Homework 3

Problem 1 (6c, 8c)

```
clear; clc;

syms x

x_vec = [0.1, 0.2, 0.3, 0.4];
f_vec = [-0.29004986, -0.56079734, -0.81401972, -1.0526302];

P_1(x) = LagrangePolynomial(f_vec(1:2), x_vec(1:2));
vpa(simplify(P_1(x)), 4)
```

ans = $-2.7075\,x - 0.0193$

```
vpa(P_1(0.18), 5)
```

ans = -0.5066

```
P_2(x) = LagrangePolynomial(f_vec(1:3), x_vec(1:3));
vpa(simplify(P_2(x)), 4)
```

ans = $0.8763\,x^2 - 2.9704\,x - 0.0018$

```
vpa(P_2(0.18), 5)
```

ans = -0.5080

```
P_3(x) = LagrangePolynomial(f_vec, x_vec);
vpa(simplify(P_3(x)), 4)
```

ans = $-0.4855\,x^3 + 1.1676\,x^2 - 3.0238\,x + 0.0011$

```
vpa(P_3(0.18), 5)
```

ans = -0.5081

```
f(x)= x^2*cos(x)-3*x
```

f(x) = $x^2\cos(x) - 3\,x$

```
vpa(f(0.18), 5)
```

ans = -0.5081

```
f_diff_2(x) = diff(f, x, 2)
```

f_diff_2(x) = $2\cos(x) - x^2\cos(x) - 4x\sin(x)$

```
f_diff_3(x) = diff(f, x, 3)
```

f_diff_3(x) = $x^2\sin(x) - 6\sin(x) - 6x\cos(x)$

```
vpa(f_diff_2(0.1), 5)
```

ans = 1.9401

```
vpa(f_diff_3(0.3), 5)
```

ans = -3.4661

```
error_1(x) = f_diff_2(0.1)/factorial(2)*(x - x_vec(1))*(x - x_vec(2));
max_x_1 = vpasolve(diff(error_1) == 0)
```

max_x_1 = 0.1500

```
vpa(error_1(max_x_1), 5)
```

ans = -0.0024

```
error_2(x) = f_diff_3(0.3)/factorial(3)*(x - x_vec(1))*(x - x_vec(2))*(x -
x_vec(3));
max_x_2 = vpasolve(diff(error_2) == 0)
```

max_x_2 =

$$\begin{pmatrix} 0.1423 \\ 0.2577 \end{pmatrix}$$

```
vpa(error_2(max_x_2), 5)
```

ans =

$$\begin{pmatrix} -2.2235\text{e-}04 \\ 2.2235\text{e-}04 \end{pmatrix}$$

```
error_1_act = vpa(abs(f(0.18) - P_1(0.18))/f(0.18), 5)
```

error_1_act = -0.0029

```
error_1_act = vpa(abs(f(0.18) - P_2(0.18))/f(0.18), 5)
```

error_1_act = $-1.4487\text{e-}04$

Problem 2 (20)

```
clear; clc;

syms x

x_vec = [0 6 10 13 17 20 28];
f_vec1 = [6.67 17.33 42.67 37.33 30.10 29.31 28.74];
f_vec2 = [6.67 16.11 18.89 15.00 10.56 9.44 8.89];

P_1(x) = LagrangePolynomial(f_vec1, x_vec);
vpa(simplify(P_1(x)), 4)
```

ans = $4.0946\text{e-}05\,x^6 - 0.0037\,x^5 + 0.1269\,x^4 - 2.0946\,x^3 + 16.1427\,x^2 - 42.6435\,x + 6.6700$

```
P_2(x) = LagrangePolynomial(f_vec2, x_vec);
vpa(simplify(P_2(x)), 4)
```

ans = $8.3616\text{e-}06\,x^6 - 7.5255\text{e-}04\,x^5 + 0.0258\,x^4 - 0.4138\,x^3 + 2.9128\,x^2 - 5.6782\,x + 6.6700$

```
x_pot_1 = vpasolve(diff(P_1) == 0, [0 28])
```

x_pot_1 =

$$\begin{pmatrix} 1.9558 \\ 10.1885 \\ 25.9642 \end{pmatrix}$$

```
x_pot_2 = vpasolve(diff(P_2) == 0, [0 28])
```

