

Homework 5

Problem 2 (10a, 10c)

```
clear; clc;

syms t y

f(t, y) = 1/t^2 - y/t - y^2;
g(t) = -1/t;
a = 1;
b = 2;
N = 20;
alpha = -1;

[t, w] = EulerMethod(f, a, b, N, alpha);

fprintf("%-15s%-15s%-15s%-15s\n", "t_i", "w_i", "y_i", "|y_i-w_i|")
```

| t_i | w_i | y_i | y_i-w_i |
|-----|-----|-----|---------|
|-----|-----|-----|---------|

```
for i = 1:length(t)

    fprintf("%-15.7g%-15.7g%-15.7g%-15.7g\n", t(i), w(i), g(t(i)), abs(g(t(i)) -
w(i)))

end
```

| | | | |
|------|------------|------------|-------------|
| 1 | -1 | -1 | 0 |
| 1.05 | -0.95 | -0.952381 | 0.002380952 |
| 1.1 | -0.9045354 | -0.9090909 | 0.004555478 |
| 1.15 | -0.8630071 | -0.8695652 | 0.00655813 |
| 1.2 | -0.8249169 | -0.8333333 | 0.008416415 |
| 1.25 | -0.7898476 | -0.8 | 0.01015245 |
| 1.3 | -0.7574466 | -0.7692308 | 0.01178416 |
| 1.35 | -0.7274145 | -0.7407407 | 0.01332622 |
| 1.4 | -0.699495 | -0.7142857 | 0.01479072 |
| 1.45 | -0.6734675 | -0.6896552 | 0.01618769 |
| 1.5 | -0.6491412 | -0.6666667 | 0.01752549 |
| 1.55 | -0.6263501 | -0.6451613 | 0.01881116 |
| 1.6 | -0.6049494 | -0.625 | 0.02005064 |
| 1.65 | -0.5848116 | -0.6060606 | 0.02124898 |
| 1.7 | -0.5658248 | -0.5882353 | 0.02241047 |
| 1.75 | -0.5478898 | -0.5714286 | 0.02353881 |
| 1.8 | -0.5309184 | -0.5555556 | 0.02463716 |
| 1.85 | -0.5148323 | -0.5405405 | 0.02570826 |
| 1.9 | -0.4995613 | -0.5263158 | 0.02675448 |
| 1.95 | -0.4850426 | -0.5128205 | 0.0277779 |
| 2 | -0.4712197 | -0.5 | 0.0287803 |

```
diff(g, 2)
```

```
ans(t) =
```

$$-\frac{2}{t^3}$$

```
syms h
```

```
L = 1
```

```
L = 1
```

```
M = 2
```

```
M = 2
```

```
eq = 0.05 == (h*M)/(2*L)*(exp(L*(t(end) - a)) - 1);
h_sol = solve(eq, h)
```

```
h_sol = 0.0291
```

Problem 4

```
clear; clc;
```

```
syms t y
```

```
f(t, y) = -5*y + 5*t^2 + 2*t
```

$$f(t, y) = 5t^2 + 2t - 5y$$

```
g(t) = t^2 + (1/3)*exp(-5*t);
```

```
a = 0;
```

```
b = 1;
```

```
N = 10;
```

```
alpha = 1/3;
```

```
[t, w] = RK4(f, a, b, N, alpha);
```

```
fprintf("%-15s%-15s%-15s%-15s\n", "t_i", "w_i", "y_i", "|y_i-w_i|")
```

```
t_i          w_i          y_i          |y_i-w_i|
```

```
for i = 1:length(t)
```

```
    fprintf("%-15.7g%-15.7g%-15.7g%-15.7g\n", t(i), w(i), g(t(i)), abs(g(t(i)) - w(i)))
```

```
end
```

```
0          0.3333333    0.3333333    0
0.1        0.212283    0.2121769    0.0001060995
```

| | | | |
|-----|-----------|-----------|--------------|
| 0.2 | 0.1627655 | 0.1626265 | 0.0001389773 |
| 0.3 | 0.1645165 | 0.1643767 | 0.0001398207 |
| 0.4 | 0.2052405 | 0.2051118 | 0.0001287441 |
| 0.5 | 0.2774767 | 0.2773617 | 0.0001149945 |
| 0.6 | 0.3766981 | 0.3765957 | 0.0001023885 |
| 0.7 | 0.5001579 | 0.5000658 | 9.215388e-05 |
| 0.8 | 0.6461896 | 0.6461052 | 8.437549e-05 |
| 0.9 | 0.8137817 | 0.813703 | 7.870457e-05 |
| 1 | 1.002321 | 1.002246 | 7.468666e-05 |

