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Coordination of multi-robot systems with bearing measurements

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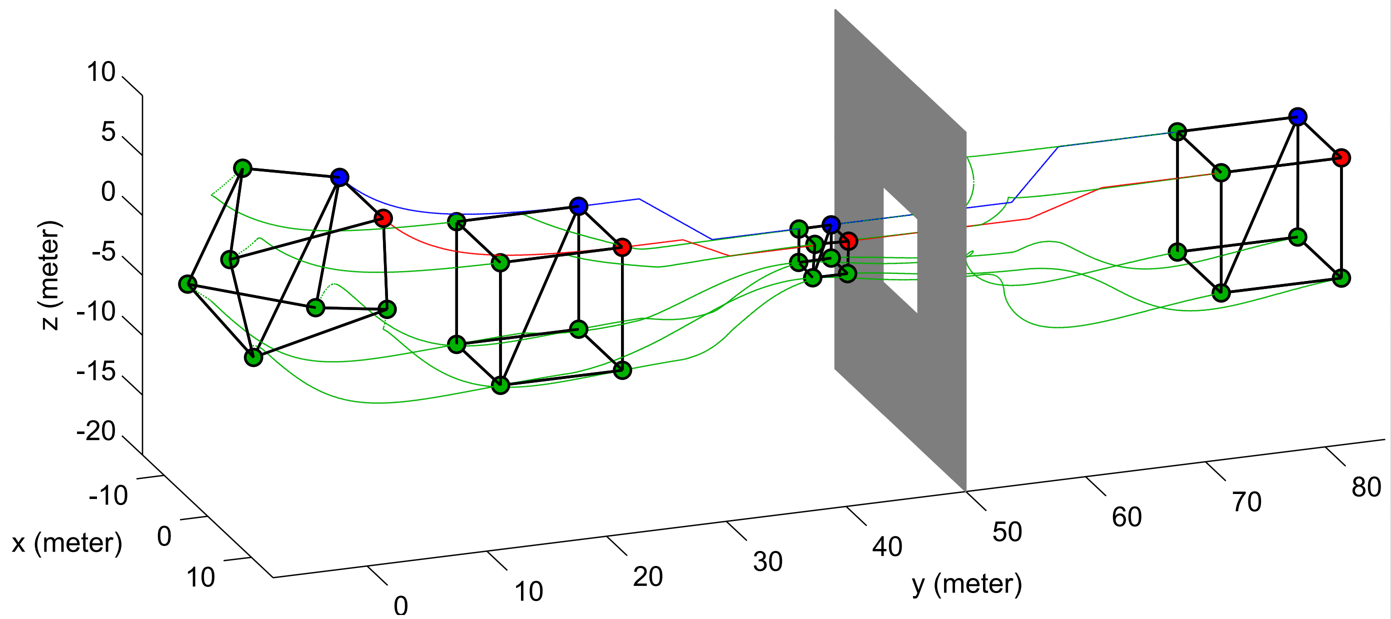
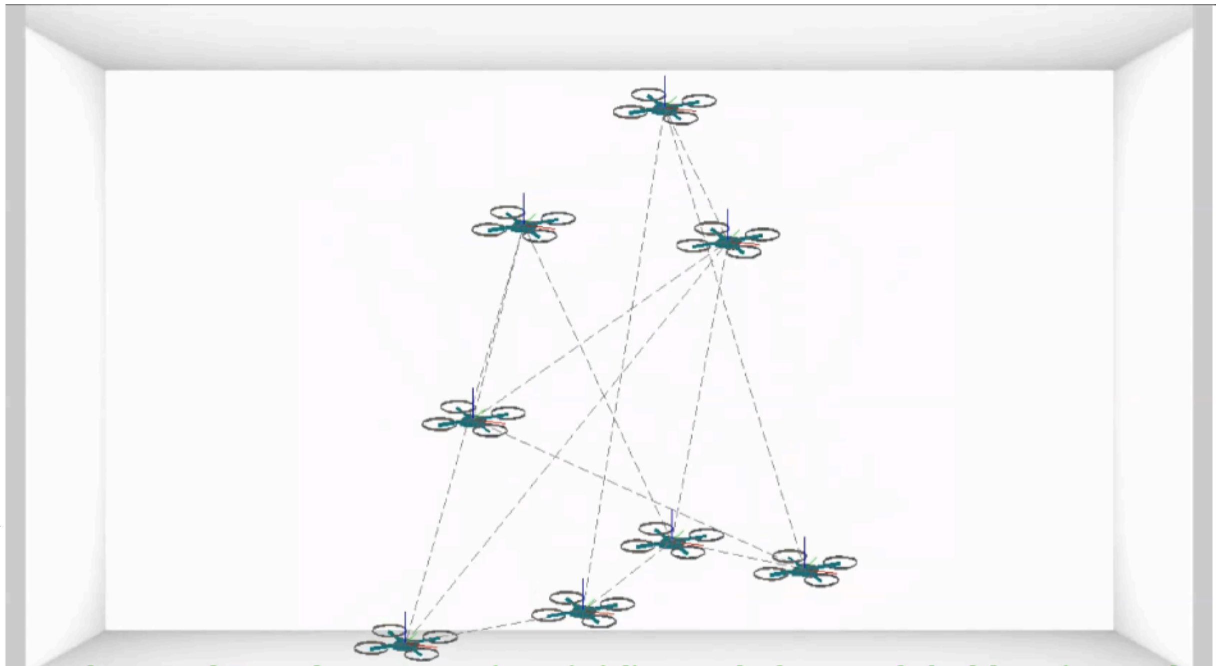
CDC 2015 - Workshop on Taxonomies of Interconnected Systems:
Partial and Imperfect Information in Multi-Agent Networks

Multi-Robot Systems

Formation Control is one of the canonical problems in multi-agent and multi-robot coordination





S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-only formation stabilization," IEEE Transactions on Automatic Control, 2015

Challenges in Multi-Robot Systems

Sensing

- GPS
- Relative Position Sensing
- Range Sensing
- Bearing Sensing

Communication

- Internet
- Radio
- Sonar
- MANet

Solutions to formation control problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

selection criteria depends on mission requirements, cost, environment…

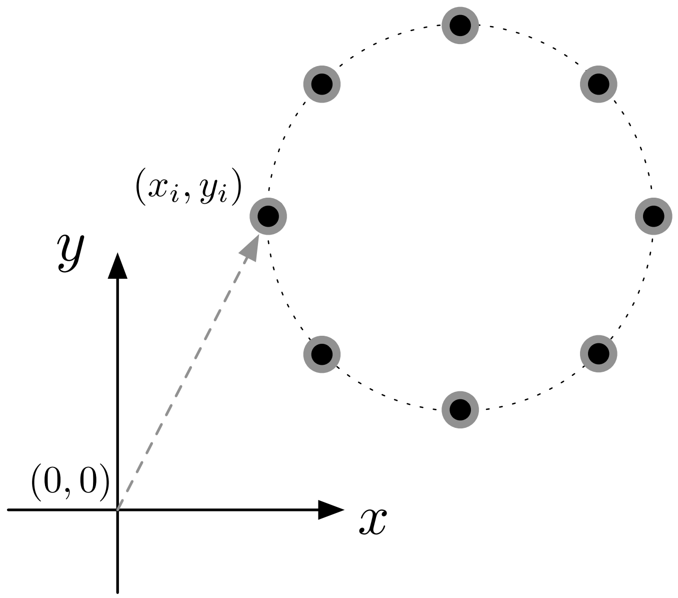
Challenges in Multi-Robot Systems

Solutions to coordination problems in multi-robot systems are *highly* dependent on the sensing and communication mediums available!

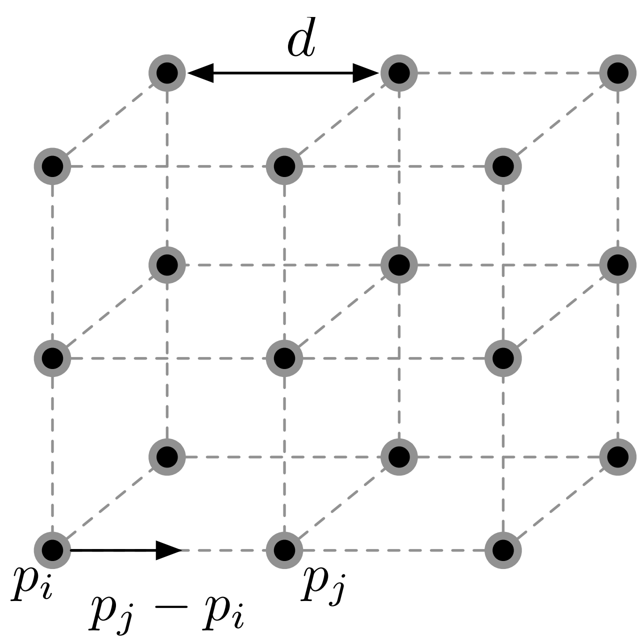
selection criteria depends on mission requirements, cost, environment…

In real-world implementations, formation control must be achieved with *impartial or imperfect information* about the state of the entire formation

Formation Control Strategies



- formation specified in a global coordinate system
- each agent assigned to a point in formation
- assumes GPS-type measurements



