

# JMcDM: A Julia package for multiple-criteria decision-making tools

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## Abstract

*JMcDM* is a *Julia* package that implements some leading multiple-criteria decision making tools for both researchers and developers. *Julia*'s REPL is well suited for researchers to perform their analysis using different methods and compare their results. The package also provides the necessary infrastructure and utility functions for writing recently published methods. The proposed package has also brought MCDA tools to a relatively new language such as *Julia* with its significant performance promises. The methods developed in the package are also designed to be familiar to users who previously used *R* and *Python* languages. The paper presents the basics of the design, example usage, and code snippets.

*Keywords:* julia, multiple-criteria, decision-making, outranking

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## Required Metadata

### Current code version

Ancillary data table required for subversion of the codebase. Kindly replace examples in right column with the correct information about your current code, and leave the left column as it is.

Nr.	Code metadata description	Please fill in this column
C1	Current code version	v0.1.5
C2	Permanent link to code/repository used for this code version	<a href="https://github.com/jbytecode/JMcDM">https://github.com/jbytecode/JMcDM</a>
C3	Code Ocean compute capsule	
C4	Legal Code License	MIT
C5	Code versioning system used	git
C6	Software code languages, tools, and services used	Julia
C7	Compilation requirements, operating environments & dependencies	Julia 1.4
C8	If available Link to developer documentation/manual	<a href="https://jbytecode.github.io/JMcDM/docs/build">https://jbytecode.github.io/JMcDM/docs/build</a>
C9	Support email for questions	mhsatman@istanbul.edu.tr

Table 1: Code metadata (mandatory)

## 1. Motivation and significance

The one-dimensional array  $a$  is in ascending order if and only if  $a_i \leq a_{i+1}$  where  $i = 1, 2, \dots, n - 1$ , and  $n$  is the length of array. In other terms, the process of ordering numbers requires the logical  $\leq$  operator to be perfectly defined. Since the operator  $\leq$  is not defined for any set of points in higher dimensions,  $\mathbb{R}^p$  for  $p \geq 2$ , there is not a unique ordering of points.

In multi-dimensional case, the binary domination operator  $\succ$  applied on points  $a$  and  $b$ ,  $a \succ b$ , is true iff each item in  $a$  is not worse than the corresponding item in  $b$  and at least one item is better than the corresponding item in  $b$  [1]. On the other hand, the more relaxed operator  $\succeq$  returns true

11 if each item in  $a$  is as good as the corresponding item in  $b$  [2]. Several  
 12 outranking methods in MCDA (Multiple-Criteria Decision Analysis) define  
 13 a unique ranking mechanism to select the best alternative among others.

14 Suppose a decision process has  $n$  alternatives and  $m$  criteria which are  
 15 either to be maximized or minimized. Each single criterion has a weight  
 16  $0 \leq w_i \leq 1$  where  $\sum_i^m w_i = 1$ .  $f_i$  is either maximum or minimum.  $g_j(\cdot)$  is  
 17 evolution function and it is taken as  $g_j(x) = x$  in many methods. A multiple  
 18 criteria decision problem can be represented using the decision table

<b>Criteria</b>	$C_1$	$C_2$	$\dots$	$C_m$
<b>Weights</b>	$w_1$	$w_2$	$\dots$	$w_m$
<b>Functions</b>	$f_1$	$f_2$	$\dots$	$f_m$
$A_1$	$g_1(A_1)$	$g_2(A_1)$	$\dots$	$g_m(A_1)$
$A_2$	$g_1(A_2)$	$g_2(A_2)$	$\dots$	$g_m(A_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$A_n$	$g_1(A_n)$	$g_2(A_n)$	$\dots$	$g_m(A_n)$

Table 2: A decision matrix in general form

19 without loss of generality. When  $A_1, A_2, \dots, A_n$  are alternatives and  $C_1, C_2,$   
 20  $\dots, C_m$  are different situations of a single criterion then the decision problem  
 21 is said to be single criterion decision problem. If  $A_i$  and  $C_j$  are strategies of  
 22 two game players then  $g_j(A_i)$  is the gain of the row player when she selects  
 23 the strategy  $i$  and the column player selects the strategy  $C_j$ .

