

# JMcDM: A Julia package for multiple-criteria decision-making tools

Mehmet Hakan Satman<sup>a</sup>, Bahadır Fatih Yıldırım<sup>b</sup>, Ersagun Kuruca<sup>c</sup>

<sup>a</sup>*Istanbul University, Department of Econometrics, Beyazit, Istanbul, Turkey*

<sup>b</sup>*Istanbul University, Department of Transportation and Logistics, Avcılar, Istanbul, Turkey*

<sup>c</sup>*Istanbul Technical University, Department of Computer Engineering, Sarıyer, Istanbul, Turkey*

---

## Abstract

*JMcDM* is a *Julia* package that implements some leading multiple-criteria decision making tools for both researchers and developers. *Julia*'s REPL is well suited for researchers to perform their analysis using different methods and compare their results. The package also provides the necessary infrastructure and utility functions for writing recently published methods. The proposed package has also brought MCDA tools to a relatively new language such as *Julia* with its significant performance promises. The methods developed in the package are also designed to be familiar to users who previously used *R* and *Python* languages. The paper presents the basics of the design, example usage, and code snippets.

*Keywords:* julia, multiple-criteria, decision-making, outranking

---

## Required Metadata

### Current code version

Ancillary data table required for subversion of the codebase. Kindly replace examples in right column with the correct information about your current code, and leave the left column as it is.

Nr.	Code metadata description	Please fill in this column
C1	Current code version	v0.1.5
C2	Permanent link to code/repository used for this code version	<a href="https://github.com/jbytecode/JMcDM">https://github.com/jbytecode/JMcDM</a>
C3	Code Ocean compute capsule	
C4	Legal Code License	MIT
C5	Code versioning system used	git
C6	Software code languages, tools, and services used	Julia
C7	Compilation requirements, operating environments & dependencies	Julia 1.4
C8	If available Link to developer documentation/manual	<a href="https://jbytecode.github.io/JMcDM/docs/build">https://jbytecode.github.io/JMcDM/docs/build</a>
C9	Support email for questions	mhsatman@istanbul.edu.tr

Table 1: Code metadata (mandatory)

## 1. Motivation and significance

The one-dimensional array  $a$  is in ascending order if and only if  $a_i \leq a_{i+1}$  where  $i = 1, 2, \dots, n - 1$ , and  $n$  is the length of array. In other terms, the process of ordering numbers requires the logical  $\leq$  operator to be perfectly defined. Since the operator  $\leq$  is not defined for any set of points in higher dimensions,  $\mathbb{R}^p$  for  $p \geq 2$ , there is not a unique ordering of points.

In multi-dimensional case, the binary domination operator  $\succ$  applied on points  $a$  and  $b$ ,  $a \succ b$ , is true iff each item in  $a$  is not worse than the corresponding item in  $b$  and at least one item is better than the corresponding item in  $b$  [1]. On the other hand, the more relaxed operator  $\succeq$  returns true

11 if each item in  $a$  is as good as the corresponding item in  $b$  [2]. Several  
 12 outranking methods in MCDA (Multiple-Criteria Decision Analysis) define  
 13 a unique ranking mechanism to select the best alternative among others.

14 Suppose a decision process has  $n$  alternatives and  $m$  criteria which are  
 15 either to be maximized or minimized. Each single criterion has a weight  
 16  $0 \leq w_i \leq 1$  where  $\sum_i^m w_i = 1$ .  $f_i$  is either maximum or minimum.  $g_j(\cdot)$  is  
 17 evolution function and it is taken as  $g_j(x) = x$  in many methods. A multiple  
 18 criteria decision problem can be represented using the decision table

<b>Criteria</b>	$C_1$	$C_2$	$\dots$	$C_m$
<b>Weights</b>	$w_1$	$w_2$	$\dots$	$w_m$
<b>Functions</b>	$f_1$	$f_2$	$\dots$	$f_m$
$A_1$	$g_1(A_1)$	$g_2(A_1)$	$\dots$	$g_m(A_1)$
$A_2$	$g_1(A_2)$	$g_2(A_2)$	$\dots$	$g_m(A_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$A_n$	$g_1(A_n)$	$g_2(A_n)$	$\dots$	$g_m(A_n)$

Table 2: A decision matrix in general form

19 without loss of generality. When  $A_1, A_2, \dots, A_n$  are alternatives and  $C_1, C_2,$   
 20  $\dots, C_m$  are different situations of a single criterion then the decision problem  
 21 is said to be single criterion decision problem. If  $A_i$  and  $C_j$  are strategies of  
 22 two game players then  $g_j(A_i)$  is the gain of the row player when she selects  
 23 the strategy  $i$  and the column player selects the strategy  $C_j$ .