Problem 5

```
clear; clc;
```

```
syms t y
```

```
f(t, y) = -5*y + 5*t^2 + 2*t
```

$$f(t, y) = 5t^2 + 2t - 5y$$

```
g(t) = t^2 + (1/3)*exp(-5*t);
```

```
a = 0;
```

```
b = 1;
```

```
N = 10;
```

```
alpha = 1/3;
```

```
[t, w_2step, w_3step, w_4step, w_5step] = AdamBashforthMethod(f, a, b, N, alpha);
```

```
w_step = [w_2step; w_3step; w_4step; w_5step]
```

```
w_step = 4x11
```

| | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|------------|
| 0.3333 | 0.2123 | 0.1739 | 0.1740 | 0.2145 | 0.2846 | 0.3823 | 0.5042 ... |
| 0.3333 | 0.2123 | 0.1628 | 0.1605 | 0.2026 | 0.2732 | 0.3747 | 0.4972 |
| 0.3333 | 0.2123 | 0.1628 | 0.1645 | 0.2066 | 0.2781 | 0.3788 | 0.4998 |
| 0.3333 | 0.2123 | 0.1628 | 0.1645 | 0.2052 | 0.2769 | 0.3765 | 0.4989 |

```
for j = 1:4
```

```
    w = w_step(j,:);
```

```
    fprintf("%-15s%-15s%-15s%-15s\n", "t_i", "w_i", "y_i", "|y_i-w_i|")
```

```
    for i = 1:length(t)
```

```
        fprintf("%-15.7g%-15.7g%-15.7g%-15.7g\n", t(i), w(i), g(t(i)), abs(g(t(i))
- w(i)))
```

```
    end
```

```
end
```

| t_i | w_i | y_i | y_i-w_i |
|-----|-----------|-----------|--------------|
| 0 | 0.3333333 | 0.3333333 | 0 |
| 0.1 | 0.212283 | 0.2121769 | 0.0001060995 |
| 0.2 | 0.1739041 | 0.1626265 | 0.0112776 |
| 0.3 | 0.1740468 | 0.1643767 | 0.009670046 |
| 0.4 | 0.2144877 | 0.2051118 | 0.009375951 |
| 0.5 | 0.2846336 | 0.2773617 | 0.007271953 |
| 0.6 | 0.3822803 | 0.3765957 | 0.005684643 |

| | | | |
|-----|-----------|-----------|--------------|
| 0.7 | 0.5042285 | 0.5000658 | 0.004162694 |
| 0.8 | 0.6491272 | 0.6461052 | 0.003021992 |
| 0.9 | 0.8158389 | 0.813703 | 0.002135924 |
| 1 | 1.003742 | 1.002246 | 0.00149555 |
| t_i | w_i | y_i | y_i-w_i |
| 0 | 0.3333333 | 0.3333333 | 0 |
| 0.1 | 0.212283 | 0.2121769 | 0.0001060995 |
| 0.2 | 0.1627655 | 0.1626265 | 0.0001389773 |
| 0.3 | 0.1605261 | 0.1643767 | 0.003850613 |
| 0.4 | 0.2026399 | 0.2051118 | 0.002471824 |
| 0.5 | 0.2732179 | 0.2773617 | 0.004143734 |
| 0.6 | 0.3747011 | 0.3765957 | 0.00189459 |
| 0.7 | 0.4972078 | 0.5000658 | 0.002857948 |
| 0.8 | 0.645264 | 0.6461052 | 0.0008412219 |
| 0.9 | 0.8119618 | 0.813703 | 0.001741164 |
| 1 | 1.002089 | 1.002246 | 0.0001565466 |
| t_i | w_i | y_i | y_i-w_i |
| 0 | 0.3333333 | 0.3333333 | 0 |
| 0.1 | 0.212283 | 0.2121769 | 0.0001060995 |
| 0.2 | 0.1627655 | 0.1626265 | 0.0001389773 |
| 0.3 | 0.1645165 | 0.1643767 | 0.0001398207 |
| 0.4 | 0.2066057 | 0.2051118 | 0.001493983 |
| 0.5 | 0.2780929 | 0.2773617 | 0.0007312636 |
| 0.6 | 0.378768 | 0.3765957 | 0.002172342 |
| 0.7 | 0.4998405 | 0.5000658 | 0.0002253161 |
| 0.8 | 0.6487176 | 0.6461052 | 0.002612367 |
| 0.9 | 0.8116247 | 0.813703 | 0.002078325 |
| 1 | 1.006412 | 1.002246 | 0.004166082 |
| t_i | w_i | y_i | y_i-w_i |
| 0 | 0.3333333 | 0.3333333 | 0 |
| 0.1 | 0.212283 | 0.2121769 | 0.0001060995 |
| 0.2 | 0.1627655 | 0.1626265 | 0.0001389773 |
| 0.3 | 0.1645165 | 0.1643767 | 0.0001398207 |
| 0.4 | 0.2052405 | 0.2051118 | 0.0001287441 |
| 0.5 | 0.2769031 | 0.2773617 | 0.0004585888 |
| 0.6 | 0.3765206 | 0.3765957 | 7.503962e-05 |
| 0.7 | 0.4988777 | 0.5000658 | 0.001188087 |
| 0.8 | 0.6471458 | 0.6461052 | 0.001040593 |
| 0.9 | 0.8107178 | 0.813703 | 0.002985168 |
| 1 | 1.007335 | 1.002246 | 0.005088809 |

```
[t, w_4step] = AdamBashforthPredictorMethod(f, a, b, N, alpha);

w = w_4step;

fprintf("%-15s%-15s%-15s%-15s\n", "t_i", "w_i", "y_i", "|y_i-w_i|")
```