- formation specified by inter-agent *distances*
- agents tasked at maintaining distances to certain neighbors
- assumes distance sensing and relative-position information in a common reference frame

Distance-Based Formation Control

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Distance-Based Formation Control Law

$$\dot{p}_i = u_i$$

$$u_i = -\sum_{j\sim i}\left(\|p_i - p_j\|^2 - d_{ij}^2\right)(p_i - p_j)$$

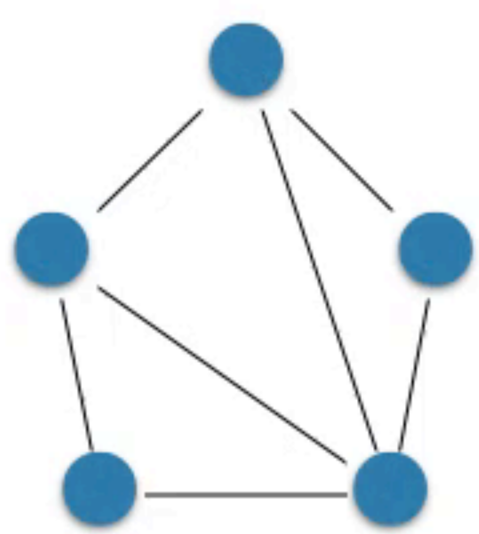
[Krick2007, Anderson2008, Dimarogonas2008, Dörfler2010]

- convergence to desired formation shape depends on the structure of the underlying sensing/communication network
- local stability analysis

Rigidity Theory

Graph Rigidity Theory

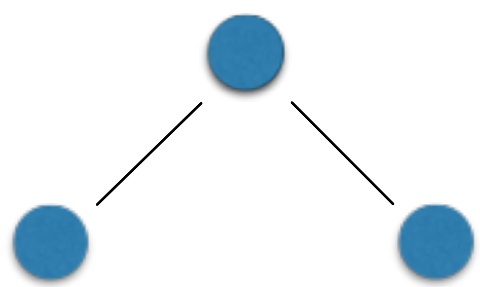
Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.



A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

Graph Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.



NOT rigid!

There is a motion that preserves distances between nodes in the graph but the shape is *not* preserved!

A *rigid* graph can only *rotate* and *translate* to ensure all distances between all nodes are preserved (i.e., preserve the shape)!

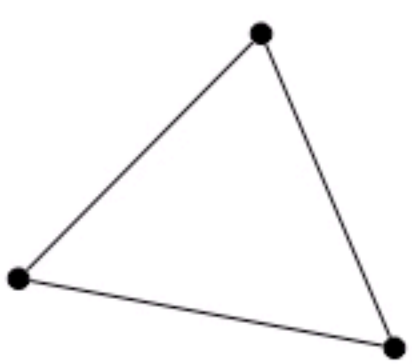
Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

Distance Rigidity

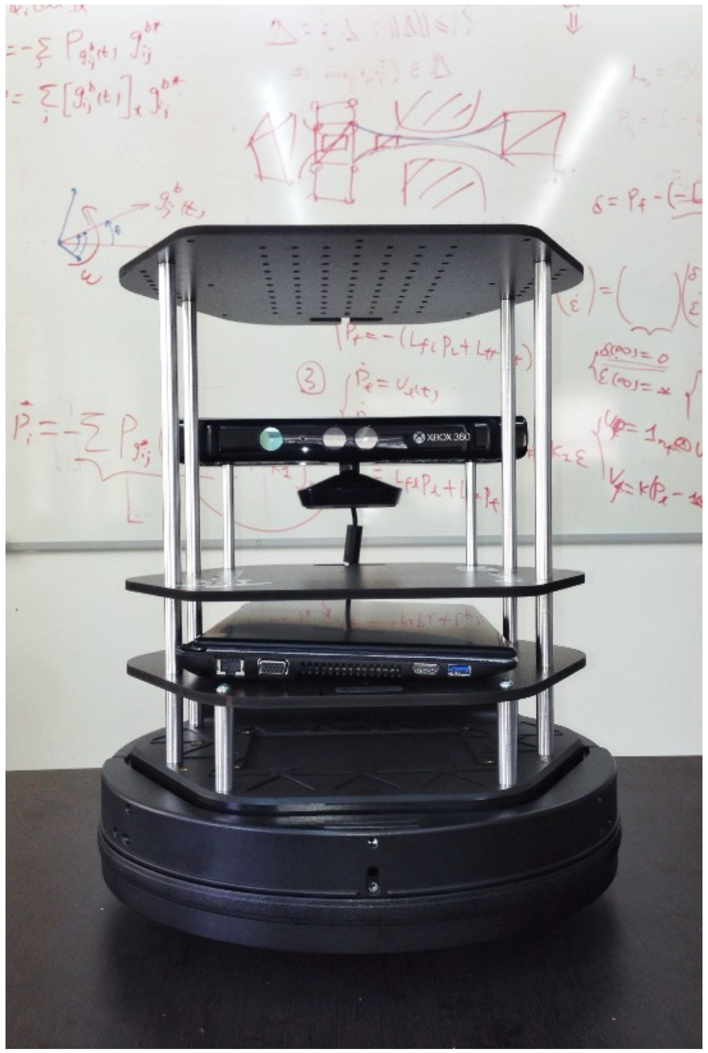
- maintain distance pairs
- rigid body translations and rotations

- rigidity for undirected graphs
- directed graph extensions - persistence [Hendrickx, Anderson, Yu]
- distance-only extensions [Cao]



- requires range sensing

Rigidity Theory - Bearing Extensions



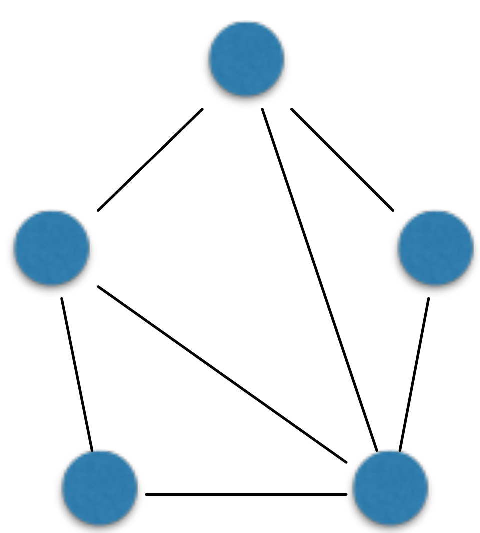
TurtleBotII with Kinect Sensor

recently, there is an interest in *bearing-based* formation control

- (relatively) cheaper sensing
  - vision-based sensors
  - angle-of-arrival sensors

Bearing Rigidity Theory

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.



A *bearing rigid* graph can *scale* and *translate* to ensure bearings between all nodes are preserved (i.e., preserve the shape)!

