Project 1

Part 1

Problem 1

Part A

Part B

```
clear; clc;

syms x
g(x) = exp(-x);
g_prime(x) = diff(g, x);

a = 1/exp(1);
b = 1;

vpa(a, 4)
```

```
ans = 0.3679
```

```
vpa(b, 4)
```

```
ans = 1.0
```

```
vpa(g(a), 4)
```

```
ans = 0.6922
```

```
vpa(g(b), 4)
```

```
ans = 0.3679
```

```
vpa(g_prime(a), 4)
```

```
ans = −0.6922
```

Part C

```
p_0 = 0.4;
TOL = 10^-6;

[p_vec, p] = fixedPoint(g, p_0, TOL);
```

```
n             p_n           |p_n-p_{n-1}|
1             0.67032       0.27032
2             0.51154       0.15878
3             0.59957       0.088024
4             0.54905       0.05052
5             0.5775        0.028451
6             0.5613        0.016199
```

```
7             0.57047        0.0091664
8             0.56526        0.0052052
9             0.56821        0.00295
10            0.56654        0.0016737
11            0.56749        0.00094903
12            0.56695        0.00053831
13            0.56725        0.00030528
14            0.56708        0.00017314
15            0.56718        9.8194e-05
16            0.56712        5.5691e-05
17            0.56715        3.1584e-05
18            0.56714        1.7913e-05
19            0.56715        1.0159e-05
20            0.56714        5.7617e-06
21            0.56714        3.2677e-06
22            0.56714        1.8533e-06
23            0.56714        1.0511e-06
24            0.56714        5.9611e-07
```

Part D

```
const = [];

for n = 1:23

    const(n) = abs(p_vec(n+1) - p)/abs(p_vec(n) - p);

end

vpa(const', 5)
```

```
ans =
```

$$\begin{pmatrix} 0.53886 \\ 0.58321 \\ 0.55804 \\ 0.57232 \\ 0.56418 \\ 0.56886 \\ 0.5661 \\ 0.56786 \\ 0.56652 \\ 0.56787 \\ 0.56606 \\ 0.56894 \\ 0.56406 \\ 0.57258 \\ 0.55768 \\ 0.58411 \\ 0.53811 \\ 0.62109 \\ 0.48028 \\ 0.74799 \\ 0.32537 \\ 1.3102 \\ 0 \end{pmatrix}$$

Problem 2

Part A

```
clear; clc;

syms x
g(x) = exp(-x);

p_0 = 0.4;
TOL = 10^-6;

[p_vec, p] = SteffensenMethod(g, p_0, TOL);
```

```
n             p_n           |p_n-p_{n-1}|
1             0.5703        0.1703
2             0.56714       0.003151
3             0.56714       1.0178e-06
4             0.56714       1.0633e-13
```

Part B

```
const = [];
```

```
for n = 1:3

    const(n) = abs(p_vec(n+1) - p)/abs(p_vec(n) - p)^2;

end

vpa(const', 5)
```

