09 - Simple Linear Regression

HCI/PSYCH 522
Iowa State University

February 17, 2022

Overview

- Simple linear regression
  - Dependent variable
  - Independent variable
  - Continuous independent variable
- Assumptions
  - Linearity
  - Normality
  - Constant Variance
  - Independence

Simple linear regression

Dependent variable

Definition

The distribution of the dependent variable depends on the values of the independent variables.

Dependent variable examples:

- Gold per minute
- Time to register
- Satisfaction

Independent variable

Definition

The independent variable affects the distribution of the dependent variable.

Independent variable examples:

- Mouse sensitivity
- Availability of a chatbot
- App being used

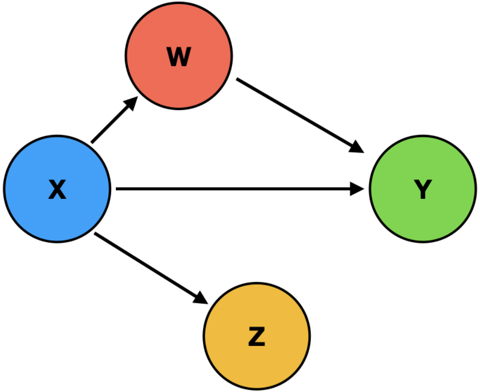
Synonyms

Terminology (all of these are [basically] equivalent):

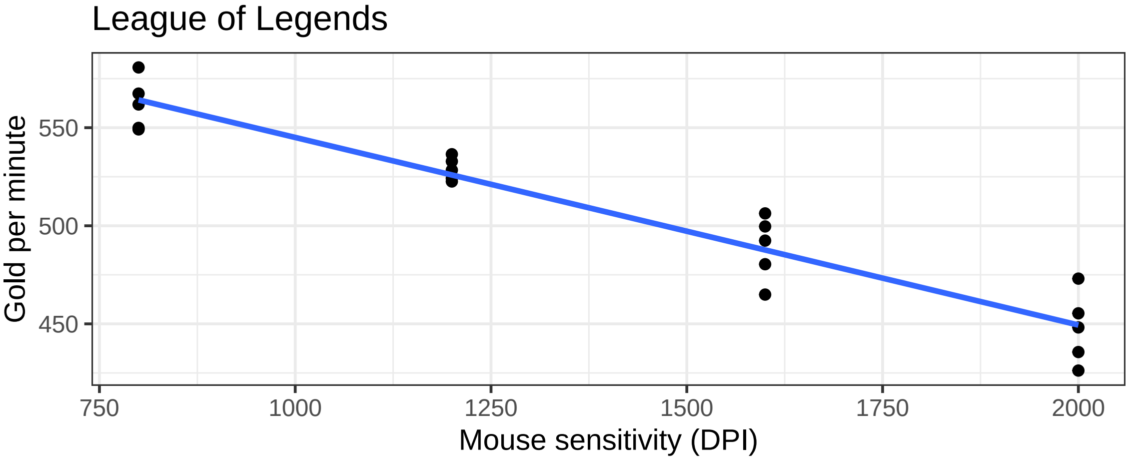
dependent	independent
response	independent
outcome	covariate
endogenous	exogenous

Independent-dependent variable

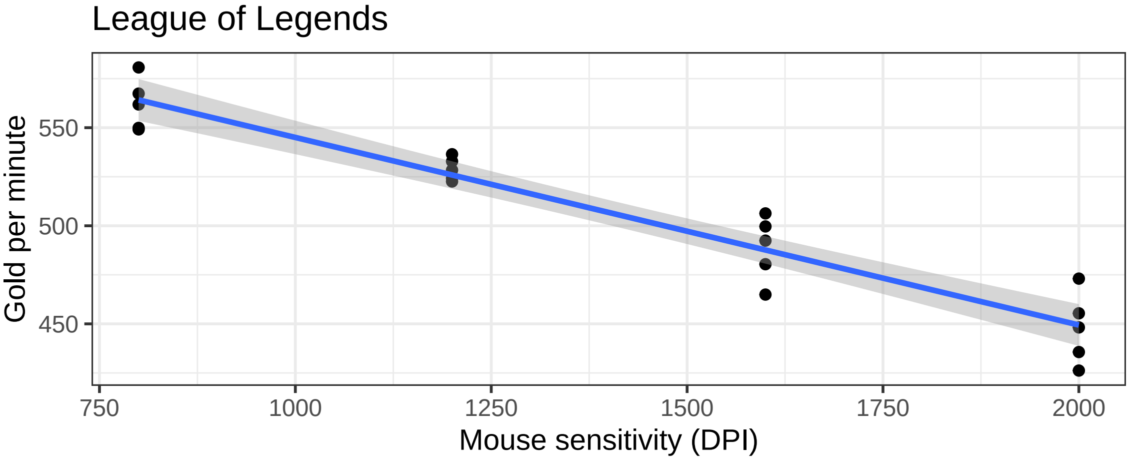
https://towardsdatascience.com/causal-inference-962ae97cefda



Continuous independent variable



Continuous independent variable



Simpe linear regression

The simple linear regression model is

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 X_i, \sigma^2)$$

where Y_i and X_i are the dependent and independent variable, respectively, for individual i.

