Lecture 4. Portfolio Theory

Thummim Cho, PhD, CFA
London School of Economics

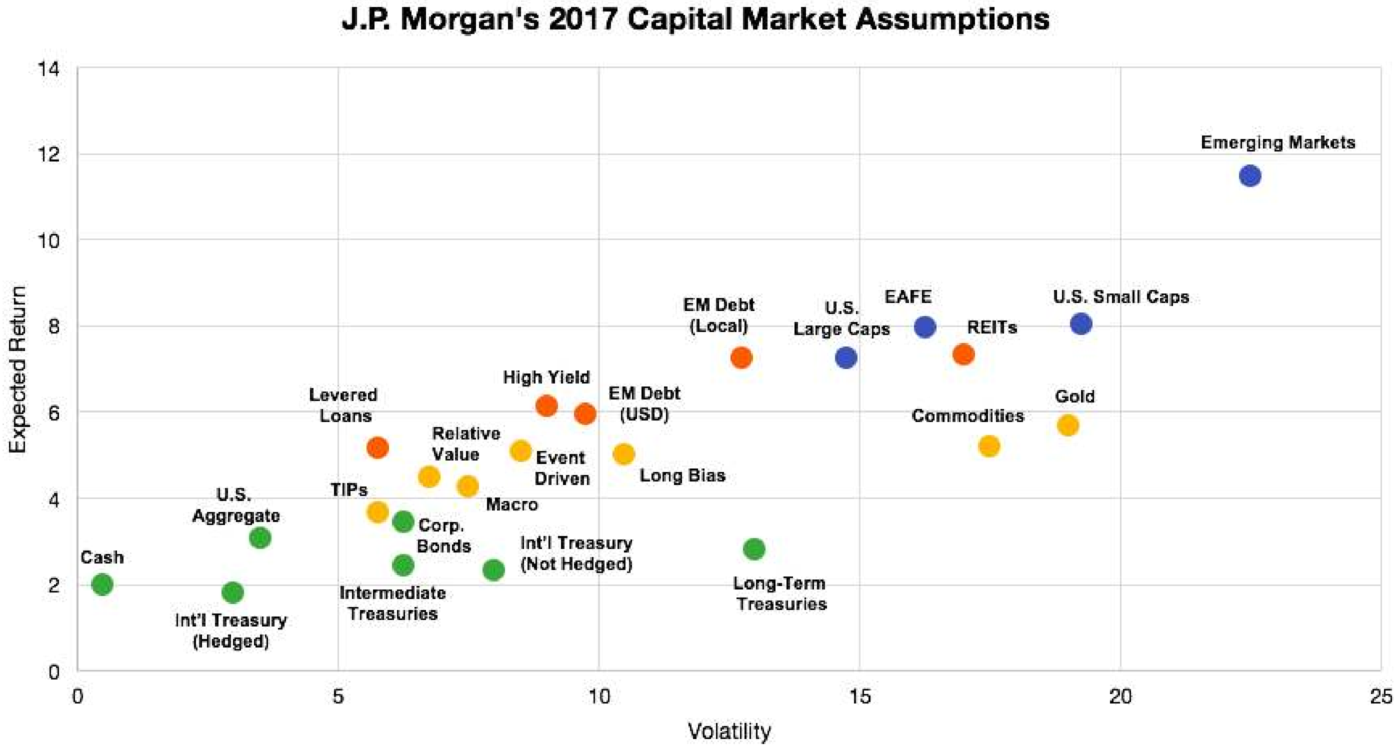
LSE Summer School

Topics Covered

- Portfolio theory
  - Calculating portfolio return and variance
  - Diversification
  - Risk aversion
  - Portfolio choice with multiple risky assets
  - Portfolio choice with a risk-free asset
- Empirical evidence: What do people actually do?

Portfolio Theory: Motivation

Now that we've studied both bonds and stocks (and know that there are many other assets out there), how should we combine them into a **diversified portfolio**?



Source: J.P. Morgan 2017 Capital Market Assumptions.

Basic Statistical Measures

Since return is a random variable for most assets, we quantify it using two statistical measures, **mean return** and **variance**.

- Expected return (mean return): $$\bar{R} = E[R] = \sum_{s=1}^{S} \pi_s R_s$$

  where s is "state of the world" next period, R_s is realized return in state s, and π_s is probability of state s happening

- Variance (tendency to deviate from the mean):

$$\sigma^2 = Var(R) = E\left[\left(R - E(R)\right)^2\right] = \sum_{s=1}^{S} \pi_s \left(R_s - E(R)\right)^2$$

- Standard deviation (volatility):

$$\sigma = Std(R) = \sqrt{Var(R)}$$

Basic Statistical Measures

Also important:

- Covariance (tendency of two random variables to move together):

$$\sigma_{i,j} = Cov(R_i, R_j) = E\left[(R_i - E[R_i])\left(R_j - E\left[R_j\right]\right)\right]$$
$$= \sum_{s=1}^{S} \pi_s (R_i - E[R_i])\left(R_j - E\left[R_j\right]\right)$$

- Correlation (normalized to be between -1 and 1):

$$\rho_{i,j} = Corr(R_i, R_j) = \frac{Cov(R_i, R_j)}{Std(R_i)Std(R_j)}$$

Portfolio Weights

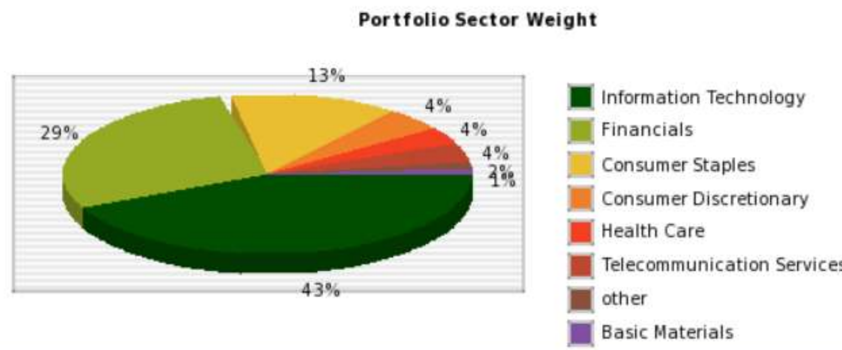
Consider *N* assets indexed by $i = 1, \dots, N$. Then, a portfolio is a basket of assets characterized by portfolio weights $i = w_1, \dots, w_N$.

- w_1 = percentage of wealth invested in asset i
- so, $w_1 + \dots + w_N = 1$

Warren Buffett

Buffett on Wikipedia
Berkshire Hathaway Site

Latest Portfolio (reported on 2021-05-17; portfolio on 2021-03-31)



Company (links to holding history)	Ticker	Value On 2021-03-31	No of Shares	% of portfolio
APPLE INC (COM)	AAPL	108,363,609,000	887,135,554	40.07%
BANK AMER CORP (COM)	BAC	39,080,793,000	1,010,100,606	14.45%
AMERICAN EXPRESS CO (COM)	AXP	21,443,817,000	151,610,700	7.92%
COCA COLA CO (COM)	KO	21,083,999,000	400,000,000	7.79%
KRAFT HEINZ CO (COM)	KHC	13,025,393,000	325,634,818	4.81%
VERIZON COMMUNICATIONS INC (COM)	VZ	9,235,649,000	158,824,575	3.41%
MOODYS CORP (COM)	MCO	7,366,643,000	24,669,778	2.72%
US BANCORP DEL (COM NEW)	USB	7,172,993,000	129,687,084	2.65%

Portfolio Mean and Variance

Let w_1 and w_2 be the portfolio weights ($w_1 + w_N = 1$) of assets 1 and 2. The expected return and variance of the portfolio (*p*) are

$$\overline{R_p} = E[w_1 R_1 + w_2 R_2] = w_1 \overline{R_1} + w_2 \overline{R_2}$$

$$\begin{aligned}\sigma_p^2 = Var(w_1 R_1 + w_2 R_2) &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{1,2} \\ &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2\end{aligned}$$

Diversification

Combining different assets allows you to achieve a lower variance.

- Example: Two risky assets 1 and 2 have the same expected return and variance: $\overline{R_1} = \overline{R_2} = \bar{R}$ and $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

  If you form a portfolio with equal weight on the two assets, what is the expected return?

  What is the return variance?

