



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V1

NOVEMBER 2014

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 10 bladsye en 1 inligtingsblad.

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat jy die vrae beantwoord.

1. Hierdie vraestel bestaan uit 12 vrae.
2. Beantwoord AL die vrae.
3. Nommer die antwoorde korrek volgens die nommeringstelsel wat in hierdie vraestel gebruik is.
4. Dui ALLE berekeninge, diagramme, grafieke, ensovoorts wat jy gebruik het om jou antwoorde te bepaal, duidelik aan.
5. Volpunte sal nie noodwendig aan slegs antwoorde toegeken word nie.
6. Jy mag 'n goedgekeurde, wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.
7. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
8. Diagramme is NIE noodwendig volgens skaal geteken NIE.
9. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
10. Skryf netjies en leesbaar.

VRAAG 11.1 Los op vir x :

1.1.1 $(x-2)(4+x)=0$ (2)

1.1.2 $3x^2 - 2x = 14$ (korrek tot TWEE desimale plekke) (4)

1.1.3 $2^{x+2} + 2^x = 20$ (3)

1.2 Los die volgende vergelykings gelyktydig op:

$x = 2y + 3$

3 $x^2 - 5xy = 24 + 16y$ (6)

1.3 Los op vir x : $(x-1)(x-2) < 6$ (4)1.4 Die wortels van 'n kwadratiese vergelyking is: $x = \frac{3 \pm \sqrt{-k-4}}{2}$ Vir watter waardes van k is die wortels reëel? (2)

[21]

VRAAG 2Gegee die rekenkundige reeks: $2 + 9 + 16 + \dots$ (tot 251 terme).

2.1 Skryf die vierde term van die reeks neer. (1)

2.2 Bereken die 251^{ste} term van die reeks. (3)

2.3 Druk die reeks in sigma-notasie uit. (2)

2.4 Bereken die som van die reeks. (2)

2.5 Hoeveel terme in die reeks is deelbaar deur 4? (4)

[12]

VRAAG 3

3.1 Gegee die kwadratiese ry: $-1 ; -7 ; -11 ; p ; \dots$

3.1.1 Skryf die waarde van p neer. (2)

3.1.2 Bepaal die n^{de} term van die ry. (4)

3.1.3 Die eerste verskil tussen twee opeenvolgende terme van die ry is 96.
Bereken die waardes van hierdie twee terme. (4)

3.2 Die eerste drie terme van 'n meetkundige ry is: $16 ; 4 ; 1$

3.2.1 Bereken die waarde van die 12^{de} term. (Laat jou antwoord in vereenvoudigde eksponensiële vorm.) (3)

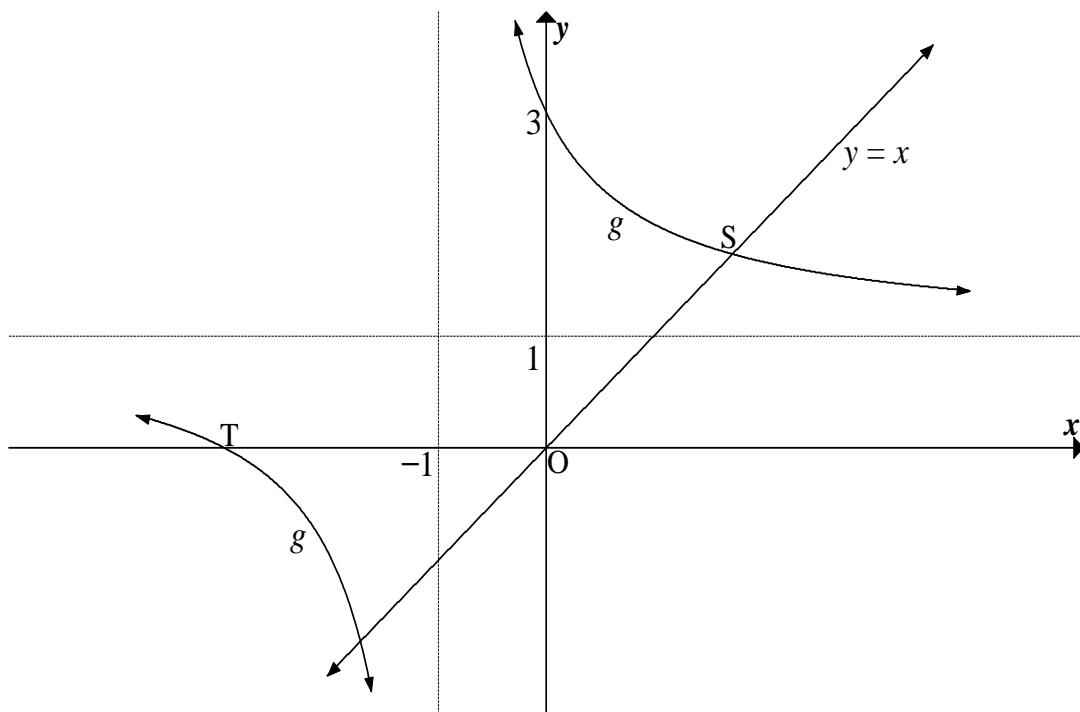
3.2.2 Bereken die som van die eerste 10 terme van die ry. (2)

3.3 Bepaal die waarde van: $\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right) \dots$ tot 98 faktore.

(4)
[19]

VRAAG 4

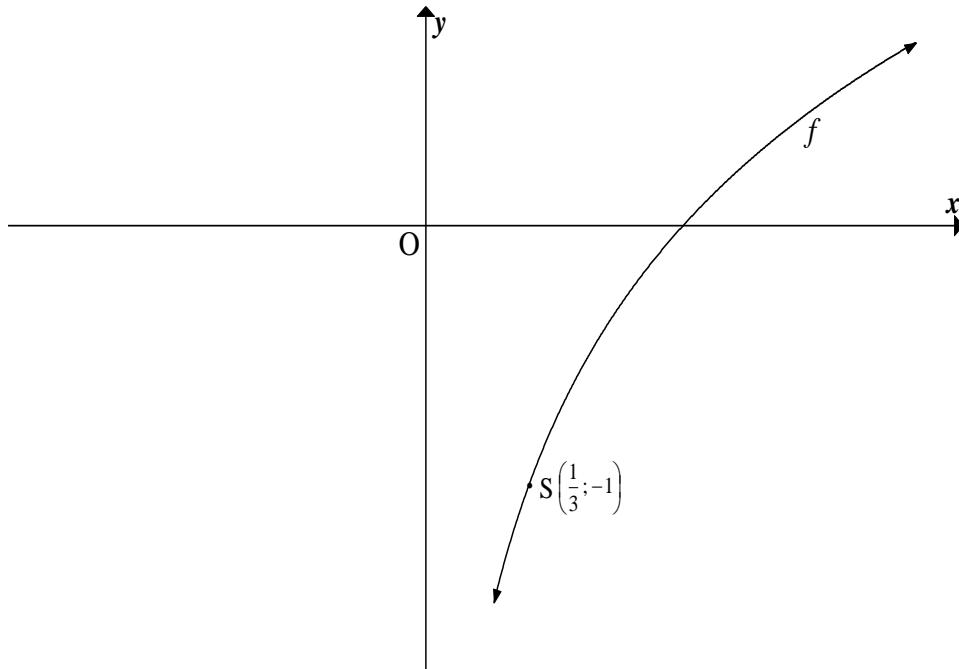
Die diagram hieronder duis die hiperbool g , gedefinieer deur $g(x) = \frac{2}{x+p} + q$ met asimptote $y = 1$ en $x = -1$ aan. Die grafiek van g sny die x -as by T en die y -as by $(0; 3)$. Die lyn $y = x$ sny die hiperbool in die eerste kwadrant by punt S .



- 4.1 Skryf die waardes van p en q neer. (2)
- 4.2 Bereken die x -koördinaat van T . (2)
- 4.3 Skryf die vergelyking van die vertikale asimptoot van die grafiek van h neer, indien $h(x) = g(x+5)$ (1)
- 4.4 Bereken die lengte van OS . (5)
- 4.5 Vir watter waardes van k sal die vergelyking $g(x) = x + k$ twee reële wortels hê wat teenoorgestelde tekens het? (1)
[11]

VRAAG 5

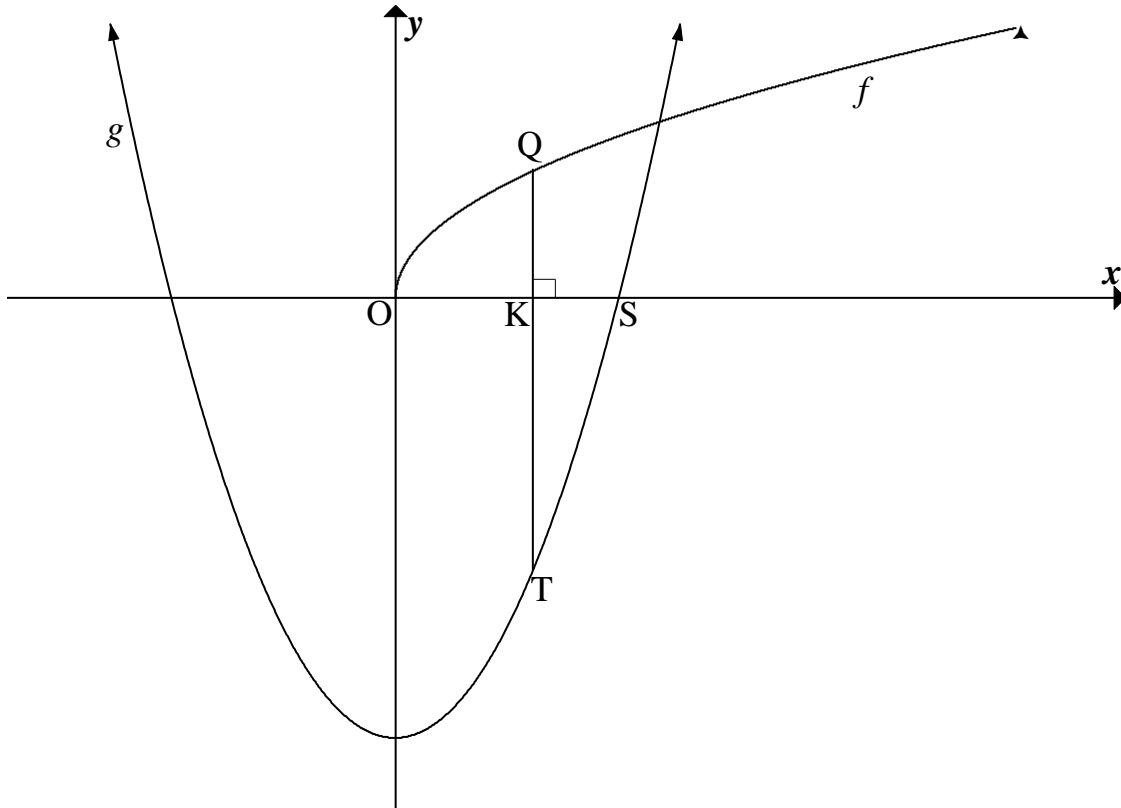
Gegee: $f(x) = \log_a x$ waar $a > 0$. S $\left(\frac{1}{3}; -1\right)$ is 'n punt op die grafiek van f .



- 5.1 Bewys dat $a = 3$. (2)
- 5.2 Skryf die vergelyking neer van h , die inverse van f , in die vorm $y = \dots$ (2)
- 5.3 Indien $g(x) = -f(x)$, bepaal die vergelyking van g . (1)
- 5.4 Skryf die definisieversameling van g neer. (1)
- 5.5 Bepaal die waardes van x waarvoor $f(x) \geq -3$. (3)
[9]

VRAAG 6

Gegee: $g(x) = 4x^2 - 6$ en $f(x) = 2\sqrt{x}$. Die grafieke van g en f is hieronder geskets. S is 'n x -afsnit van g en K is 'n punt tussen O en S . Die reguitlyn QKT met Q op die grafiek van f en T op die grafiek van g , is parallel aan die y -as.



- 6.1 Bepaal die x -koördinaat van S , korrek tot TWEE desimale plekke. (2)
- 6.2 Skryf die koördinate van die draaipunt van g neer. (2)
- 6.3
 - 6.3.1 Skryf die lengte van QKT in terme van x neer, waar x die x -koördinaat van K is. (3)
 - 6.3.2 Bereken die maksimum lengte van QT. (6)
[13]

VRAAG 7

- 7.1 Presies vyf jaar gelede het Mpume 'n nuwe motor vir R145 000 gekoop. Die huidige boekwaarde van die motor is R72 500. Indien die motor se waarde teen 'n vaste jaarlikse koers volgens die verminderendesaldo-metode depresieer, bereken die depresiasiekoers. (3)
- 7.2 Samuel het 'n huislening van R500 000 uitgeneem teen 'n rentekoers van 12% per jaar, maandeliks saamgestel. Hy beplan om hierdie lening oor 20 jaar terug te betaal, en sy eerste betaling word een maand nadat die lening toegestaan is, gemaak.
- 7.2.1 Bereken die waarde van Samuel se maandelikse paaiement. (4)
- 7.2.2 Melissa het 'n lening vir dieselfde bedrag en teen dieselfde rentekoers as Samuel uitgeneem. Melissa het besluit om R6 000 aan die einde van elke maand terug te betaal. Bereken hoeveel maande dit Melissa geneem het om haar lening volledig af te betaal. (4)
- 7.2.3 Wie betaal meer rente, Samuel of Melissa? Motiveer jou antwoord. (2)
[13]

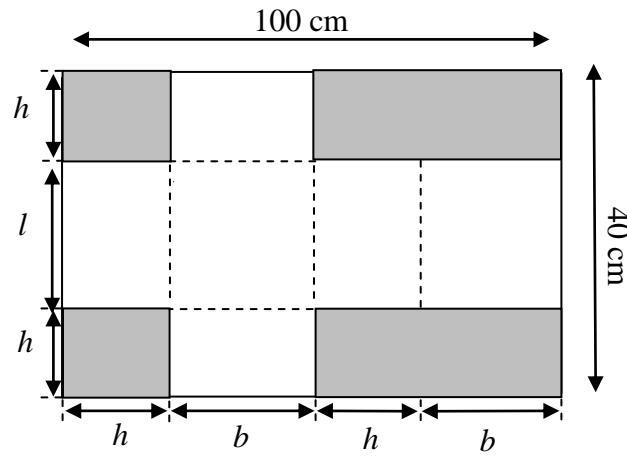
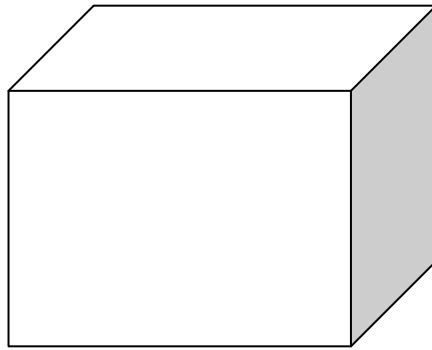
VRAAG 8

- 8.1 Bepaal $f'(x)$ vanuit eerste beginsels indien $f(x) = x^3$. (5)
- 8.2 Bepaal die afgeleide van: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ (2)
- 8.3 Indien $y = (x^6 - 1)^2$, bewys dat $\frac{dy}{dx} = 12x^5\sqrt{y}$, indien $x > 1$. (3)
- 8.4 Gegee: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Bepaal die interval waarop f konkaaf op is. (4)
[14]

VRAAG 9

Gegee: $f(x) = (x+2)(x^2 - 6x + 9)$
 $= x^3 - 4x^2 - 3x + 18$

- 9.1 Bereken die koördinate van die draapunte van die grafiek van f . (6)
- 9.2 Skets die grafiek van f en dui die afsnitte met die asse en die draapunte duidelik aan. (4)
- 9.3 Vir watter waarde(s) van x sal $x \cdot f'(x) < 0$? (3)
[13]

VRAAG 10

'n Boks word van 'n reghoekige stuk karton gemaak, 100 cm by 40 cm, deur die ingekleurde dele uit te sny en op die stipellyne, soos in die diagram hierbo aangedui, te vou.

- 10.1 Druk die lengte l in terme van die hoogte h uit. (1)
- 10.2 Bewys gevvolglik dat die volume van die boks gegee word deur $V = h(50-h)(40-2h)$ (3)
- 10.3 Vir watter waarde van h sal die volume van die boks 'n maksimum wees? (5)
[9]

VRAAG 11

'n Opname oor die vakansievoorseure van 180 personeellede is gedoen. Die opsies waaruit hulle kon kies, was om:

- Kus toe te gaan
- 'n Wildreservaat te besoek
- Tuis te bly

Die resultate is in die tabel hieronder aangeteken:

	Kus	Wild-reservaat	Tuis	Totaal
Manlik	46	24	13	83
Vroulik	52	38	7	97
Totaal	98	62	20	180

11.1 Bepaal die waarskynlikheid dat 'n personeellid wat ewekansig gekies is:

11.1.1 Manlik sal wees (1)

11.1.2 Verkies om nie 'n wildreservaat te besoek nie (2)

11.2 Is die gebeure 'manlik' en 'tuis bly' onafhanklike gebeure? Motiveer jou antwoord met nodige bewerkings.

(4)

[7]

VRAAG 12

12.1 'n Kodewoord bestaan uit vyf verskillende letters van die Engelse alfabet. Elke letter mag slegs een maal gebruik word. Hoeveel kodewoorde kan gevorm word indien:

12.1.1 Al die letters in die alfabet gebruik kan word (2)

12.1.2 Die kodewoord moet begin en moet eindig (2)

12.2 Sewe motors van verskillende vervaardigers, waarvan 3 silwer is, moet in 'n reguitlyn geparkeer word.

12.2.1 Op hoeveel verskillende maniere kan AL die motors geparkeer word? (2)

12.2.2 Indien die drie silwer motors langs mekaar geparkeer moet word, bepaal op hoeveel verskillende maniere die motors geparkeer kan word. (3)

[9]

TOTAAL: 150

INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni)$$

$$A = P(1-ni)$$

$$A = P(1-i)^n$$

$$A = P(1+i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1}; r \neq 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{oppervlakte } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE/GRAAD 12

MATHEMATICS P1/WISKUNDE VI

NOVEMBER 2014

MEMORANDUM

MARKS: 150

PUNTE: 150

**This memorandum consists of 22 pages.
Hierdie memorandum bestaan uit 22 bladsye.**

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in all aspects of the marking memorandum.