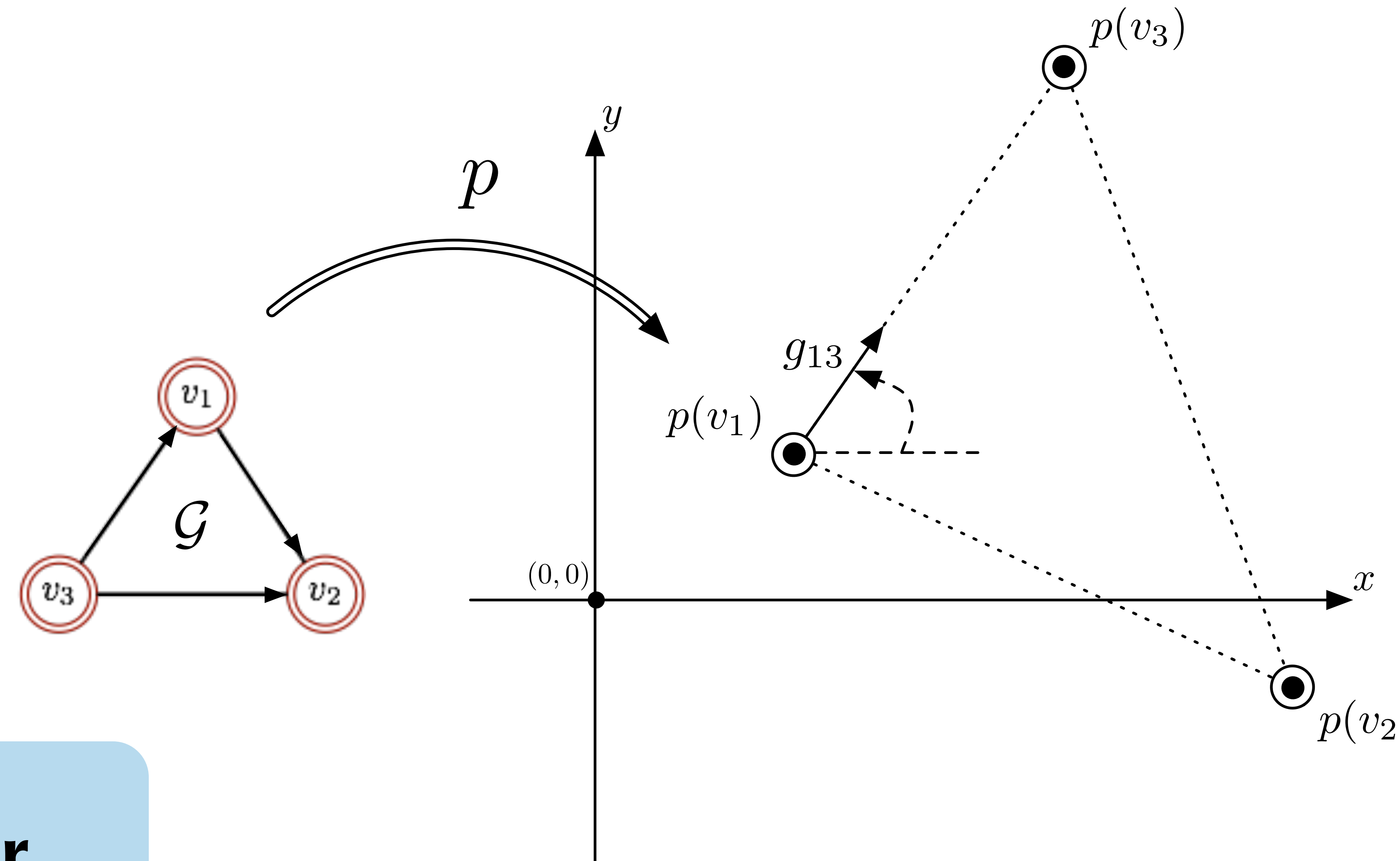
Bearing Rigidity Theory

bar-and-joint frameworks

$(\mathcal{G}, p)$

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a graph

$p : \mathcal{V} \to \mathbb{R}^2$



relative bearing vector

$$g_{ij} = \frac{p(v_j) - p(v_i)}{\|p(v_j) - p(v_i)\|}$$

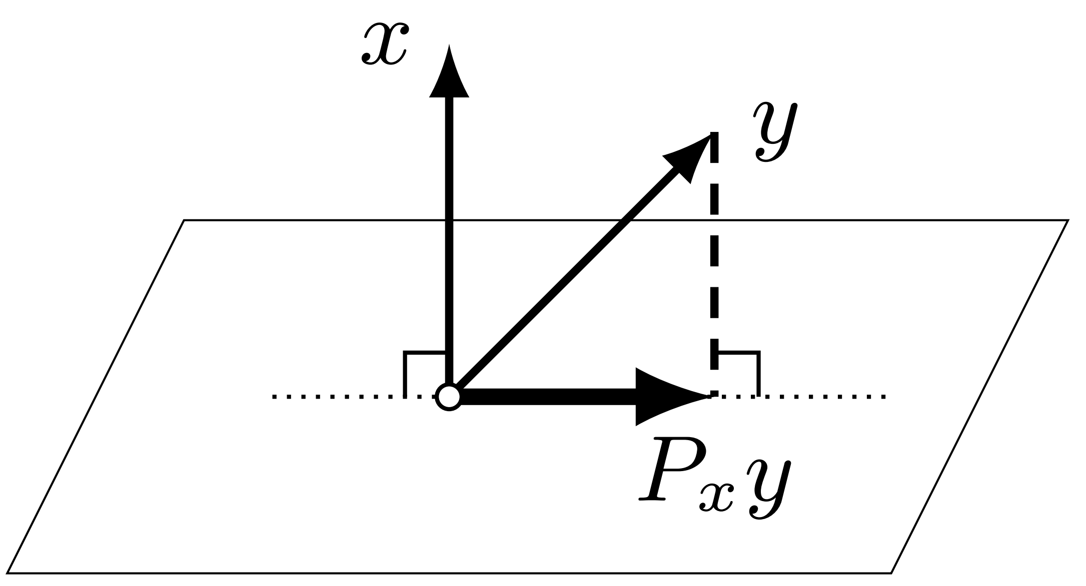
$$F_B(p) \triangleq \begin{bmatrix} g_1^{\mathrm{T}} & \cdots & g_m^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{dm}$$

When is a framework bearing rigid?

Bearing Rigidity Theory

orthogonal projection operator

$$P_x = I - \frac{1}{\|x\|^2} x x^T$$



- $\mathrm{Null}(P_x) = \mathrm{span}\{x\} \iff P_x y = 0$ iff $x \parallel y$.
- $P_x^T = P_x$ and $P_x^2 = P_x$.
- P_x is positive semi-definite.

- "parallel" vectors have the same relative bearing vectors
- *arbitrary dimensions*

Bearing Rigidity Theory

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations and scalings).

Theorem

A framework is **bearing infinitesimally rigid** if and only if the rank of the bearing rigidity matrix is *dn-d-1*.

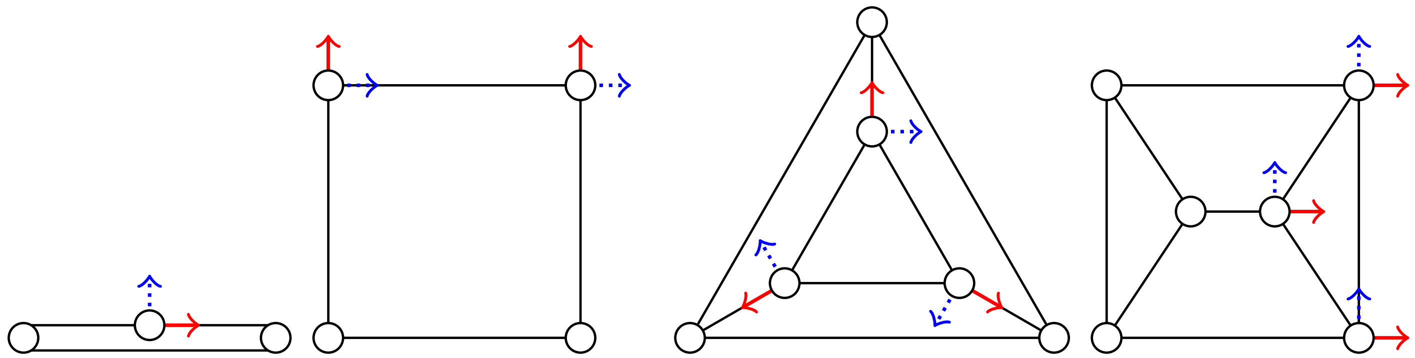
Bearing Rigidity Matrix

$$R(p(\mathcal{V})) = \frac{\partial F_{\mathcal{B}}(\mathcal{G})}{\partial p(\mathcal{V})} = \begin{bmatrix} \ddots & & \\ & \frac{P_{g_{ij}}}{\|p(v_i)-p(v_j)\|} & \\ & & \ddots \end{bmatrix} \left(E(\mathcal{G})^T \otimes I\right) \in \mathbb{R}^{md\times nd}$$

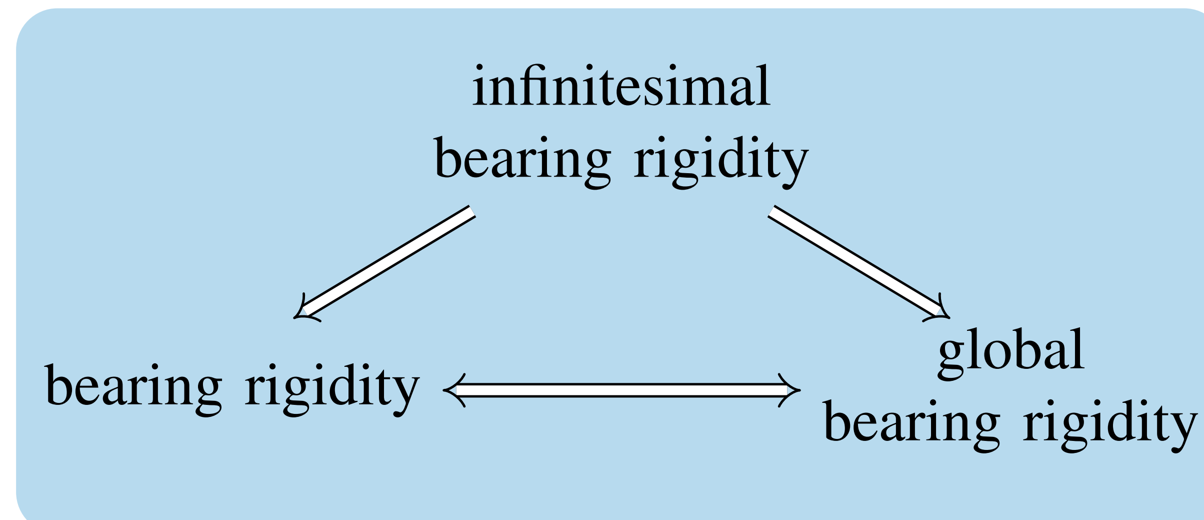
[Zhao and Zelazo, TAC2015]

