CS 310: Hashing (Part II)

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Review: Hash Table Basics

- Store objects in an array
- Use the length of the hash table in the hashing function to get a valid index

$$x_{\text{index}} = |xhc| \bmod \texttt{hta.length}$$

- `.add(T t)`: put the object `t` in the hash table
  - `hta[xindex] = x;`
- `.has(T t)`: check if `t` is in the hash table
  - `return x.equals(hta[xindex]);`
- `.remove(T t)`: delete `t` from the hash table
  - `if has(t) { hta[xindex] = null; }`
- **What's the Big-O?**

Hash Table Collision

- Motivation
  - Put `t` in the table at `hta[xindex]`
  - Problem: What if `hta[xindex]` is occupied?
  - Answer: find some other place to store `t`
- Common approaches
  - Separate chaining
  - Open addressing

Separate Chaining

- If something's already there:
  - Expand that single entry to an internal data structure
    * One which can ideally accommodate multiple objects of the same hash code
    * It should be able to grow if there are additional objects that need to be stored there

Collision Resolution

- Separate chaining: expand the single array entry into a linked list
  - Compute the integer hash code
  - "Bound" to make it a good index (just mod by the length of the table)
  - Find the list to operate on: `list = hta[xindex]`
- `.add(T t)`: put the object `t` in the hash table
  - `list.add(t);`
- `.has(T t)`: check if `t` is in the list
  - `return list.contains(t);`
- `.remove(T t)`: delete `t` from the list
  - `list.remove(t);`

Separate Chaining Analysis

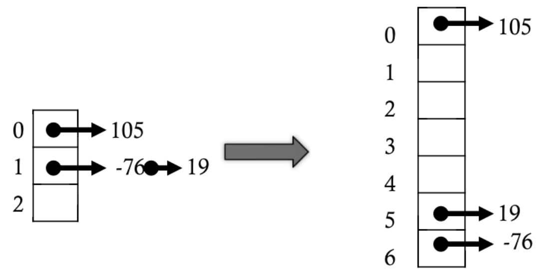
- `.add(T t)` is $O(1)$ assuming adding to the list is $O(1)$ (but that's only if duplicates are allowed)
- `.remove(T t)` and `.contains(T t)`
  - Run time depends on the number of things in each list
  - They (potentially) look through every element in the longest chain to search for `t`
    - The average case is equivalent to the number of items divided by the length of the table, which yields $O(\text{load})$
  - Worst case?
    - All the elements map to the same index: $O(\text{number of items})$
    - **How do we avoid the worst case?**

Rehashing

- The load is the number of items divided by the length of the hash table
  - A high load means a long average chain length, and a high average chain length means longer runtimes for `.add(T t)`, `.remove(T t)`, and `.has(T t)`
- Rehashing when the load is to high helps us with our average access time
  - Allows us to use a bigger array, a new hash function, and get a lower load
- Basic idea:
  - Allocate a new, larger array (the size should still be prime)
  - Copy over all the items to the new array: this *does* involve re-calculating hash-values for everything and inserting them into the new hash-table

Rehashing Example

- Invoking the following:
  - `.add(105) // 105 % 3 == 0`
  - `.add(19) // 19 % 3 == 1`
  - `.add(-76) // 76 % 3 == 1`
- Rehash when our load > 0.75
- Increase the size to the next prime that is more than double the current size
- Copy the items over, recalculating the hash using the new size of the array
- Load goes from $1 \mapsto 3/7$



Hash Table Overview

- *Separate Chaining*: expand the single array entry into a linked list
  - Compute the integer hash code from the element
  - Bound the values to allow us to easily calculate the index
  - Find the list to operate on
- `.add(T t)`: put `t` in the hash-table
  - Add the item with `list.add(t);`
  - Rehash the entire table if the load is too high
- `.has(T t)`: check if the element is already in the list
  - `return list.contains(t);`
- `.remove(T t)`: delete the element from the list
  - `list.remove(t);`

Big-O Analysis

	`.add()`*	`.has()`	`.remove()`	iteration
Best	$O(1)$	$O(1)$	$O(1)$	$O(n+m)$
Average	$O(n/m)$	$O(n/m)$	$O(n/m)$	$O(n+m)$
Worst	$O(n)$	$O(n)$	$O(n)$	$O(n+m)$

