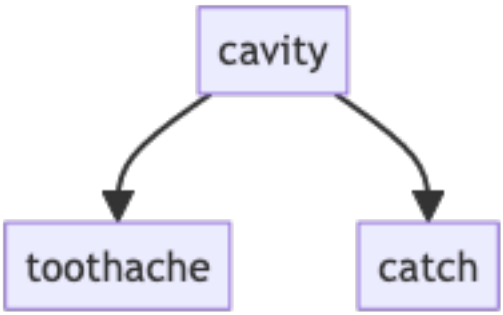
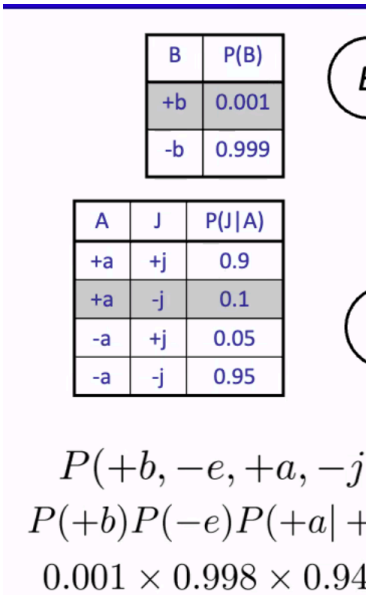
Bayes' Nets

Review

- Efficient encoding of a probabilistic model of a domain
- Set of nodes, one per variable X
- Directed, acyclic graph
- Conditional distribution for each node
- Implicitly encode joint distributions
  - To see what probability a BN gives to a full assignment, multiply all of the relevant conditionals together: $\mathbb{P}(x_1, x_2, ..., x_n) = \prod_{i=1}^{n} \mathbb{P}(x_i|\text{parents}(X_i))$
- eg:





$\mathbb{P}(+\text{cavity}, +\text{catch}, \text{-toothache}) = \mathbb{P}(+\text{cavity})\mathbb{P}(\text{-toothache}|+\text{cavity})\mathbb{P}(+\text{catch}|+\text{cavity})$ - eg2:
- Joint distribution over N variables is 2^N rows - Bayes net of N nodes with up to k parents $\in O(n * 2^{k+1})$

Extracting Conditional Independence

- Can use the chain rule and bayes' net reconstitution formula to pull apart factors and determine conditional independence
- Two variables X and Y are conditionally independent given Z iff $\mathbb{P}(X, Y|Z) = \mathbb{P}(X|Z)\mathbb{P}(Y|Z)$

D-Separation

- Study independence properties for triples

- Analyze complex cases in terms of member triples

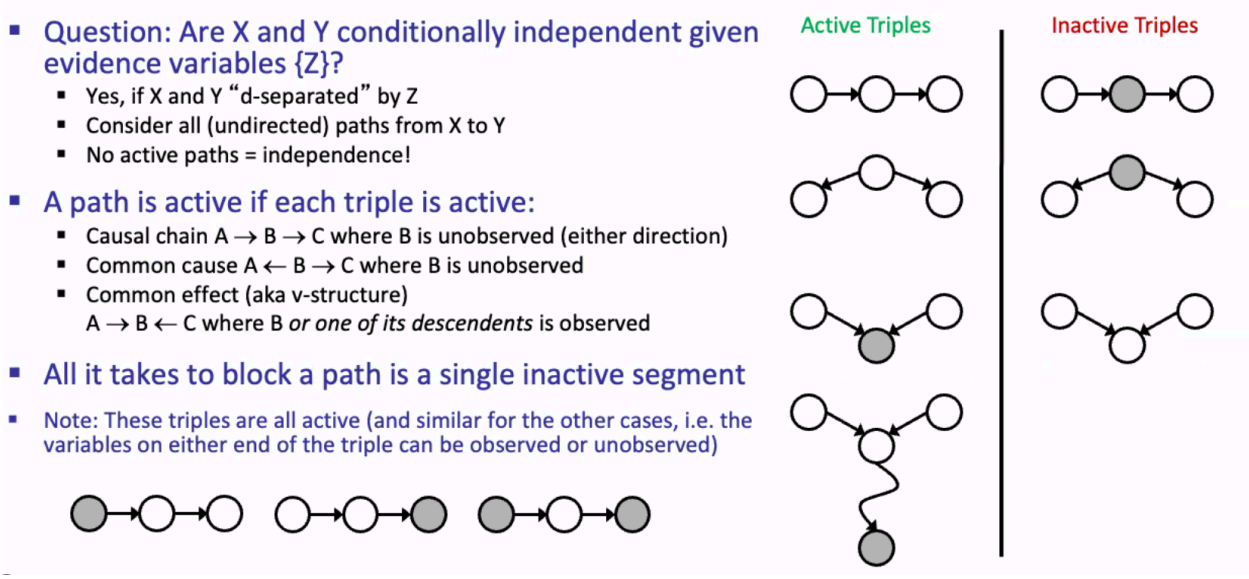


Figure 1: Screenshot_2023-10-10_at_6.05.43_PM.png

- – Shaded nodes $\iff$ observed (taken as a given value)
- To determine conditional independence between X_i and X_j, you have to check all possible (UNDIRECTED) paths $X_i \to X_j$ and see if there is a path s.t. every intermediary node is open
  - The intermediary nodes are triples along the path; open $\iff$ active

Topology Limits Distributions

- Adding arcs increases the set of distributions, but has several costs
- Full conditioning can encode any distribution