

# A model of navigation history

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**Abstract:** Navigation has been a core component of the web since its inception: users and scripts can follow hyperlinks, and can go back or forwards through the navigation history. In this paper, we present a formal model aligned with the WHATWG specification of navigation history, and investigate its properties. The fundamental property of navigation history is that traversing the history by  $\delta$  then by  $\delta'$  should be the same as traversing by  $\delta + \delta'$ . In particular, traversing by  $+1$  (forward) then by  $-1$  (back) is the same as traversing by  $0$  (doing nothing). We show that the specification-aligned model does not satisfy this property, by exhibiting a series of counter-examples, which motivate four patches to the model. We present a series of experiments, showing that browsers are inconsistent in their implementation of navigation history, but that their behaviour is closer to the patched model than to the specification-aligned model. We propose patches to the specification to align it with the patched model.

**ACM Classification:** D.2.1 Requirements/Specifications.

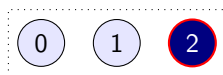
**Keywords:** Formal model, Navigation, Session history, Specification, Web browsers.

## 1. Introduction

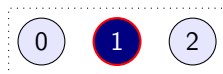
Navigation has been a core component of the web since its inception: users and scripts can follow hyperlinks, and can go back or forwards through the navigation history. Users are exposed to this functionality through following hyperlinks, and by the forward and back buttons. Scripts have many ways of accessing session history, via the navigation API [5, §7.7] and the `element.click()` method.

Prior formalizations of navigation history include [1, 2, 3, 4, 6], which predate and are not aligned with the WHATWG specification [5]. The specification of the navigation API is informal, and has complex dependencies on the rest of the HTML specification. There is little discussion of the goals of the API, and an unclear alignment with browser implementations.

In this paper, we present a formal model of navigation, aligned with the HTML specification, and investigate its properties. The starting point is that there is a total order of *documents*<sup>1</sup>, one of which is *active*, for example:



In diagrams, we use left-to-right order to indicate order, and highlight the active document. The user can *traverse* the history which changes the active document, for example pressing the back button:



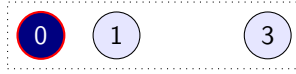
<sup>1</sup>We are eliding some of the technical details of the specification here, in particular we are conflating a *browsing context* with the document it contains, and we are ignoring issues around document loading and unloading, and around the current entry of the joint session history.



The user can also *navigate*, which replaces any document after the currently active document by a fresh active document:

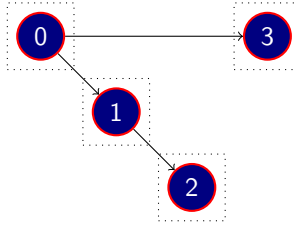


Users can also traverse the history by more than one document at a time, for example by using a pull-down menu from the back or forwards button. This is called *traversing by  $\delta$* , for instance we can traverse our running example by  $-2$  to get:

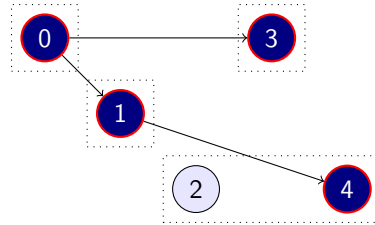


We formalize the notions of traversal and navigation in §2, and show the *fundamental property of traversal*: that traversing by  $\delta$  then by  $\delta'$  is the same as traversing by  $\delta + \delta'$ .

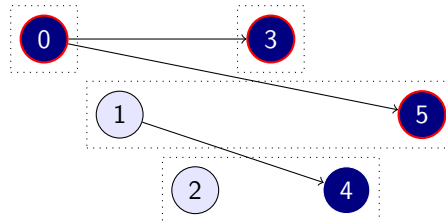
So far, the model is refreshingly simple, and corresponds well to the specification and to browser implementations. Where the problems arise is in the *hierarchical* nature of documents. HTML documents can contain `iframe` elements, which are independent documents in their own right, often used to embed third party content such as advertisements. We can treat each document as a tree, for example:



The problem comes from the ability of each document to navigate separately and maintain its own session history, but that traversal is a global operation that operates on the *joint session history*. For example if document 2 in the previous example navigates, the resulting state is:

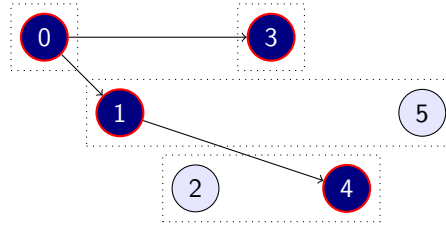


and then if document 1 navigates, the state is:



Note that node 4 here is in an unusual state: it is active, but has an inactive ancestor. The specification [5, §7.7] distinguishes between *active* documents such as 4, and *fully active* documents such as 0, 3 and 5. Active documents can become fully active by traversals

involving their ancestors. For example, after traversing by  $-1$ , document 4 is fully active:



As even a simple example like this shows, the combination of features quickly results in a complex mix of session history, ordering, and document hierarchy, which leads to the problems:

- *Formally* there is no simple model, and the model provided by the specification does not satisfy the traverse-then-traverse property.
- *Experimentally* the browsers disagree with each other, and with the HTML specification, about the semantics of navigation.

In this paper, we address these:

- §2 provides a formal model of navigation history, which is intended to align with the specification. We show, through a series of examples, that it does not satisfy the fundamental property, and give patches to the model for each example. The final model does satisfy the fundamental property.
- §3 investigates how well the patched model aligns with existing browser implementations. We show ways in which the browsers exhibit behaviours which are not aligned with the specification, and discuss how our proposed model matches these behaviours.

Finally, we propose changed wording to the specification, which would bring it in line with our patched model.

## 2. Model

In this section, we present our formal model of navigation history. §2.1 contains definitions of concepts such as tree and total order, and may be skipped by most readers. The model, together with some examples, is given in §2.2. In §2.3 we define the fundamental property of traversal, show that the model does *not* satisfy it, but can be patched to do so.

### 2.1. Preliminaries

A *directed graph*  $G = (V, \rightarrow)$  consists of:

- a set  $V$  (the *vertices*), and
- a relation  $\rightarrow \subseteq (V \times V)$  (the *edges*).

The *transitive closure* of  $\rightarrow$  is defined as  $v \rightarrow^+ v'$  whenever there exists  $v_0, \dots, v_n$  such that:

$$v = v_0 \rightarrow \dots \rightarrow v_n = v'$$

The *reflexive transitive closure* of  $\rightarrow$  is defined as  $v \rightarrow^* v'$  whenever  $v \rightarrow^+ v'$  or  $v = v'$ . A *forest* is a directed graph where:

- there is no  $v$  such that  $v \rightarrow^+ v$  (*acyclicity*)

- if  $v \rightarrow v' \leftarrow v''$  then  $v = v''$  (*at most one parent*).

A *root vertex* of a forest is a vertex  $v$  such that there is no  $w \rightarrow v$ . A *tree* is a forest with a unique root vertex. A *preorder* is a directed graph  $(V, \leq)$  such that:

- every  $v$  has  $v \leq v$  (*reflexivity*), and
- if  $v \leq v' \leq v''$  then  $v \leq v''$  (*transitivity*).

A *partial order* is a preorder such that:

- for every  $v$  and  $v'$ , if  $v \leq v'$  and  $v' \leq v$  then  $v = v'$  (*antisymmetry*).

A *total order* is a partial order such that:

- for every  $v$  and  $v'$ , either  $v \leq v'$  or  $v \geq v'$  (*totality*).

A *equivalence* is a preorder  $(V, \sim)$  such that:

- if  $v \sim v'$  then  $v' \sim v$  (*symmetry*).

## 2.2. Definitions

We can now formalize our model of navigation history, together with the operations of navigation and traversal. This formalizes the navigation history specification [5], and has a pleasant diagrammatic presentation, but as we shall see in §2.3, it has unexpected properties.

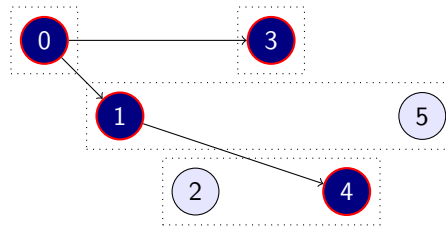
**Definition 1 (Navigation history):** A *navigation history*  $H = (D, A, \rightarrow, \leq, \sim)$  consists of:

- a finite set  $D$  (the *documents*),
- a subset  $A \subseteq D$  (the *active* documents),
- a forest  $(D, \rightarrow)$  (the *child* relationship),
- a total order  $(D, \leq)$  (the *chronological* order), and
- an equivalence relation  $(D, \sim)$  (the *same-session* equivalence).