| t_i | w_i | y_i | y_i-w_i |
|-----|-----|-----|---------|
|-----|-----|-----|---------|

```
for i = 1:length(t)

    fprintf("%-15.7g%-15.7g%-15.7g%-15.7g\n", t(i), w(i), g(t(i)), abs(g(t(i)) -
w(i)))

end
```

| | | | |
|-----|-----------|-----------|--------------|
| 0 | 0.3333333 | 0.3333333 | 0 |
| 0.1 | 0.212283 | 0.2121769 | 0.0001060995 |

| | | | |
|-----|-----------|-----------|--------------|
| 0.2 | 0.1627655 | 0.1626265 | 0.0001389773 |
| 0.3 | 0.1645165 | 0.1643767 | 0.0001398207 |
| 0.4 | 0.2048557 | 0.2051118 | 0.0002560885 |
| 0.5 | 0.2769896 | 0.2773617 | 0.0003721159 |
| 0.6 | 0.3762804 | 0.3765957 | 0.0003153045 |
| 0.7 | 0.4998012 | 0.5000658 | 0.0002645671 |
| 0.8 | 0.6458949 | 0.6461052 | 0.0002102884 |
| 0.9 | 0.8135498 | 0.813703 | 0.0001532097 |
| 1 | 1.002137 | 1.002246 | 0.0001087606 |