Portfolio Risk

Example

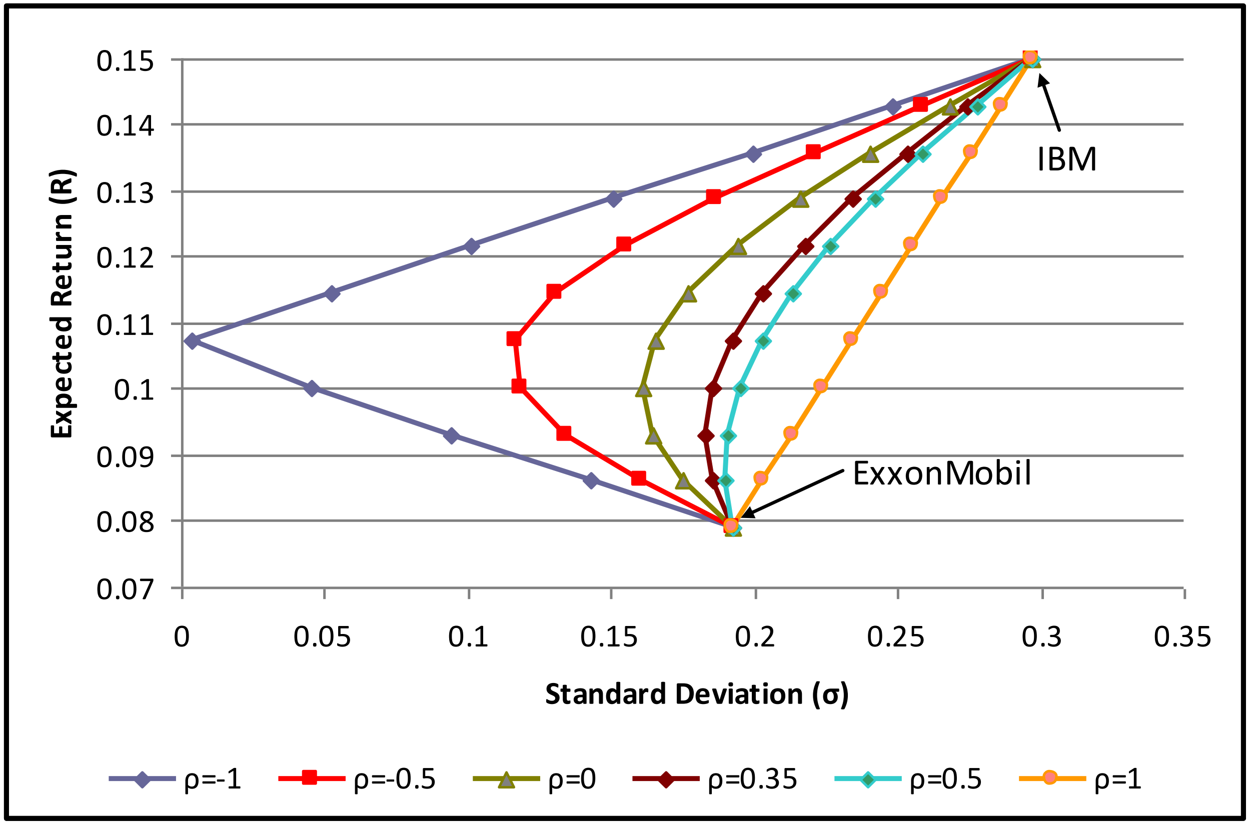
Suppose you invest 60% of your portfolio in Exxon Mobil and 40% in IBM. The expected return on your Exxon Mobil stock is 8% and on IBM is 15%. The standard deviation of their annualized daily returns are 18.2% and 27.3%, respectively. Assume a correlation coefficient of 1 and calculate the portfolio return and variance.

What if the correlation is now 0?

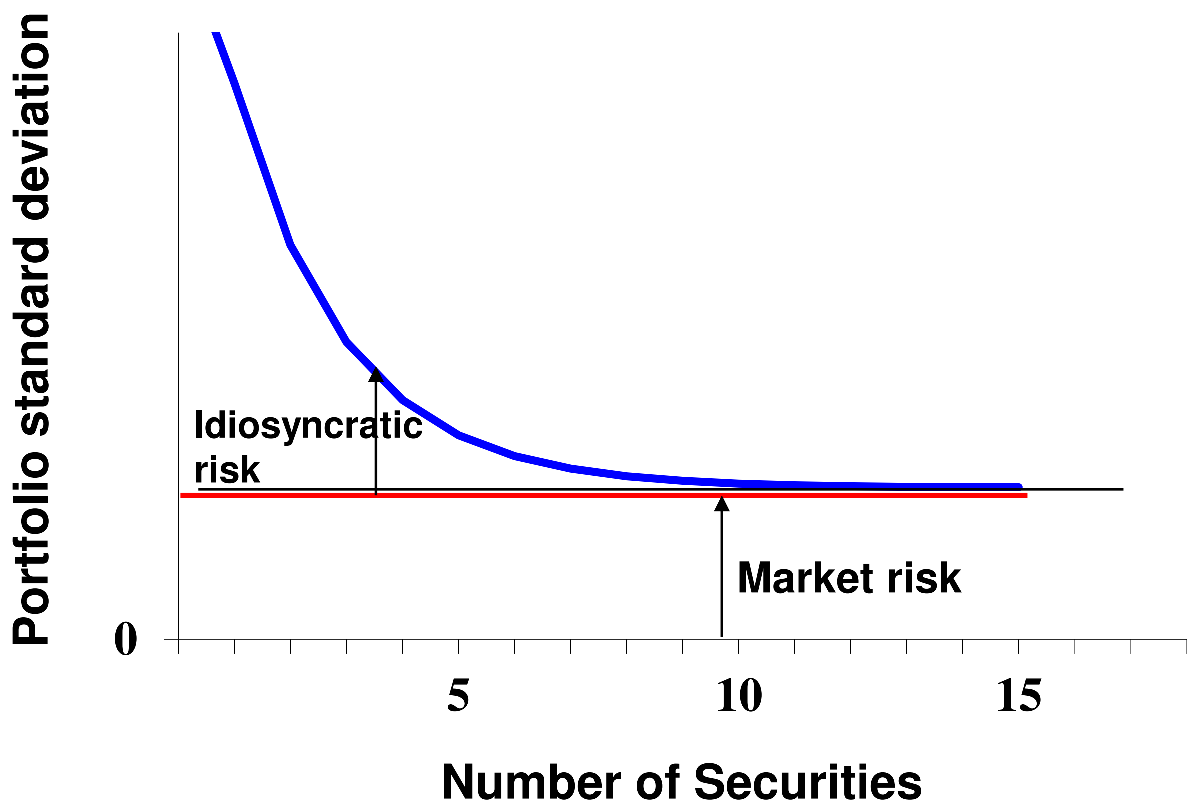
What if the correlation is now -1?

Diversification

More generally, when asset means and variances, different combined portfolios look like this (for different levels of correlation):



Diversification



Diversification

<u>Diversification</u> - Strategy designed to reduce risk by spreading the portfolio across many investments.

<u>Idiosyncratic Risk</u> - Risk factors affecting only a specific firm (e.g. death of company XYZ's CEO). Also called "diversifiable risk."

<u>Market Risk</u> - Economy-wide sources of risk that affect the overall stock market. Also called "systematic risk."

Diversification

Risk of an asset (Variance)	=	Market risk of the asset	+	Idiosyncratic risk of the asset
Risk of a portfolio	=	Market risk of the portfolio	+	Idiosyncratic risk of the portfolio

Tends to get diversified away

Topics Covered

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  - **Risk aversion**
  - Portfolio choice with multiple risky assets
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Risk Aversion

Consider two lotteries

- Lottery 1: you receive £2000 for sure
- Lottery 2: you receive £4000 with probability 0.5 and £0 with probability 0.5

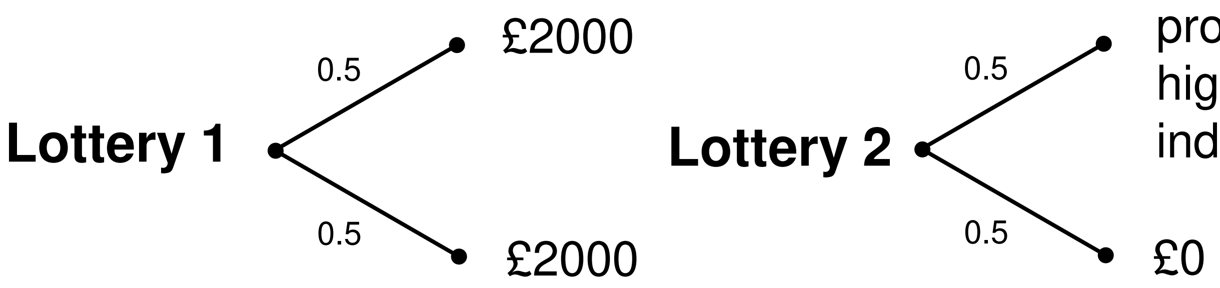
Your expected payoff is the same in both lotteries. For lottery 2, the payoff is $0.5 \times £4000 + 0.5 \times £0 = £2000$.

- Which lottery do you prefer?

Risk Aversion

- An investor is said to be:

risk averse	if chooses	lottery 1
risk neutral	if indifferent	
risk loving	if chooses	lottery 2



If you’re risk averse, you probably want this to be higher than £4000 to be indifferent.

Risk Aversion

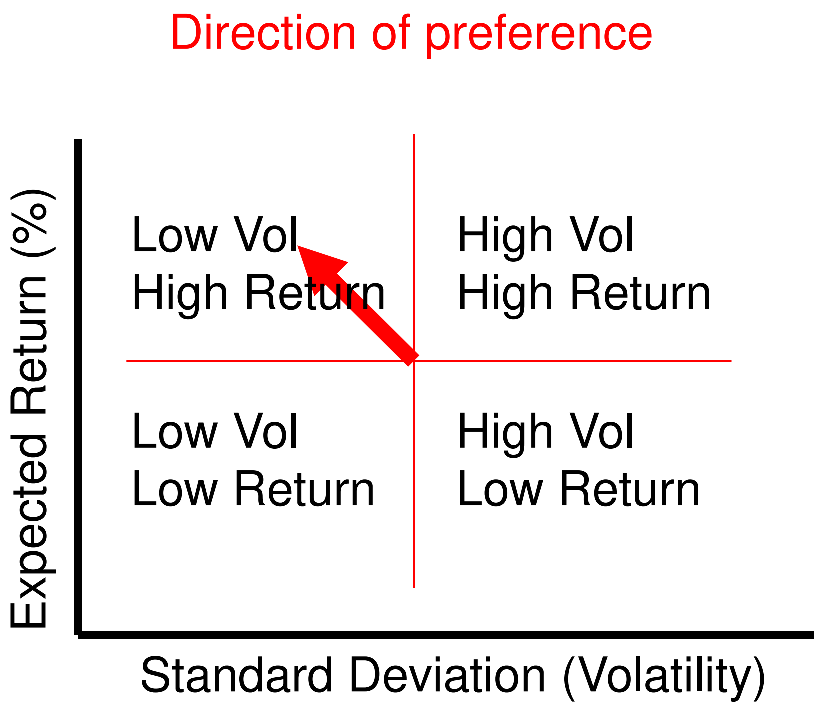
Risk aversion: Preference for a reduction in uncertainty given the same expected payoff

- A more-risk-averse person is willing to give up a larger amount of expected payoff to reduce uncertainty by a fixed amount.