Distance and Bearing Rigidity

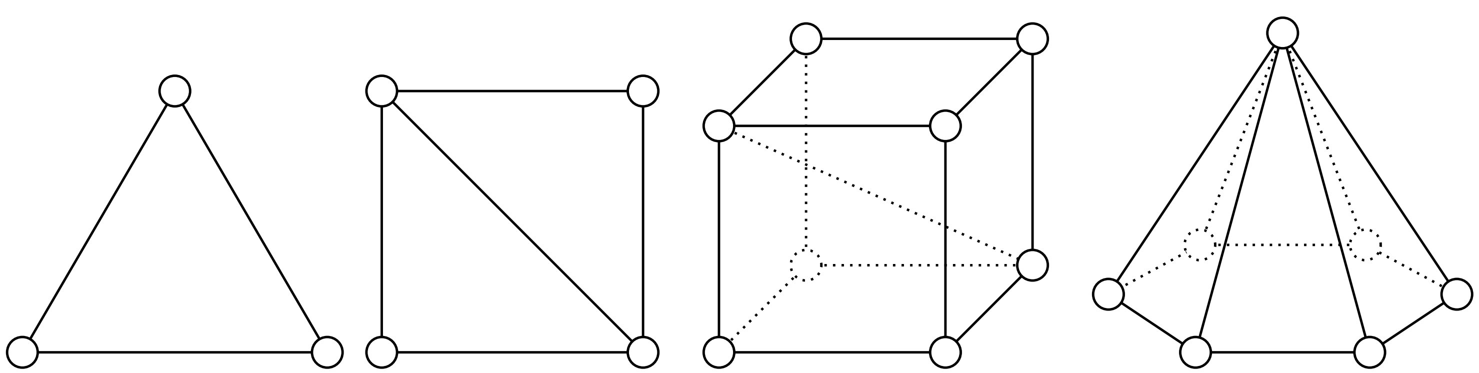
non-infinitesimally bearing rigid



(a) (b) (c) (d)



infinitesimally bearing rigid



(a) (b) (c) (d)

*this relation does *not* hold for distance rigidity

[Zhao and Zelazo, TAC2015]

Bearing-Based Formation Control

The **bearing-based formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent bearings.

A gradient controller

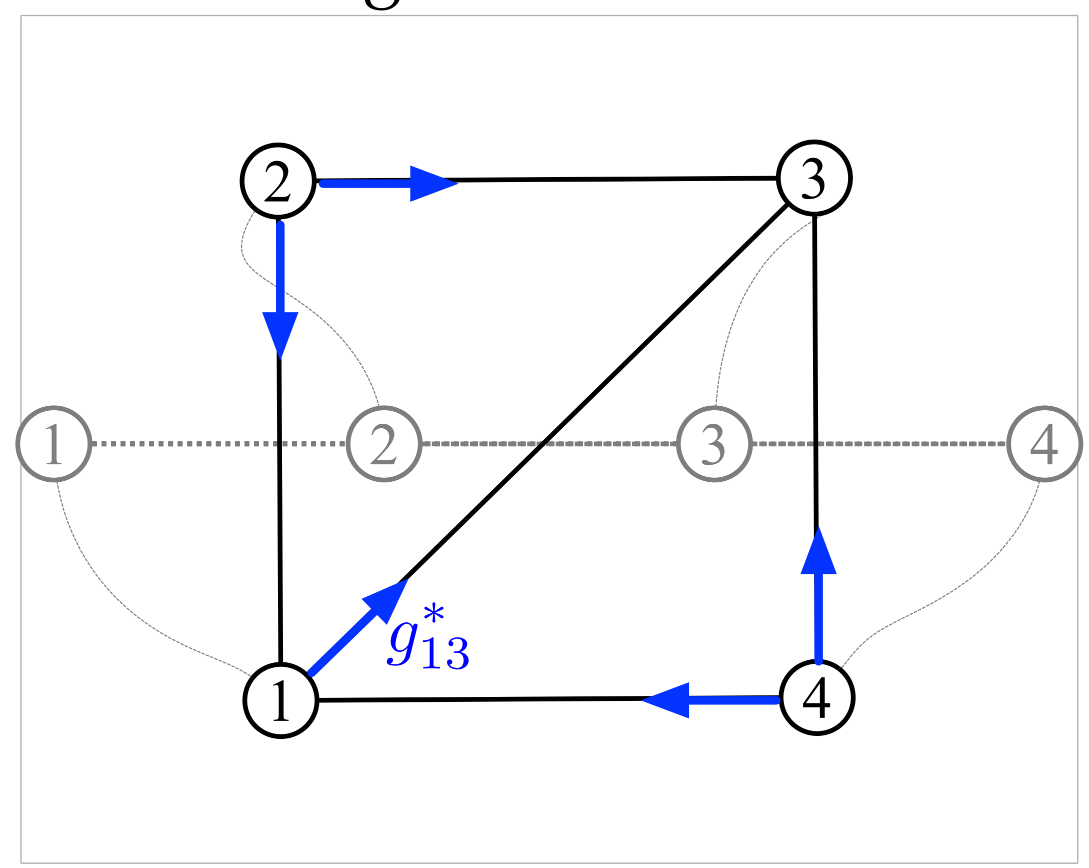
$$\Phi(p) = \sum_{\{i,j\}\in\mathcal{E}} \|g_{ij} - g_{ij}^*\|^2$$

$$u = -\nabla_p \Phi(p) = R^T(p) g^*$$

$$\dot{p}_i = -\sum_{j\sim i} \frac{1}{\|p_j - p_i\|} P_{g_{ij}} g_{ij}^*$$

- control requires bearings and *distances*!

target formation



A Bearing-Only Formation Controller

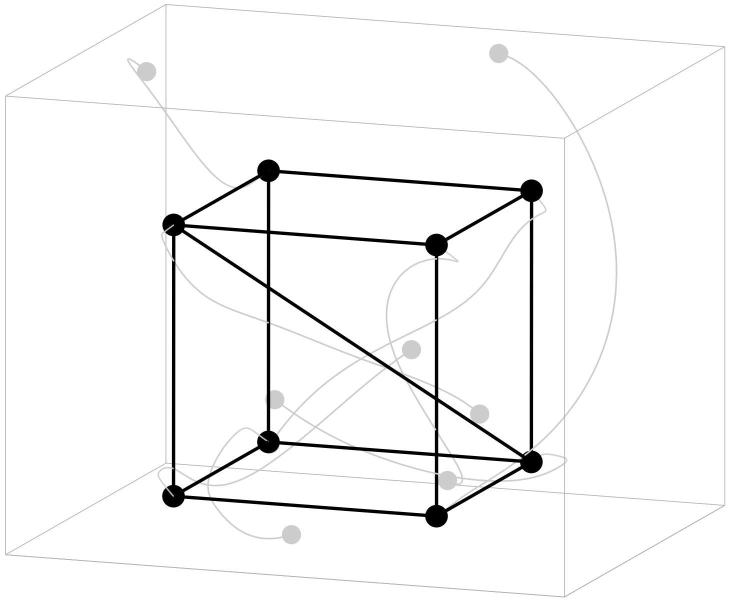
a bearing-only approach

$$\dot{p}_i(t) = -\sum_{j\sim i} P_{g_{ij}(t)} g_{ij}^*$$

stability analysis depends on the **bearing rigidity** of the formation!

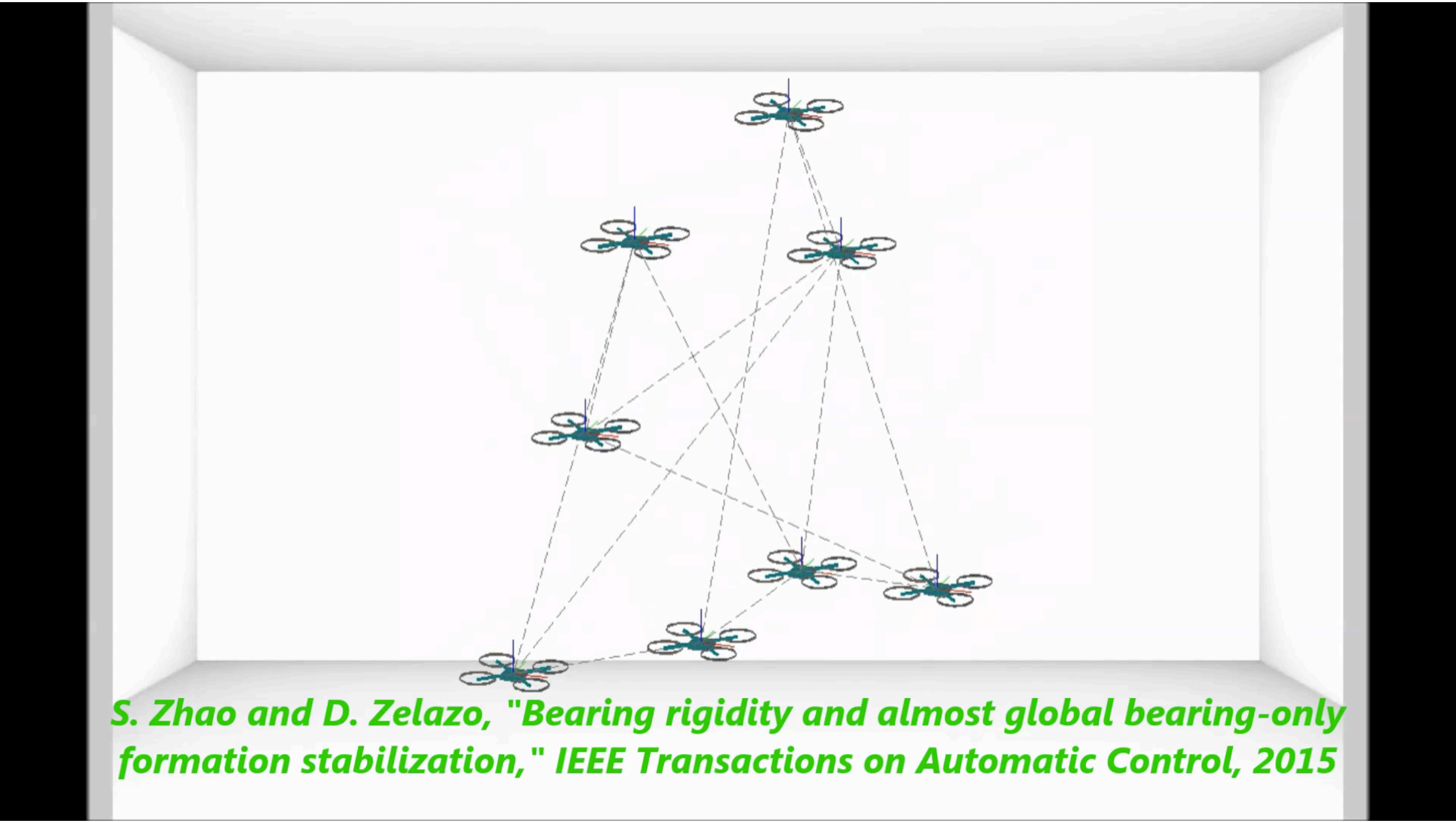
- a distributed protocol
- almost-global stability
  exponential stability
- centroid and scale invariance
- works for arbitrary dimension
- collision avoidance

