Analysis of Cascode Converter gain in Discontinuous Conduction Mode

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***Abstract*—This paper derives the formula of a Cascode Converter. A Cascode Converter is a DC-DC converter which consists of two buck-boost converters. Previous work has provided the relationship between input and output voltage of a Cascode Converter in Continuous Conduction mode(CCM) [1]. However, Discontinuous Conduction Mode(DCM) also plays an important role in converter analysis. This paper provides formulas of the a Cascode Converter in DCM, and uses software to simulate the circuit to show how it works.**

***Index Terms*—Buck-Boost converter, Cascode Converter, Discontinuous Conduction Mode(DCM).**

I. Introduction

A Cascode converter is a DC-DC converter and it consists of two buck-boost converters. It has two semi-stages, a higher and a lower semi-stage. It can be used as a Buck converter or a Boost Converter based on duty cycle [1]. Because this converter can be use as a Boost converter without high duty cycle, it has higher efficiency than other converter [2]. In this paper, the formula of Cascode converter in DCM will be shown, and verify the correctness by experiments.

II. methods

A. Notations

- V_g Input voltage
- V_{out} Output voltage
- K The dimensionless parameter K is a measure of the tendency of a converter to operate in the discontinuous conduction mode [2].
- $K_{critical}$ critical value of K
- D_1 Duty cycle: MOS ON
- D_2 Duty cycle2 : the inductor keeping conducting
- $M(D,K)$ the gain of converter operate in DCM
- MOS metal–oxide–semiconductor field-effect transistor for type 3 and type 4
- D_3 Duty cycle 3: L1 and L2 conducting simultaneously
- D_4 Duty cycle 4: indcutors stop conducting

B. Definition of DCM in a Cascode converter

Discontinuous Conduction Mode(DCM) occurred when the current of inductors which connected to the load decrease to zero in one period [2]. In Continuous Conduction Mode(CCM), using small ripple approximation derives the relationship between input and the output voltage. Small ripple approximation can greatly simplify the analysis process. Because the current of inductors decrease to zero, the ripple is not small. In DCM, small ripple approximation can not be applied to figure out the relationship between input and output voltage. So DCM occurred when process analysis that can not use small ripple approximation. A Cascode Converter has two inductors and both inductors are connected to the output node. Using the definition of DCM above, DCM in this paper is the mode when the current through inductors decrease to zero in one period.

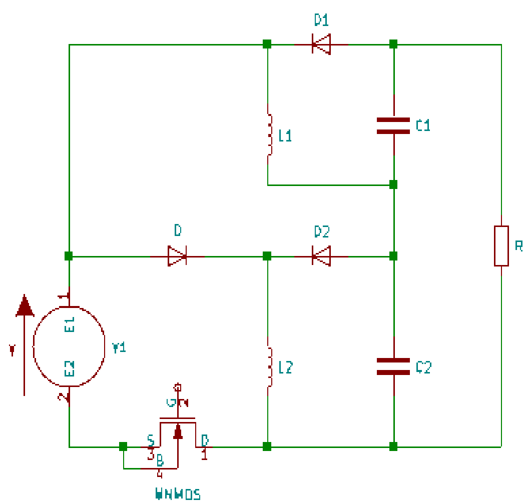

Fig. 1. Example of a figure caption.

C. Derivation gain of a Cascode Converter

In DCM, the the circuit has three intervals. When MOS is on, it is interval 1. When MOS off and L_1 and L_2 both in conducting mode is interval 2. When MOS off and either current L_1 or L_2 decrease to zero as interval 3.

In a Cascode converter, four types of interval 3 must be derived. The current of L_1 decreases to zero; The current of L_2 decreases to zero; The current of both L_1 and L_2 decrease to zero but not at the same time.

