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DarthKnight's blog

Algorithm Gym:: Graph Algorithms

By DarthKnight, 5 years ago, 313,

Welcome to the new episode of PrinceOfPersia presents: Fun with algorithms;)

You can find all the definitions here in the book "Introduction to graph theory", Douglas.B West. Important graph algorithms:

DFS

The most useful graph algorithms are search algorithms. DFS (Depth First Search) is one of them

While running DFS, we assign colors to the vertices (initially white). Algorithm itself is really simple :

```
dfs (v):
    color[v] = gray
    for u in adj[v]:
        if color[u] == white
        then dfs(u)
    color[v] = black
```

Black color here is not used, but you can use it sometimes.

Time complexity : O(n + m).

DFS tree

DFS tree is a rooted tree that is built like this:

Lemma: There is no cross edges, it means if there is an edge between v and u, then v = par[u] or u = par[v].

Starting time, finishing time

Starting time of a vertex is the time we enter it (the order we enter it) and its finishing time is the time we leave it. Calculating these are easy:

```
TIME = 0
dfs (v):
    st[v] = TIME ++
    color[v] = gray
    for u in adj[v]:
```



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```
0
```

It is useable in specially data structure problems (convert the tree into an array).

Lemma: If we run dfs(root) in a rooted tree, then v is an ancestor of u if and only if $st_v \le st_u \le ft_u \le ft_v$.

So, given arrays st and ft we can rebuild the tree.

Finding cut edges

The code below works properly because the lemma above (first lemma):

In this code, h[v] = height of vertex v in the DFS tree and d[v] = min(h[w]) where there is at least vertex u in subtree of v in the DFS tree where there is an edge between u and w).

Finding cut vertices

The code below works properly because the lemma above (first lemma):

In this code, h[v] = height of vertex v in the DFS tree and d[v] = min(h[w]) where there is at least vertex u in subtree of v in the DFS tree where there is an edge between u and w).

Finding Eulerian tours

```
It is quite like DFS, with a little change :
```

```
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```

```
0
```

```
for u in adj[v]:
        erase the edge v-u and dfs(u)
color[v] = black
push v at the end of e
```

e is the answer.

Problems: 500D - New Year Santa Network, 475B - Strongly Connected City

BFS

BFS is another search algorithm (Breadth First Search). It is usually used to calculate the distances from a vertex v to all other vertices in unweighted graphs.

Code:

Distance of vertex u from v is d[u].

Time complexity : O(n + m).

BFS tree

BFS tree is a rooted tree that is built like this:

SCC

The most useful and fast-coding algorithm for finding SCCs is Kosaraju.

In this algorithm, first of all we run DFS on the graph and sort the vertices in decreasing of their finishing time (we can use a stack).

Then, we start from the vertex with the greatest finishing time, and for each vertex v that is not yet in any SCC, do: for each u that v is reachable by u and u is not yet in any SCC, put it in the SCC of vertex v. The code is quite simple.

Problems: CAPCITY, BOTTOM

Shortest path

Shortest path algorithms are algorithms to find some shortest paths in directed or undirected graphs.

Dijkstra

This algorithm is a single source shortest path (from one source to any other vertices). Pay attention that you can't have edges with negative weight.

```
Pseudo code:
dijkstra(v) :
        d[i] = inf for each vertex i
        d[v] = 0
        s = new empty set
        while s.size() < n</pre>
                x = inf
                u = -1
                for each i in V-s //V is the set of vertices
                         if x >= d[i]
                                 then x = d[i], u = i
                insert u into s
                // The process from now is called Relaxing
                for each i in adj[u]
                         d[i] = min(d[i], d[u] + w(u,i))
There are two different implementations for this. Both are useful (C++11).
One) O(n^2)
int mark[MAXN];
void dijkstra(int v){
        fill(d,d + n, inf);
        fill(mark, mark + n, false);
        d[v] = 0;
        int u;
        while(true){
                int x = inf;
                u = -1;
                for(int i = 0; i < n; i ++)
                         if(!mark[i] and x >= d[i])
                                 x = d[i], u = i;
                if(u == -1)
                                 break;
                mark[u] = true;
                for(auto p : adj[u]) //adj[v][i] = pair(vertex, weight)
                        if(d[p.first] > d[u] + p.second)
                                 d[p.first] = d[u] + p.second;
        }
Two) O(n \log(n))
1) Using std :: set :
void dijkstra(int v){
        fill(d,d + n, inf);
        d[v] = 0;
        int u;
        set<pair<int,int> > s;
        s.insert({d[v], v});
        while(!s.empty()){
                u = s.begin() -> second;
                s.erase(s.begin());
```

```
for(auto p : adj[u]) //adj[v][i] = pair(vertex, weight)
6
                            if(d[p.first] > d[u] + p.second){
                                    s.erase({d[p.first], p.first});
                                    d[p.first] = d[u] + p.second;
                                    s.insert({d[p.first], p.first});
                            }
            }
   }
   2) Using std :: priority_queue (better):
   bool mark[MAXN];
   void dijkstra(int v){
            fill(d,d + n, inf);
            fill(mark, mark + n, false);
           d[v] = 0;
           int u;
           priority_queue<pair<int,int>,vector<pair<int,int> >,
   less<pair<int,int> > pq;
           pq.push({d[v], v});
            while(!pq.empty()){
                    u = pq.top().second;
                    pq.pop();
                    if(mark[u])
                            continue;
                    mark[u] = true;
                    for(auto p : adj[u]) //adj[v][i] = pair(vertex, weight)
                            if(d[p.first] > d[u] + p.second){
                                    d[p.first] = d[u] + p.second;
                                    pq.push({d[p.first], p.first});
                            }
            }
   }
```