- Investors are usually risk averse rather than risk loving.

- So, if we put volatility and return in the x-y diagram, they want to go northwest (NW)

Risk Aversion



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Portfolio Theory

Portfolio theory proposed by Harry Markowitz (1952).

- A theory of what portfolio an investor should hold, assuming the investor only cares about expected returns and standard deviations.
- He won the Nobel Prize partly for this.
- Milton Friedman almost denied him a Chicago PhD saying the theory is not an “economics.”

Portfolio Theory

Main conclusions

- Investors should hold mean-variance efficient portfolios: the best combination of risky assets (i.e., a particular set of portfolio weights) that offers
  - the highest expected return for a given level of risk, or equivalently
  - the lowest level of risk for a given return expectation

- When focusing only on risky assets, the **efficient frontier** contains the best possible portfolios of *risky* assets.
- When there is a risk-free asset, there is **unique portfolio of risky assets** called the **tangency portfolio** that you should combine with the risk-free asset.
- So if everyone has the same views about the mean, variances, and covariances of returns, they should hold the same risky portfolio.

Efficient Frontier

- Consider 3 stocks with the following characteristics:

Stock	Expected Return	Standard Deviation
IBM	15%	29.7%
ExxonMobil	7.9%	19.2%
Starbucks	12.3%	29.9%

and correlations:

	IBM	ExxonMobil	Starbucks
IBM	1	0.35	0.2
ExxonMobil	0.35	1	-0.1
Starbucks	0.2	-0.1	1

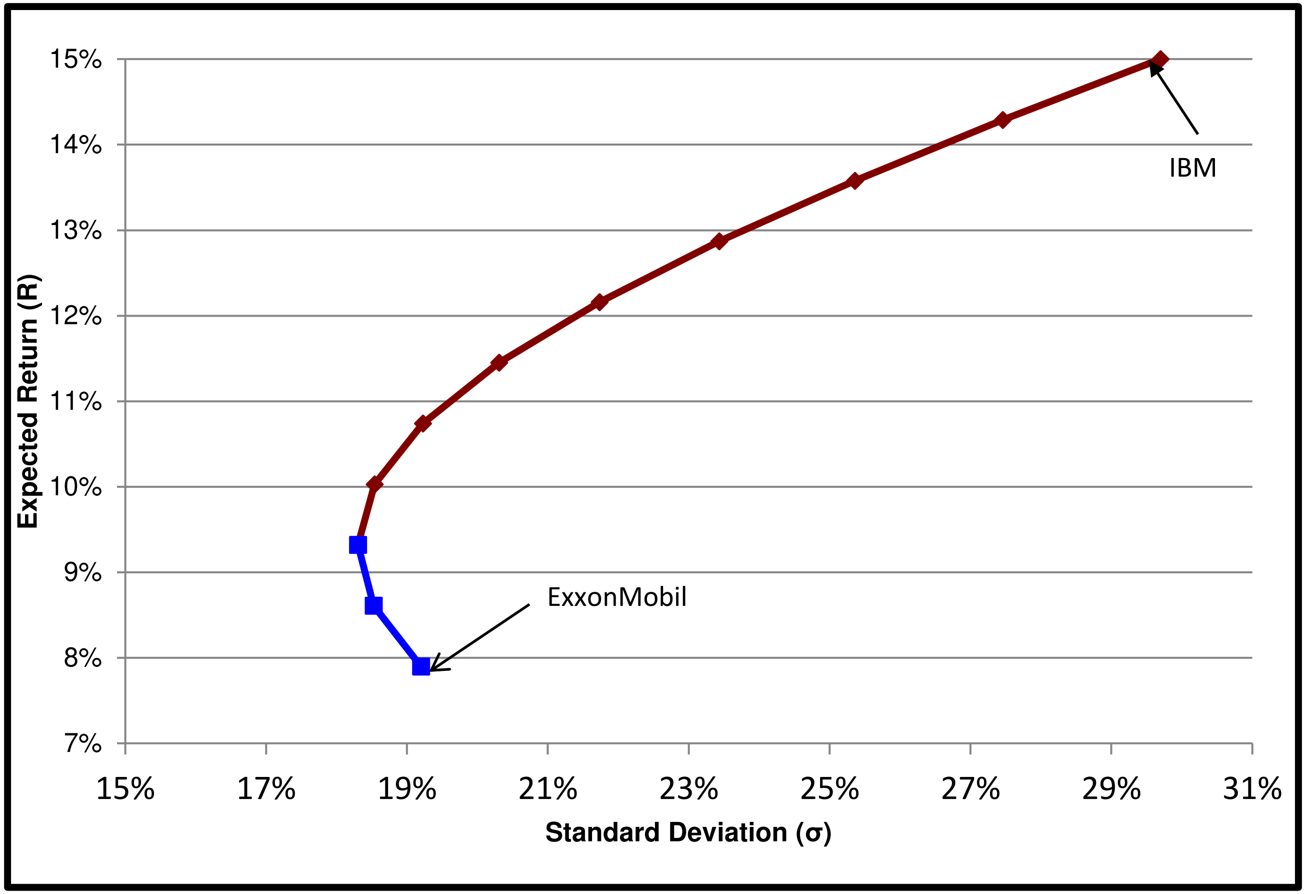
Efficient Frontier

- We consider portfolios with weights $1 \geq w_1 \geq 0$ and $1 \geq w_2 \geq 0$ such that $w_1+w_2=1$.

Portfolio Weights		mean	std
ExxonMobil	IBM		
1	0	8%	19%
0.9	0.1	9%	19%
0.8	0.2	9%	18%
0.7	0.3	10%	19%
0.6	0.4	11%	19%
0.5	0.5	11%	20%
0.4	0.6	12%	22%
0.3	0.7	13%	23%
0.2	0.8	14%	25%
0.1	0.9	14%	27%
0	1	15%	30%

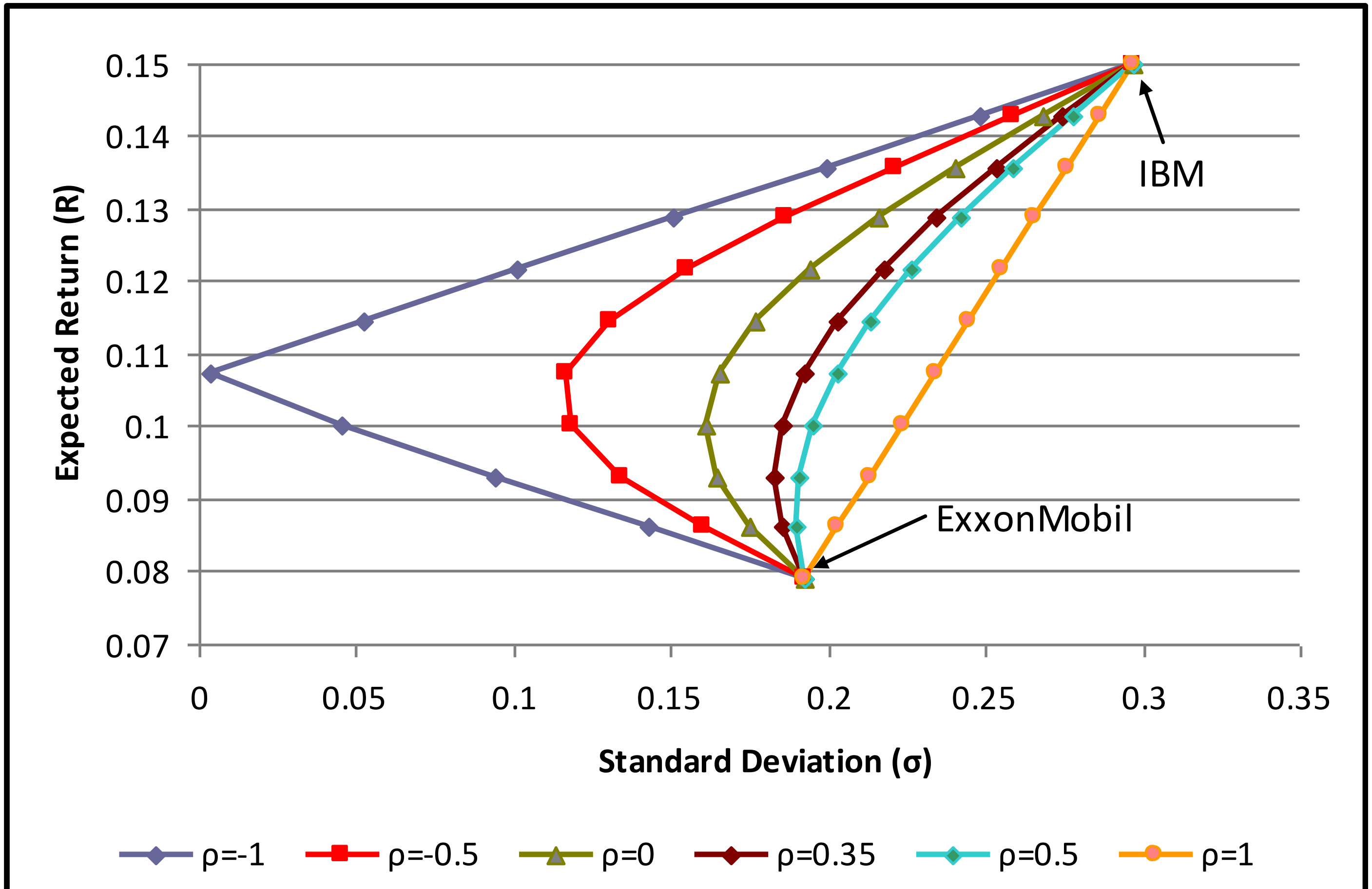
Efficient Frontier

- We can plot portfolio expected returns as a function of standard deviations – the portfolio frontier.