24 We must put here some text here like: MCDA is used in location selection  
 25 of facilities (Ref), selection of suppliers (Ref).... .

26 Multiple-criteria decision-making (MCDM) tools provide several algo-  
 27 rithms for ordering or selecting alternatives and/or determining the weights  
 28 when there is uncertainty. Although some algorithms are suitable for hand

calculations, a computer software is often required. *PyTOPS* is a Python tool for TOPSIS [3]. *Super Decisions* is a software package which is mainly focused on AHP (Analytic Hierarchy Process) and ANP (Analytic Network Process) [4]. *Visual Promethee* implements Promethee method on Windows platforms [5]. *M-BACBETH* is an other commercial software product that implements MACBETH with a easy to use GUI. [6]. *Sanna* is a standard MS Excel add-in application that supports several basic methods for multi-criteria evaluation of alternatives (WSA, TOPSIS, ELECTRE I and III, PROMETHEE I and II, MAPPAC and ORESTE) [7]. *DEA Frontier* software requires Excel add-in that can solve up to 50 DMUs with unlimited number of inputs and outputs (subject to the capacity of the standard MS Excel Solver) [8].

*JMcDM* is designed to provide a developer-friendly library for solving multiple-criteria decision problems in Julia [9]. Since Julia is a dynamic language, it is also useful for researchers that familiar with REPL environments. The package includes multi-criteria decision methods as well as a game solver for zero-sum games and methods for single criterion methods.

## 2. Software description

### 2.1. Software Architecture

*JMcDM* provides a framework for performing multi-criteria decision analysis as well as it includes utility functions for development of new methods. Each single MCDM method returns an object in subtype of `MCDMResult` which is defined as

```
abstract type MCDMResult end
```

and it is used to derive new return types. For instance, the `topsis()` function always returns a `TopsisResult` object which is defined as

```

55 struct TopsisResult <: MCDMResult
56     decisionMatrix::DataFrame
57     weights::Array{Float64,1}
58     normalizedDecisionMatrix::DataFrame
59     normalizedWeightedDecisionMatrix::DataFrame
60     bestIndex::Int64
61     scores::Array{Float64,1}
62 end

```

63 and holds many outputs in a single `struct`. Function definitions are also  
64 similar but they may differ depending on the requirements of algorithms.  
65 For instance the function `topsis` is defined as

```

66 function topsis(
67     decisionMat::DataFrame,
68     weights::Array{Float64,1},
69     fns::Array{Function,1})::TopsisResult

```

70 where `decisionMat` is the decision matrix, `weights` are weights of criteria,  
71 and `fns` is an array of functions (either `minimum` or `maximum`) that determine  
72 the optimization directions.

73 The package is registered in Julia package repository and it is available  
74 for downloading and installing using Julia's package manager.

```

75 julia> using Pkg
76 julia> Pkg.add("JMcDM")

```

77 and

```

78 julia> ]
79 (@v1.5) pkg> add JMcDM

```

80 present two distinct ways of downloading and installing the package.

## 81 2.2. Software Functionalities

82 The package implements methods for TOPSIS<sup>1</sup> [10], ELECTRE<sup>2</sup>[11],  
83 PROMETHEE<sup>3</sup> [12], DEMATEL<sup>4</sup> [13], MOORA<sup>5</sup> [14], VIKOR<sup>6</sup> [15, 16],  
84 AHP<sup>7</sup> [17], GRA<sup>8</sup> [18], NDS<sup>9</sup> [1], SAW<sup>10</sup> [19, 20], ARAS<sup>11</sup> [21], WPM<sup>12</sup> [20],  
85 WASPAS<sup>13</sup> [22], EDAS<sup>14</sup> [23], MARCOS<sup>15</sup> [24], MABAC<sup>16</sup> [25], MAIRCA<sup>17</sup>  
86 [26], COPRAS<sup>18</sup> [27], COCOSO<sup>19</sup> [28], CRITIC<sup>20</sup> [29], and Entropy[30] for  
87 multiple-criteria tools. The package also performs DEA for Data Envelop-  
88 ment Analysis [31] and includes a method for zero-sum game solver. The list  
89 of other tools and utility functions are listed and documented in the source  
90 code as well as in the online documentation.