Problem: ShortestPath Query

Floyd-Warshall

Floyd-Warshal algorithm is an all-pairs shortest path algorithm using dynamic programming.

It is too simple and undrestandable:

```
Floyd-Warshal()
       d[v][u] = \inf for each pair (v,u)
       d[v][v] = 0 for each vertex v
        for k = 1 to n
                for i = 1 to n
                        for j = 1 to n
                                d[i][j] = min(d[i][j], d[i][k] + d[k][j])
```

Time complexity : $O(n^3)$.

Bellman-Ford

Bellman-Ford is an algorithm for single source shortest path where edges can be negative (but if there is a cycle with negative weight, then this problem will be NP).

The main idea is to relax all the edges exactly n-1 times (read relaxation above in dijkstra). You can prove this algorithm using induction.

If in the n - th step, we relax an edge, then we have a negative cycle (this is if and only if).

Code:

Bellman-Ford(int v)

```
0
```

Time complexity : O(nm).

SPFA

SPFA (Shortest Path Faster Algorithm) is a fast and simple algorithm (single source) that its complexity is not calculated yet. But if $m = O(n^2)$ it's better to use the first implementation of Dijkstra.

The origin of this algorithm is unknown. It's said that at first Chinese coders used it in programming contests.

Its code looks like the combination of Dijkstra and BFS:

Time complexity : *Unknown*!.

MST

MST = Minimum Spanning Tree :) (if you don't know what it is, google it).

Best MST algorithms:

Kruskal

In this algorithm, first we sort the edges in ascending order of their weight in an array of edges.

then q.push(i)

Then in order of the sorted array, we add ech edge if and only if after adding it there won't be any cycle (check it using DSU).