Alternatively

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \qquad \epsilon_i \stackrel{ind}{\sim} N(0, \sigma^2).$$

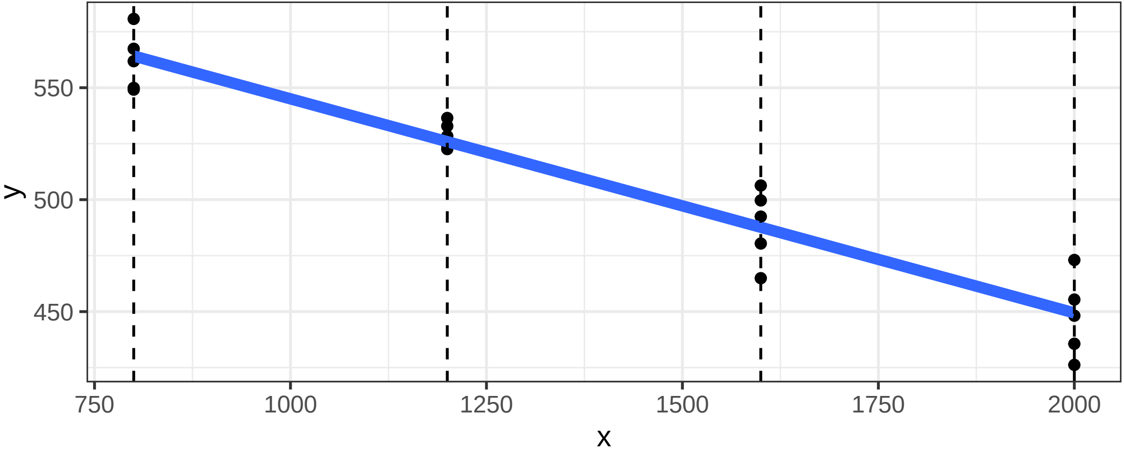
Importantly

$$E[Y_i|X_i] = \beta_0 + \beta_1 X_i$$

and

$$Var[Y_i|X_i] = \sigma^2.$$

Visualize variability



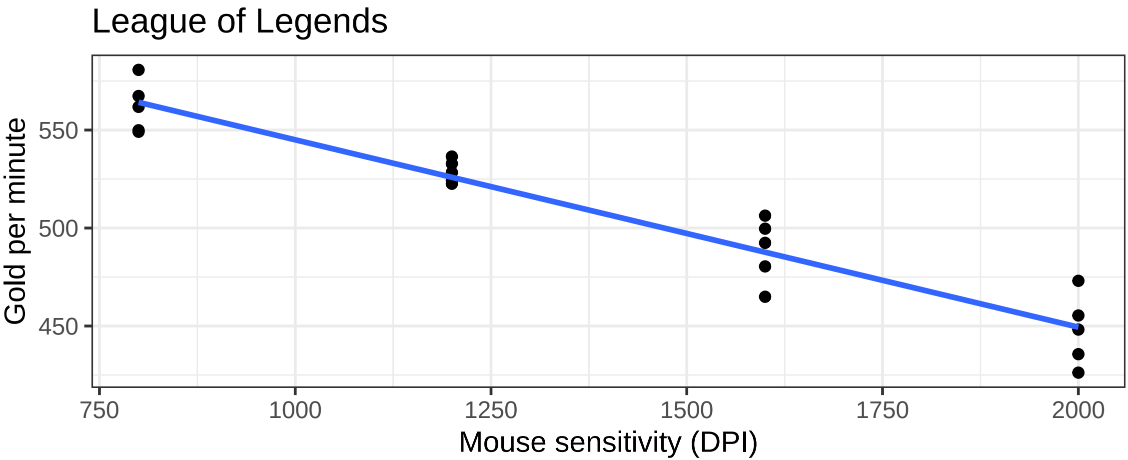
Estimate model parameters

```
m <- lm(gpm ~ sensitivity, data = mouse)
m

##
## Call:
## lm(formula = gpm ~ sensitivity, data = mouse)
##
## Coefficients:
## (Intercept)  sensitivity
##   640.63505     -0.09561
```

$$\hat{\beta}_0 = 641, \qquad \hat{\beta}_1 = -0.096$$

Fit a line


League of Legends
Gold per minute
550
500
450
750
1000
1250
1500
1750
2000
Mouse sensitivity (DPI)

