Lecture 10: Other forms of learning

Metric, Rank, Recommend, Embedding

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July 13, 2025

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Metric Learning

Most frequently used metrics of similarity (or dissimilarity) between two feature vectors are **Euclidean distance** and **cosine similarity**.

Euclidean distance

$$d^2(x, x') = \langle x - x', x - x' \rangle$$

We can modify Euclidean distance to make to parametrizable and learn metrics from data

Modified Euclidean distance

$$d_A^2(x, x') = \|x - x'\|_A = (x - x')^T A(x - x') =< x - x', x - x' >_A$$

Example

Euclidean distance A $\stackrel{\text{def}}{=} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Weighted A $\stackrel{\text{def}}{=} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Definition - Metric

1. $d(x, x') \geq 0$ nonnegativity
2. $d(x, x') \leq d(x, x') + d(x', z)$ triangle inequality
3. $d(x, x') = d(x', x)$ symmetry

In order to satisfy the definition of metric, A should be positive semi-definite. To satisfy the third condition, we can define a new metric which is $d'(x, x') = d(x, x') + d(x', x)$.

Training

We need to create two sets. The first set S is such that a pair of examples (x_i, x_k) belong to S if x_i and x_k are similar.
The second set D is such that a pair of examples (x_i, x_k) belongs to set D if x_i and x_k are dissimilar.

Optimization Problem

$$\min_{\mathrm{A}} \sum_{(\mathrm{x}_i,\mathrm{x}_k)\in\mathcal{S}} \left\|\mathrm{x}-\mathrm{x}'\right\|_{\mathrm{A}}^2 \text{ such that } \sum_{(\mathrm{x}_i,\mathrm{x}_k)\in\mathcal{D}} \left\|\mathrm{x}-\mathrm{x}'\right\|_{\mathrm{A}} \geq c$$

Riemannian Metric

Riemannian Metric is very related to this topic. They are almost defined in the same way. A Riemannian Metric defines the inner product of two vectors in the **tangent space**.

Riemannian Metric

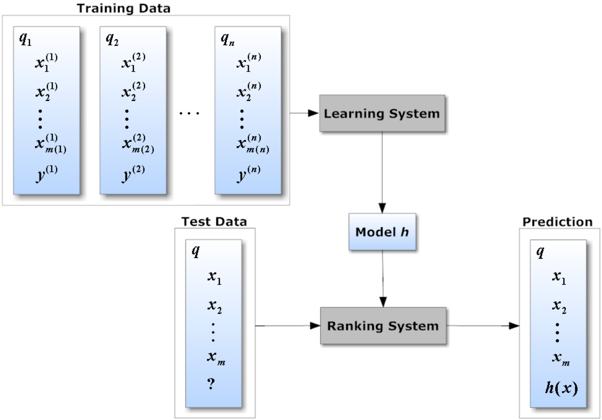
$$g(X, Y) = \sum g_{ij} x_i x_j$$

Also, a metric actually is a mapping from $R^n \times R^n$ to R, which defined as

$$g : R^n x R^n \rightarrow R.$$

This is a tensor. This may be the reason that modern machine learning frameworks use the word "tensor", such as TensorFlow and Pytorch.

Problem Settings



1. Query q: search keywords
2. Document: A paper, page, article
3. Collection of documents: A set of documents

Features of documents are different if the query is different.
The feature x_{ij} is calculated from document x_i and query q_j.
For a query q_j, we have a collection of documents X_j and the corresponding score of the documents Y_j.
Our training data contains a set of collection of documents. Problem:

Given a new query q_k and a set of documents, the task is to determine the rank of each document. (Sort the document)
The feature of document x_{ik} is known.

Methods

1. Pointwise
   Considered documents in the collection as unrelated. Then build a regression model to predict the score of each document.
2. Pairwise
   The idea is from sorting.
3. Listwise
   Build a metric, then minimize the metric.

Pairwise – Sorting

1. Bubble Sort
2. Selection Sort
3. Quick Sort

To sort an array, all we need is a function that can compare two elements in the array.
The idea here remains the same: We construct a model capable of determining which document should be given a higher rank in the list.
Training the model inside each query of documents.
What is the difference between the pointwise method and pairwise method?