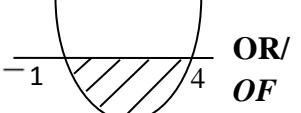
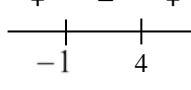
LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die memorandum van toepassing.*

QUESTION/VRAAG 1

1.1.1	$(x - 2)(4 + x) = 0$ $x = 2 \quad \text{or} \quad x = -4$	$\checkmark x = 2$ $\checkmark x = -4$ (2)
1.1.2	$3x^2 - 2x - 14 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-14)}}{2(3)}$ $= \frac{2 \pm \sqrt{172}}{6}$ $x = 2,52 \quad \text{or/of} \quad x = -1,85$	\checkmark standard form/standaardvorm \checkmark substitution into correct formula/ substitusie in korrekte formule $\checkmark \checkmark$ answers/ antwoorde (4)
	OR/OF $x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{14}{3} + \frac{1}{9}$ $\left(x - \frac{1}{3}\right)^2 = \frac{43}{9}$ $x - \frac{1}{3} = \pm \frac{\sqrt{43}}{3}$ $\therefore x = \frac{1 \pm \sqrt{43}}{3}$ $x = 2,52 \quad \text{or/of} \quad x = -1,85$	\checkmark for adding $\frac{1}{9}$ on both sides/tel $\frac{1}{9}$ by aan beide kante $\checkmark x = \frac{1 \pm \sqrt{43}}{3}$ $\checkmark \checkmark$ answers (4)

<p>1.1.3</p> $2^{x+2} + 2^x = 20$ $2^x(2^2 + 1) = 20$ $2^x = \frac{20}{5}$ $2^x = 2^2$ $\therefore x = 2$	<p>✓ common factor/gemeen faktor ✓ simplification/vereenvoudiging ✓ answer/antwoord (3)</p>
<p>OR/OF</p> $2^x \cdot 2^2 + 2^x = 2^2 \cdot 5$ $2^x(2^2 + 1) = 2^2 \cdot 5$ $2^x \cdot 5 = 2^2 \cdot 5$ $\therefore x = 2$	<p>✓ common factor/gemeen faktor ✓ simplification/vereenvoudiging ✓ answer/antwoord (3)</p>
<p>OR/OF</p> $4 \cdot 2^x + 2^x = 20$ $5 \cdot 2^x = 20$ $2^x = 4 = 2^2$ $\therefore x = 2$	<p>✓ $5 \cdot 2^x = 20$ ✓ $2^x = 4$ ✓ answer/antwoord (3)</p>
<p>1.2</p> $x = 2y + 3 \quad \dots\dots\dots(1)$ $3x^2 - 5xy = 24 + 16y \quad \dots\dots\dots(2)$ <p>(1) in (2):</p> $3(2y + 3)^2 - 5(2y + 3)y = 24 + 16y$ $3(4y^2 + 12y + 9) - 10y^2 - 15y = 24 + 16y$ $12y^2 + 36y + 27 - 10y^2 - 15y - 24 - 16y = 0$ $2y^2 + 5y + 3 = 0$ $(2y + 3)(y + 1) = 0$ $y = -\frac{3}{2} \quad \text{or} \quad y = -1$ $\therefore x = 2\left(-\frac{3}{2}\right) + 3 \quad \text{or} \quad x = 2(-1) + 3$ $x = 0 \quad \text{or} \quad x = 1$ $(0; -\frac{3}{2}) \quad \quad \quad (1; -1)$	<p>✓ substitution/substitusie ✓ simplification/vereenvoudiging ✓ standard form/standaardvorm ✓ factorisation/faktorisering ✓ y-values/y-waardes ✓ x-values/x-waardes (6)</p>

	$y = \frac{x-3}{2}$ $3x^2 - 5x\left(\frac{x-3}{2}\right) = 24 + 16\left(\frac{x-3}{2}\right)$ $3x^2 - \frac{5x^2 - 15x}{2} = 24 + \frac{16x - 48}{2}$ $\times 2: 6x^2 - 5x^2 + 15x = 48 + 16x - 48$ $x^2 - x = 0$ $x(x-1) = 0$ $x = 0 \text{ or } x = 1$ $y = -\frac{3}{2} \text{ or } y = -1$	✓ substitution/ <i>substitusie</i> ✓ simplification/ <i>vereenvoudiging</i> ✓ standard form / <i>standard vorm</i> ✓ factors/ <i>faktore</i> ✓ x- values/ <i>x-waardes</i> ✓ y-values/ <i>y-waardes</i> (6)
1.3	$(x-1)(x-2) < 6$ $x^2 - 3x + 2 < 6$ $x^2 - 3x - 4 < 0$ $(x+1)(x-4) < 0$  OR/OF  $-1 < x < 4 \text{ or } x \in (-1; 4)$	✓ standard form/ <i>standaardvorm</i> ✓ factorisation/ <i>faktorisering</i> ✓ critical values in the context of inequality / <i>kritiese waardes in die konteks van die ongelykheid</i> ✓ notation/ <i>notasie</i> (4)
1.4	$-k - 4 \geq 0$ $k \leq -4$	✓ $-k - 4 \geq 0$ ✓ answer/ <i>antwoord</i> (2) [21]

QUESTION/VRAAG 2

2.1	$T_4 = 23$	✓ 23 (1)
2.2	$\begin{aligned} T_{251} &= a + (n-1)d \\ &= 2 + (251-1)(7) \\ &= 1752 \end{aligned}$	✓ $a = 2$ and $d = 7$ ✓ subst. into correct formula /subt. in korrekte formule ✓ 1752 (3)
2.3	$\sum_{n=1}^{251} (7n-5)$ <p>OR/OF</p> $\sum_{p=0}^{250} (7p+2)$	✓ general term/ algemene term ✓ complete answer /volledige antwoord (2) ✓ general term/ algemene term ✓ complete answer / volledige antwoord (2)
2.4	$\begin{aligned} S_n &= \frac{n}{2}[a + l] \\ S_n &= \frac{251}{2}[2 + 1752] \\ &= 220127 \end{aligned}$ <p>OR/OF</p> $\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ &= \frac{251}{2}[2(2) + (251-1)(7)] \\ &= 220127 \end{aligned}$	✓ substitution/substitusie ✓ 220127 (2) ✓ substitution/substitusie ✓ 220127 (2)
2.5	<p>The new series/Die nuwe reeks is $16 + 44 + 72 + \dots + 1752$</p> $16 + 28(n-1) = 1752$ $1736 = 28(n-1)$ $62 = n - 1$ $n = 63$ <p>OR/OF</p> $2 + 9 + \underline{16} + 23 + 30 + 37 + \underline{44} + 51 + \dots + \underline{1752}$ <p>T_3 is divisible by /is deelbaar deur 4</p> <p>Then $T_7, T_{11}, T_{15}, \dots, T_{251}$ are divisible by 4, thus each 4th term is divisible by 4.</p> <p>Daarna is $T_7, T_{11}, T_{15}, \dots, T_{251}$ deelbaar deur 4, d.w.s. elke 4^{de} term is deelbaar deur 4.</p> $\therefore \text{number of terms divisible by 4 will be } = \frac{251-3}{4} + 1 = 63$ $\therefore \text{aantal terme deelbaar deur 4 sal wees } = \frac{251-3}{4} + 1 = 63$ <p>OR/OF</p>	✓✓ generating new series divisible by 4/ vorming van nuwe reeks deelbaar deur 4 ✓ $T_n = 1752$ ✓ 63 (4) ✓ T_3 is divisible by 4/ is deelbaar deur 4 ✓ identifying terms divisible by 4/ identifiseer terme deelbaar deur 4 ✓ reasoning/redenering ✓ 63 (4)

	<p>Position of terms divisible by 4: 3 ; 7 ; 11 ; ...; 247; 251 $T_n = 4n - 1 = 251$ $4n = 252$ $n = 63$</p>	<p>✓✓ generating sequence involving position of terms/<i>vorming van reeks i.t.v. posisie van terme</i> ✓ $T_n = 251$ ✓ 63 (4)</p>
		[12]

QUESTION/VRAAG 3

3.1.1	$\begin{array}{ccccccc} -1 & ; & -7 & ; & -11 & ; & p \quad ; \dots \\ \swarrow & \searrow & \swarrow & & \swarrow & & \\ -6 & & -4 & & p+11 & & \\ & \swarrow & \searrow & & & & \\ & 2 & & 2 & & & \\ p+11 - (-4) & = 2 & & & & & \\ p+15 & = 2 & & & & & \\ p & = -13 & & & & & \end{array}$ <p>OR/OF</p> $\begin{array}{ccccccc} -1 & ; & -7 & ; & -11 & ; & p \quad ; \dots \\ \swarrow & \searrow & \swarrow & & \swarrow & & \\ -6 & & -4 & & p+11 & & \\ & \swarrow & \searrow & & & & \\ & 2 & & 2 & & & \\ p+11 & = -2 & & & & & \\ p & = -13 & & & & & \end{array}$	$\checkmark p + 15 = 2$ $\checkmark p = -13$ (2)
3.1.2	$2a = 2$ $a = 1$ $3a + b = -6$ $3(1) + b = -6$ $b = -9$ $a + b + c = -1$ $1 - 9 + c = -1$ $c = 7$ $T_n = n^2 - 9n + 7$	$\checkmark a = 1$ $\checkmark b = -9$ $\checkmark c = 7$ $\checkmark \text{answer/antwoord}$ (4)
	<p>OR/OF</p> $\begin{aligned} T_n &= T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2} \\ &= -1 + (n-1)(-6) + \frac{(n-1)(n-2)(2)}{2} \\ &= -1 - 6n + 6 + \frac{2n^2 - 6n + 4}{2} \\ &= n^2 - 9n + 7 \end{aligned}$	$\checkmark \text{formula/formule}$ $\checkmark \text{substitution of first and second differences}/\text{substitusie van eerste en tweede verskille}$ $\checkmark \text{simplification/vereenvoudiging}$ $\checkmark \text{answer/antwoord}$ (4)

<p>OR/OF</p> $7; -1 ; -7 ; -11 ; p ; \dots$ $\begin{array}{ccccccc} & \swarrow & \searrow & \swarrow & \searrow & \\ -8 & & -6 & & -4 & & p+11 \\ & \searrow & \swarrow & \searrow & \swarrow & & \\ & 2 & & 2 & & 2 & \end{array}$ $T_0 = 7 = c$ $2a = 2 \therefore a = 1$ $3a + b = -6 \therefore b = -9$ $T_n = n^2 - 9n + 7$ <p>OR/OF</p> $a = \frac{1}{2}(2) = 1$ $\therefore T_n = n^2 + bn + c$ $T_1 = -1 \therefore 1 + b + c = -1 \dots\dots(1)$ $T_2 = -7 \therefore 4 + 2b + c = -7 \dots\dots(2)$ $(2) - (1): 3 + b = -6$ $\therefore b = -9$ $\text{sub in (1): } c = 7$ $\therefore T_n = n^2 - 9n + 7$	<p>✓ <i>c</i>-value/<i>c</i>-waarde ✓ <i>a</i>-value/<i>a</i>-waarde ✓ <i>b</i>-value/<i>b</i>-waarde</p> <p>✓ answer/antwoord</p> <p style="text-align: right;">(4)</p> <p>✓ <i>a</i>-value/<i>a</i>-waarde</p> <p>✓ <i>b</i>-value/<i>b</i>-waarde ✓ <i>c</i>-value/<i>c</i>-waarde</p> <p>✓ answer/antwoord</p> <p style="text-align: right;">(4)</p>
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3.1.3	<p>The sequence of first differences is/<i>Die reeks van eerste verskille is:</i></p> $-6 ; -4 ; -2 ; 0 ; \dots$ $-6+(n-1)(2) = 96$ $n = 52$ <p>∴ two terms are/<i>twee terme is:</i></p> $T_{52} = 52^2 - 9(52) + 7 = 2243$ $T_{53} = 53^2 - 9(53) + 7 = 2339$ <p>OR/OF</p> <p>The sequence of first differences is/<i>Die reeks van eerste verskille is:</i></p> $-6 ; -4 ; -2 ; 0 ; \dots$ <p>The formula for the sequence of first differences/<i>Die formule vir die reeks van eerste verskille</i> is $T_n = 2n - 8$</p> <p>1st difference/1^{ste} verskil: $2n - 8 = 96$</p> $2n = 104$ $n = 52$ <p>∴ two terms are/<i>twee terme is:</i></p> $T_{52} = 52^2 - 9(52) + 7 = 2243$ $T_{53} = 53^2 - 9(53) + 7 = 2339$ <p>OR/OF</p> $T_n - T_{n-1} = 96$ $(n^2 - 9n + 7) - [(n-1)^2 - 9(n-1) + 7] = 96$ $n^2 - 9n + 7 - n^2 + 2n - 1 + 9n - 9 - 7 = 96$ $2n = 106$ $n = 53$ $T_{52} = 52^2 - 9(52) + 7 = 2243$ $T_{53} = 53^2 - 9(53) + 7 = 2339$	$\checkmark -6+(n-1)(2) = 96$ $\checkmark 52$ $\checkmark 2\ 243$ $\checkmark 2\ 339$ (4)
	<p>OR/OF</p> $T_{n+1} - T_n = 96$ $[(n+1)^2 - 9(n+1) + 7] - [n^2 - 9n + 7] = 96$ $n^2 + 2n + 1 - 9n - 9 + 7 - n^2 + 9n - 7 = 96$ $2n = 104$ $n = 52$ $T_{52} = 52^2 - 9(52) + 7 = 2243$ $T_{53} = 53^2 - 9(53) + 7 = 2339$	$\checkmark T_{n+1} - T_n = 96$ $\checkmark 52$ $\checkmark 2\ 243$ $\checkmark 2\ 339$ (4)

3.2.1	$T_{12} = 16 \left(\frac{1}{4} \right)^{12-1}$ $= \frac{1}{4^9} \quad \text{or} \quad 4^{-9} \quad \text{or} \quad \frac{1}{2^{18}} \quad \text{or} \quad 2^{-18}$	✓ $a = 16$ and $r = \frac{1}{4}$ ✓ subst. into correct formula/ <i>subt in korrekte formule</i> ✓ answer/antwoord (3)
3.2.2	$S_{10} = \frac{16 \left(1 - \left(\frac{1}{4} \right)^{10} \right)}{1 - \frac{1}{4}}$ $= 21,33$ <p>OR/OF</p> $S_{10} = \frac{16 \left(\left(\frac{1}{4} \right)^{10} - 1 \right)}{\frac{1}{4} - 1}$ $= 21,33$	✓ substitution into correct formula / <i>substitusie in korrekte formule</i> ✓ answer/antwoord (2)
3.3	$\left(1 + \frac{1}{2} \right) \left(1 + \frac{1}{3} \right) \left(1 + \frac{1}{4} \right) \dots \left(1 + \frac{1}{99} \right)$ $= \left(\frac{3}{2} \right) \left(\frac{4}{3} \right) \left(\frac{5}{4} \right) \left(\frac{6}{5} \right) \dots \left(\frac{100}{99} \right)$ $= \left(\frac{100}{2} \right)$ $= 50$ <p>OR/OF</p> $\left(1 + \frac{1}{2} \right) \left(1 + \frac{1}{3} \right) \left(1 + \frac{1}{4} \right) \dots \left(1 + \frac{1}{99} \right)$ $T_1 = \left(1 + \frac{1}{2} \right) = \frac{3}{2}$ $T_2 = \frac{3}{2} \left(1 + \frac{1}{3} \right) = \frac{3}{2} \times \frac{4}{3} = 2$ $T_3 = 2 \left(1 + \frac{1}{4} \right) = 2 \times \frac{5}{4} = \frac{5}{2}$ $\frac{3}{2}, 2, \frac{5}{2} \dots \text{ is an arithmetic sequence with } a = \frac{3}{2} \text{ and } d = \frac{1}{2}$ $\therefore T_{98} = \frac{3}{2} + (98-1) \frac{1}{2}$ $= \frac{100}{2} = 50$	✓ improper fractions/ <i>onechte breuke</i> ✓ $\left(1 + \frac{1}{99} \right)$ or $\left(\frac{100}{99} \right)$ ✓✓ answer/antwoord (4)

QUESTION/VRAAG 4

4.1	$p = 1$ $q = 1$	✓ p value /waarde ✓ q value /waarde (2)
4.2	$0 = \frac{2}{x+1} + 1$ $-x - 1 = 2$ $x = -3$ OR/OF Reflect $(0 ; 3)$ across $y = -x$ to get $T(-3 ; 0)$ $x = -3$ <i>Reflekteer $(0 ; 3)$ om $y = -1$ om $T(-3 ; 0)$ te kry</i> $x = -3$	✓ $0 = \frac{2}{x+1} + 1$ ✓ $x = -3$ (2) ✓ reflect across/ <i>reflekteer om</i> $y = -x$ ✓ $x = -3$ (2)
4.3	Shifting g five units to the left shifts $(-1 ; 0)$ five units to the left. $x = -6$	✓ answer/antwoord (1)
4.4	$\frac{2}{x+1} + 1 = x$ $2 + x + 1 = x^2 + x$ $x^2 = 3$ $\therefore x = \sqrt{3}$ since at S, $x > 0$ $y = \sqrt{3} = 1,73\dots$ $OS^2 = x^2 + y^2 = 3 + 3 = 6$ $\therefore OS = \sqrt{6} = 2,45$ units/eenhede OR/OF	✓ equating both graphs/stel grafiese gelyk ✓ $x^2 = 3$ ✓ $x = \sqrt{3}$ and $y = \sqrt{3}$ ✓ $OS^2 = 6$ ✓ answer/antwoord (5)

	<p>Translate g one unit down and one unit to the right/<i>Transleer g een eenheid af en een eenheid na regs</i></p> <p>The new equation/<i>Die nuwe vergelyking</i> : $p(x) = \frac{2}{x}$</p> <p>Therefore the image of S is $S'(\sqrt{2}; \sqrt{2})$/ <i>Daarom is die beeld van S nou $S'(\sqrt{2}; \sqrt{2})$</i></p> <p>Now translate p back to g/<i>Transleer p terug na g</i>: $S(\sqrt{2} - 1; \sqrt{2} + 1)$</p> $OS^2 = (\sqrt{2} - 1)^2 + (\sqrt{2} + 1)^2 = 2 - 2\sqrt{2} + 1 + 2 + 2\sqrt{2} + 1$ $\therefore OS = \sqrt{6} = 2,45 \text{ units/eenhede}$	<p>✓ $p(x) = \frac{2}{x}$</p> <p>✓✓ coord. of/koörd. van S'</p> <p>✓ coord. of/koörd. van S</p> <p>✓ answer/antwoord (5)</p>
4.5	<p>$k < 3$ will give roots with opposite signs/ <i>$k < 3$ sal wortels met teenoorgestelde tekens gee</i></p>	<p>✓ $k < 3$ (1)</p> <p>[11]</p>

QUESTION 5

5.1	$y = \log_a x$ $-1 = \log_a \frac{1}{3}$ $a^{-1} = \frac{1}{3}$ $a = \left(\frac{1}{3}\right)^{-1}$ $\therefore a = 3$	✓ subt. $\left(\frac{1}{3}; -1\right)$ ✓ $a^{-1} = \frac{1}{3}$ or $a = \left(\frac{1}{3}\right)^{-1}$ (2)
5.2	$h: x = \log_3 y$ $\therefore y = 3^x$	✓ swop x and y /ruil x en y ✓ answer/antwoord (2)
5.3	$g(x) = -\log_3 x$ OR/OF $g(x) = \log_3 \frac{1}{x}$ OR/OF $g(x) = \log_{\frac{1}{3}} x$ OR/OF $x = 3^{-y}$ OR/OF $x = \left(\frac{1}{3}\right)^y$	✓ answer/antwoord (1) ✓ answer/antwoord (1) ✓ answer/antwoord (1) ✓ answer/antwoord (1) ✓ answer/antwoord (1) ✓ answer/antwoord (1)
5.4	$x > 0$ OR/OF $(0; \infty)$	✓ answer/antwoord (1) ✓ answer/antwoord (1)
5.5	$\log_3 x = -3$ $x = 3^{-3}$ $x = \frac{1}{27}$ $x \geq \frac{1}{27}$	✓ exponential form/ eksponensiële vorm ✓ simplification/vereenvoudiging ✓ answer/antwoord (3) [9]

QUESTION/VRAAG 6

6.1	$4x^2 - 6 = 0$ $x^2 = \frac{3}{2}$ $x = 1,22$ (x -coordinate of S is positive)	$\checkmark y = 0$ $\checkmark 1,22$ (2)
6.2	(0 ; -6)	$\checkmark 0$ $\checkmark -6$ (2)
6.3.1	$QT = f(x) - g(x)$ $= 2\sqrt{x} - (4x^2 - 6)$ or $= 2\sqrt{x} - 4x^2 + 6$	$\checkmark \checkmark$ correct formula/ korrekte formule \checkmark substitution/substitusie (3)
6.3.2	$QT = 2x^{\frac{1}{2}} - 4x^2 + 6$ Derivative of $QT = x^{\frac{-1}{2}} - 8x = 0$ $\frac{1}{\sqrt{x}} = 8x$ $x^{\frac{3}{2}} = \frac{1}{8}$ or $\frac{1}{x} = 64x^2$ $x = \left(\frac{1}{8}\right)^{\frac{2}{3}}$ $x = \left(\frac{1}{2}\right)^2$ or $x^3 = \frac{1}{64}$ $x = \frac{1}{4} = 0,25$ Max/Maks $QT = 2\left(\frac{1}{4}\right)^{\frac{1}{2}} - 4\left(\frac{1}{4}\right)^2 + 6$ $= 6\frac{3}{4} = 6,75$ units/eenhede	\checkmark derivative/afgeleide \checkmark derivative equal to 0/ afgeleide gelyk aan 0 $\checkmark x^{\frac{3}{2}} = \frac{1}{8}$ $\checkmark x\text{-value}/x\text{-waarde}$ \checkmark substitution/substitusie \checkmark answer/antwoord (6) [13]

QUESTION/VRAAG 7

7.1	$A = P(1 - i)^n$ $72\ 500 = 145\ 000(1 - i)^5$ $i = 1 - \sqrt[5]{\frac{72500}{145000}}$ $= 0,1294\dots$ <p>∴ Rate of interest/Rentekoers is 12,94 % p.a./p.j.</p> <p>OR/OF</p> $(1 - i)^5 = \frac{1}{2}$ $\therefore i = 1 - \left(\frac{1}{2}\right)^{\frac{1}{5}}$ $i = 0,1294$ <p>∴ Rate of interest/Rentekoers is 12,94 % p.a./p.j.</p>	✓ substitution/substitusie ✓ writing in terms of i <i>herskryf in terme van i</i> ✓ answer/antwoord (3)
7.2.1	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $500\ 000 = \frac{x \left[1 - \left(1 + \frac{0,12}{12} \right)^{-240} \right]}{\frac{0,12}{12}}$ $x = \frac{500000 \times \frac{0,12}{12}}{\left[1 - \left(1 + \frac{0,12}{12} \right)^{-240} \right]}$ $x = \text{R}5505,43$	✓ $i = \frac{0,12}{12}$ ✓ $n = 240$ ✓ substitution into correct formula ✓ answer/antwoord (4)

