**Homework 3**
**Connor Baker, February 2017**

1. Let $R$ be a relation from a nonempty set $A$ to itself. Prove that if $R$ is symmetric, transitive, and $(R) = A$, then $R$ is an equivalence relation.

*Proof.* $\square$

2. Use the Principle of Mathematical Induction to prove $3^n \geq 2^n + 1$ for all $n \in \mathbb{N}$.

*Proof.* Let $n = 1$. Then,

$$3^1 \geq 2^1 + 1. \tag{1}$$

Therefore, we must now prove that $\forall n \in \mathbb{N}$, the above inequality also holds for $n + 1$.
Let $n = k$. Then, assume the following equation to be true:

$$3^k \geq 2^k + 1. \tag{2}$$

We now try to prove that the inequality holds for $n = k + 1$:

$$3^{k+1} \geq 2^{k+1} + 1, \tag{3}$$

which is equivalent to

$$3^k * 3 \geq 2^k * 2 + 1. \tag{4}$$

Multiplying Inequality (2) by two on both sides does not destroy the inequality, but it does present us with a more useful form:

$$3^k * 2 \geq 2^k * 2 + 2. \tag{5}$$

Comparing this with Inequality (4), we find that:

$$3^k * 3 > 3^k * 2 \geq 2^k * 2 + 2 > 2^k * 2 + 1,$$

which is the same as

$$3^{k+1} > 3^k * 2 \geq 2^k * 2 + 2 > 2^{k+1} + 1. \tag{6}$$

Therefore, by the Principal of Mathematical Induciton, $3^n \geq 2^n + 1$, $\forall n \in \mathbb{N}$. $\square$

3. Use the Principle of Mathematical Induction to prove

$$\sqrt{2\sqrt{2\sqrt{2\sqrt{2\ldots}}}} \leq 2.$$

(Hint: Construct a recursively defined sequence.)

*Proof.* $\square$

4. Let $a_1 = 2, a_2 = 4$, and $a_{n+2} = 5a_{n+1} - 6a_n$ for $n \geq 1$. Prove that $a_n = 2^n$ for all natural numbers $n$.

*Proof.* $\square$

5. Use the Principle of Mathematical Induction to prove that

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + ... + n \cdot n! = (n+1)! - 1.$$

*Proof.* $\square$