Probabilities

Uncertainty

- Observed variables (evidence) $\triangleq$ agent knows certain things about the state of the world (eg: sensor readings or symptoms)
- Unobserved (hidden) variables $\triangleq$ Agent needs to reason about other aspects (eg: where an object is or what disease is present)
- Model $\triangleq$ agent knows something about how the known variables relate to the unknown variables

Random Variables

- RV $\triangleq$ some aspect of the world about which we (may) have certainty
- Commonly indicated with capital letters
- Have explicit domains ($R \in \{\text{true}, \text{false}\} \mathrel{\hat{=}} \{+r, -r\}$)
- Associate probability with each value in the domain
  - Can use a probability table for showing all of them
  - Single probability of a lower-case value is a single number
  - eg: $\mathbb{P}[W = \text{rain}] = 0.1 \iff \mathbb{P}[\text{rain}] = 0.1$
  - Each probability sum to 1 and be positive
- Size of distribution of n variables with domain sizes d is d^n
  - For every n variables, there are d possible choices
- Probabilistic model is just a joint distribution over a set of RVs

Events

- Event $\triangleq$ a set E of outcomes
- $\mathbb{P}[E] = \sum_{(x_1,...,x_n)\in E} = \mathbb{P}(x_1, ..., x_n)$
- From a joint distribution, we can calculate the probability of any event ### Marginal Distributions
- Eliminates RVs
- Marginalization $\triangleq$ summing out / combining collapsed rows by adding
- eg: $\mathbb{P}[t] = \sum_w \mathbb{P}[t, w]$ or $\mathbb{P}[w] = \sum_t \mathbb{P}[t, w]$
- Generally, $\mathbb{P}[X_1 = x_1] = \sum_{x_2} \mathbb{P}[X_1 = x_1, X_2 = x_2]$

Conditional Probabilities

- A simple relation between joint and conditional probabilities
- $\mathbb{P}(a|b) = \frac{\mathbb{P}(a,b)}{\mathbb{P}(b)}$
- 'Proportion of b where a holds'
- Normalization trick:
  - Take the entries in a table that you are stipulating and normalize the values to get the new probabilities
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Probabilistic Inference

- Compute a desired probability from other known probabilities (eg conditional from joint)
- Observing new evidence causes 'beliefs' to change
- Given $\{X_1, X_2, ..., X_n\}$, separate into
  - Evidence variables: $\{E_1, E_2, ..., E_n\}$ representing known state
  - Query variables Q representing what you care about

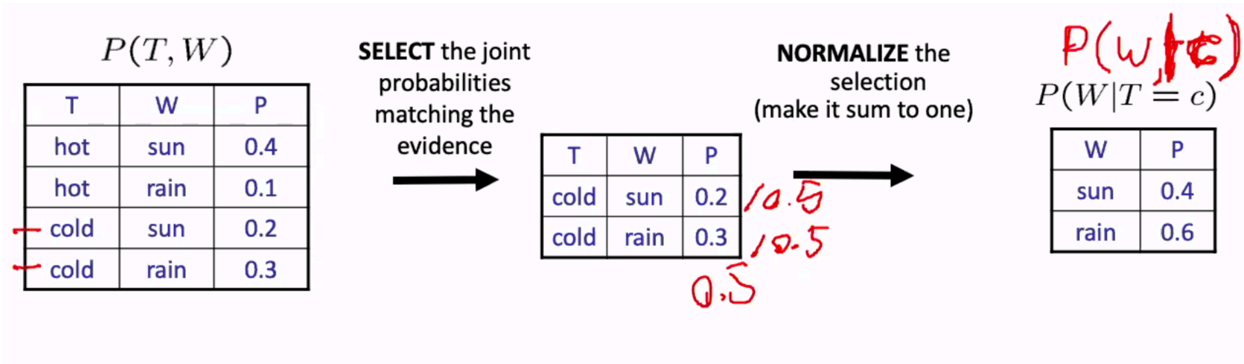

Figure 1: Screenshot_2023-10-03_at_5.49.42_PM.png

  - Hidden variables: $\{H_1, H_2, ..., H_r\}$ representing unknown variables that we don't know or care about
- Want $\mathbb{P}(Q|e_1, ..., e_k)$

Inference by Enumeration

- Steps:
  1. Select the entries consistent with the evidence
  2. Sum out H to get joint of query and evidence
  3. Normalize (multiply probabilities by $\frac{1}{Z}$)
- Worst case time complexity in $O(d^n)$
- Space complexity in $O(d^n)$ to store the joint distribution

Rules

- Product rule: $\mathbb{P}(y)\mathbb{P}(x|y) = \mathbb{P}(x, y)$
  - eg: $\mathbb{P}(x_1, x_2, x_3) = \mathbb{P}(x_1)\mathbb{P}(x_2|x_1)\mathbb{P}(x_3|x_1, x_2)$
  - Generally, $\mathbb{P}(x_1, x_2, ..., x_n) = \prod_i \mathbb{P}(x_i|x_1, x_2, ..., x_{i-1})$
- Bayes rule: $\mathbb{P}(x, y) = \mathbb{P}(x|y)\mathbb{P}(y) = \mathbb{P}(y|x)\mathbb{P}(x)$
  - Also $\mathbb{P}(x|y) = \frac{\mathbb{P}(y|x)}{\mathbb{P}(y)}\mathbb{P}(x)$
  - Lets us build one conditional from its reverse
  - Often one conditional is tricky, but the other one is simple