7.2.2	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $500000 = \frac{6000 \left[1 - \left(1 + \frac{0,12}{12} \right)^{-n} \right]}{\frac{0,12}{12}}$ $\frac{500000}{6000} \times 0,01 = 1 - (1,01)^{-n}$ $(1,01)^{-n} = 1 - \frac{5}{6}$ $-n = \frac{\log \frac{1}{6}}{\log 1,01}$ $n = 180,07$ <p>\therefore Melissa settles the loan in 181 months</p>	✓ 6000 ✓ substitute into correct formula/ <i>substitusie in korrekte formule</i> ✓ use of logs/ <i>gebruik van logs</i> ✓ answer/ <i>antwoord</i> (4)
7.2.3	<p>Samuel He is paying off his loan over a longer period thus more interest will be paid./<i>Hy betaal sy lening oor 'n langer tydperk af, dus sal hy meer rente betaal.</i></p> <p>OR/OF</p> <p>Samuel He will pay/<i>Hy betaal</i> $R5505,43 \times 240 - R500\ 000 = R821\ 303,20$ She will pay between/<i>Sy sal tussen</i> R580 000 and/<i>en</i> R586 000,00 <i>betaal</i>.</p>	✓ Samuel ✓ reason/ <i>rede</i> (2) ✓ Samuel ✓ reason/ <i>rede</i> (2) [13]

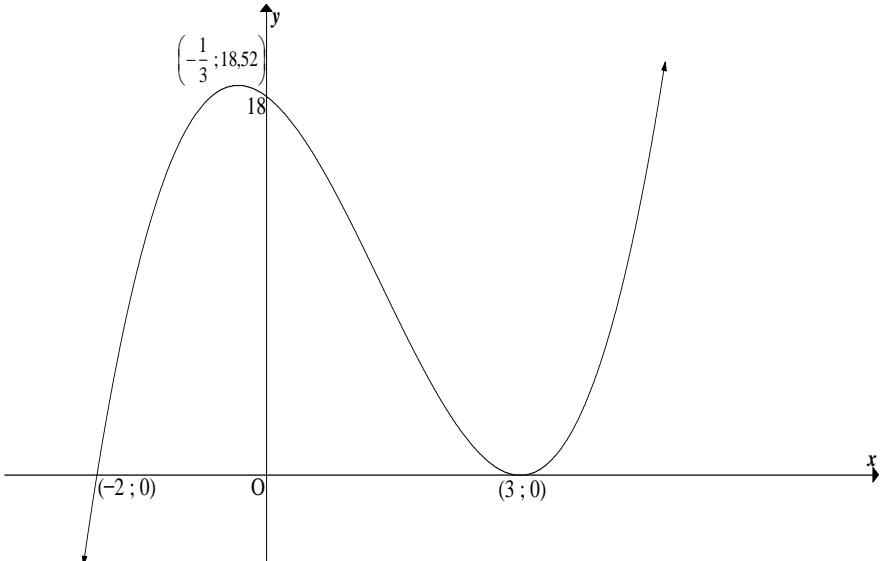
QUESTION/VRAAG 8

<p>8.1</p> $\begin{aligned} f(x+h) &= (x+h)^3 = (x^2 + 2xh + h^2)(x+h) \\ &= x^3 + x^2h + 2x^2h + 2xh^2 + h^2x + h^3 \\ &= x^3 + 3x^2h + 3xh^2 + h^3 \end{aligned}$ $\begin{aligned} f(x+h) - f(x) &= x^3 + 3x^2h + 3xh^2 + h^3 - x^3 \\ &= 3x^2h + 3xh^2 + h^3 \end{aligned}$ $\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) \\ &= 3x^2 \end{aligned}$	<p>✓ simplifying/vereenvoudiging</p> <p>✓ formula/formule</p> <p>✓ subst. into formula/subst. in formule</p> <p>✓ factorization/faktorisering</p> <p>✓ answer/antwoord</p> <p>(5)</p>
<p>OR/OF</p> $\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)(x+h)^2 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)(x^2 + 2xh + h^2) - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) \\ &= 3x^2 \end{aligned}$	<p>✓ formula/formule</p> <p>✓ subst. into formula/subst. in formule</p> <p>✓ simplifying/vereenvoudiging</p> <p>✓ factorization/faktorisering</p> <p>✓ answer/antwoord</p>
<p>OR</p>	<p>(5)</p>

	$ \begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h-x)(x^2 + 2xh + h^2 + x^2 + xh + h^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) \\ &= 3x^2 \end{aligned} $	✓ formula/formule ✓ subst. into formula/subst. in formule ✓ factorization/faktorisering ✓ simplifying/vereenvoudiging ✓ answer/antwoord (5)
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8.2	$f'(x) = 4x + 2x^3$	✓ $4x$ ✓ $2x^3$ (2)
8.3	$ \begin{aligned} y &= x^{12} - 2x^6 + 1 \\ \frac{dy}{dx} &= 12x^{11} - 12x^5 \\ &= 12x^5(x^6 - 1) \\ &= 12x^5\sqrt{y} \end{aligned} $	✓ simplification/vereenvoudiging ✓ derivative/afgeleide ✓ factors/faktore (3)
8.4	$ \begin{aligned} f(x) &= 2x^3 - 2x^2 + 4x - 1 \\ f'(x) &= 6x^2 - 4x + 4 \\ f''(x) &= 12x - 4 \\ f \text{ is concave up when} & \text{is konkaaf op as } f''(x) > 0 \\ \therefore 12x - 4 &> 0 \\ 12x &> 4 \\ x &> \frac{1}{3} \end{aligned} $	✓ first derivative/eerste afgeleide ✓ second derivative/tweede afgeleide ✓ $f''(x) > 0$ ✓ $x > \frac{1}{3}$ (4) [14]

QUESTION/VRAAG 9

9.1	$f'(x) = 3x^2 - 8x - 3 = 0$ $(3x + 1)(x - 3) = 0$ $x = -\frac{1}{3}$ or $x = 3$ $y = \frac{500}{27}$ (or $y = 18\frac{14}{27}$ or 18,52) $y = 0$ Turning points are/Draaipunte is $\left(-\frac{1}{3}; \frac{500}{27}\right)$ and $(3; 0)$	✓ derivative/afgeleide ✓ derivative/ afgeleide = 0 ✓ factors/faktore ✓ x-values/waardes ✓✓ each y- values/elke y-waarde (6)
9.2		✓ x-intercepts/afsnitte ✓ y-intercept/afsnit ✓ turning points/ draaipunte ✓ shape/vorm (4)
9.3	$x < -\frac{1}{3}$ or $0 < x < 3$ OR $(-\infty; -\frac{1}{3}) \cup (0; 3)$	✓ $x < -\frac{1}{3}$ ✓ both critical points/ beide kritieke-punte ✓ notation/notasie (3)

QUESTION/VRAAG 10

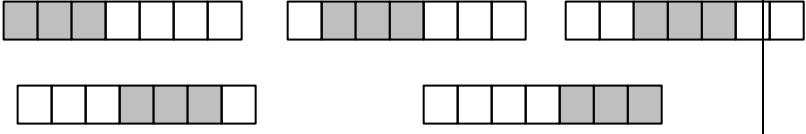
10.1	$\begin{aligned}l + 2h &= 40 \\ l &= 40 - 2h\end{aligned}$	✓ answer (1)
10.2	$\begin{aligned}2b + 2h &= 100 \\ b &= 50 - h \\ V &= lbh \\ V &= h(40 - 2h)(50 - h)\end{aligned}$	✓ $2b + 2h = 100$ ✓ $b = 50 - h$ ✓ volume formula (3)
10.3	$\begin{aligned}V &= (50h - h^2)(40 - 2h) \\ V &= 2h^3 - 140h^2 + 2000h \\ V' &= 6h^2 - 280h + 2000 = 0 \\ h &= \frac{280 \pm \sqrt{(-280)^2 - 4(6)(2000)}}{2(6)} \\ h &\neq 37,86 \text{ or } h = 8,80 \\ \therefore \text{for a box as large as possible, } h &= 8,80 \text{ cm} \\ \text{vir die grootste moontlike boks} &= 8,80 \text{ cm}\end{aligned}$	✓ simplifying/vereenvoudig ✓ derivative / afgeleide ✓ ✓ h -values in any form / h -waardes in enige vorm ✓ answer/antwoord (5) [9]

QUESTION/VRAAG 11

11.1.1	$P(\text{male/manlik}) = \frac{83}{180}$ or 0,46 or 46,11%	✓ answer/antwoord (1)
11.1.2	$\begin{aligned}P(\text{not game park/nie wildreservaat}) &= 1 - P(\text{game park/wildreservaat}) \\ &= 1 - \frac{62}{180} \\ &= \frac{59}{90} \text{ or } 0,66 \text{ or } 65,56\% \\ \textbf{OR/OF} \\ P(\text{not game park/nie wildreservaat}) &= \frac{98}{180} + \frac{20}{180} \\ &= \frac{118}{180} \\ &= \frac{59}{90} \text{ or } 0,66 \text{ or } 65,56\%\end{aligned}$	✓ $1 - \frac{62}{180}$ ✓ answer/antwoord (2) ✓ $\frac{98}{180} + \frac{20}{180}$ ✓ answer/antwoord (2)

11.2	<p>Events are independent if <i>/Gebeure is onafhanklike indien</i> $P(\text{male}) \times P(\text{home}) = P(\text{male and home})$ $P(\text{manlik}) \times P(\text{huis}) = P(\text{manlik en huis})$</p> $P(\text{male/manlik}) = \frac{83}{180}$ <p>and/en $P(\text{home/huis}) = \frac{20}{180}$ or 0,11 or 11,11%</p> $P(\text{male/manlik}) \times P(\text{home/huis})$ $= \frac{83}{180} \times \frac{20}{180}$ $= \frac{83}{1620}$ $= 0,05123 \text{ or } 5,12\%$ <p>$P(\text{male and home/manlik en huis})$</p> $= \frac{13}{180}$ $= 0,07222\dots \text{ or } 7,22\%$ <p>Therefore $P(\text{male}) \times P(\text{home}) \neq P(\text{male and home})$ Dus $P(\text{manlik}) \times P(\text{huis}) \neq P(\text{manlik en huis})$ Thus the events are not independent./<i>Dus is die gebeure nie onafhanklik nie</i></p> <p>OR/OF</p> <table border="1" data-bbox="244 1201 981 1387"> <thead> <tr> <th></th><th>Home/<i>Huis</i></th><th>Not Home/<i>Nie huis</i></th><th></th></tr> </thead> <tbody> <tr> <td>M</td><td>13</td><td>70</td><td>83</td></tr> <tr> <td>F</td><td>7</td><td>90</td><td>97</td></tr> <tr> <td></td><td>20</td><td>160</td><td>180</td></tr> </tbody> </table> <p>$P(\text{female/vroulik}) \times P(\text{not home/nie huis})$</p> $= \frac{97}{180} \times \frac{160}{180}$ $= \frac{194}{405}$ $= 0,479012345\dots \text{ or } 47,90\%$ <p>$P(\text{female and not home/vroulik en nie-huis})$</p> $= \frac{90}{180}$ $= 0,5 \text{ or } 50\%$ <p>Therefore $P(\text{female}) \times P(\text{not home}) \neq P(\text{female and not home})$ Thus the events are not independent. <i>Dus P(vroulik) \times P(\text{nie-huis}) \neq P(\text{vroulik en nie-huis})</i> <i>Dus is die gebeure nie onafhanklik nie.</i></p>		Home/ <i>Huis</i>	Not Home/ <i>Nie huis</i>		M	13	70	83	F	7	90	97		20	160	180	<p>✓ $P(m) \times P(h)$ and their values/en hulle waardes</p> <p>✓ answer of product</p> <p>✓ $P(m \text{ and/en } h)$ value/waarde</p> <p>✓ conclusion/afleiding (4)</p> <p>✓ $P(f) \times P(\text{not } h)$ and their values/en hulle waardes</p> <p>✓ answer of product</p> <p>✓ $P(f \text{ and/en not } h)$ value/waarde</p> <p>✓ conclusion/afleiding (4)</p> <p>[7]</p>
	Home/ <i>Huis</i>	Not Home/ <i>Nie huis</i>																
M	13	70	83															
F	7	90	97															
	20	160	180															

QUESTION/VRAAG 12

12.1.1	$26 \times 25 \times 24 \times 23 \times 22 \\ = 7\ 893\ 600$ <p>OR/OF</p> ${}^{26}P_5 = \frac{26!}{(26-5)!} = \frac{26!}{21!} = 7\ 893\ 600$	✓ $26 \times 25 \times 24 \times 23 \times 22$ ✓ 7 893 600 (2) ✓ formula/formule ✓ answer/antwoord (2)
12.1.2	$24 \times 23 \times 22 \\ = 12\ 144$	✓ $24 \times 23 \times 22$ ✓ 12 144 (2)
12.2.1	$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ = 5\ 040$	✓ product/produk ✓ 5 040 (2)
12.2.2	$(3 \times 2 \times 1)(5 \times 4 \times 3 \times 2 \times 1) \\ = 720$ <p>OR/OF</p> <p>The five 'units' can be parked in $5 \times 4 \times 3 \times 2 \times 1$ ways./Die vyf 'eenhede' kan op $5 \times 4 \times 3 \times 2 \times 1$ maniere geparkeer word.</p> <p>The three silver cars can be parked in $3 \times 2 \times 1$ ways./Die drie silwer motors kan op $3 \times 2 \times 1$ maniere parkeer word.</p> <p>So there are $(3 \times 2 \times 1)(5 \times 4 \times 3 \times 2 \times 1) = 720$ ways to park the cars./Dus is daar $(3 \times 2 \times 1)(5 \times 4 \times 3 \times 2 \times 1) = 720$ maniere om die motors te parkeer.</p> <p>OR/OF</p> <p>Suppose for the moment the 3 silver cars are at one end./Veronderstel die drie silwer motors is op die punt.</p> <p>The 3 cars can be arranged in $3 \times 2 \times 1 = 6$ ways./Die 3 motors kan op $3 \times 2 \times 1 = 6$ maniere gerangskik word.</p> <p>For each of them the remaining four cars can be arranged in $4 \times 3 \times 2 \times 1 = 24$ ways./Die 4 oorblywende motors kan op $4 \times 3 \times 2 \times 1 = 24$ maniere rangskik word.</p> <p>So $6 \times 24 = 144$ ways if all 3 cars at one end./Dus is daar $6 \times 24 = 144$ maniere as die 3 motors op die punt is.</p> 	✓ $3 \times 2 \times 1$ ✓ $5 \times 4 \times 3 \times 2 \times 1$ ✓ 720 (3) ✓ $5 \times 4 \times 3 \times 2 \times 1$ ✓ $3 \times 2 \times 1$ ✓ 720 (3) ✓ $6 \times 24 = 144$

TOTAL/TOTAAL: 150



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

NOVEMBER 2014

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 14 bladsye, 6 diagramvelle en 1 inligtingblad.

INSTRUKSIES EN INLIGTING

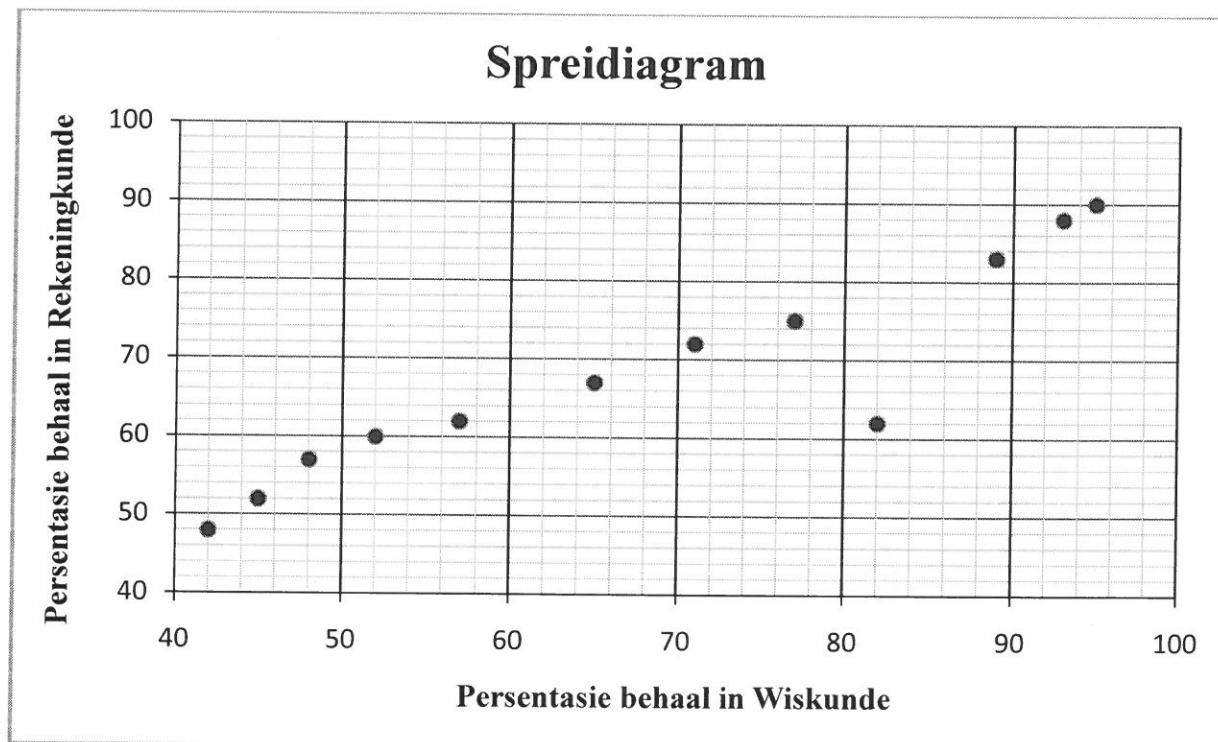
Lees die volgende instruksies aandagtig deur voordat die vrae beantwoord word.

1. Hierdie vraestel bestaan uit 10 vrae.
2. Beantwoord AL die vrae.
3. Dui ALLE berekeninge, diagramme, grafieke, ensovoorts wat jy gebruik het om die antwoorde te bepaal, duidelik aan.
4. Volpunte sal NIE noodwendig aan antwoorde alleen toegeken word NIE.
5. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.
6. Indien nodig, rond antwoorde af tot TWEE desimale plekke, tensy anders vermeld.
7. SES diagramvelle vir VRAAG 2.2.1, 2.2.2, 7.4, 8.1, 8.2, 8.3, 9.1, 9.2 en 10 is aan die einde van hierdie vraestel aangeheg. Skryf jou sentrumnommer en eksamennommer op hierdie bladsye in die ruimtes wat voorsien is en plaas dit agterin jou ANTWOORDEBOEK.
8. Diagramme is NIE noodwendig volgens skaal getekend NIE.
9. Nommer die antwoorde korrek volgens die nommeringstelsel wat in hierdie vraestel gebruik word.
10. Skryf netjies en leesbaar.

VRAAG 1

By 'n sekere skool neem slegs 12 kandidate Wiskunde en Rekeningkunde. Die punte, as 'n persentasie, wat deur hierdie kandidate in die voorbereidende eksamen in Wiskunde en Rekeningkunde behaal is, word in die tabel en spreidiagram hieronder getoon.

