Class 5

The CAPITAL ASSET PRICING MODEL CAPM

- Formalises certain aspects of portfolio theory, by making STRONG ASSUMPTIONS
- MARKET PORTFOLIO: weight of i is $w_i^M = \frac{V_i}{\sum V_k}$ → market capitalization = shares · share price

  → MATCH DEMAND and SUPPLY ⇝ MARKET PORTFOLIO = TANGENCY PORTFOLIO

- MARKET WEIGHT: determined by: $E[R]$, $SD[R]$, correlation of returns.
- For all assets J, $\frac{E[r_J]-r_F}{Cov(r_J, r_M)}$ is constant $\Rightarrow$ $\frac{E[r_M]-r_F}{Var(r_M)}$ = constant

  RETURNS, RISK-FREE, MARKET

$$\Longrightarrow E[r_i] = r_f + \beta_i \left(E[r_M] - r_f\right)$$

β of i; MARKET; RISK-FREE

$$\beta_i = \frac{Cov(r_i, r_M)}{Var(r_M)}$$

RISKYNESS, and how much premium people want to hold it

Question 1
The T-bill rate is 4% and the expected return on the market is 12%. Using the CAPM:
(a) What is the risk premium on the market?
(b) What is the required return on an investment with a beta of 1.5?
(c) If an investment with a beta of 0.8 offers an expected return of 9.8%, does it have a positive NPV?
(d) If the market expects a return of 11.2% from stock X, what is its beta?

a) The risk-premium is the extra expected return you get because of the increased risk of the market.

$$RP = E[r_M] - r_f = 12\% - 4\% = 8\% = 0.08$$

b) $$E[r_i] = r_f + \beta_i \left(E[r_M] - r_f\right)$$

$$= 0.04 + 1.5 \cdot (0.08) = 0.16 \rightsquigarrow 16\%$$

c) For an investment with $\beta_A = 0.8$, you would expect to earn

$$E[r_A] = r_f + \beta_A \left(E[r_M] - r_f\right) = 0.04 + 0.8(0.08) = 0.104$$

So the proposed investment is worse than what we can do, and has a negative NPV.

d) $r = r_f + \beta(\bar{r}_M - r_f) \rightsquigarrow 0.112 = 0.04 + \beta(0.08)$

$\rightsquigarrow \beta = 0.9$

Question 2
You are a consultant to a large manufacturing corporation that is considering a project with the following net cash flows (in millions of dollars):

Years	Cash Flow
0	-40
1-10	15

The project's beta is 1.8. Assuming that the risk free rate is 8% and the expected market return is 16%, what is the NPV of the project?

① Find the appropriate discount rate for the project.

$$r = r_f + \beta\,[\bar{r}_M - r_f] = 0.08 + 1.8 \cdot (0.16 - 0.08) = 0.224$$

This is the discount rate for the given β. So

$$NPV = -40 + \sum_{i=1}^{10} \frac{15}{1.224^i} = \dots = 18.09$$

Question 3

Consider the following table, which gives a security analyst's expected return on two stocks A and D for two different scenarios for market returns:

Market return	Return A	Return D
5%	-2%	6%
25%	38%	12%

The beta of A is 2, and the beta of B is 0.3.

(a) What is the expected return on each stock if the market return is equally likely to be 5% or 25%?

(b) If the T-bill rate is 6% and the market return is equally likely to be 5% or 25%, draw the SML for this economy.

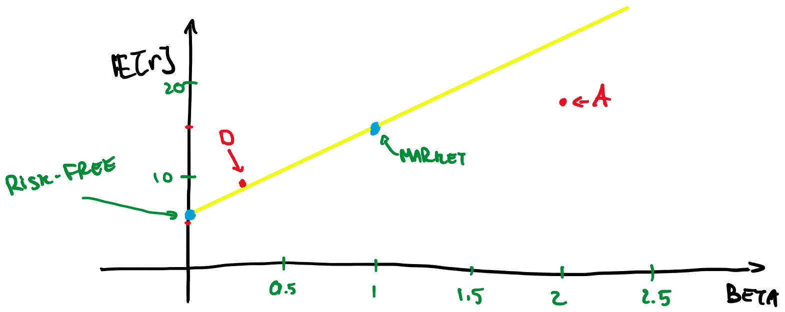
(c) Plot the two securities on the SML graph. What are the alphas of each?