such that:

- for every  $d$  there is a unique  $d' \in A$  such that  $d \sim d'$ ,
- for every  $d \rightarrow e \sim e'$  we have  $d \rightarrow e'$ , and
- for every  $d \rightarrow e$ , we have  $d \leq e$ . □

We present such navigation histories diagrammatically, using left-to-right position for chronological order, and grouping documents in the same session. Since documents in the same session must have the same parent, we only draw the document hierarchy for active children. For example the diagram:



represents:

$$\begin{aligned}
 D &= \{0, 1, 2, 3, 4, 5\} \\
 A &= \{0, 1, 3, 4\} \\
 0 &\rightarrow 1 \quad 0 \rightarrow 3 \quad 0 \rightarrow 5 \quad 1 \rightarrow 2 \quad 1 \rightarrow 4 \\
 0 &\leq 1 \leq 2 \leq 3 \leq 4 \leq 5 \\
 1 &\sim 5 \quad 2 \sim 4
 \end{aligned}$$

In such a navigation history, we define:

- $d_0$  is the unique active root document,
- $d \twoheadrightarrow e$  when  $d \rightarrow e$  and  $e \in A$  (the *active child* relationship),
- $FA = \{d \mid d_0 \twoheadrightarrow^* d\}$  (the *fully active* documents),
- $d \lesssim e$  whenever  $d \sim e$  and  $d < e$ ,
- the *session future* of  $d$  is  $\{e \mid d \lesssim e\}$ ,
- the *session past* of  $d$  is  $\{e \mid d \gtrsim e\}$ ,
- the *joint session future* is  $\{e \mid \exists d \in FA . d \lesssim e\}$ ,
- the *joint session past* is  $\{e \mid \exists d \in FA . d \gtrsim e\}$ ,

These definitions are intended to align with the specification, for example [5, 7.7.2] has the definition:

The **joint session history** of a top-level browsing context is the union of all the session histories of all browsing contexts of all the fully active **Document** objects that share that top-level browsing context, with all the entries that are current entries in their respective session histories removed except for the current entry of the joint session history.

which (eliding the concept of “current entry of the joint session history”) corresponds to the above definitions of joint session future and past. We now consider how to formalize operations on navigation histories. starting with *navigation*, which is triggered by following hyperlinks, or other actions which trigger document loading.

**Definition 2 (Navigation):** Define *deleting  $d$  from  $H$* , when  $d$  is in the joint session future to be  $H' = (D', A, \leq, \rightarrow, \sim)$  where:

- $D' = D \setminus \{e \mid d \twoheadrightarrow^* e\}$ .

Define *replacing  $d$  by  $d'$  in  $H$* , where  $d \in FA$  and  $d' \notin D$ , to be  $H' = (D', A', \leq', \rightarrow', \sim')$  where:

- $D' = D \cup \{d'\}$ ,
- $e \in A'$  whenever  $e \in A$  and  $e \neq d$ , or  $e = d'$ ,
- $e \leq' f$  whenever  $e \leq f$ , or  $f = d'$ ,
- $e \rightarrow' f$  whenever  $e \rightarrow f$ , or  $e \rightarrow d$  and  $f = d'$ , and
- $e \sim' f$  whenever  $e \sim f$ , or  $e = f$ , or  $e \sim d$  and  $f = d'$ , or  $d \sim f$  and  $e = d'$ .

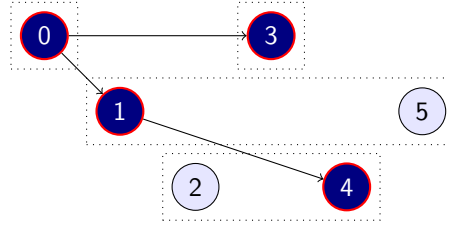
Define *navigating from  $d$  to  $d'$  in  $H$* , where  $d \in FA$  to be the result of:

- deleting the session future of  $d$ , and then

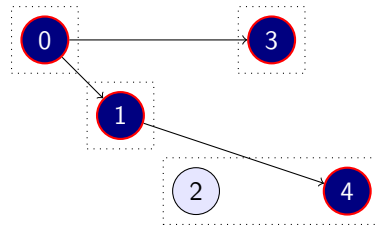
- replacing  $d$  by  $d'$ .