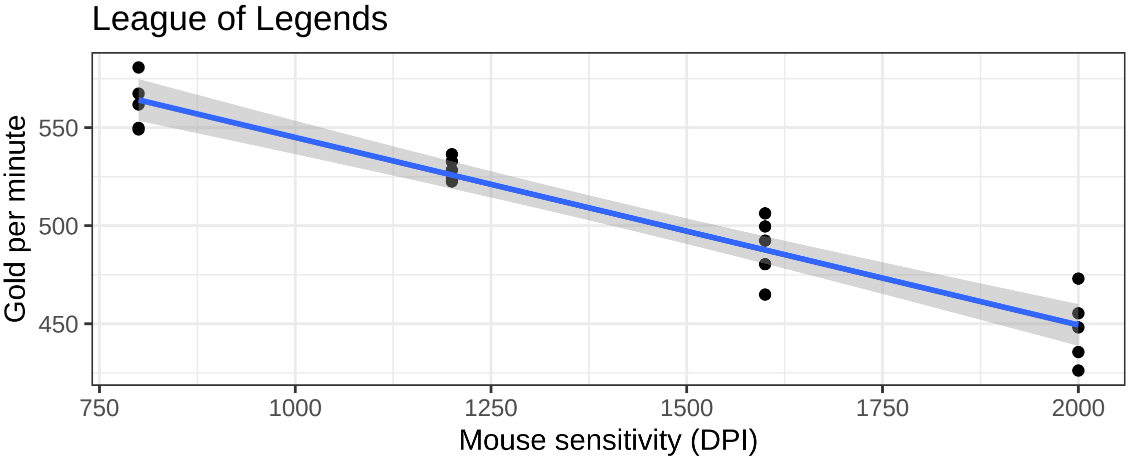
Credible intervals

```
confint(m)

##                    2.5 %       97.5 %
## (Intercept) 619.7064093 661.56368201
## sensitivity  -0.1098489  -0.08136859
```

A 95% CI for β_0 is (620, 662).
A 95% CI for β_1 is (-0.11, -0.081).

Uncertainty in the line



Interpretation

$$E[Y_i|X_i] = \beta_0 + \beta_1 X_i$$

When $X_i = 0$ (when mouse sensitivity is 0), $E[Y_i]$ (expected gold per minute) is 641 with a 95% CI of (620, 662).

For every 1 increase in X_i (mouse sensitivity increases by 1), the expected increase in Y_i (gold per minute) is -0.096 with a 95% CI of (-0.11, -0.081).

For every 400 increase in X_i (mouse sensitivity increases by 1), the expected increase in Y_i (gold per minute) is -40 with a 95% CI of (-44, -32).

Regression summary

```
summary(m)

##
## Call:
## lm(formula = gpm ~ sensitivity, data = mouse)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -23.2125  -8.8834   0.6222   7.8498  23.6453
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 640.635046   9.961643   64.31  < 2e-16 ***
## sensitivity  -0.095609   0.006778  -14.11 3.59e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.56 on 18 degrees of freedom
## Multiple R-squared:  0.917,Adjusted R-squared:  0.9124
## F-statistic:   199 on 1 and 18 DF,  p-value: 3.589e-11
```

Simple linear regression model assumptions

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \qquad \epsilon_i \stackrel{ind}{\sim} N(0, \sigma^2).$$

Assumptions:

- Linearity
- Normality
- Constant variance
- Independence

Many plots will be based off residuals:

$$r_i = \hat{\epsilon}_i = Y_i - \hat{Y}_i = Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_i.$$

Linearity assumption

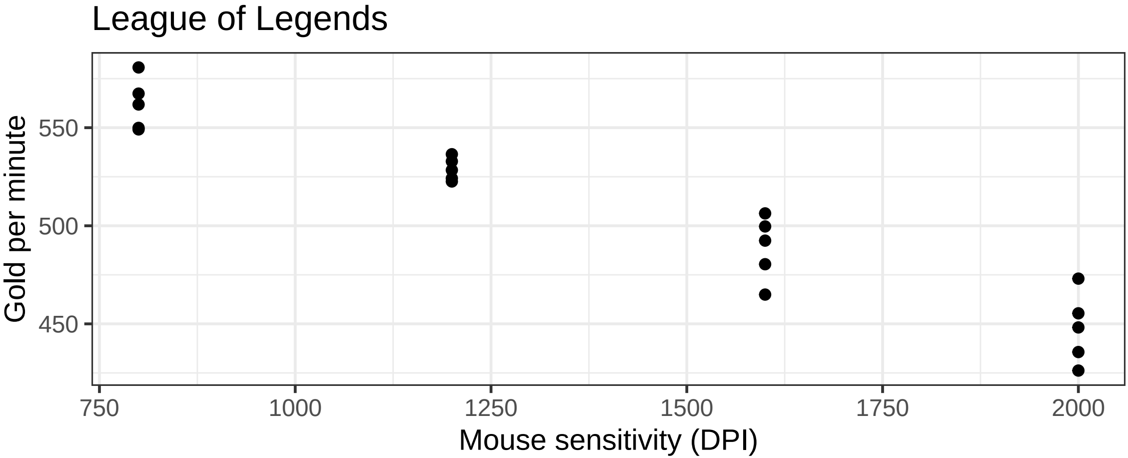
Linear relationships between expected value of the dependent variable and the independent variable:

