MA103 - Class 1

Friday 08/10/2021

Today's class: → 20 min general discussion

→ 15 min exercise correction

→ 15 min new exercises

GENERAL DISCUSSION

PERSONAL → Some info about myself

→ Quick round of names

THE COURSE → Assignments
- what are them
- how do they work?
- are they important?
- chocolate

→ Classes
- mandatory
- cold calling

→ Office hours
- Thu 11:30 - 12:30 COL 3.16
- d.mergoni@lse.ac.uk

→ Notes
- guide to common mistakes
- note uploading

PROBLEM 1 If n is a multiple of 14, then n is not a multiple of 6.

① This is not a mathematical statement

② The negation of $\forall n \in \mathbb{N}, (14 | n \Rightarrow 6 \nmid n)$

is $\exists n \in \mathbb{N}, \; 14 | n \wedge 6 | n$

to show that the second holds, we MUST show a counterexample

LESSON When we want to prove an "exists" statement, we always (in this course) MUST provide an example.

PROBLEM 2 Show that if $4|n$, then $6|9n-30$.

① Writing definitions is always a good idea
② Often by writing more, one does more mistakes.

We want to prove that $\forall n \in \mathbb{N}$, $4|n \Rightarrow 6|9n-30$.
Let n be a natural number, and assume that $4|n$. By definition, this means that there is $k \in \mathbb{Z}$ s.t. $n = 4k$.
Then we can write

$$9n - 30 = 9(4k) - 30$$
$$= 6(6k - 5)$$

since $6k-45 \in \mathbb{Z}$, we have that, by definition, $6|9n-30$.

$\forall n \in \mathbb{N}$, $4|n \Rightarrow \exists k \in \mathbb{Z}$, $n := 4k$

$\therefore 9n-30 = 6(6k-5)$

$\therefore \forall n \in \mathbb{N}$, $4|n \Rightarrow \exists k \in \mathbb{Z}$, $9n-30 = 6k$

$\Rightarrow 6|9n$ ☒

LESSON
- Less is more, too little is a pain
- do not skip steps
- start with definitions

PROBLEM 3 For which natural number n we have that $4^n - 1$ is prime?

① Start with having clear in mind what you want to prove

$$\forall n \in \mathbb{N}_{\geq 2}, \quad 4^n - 1 \text{ is not prime}$$

② Many different ways of proving it!
- $3|4^n-1$ CAUTION!
- $4^n - 1 = 2^{2n} - 1 = (2^n - 1)(2^n + 1)$ ✓
- Induction

LESSON Even if what you want to prove is true, the proof could be wrong!

PROBLEM 4 Explain what is wrong with the proof of the statement

LESSON • faulty proof $\Rightarrow$ false result !!!

EXTRA PROBLEM Here are four pairs of logically equivalent propositions. What are the pairs?

a) $(p \wedge q) \wedge t$

b) $((\neg p \wedge (t \vee \neg q)) \wedge q) \wedge p$

c) $q \vee \neg q$

d) $p \wedge q$

e) $(p \wedge (t \wedge q)) \vee (\neg t \wedge (p \wedge q))$

f) $q \Leftrightarrow \neg q$

g) $(p \vee \neg q) \Rightarrow (\neg q \wedge p)$

h) $\neg(\neg p \vee (\neg q \vee \neg t))$