□

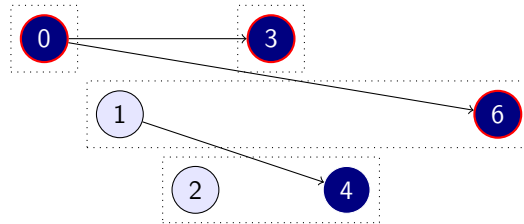
There are two parts to navigation from  $d$  to  $d'$ : deleting the session future of  $d$ , followed by replacing  $d$  by  $d'$ . For example, navigating from 1 to 6 in:



we first delete the session future of 1 (which is 5):



then replace 1 by 6:



We also define *traversing the history*, which changes the active documents.

**Definition 3 (Traversal):** Define *traversing the history to  $d$  in  $H$*  to be  $H' = (D, A', \leq, \rightarrow, \sim)$  where:

- $e \in A'$  whenever  $d \not\sim e \in A$ , or  $d = e$ .

Define  $H$  *traverses the history by  $+\delta$  to  $H'$*  when:

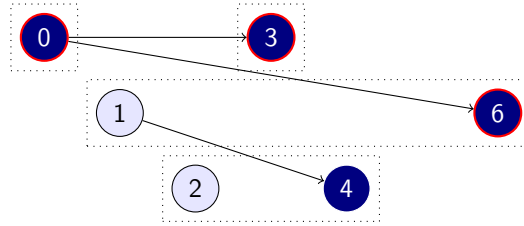
- the joint session future of  $H$  is  $d_1 < \dots < d_\delta < \dots$ ,
- $H$  traverses the history to  $d_\delta$  in  $H'$

Define  $H$  *traverses the history by  $-\delta$  to  $H'$*  when:

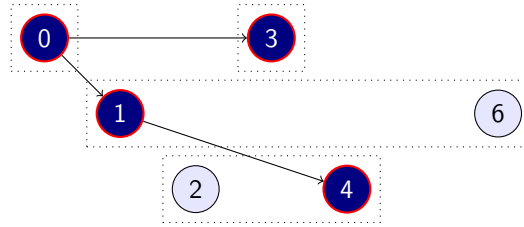
- the joint session past of  $H$  is  $d_1 > \dots > d_\delta > \dots$ ,
- $H$  traverses the history to  $d_\delta$  in  $H'$

Define  $H$  *traverses the history by 0 to  $H'$*  when  $H = H'$ .

For example, to traverse the history by  $-2$  in:



we find the joint session past (which is  $2 > 1$ ) and traverse the history to the second item (which is 1) to arrive at:



These definitions are intended to formally capture the HTML specification, for example [5, §7.7.2] includes:

To **traverse the history by a delta**  $\delta$ , the user agent must append a task to this top-level browsing context's session history traversal queue, the task consisting of running the following steps:

1. If the index of the current entry of the joint session history plus  $\delta$  is less than zero or greater than or equal to the number of items in the joint session history, then abort these steps.
2. Let *specified entry* be the entry in the joint session history whose index is the sum of  $\delta$  and the index of the current entry of the joint session history.
3. Let *specified browsing context* be the browsing context of the specified entry.
4. If the specified browsing context's active document's unload a document algorithm is currently running, abort these steps.
5. Queue a task that consists of running the following substeps [...]
  3. Traverse the history of the specified browsing context to the specified entry.

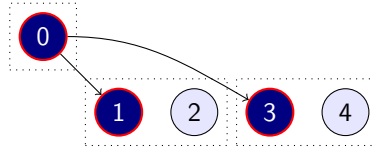
## 2.3. Properties

We now consider the fundamental property of navigation history:

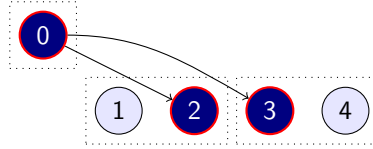
**Definition 4 (Fundamental property):**  $H$  satisfies the *fundamental property of traversal* whenever  $H$  traverses the history by  $\delta$  to  $H'$  and  $H'$  traverses the history by  $\delta'$  to  $H''$  implies  $H$  traverses the history by  $\delta + \delta'$  to  $H''$ .  $\square$

Unfortunately, navigation histories as specified do not always satisfy the fundamental property, due to ways individual session histories are combined into the joint session history. In this section, we give a series of counterexamples, and propose patches to the model to address each counterexample.