$$E[Y_i|X_i] = \beta_0 + \beta_1 X_i$$

Look at

- Independent variable vs dependent variable
- Residuals vs predicted value

Linear assumption is valid



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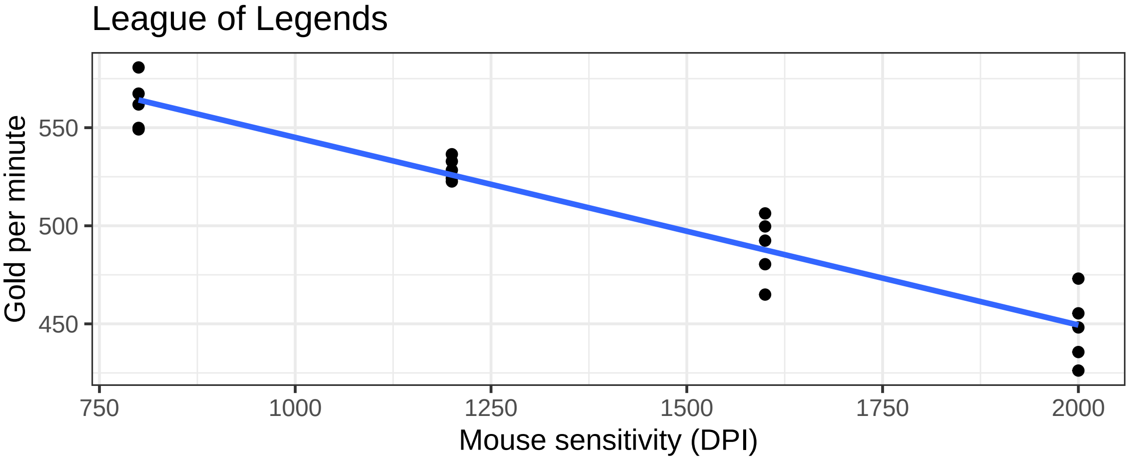
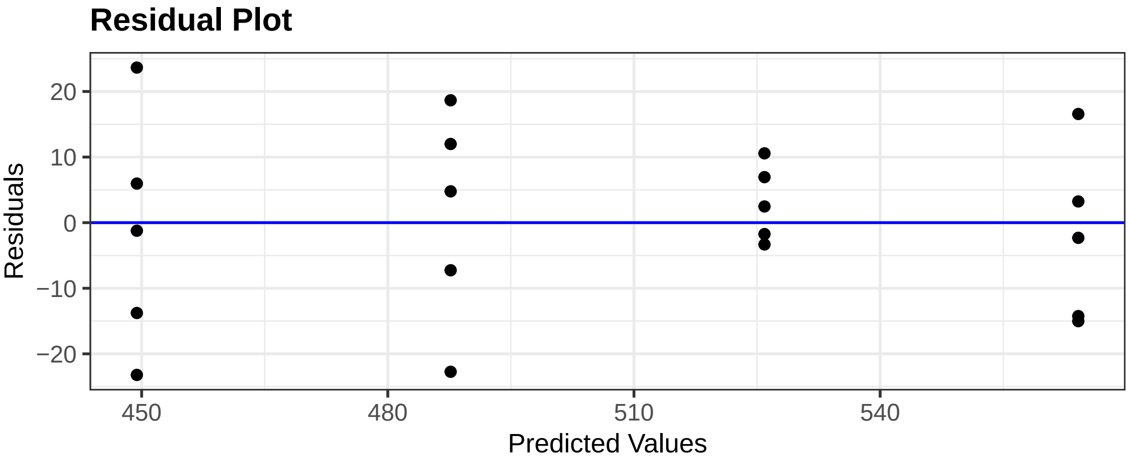
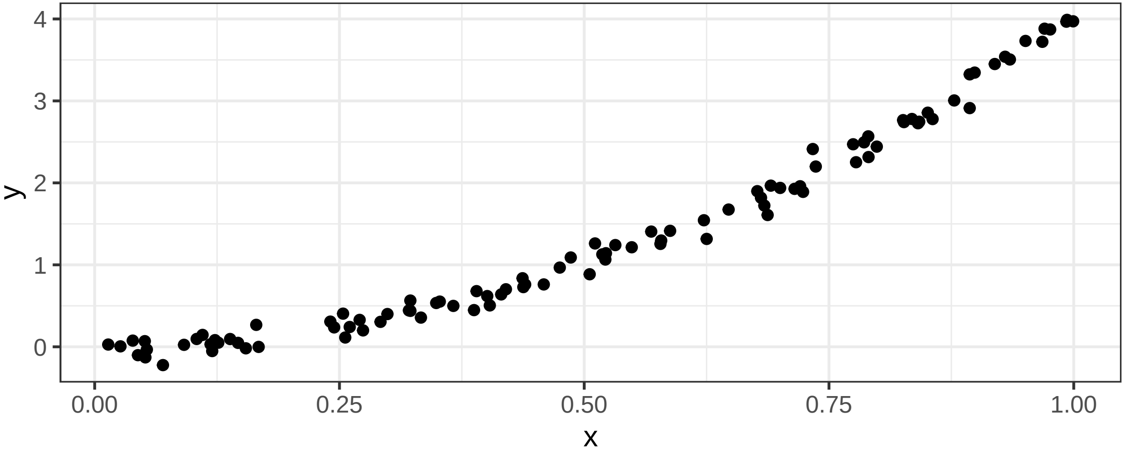