## 91 2.3. Sample code snippets analysis

92 Suppose a decision problem is given in Table 3.

---

<sup>1</sup>Technique for Order Preference by Similarity to Ideal Solutions

<sup>2</sup>Elimination and Choice Translating Reality

<sup>3</sup>Preference Ranking Organization METHod for Enrichment of Evaluations

<sup>4</sup>The Decision Making Trial and Evaluation Laboratory

<sup>5</sup>Multi-Objective Optimization By Ratio Analysis

<sup>6</sup>VlseKriterijumska Optimizacija I Kaompromisno Resenje in Serbian

<sup>7</sup>Analytic Hierarchy Process

<sup>8</sup>Grey Relational Analysis

<sup>9</sup>Non-dominated Sorting

<sup>10</sup>Simple Additive Weighting

<sup>11</sup>Additive Ratio Assessment

<sup>12</sup>Weighted Product Model

<sup>13</sup>Weighted Aggregated Sum Product ASsessment

<sup>14</sup>Evaluation based on Distance from Average Solution

<sup>15</sup>Measurement Alternatives and Ranking according to COMpromise Solution

<sup>16</sup>Multi-Attributive Border Approximation area Comparison

<sup>17</sup>Multi Attributive Ideal-Real Comparative Analysis

<sup>18</sup>Complex PROportional ASsessment

<sup>19</sup>Combined Compromise Solution

<sup>20</sup>CRiteria Importance Through Intercriteria Correlation

Criteria	Age	Size	Price	Distance	Population
Weights	0.35	0.15	0.25	0.20	0.05
Functions	min	max	min	min	max
$A_1$	6	140	150000	950	1500
$A_2$	4	90	100000	1500	2000
$A_3$	12	140	75000	550	1100

Table 3: Decision matrix

93 In this sample problem, a decision maker is subject to select an apartment  
94 by considering age of the building, size (in  $m^2$ s), price (in \$), distance to city  
95 centre (in  $ms$ ), and nearby population. The data can be entered as a two-  
96 dimensional array (matrix) or as a DataFrame object:

```
97 julia> using JMCDM, DataFrames
98 julia> df = DataFrame(
99 :age      => [6.0, 4, 12],
100 :size     => [140.0, 90, 140],
101 :price    => [150000.0, 100000, 75000],
102 :distance => [950.0, 1500, 550],
103 :population => [1500.0, 2000, 1100]);
```

104 The weight vector **w**, vector of directions **fns**, and **topsis()** function call  
105 can be performed using the *Julia* REPL.

```
106 julia> w = [0.35, 0.15, 0.25, 0.20, 0.05];
107 julia> fns = [minimum, maximum, minimum, minimum, maximum];
108 julia> result = topsis(df, w, fns);
109 julia> result.scores
110 3-element Array{Float64,1}:
```

```
111 0.5854753145549456
```

```
112 0.6517997936899308
```

```
113 0.41850223305822903
```

```
114
```

```
115 julia> result.bestIndex
```

```
116 2
```

117 In the output above it is shown that the alternative  $A_2$  has a score of  
118 0.65179 and it is selected as the best. The same analysis can be performed  
119 using `saw()` for the method of Simple Additive Weighting

```
120 julia> result = saw(df, w, fns);
```

```
121 julia> result.bestIndex
```

```
122 2
```

123 as well as using `wpm` for the method of Weighted Product Method

```
124 julia> result = wpm(df, w, fns);
```

```
125 julia> result.bestIndex
```

```
126 2
```

127 For any method, `?methodname` shows the documentation as in the same  
128 way in other Julia packages.

### 129 2.3.1. Game Solver

130 A two-player zero-sum game is not a multi-criteria decision method. On  
131 the other hand, assuming the column player's choices are natural states for  
132 the row player, the game matrix represents gains or costs for the row player  
133 depending on the alternative she plays. Table 4 represents the gains of the  
134 row player for a Rock & Paper % Scissors game. Each time the game is  
135 played, the winner takes 1 point.



