# Graphs (Part 1)

Definition of a graphs, different types of graphs and their properties

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## Graphs

- A graph is defined as the set $G = (V, E)$, where
  - $V$ is a set of vertices or nodes
  - $E$ is a set of edges, where each edge is pair $(v, w)$ and $v, w \in V$
- We checked trees before
  - Trees are DAGs (directed acyclic graphs)
  - General graphs usually have edges with more features

## Edge Features

- Edge direction
  - Directed graphs: $(u, v)$ and $(v, u)$ are not the same
- Edge weight
  - There can be a cost associated with the edge. When we think of edges as tuples, we can think of the cost $c$ as making a coordinate pair a triplet $(u, v, c)$.

## Types of Graphs

- Directed vs. Undirected
  - Presence or absence of weighted edges
- Weighted vs. Unweighted
  - Presence or absence of edge weights
- Cyclic vs. Acyclic
  - Presence or absence of cycles
- Dense vs. Sparse
  - Presence or absence of a large number of edges relative to the number of vertices
- Connected vs. Disconnected
  - Presence or absence of the property that there is a path from every vertex to every other vertex (when a graph has this property, we call it *strongly connected*)

## Dense/Sparse Graphs

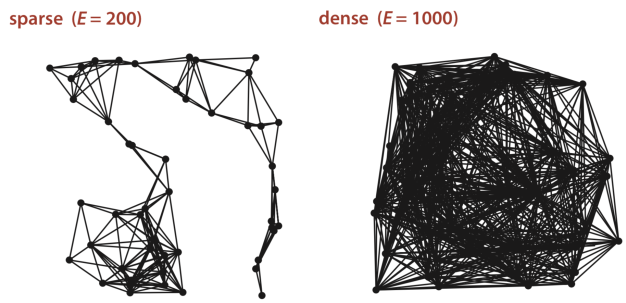


**Figure 1:** A visual comparison between a sparse and dense graph $(V = 50)$ Sedgewick et. al's Algorithms, 4th ed. Chp. 4.1

## Terms

- Nodes (also called vertices when specifically talking about graphs)
  - $|V|$ is the number of vertices
- Edges
  - $|E|$ is the number of edges
  - It is always true that $|E| \leq |V|^2$
- Connection
  - Vertex $w$ is adjacent to vertex $v$ iff. $(v, w) \in E$
    * *Directed graph*: $w$ is adjacent to $v \neq v$ is adjacent to $w$
  - A *path* is a sequence of vertices connected by edges
    * A sequence of edges $w_1, w_2, \ldots, w_n$ is a path iff. $(w_i, w_{i+1}) \in E$ for all $i$
    * *Path length*: the length of a path is defined by the number of edges divided by the sum of the edge weights

## Graph Example

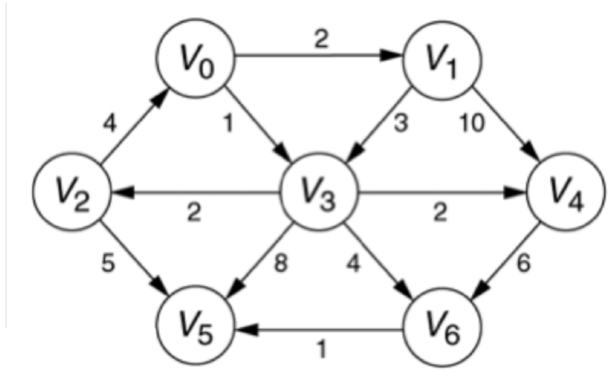


**Figure 2:** Example 1

- Directed, weighted graph
- $|V| = 7$
- $|E| = 12$
- Which vertices are adjacent to $V_3$?
  - $V_2, V_4, V_5, V_6$
- List a path from $V_3$ to $V_1$
  - $V_3, V_2, V_0, V_1$
  - Unweighted path length: 3
  - Weighted path length: 2 + 4 + 2 = 8

## Review: Graphs

### General Graph Terms

- $|V|$ and $|E|$
- Adjacent vertices
- Simple path
- Cycle
- Degree (of a vertex)

### Types of Graphs

- Directed vs. Undirected

- Weighted vs. Unweighted
- Cyclic vs. Acyclic
- Dense vs. Sparse
- Connected vs. Disconnected
- DAG

## More Terms

- *Simple path*: a path where all the vertices are distinct except the first and last, which can be the same
- *Cycle*: a path that begins and ends at the same vertex and contains at least one edge
  - Simple cycle follows from the definition of simple path above
- Vertex $v$ is reachable from vertex $2$ if there is a path from $w$ to $v$
- *Degree of a vertex*: the number of edges incident to it
  - *Indegree of* $v$: the number of incoming edges $(u, v)$
  - *Outdegree of* $v$: the number of outgoing edges $(v, w)$

## Graph Example

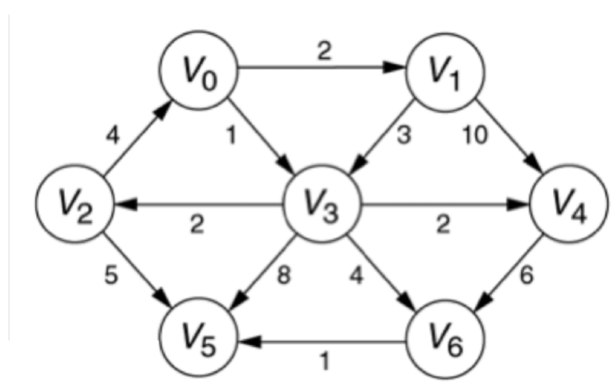


**Figure 3:** Example 2

- List a path
  - $V_3, V_2, V_0, V_3, V_6, V_5$
- List a simple path
  - $V_3, V_2, V_0, V_1, V_3$
- List a cycle

    - $V_3, V_2, V_0, V_3, V_2, V_0, V_3$
- List a simple cycle
    - $V_3, V_2, V_0, V_1, V_3$
- Is $V_0$ reachable from $V_5$?
    - No it is not ($V_5$ has outdegree zero)
- Is $V_0$ reachable from $V_1$?
    - Yes, we have the path $V_1, V_3, V_2, V_0$
- Degree of $V_3$: 6
    - Indegree: 2
    - Outdegree: 4

## Graphs Everywhere

- They can represent a wide variety of data or relations
    - Genetic distances
    - Airline flights and costs
    - Migration patterns
    - Function call graphs

## Road Map

- Graph basics
    - Definitions and terms
    - Applications
- Graph representations
    - Adjacency matrix
    - Adjacency list
- Graph algorithms
    - Graph traversal
    - Shortest path problem
    - Many more