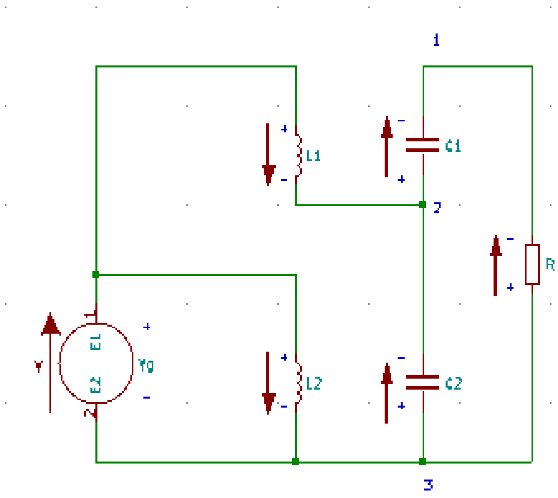

Fig. 2. Interval 1

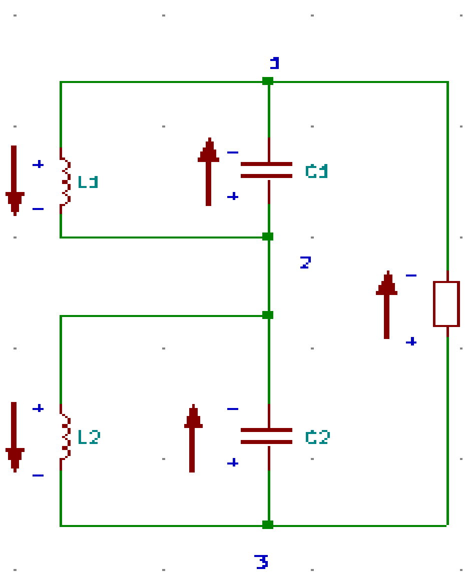

Fig. 3. Interval 2

In interval 1, MOS is on, the equivalent circuit is shown as Fig 1. Applying Kirchhoff's Voltage Law (KVL), obtain bellowing equations

$$v_{L_1} = v_g + v_{C_2} \tag{1}$$
$$v_{L_2} = v_g \tag{2}$$
$$v = v_{C_2} + v_{C_2} \tag{3}$$

Applying Kirchhoff's Current Law(KCL) at node 1,2 (figure 2), obtain bellowing equations

$$i_{C_2} = -i_o \qquad \text{node 1} \tag{4}$$
$$i_{C_2} = -i_{L_1} - i_o \qquad \text{node 2} \tag{5}$$

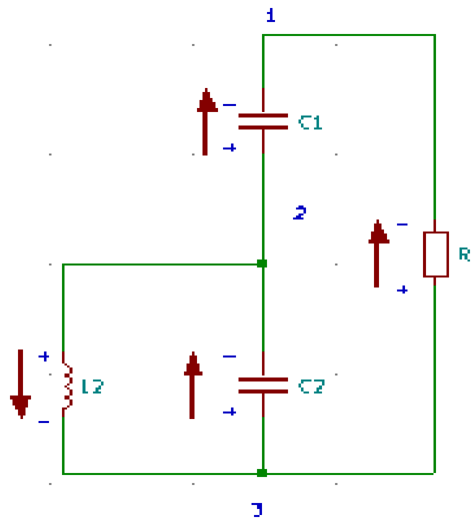

Fig. 4. the equivalent circuit of interval 3 type 1

In interval 2, MOS is off and L_1 and L_2 is conducting, using KVL, obtain following equations

$$v_{L_1} = -v_{C_2} \tag{6}$$
$$v_{L_2} = -v_{C_2} \tag{7}$$
$$v = v_{C_2} + v_{C_2} \tag{8}$$

Applying KCL at node 1,3(Fig.3)

$$i_{C_2} = i_{L_1} - i_o \qquad \text{node 1} \tag{9}$$
$$i_{C_2} = i_{L_2} - i_o \qquad \text{node 3} \tag{10}$$

Cascode converter has four cases in interval 3.

1) Type 1: Type 1 has the following characteristics. The current through L_1 decreases to zero at the begin of interval 3. L_2 keeps conducting in interval 3. The equivalent circuit is shown in Fig.4. In this case, Diode 2 is on and Diode 1 is off. The current through L_1 decrease to zero in interval 3.