	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

Table 4: Game matrix for the Rock & Paper & Scissors game

Table 4 shows that the row player wins the game if she selects **Rock** and the column player selects **Paper**. Similarly, she loses the game if she selects **Scissors** and the column player selects **Rock**. A tie has a zero gain for both players. The problem is selecting the best strategy for the row player. *JMcDM* implements the `game()` method for calculating value and the best strategy of this kind of games. The code snippet below represent the problem.

```
julia> mat = [0 -1 1; 1 0 -1; -1 1 0];
julia> dm = makeDecisionMatrix(mat);
julia> result = game(dm);
```

The `makeDecisionMatrix()` method returns a modified copy of the matrix `mat` as the minimum value of the new matrix is non-negative. The function `game()` returns a `GameResult` object which holds the value of the game and the probabilities of the alternatives for the row player.

```
julia> result.value
0.0
julia> result.row_player_probabilities
3-element Array{Float64,1}:
0.3333333333333333
0.3333333333333337
0.3333333333333333
```

```
julia> result1
```

3x7 DataFrame

Row	topsis	electre	cocoso	moora	vikor	wpm	waspas
	String	String	String	String	String	String	String
1			✓	✓	✓		
2	✓	✓				✓	✓
3		✓					

Figure 1: Results of TOPSIS, ELECTRE, VIKOR, MOORA, COCOSO, WPM, and WASPAS

156 It is shown that the value of the game is zero and the row player should  
 157 selects the alternatives with equal probability of  $\frac{1}{3}$ .

### 158 3. Illustrative Examples

159 Since *JMcDM* is designed as a software library and for REPL use, it does  
 160 not implement a significant user interface. However, the `summary()` function  
 161 provides a useful way to perform a list of methods and returns a text based  
 162 result to compare results.

```
163 julia> methods1 = [:topsis, :electre, :vikor,  
164 :moora, :cocoso, :wpm, :waspas]  
165 julia> result1 = summary(df, w, fns, methods1);
```

166 Figure 1 represents the output of the `summary()` call for methods TOP-  
 167 SIS, ELECTRE, VIKOR, MOORA, COCOSO, WPM, and WASPAS, re-  
 168 spectively.

```
169 julia> methods2 = [:aras, :saw, :edas, :marcos,  
170 :mabac, :mairca, :grey];  
171 julia> result2 = summary(df, w, fns, methods2);
```

```
julia> result2
```

3x7 DataFrame

Row	grey	aras	saw	edas	marcos	mabac	mairca
	String	String	String	String	String	String	String
1							
2		✓	✓	✓	✓		
3	✓					✓	✓

Figure 2: Results of ARAS, SAW, EDAS, MARCOS, MABAC, MAIRCA, and GREY

Figure 2 represents the output of the `summary()` call for methods ARAS, SAW, EDAS, MARCOS, MABAC, MAIRCA, and GREY, respectively.

#### 4. Impact

*JMcDM* provides a moderate number of MCDA tools and utility functions for developing new methods as well as performing decision analysis using a single function call for each method. A researcher can easily perform sequential analysis by changing the problem parameters and can compare results of many tools. Existing software packages are mainly focused on providing a small subset of methods. *JMcDM* is an all-in-one solution and has potential for increasing user productivity. Seeing the different results produced by the methods together also helps to discover which parameters the research is more sensitive to and the reasons for them.

#### 5. Conclusions

Multiple-criteria decision-making tools are used in various disciplines in the literature. Since it is difficult to calculate these tools manually, many software packages and programs have been developed. However, these packages include a selected subset of methods. *JMcDM* is a *Julia* package that

contains a significant amount of tools in the literature. Implementations of these tools are straightforward to use in *Julia* REPL for researchers. *JMcDM* also provides the necessary infrastructure for the development of new methods in the literature with its utility functions, comprehensive documentation, and unit tests compiled from many text books. The proposed package has also brought MCDA tools to a relatively new language such as Julia with significant performance promises. The methods in the package are also designed to be familiar to users who previously used R and Python languages.

## 6. Conflict of Interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

## Acknowledgements

The authors would like to thank the Editor-in-Chief, editors, reviewers for providing extremely insightful comments, and other staff of the SoftwareX Journal.

## References

- [1] K. Deb, A. Pratap, S. Agarwal, T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, *IEEE Transactions on Evolutionary Computation* 6 (2) (2002) 182–197. doi:10.1109/4235.996017.
- [2] S. Greco, J. Figueira, M. Ehrgott, *Multiple criteria decision analysis*, Vol. 37, Springer, 2016.