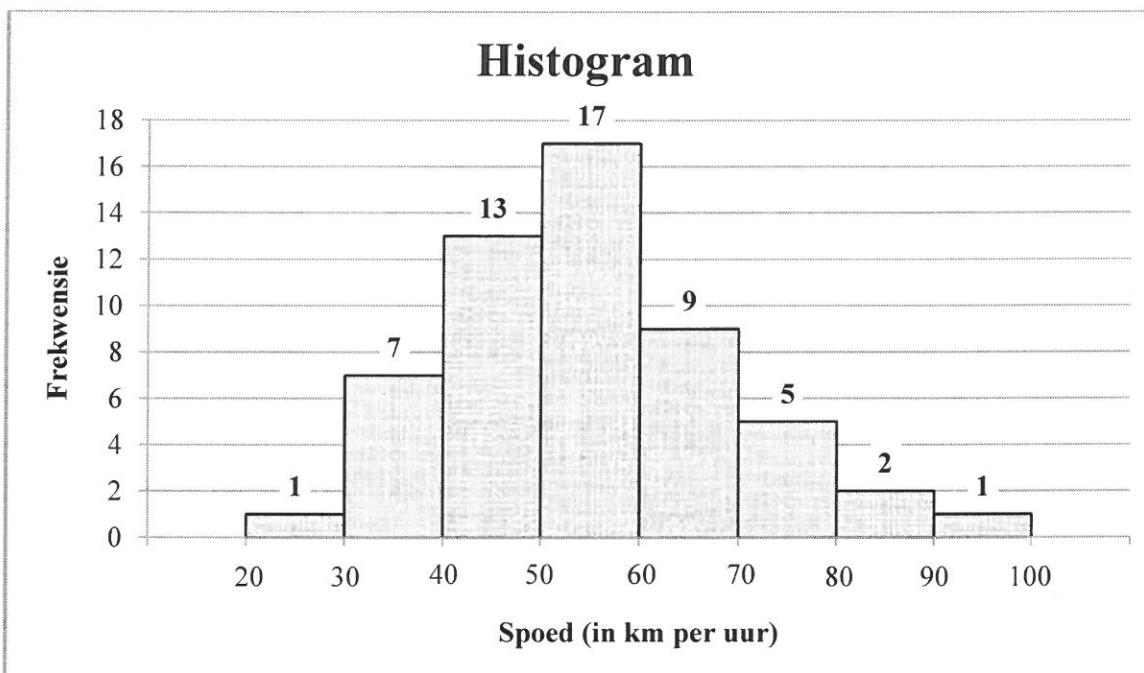
Wiskunde	52	82	93	95	71	65	77	42	89	48	45	57
Rekeningkunde	60	62	88	90	72	67	75	48	83	57	52	62



- 1.1 Bereken die gemiddelde persentasie van die Wiskunde-data. (2)
- 1.2 Bereken die standaardafwyking van die Wiskunde-data. (1)
- 1.3 Bepaal die aantal kandidate wie se persentasie in Wiskunde binne EEN standaardafwyking vanaf die gemiddelde is. (3)
- 1.4 Bereken 'n vergelyking vir die kleinstekwadrate-regressielijn (lyn van beste passing) vir die data. (3)
- 1.5 Indien 'n kandidaat uit hierdie groep 60% in die Wiskunde-eksamen behaal, maar vir die Rekeningkunde-eksamen afwesig was, voorspel, deur jou vergelyking in VRAAG 1.4 te gebruik, die persentasie wat hierdie kandidaat in die Rekeningkunde-eksamen sou behaal het. (Rond jou antwoord tot die NAASTE HEELGETAL af.) (2)
- 1.6 Gebruik die spreidiagram en identifiseer enige uitskieter(s) in die data. (1)
[12]

VRAAG 2

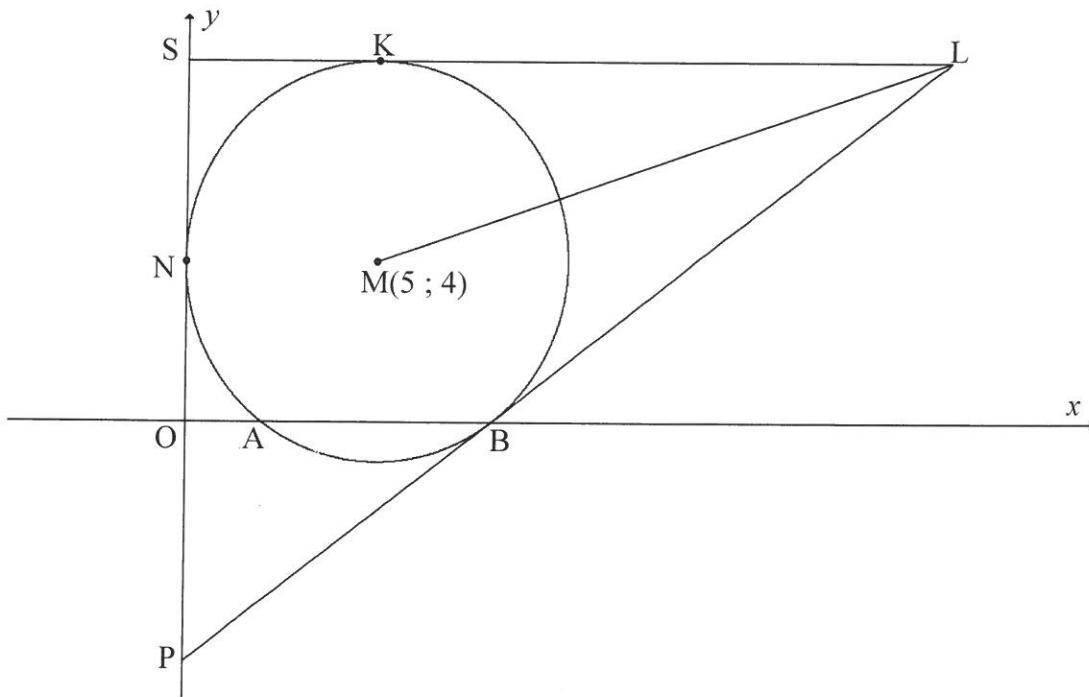
Die spoed van 55 motors wat deur 'n sekere gedeelte van 'n pad beweeg, word vir een uur gemonitor. Die spoedgrens op hierdie gedeelte van die pad is 60 km per uur. 'n Histogram is geskets om hierdie data voor te stel.



- 2.1 Identifiseer die modale klas van die data. (1)
 - 2.2 Gebruik die histogram om:
 - 2.2.1 Die kumulatiewefrekvensie-kolom in die tabel op DIAGRAMVEL 1 te voltooi (2)
 - 2.2.2 'n Ogief (kumulatiewefrekvensie-grafiek) van die data hierbo op die rooster op DIAGRAMVEL 1 te teken (3)
 - 2.3 Die verkeersdepartement stuur spoedboetes aan alle motoriste wat 'n spoed van 66 km per uur oorskry. Skat die aantal motoriste wat 'n spoedboete sal ontvang. (2)
- [8]

VRAAG 3

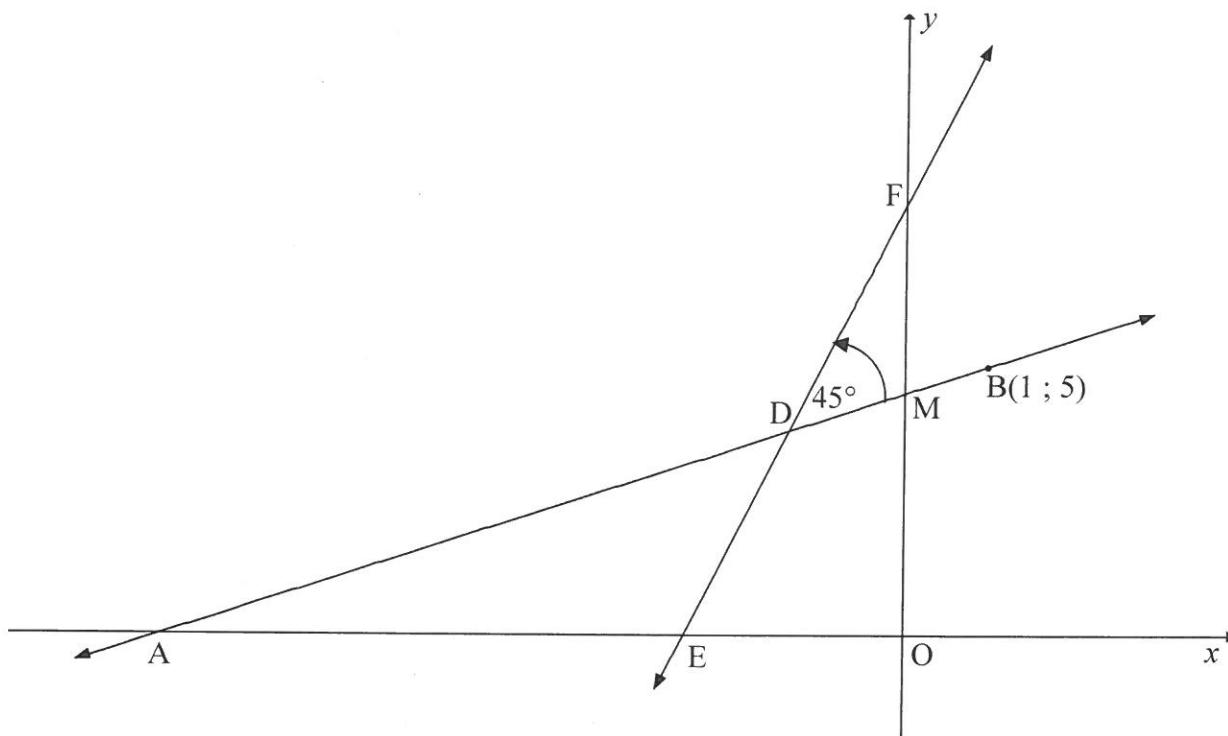
In die diagram hieronder raak 'n sirkel met middelpunt $M(5 ; 4)$ die y -as by N en sny die x -as by A en B . PBL en SKL is raaklyne aan die sirkel waar SKL ewewydig aan die x -as is. LM is getrek.



- 3.1 Skryf die lengte van die radius van die sirkel met middelpunt M neer. (1)
 - 3.2 Skryf die vergelyking van die sirkel met middelpunt M in die vorm $(x-a)^2 + (y-b)^2 = r^2$ neer. (1)
 - 3.3 Bereken die koördinate van A . (3)
 - 3.4 Indien $(8 ; 0)$ die koördinate van B is, bereken:
 - 3.4.1 Die gradiënt van MB (2)
 - 3.4.2 Die vergelyking van die raaklyn PB in die vorm $y = mx + c$ (3)
 - 3.5 Skryf die vergelyking van raaklyn SKL neer. (2)
 - 3.6 Toon aan dat L die punt $(20 ; 9)$ is. (2)
 - 3.7 Bereken die lengte van ML in wortelvorm. (2)
 - 3.8 Bepaal die vergelyking van die sirkel wat deur punt K , L en M gaan in die vorm $(x-p)^2 + (y-q)^2 = c^2$ (5)
- [21]

VRAAG 4

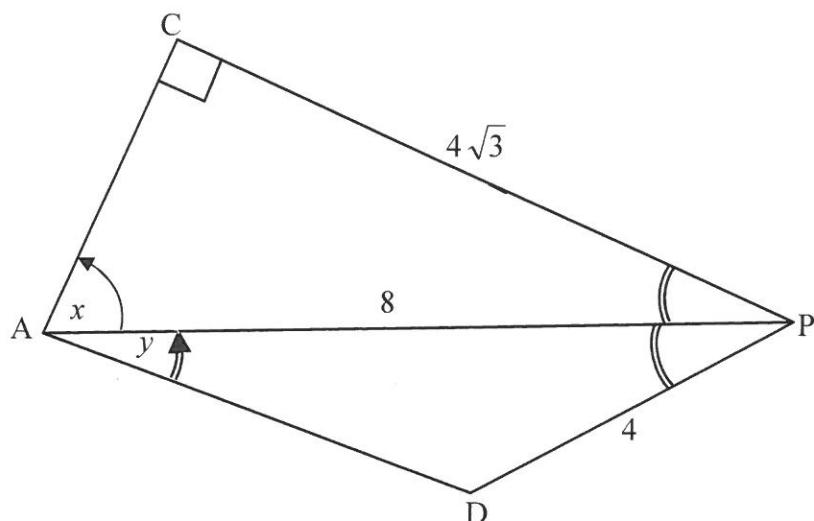
In die diagram hieronder is E en F onderskeidelik die x - en y -afsnit van die lyn met vergelyking $y = 3x + 8$. Die lyn deur B(1 ; 5) wat 'n hoek van 45° met EF vorm, soos hieronder aangetoon, het x - en y -afsnitte by A en M onderskeidelik .



- 4.1 Bepaal die koördinate van E. (2)
- 4.2 Bereken die grootte van \hat{DAE} . (3)
- 4.3 Bepaal die vergelyking van AB in die vorm $y = mx + c$. (4)
- 4.4 Indien $x - 2y + 9 = 0$ die vergelyking van AB is, bepaal die koördinate van D. (4)
- 4.5 Bereken die oppervlakte van vierhoek DMOE. (6)
[19]

VRAAG 5

In die figuur hieronder is $\triangle ACP$ en $\triangle ADP$ driehoeke met $\hat{C} = 90^\circ$, $CP = 4\sqrt{3}$, $AP = 8$ en $DP = 4$. PA halveer \hat{DPC} . Gestel $\hat{CAP} = x$ en $\hat{DAP} = y$.



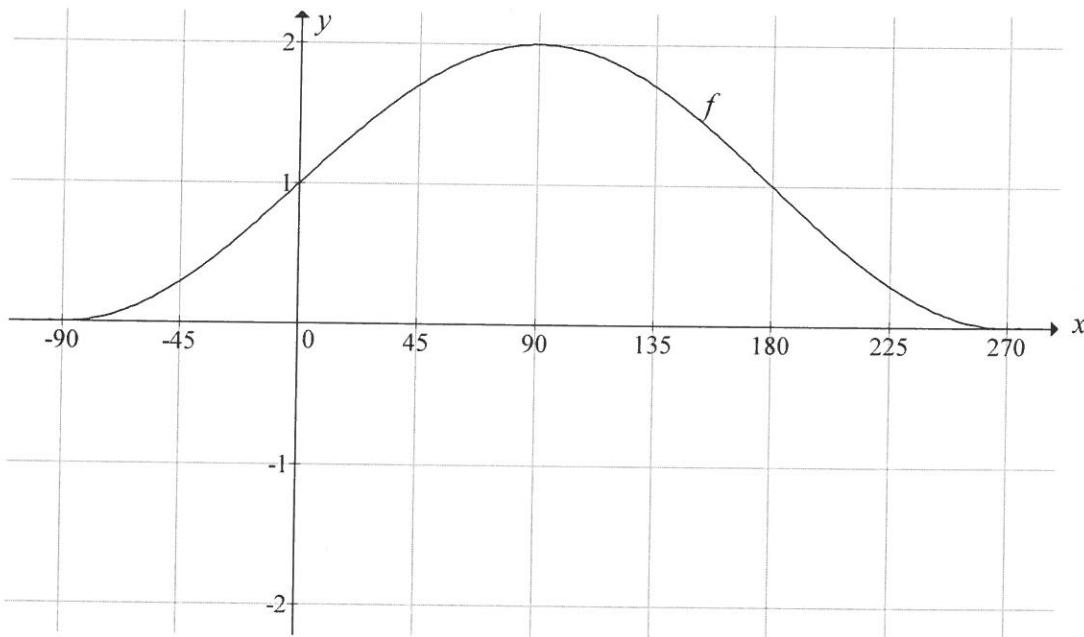
- 5.1 Toon aan, deur berekening, dat $x = 60^\circ$. (2)
- 5.2 Bereken die lengte van AD . (4)
- 5.3 Bepaal y . (3)
[9]

VRAAG 6

- 6.1 Bewys die identiteit: $\cos^2(180^\circ + x) + \tan(x - 180^\circ) \sin(720^\circ - x) \cos x = \cos 2x$ (5)
- 6.2 Gebruik $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ om die formule vir $\sin(\alpha - \beta)$ af te lei. (3)
- 6.3 Indien $\sin 76^\circ = x$ en $\cos 76^\circ = y$, toon aan dat $x^2 - y^2 = \sin 62^\circ$. (4)
[12]

VRAAG 7

In die diagram hieronder is die grafiek van $f(x) = \sin x + 1$ geskets vir $-90^\circ \leq x \leq 270^\circ$.



- 7.1 Skryf die waardeversameling van f neer. (2)
- 7.2 Toon aan dat $\sin x + 1 = \cos 2x$ as $(2 \sin x + 1) \sin x = 0$ herskryf kan word. (2)
- 7.3 Bepaal vervolgens die algemene oplossing van $\sin x + 1 = \cos 2x$. (4)
- 7.4 Gebruik die rooster op DIAGRAMVEL 2 om die grafiek van $g(x) = \cos 2x$ vir $-90^\circ \leq x \leq 270^\circ$ te teken. (3)
- 7.5 Bepaal die waarde(s) van x waarvoor $f(x + 30^\circ) = g(x + 30^\circ)$ in die interval $-90^\circ \leq x \leq 270^\circ$. (3)
- 7.6 Beskou die volgende meetkundige reeks:

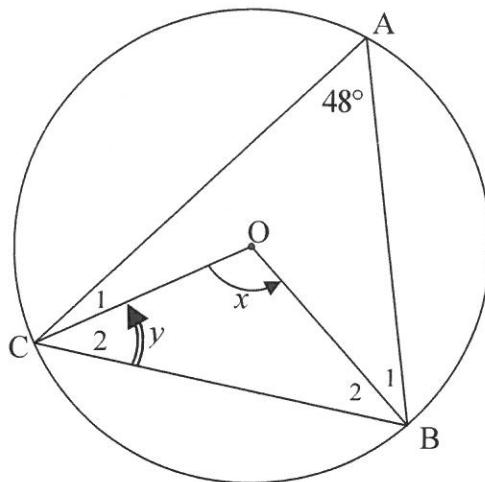
$$1 + 2 \cos 2x + 4 \cos^2 2x + \dots$$

Gebruik die grafiek van g om te bepaal vir watter waarde(s) van x in die interval $0^\circ \leq x \leq 90^\circ$ hierdie reeks sal konvergeer.

(5)
[19]

GEE REDES VIR JOU BEWERINGS IN VRAAG 8, 9 EN 10.**VRAAG 8**

- 8.1 In die diagram is O die middelpunt van die sirkel wat deur A, B en C gaan. $\hat{C}AB = 48^\circ$, $\hat{COB} = x$ en $\hat{C}_2 = y$.

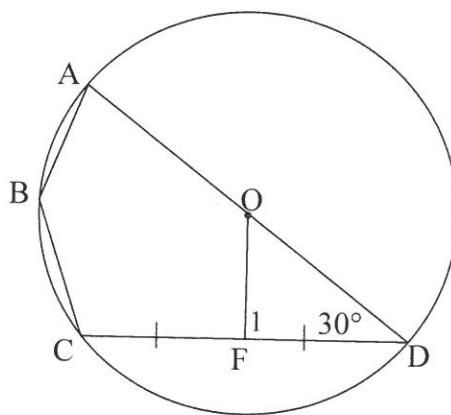


Bepaal, met redes, die grootte van:

8.1.1 x (2)

8.1.2 y (2)

- 8.2 In die diagram is O die middelpunt van die sirkel wat deur A, B, C en D gaan. AOD is 'n reguitlyn en F is die middelpunt van koord CD. $\hat{ODF} = 30^\circ$ en OF is verbind.

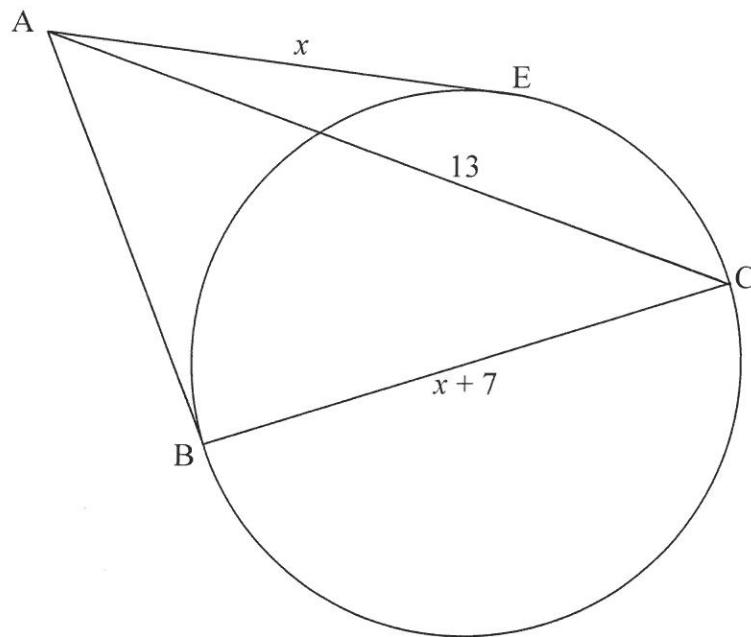


Bepaal, met redes, die grootte van:

8.2.1 \hat{F}_1 (2)

8.2.2 \hat{ABC} (2)

- 8.3 In die diagram is AB en AE raaklyne aan die sirkel by B en E onderskeidelik. BC is 'n middellyn van die sirkel. $AC = 13$, $AE = x$ en $BC = x + 7$.