24 We must put here some text here like: MCDA is used in location selection  
 25 of facilities (Ref), selection of suppliers (Ref).... .

26 Multiple-criteria decision-making (MCDM) tools provide several algo-  
 27 rithms for ordering or selecting alternatives and/or determining the weights  
 28 when there is uncertainty. Although some algorithms are suitable for hand

calculations, a computer software is often required. *PyTOPS* is a Python tool for TOPSIS [3]. *Super Decisions* is a software package which is mainly focused on AHP (Analytic Hierarchy Process) and ANP (Analytic Network Process) [4]. *Visual Promethee* implements Promethee method on Windows platforms [5]. *M-BACBETH* is an other commercial software product that implements MACBETH with a easy to use GUI. [6]. *Sanna* is a standard MS Excel add-in application that supports several basic methods for multi-criteria evaluation of alternatives (WSA, TOPSIS, ELECTRE I and III, PROMETHEE I and II, MAPPAC and ORESTE) [7]. *DEA Frontier* software requires Excel add-in that can solve up to 50 DMUs with unlimited number of inputs and outputs (subject to the capacity of the standard MS Excel Solver) [8].

*JMcDM* is designed to provide a developer-friendly library for solving multiple-criteria decision problems in Julia [9]. Since Julia is a dynamic language, it is also useful for researchers that familiar with REPL environments. The package includes multi-criteria decision methods as well as a game solver for zero-sum games and methods for single criterion methods.

## 2. Software description

### 2.1. Software Architecture

*JMcDM* provides a framework for performing multi-criteria decision analysis as well as it includes utility functions for development of new methods. Each single MCDM method returns an object in subtype of `MCDMResult` which is defined as

```
abstract type MCDMResult end
```

and it is used to derive new return types. For instance, the `topsis()` function always returns a `TopsisResult` object which is defined as

```

55 struct TopsisResult <: MCDMResult
56     decisionMatrix::DataFrame
57     weights::Array{Float64,1}
58     normalizedDecisionMatrix::DataFrame
59     normalizedWeightedDecisionMatrix::DataFrame
60     bestIndex::Int64
61     scores::Array{Float64,1}
62 end

```

63 and holds many outputs in a single `struct`. Function definitions are also  
64 similar but they may differ depending on the requirements of algorithms.  
65 For instance the function `topsis` is defined as

```

66 function topsis(
67     decisionMat::DataFrame,
68     weights::Array{Float64,1},
69     fns::Array{Function,1})::TopsisResult

```

70 where `decisionMat` is the decision matrix, `weights` are weights of criteria,  
71 and `fns` is an array of functions (either `minimum` or `maximum`) that determine  
72 the optimization directions.

73 The package is registered in Julia package repository and it is available  
74 for downloading and installing using Julia's package manager.

```

75 julia> using Pkg
76 julia> Pkg.add("JMcDM")

```

77 and

```

78 julia> ]
79 (@v1.5) pkg> add JMcDM

```

80 present two distinct ways of install the package.

## 81 2.2. Software Functionalities

82 The package implements methods for TOPSIS<sup>1</sup> [10], ELECTRE<sup>2</sup>[11],  
83 PROMETHEE<sup>3</sup> [12], DEMATEL<sup>4</sup> [13], MOORA<sup>5</sup> [14], VIKOR<sup>6</sup> [15, 16],  
84 AHP<sup>7</sup> [17], GRA<sup>8</sup> [18], NDS<sup>9</sup> [1], SAW<sup>10</sup> [19, 20], ARAS<sup>11</sup> [21], WPM<sup>12</sup> [20],  
85 WASPAS<sup>13</sup> [22], EDAS<sup>14</sup> [23], MARCOS<sup>15</sup> [24], MABAC<sup>16</sup> [25], MAIRCA<sup>17</sup>  
86 [26], COPRAS<sup>18</sup> [27], COCOSO<sup>19</sup> [28], CRITIC<sup>20</sup> [29], and Entropy[30] for  
87 multiple-criteria tools. The package also performs DEA for Data Envelop-  
88 ment Analysis [31] and includes a method for zero-sum game solver.

