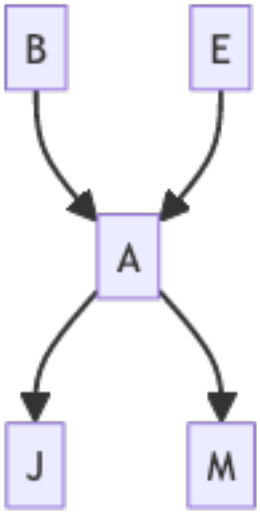
Inference

- Calculating some useful quantity from a joint probability distribution

Inference by Enumeration

- Given evidence variables $E_1...E_k = e_1...e_k$, Query* variables Q and hidden variables $H_1, ...H_r$, find $\mathbb{P}(Q|e_1...e_k)$
- Steps:
  1. Select the entries consistent with the evidence
  2. Sum out H to get joint of Q and E
  3. Normalize (multiply by $\frac{1}{Z}$)
- Given unlimited time, inference in BN is easy
- Issue: may not necessarily be directly represented in the BN
  - Computation on the BN can answer the more complicated query



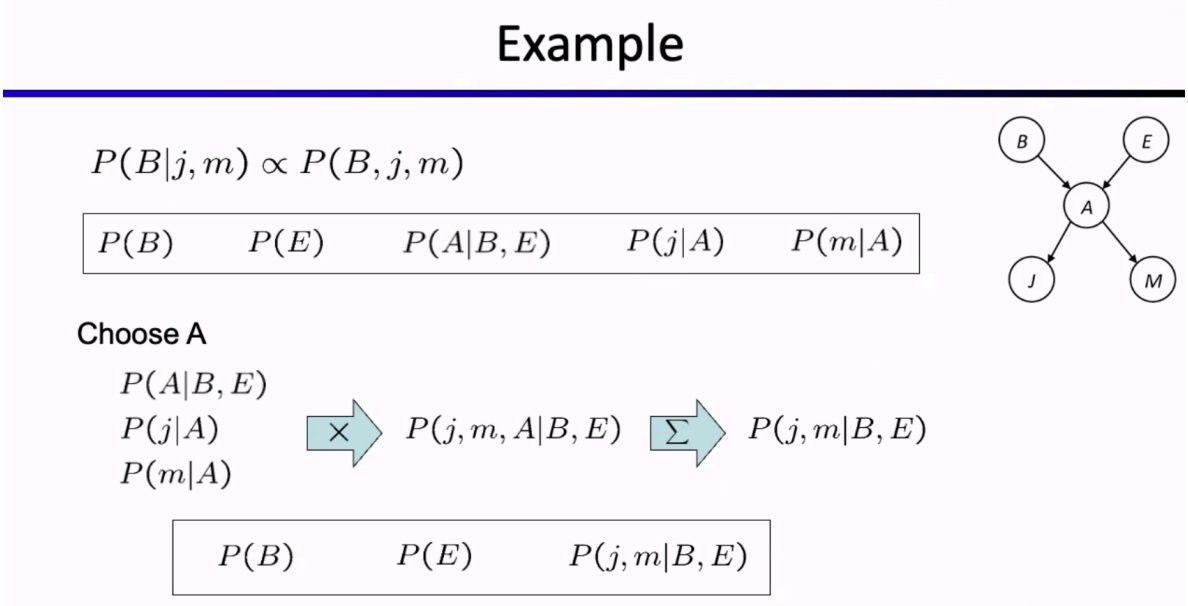
- Note: $\mathbb{P}(B|+j,+m) \propto_B \mathbb{P}(B,+j,+m)$
  - $\mathbb{P}(B,+j,+m) = \sum_{e,a} \mathbb{P}(B,e,a,+j,+m)$
  - With bayes net reconstitution formula, we can find it equal to $\sum_{e,a} \mathbb{P}(B)\mathbb{P}(e)\mathbb{P}(a|B,e)\mathbb{P}(+j|a)\mathbb{P}(+m|a)$
- Slow because you join up the whole joint distribution before you sum out the hidden variables

Variable Elimination (VE)

- Idea: interleave joining and marginalizing
- Can multiply parts of the BN together to get factors
- Example:

- Variables
  - $R \triangleq$ raining?
  - $T \triangleq$ traffic?
  - $L \triangleq$ late for class?
- $\sum_t \sum_r \mathbb{P}(L|t)\mathbb{P}(r)\mathbb{P}(t|r)$
- Implementation: join only until it is possible to eliminate, then eliminate
  - Keep track of a list of factors (initialized as the list of nodes in the BN)
  - When there is only one table with a variable, sum out the variable
    - Note: this always happens when you join on a variable
- Alg:
  1. If evidence, start with factors that select that evidence
  2. While there are still hidden variables (not Q or evidence)
     1. Pick a hidden variable h
     2. Join all factors mentioning H
     3. Eliminate (sum out) H
  3. Join all remaining factors
  4. Normalize

