MATH220/MATH240 INDIVIDUAL COURSEWORK

LEWIS MCCONKEY

1. Computing ${}^{\mathscr{C}}[T]^{\mathscr{C}}, {}^{\mathscr{B}}[T]^{\mathscr{C}}, {}^{\mathscr{C}}[T]^{\mathscr{B}}$ and ${}^{\mathscr{B}}[T]^{\mathscr{B}}$

For each of the basis vectors in $\mathscr{C}$ we apply T, and the coordinates of those vectors with respect to $\mathscr{C}$ will be the columns of ${}^{\mathscr{C}}[T]^{\mathscr{C}}$

$$T(\big(1,0\big)) = (2,1),\;\; T(\big(0,1\big)) = (-1,3) \tag{1.1}$$

Since $\mathscr{C}$ is the standard basis of $\mathbb{R}^2$ the resulting matrix is as follows:

$${}^{\mathscr{C}}[T]^{\mathscr{C}} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \tag{1.2}$$

Now finding ${}^{\mathscr{B}}[T]^{\mathscr{C}}$. We already know what we get by applying T to the $\mathscr{C}$ basis vectors by (**??**). Notice that:

$$\big(2,1\big) = \big(1,0\big) + \big(1,1\big)\,,\;\; \big(-1,3\big) = -4\,\big(1,0\big) + 3\,\big(1,1\big) \tag{1.3}$$

Hence the matrix is as follows:

$${}^{\mathscr{B}}[T]^{\mathscr{C}} = \begin{bmatrix} 1 & -4 \\ 1 & 3 \end{bmatrix} \tag{1.4}$$

For each of the basis vectors in $\mathscr{B}$ we apply T, and the coordinates of those vectors with respect to $\mathscr{C}$ will be the columns of ${}^{\mathscr{C}}[T]^{\mathscr{B}}$

$$T(\big(1,0\big)) = (2,1),\;\; T(\big(1,1\big)) = (1,4) \tag{1.5}$$

Since $\mathscr{C}$ is the standard basis of $\mathbb{R}^2$ the resulting matrix is as follows:

$${}^{\mathscr{C}}[T]^{\mathscr{B}} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \tag{1.6}$$

Now finding ${}^{\mathscr{B}}[T]^{\mathscr{B}}$. We already know what we get by applying T to the $\mathscr{B}$ basis vectors by (**??**). Notice that:

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$$\big(2,1\big) = 1\,\big(1,0\big) + 1\,\big(1,1\big)\,,\ \ \big(1,4\big) = -3\,\big(1,0\big) + 4\,\big(1,1\big) \tag{1.7}$$

Hence the matrix is as follows:

$$^{\mathscr{B}}[T]^{\mathscr{B}} = \begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix} \tag{1.8}$$

2. Computing $^{\mathscr{B}}[T \circ T]^{\mathscr{B}}$

We need to apply T twice to the $\mathscr{B}$ basis vectors and find the coordinates of these with respect to $\mathscr{B}$. These coordinates will be the columns of the matrix $^{\mathscr{B}}[T \circ T]^{\mathscr{B}}$

$$T(T(\big(1,0\big)) = T(\big(2,1\big)) = \big(3,5\big) = -2\,\big(1,0\big) + 5\,\big(1,1\big) \tag{2.1}$$

$$T(T(\big(1,1\big)) = T(\big(1,4\big)) = \big(-2,13\big) = -15\,\big(1,0\big) + 13\,\big(1,1\big) \tag{2.2}$$

Hence the matrix is as follows:

$$^{\mathscr{B}}[T \circ T]^{\mathscr{B}} = \begin{bmatrix} -2 & -15 \\ 5 & 13 \end{bmatrix} \tag{2.3}$$

3. Verifying that $(^{\mathscr{B}}[T]^{\mathscr{C}})(^{\mathscr{C}}[T]^{\mathscr{B}}) = {}^{\mathscr{B}}[T \circ T]^{\mathscr{B}} = (^{\mathscr{B}}[T]^{\mathscr{C}})(^{\mathscr{C}}[T]^{\mathscr{B}})$

$$(^{\mathscr{B}}[T]^{\mathscr{C}})(^{\mathscr{C}}[T]^{\mathscr{B}}) = \begin{bmatrix} 1 & -4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -15 \\ 5 & 13 \end{bmatrix} = {}^{\mathscr{B}}[T \circ T]^{\mathscr{B}} \tag{3.1}$$

$$(^{\mathscr{B}}[T]^{\mathscr{B}})(^{\mathscr{B}}[T]^{\mathscr{B}}) = \begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -15 \\ 5 & 13 \end{bmatrix} = {}^{\mathscr{B}}[T \circ T]^{\mathscr{B}} \tag{3.2}$$

These two equations hold true by equation (**??**). Hence:

$$(^{\mathscr{B}}[T]^{\mathscr{C}})(^{\mathscr{C}}[T]^{\mathscr{B}}) =^{\mathscr{B}} [T \circ T]^{\mathscr{B}} = (^{\mathscr{B}}[T]^{\mathscr{C}})(^{\mathscr{C}}[T]^{\mathscr{B}}) \tag{3.3}$$