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Markov chain Monte Carlo Project

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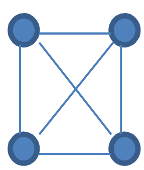
This program considers a graph containing M nodes, where M has been set to 4. The proposal distribution randomly samples from a uniform distribution to determine n, the number of edges present in the proposed state.

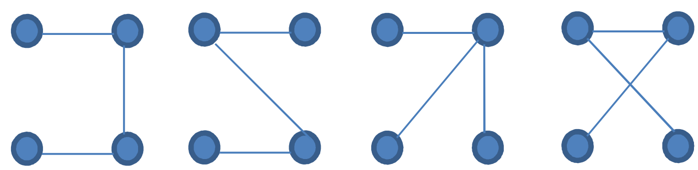
$$n = U(a, b)$$

n is the number of edges present in the proposed state, a & b are the minimum & maximum number of edges possible in the proposed state. As a graph of 4 nodes is considered, a maximum of 6 edges & a minimum of 3 edges are the bounds for a connected undirected graph.

n = 6



n = 3



n number of edges are selected randomly from a list of all edges possible between M nodes, and added to the graph. Edges selected can be written in the form:

edges in graph = ${}_{6}C_{n}$

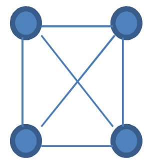
Therefore, the probability of selecting a given proposal state is the probability of selecting n # of edges (from 3to 6 possible edges) multiplied by the probability of selecting a particular combination of edges from the list of all total edges. Shown below,

$$P_{\text{proposed state}} = \frac{1}{4} * \frac{1}{6\,C\,n}$$

Therefore $q(Xi|Xj) = \frac{1}{4} * \frac{1}{6\,C\,n}$ & $q(Xj|Xi) = \frac{1}{4} * \frac{1}{6\,C\,n}$

This proposal distribution allows exploration of the entire state space. In the model, when a new proposed state is generated, before deciding whether to accept or reject the state, its connectedness is first tested. Therefore only connected & necessarily irreducible proposed states are considered for acceptance/rejection (via the Metropolis-Hastings Algorithm).

Top 1% most probable graph:



This graph occurred most frequently, 487 times in 1,000 iterations. This is likely because the probability of generating this graph is quite high. The probability of selecting n=6 is ¼, and when n=6, this is the only possible configuration of edges. This means that a proposed state with n = 6 has $\frac{q(Xi|Xj)}{q(Xj|Xi)} \geq 1$, no matter the current state. It seems this graph is therefore a "low energy" state, and the only way to break free from it is when we have a random acceptance of a proposed state.

Average Number Edges:

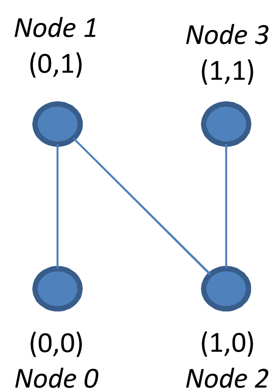
3.89

Average number of edges connected to node 0:

2.15

Considering the distance from node 0 to another node, the maximum expected shortest distance:

3.41



If the graph of nodes 1, 2, 3, & 4 is connected, the longest path from node 0 to any other node follows a "z-shape" as seen above. This distance is equal to $1+1+\sqrt{2}= 3.41$