Applying KVL, obtain following equations

$$v_{L_2} = -v_{C_2} \tag{11}$$
$$v_{C_2} + v_{C_2} = v \tag{12}$$

Applying KCL at node 1,3 in Fig 4, obtained

$$i_{C_2} = -i_o \qquad \text{node 1} \tag{13}$$
$$i_{C_2} = i_{L_2} - i_o \qquad \text{node 3} \tag{14}$$

Applying small ripple approximation on current and voltage except for i_{L_1}.

In steady-state analysis the average voltage of L_1 and L_2 are equal to zero. That is

$$(V_g + V_{C_2})D_1 - V_{C_2}D_2 = 0 \tag{15}$$
$$V_g D_1 - V_{C_2}(1 - D_1) = 0 \tag{16}$$

In steady-state the average current of C_1 and C_2 are equal to zero. That is

$$\frac{1}{2}D_2 i_{L_1peak} - I_o = 0 \tag{17}$$

$$-\frac{1}{2}D_1 i_{L_1peak} + (1-D_1)I_{L_2} - I_o = 0 \tag{18}$$

The expression of i_{L_1peak} can be obtained by using $L\frac{di}{dt} = v$

$$i_{L_1peak} = D_1 T_s \frac{V_g + V_{C_2}}{L_1} \tag{19}$$

From Eq.(15-19), the relationship of V_g,V_{C_2}, D_2, I_{L_2}, V_o can be obtained.

The relationship between the input and output voltage is

$$.V_o = \frac{(1+\sqrt{\frac{(L+2RT_s)}{L}})D_1 V_g}{2\,(1-D_1)} \tag{20}$$

The expression of D_2 is shown in Eq.(21)

$$D_2 = \frac{\sqrt{L_1\,(L_1+2RT_s)}+L_1}{RT_s} \tag{21}$$

The critical value of L_1 makes circuit operate in DCM is shown in Eq.(22)

$$L_1 \le \frac{(D_1-1)^{\,2} RT_s}{2\,(2-D_1)} \tag{22}$$

The K for Cascode converter is shown in Eq.(23)

$$K = \frac{2L_1}{RT_s} \tag{23}$$

The $K_{critical}$ of Cascode Converter in Type 1 is

$$K_{critical} = \frac{(1-D_1)^{\,2}}{(2-D_1)} \tag{24}$$

If $K_{critical} \ge K$ Cascode Converter operates in DCM.

The gain, $M(D,K)$, of Cascode Converter is shown in Eq.(25)

$$M(D,K) = \frac{(1+\sqrt{\frac{(L_1+2RT_s)}{L_1}})D_1}{2\,(1-D_1)} \tag{25}$$

and D_2 can be written as

$$D_2 = \frac{\sqrt{1+4K}+1}{2K} \tag{26}$$

2) Type 2: Type 2 is the opposite of Type 1. Type 2 has the following characteristics. The current through L_2 decrease to zero in interval 3. L_1 keep conducting in interval 3. The equivalent circuit is shown in Fig.5. In this case, Diode 2 is off and Diode 1 is on.

