STAT 404: Project Report
Battery Longevity Between Different Types of iPhones

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1 Abstract

People use their cell phones on a daily basis, and the quality of the battery has an impact on buying decisions. We have conducted an experiment to test the battery longevity of the iPhone, under varying conditions. Our focus in this experiment are the effects of low power mode on battery level reduction.

2 Introduction

We have chosen the seventh generation of iPhones to run the experiments on, the iPhone 7 and iPhone 7 Plus. We will test those iPhones by using them to play videos for a set amount of time and observe how much reduction of battery occurred. The tests will be run with varying treatment factors such as brightness, volume, and the use of the low battery option. We have a 2^{4-1} factorial design with 3 treatment factors and 1 blocking factor with 8 runs.

The following sections will cover the details of our experimental design in Section 3, followed by a statistical analysis in Section 4, and the conclusion of our experiment in Section 5. Then in the appendix in Section 6, we have Section 6.1 for tables and figures and Section 6.2 for details of our data.

3 Details of the Experimental Design

We used the iPhone 7 and iPhone 7 Plus to run 30 minutes of a YouTube video with varying treatment factors applied and recorded the reduction in battery in percentage. Below is our model followed by Table 3.1 that describes our experimental treatment factors. We also have Table 6.2.1 in the Appendix that shows how we applied treatments to our runs.

$$Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + E_{ijkl}$$
$$(i = 1, 2; j = 1, 2; k = 1, 2; l = 1, 2)$$

Figure 3.1: Our model with four main effects and two interaction effects.

Variables	Description	Levels
LPM	Low Power Mode	Off = 1, On = -1
B	Brightness	Medium (8 bars) = -1, High (12 bars) = 1
V	Volume	Medium (8 bars) = -1, High (12 bars) = 1
T	Type of iPhone	iPhone 7, iPhone 7 Plus

Table 3.1: Experimental plan for a 2^3 experiment with three treatments in two blocks of 8 runs.

Note that in Table 3.1, we are using bars as a metric in iPhones used to adjust settings such as brightness and volume. For example, a gauge with one bar would be the minimum volume while a full bar gauge would be at full volume. Eight bars is halfway through the gauge and twelve bars is three quarters of the full gauge.

Low power mode when turned on enables the iPhone to slow down the battery drain by throttling CPU performance in order to conserve battery. When turned off, performance is as normal.

The iPhone 7 and 7 Plus were chosen in this experiment because they were readily available. Both use the same processor, but the iPhone 7 Plus has slightly more pixels on its screen (1920 x 1080) vs iPhone 7 (1334 x 750) so it is possible that one phone's battery drains more than the other. Therefore we are designing this experiment with blocking on the type of iPhone. For example, Table 3.2 shows that for the the first run, we had applied low power mode on (-1 means low power mode on), set brightness and volume to high (1 means high setting).

Run	LP	B	V	Type D = 123	LPM:B	LPM:V	Power Reduction (in percentage %)
1	-1	1	1	-1	-1	-1	11
2	1	1	-1	-1	1	-1	9
3	1	-1	1	-1	-1	1	11
4	-1	-1	-1	-1	1	1	7
5	-1	-1	1	1	1	-1	11
6	1	-1	-1	1	-1	-1	11
7	-1	1	-1	1	-1	1	12
8	1	1	1	1	1	1	14

Table 3.2: Contrast matrix for a half fraction of 2^4 factorial design with two blocks.

4 Statistical Analysis

4.1 Model Setup

We have a half fraction of a 24 factorial design with 3 treatment factors spreading across 1 blocking factor. The total number of runs is 8 with a design generator of D =123. The design relation is I = 123D, which means our design falls into Resolution IV. From List 6.1.1 in the Appendix, the main effects are confounded with the three-factor interaction effects, and the two-factor interaction effects are confounded with other two-factor interaction effects. In order to do ANOVA, we need at least one degree of freedom for residuals. The solution is to eliminate one interaction effect which is not significant in our model. From Figure 6.1.3, the interaction plots of estimated B effect by V, the two lines are parallel, which indicates that the there is no interaction between factor B and V. From Figure 6.1.1 and 6.1.2, it is obvious that LPM and B, and LPM and V have interactions. Therefore, we decided to eliminate the two-factor interaction effect term B:V from our model.