- 8.3.1 Gee redes vir die bewerings hieronder.
Voltooi die tabel op DIAGRAMVEL 3.

	Bewering	Rede
(a)	$\hat{A}BC = 90^\circ$	
(b)	$AB = x$	

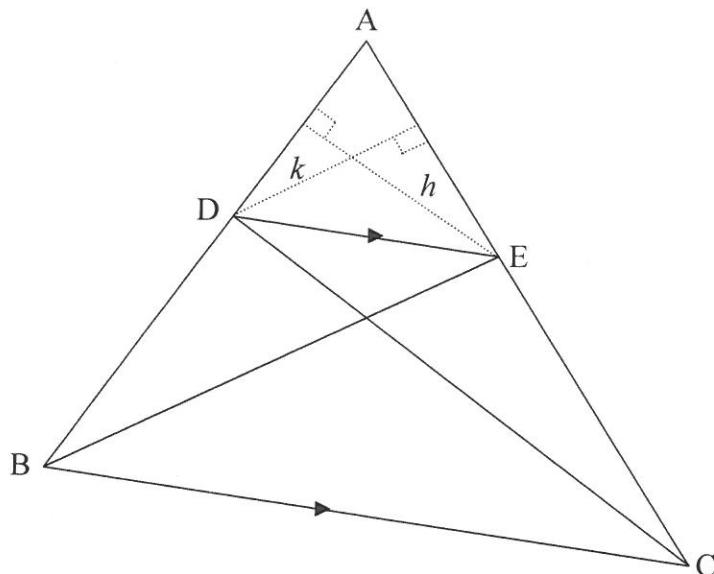
(2)

- 8.3.2 Bereken die lengte van AB .

(4)
[14]

VRAAG 9

- 9.1 In die diagram lê punte D en E op sye AB en AC van $\triangle ABC$ onderskeidelik sodat $DE \parallel BC$. DC en BE is verbind.



- 9.1.1 Verduidelik waarom die oppervlaktes van $\triangle DEB$ en $\triangle DEC$ gelyk is. (1)
- 9.1.2 Hieronder verskyn die gedeeltelik voltooide bewys van die stelling wat beweer dat indien in enige $\triangle ABC$ die lyn $DE \parallel BC$ dan is $\frac{AD}{DB} = \frac{AE}{EC}$.
- Gebruik die diagram hierbo en voltooи die bewys van die stelling op DIAGRAMVEL 4.**

Konstruksie: Konstroeer die hoogtelyne (hoogtes) h en k in $\triangle ADE$.

$$\frac{\text{oppervlakte } \triangle ADE}{\text{oppervlakte } \triangle DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \dots$$

$$\frac{\text{oppervlakte } \triangle ADE}{\text{oppervlakte } \triangle DEC} = \dots = \frac{AE}{EC}$$

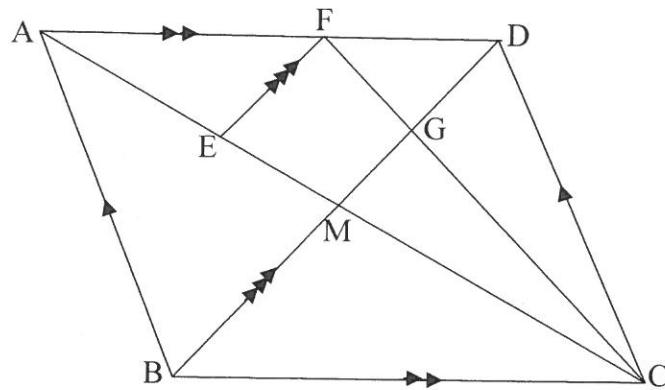
Maar oppervlakte $\triangle DEB = \dots$ (rede:)

$$\therefore \frac{\text{oppervlakte } \triangle ADE}{\text{oppervlakte } \triangle DEB} = \dots$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(5)

- 9.2 In die diagram is ABCD 'n parallelogram. Die hoeklyne van ABCD sny by M. F is 'n punt op AD sodat $AF : FD = 4 : 3$. E is 'n punt op AM sodat $EF \parallel BD$. FC en MD sny by G.



Bereken, met redes, die verhouding van:

9.2.1 $\frac{EM}{AM}$ (3)

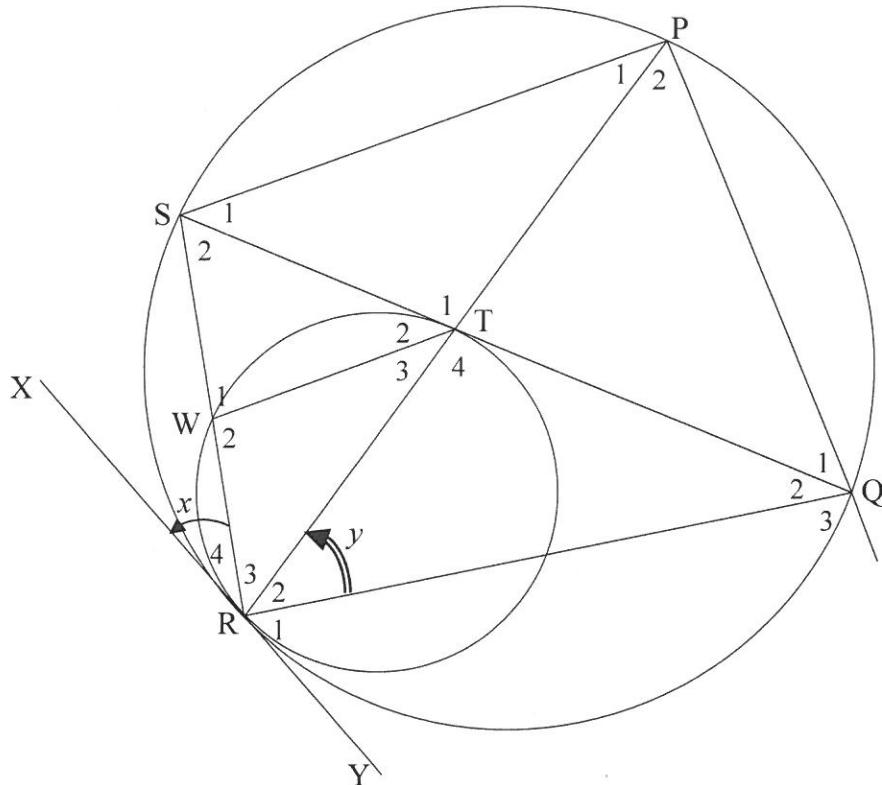
9.2.2 $\frac{CM}{ME}$ (3)

9.2.3
$$\frac{\text{oppervlakte } \triangle FDC}{\text{oppervlakte } \triangle BDC}$$
 (4)
[16]

VRAAG 10

Die twee sirkels in die diagram het 'n gemeenskaplike raaklyn XRY by R . W is enige punt op die klein sirkel. Die reguitlyn RWS ontmoet die groter sirkel by S . Die koord STQ is 'n raaklyn aan die klein sirkel, met T as die raakpunt. Koord RTP is getrek.

Gestel $\hat{R}_4 = x$ en $\hat{R}_2 = y$



10.1 Gee redes vir die bewerings hieronder.

Voltooi die tabel op DIAGRAMVEL 6.

Gestel $\hat{R}_4 = x$ en $\hat{R}_2 = y$		
	Bewering	Rede
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	$WT \parallel SP$	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

(5)

- 10.2 Bewys dat $RT = \frac{WR \cdot RP}{RS}$ (2)
- 10.3 Identifiseer, met redes, nog TWEE ander hoeke gelyk aan y . (4)
- 10.4 Bewys dat $\hat{Q}_3 = \hat{W}_2$. (3)
- 10.5 Bewys dat $\Delta RTS \parallel\parallel \Delta RQP$. (3)
- 10.6 Bewys vervolgens dat $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$. (3)
[20]

TOTAAL: **150**

SENTRUMNOMMER:

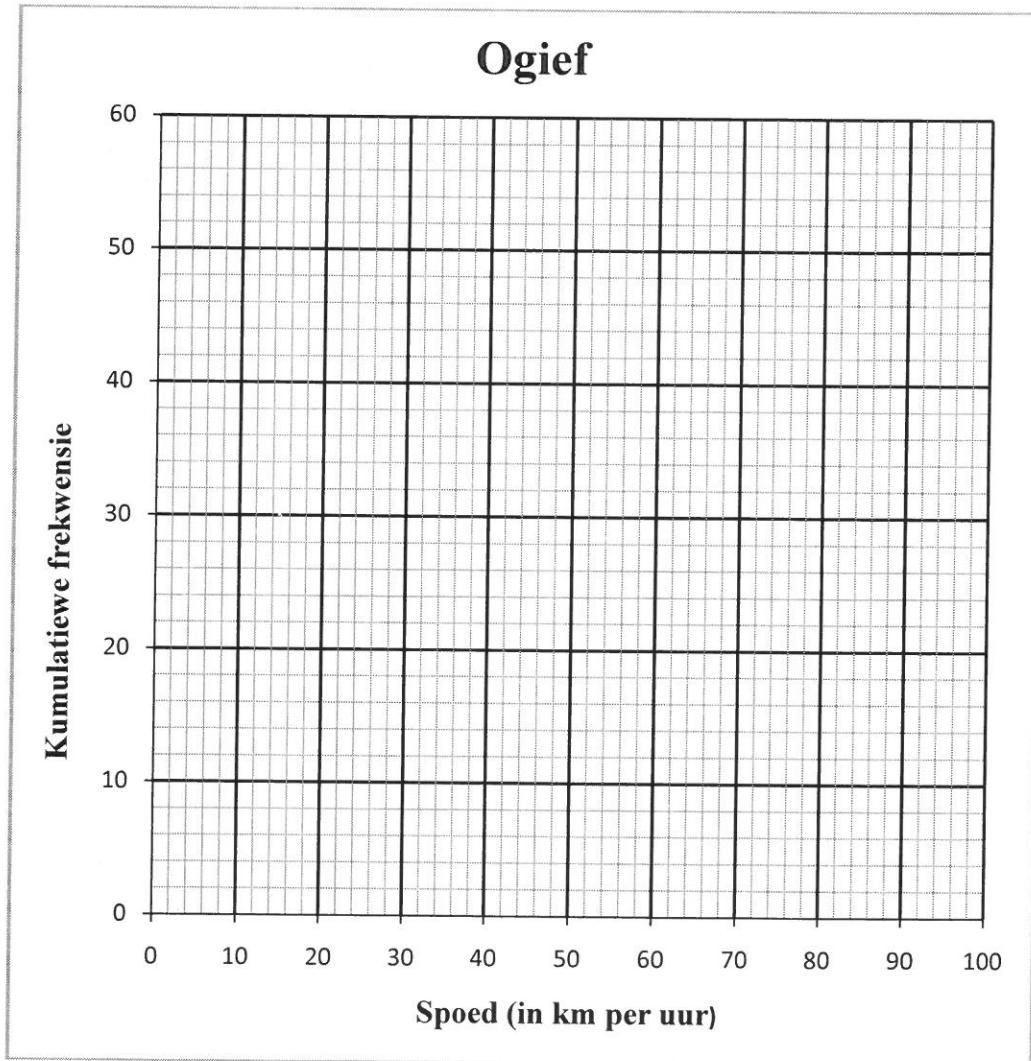
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EKSAMENNOMMER:

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DIAGRAMVEL 1**VRAAG 2.2.1**

Klas	Frekwensie	Kumulatiewe frekwensie
$20 < x \leq 30$	1	
$30 < x \leq 40$	7	
$40 < x \leq 50$	13	
$50 < x \leq 60$	17	
$60 < x \leq 70$	9	
$70 < x \leq 80$	5	
$80 < x \leq 90$	2	
$90 < x \leq 100$	1	

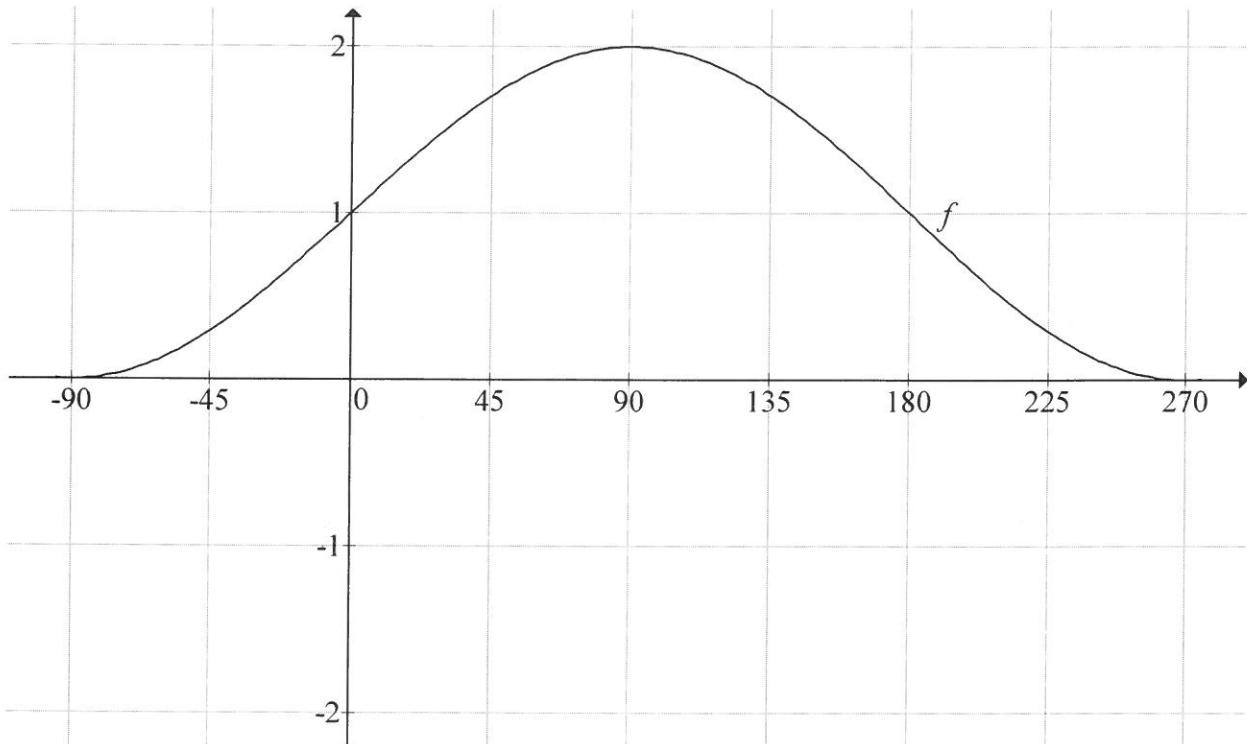
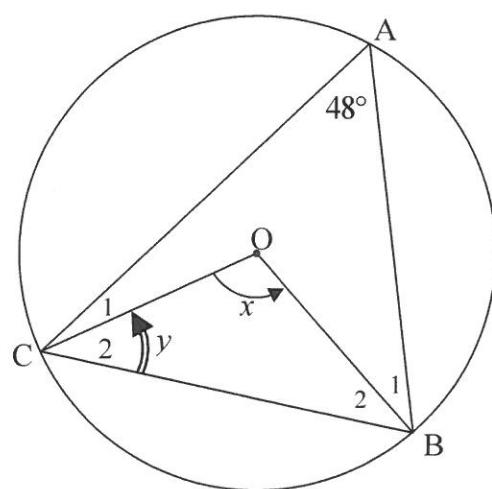
VRAAG 2.2.2

SENTRUMNOMMER:

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EKSAMENNOMMER:

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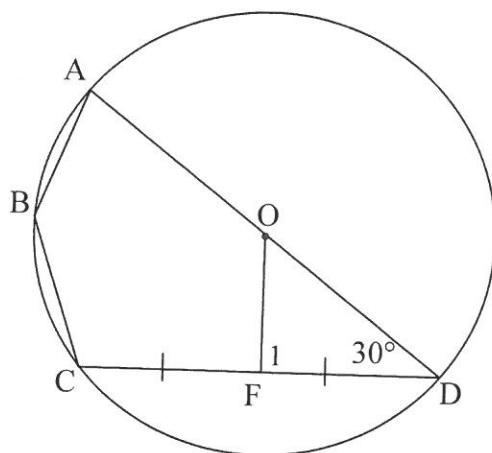
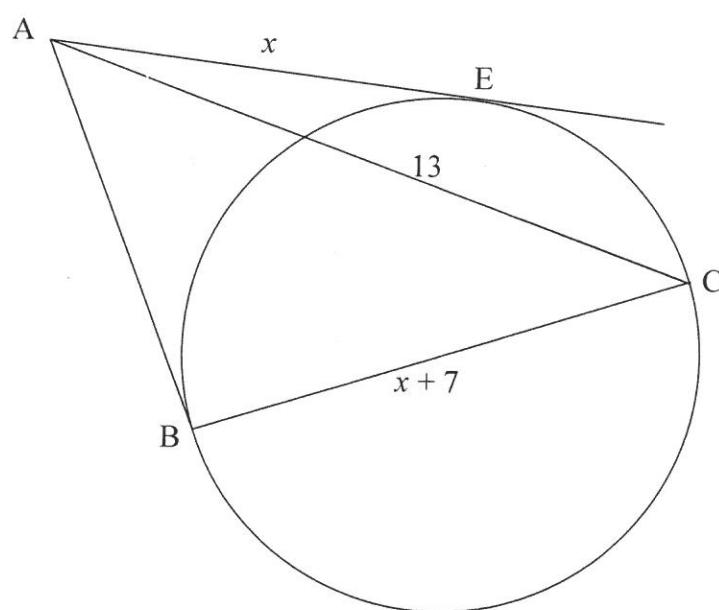
DIAGRAMVEL 2**VRAAG 7.4****VRAAG 8.1**

SENTRUMNOMMER:

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EKSAMENNOMMER:

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DIAGRAMVEL 3**VRAAG 8.2****VRAAG 8.3**

8.3.1	Bewering	Rede
(a)	$\hat{A}BC = 90^\circ$	
(b)	$AB = x$	

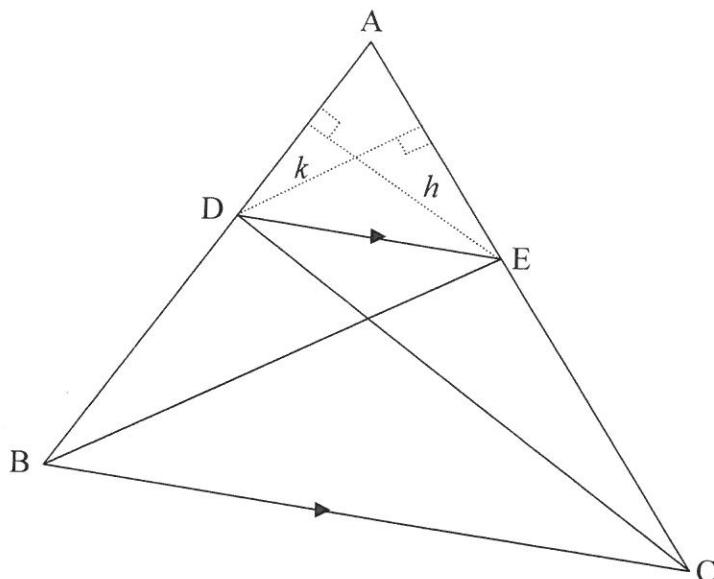
SENTRUMNOMMER:

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EKSAMENNNOMMER:

DIAGRAMVEL 4

VRAAG 9.1



9.1.2 Konstruksie: Konstrueer hoogtelyne (hoogtes) h en k in ΔADE

$$\frac{\text{oppervlakte } \Delta \text{ADE}}{\text{oppervlakte } \Delta \text{DEB}} = \frac{\frac{1}{2}(\text{AD})(h)}{\frac{1}{2}(\text{BD})(h)} = \dots$$

$$\frac{\text{oppervlakte } \Delta \text{ADE}}{\text{oppervlakte } \Delta \text{DEC}} = \dots = \frac{\text{AE}}{\text{EC}}$$

Maar oppervlakte $\Delta DEB = \dots$

(rede:)

$$\therefore \frac{\text{oppervlakte } \Delta ADE}{\text{oppervlakte } \Delta DEB} = \dots$$