ans =

$$\begin{pmatrix} 0.10244 \\ 0.10262 \\ 0 \end{pmatrix}$$

Part 2

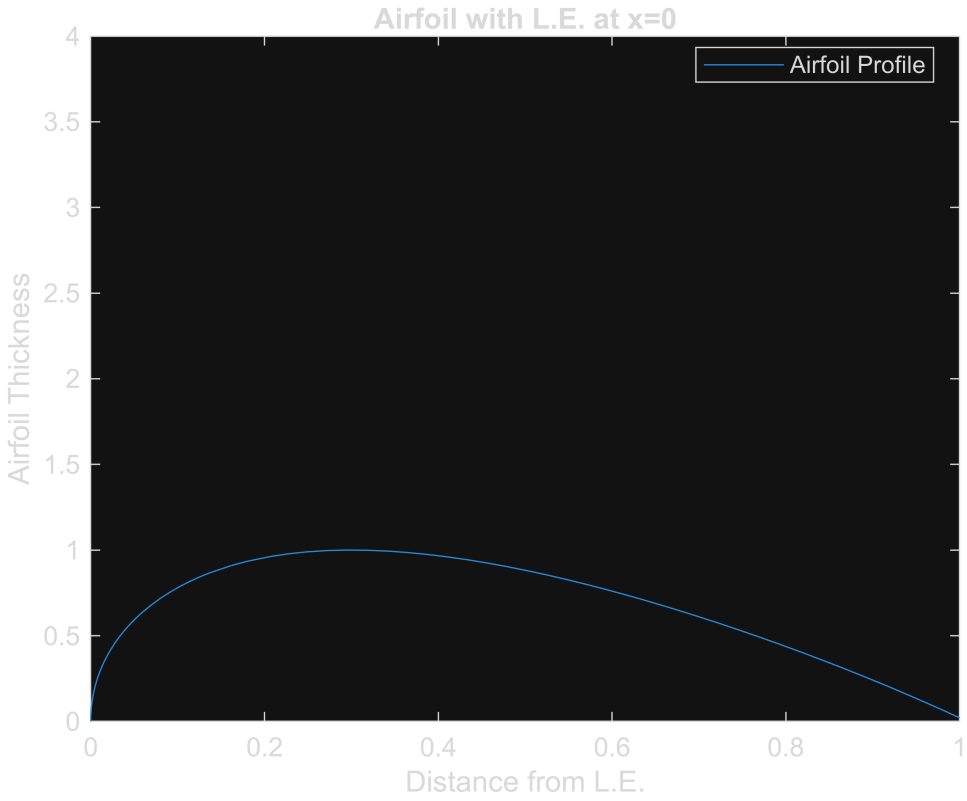
Problem 1

```
clear; clc;

syms x

y(x) = 2.969*sqrt(x) - 1.26*x - 3.516*x^2 + 2.843*x^3 - 1.015*x^4;

figure
fplot(y, [0, 1])
axis([0 1 0 4])
title("Airfoil with L.E. at x=0")
xlabel("Distance from L.E.")
ylabel("Airfoil Thickness")
legend("Airfoil Profile", "Location", "best")
```



Problem 2

Part A

```
clear; clc;

syms x

y(x) = 2.969*sqrt(x) - 1.26*x - 3.516*x^2 + 2.843*x^3 - 1.015*x^4;
y_prime(x) = diff(y, x);

vpa(y_prime)
```

```
ans(x) =
```

$$8.529\,x^2 - 7.032\,x + \frac{1.484}{\sqrt{x}} - 4.06\,x^3 - 1.26$$

Part B

```
p_0 = 0.1;
TOL = 10^-7;

p = NewtonMethod(y_prime, p_0, TOL);
```

```
n               p_n             |p_n-p_{n-1}|
1               0.1972486       0.09724864
2               0.2762193       0.0789707
```

```
3               0.2986261       0.02240674
4               0.2998248       0.001198702
5               0.2998279       3.103562e-06
6               0.2998279       2.069539e-11
```

Problem 3

Part A

Part B

```
clear; clc;

syms x

y(x) = 2.969*sqrt(x) - 1.26*x - 3.516*x^2 + 2.843*x^3 - 1.015*x^4;
y_prime(x) = diff(y, x) + x;

p_0 = 0.1;
TOL = 10^-7;

[p_vec, p] = SteffensenMethod(y_prime, p_0, TOL);
```

```
n               p_n             |p_n-p_{n-1}|
1               0.25318         0.15318
2               0.33455         0.081362
3               0.31674         0.017805
4               0.30374         0.012996
5               0.30004         0.0037032
6               0.29983         0.00021297
7               0.29983         6.4019e-07
8               0.29983         5.7534e-12
```

```
vpa(p_vec)
```

ans = $(0.2532 \quad 0.3345 \quad 0.3167 \quad 0.3037 \quad 0.3 \quad 0.2998 \quad 0.2998 \quad 0.2998)$

Part C

```
y(x) = 2.969*sqrt(x) - 1.26*x - 3.516*x^2 + 2.843*x^3 - 1.015*x^4;
y_prime(x) = diff(y, x);

f(x) = (y_prime(x + y_prime(x)) - y_prime(x))/y_prime(x);
g(x) = x - y_prime(x)/f(x);
g_prime(x) = diff(g, x);

vpa(g_prime(p_vec'))
```

ans =

$$\begin{pmatrix} -1.563 \\ 1.043 \\ 0.4613 \\ 0.1087 \\ 0.005993 \\ 1.797\text{e-}5 \\ 0.03098 \\ 0.08628 \end{pmatrix}$$