Applying KVL, obtain following equations

$$v_{L_1} = -v_{C_2} \tag{27}$$

$$v_o = v_{C_2} + v_{C_2} \tag{28}$$

Applying KCL at node 1,3 in Fig.5, obtain following equations

$$i_{C_2} = i_{L_1} - i_o \qquad \text{node 1} \tag{29}$$

$$i_{C_2} = -i_o \qquad \text{node 3} \tag{30}$$

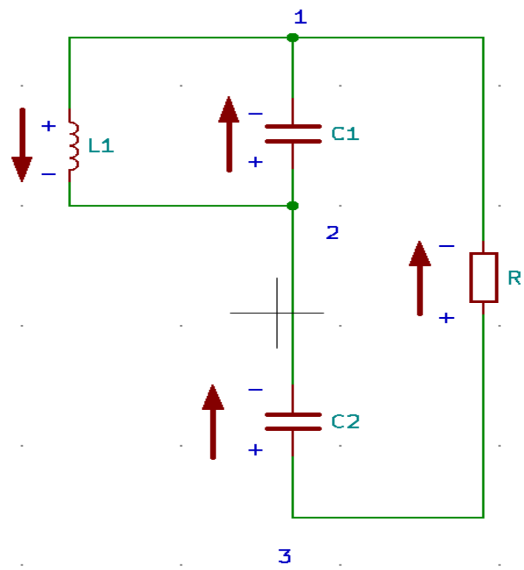

Fig. 5. Interval 3 type 2

In steady state, apply same process as Type 1 The average value of v_{L_1}, v_{L_2}, i_{C_2}, i_{C_2} equal 0. Following equations can be obtained

$$(V_g + V_{C_2})D_1 - (1-D_1)V_{C_2} = 0 \tag{31}$$

$$V_g D_1 - V_{C_2} D_2 = 0 \tag{32}$$

$$(1-D_1)I_{L_1} - I_o = 0 \tag{33}$$

$$-D_1 I_{L_1} + \frac{1}{2}D_2 i_{L_2peak} - I_o = 0 \tag{34}$$

Associated interval 1 and interval 2, obtain

$$V_{C_2} + V_{C_2} = V_o \tag{35}$$

$$i_{L_2peak} = D_1 T_s \frac{V_g}{L_2} \tag{36}$$

From Eq.(31-36), The relations between input and output voltage can be obtained

$$V_o = \frac{D_1 V_g \left(1+\sqrt{\frac{2RT_s}{L_2}(1-D_1)^2+1}\right)}{2\,(1-D_1)} \tag{37}$$

The expression of D_2 is shown in Eq.(38)

$$D_2 = \frac{(1+\sqrt{\frac{2RT_s}{L_2}(1-D_1)^2+1})L_2}{2RT_2\,(1-D_1)^2} \tag{38}$$

The critical value of L_2 makes circuit operate in DCM is shown in Eq.(39)

$$L_2 \le \frac{(1-D_1)^{\,4} RT_s}{2\,(2-D_1)} \tag{39}$$

The $K_{critical}$ of Cascode Converter in Type 2 is

$$K_{critical} = \frac{(1-D_1)^{\,4}}{(2-D_1)} \tag{40}$$

If $K_{critical} \geq K$ the converter operate in DCM

The gain, $M(D,K)$, of Cascode Converter is

$$M(D,K) = \frac{D_1\left(1+\sqrt{\frac{2RT_s}{L_2}(1-D_1)^2+1}\right)}{2\,(1-D_1)} \tag{41}$$

3) Type 3: Type 3 has the following characteristics. At the end of interval 2 the current through L_1 decrease to zero. During interval 3 the current through L_2 decrease to zero. Using same method obtain following equations

$$(V_g + V_{C_2})D_1 - V_{C_2}D_2 = 0 \tag{42}$$

$$V_g D_1 - V_{C_2}(D_2 + D_3) = 0 \tag{43}$$

$$\frac{1}{2}D_2 i_{L_1 peak} - I_o = 0 \tag{44}$$

$$-\frac{1}{2}D_1 i_{L_1 peak} + \frac{1}{2}(D_2+D_3) i_{L_2 peak} - I_o = 0 \tag{45}$$

$$\tag{46}$$

The equation of $i_{L_1 peak}$ and $i_{L_2 peak}$ are

$$i_{L_1 peak} = D_1 T_s \frac{V_g + V_{C_2}}{L_1} \tag{47}$$

$$i_{L_2 peak} = D_1 T_s \frac{V_g}{L_2} \tag{48}$$

4) Type 4: Type 4 has the following characteristics. At the end of interval 2 the current through L_2 decrease to zero. During interval 3 the current through L_1 decrease to zero. Using same method obtain following equations

$$(V_g + V_{C_2})D_1 - (D_2 + D_3)V_{C_2} = 0 \tag{49}$$

$$V_g D_1 - V_{C_2}D_2 = 0 \tag{50}$$

$$\frac{1}{2} i_{L_1 peak}(D_2 + D_3) - I_o = 0 \tag{51}$$

$$-\frac{1}{2}D_1 i_{L_1 peak} + \frac{1}{2}D_2 i_{L_2 peak} - I_o = 0 \tag{52}$$