$\underbrace{\text{SML}}$ ↳ security MARKET LINE

a) $\mathbb{E}[r_A] = 0.5 \cdot (-2\%) + 0.5 \cdot (38\%) = 18\%$

$\mathbb{E}[r_D] = 0.5 \cdot (6\%) + 0.5\,(12\%) = 9\%$

b) $\mathbb{E}[r_M] = 0.5\,(5\% + 25\%) = 15\%$



c) $\mathbb{E}[r_{\beta=2}] = 0.06 + 2 \cdot (0.15 - 0.06) = 0.24$

$\mathbb{E}[r_{\beta=0.3}] = \qquad\qquad = 0.087$

$\alpha_A = \mathbb{E}[r_A] - \mathbb{E}[r_{\beta=\beta_A}] = 0.18 - 0.24 = -0.06$

$\alpha_D = \quad \dots \quad = 0.003$

Question 2 [6 points]

Suppose the rate of return on short-term government securities (risk-free) is 5%. Suppose also that the expected return required by the market for a portfolio with a beta of 1 is 12%. According to the CAPM:

with a beta of 1 is 12%. According to the CAPM:

(i) What is the expected return on the market portfolio?

(ii) What would be the expected return on a stock with beta = 0?

(iii) Suppose you consider buying a share of stock at $40. The stock is expected to pay $3 dividends next year and you expect it to sell then for $41. If the stock risk has been evaluated at beta= -0.5, is the stock overpriced, underpriced, or correctly priced?

A. (i) 7% (ii) 0% (iii) underpriced
B. (i) 7% (ii) 0% (iii) overpriced
C. (i) 12% (ii) 0% (iii) correctly priced
D. (i) 12% (ii) 5% (iii) underpriced
E. (i) 12% (ii) 5% (iii) overpriced

iii) $\mathbb{E}[r] = \frac{41+3}{40} - 1 = 10\%$

$\mathbb{E}[r_{\beta=-0.5}] = 0.05 + (-0.5)(0.12 - 0.05) = 0.015$

so the stock is currently underpriced

(a) Suppose that investors expect next year's dividend to be $5. If you know for certain that the dividend growth rate will be 4%, that the stock will have a market beta of 0.5, that the risk-free rate will stay at 5%, and that the expected return on the market will be 11% next year, what will be the price of the stock next period (Hint: Use Gordon growth model)?

$P_1 = \frac{(1+g)D}{\mathbb{E}[r] - g} = \frac{5\% \cdot 1.04}{0.05 + 0.5 \cdot (0.11 - 0.05) - 0.04} = \dots = 130\$$

Question 4 [6 points]

The price of a stock is $50. Its expected rate of return is 10%. The risk-free rate is 6% and the expected market return is 14%. The market is efficient. What will be the price of the stock if its correlation with the market portfolio doubles (and all other variables remain unchanged—and note that covariance between X and Y equals correlation between X and Y times the standard deviation of X times the standard deviation of Y)? Assume that the stock is expected to pay a constant dividend with no growth in perpetuity.

A. $25
B. $36
B. $44
D. $48
E. $50

Answer is B. If correlation doubles, beta doubles. Since $E[R_i] - R_f = \beta_i(E[R_m] - R_f)$, this means risk premium doubles from 4% to 8% and expected return goes from 10% to 14%. This means, since constant dividend model says P = D/r, the new price is previous price of $50 times .10/.14, which is $35.7.