x_pot_2 =

$$\begin{pmatrix} 1.2956 \\ 8.7695 \\ 26.0438 \end{pmatrix}$$

```
P_1(x_pot_1)
```

ans =

$$\begin{pmatrix} -28.9002 \\ 42.7084 \\ 10.5623 \end{pmatrix}$$

```
P_2(x_pot_2)
```

ans =

$$\begin{pmatrix} 3.3729 \\ 19.4158 \\ 5.7607 \end{pmatrix}$$

Problem 3 (2c)

```
clear; clc;

x = 0.18;
x_vec = [0.1 0.2 0.3 0.4];
f_vec = [-0.29004986, -0.56079734, -0.81401972, -1.0526302];

Q_1 = NevilleMethod(x, f_vec(1:2), x_vec(1:2));
vpa(Q_1, 5)
```

ans =

$$\begin{pmatrix} 0.1000 & -0.2900 & 0 \\ 0.2000 & -0.5608 & -0.5066 \end{pmatrix}$$

```
Q_2 = NevilleMethod(x, f_vec(1:3), x_vec(1:3));
vpa(Q_2, 5)
```

ans =

$$\begin{pmatrix} 0.1000 & -0.2900 & 0 & 0 \\ 0.2000 & -0.5608 & -0.5066 & 0 \\ 0.3000 & -0.8140 & -0.5102 & -0.5080 \end{pmatrix}$$

```
Q_3 = NevilleMethod(x, f_vec, x_vec);
vpa(Q_3, 5)
```

ans =

$$\begin{pmatrix} 0.1000 & -0.2900 & 0 & 0 & 0 \\ 0.2000 & -0.5608 & -0.5066 & 0 & 0 \\ 0.3000 & -0.8140 & -0.5102 & -0.5080 & 0 \\ 0.4000 & -1.0526 & -0.5277 & -0.5084 & -0.5081 \end{pmatrix}$$

Problem 4 (10)

```
clear; clc;

x = 0;
x_vec = [-2 -1 1 2];
f_vec_wrong = [0 0 0 0];
f_vec_right = f_vec_wrong + [0 -2 3 0];

Q_wrong = NevilleMethod(x, f_vec_wrong, x_vec)
```

```
Q_wrong = 4×5
    -2     0     0     0     0
    -1     0     0     0     0
     1     0     0     0     0
     2     0     0     0     0
```

```
Q_right = NevilleMethod(x, f_vec_right, x_vec)
```

```
Q_right = 4×5
   -2.0000         0         0         0         0
   -1.0000   -2.0000   -4.0000         0         0
    1.0000    3.0000    0.5000   -1.0000         0
    2.0000         0    6.0000    2.3333    0.6667
```

Problem 5 (18)

```
clear; clc;

syms x

x_vec = [0.25 0.5 1 1.25];
f_vec = [25.2 49.2 60+36.4 60+59.4];

F = vpa(sym(dividedDifference(f_vec, x_vec)), 5)
```

```
F =
```

$$\begin{pmatrix} 25.2000 & 0 & 0 & 0 \\ 49.2000 & 96 & 0 & 0 \\ 96.4000 & 94.4000 & -2.1333 & 0 \\ 119.4000 & 92 & -3.2000 & -1.0667 \end{pmatrix}$$

```
P(x) = F(1,1) + F(2,2)*(x - x_vec(1)) + F(3,3)*(x - x_vec(1))*(x - x_vec(2)) +...
    F(4,4)*(x - x_vec(1))*(x - x_vec(2))*(x - x_vec(3));

vpa(simplify(P), 5)
```

ans(x) = $-1.0667\,x^3 - 0.2667\,x^2 + 96.6667\,x + 1.0667$

```
P(0.75)
```

ans = 72.9667

```
error = abs(73-P(0.75))/73
```

error = $4.5662\text{e-}04$

Problem 6 (9)

```
clear; clc;

syms x

x_vec = [0 3 5 8 13]
```

```
x_vec = 1×5
     0     3     5     8    13
```

```
f_vec = [0 225 383 623 993]
```

```
f_vec = 1×5
     0   225   383   623   993
```

```
f_diff_vec = [75 77 80 74 72]
```

```
f_diff_vec = 1×5
    75    77    80    74    72
```

```
F = vpa(sym(HermiteMethod(f_diff_vec, f_vec, x_vec)), 4)
```

```
F =
```

