STAT 344: Chapter 2.1

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August 2018

Definition (Random experiment)**.** An experiment that can result in different outcomes, even if it is performed the same way every time.

Definition (Sample Space)**.** The set of all possible outcomes of a random experiment, which is denoted by an S.

Definition (Discrete Sample Space)**.** A sample space in which the number of elements are finite or countably infinite. That is, $|S| \leq |\mathbb{N}|$.

Definition (Continous Sample Space)**.** A sample space in which the number of elements are uncountably infinite. That is, $|S| = |\mathbb{R}|$.

Definition (Event)**.** A subset of the sample space of a random experiment.

Definition (Union)**.** The event that consists of all outcomes that are contained in either of two events E_1 and E_2, denoted $E_1 \cup E_2$.

Definition (Intersection)**.** The event that consists of outcomes that are contained in both of two events E_1 and E_2, denoted $E_1 \cap E_2$.

Definition (Complement)**.** The set of outcomes in the sample space that are not in the event E, denoted E'.

Remark. Events are used to define outcomes of interest from a random experiment. One is often interested in the probabilities of specified events.

Definition (Mutually Exclusive Events)**.** Two events E_1 and E_2 are said to be mutually exclusive if and only if

$$E_1 \cap E_2 = \emptyset.$$

Definition (Complement (Double Negation))**.** By the definition of the complement,

$$(E')' = E.$$

Definition (Distributive Law)**.** Union and intersection are distributive as follows:

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

and

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C).$$

Definition (Multiplication Rule (for Counting Techniques)/Fundamental Counting Principle)**.** Assume that there are k steps to finish a process, and n_k is the number of ways the kth step can be completed. Then the total number of ways the process can be finished is

$$n_1 \times n_2 \times \cdots \times n_k.$$

Definition (Permutation)**.** An ordered sequence of the elements in a set used to determine the number of outcomes in events and sample spaces. The number of outcomes for n elements is given as

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1.$$

Remark. Such permutations are sometimes called linear permutations.

Definition (Permutations of Subsets)**.** The number of subsets of r elements selected form a set of n different elements is

$$P_r^n = n \times (n-1) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}.$$

Definition (Permutations of Similar Objects)**.** The number of permutations of n, where $n = n_1 + n_2 + \cdots + n_r$, objects of which n_i are each of distinct type is

$$\frac{n!}{n_1! n_2! \cdots n_r!}$$

Definition (Combinations)**.** A subset selected without replacement from a set used to determine the number of outcomes in events and sample spaces, denoted

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Definition (Sampling with Replacement)**.** A method to select samples in which items are replaced between successive selections.

Definition (Sampling without Replacement)**.** A method to select samples in which items are not replaced between successive selections.