CS 310: Trees (Part I)

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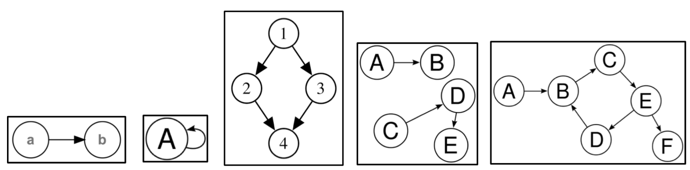
(Rooted) Trees

- A set of nodes and edges with no cycles
  - Edges point from parent to child
- One special node serves as the root
  - There is exactly one incoming edge per node per root
  - There is a unique path which traverses from the root to each node

Examples

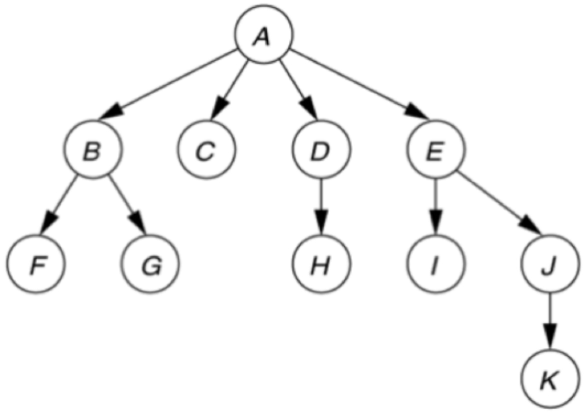


- From left to right:
  - Tree
  - Not a tree (it's cyclic)
  - Not a tree (more than one parent)
  - Not a tree (disjoint), or two trees, depending on how you look at it
  - Not a tree (more than one parent)

Tree Definitions

- Node relationship:
  - The descendants of a node x are all the nodes that we can reach by following the paths starting from x (and sometimes includes the node x itself)
  - The ancestors of a node x are the nodes on the path from the root to x (and sometimes includes the node x itself)
  - Nodes are *siblings* if they have the same parent
- Special nodes:
  - The *root node* is a node that has no parent – there is only one root node in a rooted tree
  - A *leaf node* has no children

Examples

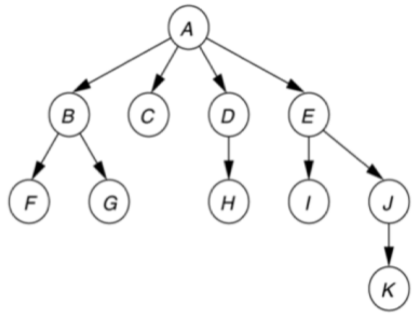


- What is the root node?
  - A
- What are the leaf nodes?
  - F, G, C, H, I and K
- What is the parent of H?
  - D
- What are the children of B?
  - F and G
- What are the ancestors of G?
  - B and A
- What are the descendants of E?
  - I, J, and K
- What are the siblings of C?
  - B, D, and E

Tree Properties

- Size (the number of nodes)
- Node height (the number of levels)
  - The *node height* is the length of the path (number of edges) from the node to the deepest leaf
  - The *tree height* is the height of the root
- Node depth (distance from the root)
  - The *depth* of a node is the length of the path from the root to the node

Example

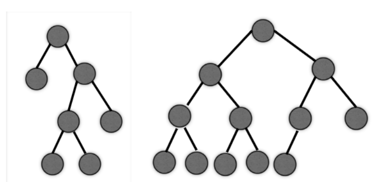


Node	Height	Depth
A	3	0
B	1	1
C	0	1
D	1	1
E	2	1
F	0	2
G	0	2
H	0	2
I	0	2
J	1	2
K	0	3

Tree Features

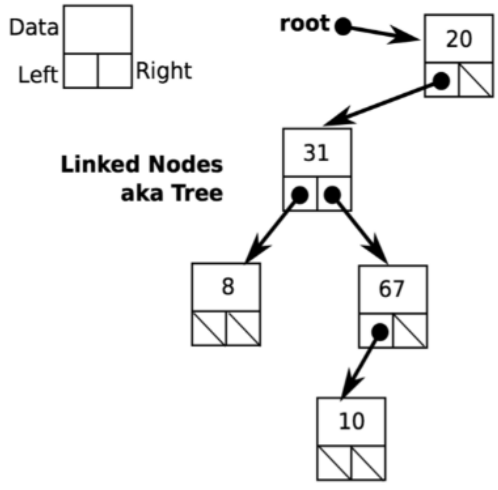
- Balanced tree
  - For a binary tree, the height of the left and right sub-trees of every node differ by 1 or 0 (as used by an AVL tree)
- Full tree
  - Every node other than the leaves has the maximum number of children
- Perfect tree
  - A full tree in which all leaves have the same depth
- Complete tree (an almost perfect tree)
  - A tree that is completely filled, with the possible exception of the bottom level, which is filled from left to right and has no missing nodes