Efficient Frontier

- The shape of the portfolios depend on the correlation between the two assets.



Efficient Frontier

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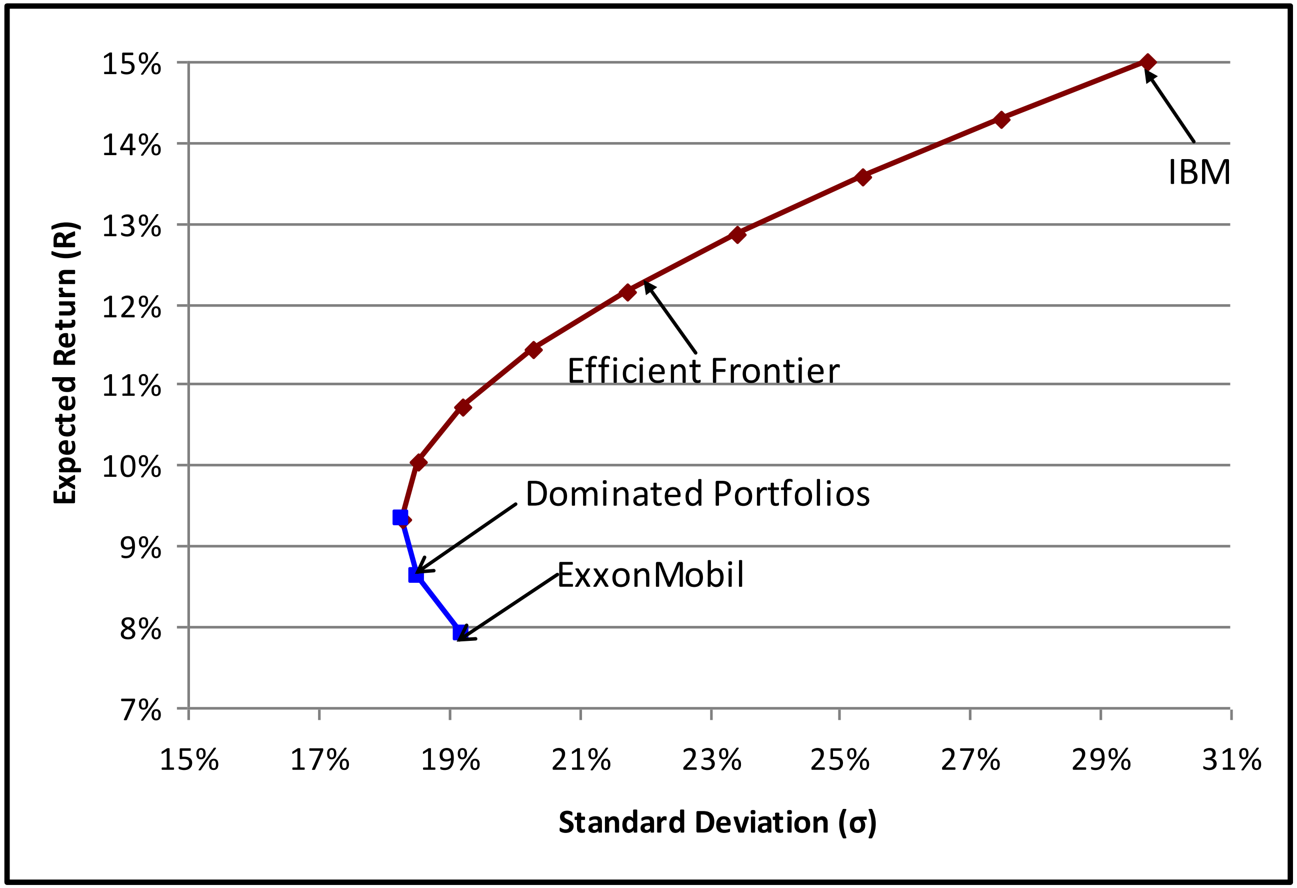
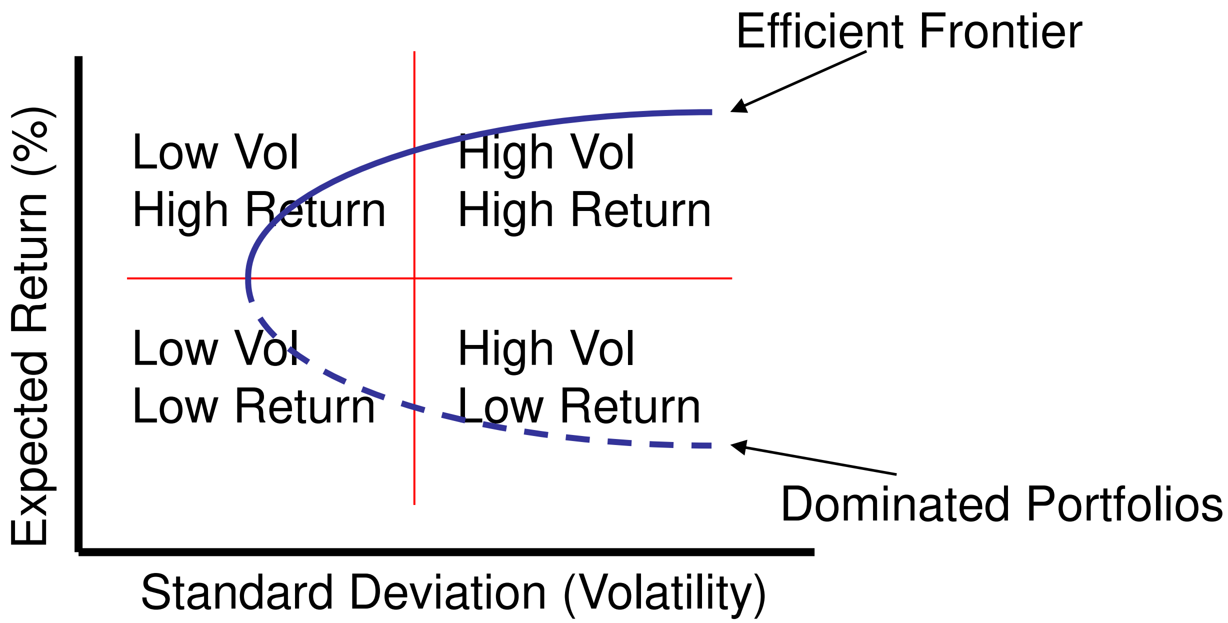


Efficient Frontier

- ExxonMobil is dominated by some linear combinations of IBM and ExxonMobil that has higher expected returns and/or lower standard deviations.
- Even though ExxonMobil is dominated by other portfolios, investors will buy it in combination with IBM because ExxonMobil provides diversification benefits.
- Investors will only buy portfolios on the upward sloping part which is called the efficient frontier since these portfolios offer the highest expected returns given risk.