4.2 Model Checking and Improvement

In order to proceed with statistical inference, we need to check our model and assumptions.

The first step is to do a Box-Cox analysis before detailed statistical inference based on ANOVA. From Figure 6.1.6, the plot of the Box-Cox transformation, $\lambda = 1$ is in the 95% confidence interval, which means that no transformation is required.

To check the normality of the random error E_{ijkl} we plot the normal Q-Q plot for residuals and plot of residuals vs fitted values. From Figure 6.1.5, all the dots are approximately on a straight line. In addition, there is no obvious pattern in Figure 6.1.7. Therefore, the assumption of E_{ijkl} following N(0, 2) is valid.

4.3 ANOVA

After checking the assumptions and our model, we did an analysis of variance on our model by using the aov function in R, and the output is in shown in Table 6.1.2 in the Appendix.

For main effect LPM:
$H_0 : \alpha_1 = \alpha_2$
$H_a : \alpha_1 \neq \alpha_2$

From Table 4.3.1, the F value of the main effect LPM is equal to $6e-16$ which is smaller than 0.01. Therefore, the main effect LPM is significant at 1% significance level. For the other two main effects B and V, their hypotheses are similar to main effect LMP's. In addition, the main effects B and V are both significant at 1% significance level (with F values equal to $4e-16$ and $3e-16$ respectively).

For blocking effect Type:
$H_0 : \delta_1 = \delta_2$
$H_a : \delta_1 \neq \delta_2$

From Table 4.3.1, the blocking effect of Type is significant at 1% significance level since its F value ($2.4e-16$) is smaller than 0.01.

For two-factor interaction effect LMP:B:
$H_0 : (\alpha\beta)_{11} = (\alpha\beta)_{12} = (\alpha\beta)_{21} = (\alpha\beta)_{22}$
$H_a :$ Not H_0

From Table 3, the two-factor interaction effect of LMP:B is significant at 1% significance level because its F value $6e-16$ is smaller than 0.01.

For the other interaction effect, LMP:V has an F value equal to $1.2e-15$, and its hypothesis is similar. It is also significant at 1% significance level.

Therefore, all the main treatment effects, blocking effects and interactions effects are significant at 1% significance level. It proves that our model is reasonable, and all the treatment and blocking factors have main effects and interactions effects that affect our response variable. According to our findings, low power mode, brightness, volume, and iPhone types all have effects and interaction effects on the longevity of the iPhone battery. However, we do not have to analyze all the variables even though they are all significant. From the half-normal plot in Figure 6.1.8, we can see that Type, B, V and LPM:B are more significant than the other effects. However, the blocking factor, type of iPhone, has no practical meaning in investigating the longevity of the battery, because the choice of buying an iphone 7 or 7 plus is based on so many other factors rather than the longevity of its battery. Therefore, we did not further explore the different types of iPhone. Instead, one of our main goals is to investigate the effectiveness of the low power mode on the iPhone. Therefore, we will keep exploring the main effect LPM and the interaction effects between LPM and B using contrasts.

4.4 Contrast

For the main effect LPM:

$$\hat{\kappa} = \frac{\Sigma_{LPM=1} y_i}{4} - \frac{\Sigma_{LPM=-1} y_i}{4} = 10.25 - 11.25 = -1$$

$$se(\tilde{\kappa}) = 2\sqrt{\frac{MS(Residuals)}{n}} \approx 0$$

Because our MS(Residuals) is approximately 0, our standard error for the contrast is also approximately 0. Our standard deviation is not giving a useful assessment of uncertainty so we cannot use it to compute the 95% confidence interval.