0	0	0	0	0	0	0	0	0	0	0
0	0	75	0	0	0	0	0	0	0	0
3	225	75	0	0	0	0	0	0	0	0
3	225	77	0.6667	0.2222	0	0	0	0	0	0
5	383	79	1	0.0667	−0.0311	0	0	0	0	0
5	383	80	0.5000	−0.2500	−0.0633	−0.0064	0	0	0	0
8	623	80	0	−0.1000	0.0300	0.0117	0.0023	0	0	0
8	623	74	−2	−0.6667	−0.1133	−0.0287	−0.0050	−9.1319e-04	0	0
13	993	74	0	0.2500	0.1146	0.0228	0.0051	7.8365e-04	1.3053e-04	0
13	993	72	−0.4000	−0.0800	−0.0412	−0.0195	−0.0042	−9.3729e-04	−1.3238e-04	−2.022

```
P(x) = F(1,2) + F(2,3)*(x - x_vec(1)) + F(3,4)*(x - x_vec(1))^2 +...
    F(4,5)*(x - x_vec(1))^2*(x - x_vec(2)) + F(5,6)*(x - x_vec(1))^2*(x -
x_vec(2))^2 +...
    F(6,7)*(x - x_vec(1))^2*(x - x_vec(2))^2*(x - x_vec(3)) +...
    F(7,8)*(x - x_vec(1))^2*(x - x_vec(2))^2*(x - x_vec(3))^2 +...
    F(8,9)*(x - x_vec(1))^2*(x - x_vec(2))^2*(x - x_vec(3))^2*(x - x_vec(4)) +...
    F(9,10)*(x - x_vec(1))^2*(x - x_vec(2))^2*(x - x_vec(3))^2*(x - x_vec(4))^2 +...
    F(10,11)*(x - x_vec(1))^2*(x - x_vec(2))^2*(x - x_vec(3))^2*(x - x_vec(4))^2*(x
- x_vec(5));

vpa(simplify(P), 4)
```

```
ans(x) =
```

$$1.0000\text{e-}37\, x\, (-2.0224\text{e+}32\, x^8 + 1.0406\text{e+}34\, x^7 - 2.1876\text{e+}35\, x^6 + 2.4304\text{e+}36\, x^5 - 1.5383\text{e+}37\, x^4 + 5.$$

```
vpa(P(10), 5)
```

```
ans =
```

$$742.5028$$

Problem 7 (25)

```
clear; clc;

x_vec = [0 6 10 13 17 20 28];
f_vec1 = [6.67 17.33 42.67 37.33 30.10 29.31 28.74];
```

```
f_vec2 = [6.67 16.11 18.89 15.00 10.56 9.44 8.89];

[a_1, b_1, c_1, d_1] = naturalSpline(f_vec1, x_vec);
[a_2, b_2, c_2, d_2] = naturalSpline(f_vec2, x_vec);

syms x

P_1(x) = piecewise(x >= 0 & x <= 6, a_1(1) + b_1(1)*x + c_1(1)*x^2 + d_1(1)*x^3,...
    x >= 6 & x <= 10, a_1(2) + b_1(2)*(x - 6) + c_1(2)*(x - 6)^2 + d_1(2)*(x -
6)^3,...
    x >= 10 & x <= 13, a_1(3) + b_1(3)*(x - 10) + c_1(3)*(x - 10)^2 + d_1(3)*(x -
10)^3,...
    x >= 13 & x <= 17, a_1(4) + b_1(4)*(x - 13) + c_1(4)*(x - 13)^2 + d_1(4)*(x -
13)^3,...
    x >= 17 & x <= 20, a_1(5) + b_1(5)*(x - 17) + c_1(5)*(x - 17)^2 + d_1(5)*(x -
17)^3,...
    x >= 20 & x <= 28, a_1(6) + b_1(6)*(x - 20) + c_1(6)*(x - 20)^2 + d_1(6)*(x -
20)^3);
vpa(P_1, 4)
```