Linear assumption is valid



Linear assumption is NOT valid



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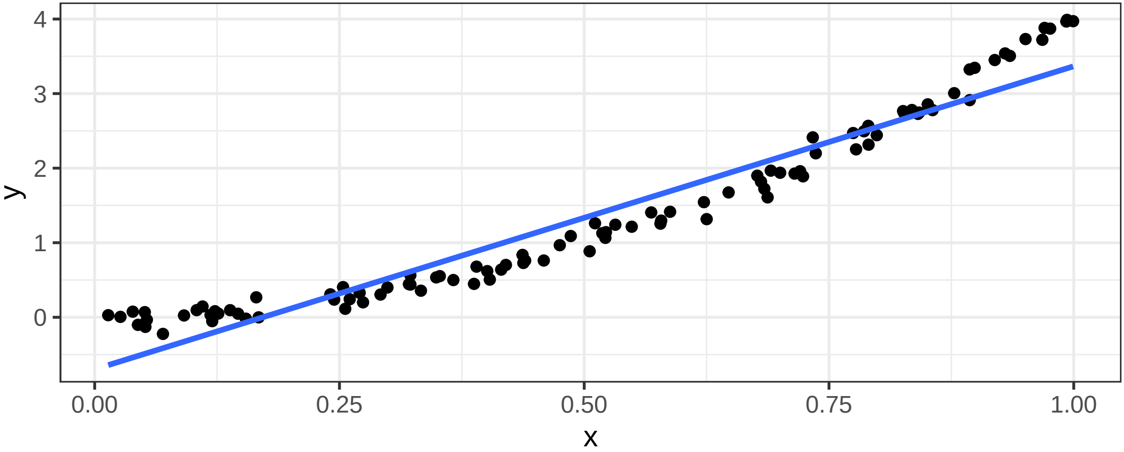
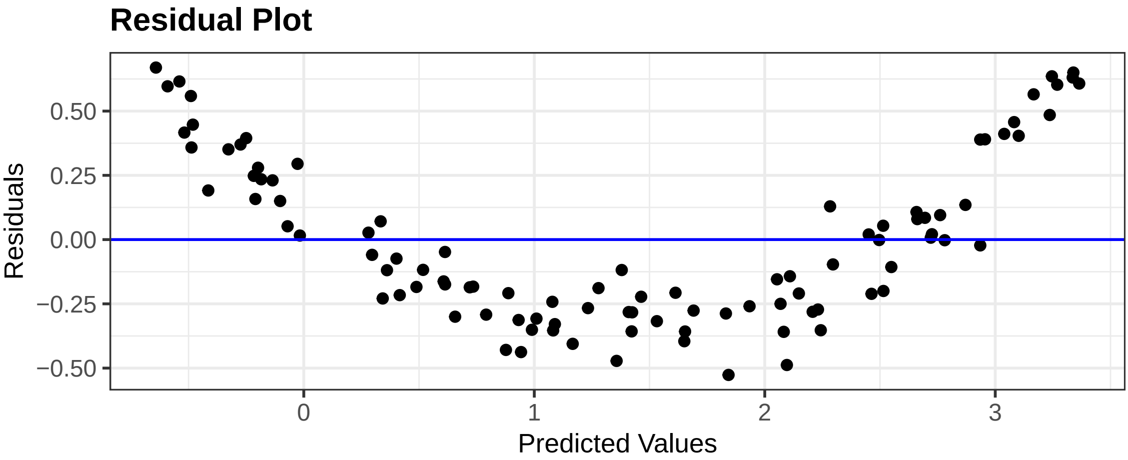