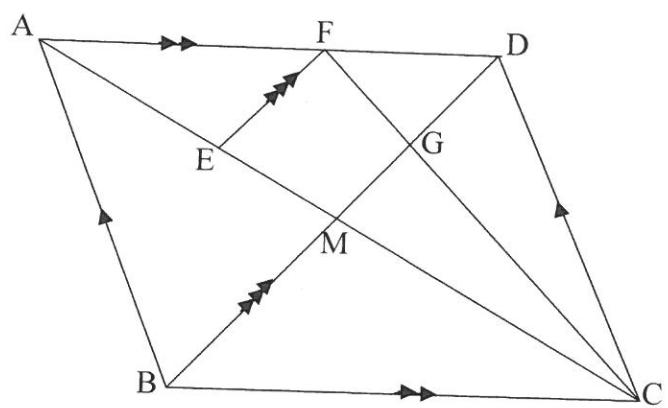
$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

SENTRUMNOMMER:

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EKSAMENNOMMER:

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DIAGRAMVEL 5**VRAAG 9.2**

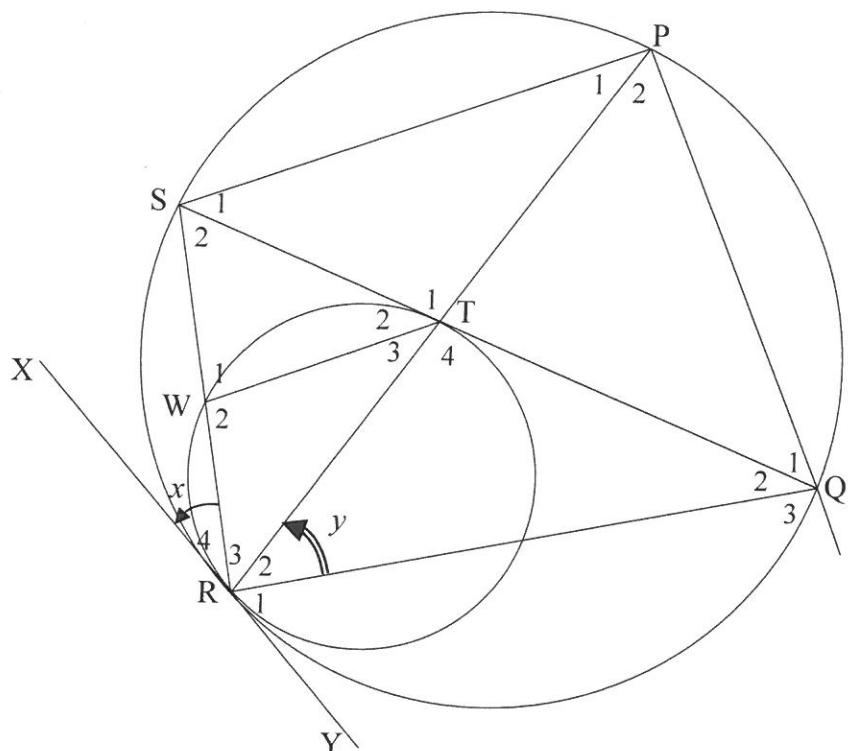
SENTRUMNOMMER:

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EKSAMENNOMMER:

DIAGRAMVEL 6

VRAAG 10



Gestel $\hat{R}_4 = x$ en $\hat{R}_2 = y$		
	Bewering	Rede
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni)$$

$$A = P(1-ni)$$

$$A = P(1-i)^n$$

$$A = P(1+i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1-r}; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{oppervlakte } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2 \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2014

MEMORANDUM

MARKS/PUNTE: 150

**This memorandum consists of 23 pages.
*Hierdie memorandum bestaan uit 23 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

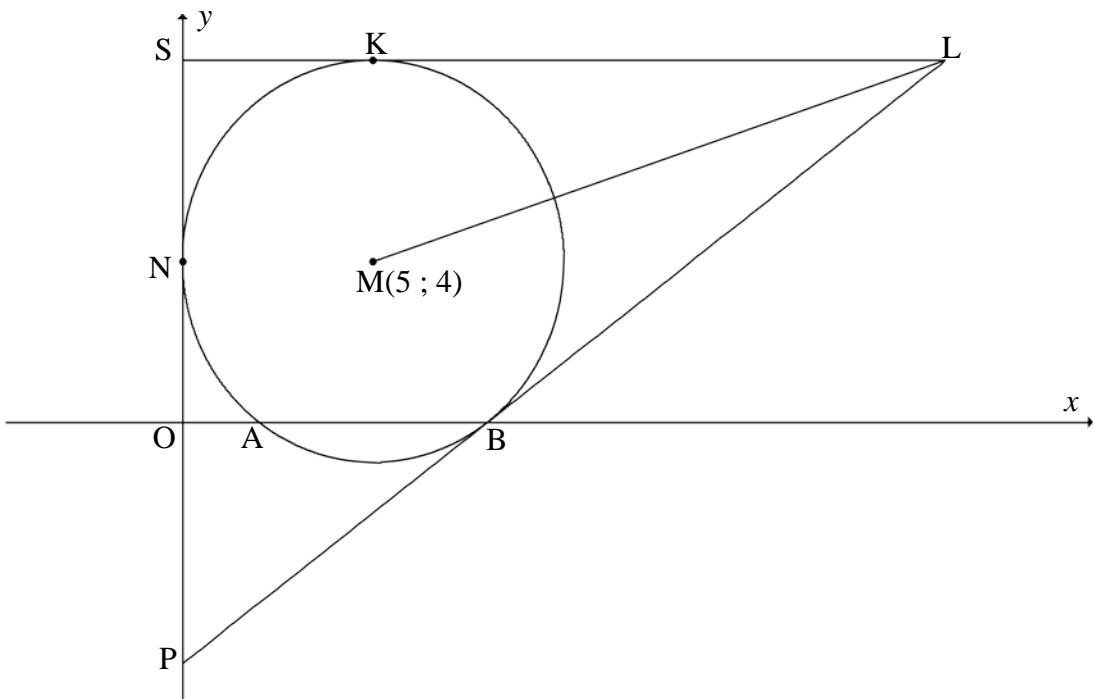
- As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.
- As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.
- Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.

QUESTION/VRAAG 1

1.1	$\bar{x} = \frac{816}{12} = 68$	✓ $\frac{816}{12}$ ✓ 68 (2)
1.2	$\sigma = 18,42$	✓ answer/antw (1)
1.3	$(68 - 18,42 ; 68 + 18,42) = (49,58 ; 86,42)$ \therefore 6 candidates had a mark within one standard deviation of the mean/6 kandidate het 'n punt binne een standaardafwyking vanaf die gemiddelde.	✓✓ interval ✓ answer/antw (3)
1.4	$a = 22,828\dots = 22,83$ $b = 0,66429\dots = 0,66$ $\therefore \hat{y} = 0,66x + 22,83$ OR/OF $\hat{y} = 22,83 + 0,66x$	✓ value of a / waarde van a ✓ value of b / waarde van b ✓ equation/vgl (3)
1.5	$\hat{y} = 0,66x + 22,83$ $y = 0,66(60) + 22,83$ $62,43\dots\% \approx 62\%$ OR/OF $62,69\% \approx 63\%$	✓ subs of 60 into equation ✓ answer/antw (2) ✓✓ answer/antw (2)
1.6	(82 ; 62)	✓ answer/antw (1) [12]

QUESTION/VRAAG 2

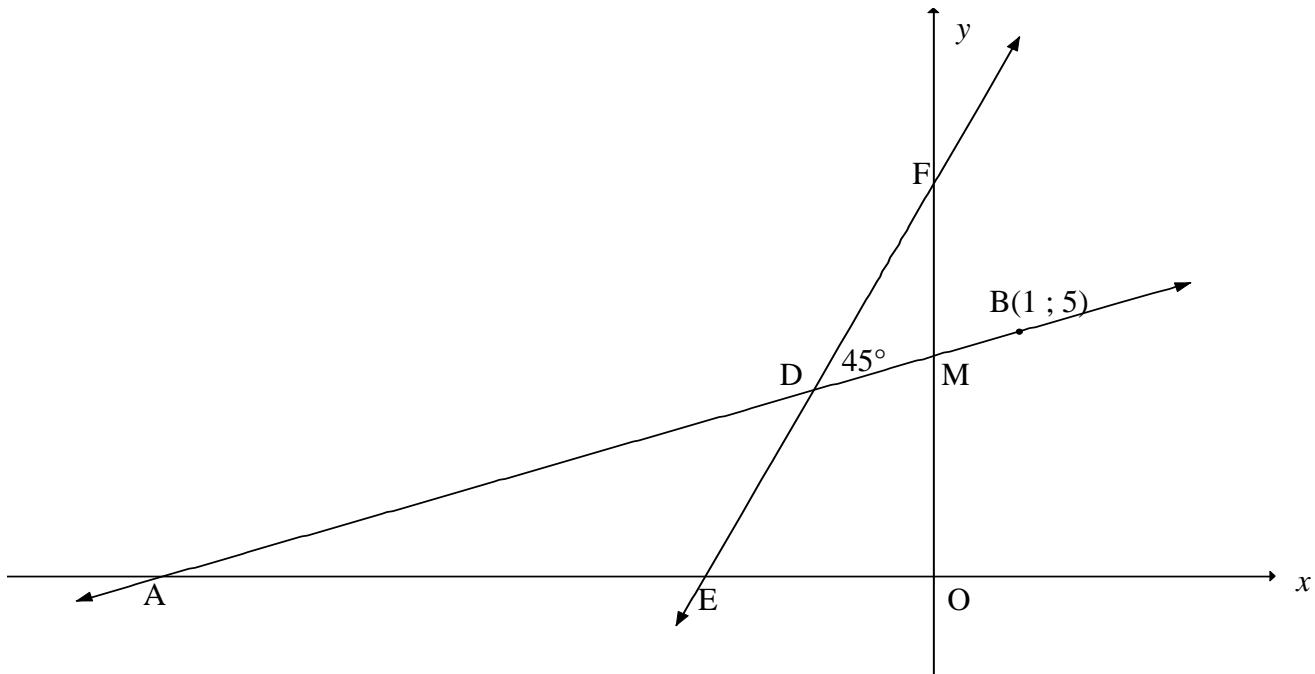
2.1	$50 < x \leq 60$ OR/OF $50 \leq x < 60$ OR/OF between 50 and 60/tussen 50 en 60	✓ answer/antw (1)																											
2.2.1	<table border="1"> <thead> <tr> <th>Class <i>Klas</i></th> <th>Frequency <i>Frekwensie</i></th> <th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th> </tr> </thead> <tbody> <tr><td>$20 < x \leq 30$</td><td>1</td><td>1</td></tr> <tr><td>$30 < x \leq 40$</td><td>7</td><td>8</td></tr> <tr><td>$40 < x \leq 50$</td><td>13</td><td>21</td></tr> <tr><td>$50 < x \leq 60$</td><td>17</td><td>38</td></tr> <tr><td>$60 < x \leq 70$</td><td>9</td><td>47</td></tr> <tr><td>$70 < x \leq 80$</td><td>5</td><td>52</td></tr> <tr><td>$80 < x \leq 90$</td><td>2</td><td>54</td></tr> <tr><td>$90 < x \leq 100$</td><td>1</td><td>55</td></tr> </tbody> </table>	Class <i>Klas</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$20 < x \leq 30$	1	1	$30 < x \leq 40$	7	8	$40 < x \leq 50$	13	21	$50 < x \leq 60$	17	38	$60 < x \leq 70$	9	47	$70 < x \leq 80$	5	52	$80 < x \leq 90$	2	54	$90 < x \leq 100$	1	55	✓ 8 ✓ 55 (2)
Class <i>Klas</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>																											
$20 < x \leq 30$	1	1																											
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$40 < x \leq 50$	13	21																											
$50 < x \leq 60$	17	38																											
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$70 < x \leq 80$	5	52																											
$80 < x \leq 90$	2	54																											
$90 < x \leq 100$	1	55																											
2.2.2	<p>The graph shows a smooth curve representing cumulative frequency against speed. The x-axis is labeled "Speed in km per hour" and "Spoed in km per uur". The y-axis is labeled "Cumulative Frequency" and "Kumulatiewe frekwensie". The curve passes through points approximately at (20, 0), (30, 1), (40, 8), (50, 22), (60, 38), (70, 47), (80, 52), (90, 55), and (100, 55). Dashed lines indicate a value of 45 on the y-axis and 65 on the x-axis.</p>	✓ grounding at (20 ; 0)/ anker by (20 ; 0) ✓ plotting at upper limits/ plot by boonste limiete ✓ smooth shape of curve/gladde kurwe (3)																											
2.3	$55 - 44$ (accept/aanvaar 43 – 45) ≈ 11 motorists/motoriste (accept/aanvaar 10 – 12 motorists/motoriste)	✓ 44 ✓ 11 (2) [8]																											

QUESTION/VRAAG 3

3.1	$r = MN = 5$	✓ answer/antw (1)	
3.2	$(x - 5)^2 + (y - 4)^2 = 25$	✓ equation/vgl (1)	
3.3	$A(x ; 0)$ $(x - 5)^2 + (0 - 4)^2 = 25$ $x^2 - 10x + 25 + 16 = 25$ $x^2 - 10x + 16 = 0$ $(x - 8)(x - 2) = 0$ $\therefore x = 8 \text{ or } x = 2$ $\therefore A(2 ; 0)$	$(x - 5)^2 + (0 - 4)^2 = 25$ $(x - 5)^2 + 16 = 25$ $(x - 5)^2 = 9$ $(x - 5) = \pm 3$ $\therefore x = 8 \text{ or } x = 2$ $\therefore A(2 ; 0)$	✓ substitute into eq/ vervang in vgl $y = 0$ ✓ standard form/ standaardvorm or perfect square form/kwadr vorm ✓ answer/antw (3)
3.4.1	$m_{MB} = \frac{4 - 0}{5 - 8}$ $= -\frac{4}{3}$	✓ subst M and B into form/vervang M and B in form ✓ $m_{MB} = -\frac{4}{3}$ (2)	

3.4.2	$m_{MB} \times m_{PB} = -1$ (tangent \perp radius/ rkl \perp radius) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ OR/OF $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$	✓ $m_{MB} \times m_{PB} = -1$ ✓ $m_{PB} = \frac{3}{4}$ ✓ equation/vgl (3)
3.5	$y_K = y_M + r = 4 + 5$ $y = 9$	✓ 9 ✓ equation/vgl (2)
3.6	At/By L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore L(20 ; 9)$	✓ equating simultaneously ✓ simplification (2)
3.7	L(20 ; 9) $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ OR/OF $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20 - 5)^2 + (9 - 4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9 + 1)}$ $= \sqrt{250}$ or / of $5\sqrt{10}$ $= \sqrt{250}$ or / of $5\sqrt{10}$	✓ correct subst into distance formula/ korrekte subst in afstand-formule ✓ answer in surd form/antw in wortelvorm (2)
3.8	MK \perp KL OR/OF $\hat{MKL} = 90^\circ$ (radius \perp tangent/radius \perp rkl) $\therefore ML$ is a diameter as it subtends a right angle/ ML is middellyn $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML /Midpt van sirkel = midpt v ML $x = \frac{5+20}{2} = \frac{25}{2} = 12,5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ Centre/midpt: (12,5 ; 6,5) Equation of the circle KLM /Vgl van sirkel KLM: $\therefore (x - 12,5)^2 + (y - 6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$ OR/OF	✓ S ✓ value of/waarde van r ✓ $x = 12,5$ ✓ $y = 6,5$ ✓ answer in correct form/ antw in korrekte vorm (5)

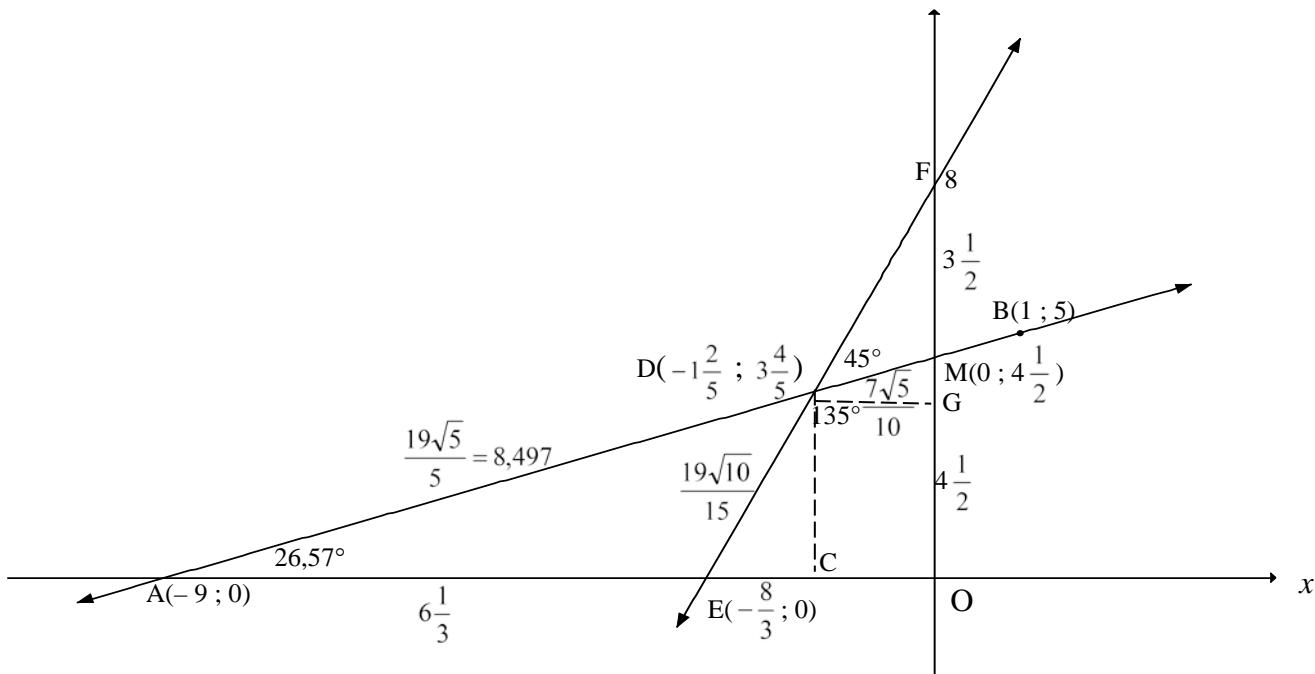
<p>MK ⊥ KL OR/OF $\hat{MKL} = 90^\circ$ (radius \perp tangent/radius $\perp rkl$) $\therefore ML$ is a diameter as it subtends a right angle/<i>ML is middellyn</i> Centre of circle = midpoint of ML/<i>Midpt van sirkel = midpt v ML</i></p> $x = \frac{5+20}{2} = \frac{25}{2} = 12,5 \quad y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ <p>Centre/<i>midpt</i>: (12,5 ; 6,5)</p> <p>Equation of the circle KLM /<i>Vgl van sirkel KLM</i>:</p> $(x - 12,5)^2 + (y - 6,5)^2 = r^2$ <p>subst (5 ; 4): $(5 - 12,5)^2 + (4 - 6,5)^2 = r^2$</p> $62,5 = r^2$ $\therefore (x - 12,5)^2 + (y - 6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$ <p>OR/OF</p> <p>By symmetry about LM/<i>deur simmetrie om LM</i>:</p> <p>MK ⊥ KL OR/OF $\hat{MKL} = 90^\circ$ (radius \perp tangent/radius $\perp rkl$) $\therefore ML$ is a diameter as it subtends a right angle/<i>ML is middellyn</i> <i>ML is a diameter /ML is 'n middellyn</i></p> $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}} \quad \text{or /of } 7,91$ <p>Centre of circle = midpoint of ML/<i>Midpt van sirkel = midpt v ML</i></p> $x = \frac{5+20}{2} = \frac{25}{2} = 12,5 \quad y = \frac{4+9}{2} = \frac{13}{2} = 6,5$ <p>Centre/<i>midpt</i>: (12,5 ; 6,5)</p> <p>Equation of the circle KLM /<i>Vgl van sirkel KLM</i>:</p> $\therefore (x - 12,5)^2 + (y - 6,5)^2 = \frac{250}{4} = \frac{125}{2} = 62,5$	<p>✓ S</p> <p>✓ $x = 12,5$ ✓ $y = 6,5$</p> <p>✓ value of/waarde van r^2</p> <p>✓ answer in correct form/antw in korrekte vorm (5)</p> <p>✓ S</p> <p>✓ value of/waarde van r</p> <p>✓ $x = 12,5$ ✓ $y = 6,5$</p> <p>✓ answer in correct form/antw in korrekte vorm (5) [21]</p>
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QUESTION/VRAAG 4

