FINDING THE ANGLE BETWEEN TWO MATRICES

LEWIS MCCONKEY

Let:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix} \tag{1}$$

First we need to calculate $\langle A, B \rangle$ with respect to the inner product:

$$\langle A, B \rangle := tr(AB^T) \tag{2}$$

In the case where A and B are the matrices stated above:

$$tr(\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}) = tr(\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -16 \\ 0 & -2 & 8 \end{bmatrix}) = 10 \tag{3}$$

Next we need to calculate $\|A\|$ and $\|B\|$:

$$\|A\| = \langle A, A \rangle = tr(AA^T) \quad \|B\| = \langle B, B \rangle = tr(BB^T) \tag{4}$$

Computing $tr(AA^T)$ and $tr(BB^T)$ in the same way as we did in (2) and in (3) by multiplying A by A transpose and B by B transpose and finding the trace of the resulting matrices we get that:

$$\|A\| = \sqrt{19} \quad \|B\| = \sqrt{66} \tag{5}$$

Therefore inputting our findings from (3) and (5) into the formula given in the lecture notes we get that:

$$\cos\theta = \frac{\langle A, B \rangle}{\|A\|\|B\|} = \frac{10}{\sqrt{19} \cdot \sqrt{66}} \tag{6}$$

Hence using a calculator we get that $\theta = 73.597$ (3 d.p.)

Date: November 16, 2022.