x assumes undirected graph
x assumes common inertial frame



[Zhao and Zelazo, TAC2015]

A Bearing-Only Formation Controller



A Bearing-Only Formation Controller

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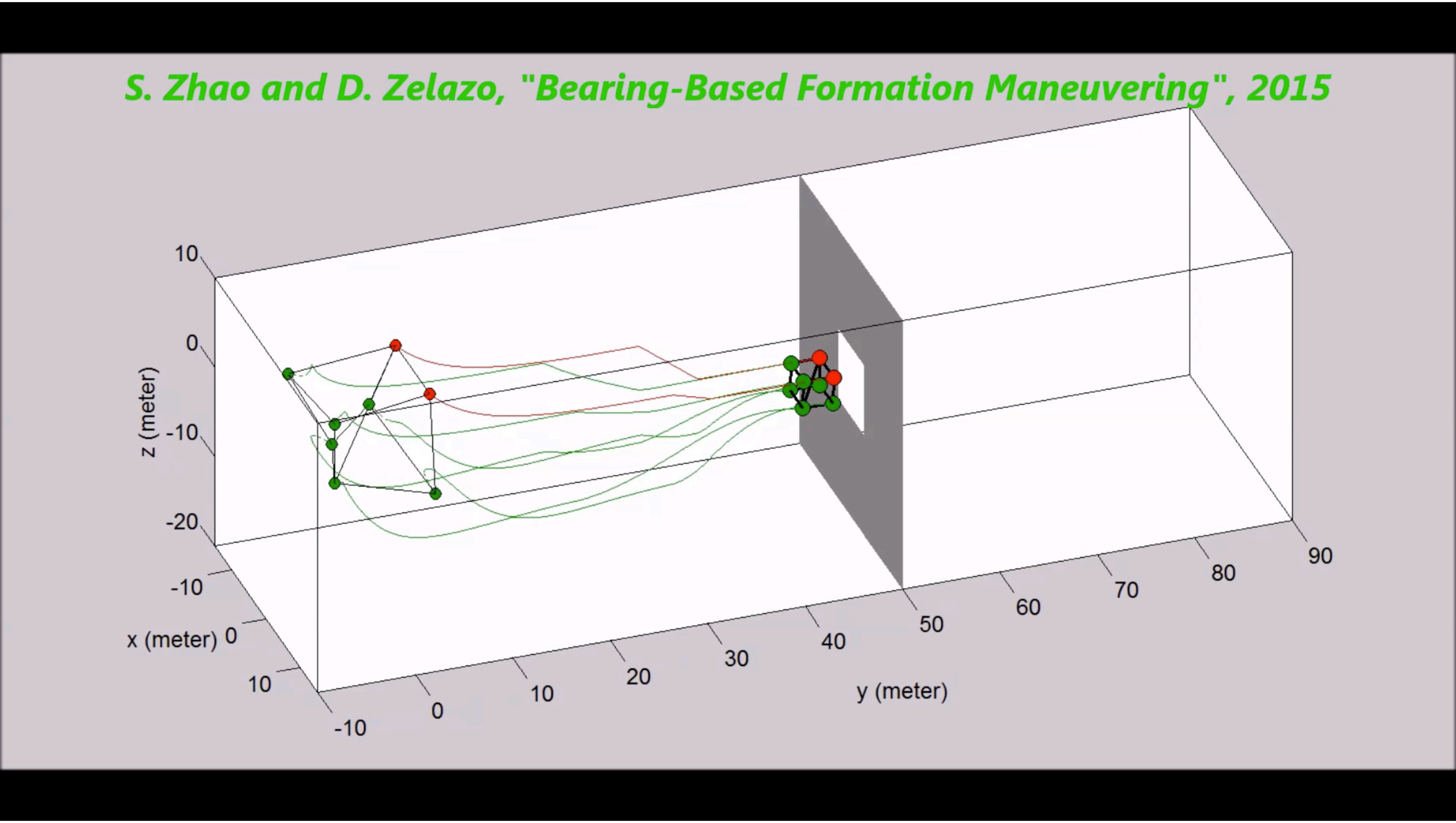
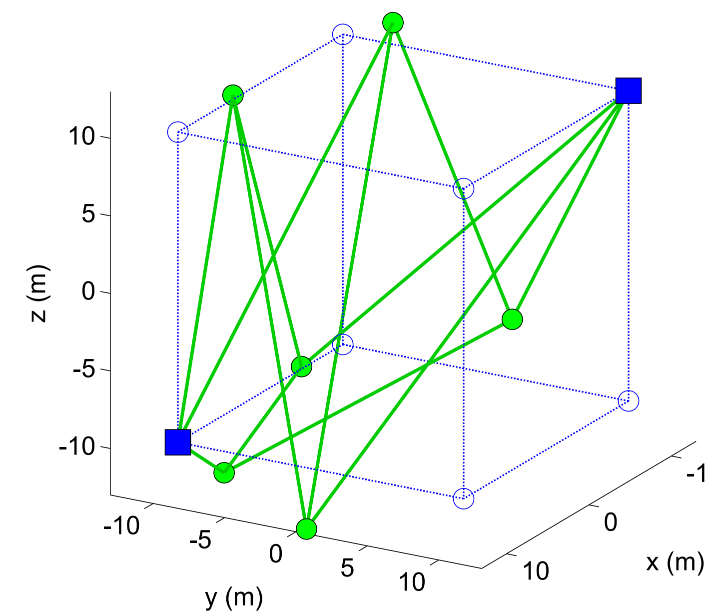


Bearing Rigidity Theory

a bearing-only approach

$$\dot{p}_i(t) = -\sum_{j\sim i} P_{g_{ij}(t)} g_{ij}^*$$



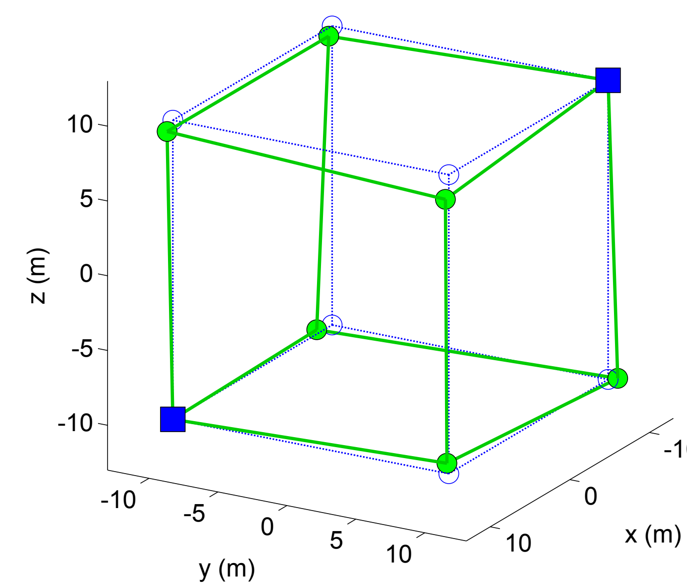
- formation maneuvering control (TCNS '15)
- leader-follower setups
- network localization problems (Automatica '15 (submitted))

