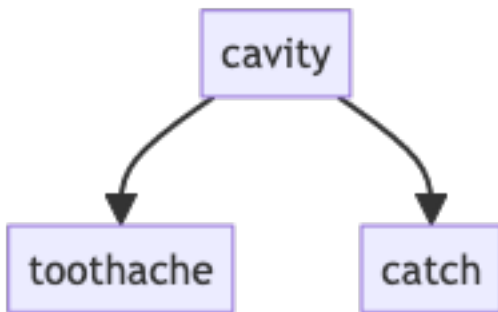


# Bayes' Nets

## Review

- Efficient encoding of a probabilistic model of a domain
- Set of nodes, one per variable  $X$
- Directed, acyclic graph
- Conditional distribution for each node
- Implicitly encode joint distributions
  - To see what probability a BN gives to a full assignment, multiply all of the relevant conditionals together:  $\mathbb{P}(x_1, x_2, \dots, x_n) = \prod_{i=1}^n \mathbb{P}(x_i | \text{parents}(X_i))$
- eg:



B	P(B)
+b	0.001
-b	0.999

A	J	P(J A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

$$P(+b, -e, +a, -j) = P(+b)P(-e)P(+a|+e)P(-j|+a, -e) = 0.001 \times 0.998 \times 0.94$$

$\mathbb{P}(+cavity, +catch, -toothache) = \mathbb{P}(+cavity)\mathbb{P}(-toothache|+cavity)\mathbb{P}(+catch|+cavity)$  - eg2:

- Joint distribution over  $N$  variables is  $2^N$  rows - Bayes net of  $N$  nodes with up to  $k$  parents  $\in O(n * 2^{k+1})$

## Extracting Conditional Independence

- Can use the chain rule and bayes' net reconstitution formula to pull apart factors and determine conditional independence
- Two variables  $X$  and  $Y$  are conditionally independent given  $Z$  iff  $\mathbb{P}(X, Y|Z) = \mathbb{P}(X|Z)\mathbb{P}(Y|Z)$

## D-Separation

- Study independence properties for triples

- Analyze complex cases in terms of member triples

■ **Question: Are X and Y conditionally independent given evidence variables {Z}?**

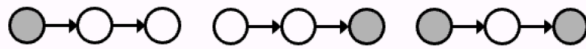
- Yes, if X and Y “d-separated” by Z
- Consider all (undirected) paths from X to Y
- No active paths = independence!

■ **A path is active if each triple is active:**

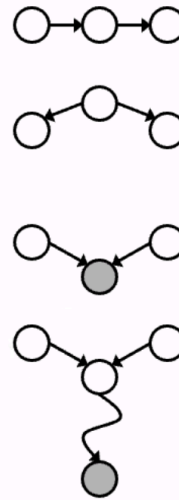
- Causal chain  $A \rightarrow B \rightarrow C$  where B is unobserved (either direction)
- Common cause  $A \leftarrow B \rightarrow C$  where B is unobserved
- Common effect (aka v-structure)  
 $A \rightarrow B \leftarrow C$  where B or one of its descendants is observed

■ **All it takes to block a path is a single inactive segment**

- **Note:** These triples are all active (and similar for the other cases, i.e. the variables on either end of the triple can be observed or unobserved)



Active Triples



Inactive Triples

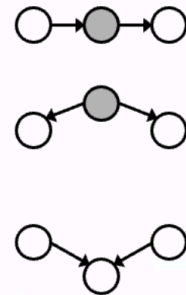


Figure 1: Screenshot\_2023-10-10\_at\_6.05.43\_PM.png

- – Shaded nodes  $\iff$  observed (taken as a given value)
- To determine conditional independence between  $X_i$  and  $X_j$ , you have to check all possible (UNDIRECTED) paths  $X_i \rightarrow X_j$  and see if there is a path s.t. every intermediary node is open
  - The intermediary nodes are triples along the path; open  $\iff$  active

## Topology Limits Distributions

- Adding arcs increases the set of distributions, but has several costs
- Full conditioning can encode any distribution