- 211 [3] V. Yadav, S. Karmakar, P. P. Kalbar, A. Dikshit, PyTOPS: A python  
212 based tool for TOPSIS, *SoftwareX* 9 (2019) 217–222. doi:10.1016/j.  
213 softx.2019.02.004.
- 214 [4] W. Adams, R. Saaty, *Super decisions software guide* (2003).  
215 URL [https://superdecisions.com/sd\\_resources/v28\\_man01.pdf](https://superdecisions.com/sd_resources/v28_man01.pdf)
- 216 [5] B. Mareschal, Y. D. Smet, Visual PROMETHEE: Developments of the  
217 PROMETHEE - GAIA multicriteria decision aid methods, in: 2009  
218 IEEE International Conference on Industrial Engineering and Engineer-  
219 ing Management, IEEE, 2009. doi:10.1109/ieem.2009.5373124.
- 220 [6] C. A. Bana e Costa, J.-M. de Corte, J.-C. Vansnick, MACBETH  
221 (Measuring Attractiveness by a Categorical Based Evaluation Tech-  
222 nique), *Wiley Encyclopedia of Operations Research and Management*  
223 *Science*, John Wiley & Sons, Inc., 2011. doi:10.1002/9780470400531.  
224 eorms0970.  
225 URL <https://doi.org/10.1002/9780470400531.eorms0970>
- 226 [7] J. Jablonsky, MS excel based software support tools for decision prob-  
227 lems with multiple criteria, *Procedia Economics and Finance* 12 (2014)  
228 251–258. doi:10.1016/s2212-5671(14)00342-6.  
229 URL <https://doi.org/10.1016%2Fs2212-5671%2814%2900342-6>
- 230 [8] J. Zhu, *Quantitative Models for Performance Evaluation and Bench-*  
231 *marking*, Springer International Publishing, 2014. doi:10.1007/  
232 978-3-319-06647-9.  
233 URL <https://doi.org/10.1007%2F978-3-319-06647-9>
- 234 [9] J. Bezanson, A. Edelman, S. Karpinski, V. B. Shah, Julia: A fresh  
235 approach to numerical computing, *SIAM Review* 59 (1) (2017) 65–98.

- doi:10.1137/141000671.  
URL <https://doi.org/10.1137/2F141000671>
- [10] C.-L. Hwang, K. Yoon, Methods for multiple attribute decision making, in: Multiple Attribute Decision Making, Springer Berlin Heidelberg, 1981, pp. 58–191. doi:10.1007/978-3-642-48318-9\_3.
- [11] B. Roy, Classement et choix en présence de points de vue multiples, RAIRO - Operations Research - Recherche Opérationnelle 2 (V1) (1968) 57–75.  
URL <http://eudml.org/doc/104443>
- [12] J.-P. Brans, P. Vincke, Note—a preference ranking organisation method: (the promethee method for multiple criteria decision-making), Management science 31 (6) (1985) 647–656.
- [13] A. Gabus, E. Fontela, World problems, an invitation to further thought within the framework of dematel, Battelle Geneva Research Center, Geneva, Switzerland (1972) 1–8.
- [14] W. K. Brauers, E. K. Zavadskas, The moora method and its application to privatization in a transition economy, Control and cybernetics 35 (2006) 445–469.
- [15] S. Opricovic, Multicriteria optimization of civil engineering systems, Faculty of Civil Engineering, Belgrade 2 (1) (1998) 5–21.
- [16] S. Opricovic, G.-H. Tzeng, Multicriteria planning of post-earthquake sustainable reconstruction, Computer-Aided Civil and Infrastructure Engineering 17 (3) (2002) 211–220. doi:10.1111/1467-8667.00269.  
URL <https://doi.org/10.1111/2F1467-8667.00269>