Code:

```
Kruskal()
```

```
solve all edges in ascending order of their weight in an array e
ans = 0
for i = 1 to m
v = e.first
u = e.second
w = e.weight
```

```
if merge(v,u) // there will be no cycle
0
                              then ans += w
   Time complexity : O(m \log(m)).
   Prim
   In this approach, we act like Dijkstra. We have a set of vertices S, in each step we add the
   nearest vertex to S, in S (distance of v from S = \min_{u \in S}(weight(u, v)) where
   weight(i, j) is the weight of the edge from i to j).
   So, pseudo code will be like this:
   Prim()
            S = new empty set
            for i = 1 to n
                     d[i] = inf
            while S.size() < n</pre>
                     x = inf
                     v = -1
                     for each i in V - S // V is the set of vertices
                              if x >= d[v]
                                      then x = d[v], v = i
                     d[v] = 0
                     S.insert(v)
                     for each u in adj[v]
                             do d[u] = min(d[u], w(v,u))
   C++ code:
   One) O(n^2)
   bool mark[MAXN];
    void prim(){
            fill(d, d + n, inf);
            fill(mark, mark + n, false);
            int x,v;
            while(true){
                     x = inf;
                     v = -1;
                     for(int i = 0; i < n; i ++)
                              if(!mark[i] and x >= d[i])
                                      x = d[i], v = i;
                     if(v == -1)
                              break;
                     d[v] = 0;
                     mark[v] = true;
                     for(auto p : adj[v]){ //adj[v][i] = pair(vertex, weight)
                              int u = p.first, w = p.second;
                              d[u] = min(d[u], w);
                     }
            }
   Two) O(m \log(n))
    void prim(){
            fill(d, d + n, inf);
            set<pair<int,int> > s;
            for(int i = 0; i < n; i ++)
                     s.insert({d[i],i});
            int v;
            while(!s.empty()){
                     v = s.begin() -> second;
                     s.erase(s.begin());
```

```
6
                    for(auto p : adj[v]){
                            int u = p.first, w = p.second;
                            if(d[u] > w){
                                    s.erase({d[u], u});
                                    d[u] = w;
                                    s.insert({d[u], u});
                            }
                    }
           }
   As Dijkstra you can use std :: priority_queue instead of std :: set .
   Maximum Flow
   You can read all about maximum flow here.
   I only wanna put the source code here (EdmondsKarp):
   algorithm EdmondsKarp
       input:
           C[1..n, 1..n] (Capacity matrix)
           E[1..n, 1..?] (Neighbour lists)
                          (Source)
                          (Sink)
           +
        output:
            f
                          (Value of maximum flow)
                          (A matrix giving a legal flow with the maximum
   value)
        f := 0 (Initial flow is zero)
        F := array(1..n, 1..n) (Residual capacity from u to v is C[u,v] -
   F[u,v])
        forever
           m. P := BreadthFirstSearch(C, E, s, t, F)
            if m = 0
               break
            f := f + m
            (Backtrack search, and write flow)
           v := t
           while v ≠ s
                u := P[v]
                F[u,v] := F[u,v] + m
                F[v,u] := F[v,u] - m
                v := u
        return (f, F)
   algorithm BreadthFirstSearch
       input:
           C, E, s, t, F
        output:
                          (Capacity of path found)
           M[t]
           Р
                          (Parent table)
       P := array(1..n)
        for u in 1..n
           P[u] := -1
        P[s] := -2 (make sure source is not rediscovered)
       M := array(1..n) (Capacity of found path to node)
       M[s] := ∞
       Q := queue()
       Q.offer(s)
       while Q.size() > 0
           u := Q.poll()
            for v in E[u]
```

```
(If there is available capacity, and v is not seen before in
search)
            if C[u,v] - F[u,v] > 0 and P[v] = -1
                P[v] := u
                M[v] := min(M[u], C[u,v] - F[u,v])
                if v ≠ t
                    Q.offer(v)
                else
                    return M[t], P
    return 0, P
EdmondsKarp pseudo code using Adjacency nodes:
algorithm EdmondsKarp
    input:
        graph (Graph with list of Adjacency nodes with
capacities, flow, reverse and destinations)
                      (Source)
        t
                      (Sink)
    output:
                         (Value of maximum flow)
        flow
    flow := 0 (Initial flow to zero)
    q := array(1..n) (Initialize q to graph length)
    while true
                           (Variable to iterate over all the
        qt := 0
corresponding edges for a source)
        q[qt++] := s (initialize source array)
        pred := array(q.length) (Initialize predecessor List with the
graph length)
        for qh=0;qh < qt && pred[t] == null</pre>
            cur := q[qh]
            for (graph[cur]) (Iterate over list of Edges)
                 Edge[] e := graph[cur] (Each edge should be associated
with Capacity)
                 if pred[e.t] == null && e.cap > e.f
                    pred[e.t] := e
                    q[qt++] : = e.t
        if pred[t] == null
        int df := MAX VALUE (Initialize to max integer value)
        for u = t; u != s; u = pred[u].s
            df := min(df, pred[u].cap - pred[u].f)
        for u = t; u != s; u = pred[u].s
            pred[u].f := pred[u].f + df
            pEdge := array(PredEdge)
            pEdge := graph[pred[u].t]
            pEdge[pred[u].rev].f := pEdge[pred[u].rev].f - df;
        flow := flow + df
    return flow
Dinic's algorithm
Here is Dinic's algorithm as you wanted.
Input: A network G = ((V, E), c, s, t).
Output: A max s - t flow.
1.set f(e) = 0 for each e in E
2.Construct G_L from G_f of G. if dist(t) == inf, then stop and output f
3.Find a blocking flow fp in G_L
```

```
4. Augment flow f by fp and go back to step 2.
    Time complexity : O(mm \log(n)).
   Theorem: Maximum flow = minimum cut.
   Maximum Matching in bipartite graphs
    Maximum matching in bipartite graphs is solvable also by maximum flow like below:
   Add two vertices S, T to the graph, every edge from X to Y (graph parts) has capacity 1, add
   an edge from S with capacity 1 to every vertex in X, add an edge from every vertex in Y with
   capacity 1 to T.
   Finally, answer = maximum matching from S to T.
   But it can be done really easier using DFS.
   As, you know, a bipartite matching is the maximum matching if and only if there is no
   augmenting path (read Introduction to graph theory).
   The code below finds a augmenting path:
   bool dfs(int v){// v is in X, it reaturns true if and only if there is an
    augmenting path starting from v
            if(mark[v])
                     return false;
            mark[v] = true;
            for(auto &u : adj[v])
                     if(match[u] == -1 or dfs(match[u])) // match[i] = the
    vertex i is matched with in the current matching, initially -1
                             return match[v] = u, match[u] = v, true;
            return false;
    }
   An easy way to solve the problem is:
    for(int i = 0;i < n;i ++)if(match[i] == -1){</pre>
            memset(mark, false, sizeof mark);
            dfs(i);
    }
   But there is a faster way:
   while(true){
            memset(mark, false, sizeof mark);
            bool fnd = false;
            fnd |= dfs(i);
            if(!fnd)
                     break;
    }
   In both cases, time complexity = O(nm).
   Problem: 498C - Array and Operations
```

Trees

Trees are the most important graphs.

