5.1

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March 2020

1 5.1:1

1.1 a

(V_1, E_1) is a subgraph of G because every vertex in V_1 is in G and every edge in E_1 is in G and connects vertices in V_1.

1.2 b

(V_2, E_2) is not a subgraph of G because cj is not an edge in G

1.3 c

(V_3, E_3) is not an induced subgraph because dh is not in E_3 but it is in G

1.4 d

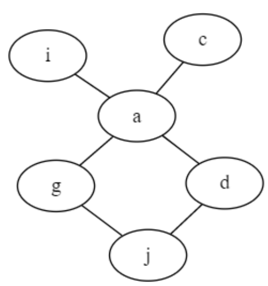

Figure 1: Induced subgraph of g, j, d, a, c, i

1.5 e

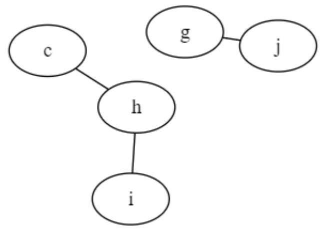

Figure 2: Induced subgraph of c, h, g, i, j

1.6 f

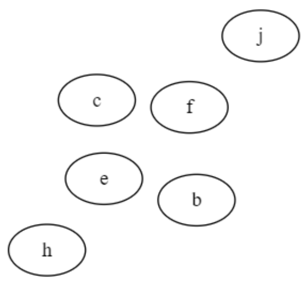

Figure 3: Non-induced subgraph of e, f, b, c, h, j

This is a non-induced subgraph because every node is in G and there exists an edge in G that connects two nodes in the non-induced subgraph that is not in the non-induced subgraph.

1.7 g

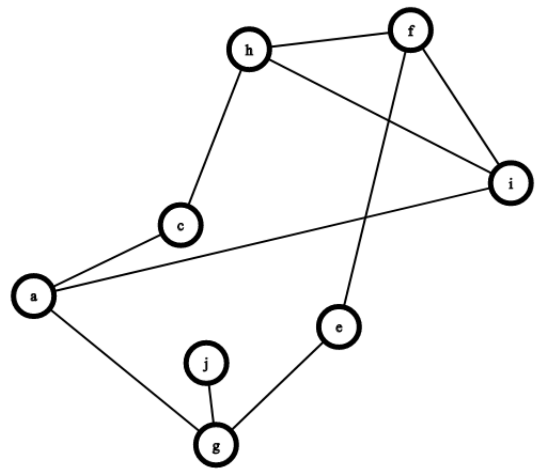

Figure 4: induced subgraph of G with 10 edges
This is an induced subgraph because every edge in G that can be in this graph is in the graph. I created this graph by counting the number of total edges, which was 14, then removing vertices with a total degree of 4, along with the corresponding edges, and created this graph.

2 5.1:2

G_1 and G_3 are isomorphic, where

$$
\begin{aligned}
v_1 &\cong w_1 \\
v_2 &\cong w_5 \\
v_3 &\cong w_4 \\
v_4 &\cong w_2 \\
v_5 &\cong w_6 \\
v_6 &\cong w_3
\end{aligned}
$$

. G_2 and G_4 are not isomorphic because all vertices in G_2 have degree greater than one, while x_2 on G_4 has degree one.

3 5.1:4

Prove that every tree on n vertices has exactly $n-1$ edges.
Assume if a tree has $n-1$ vertices, it has $n-2$ edges. We need to show that a tree with n vertices has $n-1$ edges.
Base Case: A tree with 1 vertex has 0 edges.
Induction Step:
Assume we have an n tree with k edges. if we remove a leaf node from our n tree, we have created an $n-1$ tree with $k-1$ edges. Because we stated earlier that an $n-1$ tree has $n-2$ edges, $k-1 = n-2$, or $k = n-1$ and therefore an n tree has $n-1$ edges

4 5.1:6

Prove that in any simple graph (no loops or multiedges), there are two vertices with the same degree.
Proof by contradiction, assume there exists a graph where the degree of each vertex is unique.
For a graph of n vertices, the degree of any vertex can be $0, 1, 2, \ldots, n-1$ because there are at $n-1$ possible vertices that can be connected to the vertex in question. There are n possibilities for the degree of each vertex on a graph. Because we are assuming that there exists a graph where the degree of each vertex is unique, and there are n vertices and n possibilities for the degree of each vertex, it stands to reason that, on this graph, there is one vertex with a degree of 0, one vertex with a degree of 1, one vertex with a degree of 2, and so on, until we reach a vertex with a degree of $n-1$.
If we have a vertex of degree $n-1$, it has an edge connecting to every other vertex on the graph, because there are $n-1$ other vertices to be connected to on the graph. If we have a vertex with degree 0, it is connected to no vertices

on the graph. This is a contradiction because we can not have a vertex that connects to all other vertices and also a vertex that connects to no vertices, because the vertex with degree $n-1$ needs to be connected to the vertex with degree 0, but since this defies what we stated about a vertex of degree 0, this is a contradiction, and therefore there must be at least two vertices that have the same degree on any graph.