4.5 Interpretations of Interaction Effects

The estimated effects of LPM seems to be approximately linear, and it clearly changes with the level of B. According to the half normal plot in the appendix Figure 6.1.8, LPM:B is the most significant out of the other interaction terms. In Figure 6.1.1, we can see that when the levels of LPM change from on to off, there is a decrease in the mean of our response. The change in LPM does not affect the mean of the response when brightness B is high. A favourable configuration of LPM and B is LPM at level 1 (low power mode off) and B at level -1 (medium brightness).

5 Conclusion and Further Discussion

All of the main effects and interaction effects were significant at the 1% level in our analysis of variance, so all terms have major effects on our response variable. We decided to have our focus on the effects of low power mode as opposed to the other two treatment variables volume and brightness because it has more practical value. When lower power mode is off, the brightness and volume at medium, the reduction of battery is the lowest. We did not expect low power mode to increase the reduction of battery, but our experiment could have errors due to only having 8 runs.

Our study has some limitations so our findings need to be interpreted carefully. First, due to financial constraints, we used iPhones that were borrowed from others to do the experiment instead of using brand new iPhones. Therefore, each iPhone's usage period may be different, meaning that the battery condition cannot be controlled in our experiment. Moreover, we have no replicates for each test (one degree of freedom for the residuals), which may cause experimental errors and inaccuracy.

