Diodes and the Importance of Network Orientations in Diffusively-Coupled Networks

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Abstract

Diffusively-coupled networks, a common architecture for multi-agent systems, are used in many foundational cooperative control problems, such as consensus and formation control. One reason for its widespread use is that this structure contains information on agents, edge controllers, and the latent interaction network between agents, making passivity theory a natural tool for its analysis. However, an implicit property of this structure is the symmetric feedback interconnection of the incidence matrix, which erases any orientation for the network that may be encoded in the incidence matrix. So, the diffusively-coupled structure is useful when modeling undirected networks, and the existing passivity-based analytical framework is primarily effective for undirected networks. These present the challenge of how to model directed networks effectively as an unresolved issue.

In this work, inspired by the single conductance property of a diode, we explore a special instance of diffusive networks where the edge controllers follow the model of an ideal diode. The diffusive diode networks highlight the importance of the chosen edge orientation with respect to the incidence state values and exhibit system behaviors that correspond to directed networks or undirected networks. To enable a more direct comparison to the undirected and directed consensus protocols, we focus on the case where the agents are modeled by integrator dynamics.

We begin with a basic analysis of the networked system with diodes. We show that the generated trajectories are bounded and continuous, and the closed-loop system can be expressed as a linear consensus protocol over a time-varying graph. This is followed by an example illustrating the complexity of diffusive diode networks. With these observations, we present a sufficient condition on when the diffusive diode networks can achieve average consensus, which does not highlight the role of the initial conditions or the orientation of the graphs. Then, for some special directed graphs (i.e., paths, cycles, rooted out-trees), we discuss how the system state together with the graph orientation, influences the network steady-state. We explore sufficient conditions on the graph and the initial conditions of the network that lead to consensus. We also provide a negative result providing a necessary condition that graphs containing rooted out-trees can not achieve consensus. Our results are supported by numerous numerical examples.

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