Efficient Frontier

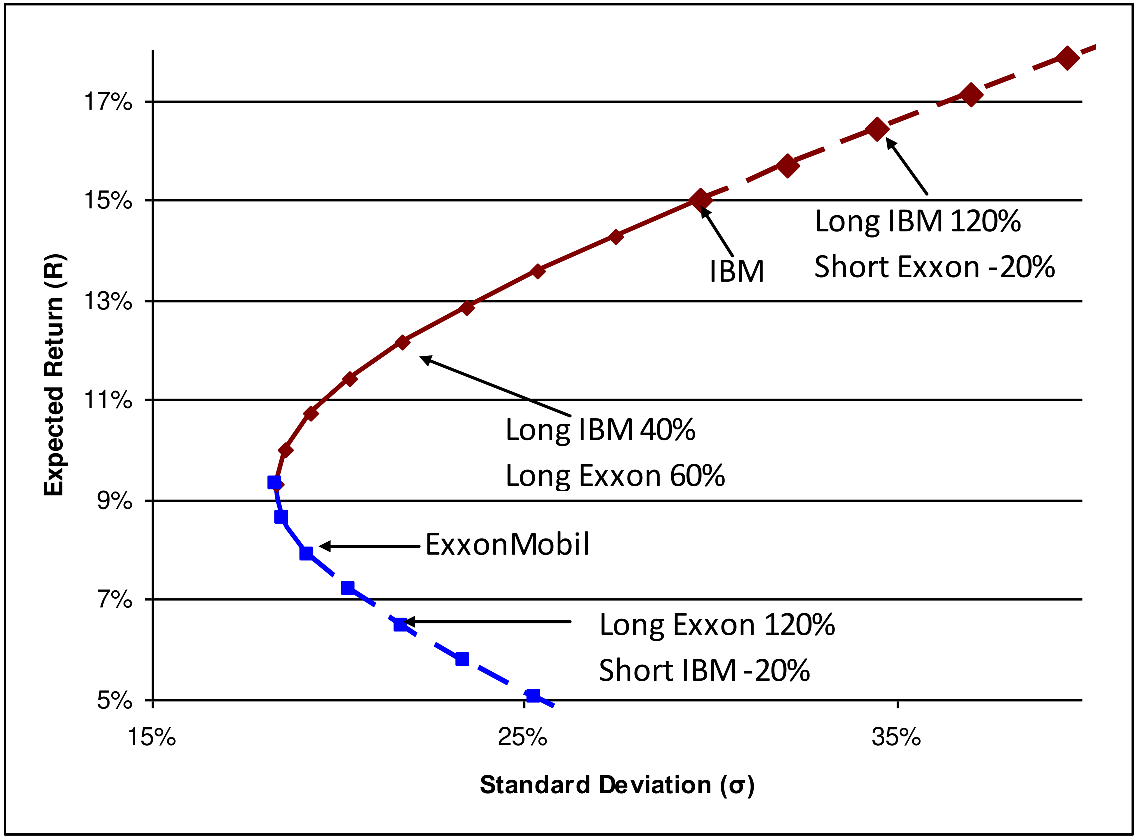


Efficient Frontier with Short Sales

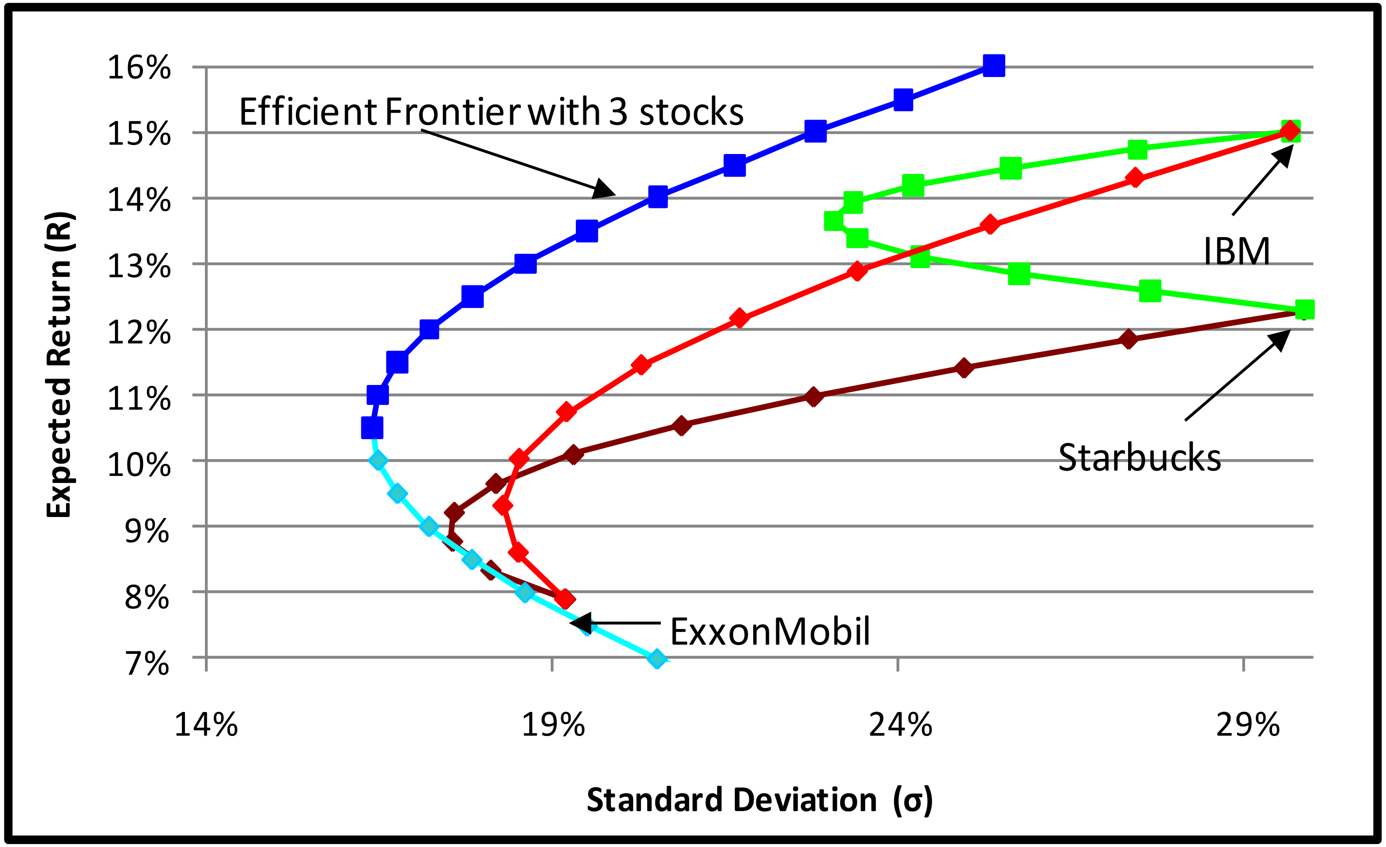
- We have so far considered the returns on portfolios of two stocks with <u>positive portfolio weights</u>.
- We can also have negative portfolio weights
  - A positive investment in a security (e.g. $w_1 > 0$) is a **long position**,
  - A negative investment (e.g. $w_1 < 0$) is a **short position**.
- **Short-selling** refers to an activity when you sell a stock that you do not own!
  - you borrow a stock from someone (usually via your broker) and sell it;
  - In the future, you buy it in the market, and return to the person from whom you borrowed it;
  - you also need to pay all the dividends that the stock generates.

Efficient Frontier with Short Sales

- Short-selling allows us to expand the frontier of portfolios beyond IBM and ExxonMobil points:

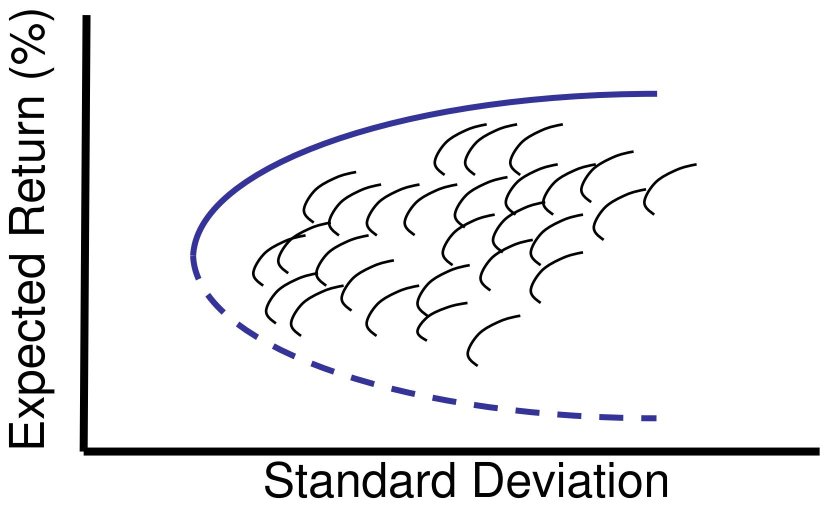


Efficient Frontier with >2 stocks

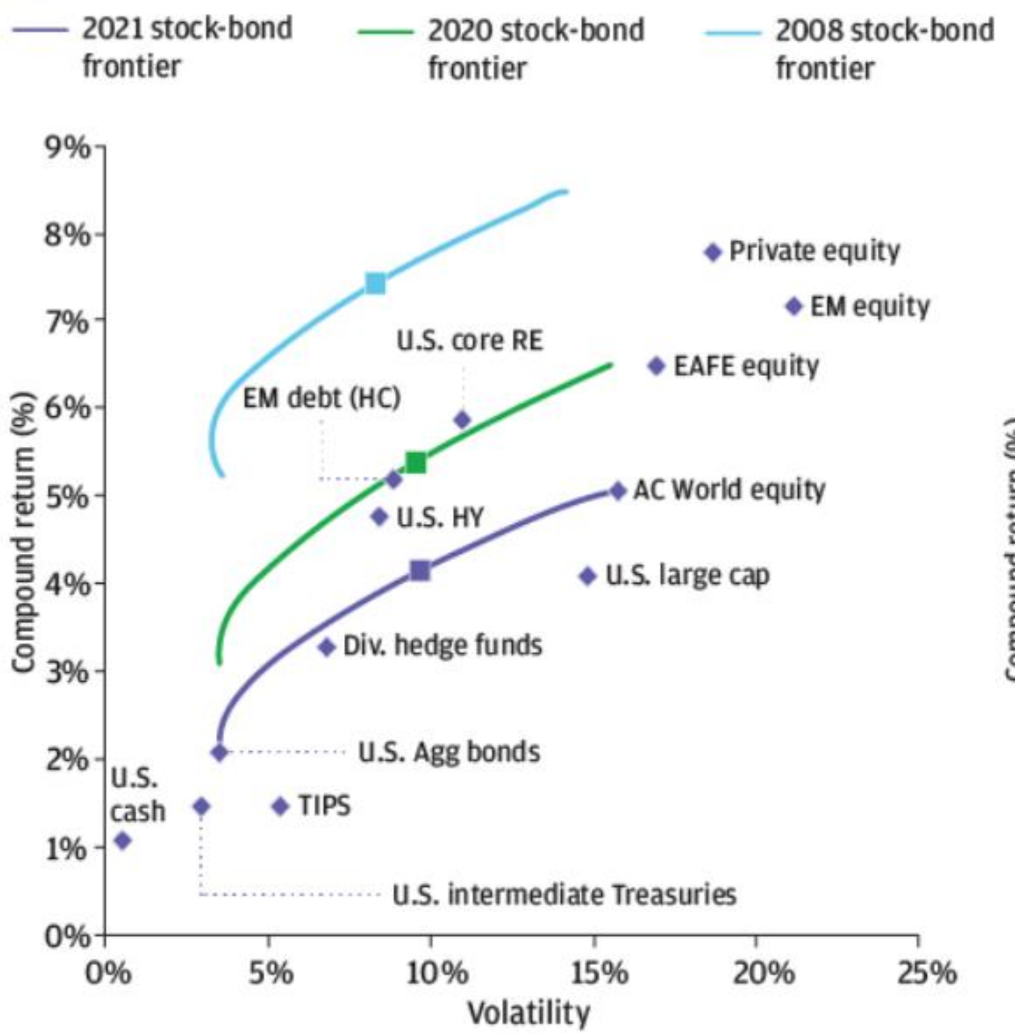


Efficient Frontier with >2 stocks

- Two important features of efficient frontiers
  - If portfolios A and B lie on the frontier, then all portfolios that invest *w*% in A and (*1-w*)% in B also lie on the frontier. In other words, a linear combination of two efficient portfolios is also efficient!
  - Portfolios on the *N*-stock efficient frontier dominate the portfolios on 2-, 3-, … , *N*-1 stock frontiers:



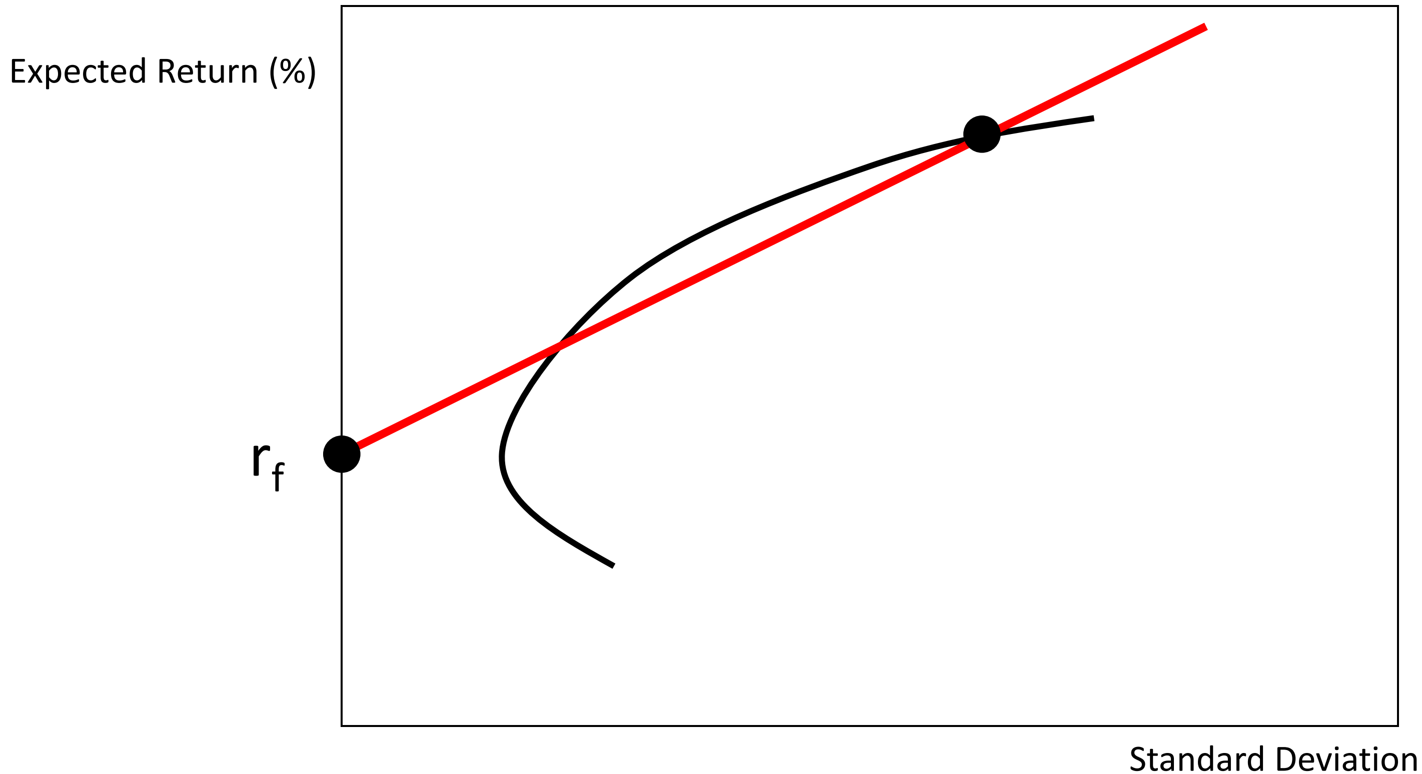
Efficient Frontier with Asset Classes



JP Morgan

Efficient Frontier with a Risk Free Asset

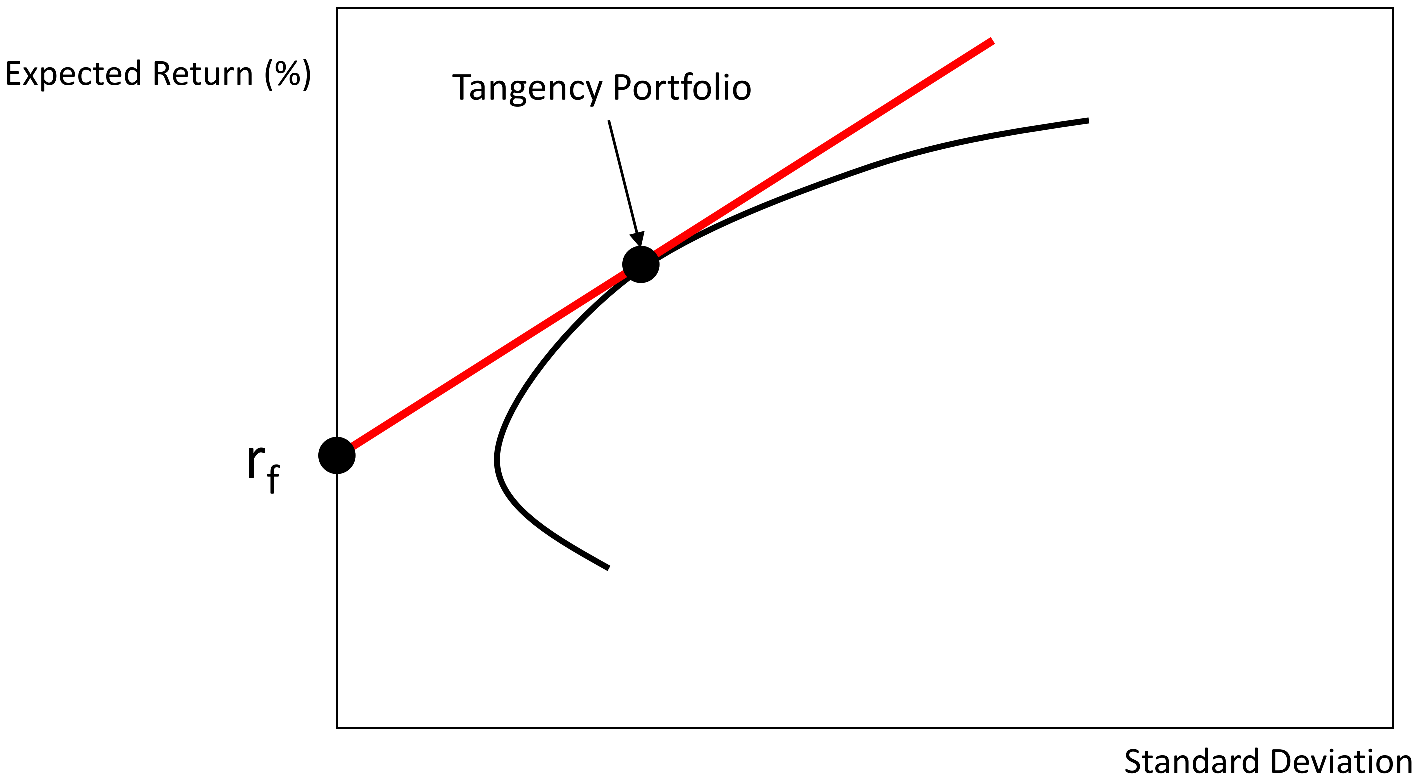
If risk-free asset with return R_f is available, we can combine it with a portfolio *p* on the efficient frontier.



The combinations form a line since expected return $= (1 - w_p)R_f + w_p\bar{R}_p$ and SD = $w_p\sigma_p$ (where bar denotes mean and sigma denotes stdev).

Efficient Frontier with a Risk Free Asset

The best combination is through the “tangency portfolio.”



Efficient Frontier with a Risk Free Asset

- Consider any portfolio *P* on the N-stock efficient frontier.
  - The slope of straight line is given by $(\bar{R}_P - R_f)/\sigma_P$.
  - This ratio is called the Sharpe ratio. It quantifies the trade-off between a higher risk premium and a higher stock return volatility.
  - The Sharpe ratio is larger when the risk premium $\bar{R}_P - R_f$ is larger or the stock return standard deviation σ_P is smaller.
- Investors prefer portfolios with higher Sharpe ratios!
  - Investors pick the one with the largest Sharpe ratio.
  - i.e., the line that passes through the tangency portfolio
  - That means all investors hold the same risky portfolio

Portfolio Theory Conclusion

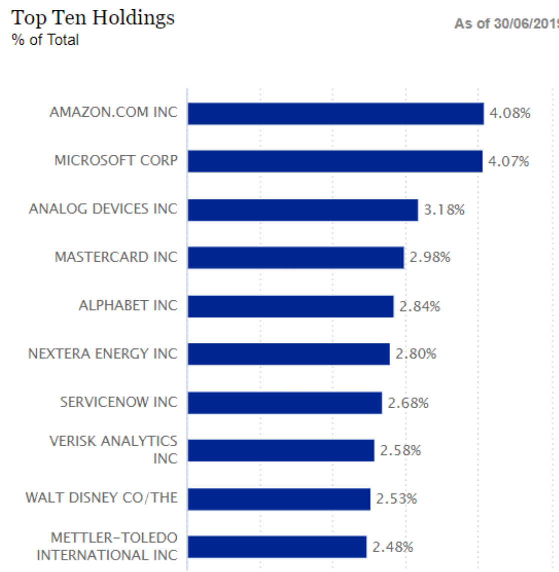
- Investors should hold a diversified portfolio.
- If there is a risk-free asset and the investors agree on the mean return and variance/covariance forecasts, all investors must hold the same risky portfolio with the highest Sharpe ratio regardless of their risk aversion.
- They then combine this **optimal risky portfolio** (**tangency portfolio**) with the risk-free asset to form a "**complete portfolio**," which could be different by investor due to differences in risk aversion.

