assignment9

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1 Summary of Problem Specification

1.1 Abstract

Using the inverse of a matrix find the solution to a system of linear equations.

1.2 Algorithm

Though very slow, the algorithm that we'll use for finding the inverse involves the use of the determinant and the adjugate (and thus cofactor) matrices. This kind of algorithm is best expressed through an example:
Consider the system:

$$x_1 + 3x_2 - 2x_3 = 5$$
$$2x_1 + 5x_3 = 10$$
$$x_1 - x_2 - 10x_3 = 2$$

We can represent this linear system of equations as an augmented matrix:

$$A = \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 2 & 0 & 5 & 10 \\ 1 & -1 & -10 & 2 \end{array}\right]$$

1. Look only at the coefficient matrix, and begin to calculate the cofactor matrix. Let our coefficient matrix be C. Then:

   $$C = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 0 & 5 \\ 1 & -1 & -10 \end{bmatrix}$$

   And our cofactor matrix C' will be the result of:

   $$C' = \begin{bmatrix} +\begin{vmatrix} 0 & 5 \\ -1 & -10 \end{vmatrix} & -\begin{vmatrix} 2 & 5 \\ 1 & -10 \end{vmatrix} & +\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} \\ -\begin{vmatrix} 3 & -2 \\ -1 & -10 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ 1 & -10 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} \\ +\begin{vmatrix} 3 & -2 \\ 0 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ 2 & 5 \end{vmatrix} & +\begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} \end{bmatrix}$$

2. Evaluating C' yields the matrix:

$$C' = \begin{bmatrix} 5 & 25 & -2 \\ 32 & -8 & 4 \\ 15 & -9 & -6 \end{bmatrix}$$

3. We then take the transpose of C', called the adjugate of C:

$$adj(C) = \begin{bmatrix} 5 & 32 & 15 \\ 25 & -8 & -9 \\ -2 & 4 & -6 \end{bmatrix}$$

4. Now that we've found the adjugate of C, we simply multiply that matrix by the reciprocal of the determinant to find the inverse of C.

$$C^{-1} = \frac{1}{det(C)} * adj(C) = \begin{bmatrix} 5/69 & 32/69 & 15/69 \\ 25/69 & -8/69 & -3/23 \\ -2/69 & 4/69 & -2/23 \end{bmatrix}$$

5. Finally, we need only multiply the inverse by the solutions vector to find values of x.

$$Cx = b$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 0 & 5 \\ 1 & -1 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 2 \end{bmatrix}$$

$$C'Cx = C'b$$

$$Ix = C'b$$

$$x = C'b$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 2 \end{bmatrix} \begin{bmatrix} 5/69 & 32/69 & 15/69 \\ 25/69 & -8/69 & -3/23 \\ -2/69 & 4/69 & -2/23 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 125/23 \\ 9/23 \\ 6/23 \end{bmatrix}$$