In the last lectures we talked about segment trees on trees and heavy-light decomposition.

Partial sum on trees

0

We can also use partial sum on trees.

Example: Having a rooted tree, each vertex has a value (initially 0), each query gives you numbers v and u (v is an ancestor of u) and asks you to increase the value of all vertices in the path from u to v by 1.

So, we have an array p, and for each query, we increase p[u] by 1 and decrease p[par[v]] by 1. The we run this (like a normal partial sum):

DSU on trees

We can use DSU on a rooted tree (not tree DSUs, DSUs like vectors).

For example, in each node, we have a vector, all nodes in its subtree (this can be used only for offline queries, because we may have to delete it for memory usage).

Here again we use DSU technique, we will have a vector V for every node. When we want to have V[v] we should merge the vectors of its children. I mean if its children are $u_1, u_2, ..., u_k$ where $V[u_1].size() \le V[u_2].size() \le ... \le V[u_k].size()$, we will put all elements from $V[u_i]$ for every $1 \le i < k$, in V[k] and then, $V[v] = V[u_k]$.

Using this trick, time complexity will be $O(n \log(n))$.

C++ example (it's a little complicated) :

```
typedef vector<int> vi;
vi *V[MAXN];
void dfs(int v, int par = -1){
        int mx = 0, chl = -1;
        for(auto u : adj[v])if(par - u){
                dfs(u,v);
                if(mx < V[u]->size()){
                        mx = V[u]->size();
                        chl = u;
                }
        for(auto u : adj[v])if(par - u and chl - u){
                for(auto a : *V[u])
                        V[chl]->push_back(a);
                delete V[u];
        if(chl + 1)
                V[v] = V[chl];
        else{
                V[v] = new vi;
                V[v]->push_back(v);
        }
}
```

LCA

 $\ensuremath{\mathsf{LCA}}$ of two vertices in a rooted tree, is their lowest common ancestor.

There are so many algorithms for this, I will discuss the important ones.

Each algorithm has complexities < O(f(n)), O(g(n)) >, it means that this algorithm's preprocess is O(f(n)) and answering a query is O(g(n)).

In all algorithms, h[v] = height of vertex v.

```
One) Brute force < O(n), O(n) >
The simplest approach. We go up enough to achieve the goal.
Preproccess:
void dfs(int v,int p = -1){
         if(par + 1)
                  h[v] = h[p] + 1;
         par[v] = p;
         for(auto u : adj[v])    if(p - u)
                 dfs(u,v);
}
Query:
int LCA(int v,int u){
         if(v == u)
                  return v;
         if(h[v] < h[u])
                  swap(v,u);
         return LCA(par[v], u);
}
Two) SQRT decomposition < O(n), O(\sqrt{n}) >
I talked about SQRT decomposition in the first lecture.
Here, we will cut the tree into \sqrt{H} (H = height of the tree), starting from 0, k - th of them
contains all vertices with h in interval [k\sqrt{H}, (k+1)\sqrt{H}].
Also, for each vertex v in k - th piece, we store r[v] that is, its lowest ancestor in the piece
number k - 1.
Preprocess:
void dfs(int v,int p = -1){
         if(par + 1)
                  h[v] = h[p] + 1;
         par[v] = p;
         if(h[v] % SQRT == 0)
                  r[v] = p;
         else
                  r[v] = r[p];
         for(auto u : adj[v])
                                   if(p - u)
                  dfs(u,v);
}
Query:
int LCA(int v,int u){
         if(v == u)
                  return v;
         if(h[v] < h[u])
                  swap(v,u);
         if(h[v] == h[u])
                  return (r[v] == r[u] ? LCA(par[v], par[u]) : LCA(r[v],
r[u]));
         if(h[v] - h[u] < SQRT)
                  return LCA(par[v], u);
         return LCA(r[v], u);
}
```

