BEARING-ONLY FORMATION CONTROL WITH DIRECTED SENSING

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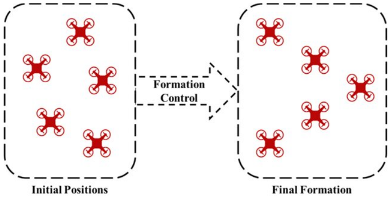
TASP | TECHNION AUTONOMOUS SYSTEMS PROGRAM

CoNeCt Cooperative Networks and Controls Lab

Given a group of autonomous agents operating in a common environment, design a distributed control strategy for each agent such that the agents **achieve and maintain a desired spatial arrangement or target formation**.

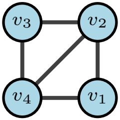


FORMATION CONTROL OF ROBOTS

SENSING CONDITIONS

Undirected sensing:

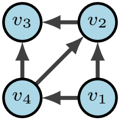
- The sensing between agents is symmetric.
- unrealistic



Undirected sensing graph

Directed sensing:

- The sensing between every couple of agents is not necessarily symmetric.
- physically motivated

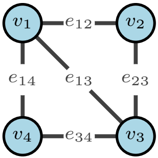


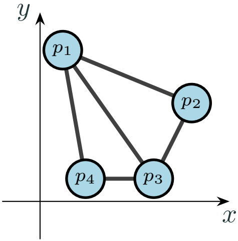
Directed sensing graph

Set up the formation control system

- Distributive control strategy
- Single integrator
  $\dot{p}_i = u_i$
- Sensing graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$
- Control objective
  Drive the system to target formation asymptotically

A framework is formed by mapping the agents in underlying graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ (with $|\mathcal{V}| = n,\ |\mathcal{E}| = m$) to a configuration $p = [p_1^T, p_2^T, \cdots, p_n^T]^T$ ($p_i \in \mathbb{R}^{\mathrm{d}}$).





Displacement measurement:

$z_{ij} = p_j - p_i$

Distance measurement:

$d_{ij} = ||z_{ij}||$

Bearing measurement:

$g_{ij} = \frac{z_{ij}}{d_{ij}}$

Bearing vector:

$g = [g_1^T, \cdots, g_m^T]^T$

Bearing function$F_B : \mathbb{R}^{\mathrm{d}n} \to \mathbb{R}^{\mathrm{d}m}$:

$F_B(p) = g$

Bearing Formation (corresponding to framework):

$(\mathcal{G}, g)$

FORMULATION OF BEARING FORMATION CONTROL SYSTEM

Set up the formation control system

- Distributive control strategy
- Single integrator
  $\dot{p}_i = u_i$
- Sensing graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$
- Control objective
  Drive the system to target formation asymptotically

Especially for bearing formation control:

- Target shape is described by Target bearing formation $(\mathcal{G}, \mathrm{g})$.
- The design of control strategy mainly depends on the current bearing measurement g.
- The control objective is $g(t) = F_B(p(t)) \to \mathrm{g}$ when $t \to \infty$.

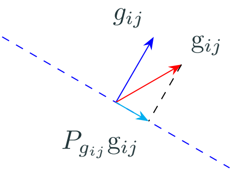
The bearing-only formation control [Zhao '2016] for undirected sensing:

$$\boxed{u_i = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}} \mathbf{g}_{ij}, \forall i \in \mathcal{V}}$$

Projection Matrices

$$P_{g_{ij}} = P(g_{ij}) = I_{\mathsf{d}} - g_{ij} g_{ij}^T$$

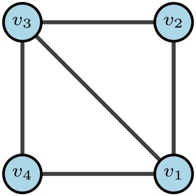
$$P_{g_{ij}} \mathrm{g}_{ij} = 0, \text{ when } g_{ij} \parallel \mathrm{g}_{ij}$$



The bearing-only formation control [Zhao '2016] for undirected sensing:

$$u_i = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}} \mathbf{g}_{ij}, \forall i \in \mathcal{V}$$

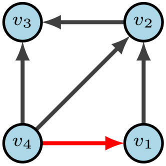
The MAS converges to the target formation almost globally asymptotically.



