# Search Problems

## Uniform Cost Search

- Type of uninformed search
- Explores in all directions, so it isn't optimal

## Informed Search

- Heuristic := a function that estimates how close a state is to a goal
  - designed for a particular search problem
  - Takes in a state and outputs a guess for how close you are to the goal
- Manhattan distance := distance from a point making only one $90^o$ turn
  - Possible heuristic

## Greedy Search

- Uninformed? search alg
- Fringe based algorithm
- Expand the node that seems closest with respect to some heuristic
- Not always optimal
- Somewhat like DFS because it explores a full path at a time

A* Search

- Considers both cost and heuristic value
- Like a combination of greedy and UCS
- Heuristic:

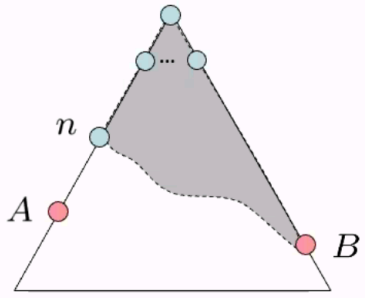
$$f(n) = g(n) + h(n) \Big| \begin{cases} g(n) := \text{distance traveled so far} \\ h(n) := \text{heuristic value for a point} \end{cases}$$

- Note: you must evaluate the goal before you can declare success
- Not optimal b/c weightings of $g(n)$ and $h(n)$ can vary
  - Need optimal heuristic for optimality
- Expands mainly toward goal, but does hedge its bets to ensure optimality

Optimality

**Proof:**

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal




Proof: If $A$ is an optimal goal where $\{A, B\} \in$ goals, $A$ must be popped off the fringe before $B$ - Claim:

$B \in$ fringe $\implies \exists$ Ancestor of A $\in$ fringe - Argument: Ancestor of A will be popped off before $B$ - Known:

$$n \in \text{ancestors}_A \implies f(n) \leq f(A)$$
$$B \in \text{goals}_{\text{suboptimal}} \implies f(A) < f(B)$$

The heuristic is admissible, so it must be an underestimate $\implies h(A) = h(B) = 0 \therefore \{f(A) = g(A), f(B) = g(B)\}$

Given that $B$ is suboptimal, we know the path cost to A $<$ the path cost to B: $g(A) < g(B)$

$\therefore f(n) \leq f(A) < f(B)$ so $n$ must be expanded before $B \implies$ all ancestors of $A$ are expanded before $B \therefore$ A* is optimal.

## Heuristics

- Two types:
  1. Optimistic
  2. Pessimistic
- Admissible (optimistic) iff $0 \leq h(n) \leq h^*(n)|_{h^*(n) := \text{true cost to the nearest goal}}$

Pessimistic (not admissible)

- Regardless of $h(n)$, consider $g(n)$ as more important
- Breaks optimality by trapping good plans on the fringe
- We thought the plan was bad, but it ended up being a good plan

Optimistic (admissible)

- Gives better estimates than the reality
- Ensures optimality because it forces excess computation (explore things that are wrong)

Creating An Admissible Heuristic

- Most of the hard work is for a search problem comes down to the heuristic
- Often we evaluate the cost of an easier problem as our heuristic
  - Manhattan distance, straight-line distance