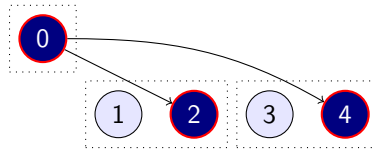
**Counterexample 1:** Let  $H$  be:



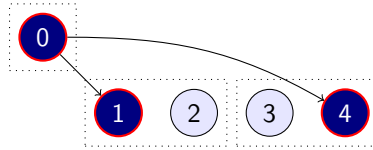
which traverses the history by 1 to:



which traverses the history by 1 to:



but  $H$  traverses the history by 2 to:



□

This counterexample is caused by the definition of ‘traverses the history by  $\delta$ ’ which only traverses one document’s session history. Instead, we should traverse the history of all  $\delta$  documents.

**Patch 1 (Traverse intermediaries):** Define  $H$  traverses the history by  $+\delta$  to  $H'$  when:

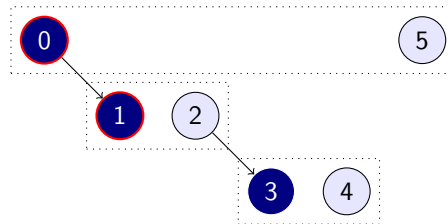
- the joint session future of  $H$  is  $d_1 < \dots < d_\delta < \dots$ ,
- there is some  $H = H_0, \dots, H_\delta = H'$ , such that
- $H_{i-1}$  traverses the history to  $d_i$  in  $H_i$  for each  $1 \leq i \leq \delta$ .

Define  $H$  traverses the history by  $-\delta$  to  $H'$  when:

- the joint session past of  $H$  is  $d_1 > \dots > d_\delta > \dots$ ,
- there is some  $H = H_0, \dots, H_\delta = H'$ , such that
- $H_{i-1}$  traverses the history to  $d_i$  in  $H_i$  for each  $1 \leq i \leq \delta$ .

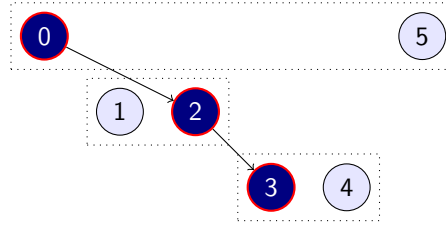
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**Counterexample 2:** Let  $H$  be:

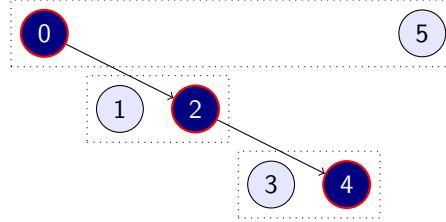




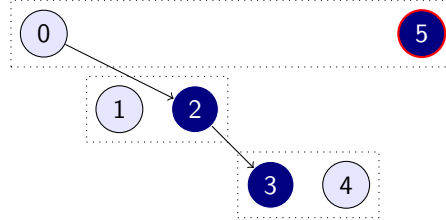
which traverses the history by 1 to:



which in turn traverses the history by 1 to:



but  $H$  traverses the history by 2 to:



□

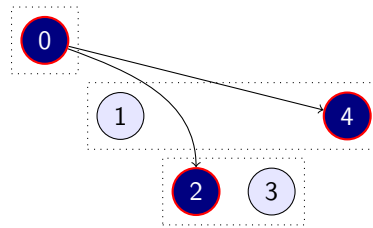
The problem this time is that the definition of ‘joint session history’ only includes the fully active documents, not all active documents.

**Patch 2 (Active joint session history):** Define:

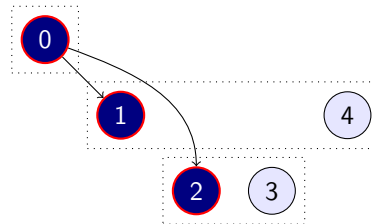
- the *joint session future* is  $\{e \mid \exists d \in A . d \lesssim e\}$ , and
- the *joint session past* is  $\{e \mid \exists d \in A . d \gtrsim e\}$ .

