Homework 1
Connor Baker, January 2017

1. Prove by contradiction that if $a - b$ is odd, then $a + b$ is odd.

Assumptions Assume that $a, b \in \mathbb{Z}$, $a - b$ is even, and $a + b$ is odd.

Lemma Let $x, y, z \in \mathbb{Z}$, $x = 2z$, $y = 2z + 1$. For all z, x is even (any number with a factor of two is even), and y is odd, (any even number plus one is odd).

Proof If $a-b$ is even, then by the previous lemma, $a-b = 2z$ for some unknown number z. If $a + b$ is odd, then by the previous lemma, $a + b = 2z + 1$ for some unknown number z. Substituting $2z + b$ for a (from the first equation) into $a+b = 2z+1$ yields $2z+b+b = 2z+1$. Simplifying yields $b = 1/2$, which means that b is *not* in the set of integers. Therefore, the original assumption is a contradiction. As a result, if $a - b$ is odd, then $a + b$ must odd.

2. Write a proof by contrapositive to show that if xy is odd, then both x and y are odd.

3. Prove that there do not exist integers m and n such that $12m+15n = 1$.

4. Prove there is a natural number M such that for every natural number n, $\frac{1}{n} < M$.

5. Prove that if $-2 < x < 1$ or $x > 3$, then $\frac{(x-1)(x+2)}{(x-3)(x+4)} > 0$.