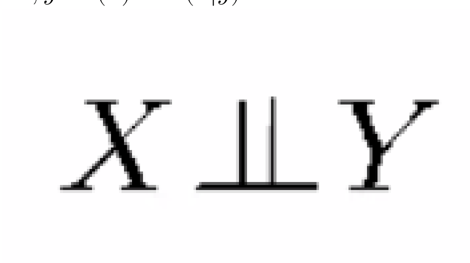
Probabilistic Models

- Models describe how (a portion of) the world works
- Models are always simplifications
  - May not account for every variable
  - May not account for all interactions between variables

Probability

Independence

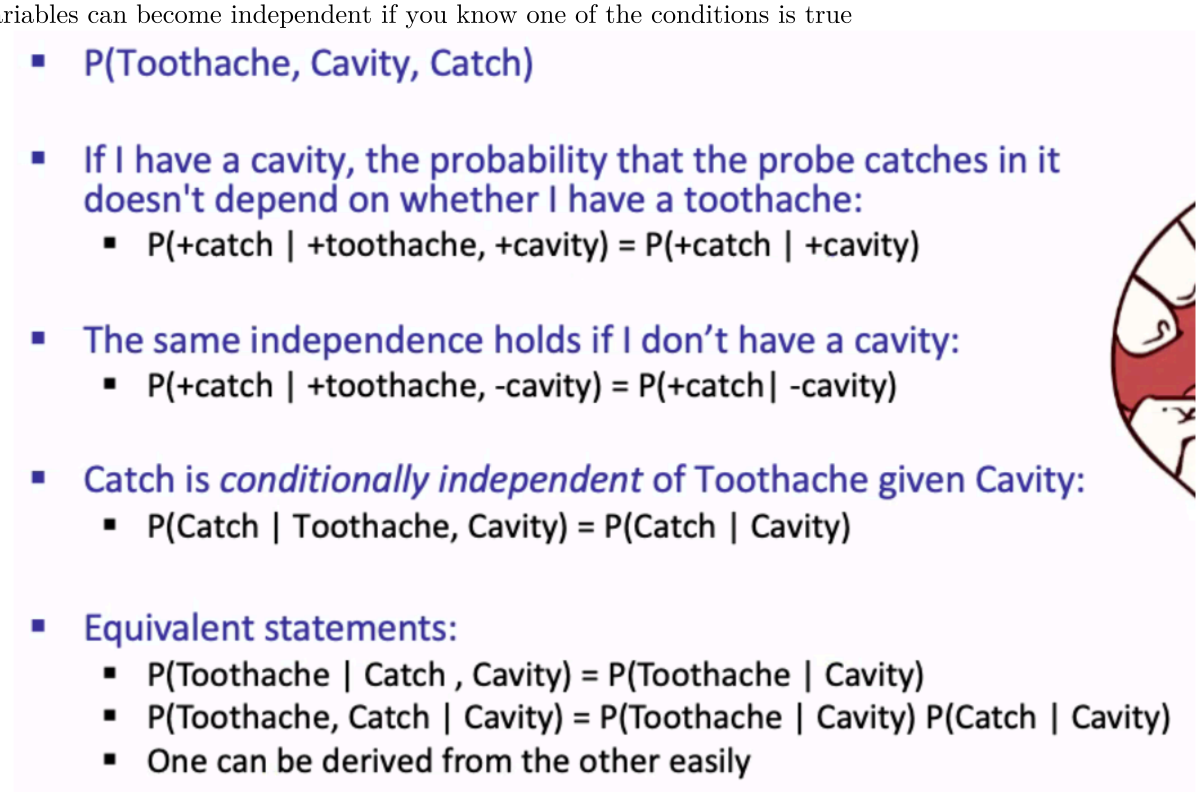
- $\forall x, y : \mathbb{P}(x, y) = m\mathbb{P}(x)\mathbb{P}(y)$
  - $\forall x, y : \mathbb{P}(x|y) = \mathbb{P}(x)$
- Also true if $\forall x, y : \mathbb{P}(x) = \mathbb{P}(x|y)$



- Written as:
- Independent actions can be considered individually; do not need to keep a massive table with independent probabilities
- Pretty rare (especially in models)