ans(x) =

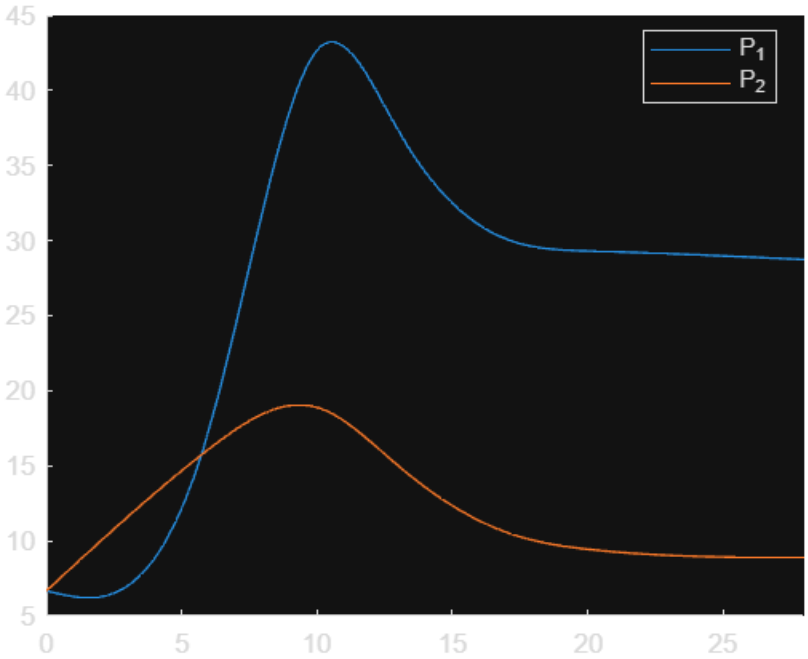
$$\begin{cases} 0.0618\,x^3 - 0.4469\,x + 6.6700 & \text{if } x \in [0,6] \\ 6.2237\,x + 1.1118\,(x-6)^2 - 0.2710\,(x-6)^3 - 20.0124 & \text{if } x \in [6,10] \\ 2.1104\,x - 2.1401\,(x-10)^2 + 0.2811\,(x-10)^3 + 21.5655 & \text{if } x \in [10,13] \\ 0.3897\,(x-13)^2 - 3.1406\,x - 0.0141\,(x-13)^3 + 78.1580 & \text{if } x \in [13,17] \\ 0.2204\,(x-17)^2 - 0.7002\,x - 0.0249\,(x-17)^3 + 42.0035 & \text{if } x \in [17,20] \\ 1.6070\text{e-}04\,(x-20)^3 - 0.0039\,(x-20)^2 - 0.0507\,x + 30.3236 & \text{if } x \in [20,28] \end{cases}$$

```
P_2(x) = piecewise(x >= 0 & x < 6, a_2(1) + b_2(1)*x + c_2(1)*x^2 + d_2(1)*x^3,...
    x >= 6 & x < 10, a_2(2) + b_2(2)*(x - 6) + c_2(2)*(x - 6)^2 + d_2(2)*(x -
6)^3,...
    x >= 10 & x < 13, a_2(3) + b_2(3)*(x - 10) + c_2(3)*(x - 10)^2 + d_2(3)*(x -
10)^3,...
    x >= 13 & x < 17, a_2(4) + b_2(4)*(x - 13) + c_2(4)*(x - 13)^2 + d_2(4)*(x -
13)^3,...
    x >= 17 & x < 20, a_2(5) + b_2(5)*(x - 17) + c_2(5)*(x - 17)^2 + d_2(5)*(x -
17)^3,...
    x >= 20 & x <= 28, a_2(6) + b_2(6)*(x - 20) + c_2(6)*(x - 20)^2 + d_2(6)*(x -
20)^3);
vpa(P_2, 4)
```