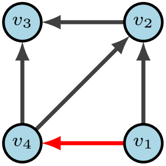
The target formation

Bearing-only formation control with directed sensing.

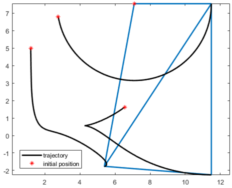
$$\boxed{u_i = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}} \mathbf{g}_{ij}, \forall i \in \mathcal{V}}$$



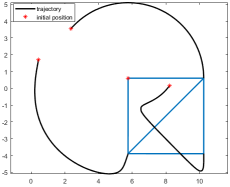
The target formation



The target formation

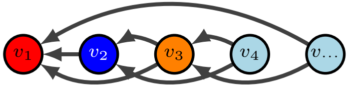


Trajectory



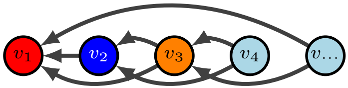
Trajectory

Trinh '2018 is the first to work on the directed sensing. The directed sensing graph is called Leader first follower (LFF) graph generated from Henneberg construction.



Property:

- Orderliness
- LFF structure
  - Leader: agent with no outgoing edge
  - First follower: agent with only one outgoing edge towards the leader.
- Exactly two outgoing edges for agents except LFF



The control input of agent v_i is a function of its neighbour and itself:

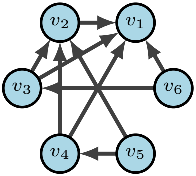
$$\dot{p}_i = u_i = -\sum_{j \in \mathcal{N}_i} P_{g_{ij}} \mathbf{g}_{ij}$$

The control system with the specific directed sensing:

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \vdots \\ \dot{p}_n \end{bmatrix} = \begin{bmatrix} u_1(p_1) \\ u_2(p_1, p_2) \\ u_3(p_1, p_2, p_3) \\ \vdots \\ u_n(p_1, p_2, p_3, \cdots, p_{n-1}, p_n) \end{bmatrix}$$

Theorem [Trinh '2018]

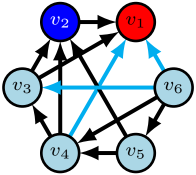
For the MAS whose sensing graph is LFF graph, bearing-only formation control asymptotically drives the MAS to a final configuration satisfying the target framework from almost any initial configuration.



The target formation

How can we expand the LFF formation?

- Ordered structure kept
- Leader and First follower kept
- Exactly two outgoing edges for the agents except LFF extended



The target formation

Theorem

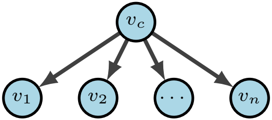
If the sensing graph satisfies the following conditions,

- There is a leader and first follower.
- The structure is ordered.
- Every vertex other than the LFF has at least two outgoing edges.

Then the bearing-only formation control drives the MAS to target formation.

The subsystem of agents with more than two outgoing edges.

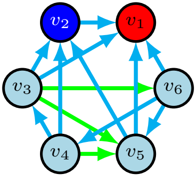
- Equilibrium analysis is strongly nonlinear.
  Defining a corresponding linear problem.
- Null space analysis
- Geometrical constraints



Sensing graph for the subsystem

MOTIVATION ON GRAPH EXPANSION

- Ordered structure extended
- Leader and First follower kept
- Exactly two outgoing edges for the agents except LFF extended



The target formation

Theorem

If the sensing graph satisfies the following conditions:

- There exists a leader and a first follower
- It contains a subgraph which is LFF graph generated from Henneberg construction

Then the MAS controlled by bearing-only formation control has only two equilibrium: $g^* = \pm \mathrm{g}$, including the target formation.

Conjecture

The equilibrium $-\mathrm{g}$ is unstable, while the simulation shows the equilibrium g is asymptotically stable.

PROOF SKETCH

- The cascade structure disappears.
  Analyze the whole system directly.
- Equilibrium analysis is strongly nonlinear.
  The same trick as previous.

FUTURE WORK

- Bearing rigidity theory on frameworks with directed underlying graphs
- Stability analysis for the disordered LFF formation
- Further expansion on disordered LFF formation
- Bearing-only formation with dynamic sensing condition

Thank-You!

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