Conditional Independence

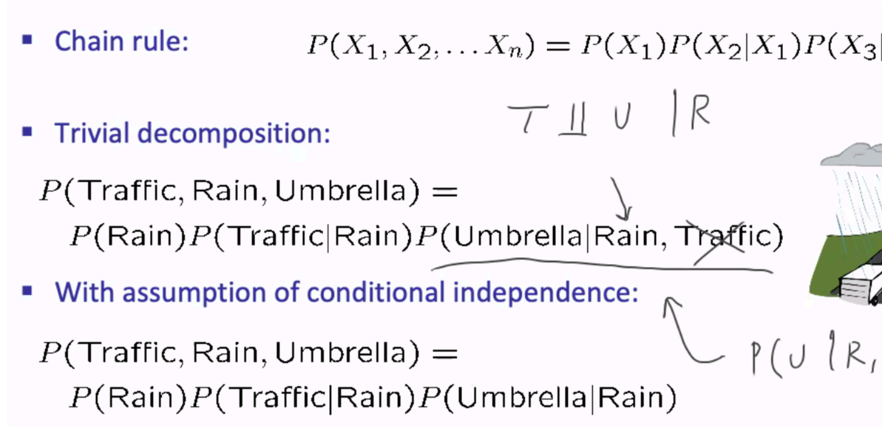
- Variables can become independent if you know one of the conditions is true



- eg:
- $\forall x, y, z : \mathbb{P}(x, y|z) = \mathbb{P}(x|z)\mathbb{P}(y|z)$

- $\forall x, y, z : \mathbb{P}(x|z, y) = \mathbb{P}(x, z)$

- Chain Rule with conditional independence:



  - Note the necessity of the conditional independence assumption
  - Can be represented with Bayes' nets

Bayes' Nets

- A technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)

- Good because sometimes the tables get too big

- Hard to learn full table empirically about tables with more than a few variables

- Type of graph

  - Nodes: variables (with domains)
    - Can be assigned (observed) or unassigned (unobserved)
  - Arcs: interactions
    - Similar to CSP constraints
    - Indicate 'direct influence' between variables
    - Formally: encode conditional independence

- Given a set of nodes, one per variable X, it is a directed, acyclic graph with a conditional distribution for each node

  - A collection of distributions over X, one for each combination of parents' values: $\mathbb{P}(X|a_1, ...a_n)$
  - CPT: conditional probability table

- To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together: $\mathbb{P}(x_1, x_2, ..., x_n) = \prod_{i=1}^{n} \mathbb{P}(x_i|\text{parents}(X_i))$

  - Good because it assumes conditional independence

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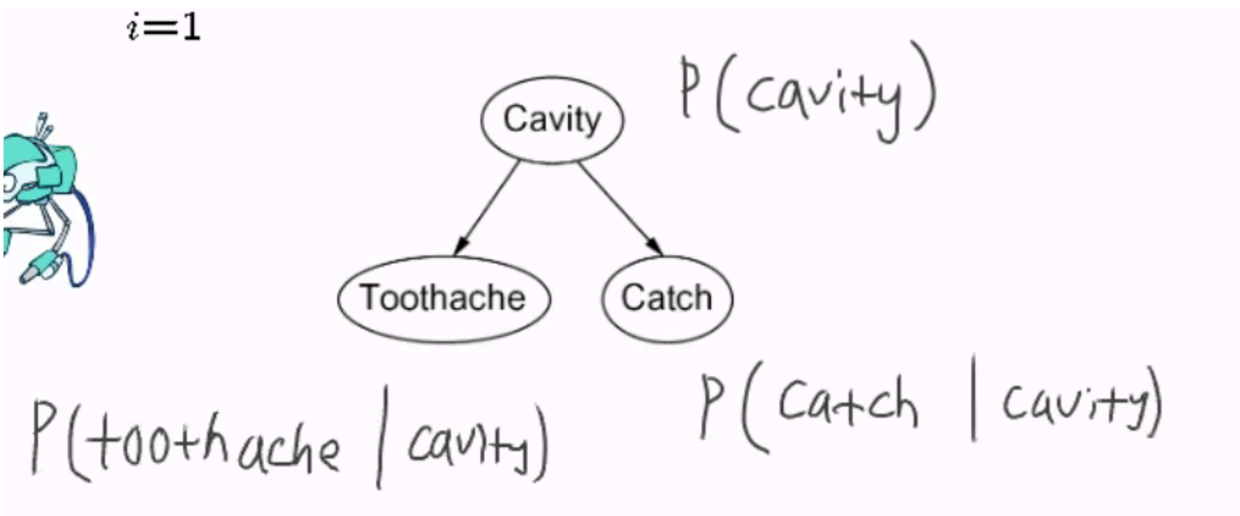

Figure 1: Screenshot_2023-10-05_at_6.19.03_PM.png