Examples (Binary Trees)



- Left: a balanced and full tree
- Right: a complete and perfect tree

Trees Implemented with Linked Nodes



- Node structures are typically used for linked lists (owing to the singly-linked `next`/`data` idea)
- Trees also use nodes
  - Data, pointers to children, possibly pointers to parents
  - Binary trees have left and right children:

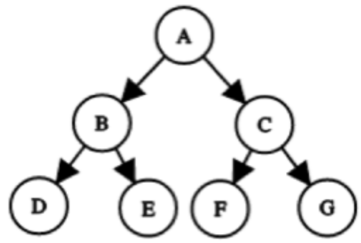
```
class Node<T> {
    T data;
    Node<T> left, right;
}
```

- Trees must have a finite number of children – however, the maximum number of children is arbitrary

Trees Implemented with Arrays

- Store nodes in an array in level order
  - Start with the root
  - Left to right for each level
  - Reserve space for missing nodes
- Used to represent k-ary tree
  - Each parent can have at most k children
  - Binary tree when $k = 2$
- Works the best with trees of a regular structures
  - Trees that are perfect or complete don't have (or have small) gaps
  - There's less wasted memory then

Binary Tree with an Array



A	B	C	D	E	F	G
0	1	2	3	4	5	6

- Root is at index 0
- Parent at index p:
  - Left child: $2p + 1$
  - Right child: $2p + 2$
- Child at index c:
  - Parent at index $\lfloor (c - 1)/2 \rfloor$

Binary Tree with an Array

- Complete binary trees
  - The array is filled left to right
- Arbitrary binary trees
  - Some elements of the array can be `null`

Tree Implementations

- Arrays
  - Fast memory access
  - Must have a fixed branching factor
  - Inefficient use of memory
  - Common for regular and stable structures like complete binary trees
- Linked nodes
  - Easy to move around
  - Easier to deal with arbitrary trees

Tree Applications

- Model hierarchical structures
  - File systems, class inheritance
- Store sorted data and support efficient search and insertion
  - Search trees or ordered trees in $O(\log(n))$ time
  - Sorted lists and hash tables also come into play

- Many others
  - Expression trees
  - Huffman coding

Common Tree Operations

- Searching for an item
- Adding items
- Deleting items
- Balancing
- Iterating through the tree
  - Selecting part of the tree

Recursion

- *Recursion*: something defined in terms of itself
- *Function recursion*: when a function invokes itself
  - Two types of function recursion:
    * Direct: a function invokes itself
    * Indirect: `foo` invokes `bar`, and `bar` invokes `foo`

Example: Factorial

- Mathematically:

$$n! = \begin{cases} 1 & n \leq 1 \\ (n-1)! & n > 1 \end{cases}$$

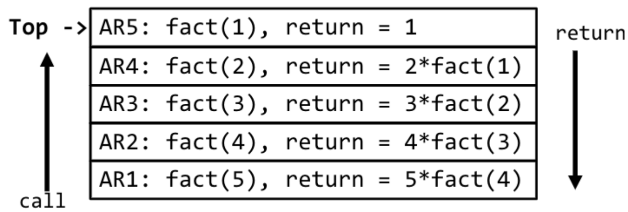
- Programatically:

```
public static int factorial(int n) {
    return (n <= 1) ? 1 : n * factorial(n - 1);
}
```

Recursion Basics

- Think of your task in terms of a “base case” and a “recursive case”
- The *base case* is the answer which is directly calculated with no recursion
- The *recursive case* is the answer to be calculated by repeated application
  - It is crucial that repeated application of the definition make progress towards the base case

Recursive Function Execution



- Each recursive call is distinct
  - There is a separate frame or activation record on the runtime stack for each call
  - It's as if they're separate functions with the same implementation

Common Issues with Recursion

- The base case is unreachable
  - If the recursive definition never triggers the base case, it will never stop
  - Don't put the base case after the recursive call – it'll never be reached
- The recursive case doesn't make progress towards the base case
- Space or time efficiency matters

Example

- Consider the following definition of the Fibonacci numbers

```
public static int fib(int n) {
    return (n <= 2) ? 1 : fib(n - 1) + fib(n - 2);
}
```

- For a call to `fib(5)`, how many additional calls are made?
  - Nine calls
  - It's $O(2^n)$