6 Appendix

6.1 Tables and Figures

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Design generator D = 123.
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1 = LPM
2 = B
3 = V
D = T
Aliases
I = 123D
1 = 23D
2 = 13D
3 = 12D
12 = 3D
13 = 2D
23 = 1D
123 = D
```

List 6.1.1: Design generators

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
LPM	1	2.0	2.0	1.127e+30	6.0e−16	***
B	1	4.5	4.5	2.535e+30	4.0e−16	***
V	1	8.0	8.0	4.507e+30	3.0e−16	***
Type	1	12.5	12.5	7.043e+30	2.4e−16	***
LPM:B	1	2.0	2.0	1.127e+30	6.0e−16	***
LPM:V	1	0.5	0.5	2.817e+29	1.2e−15	***
Residuals	1	0.0	0.0			

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Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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Table 6.1.2: ANOVA table

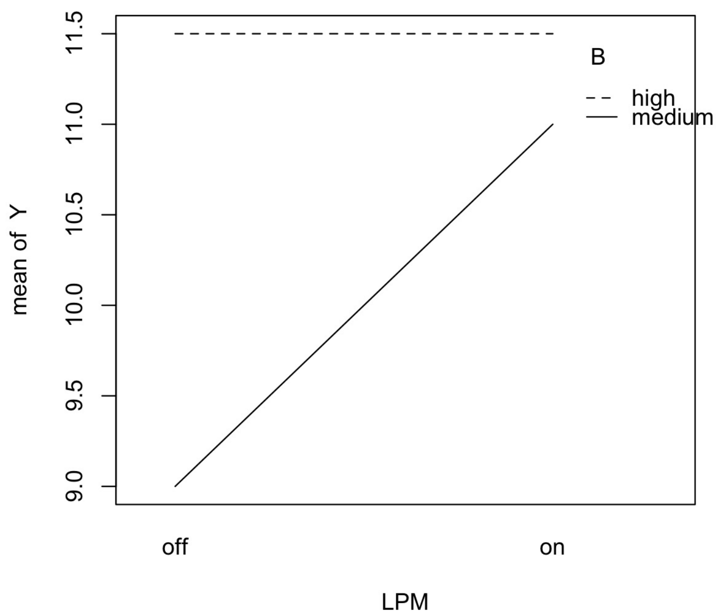

