MATH220/MATH240 INDIVIDUAL COURSEWORK

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$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \tag{1}$$

Since the matrix A is upper triangular the eigenvalues are the entries of the diagonals. The characteristic polynomial is $(2-\lambda)^2(1-\lambda)$. Hence the eigenvalues are $\lambda = 1, \lambda = 2$ (repeated).

In the case when $\lambda = 1$:
Finding all vectors such that $(A - I_3)\overrightarrow{x} = \overrightarrow{0}$. In other words:

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

From equation (**??**) we get $x+y-z = 0, y = 0, z = 0$. So $x = y = z = 0$. Hence the solution set to this system of equations is the eigenspace:

$$V_1 = \{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x = y = z = 0\} = span_{\mathbb{R}}\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \} \tag{3}$$

In the case when $\lambda = 2$:
Finding all vectors such that $(A - 2I_3)\overrightarrow{x} = \overrightarrow{0}$. In other words:

$$\begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

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From equation (**??**) we get $x-y=0, -y+z=0, 0=0$. So $x=y=z$. Hence the solution set to this system of equations is the eigenspace

$$V_2 = \{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x \in \mathbb{R} \} = span_{\mathbb{R}} \{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \} \tag{5}$$

The two vectors (1,1,1) and (0,0,0) are both eigenvectors. However they are not linearly independent so by theorem 1.44. they don't form a basis of $\mathbb{R}^3$.