Linear assumption is NOT valid

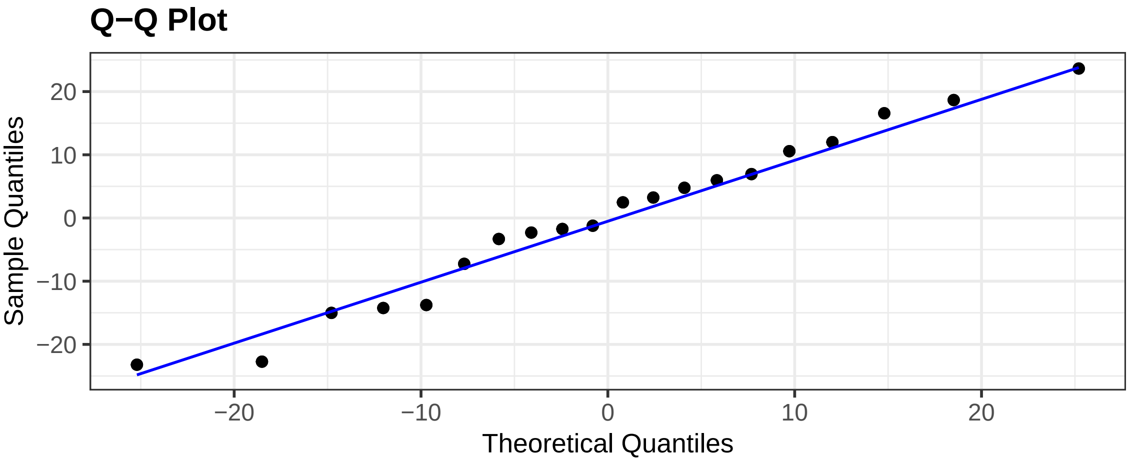


Normality

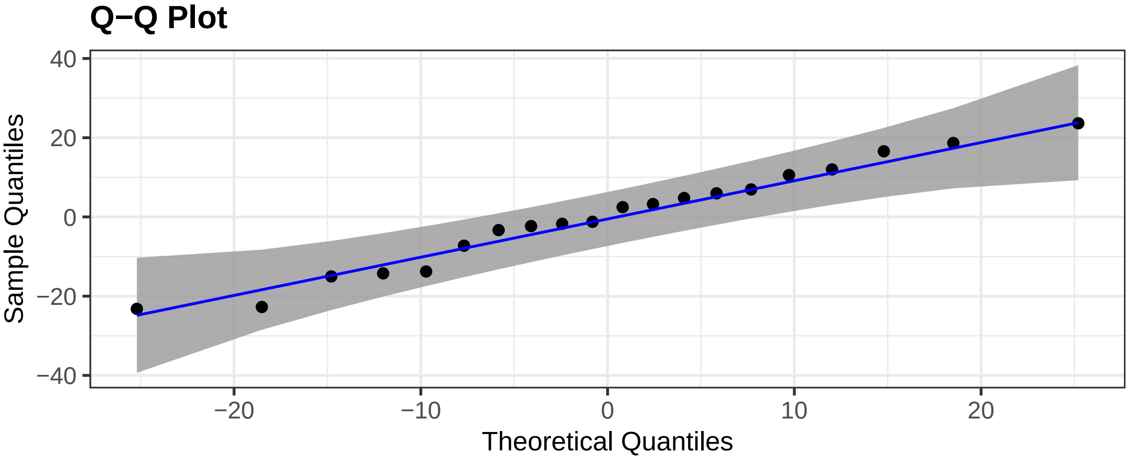
$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \qquad \epsilon_i \stackrel{ind}{\sim} N(0, \sigma^2).$$

Best diagnostic is a QQ-plot

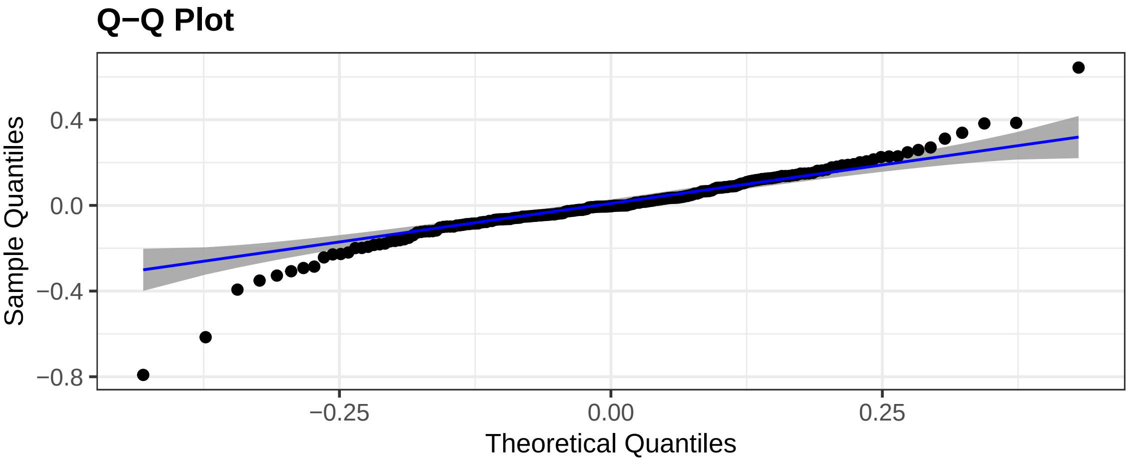
QQ-plot (normality is valid)