Figure 6.1.1: Mean of power reduction (%) plotted against the two levels of the treatment factor Brightness and the two levels of the treatment factor Low Power Mode.

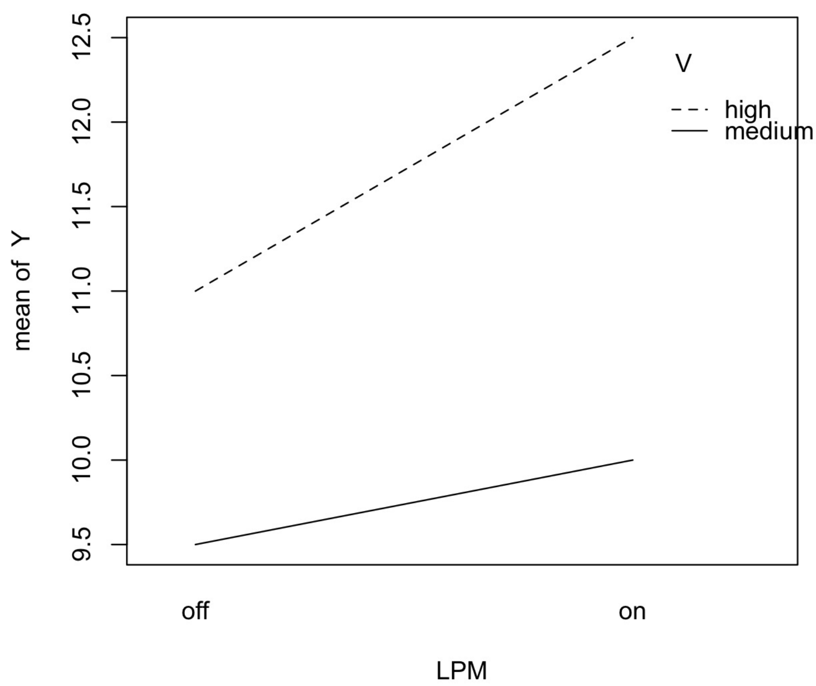

Figure 6.1.2: Mean of power reduction (%) plotted against the two levels of the treatment factor Volume and the two levels of the treatment factor Low Power Mode

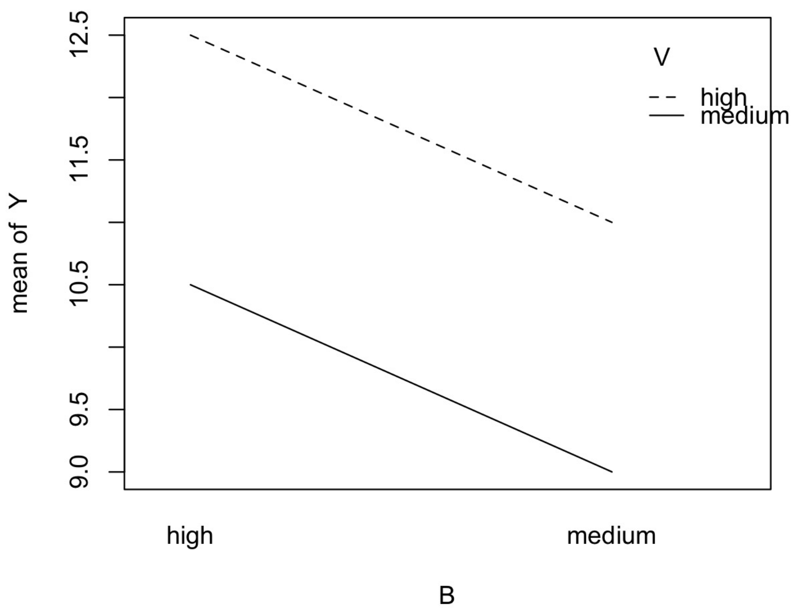

Figure 6.1.3: Mean of power reduction (%) plotted against the two levels of the treatment factor Volume and the two levels of the treatment factor Brightness.

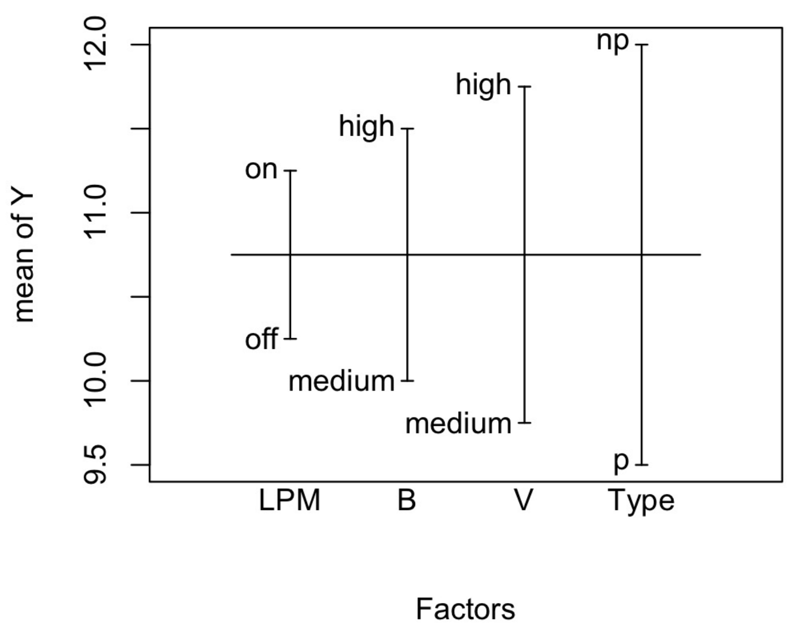

Figure 6.1.4: Design plot: mean of power reduction (%) plotted against the two levels of the treatment factor Low Power Mode, the two levels of the treatment factor Brightness, the two levels of the treatment factor Volume, and the two levels of the blocking factor Types of iPhones.