□

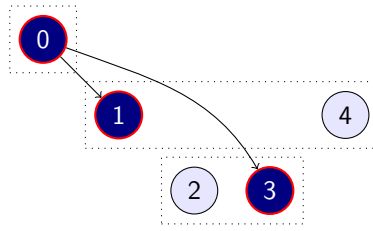
**Counterexample 3:** Let  $H$  be:



which traverses the history by  $-1$  to:



which traverses the history by 1 to:



which is not the same as  $H$ . □

This counterexample is caused by an asymmetry in the definition of traversal: it is defined in terms of navigating *to* a document  $d$ , and not navigating *from* a document. We fix this by making the definition symmetric:

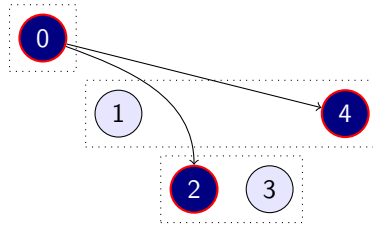
**Patch 3 (Symmetric traversal):** Define  $H$  *traverses the history from*  $d'$  when there is some  $d$  such that:

- $d \lesssim d'$ ,
- for any  $e \lesssim d'$  we have  $e \leq d$ , and
- $H$  traverses the history to  $d$ .

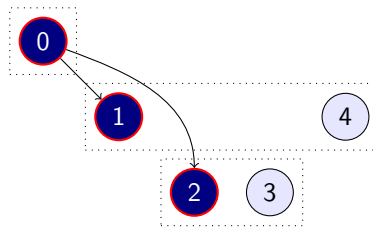
Define  $H$  *traverses the history by*  $-\delta$  *to*  $H'$  when:

- the joint session past and active documents of  $H$  are  $d_1 > \dots > d_\delta > \dots$ ,
- there is some  $H = H_0, \dots, H_\delta = H'$ , such that
- $H_{i-1}$  traverses the history from  $d_i$  in  $H_i$  for each  $1 \leq i \leq \delta$ . □

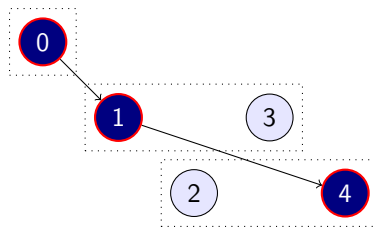
For example, to traverse the history by  $-1$  from:



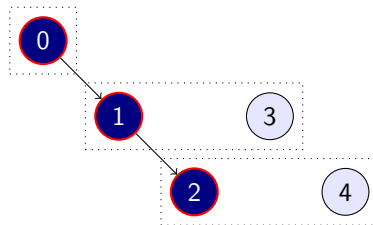
we find the joint session past and active documents (which is  $4 > 2 > 1 > 0$ ) and traverse the history from the first item (which is 4) which is the same as traversing the history to 1:



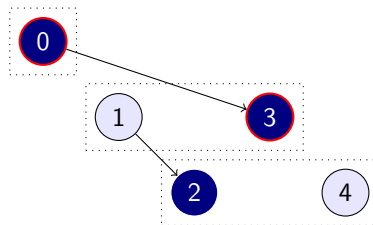
**Counterexample 4:** Let  $H$  be:



which traverses the history by  $-1$  to:

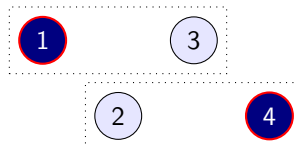


which traverses the history by  $1$  to:

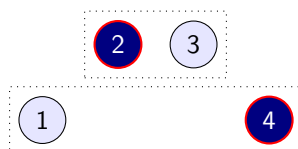


which is not the same as  $H$ . □

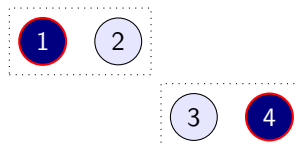
The problem here is not the definition of ‘traversing by  $\delta$ ’, but the definition of navigation histories themselves. They allow for states such as  $H$  from Counterexample 4, which includes the problematic documents:



There are similar problems with documents:



and with documents:

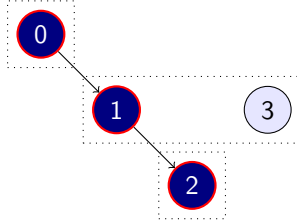


It turns out that these are the only remaining cause of counterexamples, and we will call examples like this not *well-formed*.