Problem settings: We have user information and their preference. Give a new thing, will they like it? Youtube, Tiktok.

	user				movie					rated movies								
	Ed	Al	Zak	···	It	Up	Jaws	Her		It	Up	Jaws	Her					
	x_1	x_2	x_3	···	x_{21}	x_{22}	x_{23}	x_{24}	···	x_{40}	x_{41}	x_{42}	x_{43}	···	x_{99}	x_{100}	y	
$\mathbf{x}^{(1)}$	1	0	0	···	1	0	0	0	···	0.2	0.8	0.4	0	···	0.3	0.8	1	$y^{(1)}$
$\mathbf{x}^{(2)}$	1	0	0	···	0	1	0	0	···	0.2	0.8	0.4	0	···	0.3	0.8	3	$y^{(2)}$
$\mathbf{x}^{(3)}$	1	0	0	···	0	0	1	0	···	0.2	0.8	0.4	0.7	···	0.3	0.8	2	$y^{(3)}$
$\mathbf{x}^{(4)}$	0	1	0	···	0	0	1	0	···	0	0	0.7	0.1	···	0.35	0.78	3	$y^{(4)}$
$\mathbf{x}^{(5)}$	0	1	0	···	0	0	0	1	···	0	0	0.7	0.1	···	0.35	0.78	1	$y^{(5)}$
$\mathbf{x}^{(6)}$	0	0	1	···	1	0	0	0	···	0.8	0	0	0.6	···	0.5	0.77	4	$y^{(6)}$
···	···	···	···	···	···	···	···	···	···	···	···	···	···	···	···	···	···	···
$\mathbf{x}^{(D)}$	0	0	0	···	0	0	1	0	···	0	0	1	0	···	0.95	0.85	5	$y^{(D)}$

Figure 1: Example for sparse feature vectors $\mathbf{x}$ and their respective labels y.

Factorization Machines

Model

The factorization machine model is defined as follows:

$$f(\mathrm{x}) \stackrel{\text{def}}{=} b + \sum_{i=1}^{D} w_i x_i + \sum_{i=1}^{D} \sum_{j=i+1}^{D} (\mathrm{v}_i \mathrm{v}_j)\, x_i x_j$$

where b and $w_i, i = 1, \ldots D$ are scalar parameters similar to those used in linear regression. Vectors $v_i, i = 1, \ldots D$, are k-dimensional vectors of factors. k is a hyperparameter and is usually much smaller than D.

Loss Function

The loss function can be squared error loss (for regression) or hinge loss. For classification with $y \in \{-1, +1\}$, with hinge loss or logistic loss the prediction is made as $y = sign(f(x))$.

Denoising Autoencoders (DAE)

1. A Denoising autoencoder is a neural network that reconstruct its input from bottleneck layer.
2. For recommander model, the denoising autoencoder is to reconstruct those removed item.
3. Imagine you possess a list encompassing the preferences of various users. Presently, you are faced with a situation where you only possess partial data related to a new user.

Word Embeddings

1. word2vec – skip-gram or CBOW
2. The dataset are generated from unlabeled text.

Example: "I almost finished reading the book on machine learning."
Let's denote words in a skip-gram like this: $[x_{-3}, x_{-2}, x_{-1}, x, x_{+1}, x_{+2}, x_{+3}]$. In our above example of the sentence, x_{-3} is the one-hot vector for "finished," x_{-2} corresponds to "reading," x is the skipped word $(\cdot)$, x_{+1} is "on" and so on. A skip-gram with window size 5 will look like this: $[x_{-2}, x_{-1}, x, x_{+1}, x_{+2}]$.

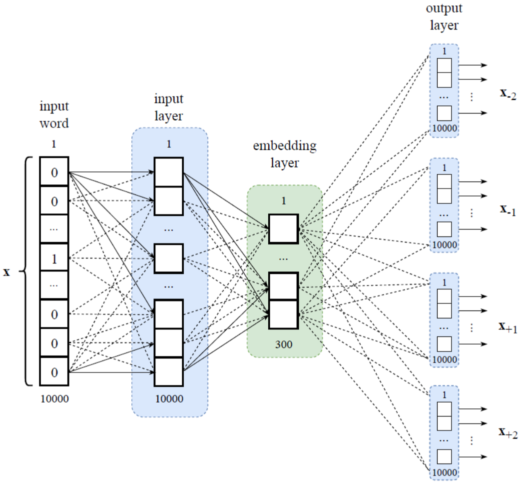
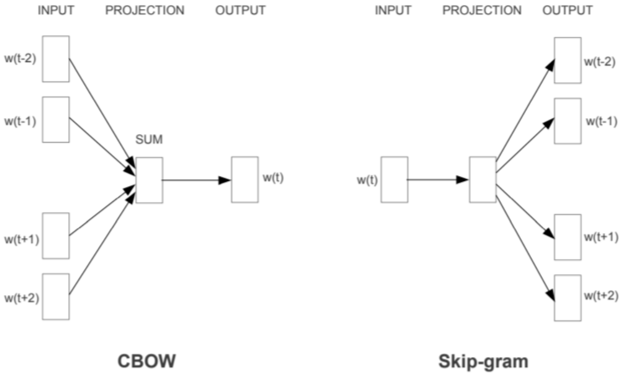


Figure 2: The skip-gram model with window size 5 and the embedding layer of 300 units.


INPUT
PROJECTION
OUTPUT
w(t-2)
w(t-1)
SUM
w(t)
w(t+1)
w(t+2)
CBOW
INPUT
PROJECTION
OUTPUT
w(t)
w(t-2)
w(t-1)
w(t+1)
w(t+2)
Skip-gram