Report

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1 Week 1, Day 1: NNG Tutorial World

In this report, we will explore the concept of numbers as defined in Lean, specifically focusing on the level titled "The Birth of Number" from the project Tut. World.

1.1 The Birth of Number

Numbers in Lean are defined by two fundamental rules:

- **0 is a number.**
- **If n is a number, then the successor succ n of n is a number.**

The successor of n is the number that comes immediately after n. This concept allows us to start counting and define basic numbers.

1.1.1 Counting to Four

Starting from 0, we can define the first few numbers as follows:

$$\begin{aligned}1 &= \text{succ } 0,\\ 2 &= \text{succ } 1,\\ 3 &= \text{succ } 2,\\ 4 &= \text{succ } 3.\end{aligned}$$

1.1.2 Proof: $2 = \textbf{succ } 1$

The proof that $2 = \text{succ } 1$ is provided in Lean under the lemma named `two_eq_succ_one`. Let's prove that 2 is indeed the number after the number after zero.

Step-by-Step Proof To begin, we start by rewriting 2 using the lemma `two_eq_succ_one`:

```
rw [two_eq_succ_one]
```

Next, we rewrite 1 as succ 0 using:

```
rw [one_eq_succ_zero]
```

Finally, we verify that the equation holds by using the reflexivity command:

```
rfl
```

1.1.3 Alternative Approach

The proof can also be completed in a more concise manner by combining the rewrite steps:

```
rw [two_eq_succ_one, one_eq_succ_zero]
rfl
```