The expression of $i_{L_1 peak}$ and $i_{L_2 peak}$ are

$$i_{L_1 peak} = D_1 T_s \frac{V_g + V_{C_2}}{L_1} \tag{53}$$

$$i_{L_2 peak} = D_1 T_s \frac{V_g}{L_2} \tag{54}$$

If trying to solve the equations shows in Type 3 and Type 4, they will produce 4th order equations. It is very hard to give the expression of $M(D,K)$ from 4th order equations except give the real number.

So the expression of $M(D,K)$, D_2, D_3 and $K_{critical}$ of type 3 and type 4 do not provided in this paper.

III. RESULT

The previous sections give the expression of $M(D,K)$ and the value of L_1, L_2 of this circuit.

In this section, PSIM will be used to do simulation and shows the relationship between input and output voltage. Using PSIM to do the simulation, the current waveform of inductors can be obtained. To inspect the current waveform of inductors shows that circuit working in DCM. The value obtained using PSIM is very close to the theoretical value.

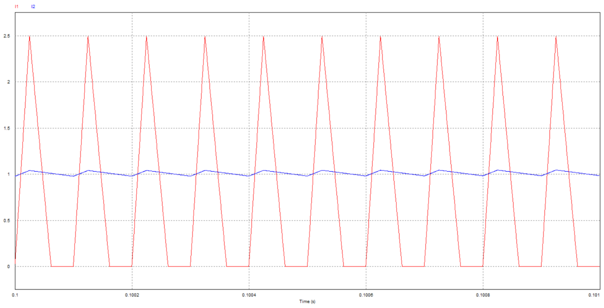

Fig. 6. the current through L_1 and L_2 for type 1

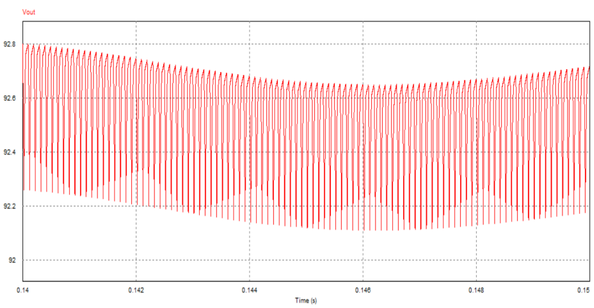

Fig. 7. output voltage of type 1

A. Simulate for type 1

For type 1, do simulate using following parameters.

$$V_g = 75\ V \tag{55}$$

$$R = 200\ \Omega \tag{56}$$

$$D = 0.25 \tag{57}$$

$$L_1 = 1\ mH \tag{58}$$

$$L_2 = 30\ mH \tag{59}$$

$$C_1 = 100\ uF \tag{60}$$

$$C_2 = 100\ uF \tag{61}$$

$$f = 10kHz \tag{62}$$

Using equations shows in Type 1, the output voltage is

$$V_{out} = 92.5391 \tag{63}$$

$K = 0.1 < K_{critical} = 0.321429$, the circuit operate in DCM.

Using PSIM, the output voltage is shown in Fig. 7.

In Fig.7, the output voltage can be found. It is similar to the calculation.

B. Simulate for type 2

For type 2, do simulate using following parameters.