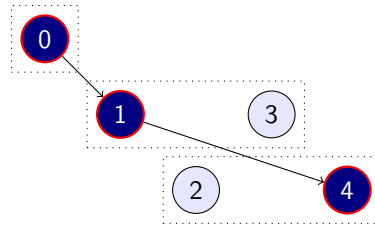
**Definition 5 (Well-formed):** A navigation history is *well formed* whenever for any  $a \lesssim b$  and  $c \lesssim d$ , if  $a \in A$  and  $d \in A$  then  $d \leq b$ .

We have that traversal preserves being well-formed: if  $H$  is well-formed, and  $H$  traverses by  $\delta$  to  $H'$ , then  $H'$  is well-formed. Unfortunately, this is not true for navigation, because of the way it clears the session future.

**Counterexample 5:** Let  $H$  be the well-formed history:



which navigates from 2 to:



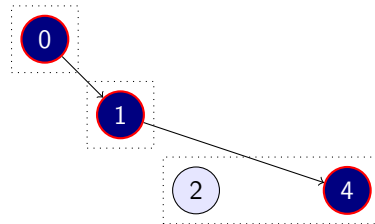
which is not well-formed. □

Fortunately, we can patch navigation to address this, by requiring that we clear the entire joint session future, not just the session future of the document being navigated from.

**Patch 4 (Navigation deletes joint session future):** Define *navigating from  $d$  to  $d'$  in  $H$* , where  $d \in FA$  to be the result of:

- deleting the joint session future, and then
- replacing  $d$  by  $d'$ . □

For example in Counterexample 5, navigation from 2 now results in the well-formed history:



With these patches, we can prove the fundamental property of traversal.

**Theorem 1:** For any well-formed navigation history  $H$ , if  $H$  traverses the history by  $\delta$  to  $H'$  and  $H'$  traverses the history by  $\delta'$  to  $H''$  then  $H$  traverses the history by  $\delta + \delta'$  to  $H''$ .

**Proof:** In this paper, we give a proof sketch. The full details have been Mechanically verified in Agda [7]. Define:

- the *back target* of a navigation history  $H$  to be the  $\leq$ -largest  $b \in A$  such that there is some  $a \in D$  with  $a \lesssim b$ ,
- the *forward target* of  $H$  to be the  $\leq$ -smallest  $f \in D$  such that there is some  $e \in A$  with  $e \lesssim f$ .

We then show some lemmas:

1.  $H$  traverses by  $+(\delta + 1)$  to  $H'$  if and only if  $H$  traverses to  $f$ , then traverses by  $+\delta$  to  $H'$ .
2.  $H$  traverses by  $-(\delta + 1)$  to  $H'$  if and only if  $H$  traverses from  $b$ , then traverses by  $-\delta$  to  $H'$ .
3. If  $H$  is well-formed and  $H$  traverses to  $f$  with result  $H'$ , then  $f$  is the back target of  $H'$ , and  $H'$  traverses from  $f$  with result  $H$ .
4. If  $H$  is well-formed and  $H$  traverses from  $b$  with result  $H'$ , then  $b$  is the forward target of  $H'$ , and  $H'$  traverses to  $b$  with result  $H$ .
5. If  $H$  is well-formed and  $H$  traverses to  $f$  to  $H'$ , then  $H'$  is well-formed.
6. If  $H$  is well-formed and  $H$  traverses from  $b$  to  $H'$ , then  $H'$  is well-formed.

The result is then an induction on  $\delta$ . □

### 3. Experiments

In this section, we summarize our experiments to validate the conformance of browser implementations with respect to the WHATWG specification, to our proposed changes, and to each other. The full details are in Appendix A.

[Present one experiment in detail.]

### 4. Specification

[TODO: wordsmith the first patch.]

In §7.7.2, remove the definition of “joint session history” and “current entry of the joint session history”, and add the following:

The **session past** of a browsing context is the entries of the session history added before the current entry (and does not include the current entry).

The **session future** of a browsing context is the entries of the session history added after the current entry (and does not include the current entry).

If an entry has a next entry in the chronologically ordered session history, it is its **successor**.

**Note:** every entry in the session past has a successor.

If an entry has a previous entry in the chronologically ordered session history, it is its **predecessor**.

**Note:** every entry in the session future has a predecessor.

The **joint session past** of a top-level browsing context is the union of all the session pasts of all browsing contexts that share that top-level browsing context.

