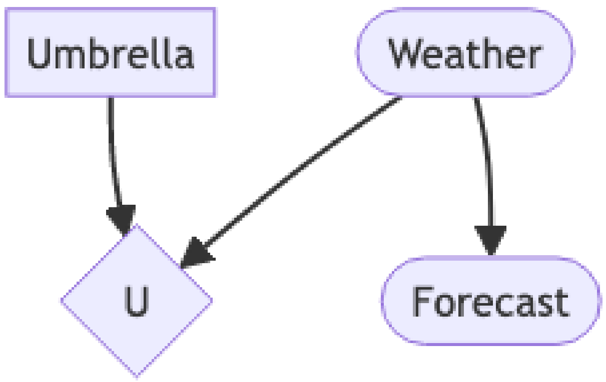
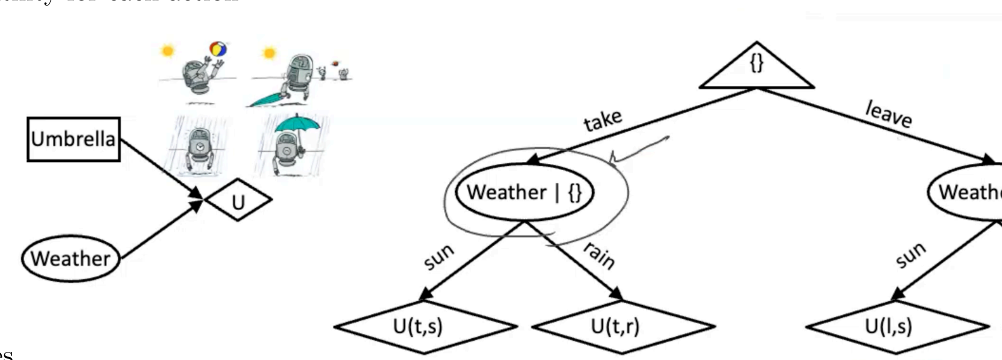
Decision Networks and Value of Information

Decision Networks

- eg:



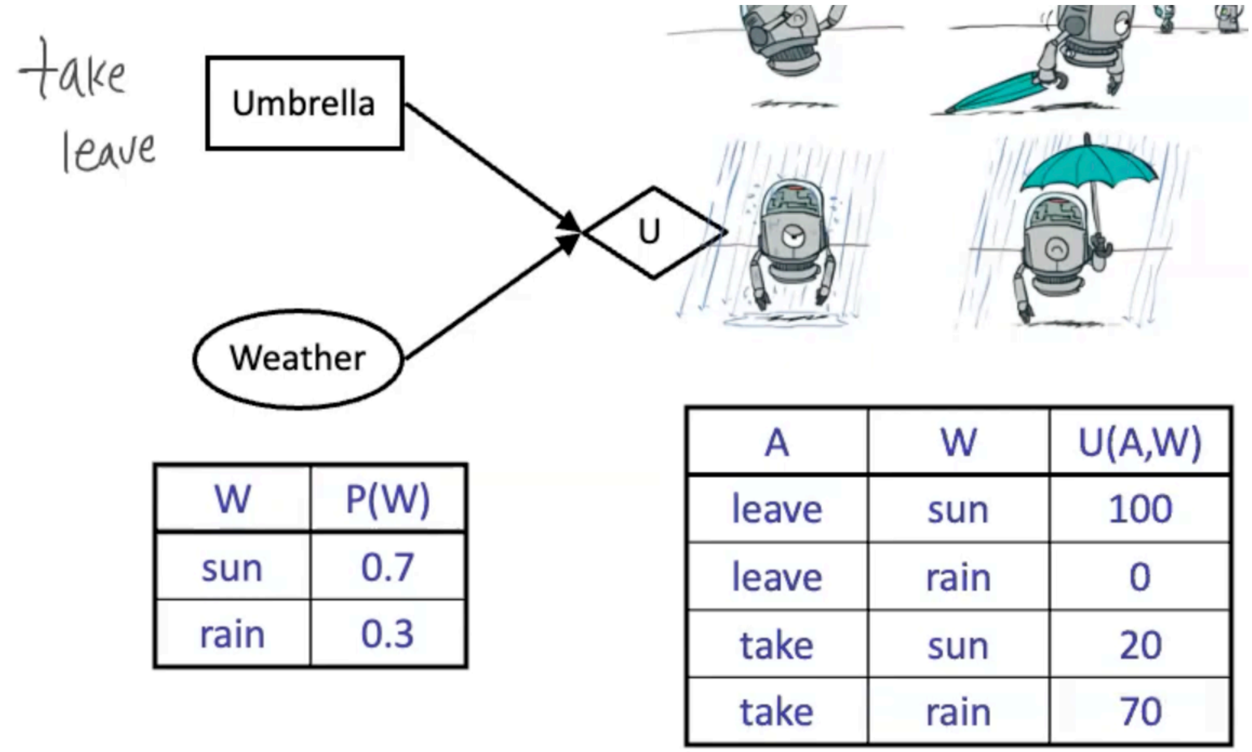
- Rectangle $\triangleq$ choice of actions
  - Umbrella node is that the agent can choose whether to take the umbrella
  - Cannot have parents
  - Act as observed evidence
- Diamond $\triangleq$ utility of the outcome
  - Depends on the action and chance nodes
  - Has to be specified as a function of all of its parents
  - Have utility values for each action instead of probabilities
- Oval $\triangleq$ chance nodes (just like BNs)
- BN but with nodes for utility and actions
- Lets us calculate the expected utility for each action