Bearing-Based Formation Stabilization with Directed Interaction Topologies

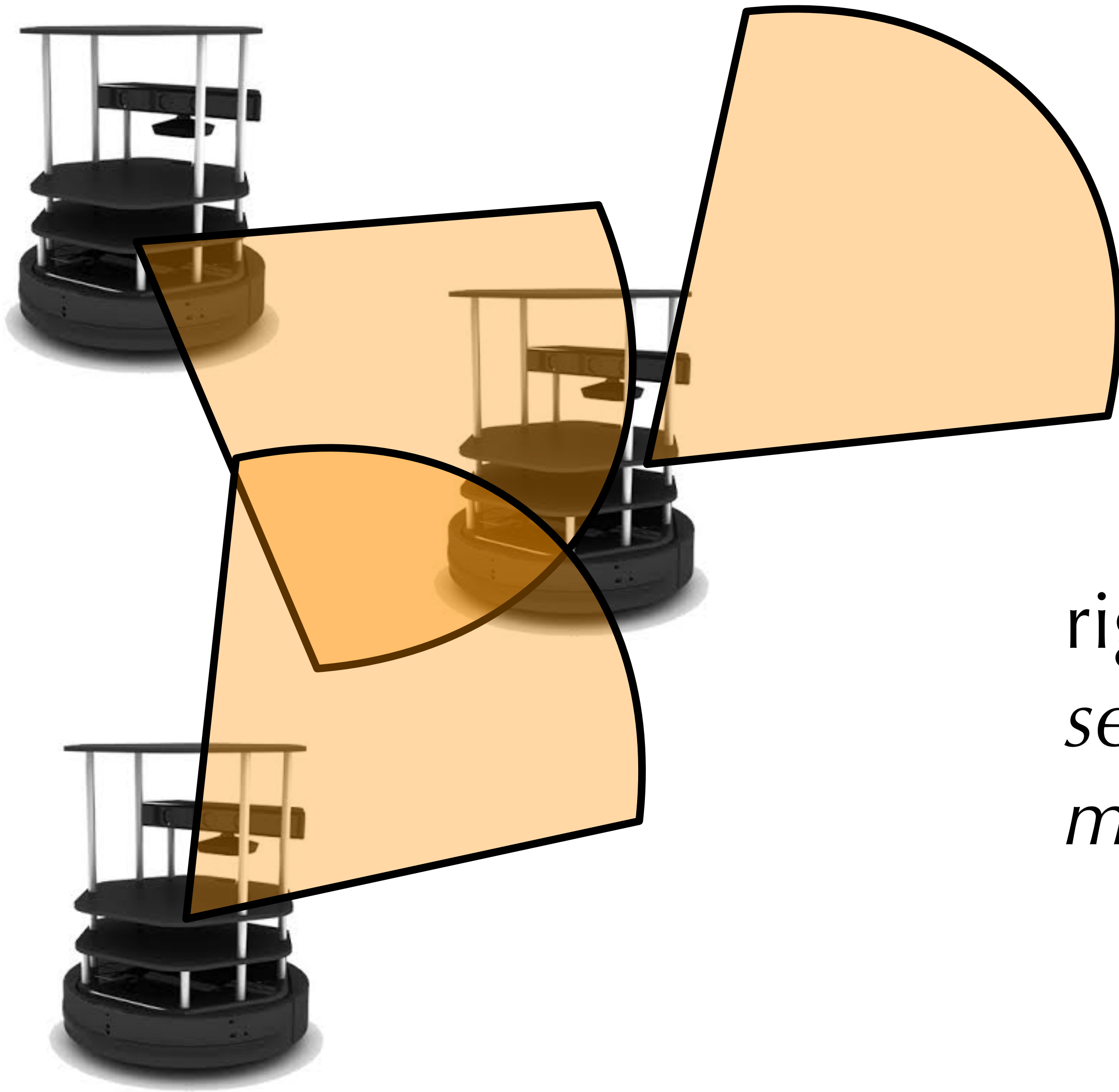
Friday A07

9:30 - 9:50

Zhao, Zelazo



Formation Control in Local Coordinates



- sensing is typically *physically attached to the body frame* of the robot
- sensing is inherently directed
- knowledge of common inertial frame is *not* a realistic assumption

rigidity theory extensions for *directed sensing graphs* and *local (body-frame) measurements*

SE(2) Rigidity Theory

SE(2) Rigidity Theory

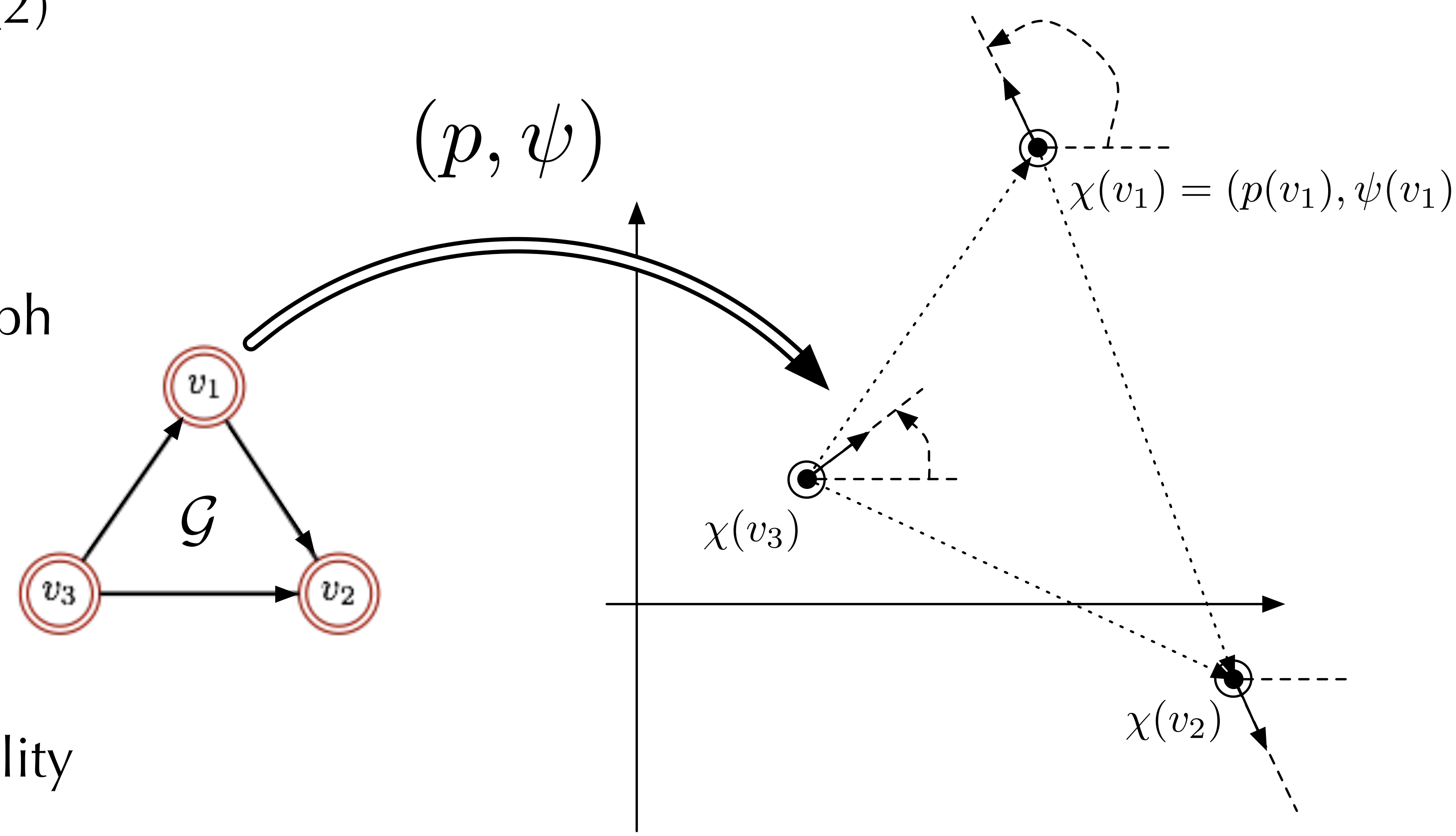
bar-and-joint frameworks in SE(2)

$$(\mathcal{G}, p, \psi)$$

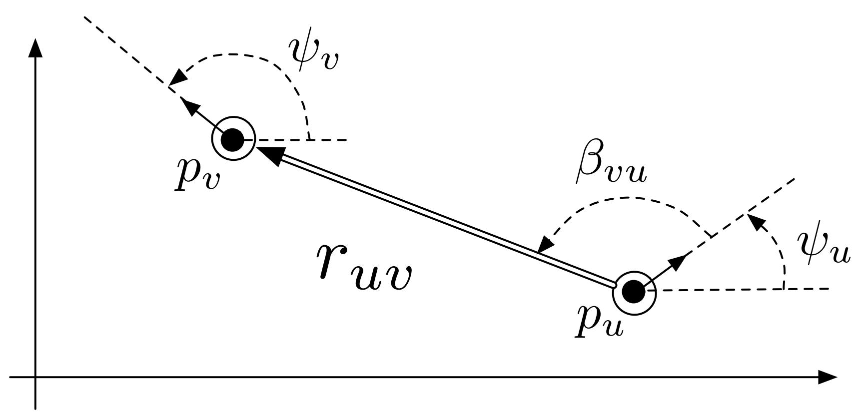
$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a *directed* graph

$$p : \mathcal{V} \to \mathbb{R}^2$$

$$\psi : \mathcal{V} \to \mathcal{S}^1$$



a directed edge indicates availability of relative bearing measurement



$$r_{uv} = \begin{bmatrix} \cos(\psi_u) & \sin(\psi_u) \\ -\sin(\psi_u) & \cos(\psi_u) \end{bmatrix} \frac{p_v - p_u}{\|p_v - p_u\|}$$