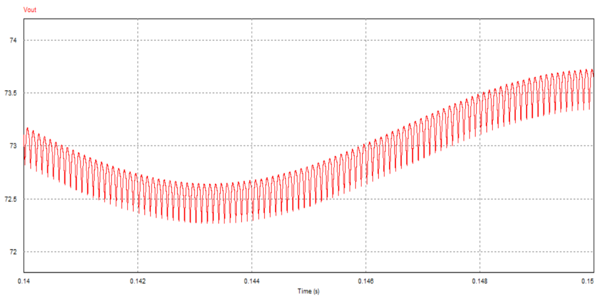

Fig. 8. output voltage of type 2

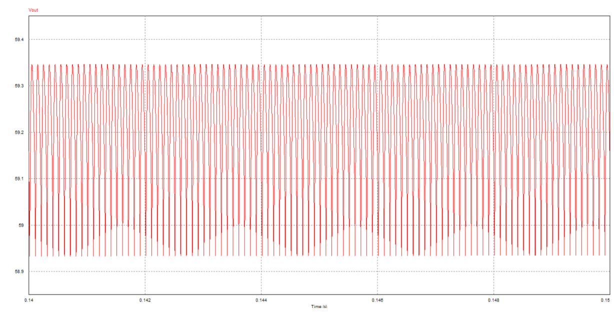

Fig. 10. output voltage of type 3

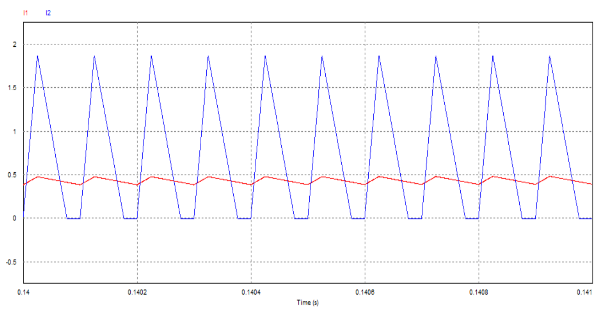

Fig. 9. the current through L_1 and L_2 for type 2

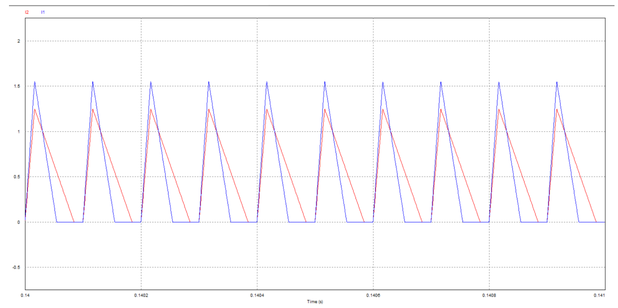

Fig. 11. the current through L_1 and L_2 for type 3

$$V_g = 75\ V \tag{64}$$
$$R = 200\ \Omega \tag{65}$$
$$D = 0.25 \tag{66}$$
$$L_1 = 30\ mH \tag{67}$$
$$L_2 = 1\ mH \tag{68}$$
$$C_1 = 100\ uF \tag{69}$$
$$C_2 = 100\ uF \tag{70}$$
$$f = 10kHz \tag{71}$$

Using equations shows in Type 2, the output voltage is

$$V_{out} = 73.096\ V \tag{72}$$

$K = 0.1 < K_{critical} = 0.180804$, the circuit operate in DCM. Using PSIM, the output voltage is shown in Fig.8. In Fig 8, the output voltage can be found. It is similar to the calculation.

C. Simulate for Type 3 and Type 4

As shown before, in Type 3 and Type 4 is very hard to derive the gain of Cascode converter except using real number.

Using real numbers to do simulate and also using these numbers to solve equations.

For type 3, do simulate using following parameters. The current through L_1 decrease to zero first.

$$V_g = 75\ V \tag{73}$$
$$R = 200\ \Omega \tag{74}$$
$$D = 1/6 \tag{75}$$
$$L_1 = 1\ mH \tag{76}$$
$$L_2 = 1\ mH \tag{77}$$
$$C_1 = 100\ mF \tag{78}$$
$$C_2 = 100\ mF \tag{79}$$
$$f = 10kHz \tag{80}$$

Use the formula in Type 3 and substitute the number, the output voltage has been found. $V_o = 59.2223\ V$. The result of simulating is shown in Fig.10. Using PSIM, the output voltage is shown in Fig.10. In Fig.10, the output voltage can be found. It is similar to the calculation.