- Can be drawn as expectimax trees

Maximum Expected Utility (MEU)

- Choose the action which maximizes the expected utility given the evidence
- Can directly operationalize this with decision networks
-
- $\mathbb{EU}(\text{action}) := \sum_{\text{chance}} \mathbb{P}(\text{chance})\mathbb{U}(\text{action}, \text{chance})$
  - $\mathbb{EU}(\text{leave}) = \sum_w \mathbb{P}(w)\mathbb{U}(\text{leave}, w) = 0.7 * 100 + 0.3 * 0 = 70$
  - $\mathbb{EU}(\text{take}) = 0.7 * 20 + 0.3 * 70 = 35$
  - The MEU is 70 if we leave the umbrella, so that's the optimal decision



W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Figure 1: 375

- $MEU(K) := \max_a \mathbb{E}\mathbb{U}(a|K)|_{K:=\{\text{knowledge}\}}$
  - K can be $\emptyset$
- Have to calculate the probability of all connected chance nodes dependent on given information

Action Selection

1. Instantiate all evidence
2. Set action node(s) each possible way
3. Calculate posterior for all parents of utility node, given the evidence
   - Use BN rules (eg variable elimination)
4. Calculate expected utility for each action
5. Choose maximizing action by calculating MEUs

Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network

1. Calculate MEU(known evidence)
2. Calculate MEU(given evidence $\cup$ aquired evidence)
3. VPI(aquired evidence) := MEU(given evidence $\cup$ aquired evidence) $-$ MEU(given evidence)
   - VPI := value price of information
   - Need to calculate a distribution only on only the given evidence as well as the corresponding MEU values for each and then weight them for the VPI
     - This is because we don't know the actual value, we just know that that information will be known

- **MEU with no evidence**

  $$\mathrm{MEU}(\emptyset) = \max_a \mathrm{EU}(a) = 70$$

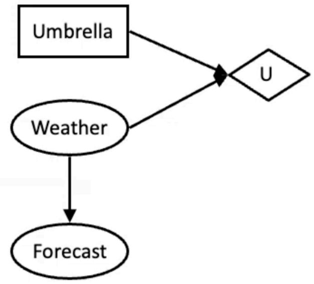
  MEU if forecast is bad

  $$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

  MEU if forecast is good

  $$\mathrm{MEU}(F = \mathrm{good}) = \max_a \mathrm{EU}(a|\mathrm{good}) = 95$$

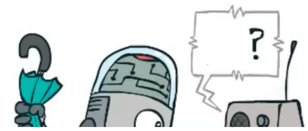
  Forecast distribution

A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

F	P(F)
good	0.59
bad	0.41

  $$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$VPI(E'|e) := \left(\sum_{e'} \mathbb{P}(e'|e) MEU(e, e') \right) - MEU(e) \Bigg|$$

E' := aquired evidence
e := evidence already known

- eg: Prize is \$0 or \$100. You can play any number between 1 and 100 (chance of winning is 1%); $VPI = 100 - 1 = 99$
- Nonnegative: $\forall E', e : VPI(E'|e) \geq 0$
- Nonadditive: $VPI(E_j, E_k|e) \neq VPI(E_j|e) + VPI(E_k|e)$
- Order-independent: $VPI(E_j, E_k|e) = VPI(E_j|e) + VPI(E_k|e, E_j) = VPI(E_k|e) + VPI(E_j|e, E_k)$
- VPI is 0 if the utility is independent of new information

Value of Imperfect Information

- Can always model as a perfect indicator
- Even if something is noisy, we can model it as a perfect indicator of a guess