- [17] T. L. Saaty, A scaling method for priorities in hierarchical structures, Journal of Mathematical Psychology 15 (3) (1977) 234–281. doi:10.1016/0022-2496(77)90033-5.  
URL <https://doi.org/10.1016%2F0022-2496%2877%2990033-5>
- [18] D. Ju-Long, Control problems of grey systems, Systems & Control Letters 1 (5) (1982) 288–294. doi:10.1016/s0167-6911(82)80025-x.  
URL <https://doi.org/10.1016%2Fs0167-6911%2882%2980025-x>
- [19] C. W. Churchman, R. L. Ackoff, An approximate measure of value, Journal of the Operations Research Society of America 2 (2) (1954) 172–187. doi:10.1287/opre.2.2.172.  
URL <https://doi.org/10.1287%2Fopre.2.2.172>
- [20] E. Triantaphyllou, S. H. Mann, An examination of the effectiveness of multi-dimensional decision-making methods: A decision-making paradox, Decision Support Systems 5 (3) (1989) 303–312. doi:10.1016/0167-9236(89)90037-7.  
URL <https://doi.org/10.1016%2F0167-9236%2889%2990037-7>
- [21] E. K. Zavadskas, Z. Turskis, A new additive ratio assessment (aras) method in multicriteria decision-making, Technological and Economic Development of Economy 16 (2) (2010) 159–172.
- [22] E. K. Zavadskas, Z. Turskis, J. Antucheviciene, Optimization of weighted aggregated sum product assessment, Electronics and Electrical Engineering 122 (6) (jun 2012). doi:10.5755/j01.eee.122.6.1810.  
URL <https://doi.org/10.5755%2Fj01.eee.122.6.1810>
- [23] M. K. Ghorabae, E. K. Zavadskas, L. Olfat, Z. Turskis, Multi-criteria inventory classification using a new method of evaluation based on dis-

- 285 tance from average solution (EDAS), *Informatica* 26 (3) (2015) 435–451.  
 286 doi:10.15388/informatica.2015.57.  
 287 URL <https://doi.org/10.15388%2Finformatica.2015.57>
- 288 [24] Ž. Stević, D. Pamučar, A. Puška, P. Chatterjee, Sustainable supplier se-  
 289 lection in healthcare industries using a new MCDM method: Measure-  
 290 ment of alternatives and ranking according to COmpromise solution  
 291 (MARCOS), *Computers & Industrial Engineering* 140 (2020) 106231.  
 292 doi:10.1016/j.cie.2019.106231.  
 293 URL <https://doi.org/10.1016%2Fj.cie.2019.106231>
- 294 [25] D. Pamučar, G. Čirović, The selection of transport and handling re-  
 295 sources in logistics centers using multi-attributive border approximation  
 296 area comparison (MABAC), *Expert Systems with Applications* 42 (6)  
 297 (2015) 3016–3028. doi:10.1016/j.eswa.2014.11.057.  
 298 URL <https://doi.org/10.1016%2Fj.eswa.2014.11.057>
- 299 [26] D. Pamučar, L. Vasin, L. Lukovac, Selection of railway level crossings for  
 300 investing in security equipment using hybrid dematel-marica model, in:  
 301 XVI international scientific-expert conference on railway, railcon, 2014,  
 302 pp. 89–92.
- 303 [27] E. K. Zavadskas, A. Kaklauskas, V. Sarka, The new method of multi-  
 304 criteria complex proportional assessment of projects, *Technological and*  
 305 *economic development of economy* 1 (3) (1994) 131–139.
- 306 [28] M. Yazdani, P. Zarate, E. K. Zavadskas, Z. Turskis, A combined compro-  
 307 mise solution (CoCoSo) method for multi-criteria decision-making prob-  
 308 lems, *Management Decision* 57 (9) (2019) 2501–2519. doi:10.1108/



- 309 md-05-2017-0458.  
310 URL <https://doi.org/10.1108%2Fmd-05-2017-0458>
- 311 [29] D. Diakoulaki, G. Mavrotas, L. Papayannakis, Determining objective  
312 weights in multiple criteria problems: The critic method, Comput-  
313 ers & Operations Research 22 (7) (1995) 763–770. doi:10.1016/  
314 0305-0548(94)00059-h.  
315 URL <https://doi.org/10.1016%2F0305-0548%2894%2900059-h>
- 316 [30] C. E. Shannon, A mathematical theory of communication, Bell System  
317 Technical Journal 27 (3) (1948) 379–423. doi:10.1002/j.1538-7305.  
318 1948.tb01338.x.  
319 URL <https://doi.org/10.1002%2Fj.1538-7305.1948.tb01338.x>
- 320 [31] A. Charnes, W. Cooper, E. Rhodes, Measuring the efficiency of decision  
321 making units, European Journal of Operational Research 2 (6) (1978)  
322 429–444. doi:10.1016/0377-2217(78)90138-8.  
323 URL <https://doi.org/10.1016%2F0377-2217%2878%2990138-8>