4.1	$y = 0: 3x + 8 = 0$ $x = -\frac{8}{3}$ $\therefore E\left(-2\frac{2}{3}; 0\right)$ OR/OF $E\left(-\frac{8}{3}; 0\right)$	✓ y-value/waarde ✓ x-value/waarde (2)
4.2	$\tan \hat{D}\hat{E}O = m_{DE} = 3$ $\therefore \hat{D}\hat{E}O = 71,565\dots = 71,57^\circ$ $\hat{D}\hat{A}E = 71,565\dots^\circ - 45^\circ$ $= 26,57^\circ$	✓ $\tan \hat{D}\hat{E}O = 3$ ✓ $71,565\dots^\circ$ ✓ $26,57^\circ$ (3)
4.3	$m_{AB} = \tan 26,57^\circ$ $= \frac{1}{2}$ $y = \frac{1}{2}x + c$ OR/OF $y - y_1 = \frac{1}{2}(x - x_1)$ $5 = \frac{1}{2}(1) + c$ $y - 5 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x + 4\frac{1}{2}$ $y = \frac{1}{2}x + \frac{9}{2}$	✓ $m_{AB} = \tan 26,57^\circ$ ✓ $m_{AB} = \frac{1}{2}$ ✓ subst of m and $(1; 5)$ into formula/ subst m en $(1; 5)$ in formule ✓ equation/vgl (4)

<p>4.4 Solve $x - 2y + 9 = 0$ and $y = 3x + 8$ simultaneously:</p> $x - 2(3x+8) + 9 = 0$ $x - 6x - 16 + 9 = 0$ $-5x = 7$ $x = -1\frac{2}{5}$ $\therefore y = 3(-1\frac{2}{5}) + 8 \quad \text{OR/OF} \quad -1\frac{2}{5} - 2y + 9 = 0$ $y = 3\frac{4}{5} \quad y = 3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}; 3\frac{4}{5})$ <p>OR/OF</p> $x = 2y - 9$ $y = 3(2y - 9) + 8$ $y = 6y - 27 + 8$ $\therefore y = 3\frac{4}{5}$ $x = 2(3\frac{4}{5}) - 9 \quad \text{OR/OF} \quad 3\frac{4}{5} = 3x + 8$ $x = -1\frac{2}{5} \quad x = -1\frac{2}{5}$ $\therefore D(-1\frac{2}{5}; 3\frac{4}{5})$ <p>OR/OF</p> $3x + 8 = \frac{1}{2}x + 4\frac{1}{2}$ $6x + 16 = x + 9$ $5x = -7$ $\therefore x = -1\frac{2}{5}$ $\therefore y = 3(-1\frac{2}{5}) + 8 \quad \text{OR/OF} \quad y = \frac{1}{2}(-1\frac{2}{5}) + 4\frac{1}{2}$ $y = 3\frac{4}{5} \quad y = 3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}; 3\frac{4}{5})$ <p>OR/OF</p>	<p>✓ subst/vervang</p> <p>✓ x-value/waarde</p> <p>✓ subst/vervang</p> <p>✓ y-value/waarde (4)</p> <p>✓ subst/vervang</p> <p>✓ y value/waarde</p> <p>✓ subst/vervang</p> <p>✓ x-value/waarde</p> <p>(4)</p> <p>✓ equating/gelyk stel</p> <p>✓ x value/waarde</p> <p>✓ subst/vervang</p> <p>✓ y-value/waarde (4)</p>
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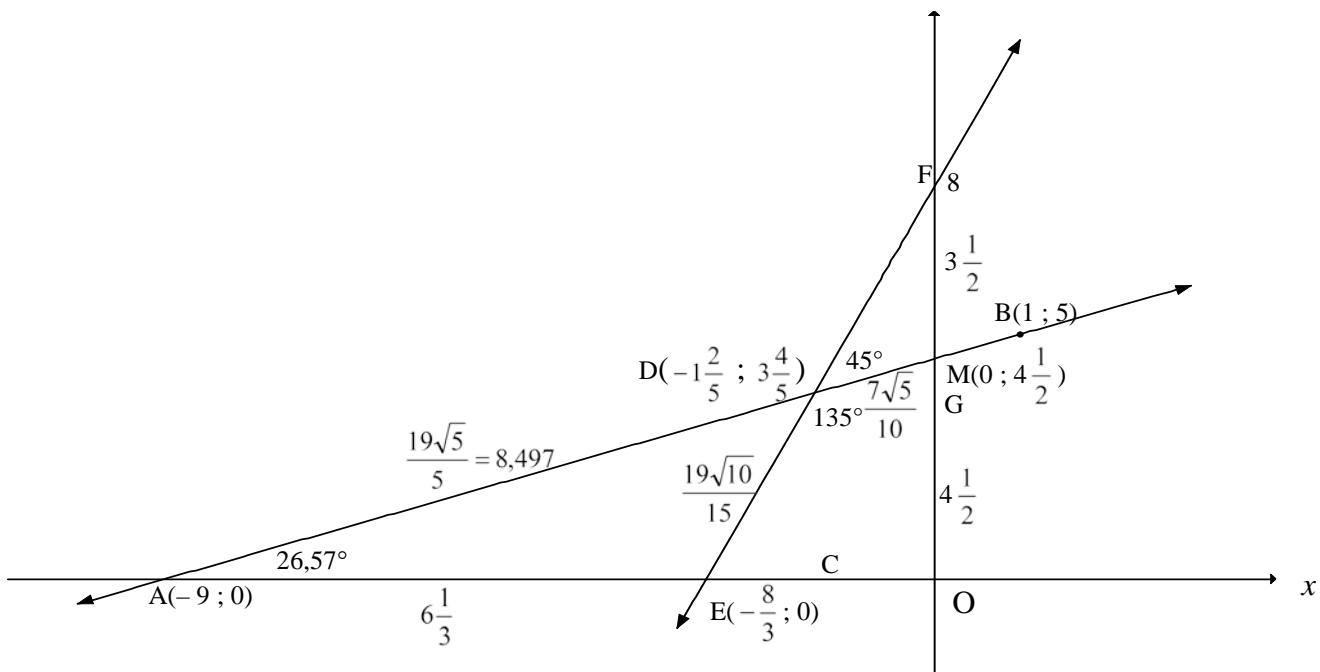
	$\begin{aligned}x - 2y &= -9 \dots\dots\dots(1) \\ -6x + 2y &= 16 \dots\dots\dots(2)\end{aligned}$ <p>(1) + (2):</p> $\begin{aligned}-5x &= 7 \\ \therefore x &= -1\frac{2}{5}\end{aligned}$ $\therefore -1\frac{2}{5} - 2y = -9 \quad \text{OR/OF} \quad y = 3(-1\frac{2}{5}) + 8$ $\begin{aligned}y &= 3\frac{4}{5} \\ y &= 3\frac{4}{5}\end{aligned}$ $\therefore D(-1\frac{2}{5}; 3\frac{4}{5})$ <p>OR/OF</p> $\begin{aligned}y &= 3x + 8 \dots\dots\dots(1) \\ 6y &= 3x + 27 \dots\dots\dots(2)\end{aligned}$ <p>(1) - (2):</p> $\begin{aligned}-5y &= -19 \\ \therefore y &= 3\frac{4}{5}\end{aligned}$ $\begin{aligned}3\frac{4}{5} &= 3x + 8 \quad \text{OR/OF} \quad x = 2(3\frac{4}{5}) - 9 \\ x &= -1\frac{2}{5} \quad x = -1\frac{2}{5}\end{aligned}$ $\therefore D(-1\frac{2}{5}; 3\frac{4}{5})$	<ul style="list-style-type: none"> ✓ adding/<i>optelling</i> ✓ <i>x</i>-value/<i>waarde</i> ✓ subst/<i>vervang</i> ✓ <i>y</i>-value/<i>waarde</i>
		(4)

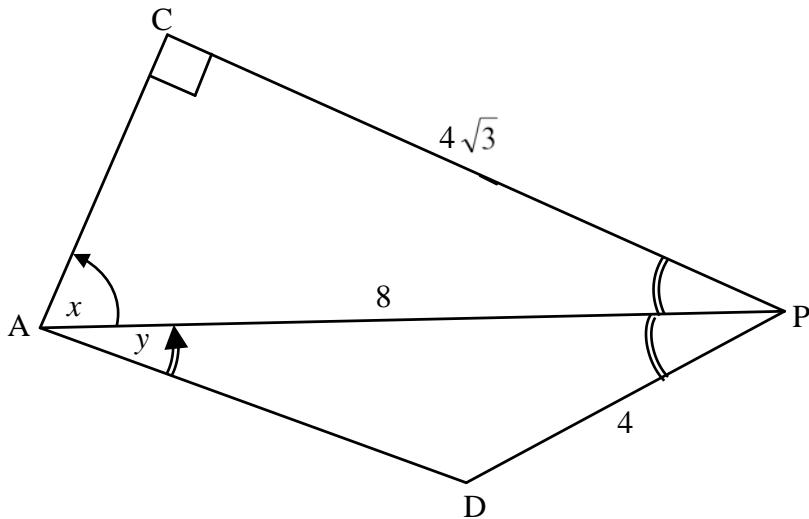


4.5	<p>area DMOE = area ΔAMO – area ΔADE</p> $x_A = 2(0) - 9 \quad \therefore A(-9; 0)$ <p>area ΔAMO area ΔADE</p> $= \frac{1}{2} \cdot AO \cdot OM$ $= \frac{1}{2} \cdot (9) \cdot (4 \frac{1}{2})$ $= 20,25$ $= \frac{1}{2} \cdot AE \cdot y_D$ $= \frac{1}{2} \cdot (AO - EO) \cdot y_D$ $= \frac{1}{2} \left(9 - 2 \frac{2}{3} \right) \left(3 \frac{4}{5} \right)$ $= 12,03$ <p>OR/OF</p> <p>area ΔADE</p> $= \frac{1}{2} AD \cdot AE \cdot \sin DAE$ $= \frac{1}{2} \left(\frac{19\sqrt{5}}{5} \right) \cdot 6 \frac{1}{3} \cdot \sin 26,57^\circ$ $= 12,03$ <p>\therefore area DMOE = 8,22 square units/vk eenh</p> <p>OR/OF</p>	<p>✓ correct method/ korrekte metode</p> <p>✓ $x_A = -9$</p> <p>✓ $\frac{1}{2}(9)(4 \frac{1}{2})$</p> <p>✓ $AE = 9 - 2 \frac{2}{3} = 6 \frac{1}{3}$</p> <p>✓ $y_D = 3 \frac{4}{5}$</p> <p>OR/OF</p> <p>✓ $AD = \frac{19\sqrt{5}}{5}$</p> <p>✓ $AE = 6 \frac{1}{3}$</p> <p>✓ answer/antw</p>
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	<p>area DMOE = area rectangle DCOG + area ΔDMG + area ΔDEC</p> $= \left(1\frac{2}{5} \times 3\frac{4}{5}\right) + \frac{1}{2}\left(1\frac{2}{5}\right)\left(\frac{7}{10}\right) + \frac{1}{2}\left(3\frac{4}{5}\right)\left(\frac{19}{15}\right)$ $= 8,22 \text{ square units/vk eenh}$	<ul style="list-style-type: none"> ✓ correct method/ korrekte metode ✓ $3\frac{4}{5}$ ✓ $1\frac{2}{5}$ ✓ 0,7 ✓ $\frac{19}{15}$ ✓ answer
	OR/OF	(6)
	<p>area DMOE = area ΔEZO + area ΔODM</p> $= \frac{1}{2}(EO \times y_D) + \frac{1}{2}(OM \times -x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times \frac{19}{5}\right) + \left(\frac{9}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{304 + 189}{30}\right)$ $= \frac{493}{60} \text{ or/of } 8\frac{13}{60} \text{ or/of } 8,22 \text{ square units/vk eenh}$	<ul style="list-style-type: none"> ✓ correct method/ korrekte metode ✓ $y_D = \frac{19}{5}$ or $3\frac{4}{5}$ ✓ $EO = \frac{8}{3}$ ✓ $-x_D = \frac{7}{5}$ ✓ $OM = \frac{9}{2}$ or $4\frac{1}{2}$ ✓ answer/antw
	OR/OF	(6)
	<p>area DMOE = area ΔEOF – area ΔDMF</p> $= \frac{1}{2}(EO \times OF) - \frac{1}{2}(OF - OM)(-x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times 8\right) + \left(\frac{7}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{640 - 147}{30}\right)$ $= \frac{493}{60} \text{ or } 8\frac{13}{60} \text{ or } 8,22 \text{ square units/vk eenh}$	<ul style="list-style-type: none"> ✓ correct method/ korrekte metode ✓ $y_F = 8$ ✓ $EO = \frac{8}{3}$ ✓ $-x_D = \frac{7}{5}$ ✓ $FM = 3\frac{1}{2}$ ✓ answer/antw
	OR/OF	(6)

$\begin{aligned} \text{area } \Delta EOM &= \frac{1}{2}(EO \times OM) \\ &= \frac{1}{2}\left(\frac{8}{3} \times \frac{9}{2}\right) \\ &= 6 \text{ sq units/vk eenh} \end{aligned}$ $\begin{aligned} ED &= \sqrt{\left(-\frac{7}{5} + \frac{8}{3}\right)^2 + \left(\frac{19}{5}\right)^2} \quad \text{and } DM = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{9}{2} - \frac{19}{5}\right)^2} \\ &= \frac{19\sqrt{10}}{15} \text{ or } 4,005\dots \quad = \frac{7\sqrt{5}}{10} \text{ or } 1,565\dots \end{aligned}$ $\begin{aligned} \text{area } \Delta EDM &= \frac{1}{2}(ED \times DM \times \sin E\hat{D}M) \\ &= \frac{1}{2}\left(\frac{19\sqrt{10}}{15}\right)\left(\frac{7\sqrt{5}}{10}\right)\sin 135^\circ \\ &= \frac{133}{60} \text{ or } 2,216\dots \end{aligned}$ <p>$\therefore \text{area DMOE} = \text{area } \Delta EOM + \text{area } \Delta EDM$</p> $\begin{aligned} &= 6 + 2,216\dots \\ &= \frac{493}{60} \text{ or/of } 8\frac{13}{60} \text{ or/of } 8,22 \text{ square units/eenh}^2 \end{aligned}$	✓ area ΔEOM ✓ $ED = \frac{19\sqrt{10}}{15}$ ✓ $DM = \frac{7\sqrt{5}}{10}$ ✓ area ΔEDM ✓ correct method/ <i>korrekte metode</i> ✓ answer/ <i>antw</i>
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(6)
[19]

QUESTION/VRAAG 5

5.1	$\sin C\hat{A}P = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60^\circ$ OR/OF $\frac{\sin 90^\circ}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60^\circ$	<ul style="list-style-type: none"> ✓ correct sine ratio/ korrekte sin-verh ✓ $\frac{\sqrt{3}}{2}$ <p>(2)</p> <ul style="list-style-type: none"> ✓ correct sine ratio/ korrekte sin-verh ✓ $\frac{\sqrt{3}}{2}$ <p>(2)</p>
5.2	$C\hat{P}A = D\hat{P}A = 30^\circ$ (APbisects DPC) $AD^2 = AP^2 + DP^2 - 2(AP)(DP)\cos A\hat{P}D$ $= 8^2 + 4^2 - 2(8)(4)\cos 30^\circ$ $= 8^2 + 4^2 - 2(8)(4)\left(\frac{\sqrt{3}}{2}\right)$ $= 24,57\dots$ $AD = 4,96$	<ul style="list-style-type: none"> ✓ $D\hat{P}A = 30^\circ$ ✓ correct subst into cosine rule/ korrekte subst in cos-reël ✓ 24,57\dots ✓ 4,96 <p>(4)</p>

<p>5.3</p> $\frac{\sin D\hat{A}P}{DP} = \frac{\sin A\hat{P}D}{AD}$ $\frac{\sin y}{4} = \frac{\sin 30^\circ}{4,96}$ $\sin y = \frac{4 \sin 30^\circ}{4,96}$ $= 0,403\dots$ $y = 23,78^\circ$ <p>OR/OF</p> $AD^2 = AP^2 + DP^2 - 2 \cdot AP \cdot DP \cdot \cos D\hat{A}P$ $4^2 = 8^2 + (4,96)^2 - 2(8)(4,96) \cdot \cos y$ $\cos y = \frac{8^2 + (4,96)^2 - 4^2}{2(8)(4,96)}$ $\cos y = 0,9148\dots$ $y = 23,82^\circ$	<ul style="list-style-type: none"> ✓ correct subst into sine rule/ <i>korrekte subst in sin-reël</i> ✓ $\sin y$ subject ✓ $23,78^\circ$ <p>(3)</p>
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QUESTION/VRAAG 6

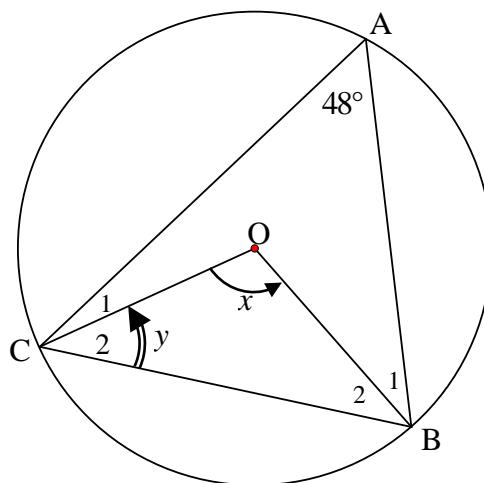
6.1	$\begin{aligned} & \cos^2(180^\circ + x) + \tan(x - 180^\circ) \sin(720^\circ - x) \cos x \\ &= (-\cos x)^2 + [-(-\tan x)](-\sin x)(\cos x) \\ &= \cos^2 x + \left(\frac{\sin x}{\cos x}\right)(-\sin x)(\cos x) \\ &= \cos^2 x - \sin^2 x \\ &= \cos 2x \end{aligned}$	<ul style="list-style-type: none"> ✓ $(-\cos x)^2$ or $\cos^2 x$ ✓ $\tan x$ or $-(-\tan x)$ ✓ $-\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\cos^2 x - \sin^2 x$ <p style="text-align: right;">(5)</p>
6.2	$\begin{aligned} & \sin(\alpha - \beta) \\ &= \cos[90^\circ - (\alpha - \beta)] \\ &= \cos[(90^\circ - \alpha) + \beta] \\ &= \cos(90^\circ - \alpha) \cos \beta - \sin(90^\circ - \alpha) \sin \beta \\ &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} & \sin(\alpha - \beta) \\ &= \cos[90^\circ - (\alpha - \beta)] \\ &= \cos[(90^\circ + \beta) + (-\alpha)] \\ &= \cos(90^\circ + \beta) \cos(-\alpha) - \sin(90^\circ + \beta) \sin(-\alpha) \\ &= (-\sin \beta) \cos \alpha - \cos \beta (-\sin \alpha) \\ &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \end{aligned}$	<ul style="list-style-type: none"> ✓ rewrite as/herkryf $\cos[(90^\circ - \alpha) + \beta]$ ✓ expansion/ <i>uitbreiding</i> ✓ simpl/vereenv <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> ✓ rewrite as/herkryf $\cos[(90^\circ + \beta) + (-\alpha)]$ ✓ expansion/ <i>uitbreiding</i> ✓ simpl/vereenv <p style="text-align: right;">(3)</p>
6.3	$\begin{aligned} & x^2 - y^2 \\ &= \sin^2 76^\circ - \cos^2 76^\circ \\ &= -(\cos^2 76^\circ - \sin^2 76^\circ) \\ &= -\cos 2(76^\circ) \\ &= -\cos 152^\circ \\ &= -(-\cos 28^\circ) \quad \textbf{OR/OF} \quad = -\cos(90^\circ + 62^\circ) \\ &= \cos 28^\circ \quad = -(-\sin 62^\circ) \\ &= \cos(90^\circ - 62^\circ) \quad = \sin 62^\circ \\ &= \sin 62^\circ \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} & x^2 - y^2 \\ &= \sin^2 76^\circ - \cos^2 76^\circ \\ &= \sin 76^\circ \sin 76^\circ - \cos 76^\circ \cos 76^\circ \\ &= \sin 76^\circ \cos 14^\circ - \cos 76^\circ \sin 14^\circ \\ &= \sin(76^\circ - 14^\circ) \\ &= \sin 62^\circ \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} & x^2 - y^2 \\ &= \sin^2 76^\circ - \cos^2 76^\circ \\ &= \cos^2 14^\circ - \sin^2 14^\circ \\ &= \cos 2(14^\circ) \\ &= \cos 28^\circ \\ &= \sin 62^\circ \end{aligned}$	<ul style="list-style-type: none"> ✓ $-(\cos^2 76^\circ - \sin^2 76^\circ)$ ✓ recognition of cos double angle ✓ $-\cos 152^\circ$ ✓ $\cos 28^\circ$ <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ $\cos 14^\circ$ ✓ $\sin 14^\circ$ ✓ recognition of sine compound angle ✓ $\sin(76^\circ - 14^\circ)$ <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ $\cos^2 14^\circ$ ✓ $\sin^2 14^\circ$ ✓ recognition of cos double angle ✓ $\cos 28^\circ$ <p style="text-align: right;">(4) [12]</p>

