① $X_0 = [1\ 1\ 1]^T$ stopping Threshold $\epsilon = 10^{-6}$

a)

$f(x) = (x_1+5)^2 + (x_2+8)^2 + (x_3+7)^2 + 2x_1^2x_2^2 + 4x_1^2x_3^2$

$(x_1+5)(x_1+5) \quad (x_2+8)(x_2+8) \quad (x_3+7)(x_3+7)$

$x_1^2 + 5x_1 + 5x_1 + 25 + x_2^2 + 8x + 8x + 64 + x_3^2 + 7x_3 + 7x_3 + 49 + 2x_1^2x_2^2 + 4x_1^2x_3^2$

$x_1^2 + 10x_1 + 25 + x_2^2 + 16x_2 + 64 + x_3^2 + 14x_3 + 49 + 2x_1^2x_2^2 + 4x_1^2x_3^2$

Let $A = X_1$
Let $B = X_2$
Let $C = X_3$
} Better Readability

$f(A,B,C): A^2 + 10A + 25 + B^2 + 16B + 64 + C^2 + 14C + 49 + 2A^2B^2 + 4A^2C^2$

$\frac{\partial}{\partial A} = 2A + 10 + 4B^2A + 8C^2A \qquad \frac{\partial^2}{\partial A^2} = 2 + 4B^2 + 8C^2$

$\frac{\partial}{\partial B} = 2B + 16 + 4A^2B \qquad \frac{\partial^2}{\partial B^2} = 2 + 4A^2$

$\frac{\partial}{\partial C} = 2C + 14 + 8A^2C \qquad \frac{\partial^2}{\partial C^2} = 2 + 8A^2$

The final solution satisfies the second order necessary conditions for a minimum.

Find critical numbers

$\frac{\partial}{\partial A} = 2A + 10 + 4B^2A + 8C^2A = 0$

$-10 \qquad -10$

$2A + 4B^2A + 8C^2A = -10$

solve for A

$2A + 4(-32 - 64A^2)^2 A + 8(-7 - 28A^2)^2 A = -10$

$2A + 4(1024 + 4096a^2 + 4096a^4)A + 8(49 + 392a^2 + 784a^4)$

$2A + 4096a + 16384a^3 + 16384a^5 + 392a + 3136a^3 + 6272a^5$

$22656a^5 + 19520a^3 + 4490a = -10$

$\boxed{a \approx -0.00222}$

$\frac{\partial}{\partial B} = 2B + 16 + 4A^2B = 0$

$2B + 4A^2B = -16$

$\frac{B(2 + 4A^2)}{B} = \frac{-16}{B}$

$-16\left(2 + 4A^2 = \frac{-16}{B}\right)$

$-32 - 64A^2 = B$

$A = 70$

$2B + 4(0.00222)^2 B = -16$

$\boxed{B = -7.9999}$

Solve system of equations

$\frac{\partial}{\partial c} = 2C + 14 + 8A^2C = 0$

$-14 \qquad -14$

$2C + 8A^2C = -14$

$\frac{2C(1 + 4A^2)}{2C} = \frac{-14}{2C}$

$-7\left(1 + 4A^2 = \frac{-7}{C}\right)$

$-7 - 28A^2 = C$

$C = -7 - 28(0.0022)^2$

$\boxed{C = -7.00}$