0).

- CRITERION: Invest $\iff NPV(C) > 0$

  Rem Can also be done with rate of return.

  ↳ It kind of links to (1).
  RR is the r we get when fixing PV and solving for r.

- EXAMPLE: Perpetuity. Same thing. Just do the calculations.

$$\dots \quad PV_0(P) = \frac{C}{r}$$

Question 1

You own an empty parcel of land that you do not plan to sell. For an additional $1.3 million you can build a motel on the property. The motel will be worth extra $1.5 million next year. If common stocks with the same risk as the investment in the motel offer a 10% expected return, would you construct the motel?

$$\left.\begin{aligned} C_0 &= -1.3 \\ C_1 &= 1.5 \\ r &= 0.1 \end{aligned}\right] \rightarrow \quad NPV(C) = \sum_t \frac{1}{(1+r)^t} C_t = -1.3 + \frac{1.5}{1+0.1} > 0$$

```
def NPV(r, C):
    #Calculate the Net Present Value of a sequence (array) C of cash flows
    #with discount rate r.
    NPV = 0
    for i in range(len(C)):
        NPV += C[i]/((1+r)**i)
    return NPV
```

```
print(NPV(0.1, [-1.3, 1.5]))
```

```
0.06363636363636349
```

Question 2

Calculate the NPV and rate of return for each of the following investments. The required rate of return is 20% for all four investments.

Investment	Initial CF, C_0	CF in year 1, C_1
1	-10,000	+18,000
2	-5,000	+9,000
3	-5,000	+5,700
4	-2,000	+4,000

Suppose each investment would require use of the same parcel of land and therefore you can take only one. Which one would you take and why?

NPV: $C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots$

$r = 0.2$

① $-10 + \frac{18}{1.2} = 5$

② $-5 + \frac{9}{1.2} = 2.5$

③ $-5 + \frac{5.7}{1.2} = -0.25$

Rate of return: $\frac{C_1}{P_0} - 1 = RR$

$\frac{18}{10} - 1 = 0.8$

$\frac{9}{5} - 1 = 0.8$

$\frac{5.7}{5} - 1 = 0.14$

④ $\left|-2\right| + \left|\frac{4}{1.2}\right| = 1.333\ldots$ $\frac{4}{2} - 1 = 1$

```
def RR(P0, C1):
    #We are assuming there is just one time-step
    return C1/P0-1
```

```
RR(10,18)
```

```
0.8
```

Question 3
For an outlay of €8 million SBUX can finally open stores in Estonia. Unfortunately, the market value of these stores next year (assuming it can be measured) will be very sensitive to the growth rate of the Estonian economy:

Slump	Normal	Boom
€8 million	€12 million	€16 million

(a) What is the expected cash flow? Assume the outcomes for the economy are equally likely.
(b) What is the expected rate of return on the investment in the project?
(c) One share of the Estonian stock market index is selling for €10. The stock has the following payoffs after one year:

Slump	Normal	Boom
€8	€12	€16

Calculate the expected rate of return offered by the index. Explain why this is the opportunity cost of capital for SBUX's expansion to Estonia.
(d) Calculate the project's NPV. Is the project a good investment? Why?

a) $\mathbb{E}[X] = \sum_c \mathbb{P}[X=c]\cdot c = \frac{1}{3}\cdot 8 + \frac{1}{3}\cdot 12 + \frac{1}{3}\cdot 16 = 12$

b) $RR = \frac{12}{8} - 1 = 0.5$

c) Same but with 10 \$ instead of 8M and we get $r = 0.2$

d) $C_0 = -8$ $C_1 = 12$ $r = 0.2$!

and $NPV = 2 > 0$

```
def NPV_complicated(r, C):
    #Example of C would be: [[C_0, p_0], [C_1, p_1], ...]
    T=len(C)
    E=[0 for i in range(T)]
    for i in range(T):
        C_i, p_i = C[i]
        if (sum(p_i)!=1) or (len(C_i)!=len(p_i)) or any([p_i[j]<0 for j in range(len(p_i))]):
```

```
            print("ERROR")
            return -1
        for j in range(len(C_i)):
            E[i]+=C_i[j]*p_i[j]
    return NPV(r, E)
```

```
NPV_complicated(0.2, [[[-8], [1]], [[8, 12, 16], [1/3, 1/3, 1/3]]])
```

```
2.0
```

Question 4

As winner of a hotdog eating contest, you can choose one of the following prizes, which will all happen with certainty:

(a) $100,000 now.
(b) 180,000 at the end of five years
(c) 11,400 a year forever
(d) 19,000 for each of 10 years

If risk-free rate is 12%, which is the most valuable prize?

Do the same calculations again

```
###### Perpetuity
n = 50
P = 11.4
r = 0.12
C = [0]+[P for i in range(n)]
limit = [NPV(r, C[:i+1]) for i in range(n)]
plt.plot(limit)
plt.plot([P/r for i in range(n)])
```

```
[<matplotlib.lines.Line2D at 0x7fd65064c1f0>]
```