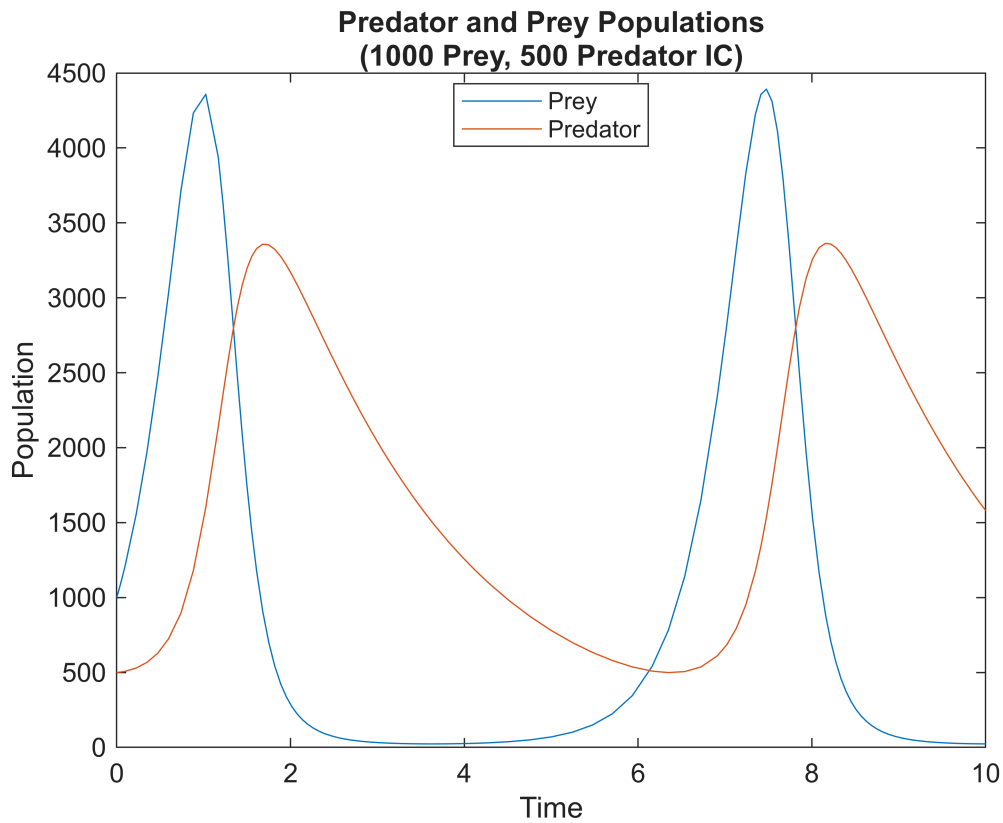
Problem 6

```
clear; clc;

k1 = 3;
k2 = 0.002;
k3 = 0.0006;
k4 = 0.5;
tspan = [0 10];
x0 = [1000 500];

[t, x] = ode45(@(t,x) odefcn(t, x, k1, k2, k3, k4), tspan, x0);

figure
plot(t, x(:,1), t, x(:,2))
xlabel("Time")
ylabel("Population")
legend("Prey", "Predator", "Location", "best")
title(["Predator and Prey Populations", "(1000 Prey, 500 Predator IC)"])
```



```
syms x1 x2
```

```
eq1 = 0 == k1*x1 - k2*x1*x2;
```

```
eq2 = 0 == k3*x1*x2 - k4*x2;
```

```
[x1_sol, x2_sol] = solve([eq1, eq2], [x1, x2])
```

```
x1_sol =
```

```
( 0
 833.3333)
```

```
x2_sol =
```

```
( 0
1500)
```

```
x0 = [1 0];
```

```
[t, x] = ode45(@(t,x) odefcn(t, x, k1, k2, k3, k4), tspan, x0);
```

```
figure
```

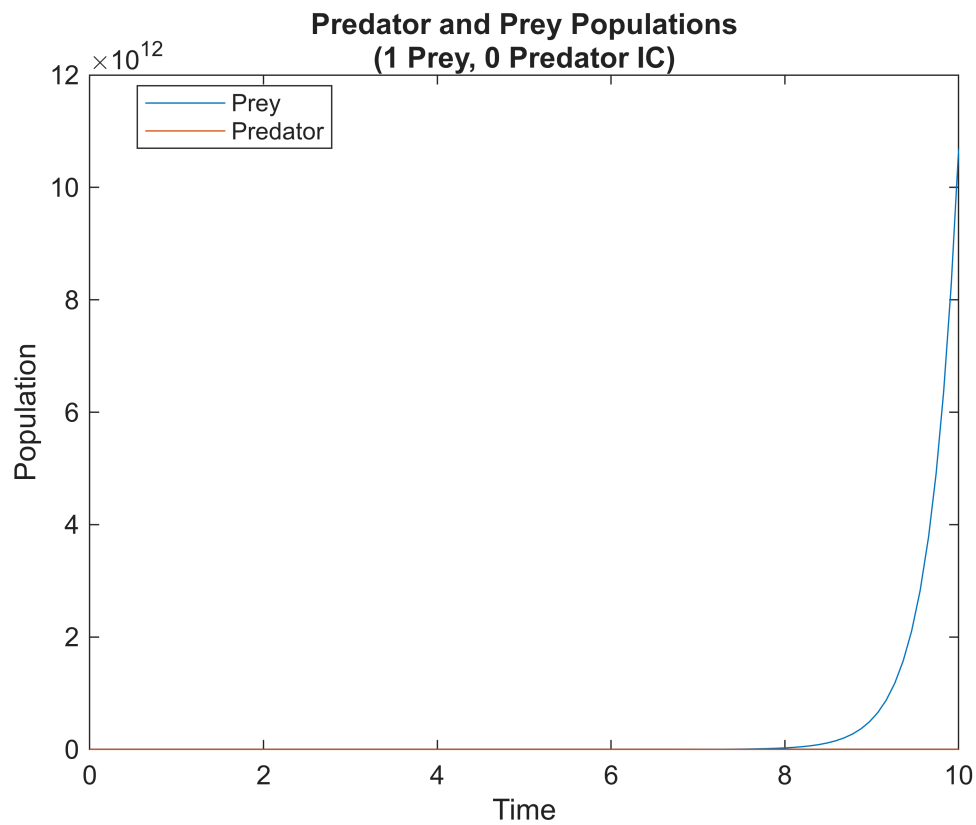
```
plot(t, x(:,1), t, x(:,2))
```

```
xlabel("Time")
```

```
ylabel("Population")
```

```
legend("Prey", "Predator", "Location", "best")
```