QQ-plot (normality is valid)



QQ-plot (normality is NOT valid)

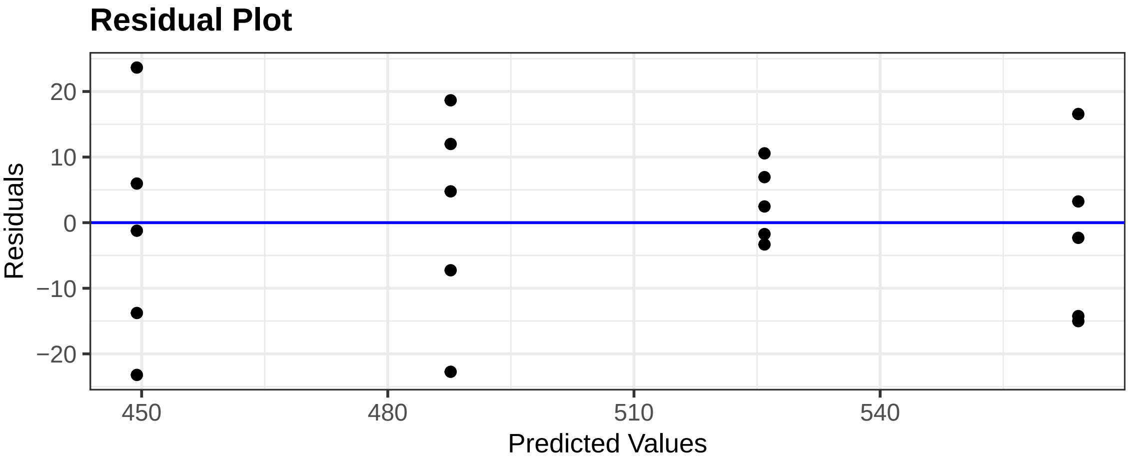


Constant variance

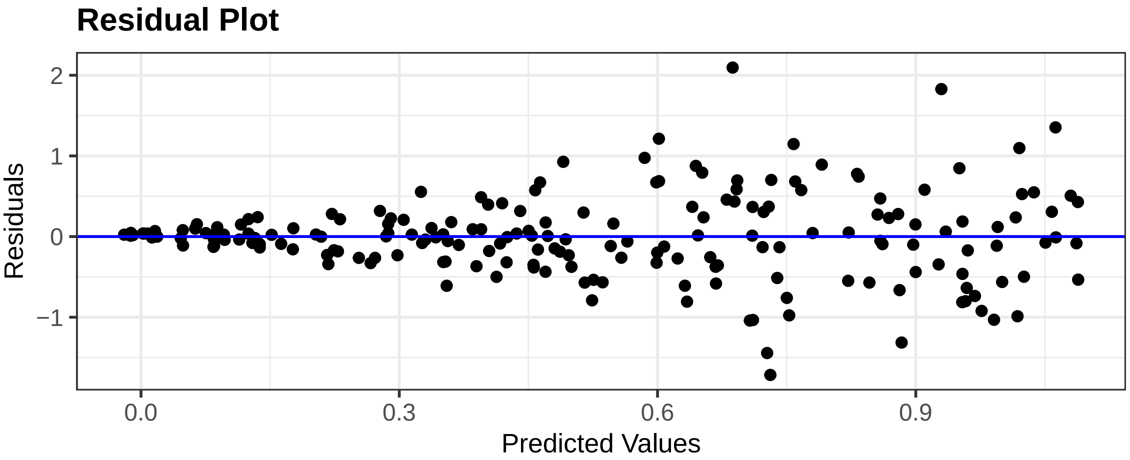
$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \qquad \epsilon_i \stackrel{ind}{\sim} N(0, \sigma^2).$$

