Homework 6
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1. An algebraic number is any number that is a root of a polynomial with rational coefficients. Prove that the algebraic numbers are countable. A number is transcendental if it is not algebraic. Prove there are uncountable many transcendental numbers.

2. Let A be the set of all functions $f : \mathbb{N} \to \{0, 1\}$. Find the cardinality of A.

3. Let A be the set of all functions $f : \mathbb{N} \to \{0, 1\}$ that are "eventually zero" (We say that f is eventually zero if there is a positive integer N such that $f(n) = 0$ for all $n \geq N$). Find the cardinality of A.

4. Use the axiom of choice to prove that if there exists $f : A \to B$ that is onto, then there exists a function $g : B \to A$ that is one-to-one.

5. We say that $|A| \geq |B|$ if there exists a function $f : A \to B$ which is onto. Prove that if $|A| \geq |B|$, and $|B| \geq |A|$, then $|A| = |B|$. (Hint: Use 4).