ans(x) =

$$\begin{cases} -0.0025\,x^3 + 1.6629\,x + 6.6700 & \text{if } x \in [0, 6) \\ 1.3943\,x - 0.0448\,(x-6)^2 - 0.0325\,(x-6)^3 + 7.7445 & \text{if } x \in [6, 10) \\ 0.0592\,(x-10)^3 - 0.4349\,(x-10)^2 - 0.5244\,x + 24.1342 & \text{if } x \in [10, 13) \\ 0.0976\,(x-13)^2 - 1.5365\,x + 0.0023\,(x-13)^3 + 34.9739 & \text{if } x \in [13, 17) \\ 0.1247\,(x-17)^2 - 0.6473\,x - 0.0111\,(x-17)^3 + 21.5644 & \text{if } x \in [17, 20) \\ 0.0245\,(x-20)^2 - 0.1996\,x - 0.0010\,(x-20)^3 + 13.4311 & \text{if } x \in [20, 28] \end{cases}$$

```
figure
hold on
fplot(P_1)
fplot(P_2)
xlim([0 28])
legend("P_1", "P_2", "location", "best")
```




```
x_sol_1 = vpasolve(diff(a_1(3) + b_1(3)*(x - 10) + c_1(3)*(x - 10)^2 + d_1(3)*(x -
10)^3) == 0, [10, 13])
```

x_sol_1 = 10.5534

```
x_sol_2 = vpasolve(diff(a_2(2) + b_2(2)*(x - 6) + c_2(2)*(x - 6)^2 + d_2(2)*(x -
6)^3) == 0, [6, 10])
```

x_sol_2 = 9.3496

```
subs(a_1(3) + b_1(3)*(x - 10) + c_1(3)*(x - 10)^2 + d_1(3)*(x - 10)^3, x, x_sol_1)
```

ans = 43.2302

```
subs(a_2(2) + b_2(2)*(x - 6) + c_2(2)*(x - 6)^2 + d_2(2)*(x - 6)^3, x, x_sol_2)
```

ans = 19.0561

Problem 8 (3)

```
clear; clc;

syms x

x_i = [1, 1.1, 1.3, 1.5, 1.9, 2.1];
y_i = [1.84, 1.96, 2.21, 2.45, 2.94, 3.18];

syms a_0 a_1

eq1 = a_0*length(x_i) + a_1*sum(x_i) == sum(y_i);
eq2 = a_0*sum(x_i) + a_1*sum(x_i.^2) == sum(x_i.*y_i);
[a_0_sol, a_1_sol] = solve([eq1, eq2], [a_0, a_1]);
P_1(x) = a_1_sol*x + a_0_sol;
vpa(simplify(P_1), 4)
```

ans(x) = $1.2196\,x + 0.6209$

```
syms a_2

eq1 = a_0*length(x_i) + a_1*sum(x_i) + a_2*sum(x_i.^2) == sum(y_i);
eq2 = a_0*sum(x_i) + a_1*sum(x_i.^2) + a_2*sum(x_i.^3) == sum(x_i.*y_i);
eq3 = a_0*sum(x_i.^2) + a_1*sum(x_i.^3) + a_2*sum(x_i.^4) == sum(x_i.^2.*y_i);
[a_0_sol, a_1_sol, a_2_sol] = solve([eq1, eq2, eq3], [a_0, a_1, a_2]);
P_2(x) = a_2_sol*x^2 + a_1_sol*x + a_0_sol;
vpa(simplify(P_2), 4)
```

ans(x) = $-0.0109\,x^2 + 1.2533\,x + 0.5966$

```
syms a_3

eq1 = a_0*length(x_i) + a_1*sum(x_i) + a_2*sum(x_i.^2) + a_3*sum(x_i.^3) ==
sum(y_i);
eq2 = a_0*sum(x_i) + a_1*sum(x_i.^2) + a_2*sum(x_i.^3) + a_3*sum(x_i.^4) ==
sum(x_i.*y_i);
eq3 = a_0*sum(x_i.^2) + a_1*sum(x_i.^3) + a_2*sum(x_i.^4) + a_3*sum(x_i.^5) ==
sum(x_i.^2.*y_i);
eq4 = a_0*sum(x_i.^3) + a_1*sum(x_i.^4) + a_2*sum(x_i.^5) + a_3*sum(x_i.^6) ==
sum(x_i.^3.*y_i);
[a_0_sol, a_1_sol, a_2_sol, a_3_sol] = solve([eq1, eq2, eq3, eq4], [a_0, a_1, a_2,
a_3]);
```

```
P_3(x) = a_3_sol*x^3 + a_2_sol*x^2 + a_1_sol*x + a_0_sol;
vpa(simplify(P_3), 4)
```