Three) Sparse table $< O(n \log(n)), O(1) >$

Let's introduce you an order of tree vertices, haas and I named it Euler order. It is like DFS

```
order, but every time we enter a vertex, we write it's number down (even when we come from a
child to this node in DFS).
Code for calculate this:
vector<int> euler;
void dfs(int v,int p = -1){
         euler.push_back(v);
          for(auto u : adj[v])
                                      if(p - u)
                   dfs(u,v), euler.push_back(v);
}
If we have a | \text{vector} < \text{pair} < \text{int, int} > |  instead of this and push | \{h[v], v\} |  in the
vector, and the first time \{h[v], v\} is appeared is s[v] and s[v] < s[u] then LCA(v, u) = (min_{i=s[v]}^{s[u]}euler[i]).second.
For this propose we can use RMQ problem, and the best algorithm for that, is to use Sparse
Four) Something like Sparse table :) < O(n \log(n)), O(\log(n)) >
This is the most useful and simple (among fast algorithms) algorithm.
For each vector v and number i, we store its 2^i-th ancestor. This can be done in O(n \log(n))
. Then, for each query, we find the lowest ancestors of them which are in the same height, but
different (read the source code for understanding).
Preprocess:
int par[MAXN][MAXLOG]; // initially all -1
void dfs(int v,int p = -1){
         par[v][0] = p;
         if(p + 1)
                   h[v] = h[p] + 1;
          for(int i = 1; i < MAXLOG; i ++)
                   if(par[v][i-1] + 1)
                             par[v][i] = par[par[v][i-1]][i-1];
          for(auto u : adj[v])
                                    if(p - u)
                   dfs(u,v);
}
Query:
int LCA(int v,int u){
         if(h[v] < h[u])
                   swap(v,u);
          for(int i = MAXLOG - 1;i >= 0;i --)
                   if(par[v][i] + 1 and h[par[v][i]] >= h[u])
                            v = par[v][i];
          // now h[v] = h[u]
         if(v == u)
                   return v:
          for(int i = MAXLOG - 1;i >= 0;i --)
                   if(par[v][i] - par[u][i])
                             v = par[v][i], u = par[u][i];
          return par[v][0];
}
Five) Advance RMQ < O(n), O(1) >
In the third approach, we said that LCA can be solved by RMQ.
When you look at the vector euler you see that for each i that 1 \le i \le euler.size(),
|euler[i].first - euler[i + 1].first| = 1.
So, we can convert the euler from its size(we consider its size is n + 1) into a binary sequence
```



of length n (if euler[i].first - euler[i+1].first = 1 we put 1 otherwise 0).

So, we have to solve the problem on a binary sequence \boldsymbol{A} .

To solve this restricted version of the problem we need to partition A into blocks of size $l=\left[\frac{\log(n)}{2}\right]$. Let A'[i] be the minimum value for the i - th block in A and B[i] be the position of this minimum value in A. Both A and B are $\frac{n}{l}$ long. Now, we preprocess A' using the Sparse Table algorithm described in lecture 1. This will take $O(\frac{n}{l}\log(\frac{n}{l}))=O(N)$ time and space. After this preprocessing we can make queries that span over several blocks in O(1). It remains now to show how the in-block queries can be made. Note that the length of a block is $l=\left[\frac{\log(n)}{2}\right]$, which is quite small. Also, note that A is a binary array. The total number of binary arrays of size l is $2^l=\sqrt{n}$. So, for each binary block of size l we need to lock up in a table l0 the value for l1 l2 between every pair of indices. This can be trivially computed in l3 and store it in array l4 l5. The block type is a binary number obtained by replacing - 1 with l3 and l4 l5 with l6 (as described above).

Now, to answer RMQA(i, j) we have two cases:

- i and j are in the same block, so we use the value computed in P and T
- i and j are in different blocks, so we compute three values: the minimum from i to the end of i's block using P and T, the minimum of all blocks between i's and j's block using precomputed queries on A' and the minimum from the beginning of j's block to j, again using T and P; finally return the position where the overall minimum is using the three values you just computed.

Six) Tarjan's algorithm O(na(n)) (a(n) is the inverse ackermann function)

Tarjan's algorithm is offline; that is, unlike other lowest common ancestor algorithms, it requires that all pairs of nodes for which the lowest common ancestor is desired must be specified in advance. The simplest version of the algorithm uses the union-find data structure, which unlike other lowest common ancestor data structures can take more than constant time per operation when the number of pairs of nodes is similar in magnitude to the number of nodes. A later refinement by Gabow & Tarjan (1983) speeds the algorithm up to linear time.

The pseudocode below determines the lowest common ancestor of each pair in P, given the root r of a tree in which the children of node n are in the set n.children. For this offline algorithm, the set P must be specified in advance. It uses the MakeSet, Find, and Union functions of a disjoint-set forest. MakeSet(u) removes u to a singleton set, Find(u) returns the standard representative of the set containing u, and Union(u, v) merges the set containing u with the set containing v. TarjanOLCA(r) is first called on the root r.

Each node is initially white, and is colored black after it and all its children have been visited. The lowest common ancestor of the pair $\{u,v\}$ is available as Find(v).ancestor immediately (and only immediately) after u is colored black, provided v is already black. Otherwise, it will be available later as Find(u).ancestor, immediately after v is colored black.

```
function MakeSet(x)
    x.parent := x
    x.rank := 0

function Union(x, y)
```





5 years ago, # |

what about bidirectional component searching algorithm?

→ Reply

(Ost1a

5 years ago, # |

← Rev. 3 _____ +18 ___

△ +3 ▼





→ Reply

5 years ago, # |

▲ 0 🐺



It is modification of Ford-Bellman, so worst case complexity is O(nm), isn't it?