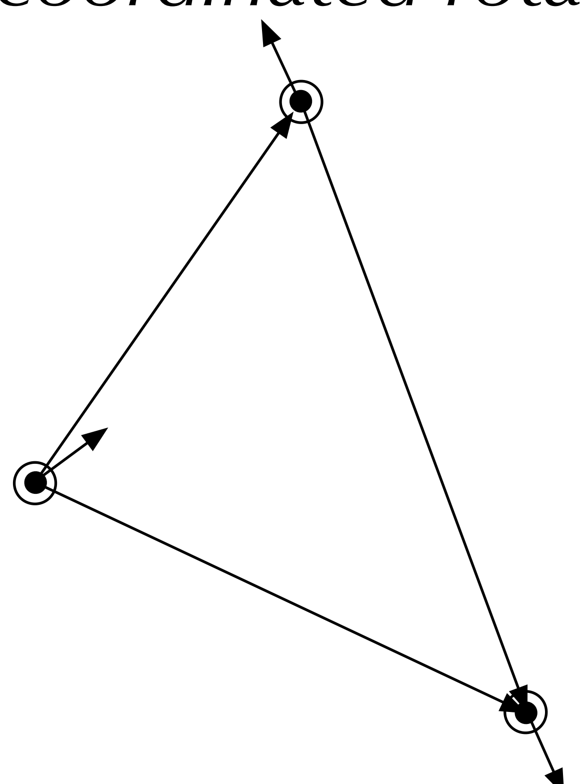
$$b_{\mathcal{G}}(p, \psi) = \begin{bmatrix} r_{e_1}^T & \cdots & r_{e_{|\mathcal{E}|}} \end{bmatrix}^T$$

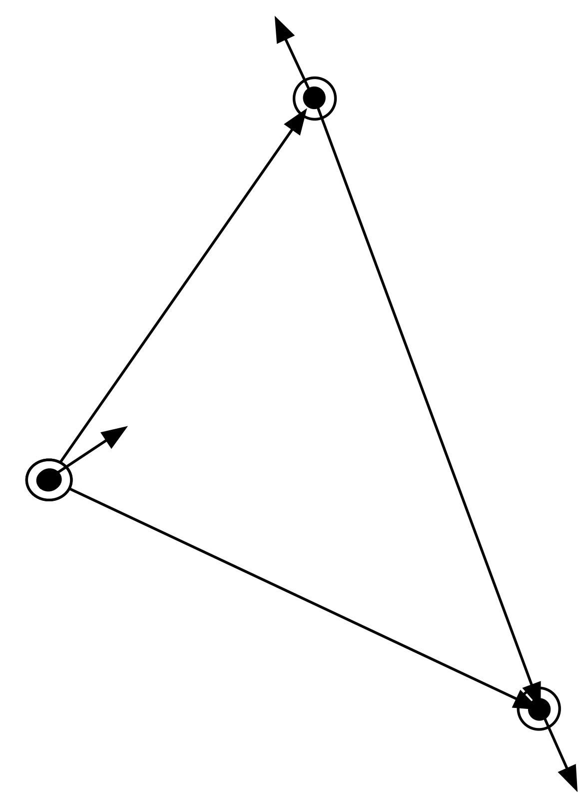
Infinitesimal Motions in SE(2)

Rigidity is a combinatorial theory for characterizing the "stiffness" or "flexibility" of structures formed by rigid bodies connected by flexible linkages or hinges.

SE(2) Rigidity

- maintain bearings in *local* frame
- rigid body rotations and scaling + *coordinated rotations*





Rigidity Theory

A framework is **infinitesimally rigid** if all the infinitesimal motions are *trivial* (i.e., translations, scalings, coordinated rotations).

Theorem

A framework is **SE(2) infinitesimally rigid** if and only if the rank of the directed bearing rigidity matrix is *3n-4*.

Directed Bearing Rigidity Matrix

$$\mathcal{B}_{\mathcal{G}}(p,\psi) = \nabla_{(p,\psi)} b_{\mathcal{G}}(p,\psi)$$

$$= \left[\begin{array}{cc} -\mathrm{diag}\left(\frac{P_{r_{vu}}}{\|p_u - p_v\|} T(\psi_v)^T \right) (E^T \otimes I) & -\mathrm{diag}(r_{vu}^{\perp}) E_{out}^T \end{array} \right]$$

Rigidity Theory

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Directed Bearing Rigidity Matrix

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$$\frac{\partial r_{vu}}{\partial \chi_v} = \left[\begin{array}{cc} -\frac{r_{vu}^{\perp}(r_{vu}^{\perp})^T}{\|p_u - p_v\|} T(\psi_v)^T & -r_{vu}^{\perp} \end{array} \right]$$

$$\frac{\partial r_{vu}}{\partial \chi_u} = \left[\begin{array}{cc} \frac{r_{vu}^{\perp}(r_{vu}^{\perp})^T}{\|p_u - p_v\|} T(\psi_v)^T & \mathbf{0} \end{array} \right]$$

SE(2) Formation Control

The **SE(2) bearing-based formation control** problem is to design a (distributed) control law that drives the agents to a desired spatial configuration determined by interagent bearings measured in the local body frame of each agent.

A gradient controller

$$\Phi(p,\psi) = \sum_{(i,j)\in\mathcal{E}} \|r_{ij} - r_{ij}^*\|^2$$

$$\begin{bmatrix} \dot{p} \\ \dot{\psi} \end{bmatrix} = -\nabla_{(p,\psi)}\Phi(p,\psi) = \mathcal{B}_{\mathcal{G}}(p,\psi)^T b_{\mathcal{G}}^*$$

$$\dot{p}_i = \sum_{(i,j)\in\mathcal{E}} \frac{P_{r_{ij}}}{\|p_j - p_i\|} r_{ij}^d + \sum_{(j,i)\in\mathcal{E}} T(\psi_j - \psi_i)\frac{P_{r_{ji}}}{\|p_i - p_j\|} r_{ji}^d$$

$$\dot{\psi}_i = -\sum_{(i,j)\in\mathcal{E}} (r_{ij}^{\perp})^T r_{ij}^d$$

x requires distances
x requires communication
x requires relative orientation

A Bearing-Only Formation Controller

a scale-free SE(2) formation control

$$T(\psi_i)^T \dot{p}_i = -\sum_{(i,j)\in\mathcal{E}} P_{r_{ij}} r_{ij}^d + \sum_{(j,i)\in\mathcal{E}} T(\psi_i - \psi_j)^T P_{r_{ji}} r_{ji}^d$$

$$\dot{\psi}_i = -\sum_{(i,j)\in\mathcal{E}} (r_{ij}^{\perp})^T r_{ij}^d,$$

stability analysis depends on the **SE(2) bearing rigidity** of the formation!

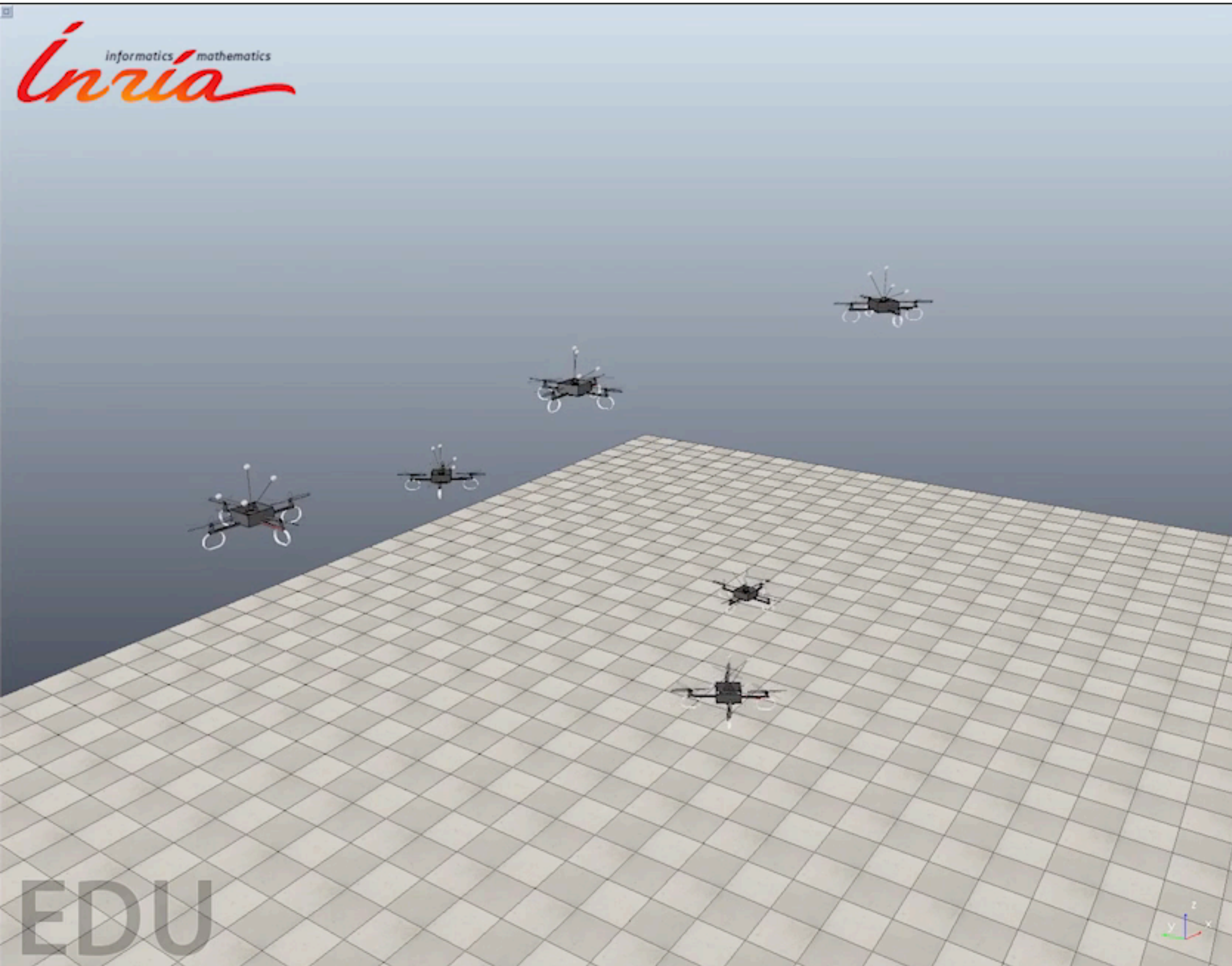
Bearing-Only Formation Control Using an SE(2) Rigidity Theory

