Deterministic Games

- States: S (starts at s_0)
- Players: $P = 1...N$ (usually take turns)
- Actions: A (may depend on player / state)
- Transition function: Sx$A \longrightarrow$ S
- Terminal Test: $S \longrightarrow \{\text{true}, \text{false}\}$
- Terminal Utilities: Sx$P \longrightarrow$ R
- Solution for a player is a policy: $S \longrightarrow A$

Zero-Sum Games

- Agents have opposite utilities (values on outcomes)
- Think of a single value that one maximizes and the other minimizes
- Adversarial, pure competition

Solving Zero-Sum Games

- Have to think about how the other agent will respond to a move
- Each state has a value which is the best possible outcome at that state
  - In a tree, it is the highest value of all child states
- terminality $\implies V(s) \mathrel{\hat{=}}$ known
- nonterminal $\implies V(s) = \max(V(s') \,\forall\, s' \in \text{successors}(s))$
  - The opponent tries to minimize $V(s)$ instead of maximize it

Adversarial Search (Minimax)

- Minimax value := the best achievable utility against a rational (optimal) adversary

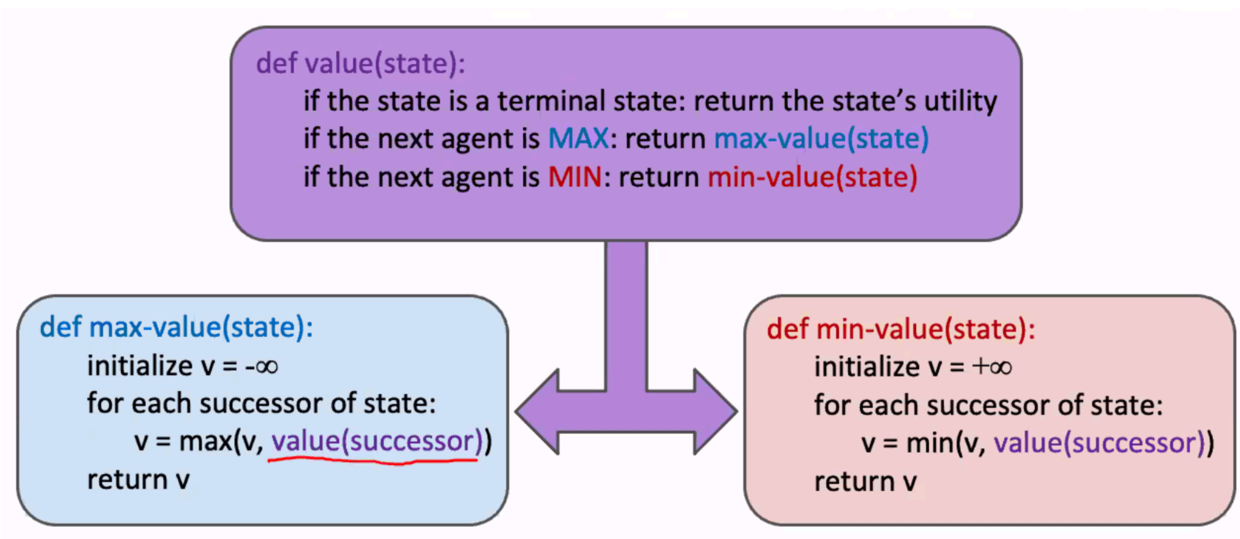

Figure 1: Screenshot_2023-09-12_at_5.42.27_PM.png

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- Optimal against a perfect player but not imperfect players
- b := branching factor
- m := maximum depth of game

- Time $\in O(b^m)$
- Space $\in O(bm)$

Alpha-Beta Pruning

- Used to minimize excess calculation
- Once you have a possible value for a parent node, only evaluate children if their value could be propagated with optimality
- Keep track of α in maximizer nodes; if the value $\leq$ α for a node, ignore node and children
- Keep track of β in minimizer nodes; if the value $\geq$ β for a node, ignore the node and children

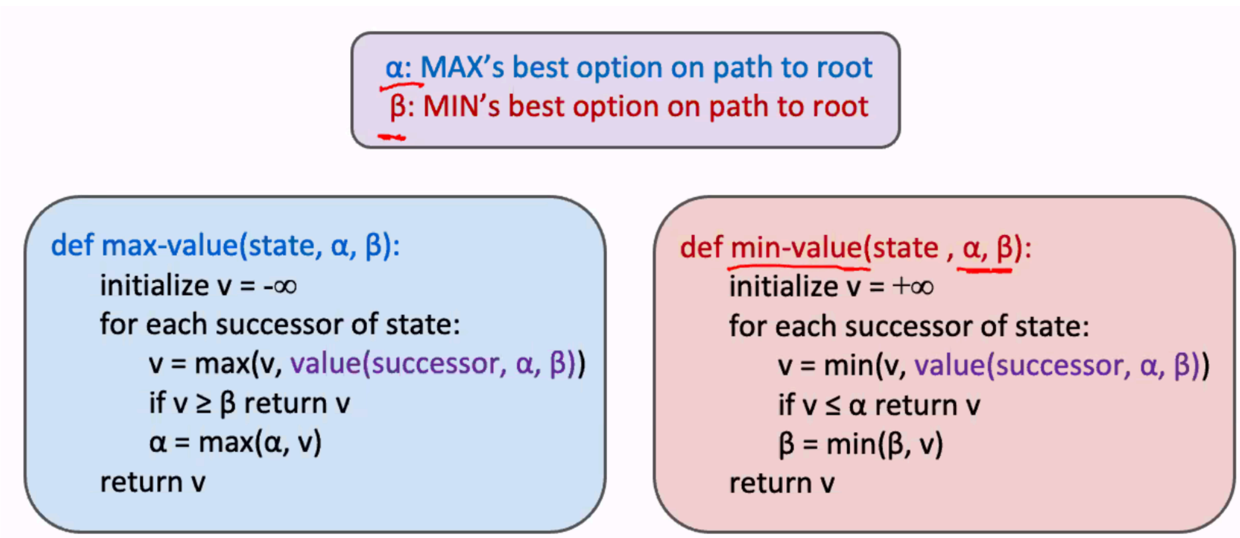

Figure 2: Screenshot_2023-09-12_at_6.02.01_PM.png

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- Has no effect on the minmax value computed for the root
- Values of intermediate nodes might be wrong
- Good child ordering improved effectiveness of pruning
- Time with perfect ordering $\in O(b^{m/2})$
  - Doubles solvable depth for a given computational power
- Simple example of metareasoning (computing about what to compute)

Depth-limited Search

- Search only to a limited depth in the tree
- Replace terminal utilities with an evaluation function for non-terminal positions
- Can get in an infinite loop if evaluation function is not specific enough
  - Usually a weighted linear sum of features

General Games

- Agents have independent utilities (values on outcomes)
- Cooperation, indifference, competition, and more are all possible