Empirical Evidence 1: Mutual Fund Holdings

Berkshire Hathaway (2019Q1)

	Symbol	Holdings	Mkt. price	Value
TOTAL				$216,815,915,923
Apple Inc.	AAPL	249,589,329	$208.26	$51,979,473,658
Bank of America Corp	BAC	896,167,600	$30.58	$27,404,805,208
The Coca-Cola Co	KO	400,000,000	$53.98	$21,592,000,000
Wells Fargo & Co	WFC	409,803,773	$48.63	$19,928,757,481
American Express Company	AXP	151,610,700	$126.54	$19,184,817,978
Kraft Heinz Co	KHC	325,634,818	$31.87	$10,377,981,650
U.S. Bancorp	USB	129,308,831	$57.16	$7,391,292,780
JPMorgan Chase & Co.	JPM	59,514,932	$115.99	$6,903,136,963
Moody's Corporation	MCO	24,669,778	$203.13	$5,011,172,005
Delta Air Lines, Inc.	DAL	70,910,456	$62.53	$4,434,030,814

Franklin Select U.S. Equity



BlackRock Advantage U.S. Total Market Fund

TOP HOLDINGS (%)[2]	
AMAZON.COM INC	3.16
JOHNSON & JOHNSON	2.34
MICROSOFT CORPORATION	2.33
APPLE INC	2.27
BERKSHIRE HATHAWAY INC	1.91
MERCK & CO INC	1.85
FACEBOOK INC	1.83
JPMORGAN CHASE & CO	1.54
AMGEN INC	1.48
SALESFORCE.COM INC.	1.45
Total of Portfolio	20.16

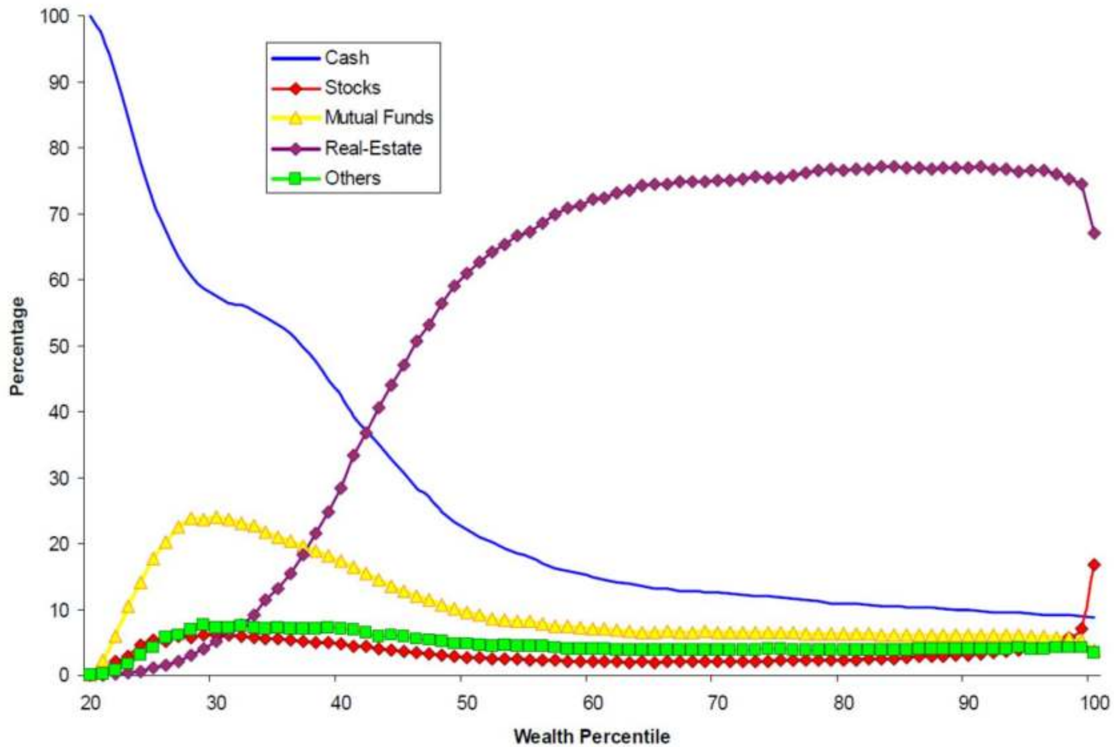
Empirical Evidence 2: Swedish Data

- "Down or Out: Assessing the Welfare Costs of Household Investment Mistakes" by Calvet, Campbell, and Sodini (2007)
- Data collected by Statistics Sweden to levy caiptal income and wealth taxes
  - All financial asset holdings including bank accounts, mutual funds, stocks, pension savings, and debt outstanding.
  - Demographic information like age, gender, education.

Empirical Evidence 2: Swedish Data

FIGURE 2. COMPOSITION OF FINANCIAL AND REAL ESTATE PORTFOLIO

A. Variation with Gross Wealth

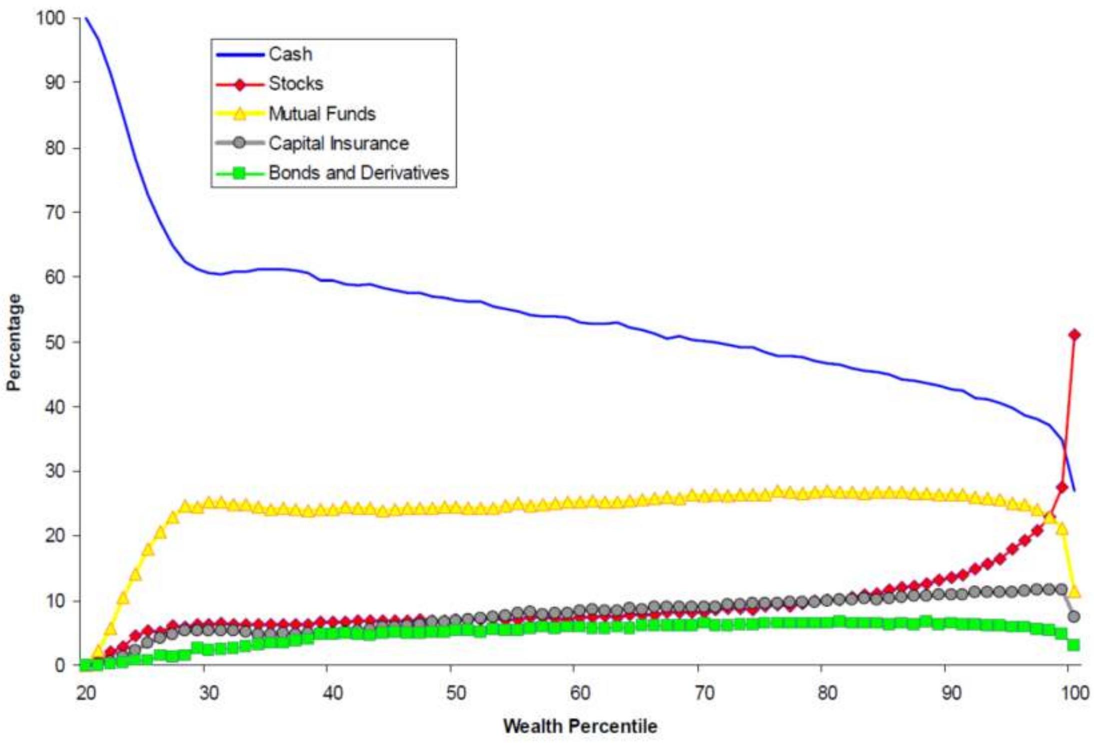


- What is this "real estate"?