Following are parameters for type 4. The current through L_2 decrease to zero first.

$$V_g = 75\ V \tag{81}$$
$$R = 200\ \Omega \tag{82}$$
$$D = 1/6 \tag{83}$$
$$L_1 = 2\ mH \tag{84}$$
$$L_2 = 1\ mH \tag{85}$$
$$C_1 = 100\ mF \tag{86}$$
$$C_2 = 100\ mF \tag{87}$$
$$f = 10kHz \tag{88}$$

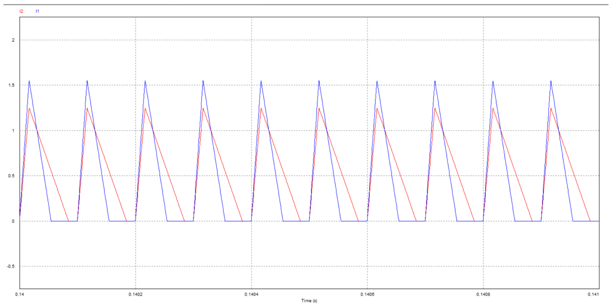

Fig. 12. the current through L_1 and L_2 for type 3

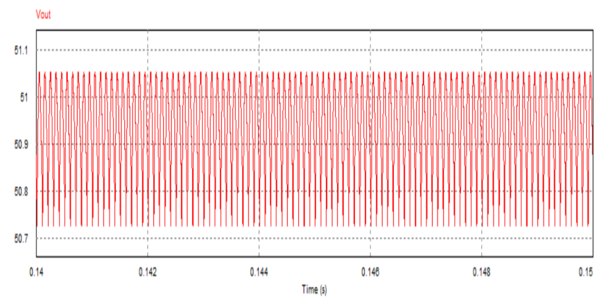

Fig. 13. output voltage of type 4

Use the formula in Type 4 and substitute the number, the output voltage has been found. $V_o = 50.9437\ V$. The result of simulating is shown in Fig.13. Using PSIM, the output voltage is shown in Fig.13. In Fig.13, the output voltage can be found. It is similar to the calculation.

IV. DISCUSSIONS

This paper shows all formulas about the Cascode Converter in DCM. For Type 1 and Type 2, the relationship between input and output voltage is given in Eq.(25) and Eq.(37). For Type 3 and Type 4, the relationship between input and output voltage is too difficult to expression by a simple formula. So the equations are listed. At the same time this paper shows two formulas to judging DCM. In type 1, when $K < K_{critical}$ the current in L_1 will reach zero at the end of interval 2. Then the converter operates in DCM. Type 2 is similar with Type 1. In Type 3 and Type 4, the relationship between input and output voltage can be obtained by solving equations. When $K < K_{critical}$ occurred both in L_1 and L_2, the converter operate in type 3 or type 4. In order to determine the converter operate in which type, it is necessary to solve the equations mentioned in chapter II. Only one of them will get a reasonable answer. In conclusion, this paper shows the method and equations to analyze Cascode Converter in DCM.

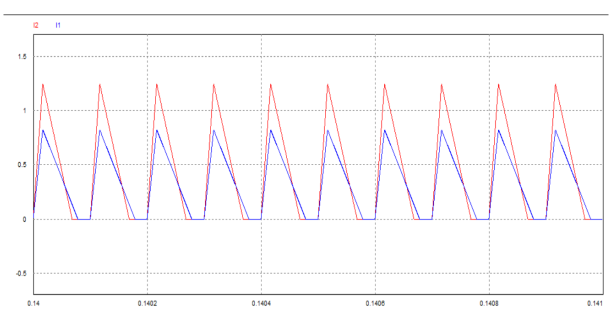

Fig. 14. the current through L_1 and L_2 for type 4

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