Plot residuals vs predicted values and look for a “horn” shape pattern

Constant variance assumption is valid



Constant variance assumption is NOT valid



Independence

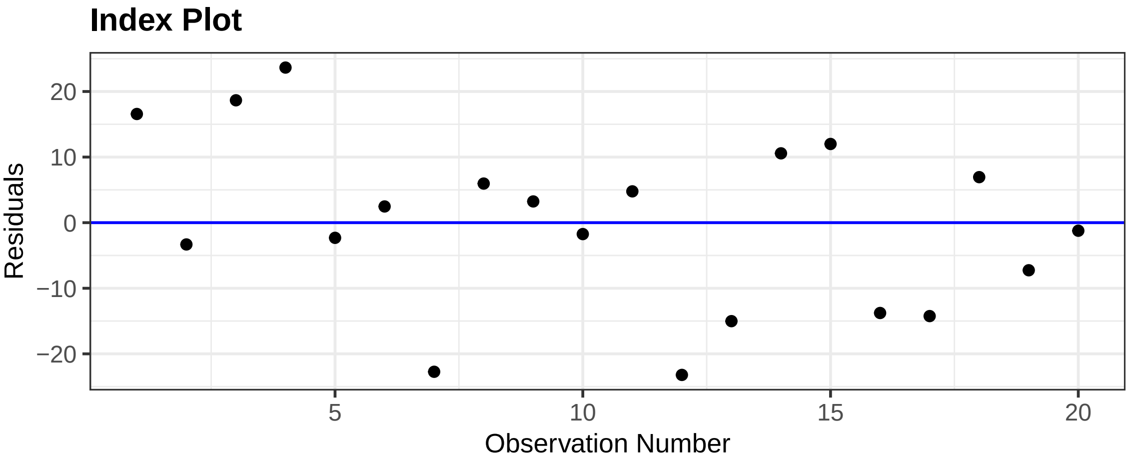
$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \qquad \epsilon_i \stackrel{ind}{\sim} N(0, \sigma^2).$$

No great way to assess this assumption other than subject matter knowledge.

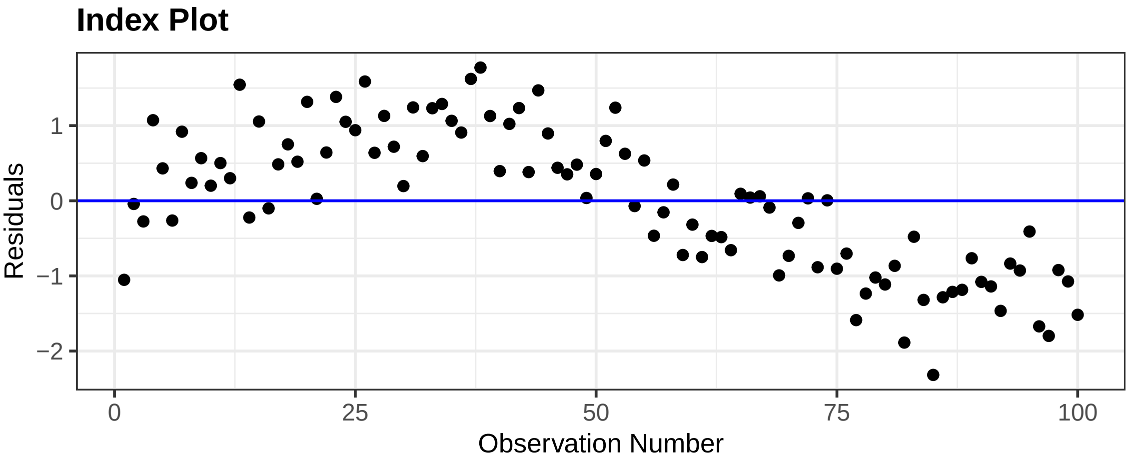
Main causes for dependence are

- temporal (residuals vs index might help)
- spatial
- clustering

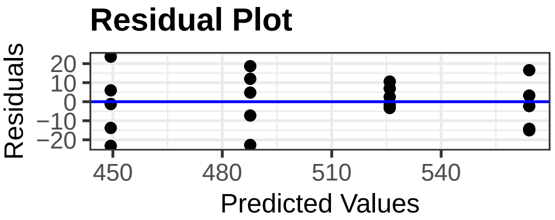
Residuals vs index (independence assumption is valid)

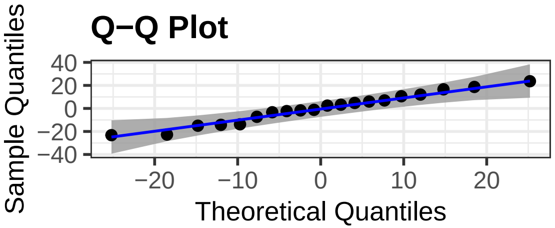


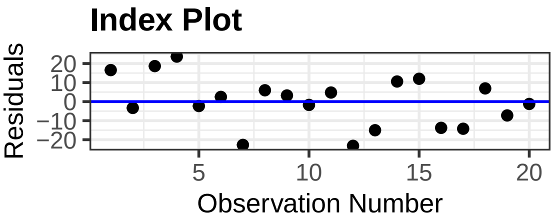
Residuals vs index (independence assumption is NOT valid)



All plots together







Summary

Simple linear regression model:

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \qquad \epsilon_i \stackrel{ind}{\sim} N(0, \sigma^2).$$

Assumptions:

- Linearity
- Normality
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