ans(x) = $-0.0100\,x^3 + 0.0353\,x^2 + 1.1850\,x + 0.6290$

```
error_1 = vpa(sum((y_i - P_1(x_i)).^2), 5)
```

error_1 = 2.7194e-05

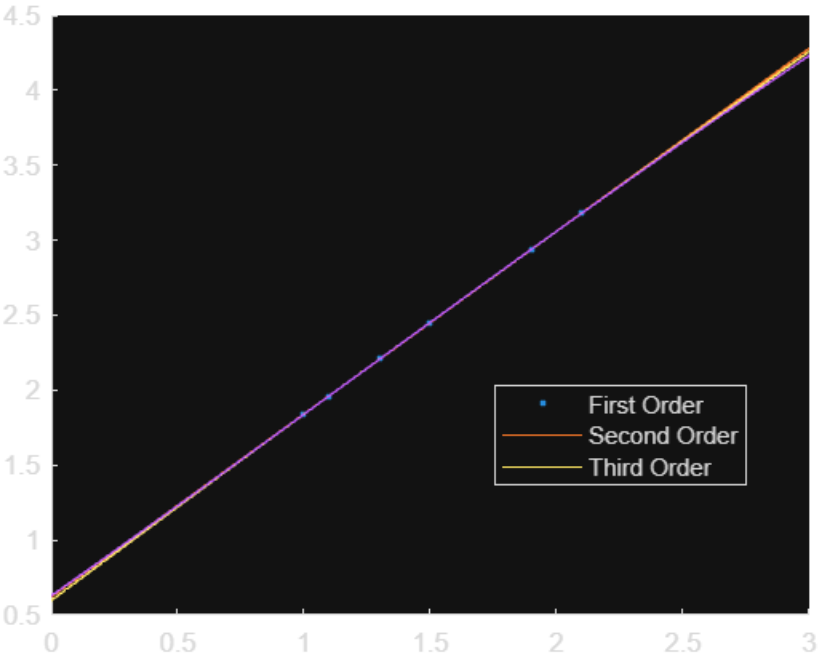
```
error_2 = vpa(sum((y_i - P_2(x_i)).^2), 5)
```

error_2 = 1.8015e-05

```
error_3 = vpa(sum((y_i - P_3(x_i)).^2), 5)
```

error_3 = 1.7407e-05

```
figure
hold on
plot(x_i, y_i, ".")
fplot(P_1)
fplot(P_2)
fplot(P_3)
xlim([0, 3])
legend("First Order", "Second Order", "Third Order", "Location", "best")
```



Probme 9 (1b, 3b, 5b)

```
clear; clc;

syms x

x_int = [0, 2];
f(x) = x^3;

syms a_0 a_1
eq1 = a_0*int(1, x, x_int(1), x_int(end)) + a_1*int(x, x, x_int(1), x_int(end))
==...
    int(f, x, x_int(1), x_int(end));
eq2 = a_0*int(x, x, x_int(1), x_int(end)) + a_1*int(x^2, x, x_int(1), x_int(end))
==...
    int(x*f, x, x_int(1), x_int(end));
[a_0_sol, a_1_sol] = solve([eq1, eq2], [a_0, a_1]);
P_1(x) = a_1_sol*x + a_0_sol;
vpa(simplify(P_1), 4)
```