Also you didn't mention, but this algorithm also works for graphs with negative edges :)

→ Reply



5 years ago, # ^ |

its complexity is not calculated

+3

The worst case is obvious, but important thing is its exact complexity(or at least average).

→ Reply

5 years ago, # ^ |

← Rev. 3

A +13

As mentioned above, it's easy to see, that this algo is just accurately written ford-bellman. So complexity is O(nm). It's easy to construct graph on which its complexity is C*n*m. For example full graph with

 $a[i][j] = \begin{cases} 2|i-j|, & |i-j| > 1\\ 1, & |i-j| = 1 \end{cases}.$



If you would go through edges always in decreasing order, then on every iteration every not satisfied vertex will be added to queue.

romanandree

Once burunduk1 told me that on average this algo works in \$O(m log n)\$, but i think this bound is from experience, i.e. not proved.

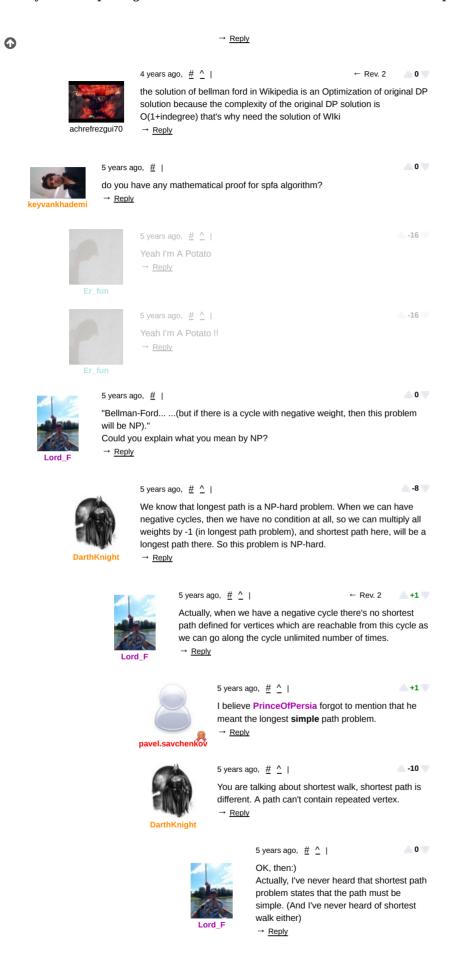
P.S. We always used this algo in finding MaxFlowMinCost, because it's easier to write than Dijkstra with potentials. And it works fast, because graph is changed on every step, so it's almost impossible to create anti-test(although i think once we've got TL...).

→ Reply



5 years ago, # ^ |

Could you show you MaxFlowMinCost code? I try to find something faster than $O(V^3 * E)$ and really dijkstra with potentials not to pleasant=)







5 years ago, # |

The "Finding cut vertices" pseudocode claims that the root of dfs is cut vertice in all cases where the root has 2 or more adjacent vertices. Instead, it should be that the root of dfs is cut vertice iff it has 2 or more children in the dfs tree.

→ Reply



5 years ago, # ^ | Is there any difference ???



5 years ago, # ^ |

Think of a graph which is a cycle. Every vertice has 2 adjacent vertices, but every vertice has 1 child in dfs tree, except the last vertice reached by dfs which has no children in dfs tree.

→ Reply



The graph you are talking about doesn't have any cut vertex. My algorithm works fine.

← Rev. 2

A +3

<u></u> 0 🔻

A 0 =

A 0

A 0

▲ +3 ▼

0

P.S: I meant adj[v].size() is the number of children. → Reply



5 years ago, # ^ |

No, your algorithm returns that the root of the dfs is cut vertex, because it has 2 adjacent vertices.

→ Reply



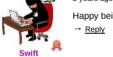
5 years ago, # ^ |

Ok, you should clarify that adj[v].size() is the number of children in dfs tree because clearly adj[v] is the adjacency list of v in the graph, not adjacency list of v in the dfs tree.

→ Reply



5 years ago, # | Happy being in top contributors!



5 years ago, # |





http://en.m.wikipedia.org/wiki/Tarjan%27s_offline_lowest_common_ancestors_algorithm

Tarjan's Offline LCA algorithm is the LCA algorithm I usually use. It is based on dsu hence it costs O(n * a(n), O(1)), where a(n) is the inversw ackermann function. It is very essential and should be added to this post.

→ Reply



helow.

5 years ago, # ^ |

Thank you for your update. I'm surprised by your high efficiency!

For competitive programmers, I recommend the implementation of dsu



s[x] denotes parent of x

function f(x) finds the root of x



△ 0 **▼**

<u></u> 0 🔻

0

<u>0</u> 0

A 0 🔻

A 0





3 months ago, # ^ |



5 years ago, # |

Is a DFS tree equivalent to a Union Find structure created using given edge pairs ${\it PS}$ am relatively new to graphs and find them similar:)

→ Reply



Manikantan\

→ Reply



5 years ago, # |



→ Reply



5 years ago, # |

This is great man.