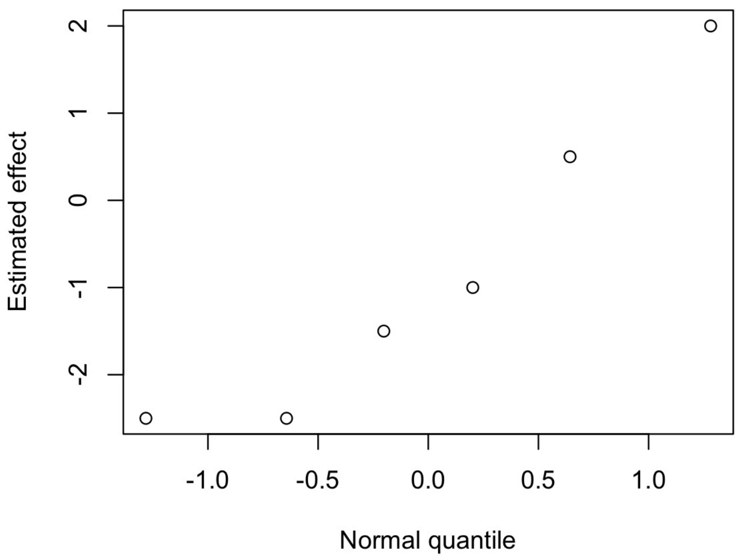

Figure 6.1.5: Normal Q-Q plot of the residuals.

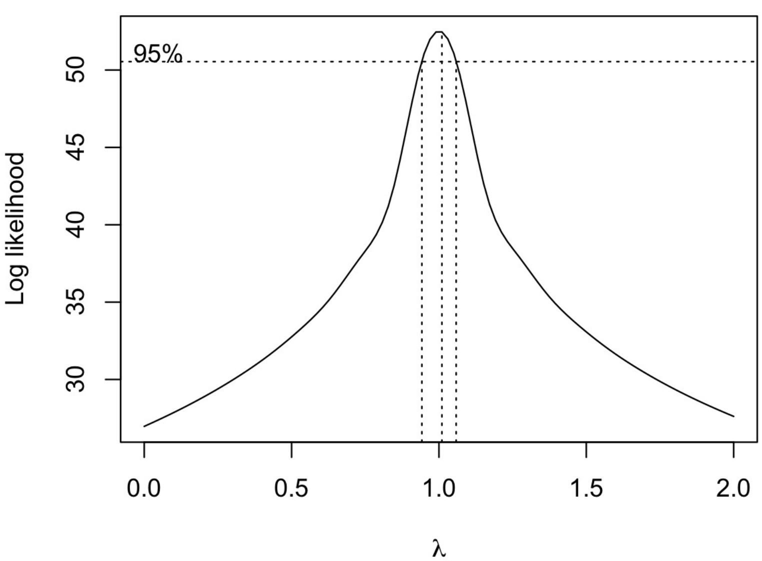

Figure 6.1.6: Box-Cox analysis applied to our model

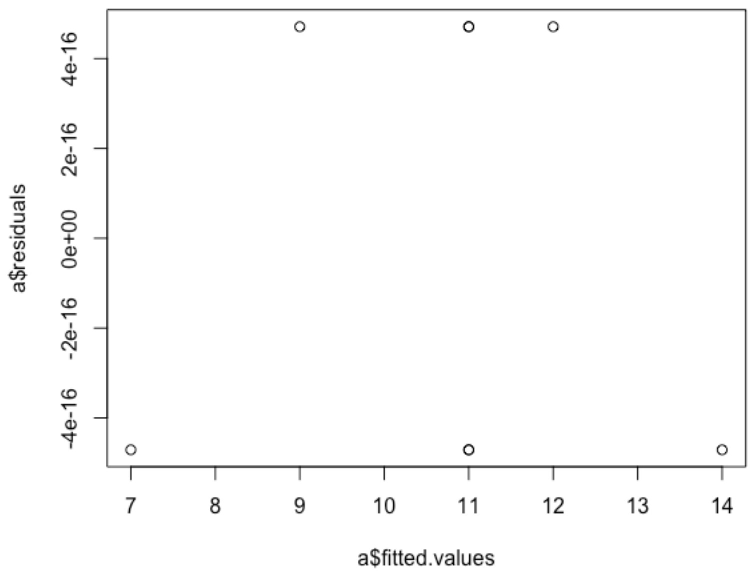

Figure 6.1.7: Residuals vs Fitted values of our model.

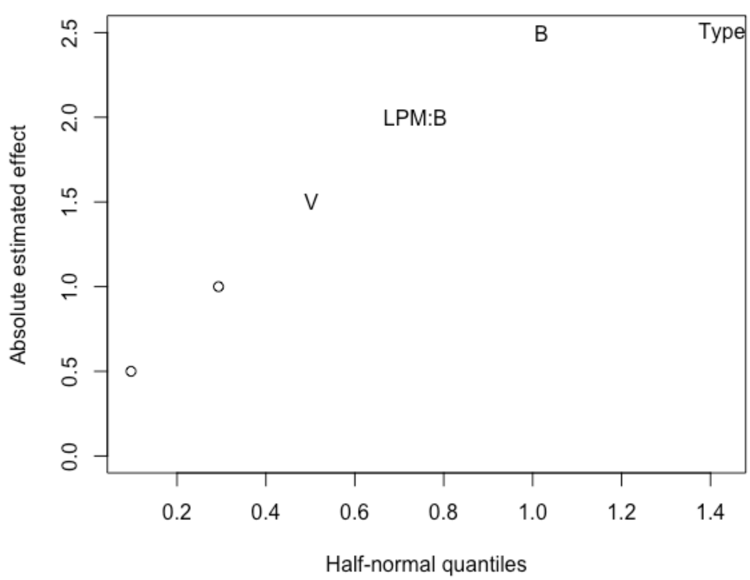

Figure 6.1.2: The half-normal plot of our model.

6.2 Data Appendix

Type of iPhone	LPM	B	V	Power Reduction (in percentage %)
iPhone 7 Plus	off	high	high	11
iPhone 7 Plus	on	high	medium	9
iPhone 7 Plus	on	medium	high	11
iPhone 7 Plus	off	medium	medium	7
iPhone 7	off	medium	high	11
iPhone 7	on	medium	medium	11
iPhone 7	off	high	medium	12
iPhone 7	on	high	high	14

Table 6.2.1: Table of combination of factors for our experimental runs.