# CS 310: Hashing (Part I)

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| Implementation | get/set | add/del at end | add/del start | add/del mid | search | can grow? |
|---|---|---|---|---|---|---|
| Static Array | 1 | 1 | $N$ | $N$ | $N$ | no |
| Dynamic Array | 1 | **1***  | $N$ | $N$ | $N$ | no |
| Singly-Linked | $N$ | 1, $N$ | 1 | $N$ | $N$ | yes |
| Doubly-Linked | $N$ | 1 | 1 | $N$ | $N$ | yes |
| Stack | 1 (top) | 1 (pop) | 1 (push) | - | - | yes |
| Queue | 1 (getFront) | 1 (enqueue) | 1 (dequeue) | - | - | yes |

*Amortized analysis

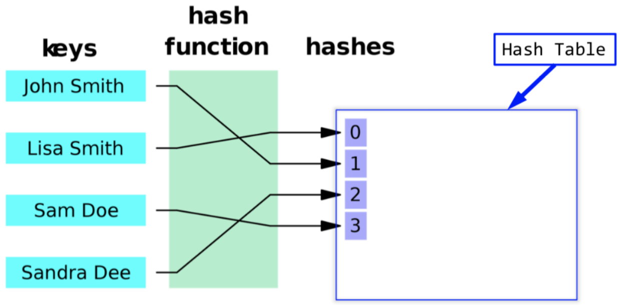
## Motivation

- Task: suggest a data structure that can support retrieval and deletion of any value in constant time
  - `.add(T t)`, `.remove(T t)`, and `.has(T t)` are $O(n)$
- What should we do if the value we want to remove is in some range, $[a, b]$?
  - An array would work best here, because we have a contiguous range
  - Could use something like `myArray[b-a]` where `myArray[i-a]` indicates whether we have added an integer $i$ into our storage
    - Initialize the entire array to zero to make it clear that it contains no elements
    - How would we perform `.add(T t)`, `.remove(T t)`, or `.has(T t)`?
- So an array works to tell us if something is in the set, because we can use the value at each index as a true or false boolean that tells us whether we have the element at the index
- What we would like *in general* is a mapping from any `Object` to some small manageable integer
  - We call this mapping a *hashing*

## Terminology

- *Hash code*: the integer computer for any object
- *Hash function*: the function that computes a hash code for any object
- *Hash table*
  - Storage of objects based on their hash codes
    - Use a has code to perform a fast table lookup
  - Looks a lot like an array or a linked list
  - Usually we do not keep duplicates – duplicate elements get mapped to the same place in the hash table

## Hash Table Example




## Hash Table

- Store objects in an array in a retrievable way
- A simplified view of `.add(T t)`:
  - Compute the integer hash code `xhc` from `x`
  - Put `x` in an array `hta` at the index `xhc`
- Things to consider
  - How do you compute `xhc`? Where should that code exist?
  - What if `xhc` is too big (it's larger than the length of the hash table)?
  - What if the current index is occupied?
    - We call this a *hash collision*

## Roadmap

- Basic ideas of hashing
- Hash code computation
- Using hash codes
  - Bounding the range
  - Hash table collision resolution and other strategies

## Hash Code

- An integer computed for an object
- Every object in Java has a `.hashCode()` method
  - `int hc = thing.hashCode();`
  - By default, `Object`'s `.hashCode()` is invoked (since all objects extend `Object`), which typically converts the memory address to an integer and uses that (though this behavior isn't required by the JVM)
  - Official documentation

## Overriding `.hashCode()`

- We can override `.hashCode()` to do something special on a per-class basis
  - For your own classes, override the default `.hashCode()`
  - Compute the hash based on the internal data of an object
  - Follow the hash contract

```java
public class Student {
    Integer gnumber;
    String name;
    Department dept;

    // Other methods omitted

    @Override
    public int hashCode() { return gnumber.hashCode(); }
}
```

## Hash Contract

- `x.equals(y)` $\Longrightarrow$ `x.hashCode() == y.hashCode()`
  - However, *different objects can have the same hash code*
- We can revisit our code snippet from above and make the hash more resilient

```java
public class Student {
    Integer gnumber;
    String name;
    Department dept;

    // Other methods omitted

    @Override
    public int hashCode() { return gnumber.hashCode() + dept.hashCode(); }
}
```

## Picking a Good Hash Function

- Adhere to the *Hash Contract*
  - If $x = y$ then they must have the same hash
- *Distribute* different objects "fairly" across the integers
  - We get reduced collision as a benefit of this
- Compute the hash code as *quickly* as possible
  - Since we'll be doing a lot of hashing, we can greatly speed up the hash table with a fast hash function
- *Note*: We can't usually get all three of these, so we have to decide which, based on the problem, need to be prioritized

## Hash Table (a Simplified View)

- Store objects in an array `hta` in a retrievable way
- `.add(T t)`: put the object `t` in the hash table
  - Compute the hash code `thc` from `t`
  - Use `thc` as the index of `hta` in which we will store `t`
- `.has(T t)`: check if `t` is in the hash table
  - Compute the hash code `thc` from `t`
  - `return t.equals(hta[thc]);`
- `.remove(T t)`: delete `t` from the hash table
  - Compute the hash code `thc` from `t`
  - `if has(t) { hta[thc] = null;}`