ans(x) = $3.6000\,x - 1.6000$

```
syms a_2
eq1 = a_0*int(1, x, x_int(1), x_int(end)) + a_1*int(x, x, x_int(1), x_int(end)) +...
    a_2*int(x^2, x, x_int(1), x_int(end)) == int(f, x, x_int(1), x_int(end));
eq2 = a_0*int(x, x, x_int(1), x_int(end)) + a_1*int(x^2, x, x_int(1), x_int(end))
+...
    a_2*int(x^3, x, x_int(1), x_int(end)) == int(f*x, x, x_int(1), x_int(end));
eq3 = a_0*int(x^2, x, x_int(1), x_int(end)) + a_1*int(x^3, x, x_int(1), x_int(end))
+...
    a_2*int(x^4, x, x_int(1), x_int(end)) == int(f*x^2, x, x_int(1), x_int(end));
[a_0_sol, a_1_sol, a_2_sol] = solve([eq1, eq2, eq3], [a_0, a_1, a_2]);
P_2(x) = a_2_sol*x^2 + a_1_sol*x + a_0_sol;
vpa(simplify(P_2), 4)
```

ans(x) = $3\,x^2 - 2.4000\,x + 0.4000$

```
error_1 = vpa(int((f - P_1).^2, x, x_int(1), x_int(end)), 5)
```

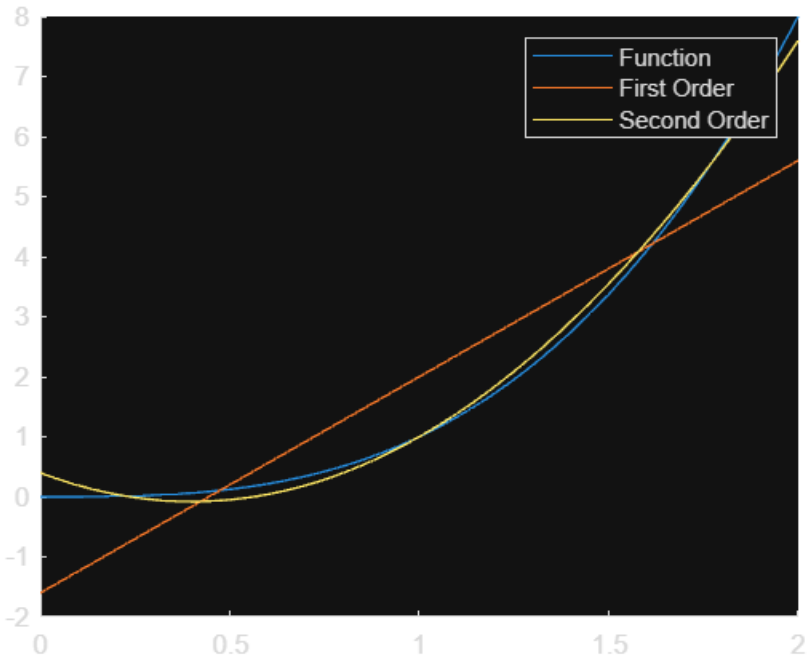
error_1 = 1.6457

```
error_2 = vpa(int((f - P_2).^2, x, x_int(1), x_int(end)), 5)
```

error_2 = 0.0457

```
figure
hold on
fplot(f)
fplot(P_1)
fplot(P_2)
```

```
xlim([x_int(1), x_int(end)])
legend("Function", "First Order", "Second Order")
```



Problem 10 (2a)

```
clear; clc;

syms x

f(x) = exp(x);

x_vec = [-1, 1, 0, sqrt(2)/2, -sqrt(2)/2, sqrt(3)/2, -sqrt(3)/2, sqrt(2 +
sqrt(2))/2, sqrt(2 - sqrt(2))/2, -sqrt(2 + sqrt(2))/2, -sqrt(2 - sqrt(2))/2]';
x_vec = sort(x_vec);
f_vec = f(x_vec);
f_diff_vec = f_vec;

P_Lagrange = LagrangePolynomial(f_vec, x_vec);
vpa(expand(P_Lagrange), 4)
```

ans = $2.8376\text{e-}07\,x^{10} + 2.8390\text{e-}06\,x^{9} + 2.4794\text{e-}05\,x^{8} + 1.9831\text{e-}04\,x^{7} + 0.0014\,x^{6} + 0.0083\,x^{5} + 0.0417\,x^{4} + 0.$

```

F = sym(HermiteMethod(f_diff_vec, f_vec, x_vec));
P_Hermite(x) = F(1, 2);
for i = 2:length(F)-1
    G(x) = x/x;
    for j = 1:i-1
        G(x) = G(x)*(x - F(j,1));
```

```
    end
    P_Hermite(x) = P_Hermite(x) + F(i, i+1)*G(x);
end
vpa(expand(P_Hermite), 4)
```