→ Reply



5 years ago, #

I think there's no need to use color array in finding eulerian tour. You can visit each vertex several times.

→ Reply



What you mean by "Finding Eulerian tours"? I have some problems with terminology:)

Your code makes no sense for me while solving this problem — and one may call it Euler tour problem. Is it actually related to this technique instead? For a tree with 5 vertices and edges



I_love_Tanya_Romanova

1 2

1 -

your algorithm is printing 3-2-5-4-1. Can you please explain how is it related to Eulerian path/Eulerian tour/Eulerian cycle?

→ Reply

5 years ago, # |

← Rev. 2

For SPFA you wrote

Time complexity : Unknown!.



This algorithm is modification of Bellman–Ford algorithm, and worst-case running time is $O(V^*E)$. It runs in O(E) on random graphs, but problem setter can actually make it run in $O(V^*E)$ if he isn't lazy to prepare good testcases: D

P.S. Thanks to **Burunduk1** for teaching me how to actually create tests to make SPFA work in O(V*E):)

→ Reply





← Rev 3 A 0 🔻 5 years ago, # ^ |

4 years ago, # ^ | ← Rev. 2

If SPEA work in O(V*E) in worst-case, then it's better to use Bellman-Ford algorithm :) (when problemsetter like you!)



Akin

Worst Test Case: Suppose one wants the shortest path from vertex 1 to vertex n. Then we can add edge (i, i + 1) with a small random weight for $1 \leq i < n$ (thus the shortest path should be 1-2-...-n), and randomly add 4n other heavy edges. For this case, the so-called SPFA algorithm will be very slow.

→ Reply

2 years ago, # ^ |



0

A 0 🔻

<u>0</u> 0



How to make worst case tests for SFPA? And is it recommended to use SFPA in general on Shortest Path

→ Reply



2 years ago, # ^ | ← Rev. 2 Yes, I accepted an $\mathcal{O}(N^4)$ solution with N = 500 as I

→ Reply

0 5 years ago, # |

remember. Actually the tests was bad :D.



very very good tutorial

can you please help us by writing nice article about Dynamic Programming & Greedy ?:)

→ Reply



5 years ago, # |

<u></u> 0 🐙 Can anyone link problems using tree sqrt decomposition

→ Reply

Ahmed_Morsy

welcome to the Top 10 contribution list PrinceOfPersia:D



can't wait for more posts like this one

OmaeWaMouShenDeiru

Nice work you've done :D

5 years ago, # |

→ Reply



Thank you! Also it will be nice to see MaxFlowMinCost tutorial here. → Reply

5 years ago, # |

I am going to learn a LCA algorithm and I have a question. @PrinceOfPersia, you wrote that Four) fourth method called: "Something like Sparse table" is "the most useful and simple (among fast algorithms) algorithm." Why it is better than third method where query takes O(1) instead of log(n)? Is it because 3rd solution is

hard to implement?



Using 4-th method , what about adding new vertices to a tree ? After adding a vertex I have to recalculate everything, right?

→ Reply

```
0
                                 5 years ago, # ^ |
                                                                                      ← Rev 2
                                                                                                  0
                                 Because its code is much shorter and it's faster to type.
                        4 years ago, # |
                                                                                                   A 0 🔻
                        The big bang theory every where :D **fun with flags vs Fun with algorithms **
    letsLoseOnceAgain
                             4 years ago, # |
                              I think Path-based SC is also simple and it performs one DFS so it's faster.
                       4 years ago, # |
                                                                                      ← Rev 2
                                                                                                  A 0
                       In implementation of Dijkstra with priority, doesn't the size of queue grow much?
       Shavan.To
                       4 years ago, # |
                                                                                      ← Rev. 2
                        I guess there is a typo while writing prim's algorithm.
                        for each i in V — S // V is the set of vertices if x \ge d[v] then x = d[v], v = i
                       Here it should be if x \ge d[i] then x = d[i], y=i
                        → Reply
                                                                                                   <u></u> 0 🐙
                       4 years ago, # |
                         So, given arrays st and ft we can rebuild the tree.
                        How we can build the tree?
                        How is the algorithm?
                        → Reply
                                                                                                   <u></u> 0 🕎
                       4 years ago, # |
                        Открытую ссылку на условия задач я не нашел. Если у вас есть логин для
                       opencup в ejudge или Яндекс.Контест их можно найти.
                        → Reply
       aboodmanna
                                                                                                   A 0
                       4 years ago, # |
                        Could someone post practice problems that use the ideas behind each of those
                        algorithms? Or perhaps link an existing post?
                        Thanks!
                        → Reply
                       4 years ago, # |
                                                                                      ← Rev. 4 0
                       Hello, thank you for sharing this.
                       In LCA, you do:
                        void dfs(int v,int p = -1){
                                 if(par + 1)
                                           h[v] = h[p] + 1;
                        I assume that:
       r0drigopaes
                         • There is an array of parents: par[]
                         • The first you call is: dfs(0)
                       If these are correct, then:
                         • What does the "if" mean? If par is an allocated array, (par + 1) is just a pointer
```



operation to one position further from the beginning of the array, in other words, it will be a memory area that is not NULL and therefore will always be true.

