CMPT 440 – Spring 2019: Quantum Finite Automata

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Due Date: 30/4/2020

Theoretical Background

In quantum computing, quantum finite automata, also known as quantum state machines are used to provide an abstraction of any transitions that may occur during computation. It is believed that certain problems can be solved faster and with greater efficiency by a QFA than it could by any other traditional finite automata. The QFA can be defined by the 15-tuple that is as follows:
T = (Σ, Q, q_0, P, α).

$$P = \frac{1}{R}\sum_{i-0}^{N} x - \delta(1,0)$$

There exist two types of QFAs, 1-way QFAs and 2-way QFAs. Superposition is pretty much the system's capability to be in many states at one time. 2-way QFAs are more powerful, mostly due to the fact that it allows superpositions where different parts of the superposition have the head of QFA at different locations. So in 2-way QFAs, the heads can move to the right, left, or remain where they are. In a 1-way QFA on the other hand, the head can only move one cell to the right for every iteration.

An Example

According to Tian et al, in a general QFA transition diagram, the starting state is represented by $|\psi_0\rangle$. Also, $|0\rangle$, $|1\rangle$, …, $|m\text{-}2\rangle$, $|m\text{-}1\rangle$ are the orthonormal basis states, some of which are accepted states. For every one of the σ_n input symbols, a corresponding unitary operator gets assigned as $U\sigma_n$. $U\sigma_n$ is applied to the quantum state. After reading the last input symbol the quantum state gets measured and projected to a state that is either accepting or rejecting.

References

A. Ambainis and R. Freivalds. 1-way quantum finite automata: strengths, weaknesses and generalizations. In Proceedings 39th Annual Symposium on Foundations of Computer Science (Cat. No. 98CB36280), pages 332–341. IEEE, 1998.

Tian, Y., Feng, T., Luo, M. *et al.* Experimental demonstration of quantum finite automaton. *npj Quantum Inf* 5, 56 (2019). https://doi.org/10.1038/s41534-019-0163-x