ans(x) = $-1.0265\text{e-}11\,x^{21} - 4.0858\text{e-}11\,x^{20} + 5.8513\text{e-}11\,x^{19} + 2.3250\text{e-}10\,x^{18} - 1.4328\text{e-}10\,x^{17} - 5.6961\text{e-}10\,x^{16} +$

```
[a, b, c, d] = naturalSpline(f_vec, x_vec);
funcs = cell(1, length(x_vec)-1);
for i = 1:length(x_vec)-1
    poly(x) = a(i) + b(i)*(x - x_vec(i)) + c(i)*(x - x_vec(i))^2 + d(i)*(x -
x_vec(i))^3;
    funcs{i} = poly(x);
end
P_Spline = piecewise(x >= x_vec(1) & x <= x_vec(2), funcs{1},...
    x >= x_vec(2) & x <= x_vec(3), funcs{2},...
    x >= x_vec(3) & x <= x_vec(4), funcs{3},...
    x >= x_vec(4) & x <= x_vec(5), funcs{4},...
    x >= x_vec(5) & x <= x_vec(6), funcs{5},...
    x >= x_vec(6) & x <= x_vec(7), funcs{6},...
    x >= x_vec(7) & x <= x_vec(8), funcs{7},...
    x >= x_vec(8) & x <= x_vec(9), funcs{8},...
    x >= x_vec(9) & x <= x_vec(10), funcs{9},...
    x >= x_vec(10) & x <= x_vec(11), funcs{10});
vpa(expand(P_Spline), 4)
```

ans =

$$\begin{cases} 1.1048\,x^3 + 3.3144\,x^2 + 3.6902\,x + 1.8485 & \text{if } x \in [-1, -0.9239] \\ -0.2862\,x^3 - 0.5411\,x^2 + 0.1282\,x + 0.7516 & \text{if } x \in [-0.9239, -0.8660] \\ 0.0920\,x^3 + 0.4416\,x^2 + 0.9793\,x + 0.9972 & \text{if } x \in [-0.8660, -0.7071] \\ 0.0927\,x^3 + 0.4432\,x^2 + 0.9804\,x + 0.9975 & \text{if } x \in [-0.7071, -0.3827] \\ 0.1374\,x^3 + 0.4944\,x^2 + 0.9999\,x + 1 & \text{if } x \in [-0.3827, 0] \\ 0.1986\,x^3 + 0.4944\,x^2 + 0.9999\,x + 1 & \text{if } x \in [0, 0.3827] \\ 0.3039\,x^3 + 0.3735\,x^2 + 1.0463\,x + 0.9941 & \text{if } x \in [0.3827, 0.7071] \\ 0.2375\,x^3 + 0.5145\,x^2 + 0.9466\,x + 1.0176 & \text{if } x \in [0.7071, 0.8660] \\ 3.0295\,x^3 - 6.7396\,x^2 + 7.2287\,x - 0.7959 & \text{if } x \in [0.8660, 0.9239] \\ -7.2571\,x^3 + 21.7714\,x^2 - 19.1119\,x + 7.3160 & \text{if } x \in [0.9239, 1] \end{cases}$$

```
Lagrange_int = int(P_Lagrange, x, -1, 1);
Hermite_int = int(P_Hermite, x, -1, 1);
Spline_int = int(P_Spline, x, -1, 1);
Actual_int = int(f, x, -1, 1);

vpa(Lagrange_int, 16)
```

```
ans = 2.3504
```

```
vpa(Hermite_int, 16)
```

```
ans = 2.3504
```

```
vpa(Spline_int, 16)
```

```
ans = 2.3504
```

```
vpa(Actual_int, 16)
```

```
ans = 2.3504
```

```
rel_err_L = abs(Actual_int - Lagrange_int)
```

```
rel_err_L = 8.6541e-14
```

```
rel_err_H = abs(Actual_int - Hermite_int)
```

```
rel_err_H = 7.6395e-16
```

```
rel_err_S = abs(Actual_int - Hermite_int)
```

```
rel_err_S = 7.6395e-16
```