## 89 2.3. Sample code snippets analysis

90 Suppose a decision problem is given in Table 3.

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<sup>1</sup>Technique for Order Preference by Similarity to Ideal Solutions

<sup>2</sup>Elimination and Choice Translating Reality

<sup>3</sup>Preference Ranking Organization METHod for Enrichment of Evaluations

<sup>4</sup>The Decision Making Trial and Evaluation Laboratory

<sup>5</sup>Multi-Objective Optimization By Ratio Analysis

<sup>6</sup>VlseKriterijumska Optimizacija I Kaompromisno Resenje in Serbian

<sup>7</sup>Analytic Hierarchy Process

<sup>8</sup>Grey Relational Analysis

<sup>9</sup>Non-dominated Sorting

<sup>10</sup>Simple Additive Weighting

<sup>11</sup>Additive Ratio Assessment

<sup>12</sup>Weighted Product Model

<sup>13</sup>Weighted Aggregated Sum Product ASsessment

<sup>14</sup>Evaluation based on Distance from Average Solution

<sup>15</sup>Measurement Alternatives and Ranking according to COmpromise Solution

<sup>16</sup>Multi-Attributive Border Approximation area Comparison

<sup>17</sup>Multi Attributive Ideal-Real Comparative Analysis

<sup>18</sup>Complex PROportional ASsessment

<sup>19</sup>Combined Compromise Solution

<sup>20</sup>CRiteria Importance Through Intercriteria Correlation

Criteria	Age	Size	Price	Distance	Population
Weights	0.35	0.15	0.25	0.20	0.05
Functions	min	max	min	min	max
$A_1$	6	140	150000	950	1500
$A_2$	4	90	100000	1500	2000
$A_3$	12	140	75000	550	1100

Table 3: Decision matrix

91 In this sample problem, a decision maker is subject to select an apartment  
92 by considering age of the building, size (in  $m^2$ s), price (in \$), distance to city  
93 centre, and nearby population. The data can be entered as a two-dimensional  
94 array (matrix) or as a DataFrame object:

```

95 julia> using JMCDM, DataFrames
96 julia> df = DataFrame(
97 :age      => [6.0, 4, 12],
98 :size     => [140.0, 90, 140],
99 :price    => [150000.0, 100000, 75000],
100 :distance => [950.0, 1500, 550],
101 :population => [1500.0, 2000, 1100]);
102
103 julia> w = [0.35, 0.15, 0.25, 0.20, 0.05];
104 julia> fns = [minimum, maximum, minimum, minimum, maximum];
105 julia> result = topsis(df, w, fns);
106 julia> result.scores
107 3-element Array{Float64,1}:
108 0.5854753145549456
109 0.6517997936899308

```

```
110 0.41850223305822903
```

```
111
```

```
112 julia> result.bestIndex
```

```
113 2
```

114 In the output above it is shown that the alternative  $A_2$  has a score of  
115 0.65179 and it is selected as the best. The same analysis can be performed  
116 using `saw()` for the method of Simple Additive Weighting

```
117 julia> result = saw(df, w, fns);
```

```
118 julia> result.bestIndex
```

```
119 2
```

120 as well as using `wpm` for the method of Weighted Product Method

```
121 julia> result = wpm(df, w, fns);
```

```
122 julia> result.bestIndex
```

```
123 2
```

124 For any method, `?methodname` shows the documentation as in the same  
125 way in other Julia packages.

### 126 2.3.1. Game Solver

127 A two-player zero-sum game is not a multi-criteria decision method. On  
128 the other hand, assuming the column player's choices are natural states for  
129 the row player, the game matrix represents gains or costs for the row player  
130 depending on the alternative she plays. Table 4 represents the gains of the  
131 row player.