## Hash Table: Issues

```
class HashSet<T> {
    T hta[];
    int size;

    void add(T x) {
        int xhc = x.hashCode();
        // What if xhc is out of bounds?
        // What if this entry is already occupied?
        hta[xhc] = x;
        size++;
    }

    boolean has(T x) {
        int xhc = x.hashCode();
        return x.equals(this.hta[xhc]);
    }
}
```

## Hash Code Processing

- Modulo (remainder) is the typical way to bound the hash code based on the hash table length:

  ```
  int n = hta.length;
  hta[abs(xhc) % n] = x;
  ```

- For math-related reasons, it's usually best to make $n$ a larger *prime* number
  - We do this so that multiples of the same number modulo $n$ are less likely to collide

    ```
    30 % 17 = 13
    300 % 17 = 11
    3000 % 17 = 8
    30000 % 17 = 12
    ```

    ```
    30 % 9 = 3
    300 % 9 = 3
    3000 % 9 = 3
    30000 % 9 = 3
    ```

- Now we have bounding issues resolved, but we've shortened the range
  - It's more likely now that we'll have different objects mapping to the same entry

## Hash Table Collisions

- Motivation
  - Put `x` in a table at `hta[xhc]`
  - **Problem**: what if `hta[xhc]` is occupied?
  - **Answer**: find some other storage for `x`
- Common approaches
  - Separate chaining
  - Open addressing

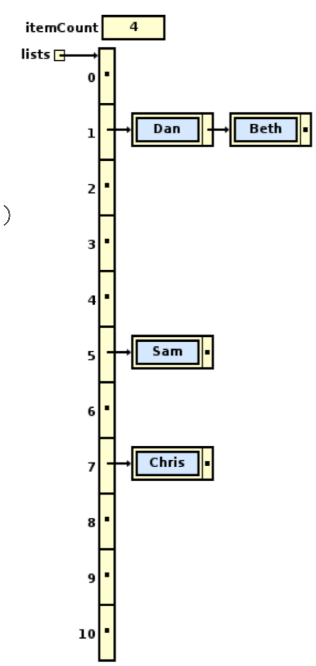
## Separate Chaining

- Something already there?
  - Expand that single entry into an internal data structure
    * This way we can accommodate multiple objects which have the same hash code
    * We can also grow this structure if we get more collisions
  - We have a data structure for this... what is it?
    * A linked list (or an array list)
  - **What's the worst case complexity?**
    * **How can we avoid that case?**

## Separate Chaining Examples

### Example 1

```
String [] sa1 = new String[] {
    "Chris",
    "Sam",
    "Beth",
    "Dan"
};
SeparateChainHS<String> h = new SeparateChainHS<String>(11)
for(String s : sa1) {
    h.add(s);
}
// String("Chris").hashCode() % 11=65087095 % 11 = 7
// String("Sam").hashCode() % 11=82879 % 11 = 5
// String("Beth").hashCode() % 11=68465 % 11 = 1
// String("Dan").hashCode() % 11=2066967 % 11 = 1
```

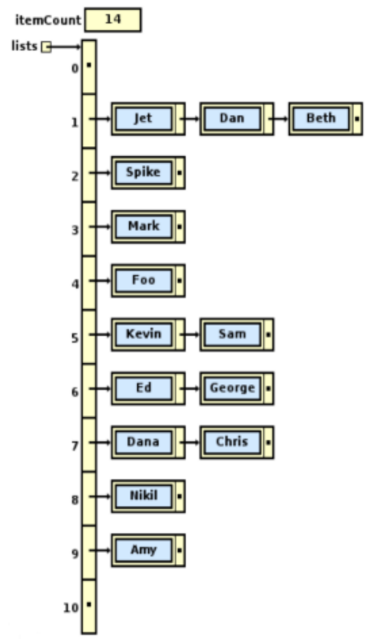



- Every table entry is a linked list
  - `hta[i]` stores all the values mapping to index `i`

- Example:
  - Table length = 11
  - Number of items = 4
  - Load:

$$\frac{\text{item count}}{\text{hash table length}} = \frac{4}{11} \approx 0.\overline{36}$$

**Example 2**




- Table length = 11
- Number of items = 4
- Load:

$$\frac{\text{item count}}{\text{hash table length}} = \frac{14}{11}$$

$$\frac{14}{11} =\approx 1.\overline{27}$$

**Discussion**

- Why use an array of linked lists?
- How can we calculate a good load factor?

## Separate Chaining Analysis

- `.add(T t)` is $O(1)$ assuming adding to a list is $O(1)$ and that duplicates are allowed
  - If duplicates aren't allowed, the time jumps to $O(n)$ since we need to search the chain to make sure that we don't already have the element in the chain
- `.remove(T t)`/`.contains(T t)`
  - The run time depends on the number of things in each list
  - `.remove(T t)`/`.contains(T t)` must potentially look through all elements in the longest chain in the hash table to see if `t` is present
    - The average case is $O(\text{average chain length}) = O(\texttt{itemCount}/\texttt{tableLength}) = O(\text{load})$
    - The worst case time is $O(\texttt{itemCount})$
    - How do we avoid the worst case?

## Separate Chaining is Viable in Practice

- It's relatively simple to implement (see Weiss Fig. 20.20)
- It's reasonably efficient
- Java's built-in hash tables use it
    - `java.util.HashSet`, `java.util.HashMap`, and `java.util.Hashtable` all use separate chaining