- Hash table with separate chaining (*assuming no duplicates are allowed, and not considering rehashing overhead)
  - The load is n/m
  - n is the number of values in the hash-table
  - m is the number of entries (array capacity)

Separate Chaining is Viable in Practice

- Simple to implement
  - Weiss Figure 20.20
- Reasonably efficient
- A "chain" can be implemented as a different data structure
  - We can use trees, an `ArrayList`, or others
  - Binary search trees can be used if there are no duplicates allowed
- Java's built-in hash tables use separate chaining
  - `java.util.HashSet`, `java.util.HashMap`, and `java.util.Hashtable` all use separate chaining
  - `java.util.HashMap` uses red-black trees when the number of values in one chain is more than eight

Hash Code Review

- We define a hash function to map any object to a manageable integer as its hash code
  - `.hashCode()` defined in `java.lang.Object`
- Hash contract: objects that are identical must have the same hash code
- We need hash functions that are easy to compute and distribute well
- Java provides implementations for built-in types

Hash Code in Java

- Built in for boxed and container types (`Integer`, `Integer[]`, etc.)
- The output is a 32-bit integer
- Straightforward for types with a size of no more than 32 bits
  - If that's true, it's trivial to map every unique value to a different integer
    - This works for our `Integer`, `Boolean`, and `Character` types
  - Types with a larger size use the following trick
    - Exclusive OR the two halves to reduce the reference from 8 bytes to 4 bytes
      - `(int) (this.longValue() ^ (this.longValue() >>> 32))` (that's the logical shift right)

Aggregate Type: String

- The hash code for a string object is computed by taking the β-expansion where the most-significant-place is the ASCII value of the first index of the string (and the base is 31)
  - It follows then that the hash value of the empty string is zero
- Discussion
  - Is 31 special?

    - * "The value 31 was chosen because it is an odd prime. If it were even and the multiplication overflowed, information would be lost, as multiplication by 2 is equivalent to shifting. The advantage of using a prime is less clear, but it is traditional. A nice property of 31 is that the multiplication can be replaced by a shift and a subtraction for better performance:
      `31 * i == (i << 5) - i`.
      Modern VMs do this sort of optimization automatically."
      (Taken from `https://stackoverflow.com/a/299748`)
    - * Getting more into the math, using a prime number for the modulo is important because (I think – my abstract algebra is weak to non-existent) the Integers modulo a prime number are a field, whereas with other non-prime numbers they are only a ring.
      This is also typically why we use addition and multiplication – the integers under modulo are closed under these operations.
      An added benefit of using a prime number for both hash function and the modulo is that they are *highly* unlikely to share factors in common. (Common factors in the hash function and the modulo result in more collisions.)
  - **Can you write code to compute the hash code faster instead of following the formula strictly?**
    - * I mean, probably. I like the trick with exclusive or for hashing references (which are typically 64 bits on most modern machines).

Polynomial Hash Code

- String uses a polynomial hash code

$$a_0\beta^{n-1} + \cdots + a_{n-1}\beta^0$$

  - Java uses $\beta = 31$ for strings
- Optimize
  - We can regroup a polynomial of any degree and reduce the power calculation to multiplication
  - Example of regrouping a degree 3 polynomial so that we don't have to use exponentiation:

$$a_0\beta^3 + a_1\beta^2 + a_2\beta + a_3$$

    becomes

$$(((a_0\beta)\beta + a_1)\beta + a_2)\beta + a_3$$

Aggregate Type Hashing

- Other aggregate types
  - Use the String approach: create a polynomial has code using each of the elements, computing each element's hash individually
- The poor man's strategy: `x.toString().hashCode()`

Object Hash Code Example

```java
// Composite hashCode from all attributes
public class Student {
    private String name;
    private int age;
    private double grade;
    ...
    // Note: Default is memory address, not proper hashCode!!!
```

```java
    @Override
    public int hashCode( ) {
        int hash = 17; // pick prime constants
        hash = 31 * hash + name.hashCode();
        hash = 31 * hash + ((Integer) age).hashCode();
        hash = 31 * hash + ((Double) grade).hashCode();
        return hash;
    }
}
```