QUESTION/VRAAG 7

7.1	$0 \leq y \leq 2$ or $y \in [0 ; 2]$	✓ critical values/ kritieke waardes ✓ notation/notasie (2)
7.2	$\sin x + 1 = \cos 2x$ $\sin x + 1 = 1 - 2\sin^2 x$ $2\sin^2 x + \sin x = 0$ $\sin x(2\sin x + 1) = 0$	✓ $1 - 2\sin^2 x$ ✓ st form/st vorm (2)
7.3	$\sin x(2\sin x + 1) = 0$ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ $x = 0^\circ + k \cdot 360^\circ$ or $x = 210^\circ + k \cdot 360^\circ$ or $x = 180^\circ + k \cdot 360^\circ$ or $x = 330^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ OR/OF $x = k \cdot 180^\circ, k \in \mathbb{Z}$	✓ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ ✓ $0^\circ ; 180^\circ$ OR/OF $x = k \cdot 180^\circ$ ✓ $210^\circ ; 330^\circ$ ✓ $k \cdot 360^\circ, k \in \mathbb{Z}$ (4)
7.4		✓ y-intercept/afsnit ✓ x-intercepts/afsnitte ✓ min/max points/min/maks punte (3)
7.5	$f(x) = g(x)$ at/by: $x = -30^\circ ; 0^\circ ; 180^\circ ; 210^\circ$ $\therefore f(x + 30^\circ) = g(x + 30^\circ)$ at/by: $x = -60^\circ ; -30^\circ ; 150^\circ ; 180^\circ$	✓ $-30^\circ ; 0^\circ ; 180^\circ ; 210^\circ$ ✓✓ $-60^\circ ; -30^\circ ; 150^\circ ; 180^\circ$ (3)
7.6	Series will converge if/Reeks sal konvergeer as: $-1 < r < 1$ $-1 < 2\cos 2x < 1$ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ $\therefore 30^\circ < x < 60^\circ$ or $x \in (30^\circ ; 60^\circ)$	✓ $-1 < r < 1$ ✓ $r = 2\cos 2x$ ✓ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ ✓✓ $30^\circ < x < 60^\circ$ (5) [19]

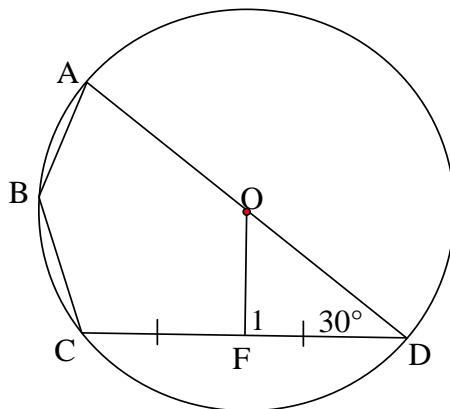
QUESTION/VRAAG 8

8.1



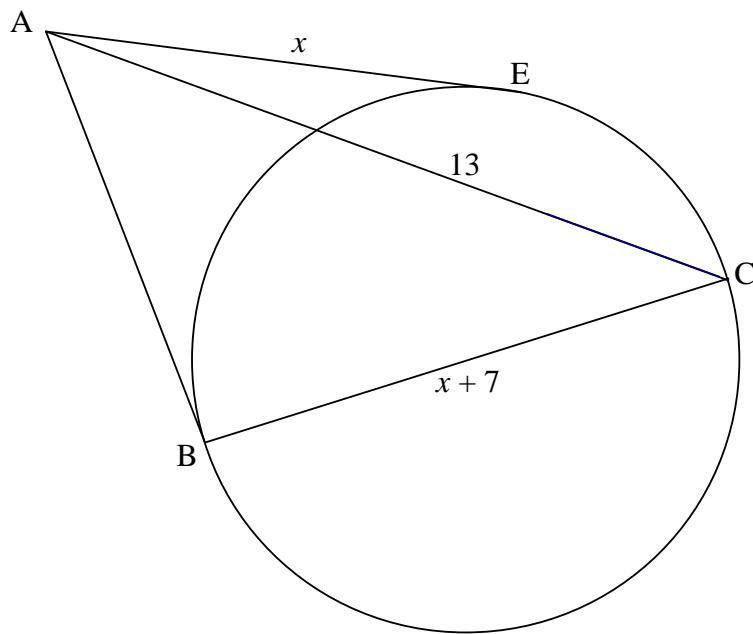
8.1.1	$x = 96^\circ$ (\angle at centre = $2 \angle$ at circumference/ \angle by midpt = $2 \angle$ by omtrek)	✓ S ✓ R (2)
8.1.2	$\hat{C}_2 + \hat{B}_2 = 180^\circ - 96^\circ = 84^\circ$ (sum of \angle s in Δ / som v \angle e in Δ) $y = \hat{B}_2 = 42^\circ$ (\angle s opp = sides/ \angle e teenoor = sye)	✓ S ✓ S (2)

8.2



8.2.1	$\hat{F}_1 = 90^\circ$ (line from centre to midpt chord/ lyn vanaf midpt na midpt kd)	✓ S ✓ R (2)
8.2.2	$\hat{ABC} = 150^\circ$ (opposite \angle s of cyclic quad/ tos \angle e v koordevh)	✓ S ✓ R (2)

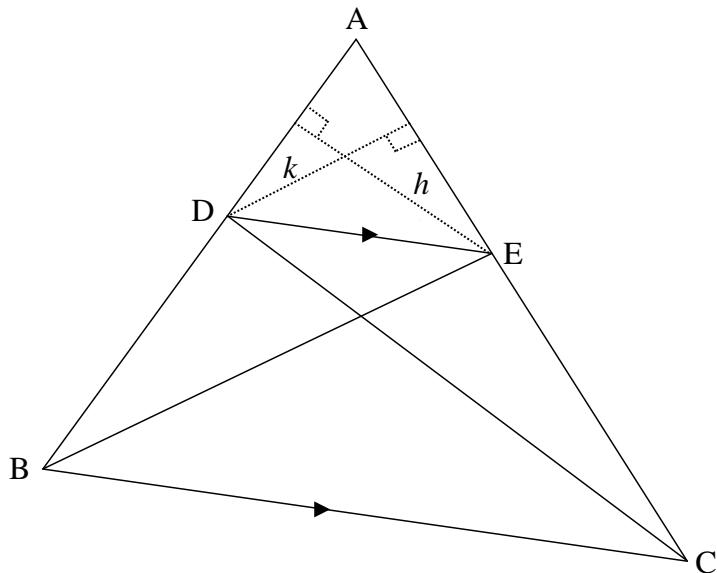
8.3



8.3.1 (a)	tangent \perp radius/diameter / raaklyn \perp radius/middellyn	\checkmark R (1)
8.3.1 (b)	tangents from common pt OR tangents from same pt / raaklyne v gemeensk pt OF raaklyne vanaf dies pt	\checkmark R (1)
8.3.2	$\begin{aligned} AB^2 + BC^2 &= AC^2 \\ x^2 + (x + 7)^2 &= 13^2 \quad (\text{Theorem of/Stelling van Pythagoras}) \\ x^2 + x^2 + 14x + 49 &= 169 \\ 2x^2 + 14x - 120 &= 0 \\ x^2 + 7x - 60 &= 0 \\ (x - 5)(x + 12) &= 0 \\ x = 5 \quad (x \neq -12) \end{aligned}$	\checkmark $AB^2 + BC^2 = AC^2$ \checkmark $x^2 + (x + 7)^2 = 13^2$ \checkmark standard form \checkmark answer (4) [14]

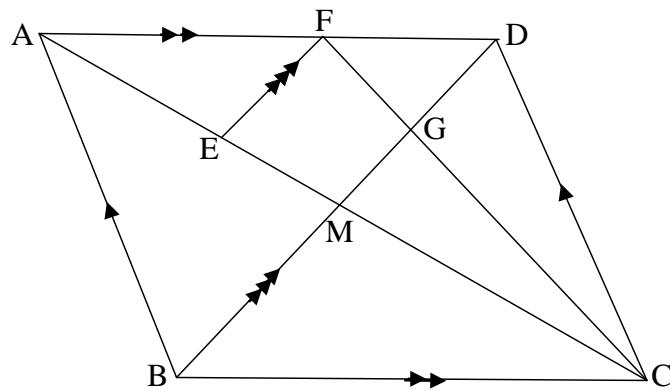
QUESTION/VRAAG 9

9.1



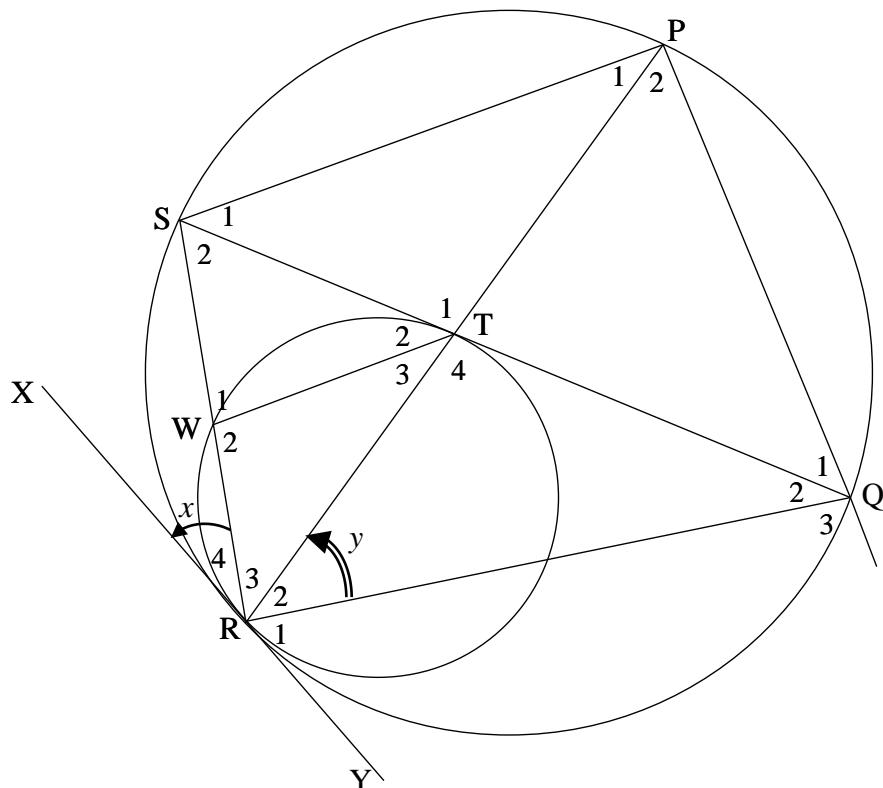
9.1.1	Same base (DE) and same height (between parallel lines) <i>Dieselde basis (DE) en dieselde hoogte (tussen ewewydige lyne)</i>	✓ same base/dies basis between lines/ <i>tussen / / lyne</i> (1)
9.1.2	$\frac{AD}{DB}$ $\frac{1}{2} AE \times k$ $\frac{1}{2} EC \times k$ But/Maar area $\Delta DEB =$ area ΔDEC (Same base and same height/dieselde basis en dieselde hoogte) $\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ S ✓ S ✓ S ✓ R ✓ S (5)

9.2

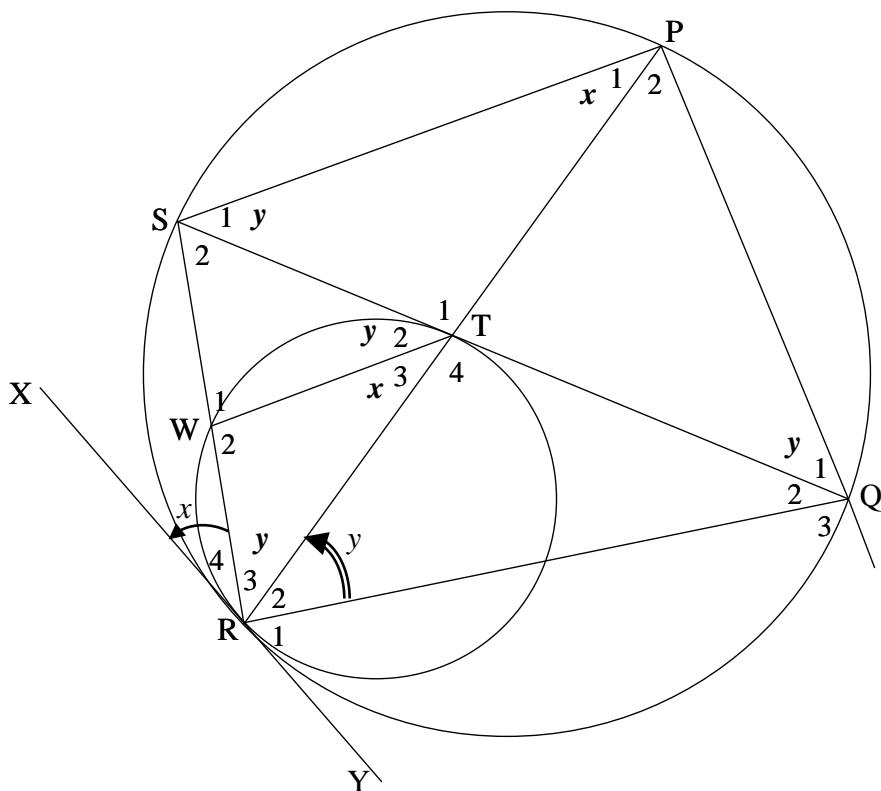


9.2.1	$\frac{EM}{AM} = \frac{FD}{AD}$ <p>(Line parallel one side of \triangle OR prop th; $EF \parallel BD$) (Lyn ewewydig aan sy v \triangle OF eweredigst; $EF \parallel BD$)</p> $\frac{EM}{AM} = \frac{3}{7}$	✓ S ✓R ✓ answer/antw (3)
9.2.2	$CM = AM$ $\frac{CM}{ME} = \frac{AM}{ME} = \frac{7}{3}$ <p>(diags of parm bisect/hoekl parm halv) (from 9.2.1/vanaf 9.2.1)</p>	✓ S ✓R ✓ answer/antw (3)
9.2.3	$h \text{ of } \triangle FDC = h \text{ of } \triangle BDC \quad (\text{AD} \parallel \text{BC})$ $\frac{\text{area } \triangle FDC}{\text{area } \triangle BDC} = \frac{\frac{1}{2} FD.h}{\frac{1}{2} BC.h}$ $= \frac{FD}{AD} \quad (\text{opp sides of parm} =)$ $= \frac{3}{7} \quad (\text{tos sye v parm} =)$ <p>OR/OF</p> $\frac{\text{area } \triangle FDC}{\text{area } \triangle ADC} = \frac{FD}{AD} = \frac{3}{7} \quad (\text{same heights})$ $\text{But Area } \triangle ADC = \text{Area } \triangle BDC \text{ (diags of parm bisect area)} \\ (\text{hoekl v parm halv opp})$ $\frac{\text{area } \triangle FDC}{\text{area } \triangle BDC} = \frac{3}{7}$	✓ AD BC ✓ subst into area form/ subst in opp formule ✓ S ✓ answer/antw (4)

QUESTION/VRAAG 10



10.1.1	Tangent chord theorem/Raaklyn-koordstelling	✓ R (1)
10.1.2	Tangent chord theorem/Raaklyn-koordstelling	✓ R (1)
10.1.3	Corresponding angles equal/Ooreenkomsstige \angle e gelyk	✓ R (1)
10.1.4	\angle s subtended by chord PQ OR \angle s in same segment \angle e onderspan deur dieselfde koord OF \angle e in dieselfde segment	✓ R (1)
10.1.5	alternate \angle s/verwisselende \angle e ; WT SP	✓ R (1)
10.2	$\frac{RW}{RS} = \frac{RT}{RP}$ (Line parallel one side of Δ OR $\therefore RT = \frac{WR \cdot RP}{RS}$ prop th; WT SP) $(\text{Lyn ewewydig aan sy v } \Delta \text{ } OF \text{ eweredighst: WT // SP})$ OR/OF $\Delta RTW \Delta RPS$ (\angle ; \angle ; \angle) $\therefore \frac{RW}{RS} = \frac{RT}{RP}$ ($\Delta RTW \Delta RPS$) $\therefore RT = \frac{RW \cdot RP}{RS}$	✓ S ✓ R (2)
10.3	$y = \hat{T}_2 = \hat{R}_3$ (tan chord theorem/Rkl-koordst) $y = \hat{R}_3 = \hat{Q}_1$ (\angle s in same segment/ \angle e in dieselfde segment)	✓ S ✓ R ✓ S ✓ R (4)



10.4	$\hat{Q}_3 = \hat{P}\hat{S}\hat{R}$ (ext \angle of cyc quad/buite \angle v kdvh) $\hat{P}\hat{S}\hat{R} = \hat{W}_2$ (corresp \angle s/ooreenk \angle e ; WT SP) $\therefore \hat{Q}_3 = \hat{W}_2$ OR/OF $\hat{Q}_2 = x$ (\angle s in same segment/ \angle e in dies segment) $\hat{Q}_3 = 180^\circ - (x + y)$ (\angle s on straight line/ \angle e op reguitlyn) $\hat{W}_2 = 180^\circ - (x + y)$ (\angle s of $\Delta WRT/\angle$ e v ΔWRT) $\therefore \hat{Q}_3 = \hat{W}_2$	✓ S ✓ R ✓ S ✓ R ✓ S ✓ S ✓ S
10.5	In ΔRTS and ΔRQP : $\hat{R}_3 = \hat{R}_2 = y$ (proven above/hierbo bewys) $\hat{S}_2 = \hat{P}_2$ (\angle s in same segment/ \angle e in dies segment) $R\hat{T}\hat{S} = R\hat{Q}\hat{P}$ (3 rd angle of Δ) $\therefore \Delta RTS \parallel\mid\mid \Delta RQP$ (\angle ; \angle ; \angle)	✓ S ✓ S/R ✓ S OR/OF (\angle ; \angle ; \angle)

<p>10.6</p> $\frac{RT}{RQ} = \frac{RS}{RP} \quad (\Delta RTS \Delta RQP)$ $\frac{RS}{RP} \times \frac{RS}{RP} = \frac{RT}{RQ} \times \frac{RS}{RP}$ $\left(\frac{RS}{RP}\right)^2 = \left(\frac{RT}{RP}\right) \left(\frac{RS}{RQ}\right)$ $= \left(\frac{RW}{RS}\right) \left(\frac{RS}{RQ}\right) \quad (\text{proven in 10.2/bewys in 10.2})$ $= \frac{RW}{RQ}$ <p>OR/OF</p> $\frac{RT}{RQ} = \frac{RS}{RP} \quad (\Delta RTS \Delta RQP)$ <p>But $RT = \frac{WR.RP}{RS}$ $(\text{proven in 10.2/bewys in 10.2})$</p> $\therefore \frac{RT}{RQ} = \frac{WR.RP}{RQ.RS} = \frac{RS}{RP}$ $WR.RP^2 = RQ.RS^2$ $\therefore \frac{WR}{RQ} = \frac{RS^2}{RP^2}$ <p>OR/OF</p> $\frac{RT}{RS} = \frac{RQ}{RP} \quad (\Delta RTS \Delta RQP)$ $RQ = \frac{RT.RP}{RS}$ <p>and $WR = \frac{RT.RS}{RP}$ $(\text{proven in 10.2/bewys in 10.2})$</p> $\frac{WR}{RQ} = \frac{\frac{RT.RS}{RP}}{\frac{RT.RP}{RS}}$ $= \frac{RT.RS}{RP} \times \frac{RS}{RT.RP}$ $= \frac{RS^2}{RP^2}$	<p>✓ S</p> <p>✓ $\times \frac{RS}{RP}$ on both sides</p> <p>✓ $\left(\frac{RT}{RP}\right) \left(\frac{RS}{RQ}\right)$ (3)</p> <p>✓ S</p> <p>✓ $RT = \frac{WR.RP}{RS}$</p> <p>✓ multiplication/ vermenigvuldig (3)</p> <p>✓ S</p> <p>✓ $WR = \frac{RT.RS}{RP}$</p> <p>✓ simplification/ vereenvoudiging (3) [20]</p>
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TOTAL/TOTAAL: 150