

$$f(x) = (x_1 - 1)^2 + 2(x_2 - 2)^2$$

$$\text{Let } A = x_1$$

$$\text{Let } B = x_2 \quad (\text{without Barrier})$$

$$f(a, b) = (A - 1)^2 + 2(B - 2)^2$$

$$f(a, b) = A^2 - 2A + 1 + 2(B^2 - 4B + 4)$$

$$f(a, b) = A^2 - 2A + 1 + 2B^2 - 8B + 8$$

$$\frac{\partial}{\partial A} = 2A - 2$$

$$\frac{\partial}{\partial B} = 4B - 8$$

$$\frac{\partial^2}{\partial A^2} = 2$$

$$\frac{\partial^2}{\partial B^2} = 4$$

Find critical points

$$\begin{array}{r} 2A - 2 = 0 \\ +2 \quad +2 \end{array}$$

$$\begin{array}{r} 4B - 8 = 0 \\ +8 \quad +8 \end{array}$$

$$\frac{2A = 2}{2}$$

$$\frac{4B = 8}{4}$$

$$A = 1$$

$$B = 2$$

Objective function

$$f(1, 2) = (1 - 1)^2 + 2(2 - 2)^2$$

$$f(1, 2) = 0$$