Empirical Evidence 2: Swedish Data

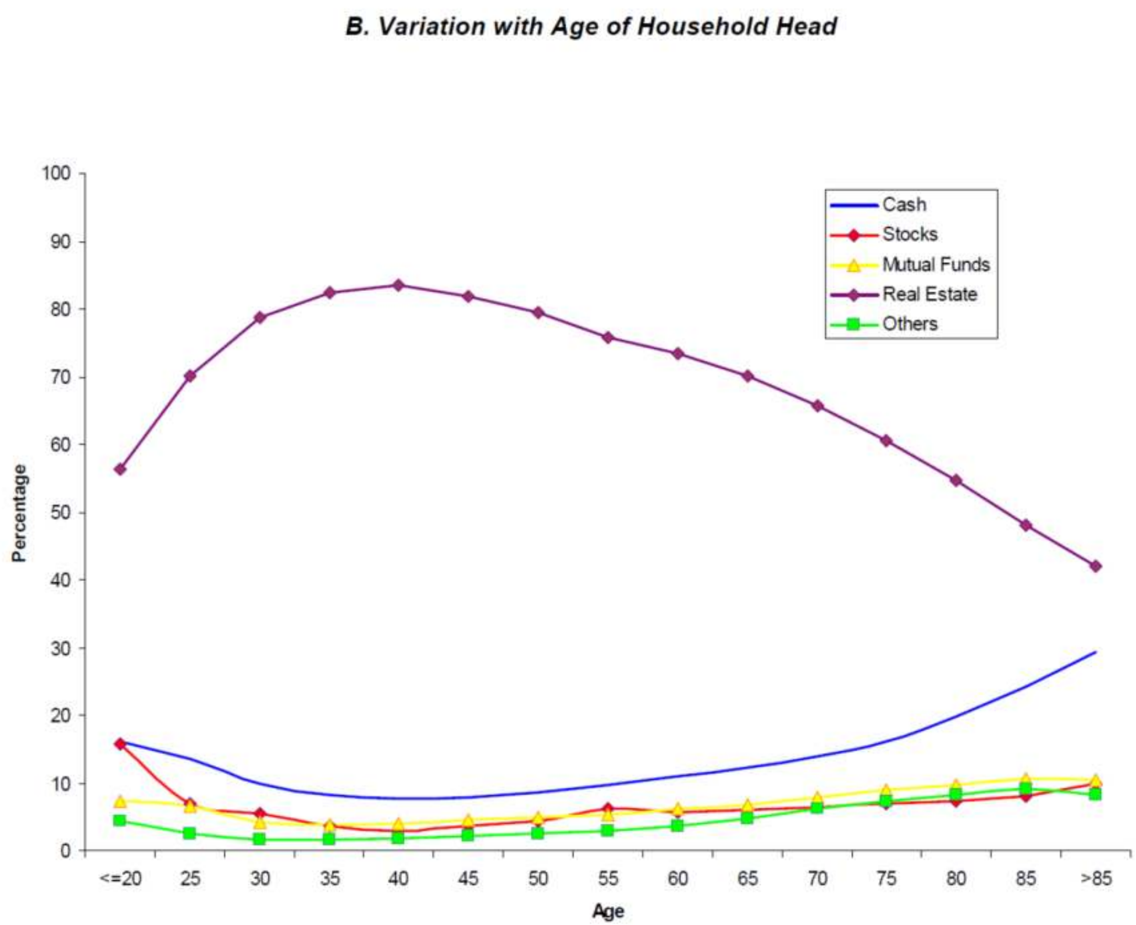
FIGURE 3. COMPOSITION OF FINANCIAL PORTFOLIO

A. Variation with Gross Wealth



Households hold diversified portfolios except in the extreme percentiles.

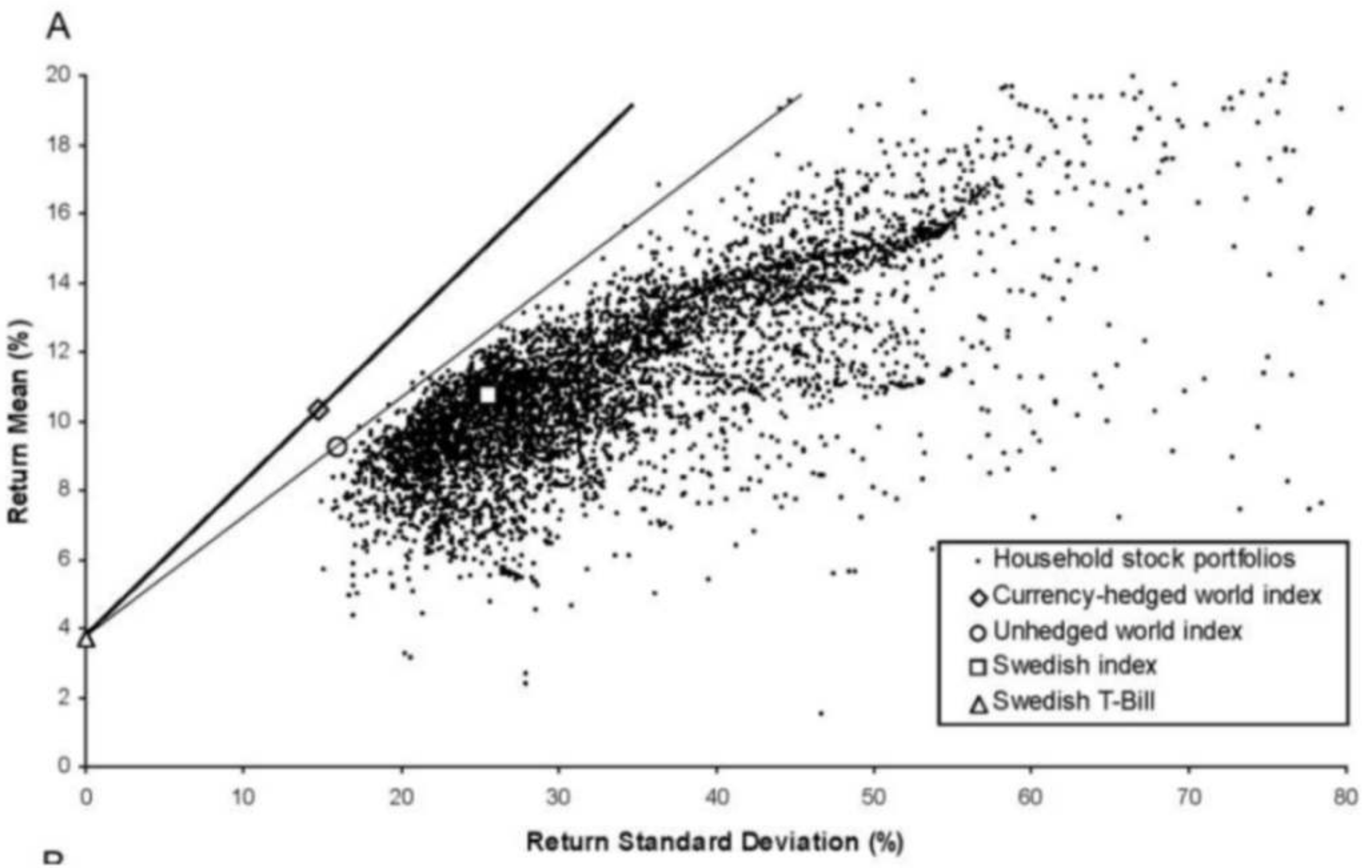
Empirical Evidence 2: Swedish Data



From optimal portfolio perspective, why might younger people hold more stocks?

Empirical Evidence 2: Swedish Data

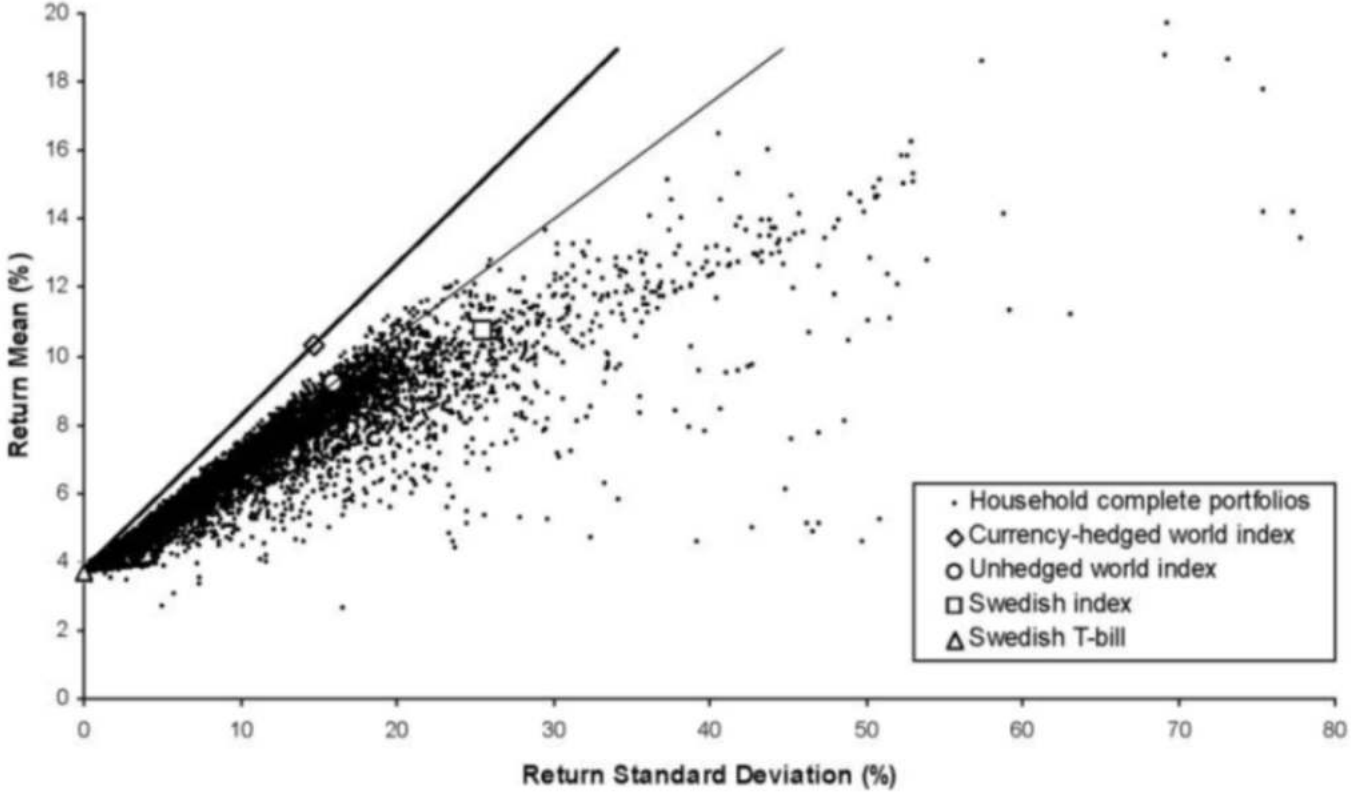
- Household risky portfolios:



- What was the prediction of modern portfolio theory here?

Empirical Evidence 2: Swedish Data

- Household complete portfolios:



- Why is it looking “better” here than before?