	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

Table 4: Game matrix for the Rock & Paper & Scissors game

Table 4 shows that the row player wins the game if she selects **Rock** and the column player selects **Paper**. Similarly, she loses the game if she selects **Scissors** and the column player selects **Rock**. A tie has a zero gain for both players. The problem is selecting the best strategy for the row player. *JMcDM* implements the `game()` method for calculating value and the best strategy of this kind of games. The code snippet below represent the problem.

```
julia> mat = [0 -1 1; 1 0 -1; -1 1 0];
julia> dm = makeDecisionMatrix(mat);
julia> result = game(dm);
```

The `makeDecisionMatrix()` method returns a modified copy of the matrix `mat` as the minimum value of the new matrix is non-negative. The function `game()` returns a `GameResult` object which holds the value of the game and the probabilities of the alternatives for the row player.

```
julia> result.value
0.0
julia> result.row_player_probabilities
3-element Array{Float64,1}:
0.3333333333333333
0.3333333333333337
0.3333333333333333
```

```
julia> result1
```

3x7 DataFrame

Row	topsis	electre	cocoso	moora	vikor	wpm	waspas
	String	String	String	String	String	String	String
1			✓	✓	✓		
2	✓	✓				✓	✓
3		✓					

Figure 1: Results of TOPSIS, ELECTRE, VIKOR, MOORA, COCOSO, WPM, and WASPAS

152 It is shown that the value of the game is zero and the row player should  
 153 selects the alternatives with equal probability of  $\frac{1}{3}$ .

### 154 3. Illustrative Examples

155 Since *JMcDM* is designed as a software library and for REPL use, it does  
 156 not implement a significant user interface. However, the `summary()` function  
 157 provides a useful way to perform a list of methods and returns a text based  
 158 result to compare results.

```
159 julia> methods1 = [:topsis, :electre, :vikor,  
160 :moora, :cocoso, :wpm, :waspas]  
161 julia> result1 = summary(df, w, fns, methods1);
```

162 Figure 1 represents the output of the `summary()` call for methods TOP-  
 163 SIS, ELECTRE, VIKOR, MOORA, COCOSO, WPM, and WASPAS, re-  
 164 spectively.

```
165 julia> methods2 = [:aras, :saw, :edas, :marcos,  
166 :mabac, :mairca, :grey];  
167 julia> result2 = summary(df, w, fns, methods2);
```

```
julia> result2
```

3x7 DataFrame

Row	grey	aras	saw	edas	marcos	mabac	mairca
	String	String	String	String	String	String	String
1							
2		✓	✓	✓	✓		
3	✓					✓	✓

Figure 2: Results of ARAS, SAW, EDAS, MARCOS, MABAC, MAIRCA, and GREY

Figure 2 represents the output of the `summary()` call for methods ARAS, SAW, EDAS, MARCOS, MABAC, MAIRCA, and GREY, respectively.

#### 4. Impact

*JMcDM* provides a moderate number of MCDA tools and utility functions for developing new methods as well as performing decision analysis using a single function call for each method. A researcher can easily perform sequential analysis by changing the problem parameters and can compare results of many tools. Existing software packages are mainly focused on providing a small subset of methods. *JMcDM* is an all-in-one solution and has potential for increasing user productivity. Seeing the different results produced by the methods together also helps to discover which parameters the research is more sensitive to and the reasons for them.

#### 5. Conclusions

Multiple-criteria decision-making tools are used in various disciplines in the literature. Since it is difficult to calculate these tools manually, many software packages and programs have been developed. However, these packages include a selected subset of methods. *JMcDM* is a *Julia* package that

contains a significant amount of tools in the literature. Implementations of these tools are straightforward to use in *Julia* REPL for researchers. *JMcDM* also provides the necessary infrastructure for the development of new methods in the literature with its utility functions, comprehensive documentation, and unit tests compiled from many text books. The proposed package has also brought MCDA tools to a relatively new language such as Julia with significant performance promises. The methods in the package are also designed to be familiar to users who previously used R and Python languages.

## 6. Conflict of Interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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