- eg:



  - Factors for each step are boxed

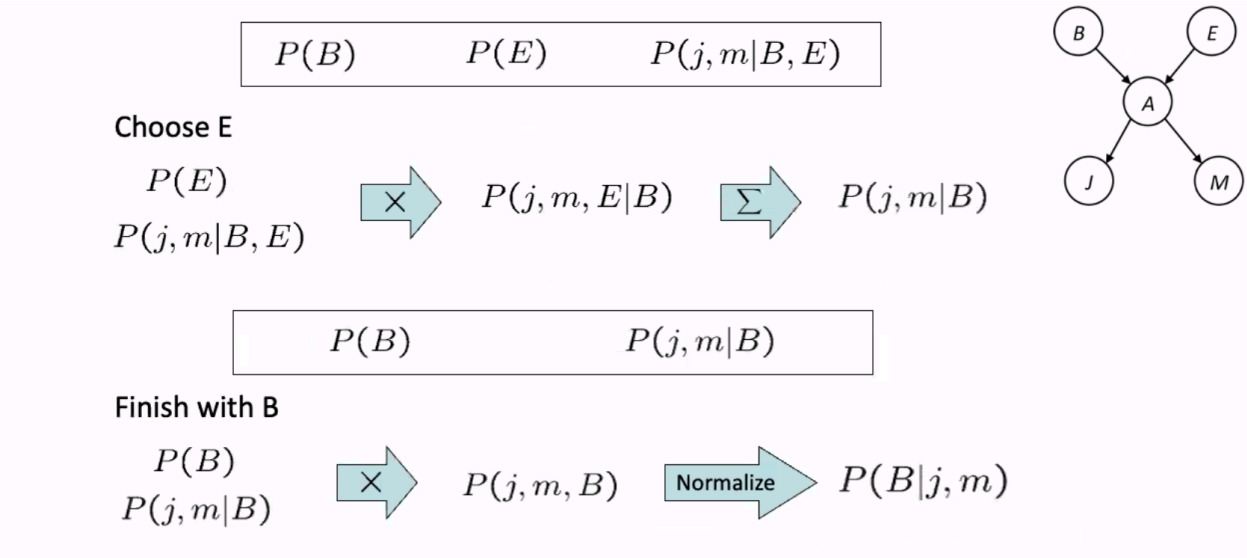


Figure 1: 450

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- Can choose elimination by whatever order you want (but it will affect runtime)
  - Always choose to join and eliminate variables that prevent the result from being large #### Factors
- Represent multiplying smaller things together
- When we write $\mathbb{P}(Y_1...Y_N|X_1...X_M)$
  - It's a factor of a multidimensional array
  - Its values are $\mathbb{P}(y_1...y_N|x_1...x_M)$
  - Any assigned (lower-case) X or Y is a dimension missing selected form the array

- Joint distribution $\triangleq \mathbb{P}(X, Y)$
  - Entries of $\mathbb{P}(x, y) \forall x, y$
  - Sums to 1
- Selected joint $\triangleq \mathbb{P}(x, Y)$
  - A slice of the joint distribution
  - Entries of $\mathbb{P}(x, y)$ for fixed x, all y
  - Sums to $\mathbb{P}(x)$
- Single conditional: $\mathbb{P}(Y|x)$
  - Entries of $\mathbb{P}(y|x)$ for fixed x
  - Sums to 1
- Family of conditionals $\triangleq \mathbb{P}(Y|X)$
  - Multiply conditionals
  - Entries of $\mathbb{P}(y|x)$ for all x and y
  - Sums to $|X|$
- Specified family $\triangleq \mathbb{P}(y|X)$
  - Entries of $\mathbb{P}(y|x)$ for fixed y
  - Sums is variable

Operations

Joins

- Combine like factors (multiply their probs)
- Just like a db join
- Get all factors over the joining variable
- Build a new factor over the union of the variables involved
- Joining on a variable $\triangleq$ join all tables with that variable
- Multiple joins $\triangleq$ you have to do a pointwise multiplication
  - You add variables that are necessary to describe all of the rows of all of the joined tables

Marginalization (Elimination)

- Take a factor and sum out a variable
- Shrinks a factor to a smaller one
- Type of 'projection' operation
- Sum values over rows that differ in the variable being eliminated