

Time Complexity Analysis

Note: I also put comments before algorithms and after statements defining the time complexity of each method. The comments stating time complexity after each statement show how I determined the values to add for each algorithm to get the total time complexity.

Question 1-Longest Common Subsequence

allSubsequences(String str) algorithm -

$n = \text{str.length}();$

$O(1) + O(1) + O(1) + O(2^n * (1 + 2n + 1)) + O(1)$

$O(2 * 2^n + 2n * 2^n)$

$O(2n * 2^n)$

$O(n * 2^n)$

Total Time Complexity- $O(n * 2^n)$

longestCommonSubsequences(String word1, String word2) algorithm -

$p = \text{word1.length}();$

$k = \text{word2.length}();$

$O(p * 2^p) + O(k * 2^k) + O(1) + O(1) + O(1) + O(p * 2^p * (2 * (k * 2^k))) + O(1)$

$O(p * 2^p * 2k * 2^k)$

$O(p * 2^p * k * 2^k)$

So , the worst case time complexity is $O(p * 2^p * k * 2^k)$

Since the for loops always iterates the same amount of times and there is not a break in the for loop the best case time complexity is also $\Omega(p * 2^p * k * 2^k)$

Question 2-Longest Common Substring

$n = \text{text1.length}();$

$m = \text{text2.length}();$

i is the firstIteratorVariable in the first loop

j is the secondIterator variable in the second loop

k is the third iterator variable in the third loop

l is the fourth iterator variable in the fourth loop

$O(1) + O(1) + O(1) + O(1) + O(1) + O(1) + O(1) + O(1) + O((n(n * (j-i) * (m(m * (l-k + 4) - 1) / 2) - 1)) / 2) + O(1)$

$$\begin{aligned}
&O((n * (j-i) * (m * (l - k + 4) - 1) / 2) - 1)) / 2) \\
&O((n * (j-i) * (m * (ml - mk + 4) - 1) / 2) - 1)) / 2) \\
&O((n * (j-i) * (m^{2l} - m^{2k} + 3m) / 2) - 1)) / 2) \\
&O((n * (nj - ni * (m^{2l} - m^{2k} + 3m) - 1)) / 2) \\
&O((n * (nj - ni * (m^{2l} - m^{2k} + 3m))) / 2) \\
&O((n^{2j} - n^{2i} * (m^{2l} - m^{2k} + 3m))) / 2) \\
&O((n^{2j} - n^{2i} * (m^{2l} - m^{2k}))) / 2) \\
&O((n^{2j} * m^{2l} - n^{2j} * m^{2k} - n^{2i} * m^{2l} + n^{2i} * m^{2k}) / 2) \\
&O(n^{2j} * m^{2l} - n^{2j} * m^{2k} - n^{2i} * m^{2l} + n^{2i} * m^{2k})
\end{aligned}$$

So , the worst case time complexity is $O(n^{2j} * m^{2l} - n^{2j} * m^{2k} - n^{2i} * m^{2l} + n^{2i} * m^{2k})$

Since the for loops always iterates the same amount of times and there is not a break in any of the for loops the best case time complexity is also $OMEGA(n^{2j} * m^{2l} - n^{2j} * m^{2k} - n^{2i} * m^{2l} + n^{2i} * m^{2k})$ and since the bounds are equal we can define a bound for theta as $THETA(n^{2j} * m^{2l} - n^{2j} * m^{2k} - n^{2i} * m^{2l} + n^{2i} * m^{2k})$

Question 3-notFibonacci

$$\begin{aligned}
&O(1) + O(1) \quad O(1) + O(1) + O(1) + O((n - 2) * (1 + 1)) \\
&O((n - 2) * (2)) \\
&O((n - 2) * (2)) \\
&O(2n - 4) \\
&O(n)
\end{aligned}$$

So the total worst-case time complexity $O(n)$

The loops have no conditional breaks in them besides $i < n$, so the best case is the same as the worst case making $OMEGA(n)$ the best case time complexity. Additionally, since the worst case and best case time complexity are the same theta can be defined as $THETA(n)$.

Question 4-Where In Sequence

$$\begin{aligned}
&n = \text{notFibonacci.length} \\
&O(1) + O(1) + O(1) + O(1) + O(1) + O(1) + O(\text{target}) + O(n) \\
&O(\text{target} + n)
\end{aligned}$$

So the total worst case time complexity is, $O(\text{target} + n)$.

The best case time complexity would be if $\text{target} == 0$ in which the first if statement would be true and 1 would be returned, then the time complexity would be $\Omega(1)$.

Problem 5-Remove Element

$n = \text{nums.length}$

$m = \text{numsWithoutVal.length}$

$k = \text{occurrencesOfVal}$

$O(1) + O(n^2) + O(1) + O(1) + O(m(k + 1 + 1)) + O(n(1+1))$

$O(2n) + O(mk+2m) + O(2n)$

$O(mk + 4n + 2m)$

$O(mk + n + m)$

So the worst case time complexity is $O(mk + n + m)$

The best case time complexity would be $\Omega(1)$ which would happen if nums was an empty array,

which would imply that $n=0$, $m=0$, and $k=0$

which would lead to the following time complexity $\Omega(1) + \Omega(n^2) +$

$\Omega(1) + \Omega(1) + \Omega(m(k + 1 + 1)) +$

$\Omega(n^2)=\Omega(6)=\Omega(1)$