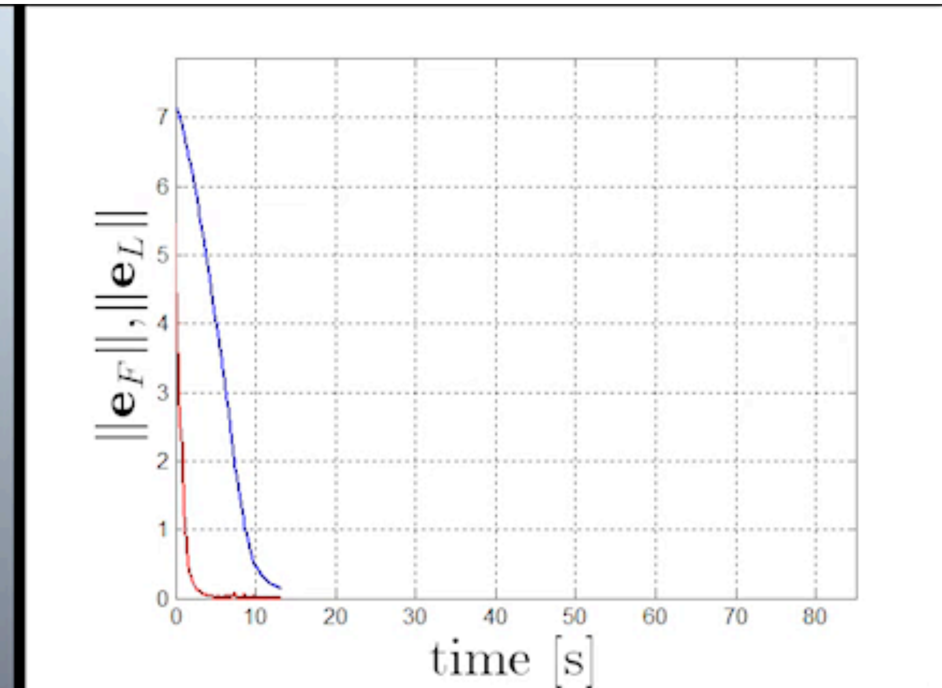
Friday A07

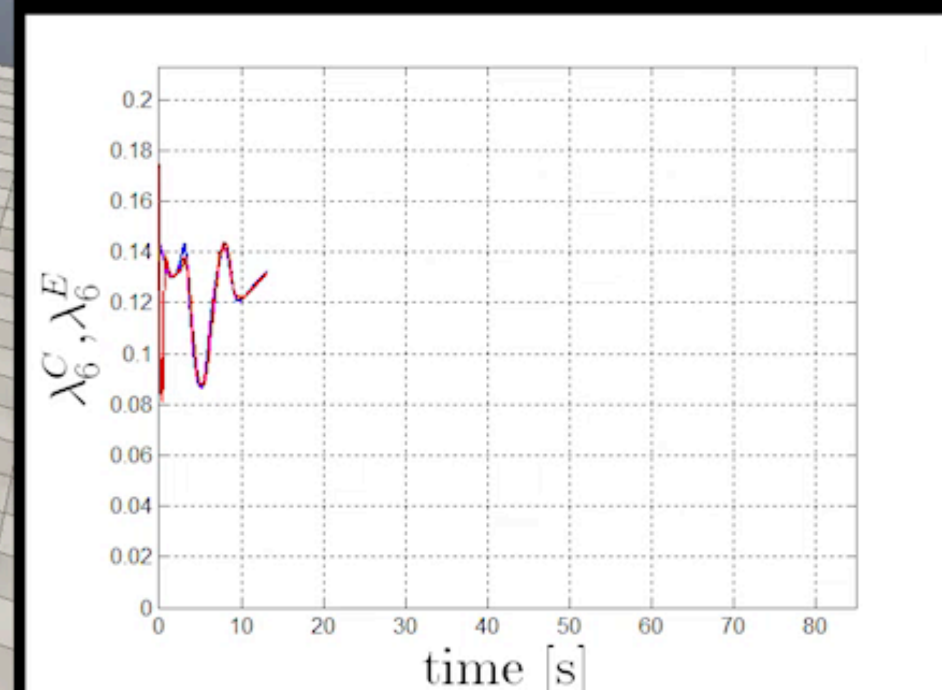
9:50 - 10:10

Zelazo, Robuffo Giordano, Franchi

An SE(2) Formation Controller







The formation reaches the desired bearings

Conclusions and Outlook

- coordination methods for multi-agent systems depend on sensing and communication mediums

- *rigidity theory* is a powerful framework for handling high-level multi-agent objectives under different sensing and communication constraints
  - bearing rigidity
  - SE(2) rigidity
  - SE(n) rigidity

- directed sensing still has many open challenges

Invited Session Advertisement

Rigidity Theory for Problems in Multi-Agent Coordination

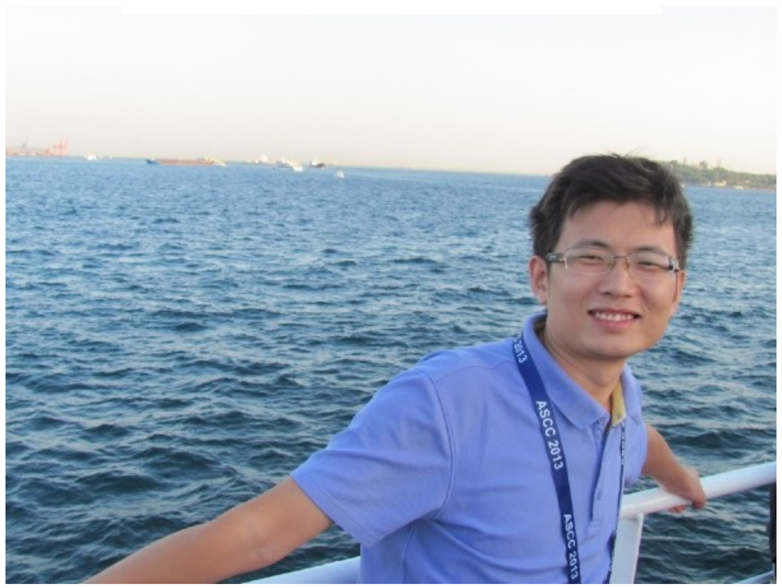
Friday A07
8:30 - 10:30

Organizers
Daniel Zelazo
Paolo Robuffo-Giordano
Antonio Franchi

Speakers

- Z. Sun, U. Helmke, B.D.O. Anderson
  Rigid Formation Shape Control in General Dimensions: An Invariance Principle and Open Problems
- R. Williams, A. Gasparri, M. Soffietti, G. Sukhatme
  Redundantly Rigid Topologies in Decentralized Multi-Agent Networks
- T. Eren
  Combinatorial Measures of Rigidity in Wireless Sensors and Robot Networks
- S. Zhao, D. Zelazo
  Bearing-based Formation Stabilization with Directed Interaction Topologies
- D. Zelazo, P. Robuffo Giordano, A. Franchi
  Bearing-only Formation Control Using an SE(2) Rigidity Theory
- I. Shames, T. Summers, F. Farokhi, R.C. Shekhar
  Conditions and Strategies for Uniqueness of the Solutions to Cooperative Localization and Mapping Using Rigidity Theory

Acknowledgements



Dr. Shiyu Zhao

Dr. Paolo Robuffo Giordano

Dr. Antonio Franchi

Fabrizio Schiano

Questions?