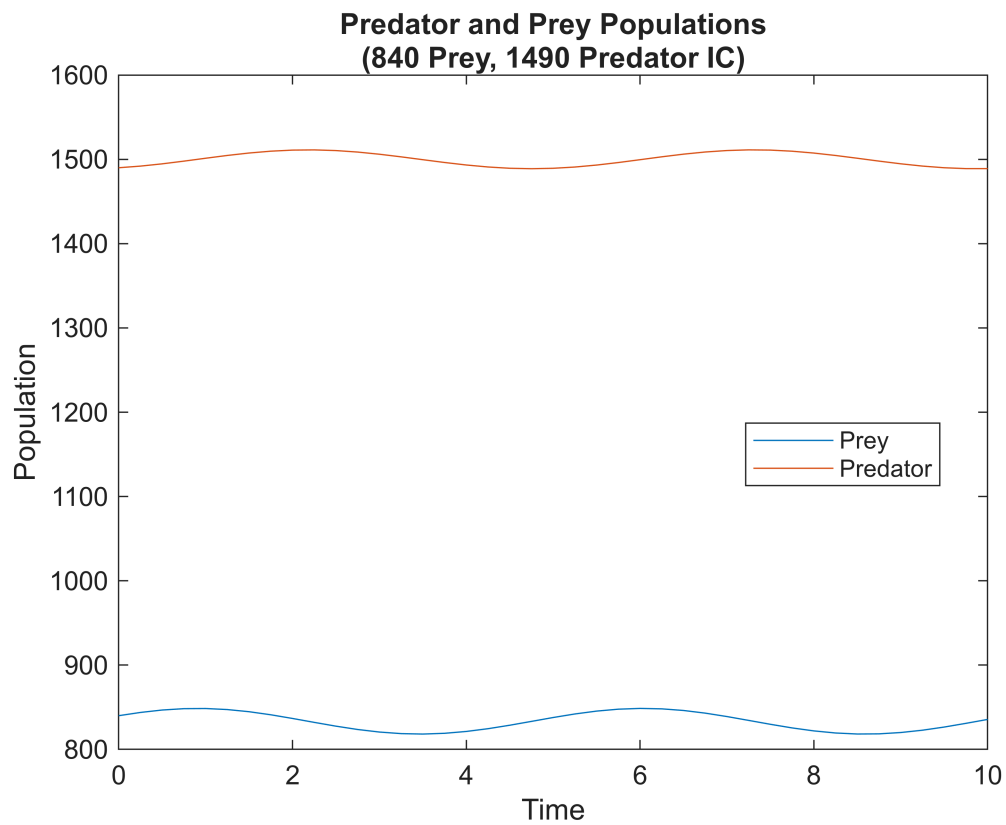
```
title(["Predator and Prey Populations", "(1 Prey, 0 Predator IC)"])
```



```
x0 = [840 1490];

[t, x] = ode45(@(t,x) odefcn(t, x, k1, k2, k3, k4), tspan, x0);

figure
plot(t, x(:,1), t, x(:,2))
xlabel("Time")
ylabel("Population")
legend("Prey", "Predator", "Location", "best")
title(["Predator and Prey Populations", "(840 Prey, 1490 Predator IC)"])
```



```
function dxdt = odefcn(t, x, k1, k2, k3, k4)

    dxdt = zeros(2,1);
    dxdt(1) = k1*x(1) - k2*x(1)*x(2);
    dxdt(2) = k3*x(1)*x(2) - k4*x(2);

end
```