• The first time, v is 0 an p is -1, then h[0] = h[-1]+1 ... it will be an out of range array access, due to h[-1].

Probably I'm missing something. Could you please, help me to understand? → Reply

4 years ago, # |

A 0

0

0

<u>0</u>

A 0

+4

A+1 T

← Rev. 2

PrinceOfPersia In the dijkstra's priority queue code, what is the use of MARK



If I am correct, it is kept for keeping track of already used vertices. But when I used that, My soln to this problem EZDIJKST gave me a Wrong answer verdict. However it was accepted after removing the visited array. Can you point out what is wrong here?

WA Code ACC Code

→ Reply

→ Reply



4 years ago, # ^ | PrinceOfPersia in the code, it should be greater< pair<int, int> > and not less



3 years ago, # ^ | Yes, I find the same issue. I code according to the tutorial and get WA. Finally find that it should use greater instead of less. Please correct it.

→ Reply



4 years ago, # |

4 years ago, # |

Can someone explain how complexity of DSU on trees is O(nlogn)? → Reply

can i anyone tell me how to run maximum spanning tree algorithm, and i google



https://www.quora.com/What-is-the-maximum-spanning-tree-algorithm, i still not

figure it out ==b

→ Reply



4 years ago, # | easy e-maxx, easy life



4 years ago, # ^ | what do you mean?? → Reply

WannabeStronger



4 years ago, # ^ | Real fan of Justin Bieber → Reply

4 years ago, # | Nice post!

It was very useful for me. I hope that there are more users like me. Thanks very



much..:) → Reply

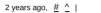


3 years ago, # |

Please someone explain type 3(Sparse table), along with complexity, for finding

A sample code with pre-process and guery would be of great help.

→ Reply



A +3

In type 3 after making euler vector we will find the node with minimum height such that it occur in euler vector between the position at which query node a & b comes first time in euler vector. logic behind this is if you observe in euler vector when query node a & b(let suppose a comes first in euler vector than b) then firstly there will be node of its child (in this case node with minimum height will be a if a is ancestor of b) and after completing dfs of its own it will go through the parent from where dfs of child is processing until b comes and in between a and b the node with minimum height will be LCA of a & b. there can not be any node b/w a & b with height less than LCA of a & b because a & b came into the dfs of LCA and parent of LCA came before starting the dfs of LCA and it will come again after completing dfs of LCA so it can't be present b/w a & b.



Time complexity will be O(n) for dfs and O(nlogn) for pre-processing of sparse table and query will be in O(1).

Hope it help here is my accepted code for problem LCA code i used the same logic with only one diff i used vector instead of vector<pair<int,int> > to store euler vector

→ Reply



3 years ago, # |

Thanks for great post!

→ Reply



3 years ago, # |

0

+4

0

Can anyone please elaborate on Type 4 Method of finding LCA? → Reply



A 0 3 years ago, # |

Can someone explain why DSU on tree has O(nlogn) complexity... I understood the concept of DSU (in Kruskal) but could not have any clues about DSU on tree (in this post and http://www.codeforces.com/blog/entry/44351)

→ Reply



3 years ago, # | In **Dijkstra** Algorithm, using std :: priority_queue



Declaration is priority_queue<pair<int,int>,vector<pair<int,int> >, less<pair<int,int>

and during pushing in queue is pq.push({d[p.first], p.first});

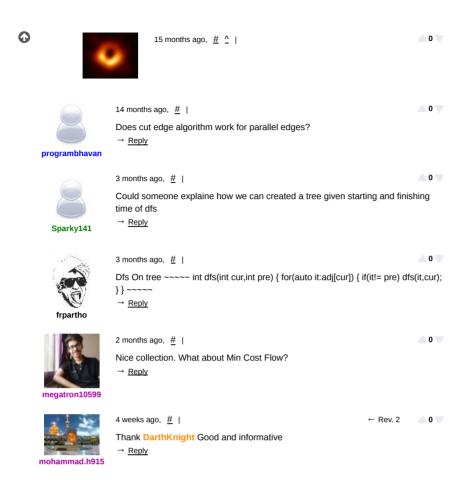
Shouldn't it be greater<pair<int,int> instead of less<pair<int,int> , for getting minimum distance node first? → Reply



0 1 7 0 2 8 1 2 1 0 3 1 1 3 5 1 4 1 3 4 8 i am getting wrong output for this it should be greater instead of less

→ Reply





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The only programming contests Web 2.0 platform
Server time: Sep/04/2019 11:11:51^{uTC+5.5} (g2).
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