Entries in the joint session past are in decreasing chronological order of the time they were added to their respective session histories. Each entry has an index;

the latest entry has index  $-1$ , and the subsequent entries are numbered with consecutively decreasing integers ( $-2$ ,  $-3$ ,  $-4$ , etc).

The **joint session future** of a top-level browsing context is the union of all the session futures of all browsing contexts that share that top-level browsing context.

Entries in the joint session future are in increasing chronological order of the time their predecessor were added to their respective session histories. Each entry has an index; the earliest entry has index 1, and the subsequent entries are numbered with consecutively increasing integers (2, 3, 4, etc).

The definition of “traverse the history by a delta” begins:

To **traverse the history by a delta** *delta*, the user agent must append a task to this top-level browsing context’s session history traversal queue, the task consisting of running the following steps:

1. If the index of the current entry of the joint session history plus *delta* is less than zero or greater than or equal to the number of items in the joint session history, then abort these steps.
2. Let *specified entry* be the entry in the joint session history whose index is the sum of *delta* and the index of the current entry of the joint session history. . .

We propose replacing this text with the following:

To **traverse the history by a delta** *delta*, the user agent must append a task to this top-level browsing context’s session history traversal queue, the task consisting of running the following steps:

1. If *delta* is zero, then abort these steps.
2. If *delta* is positive, let *specified entry* be the entry in the joint session future whose index is *delta*. If there is no such entry, then abort these steps.
3. If *delta* is negative, let *specified entry* be the entry in the joint session past whose index is *delta*. If there is no such entry, then abort these steps. . .

## 5. Conclusion

[We did stuff.]

## References

- [1] Minmin Han and Christine Hofmeister. Modeling and verification of adaptive navigation in web applications. In *Proc. Int. Conf. Web Engineering*, pages 329–336, 2006.
- [2] May Haydar. Formal framework for automated analysis and verification of web-based applications. In *Proc. IEEE Int. Conf. Automated Software Engineering*, pages 410–413, 2004.
- [3] May Haydar, Alexandre Petrenko, and Houari Sahraoui. Formal verification of web applications modeled by communicating automata. In *Proc. Int. Conf. Formal Techniques for Networked and Distributed Systems*, pages 115–132, 2004.

- [4] Karl R. P. H. Leung, Lucas Chi Kwong Hui, S. M. Yiu, and Ricky W. M. Tang. Modeling web navigation by statechart. In *Proc. Int. Computer Software and Applications Conf.*, pages 41–47, 2000.
- [5] WHATWG. HTML living standard. <https://html.spec.whatwg.org/>, 2016.
- [6] Marco Winckler and Philippe Palanque. StateWebCharts: A formal description technique dedicated to navigation modelling of web applications. In *Proc. Int. Workshop Interactive Systems. Design, Specification, and Verification*, pages 61–76, 2003.
- [7] Mechanical verification of navigation history. Agda formalization, 2016. <https://github.com/ConnorGBrewster/ServoNavigation/tree/master/notes>.

## A. Experiments

**Experiment 1:** In this experiment Goal ?? is tested.

- $H$  traverses the history by  $-4$  to  $H'$
- $H'$  traverses the history by  $+4$  to  $H''$

By Goal ??, these traversals should be the same thing as  $H$  traversing by 0 which is a no-op; therefore,  $H = H''$ .

Firefox:

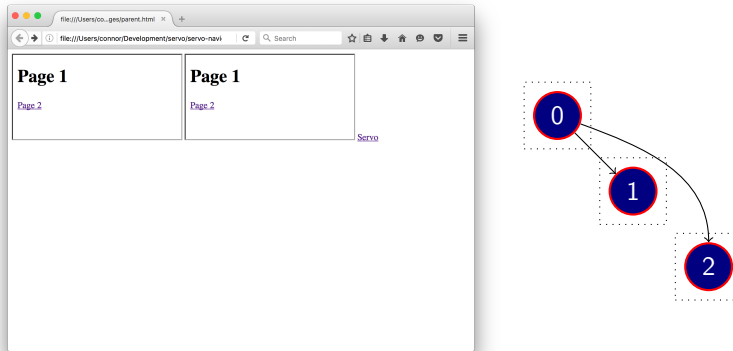


Figure 1: Initial State

Navigate document 1 to Page 2:

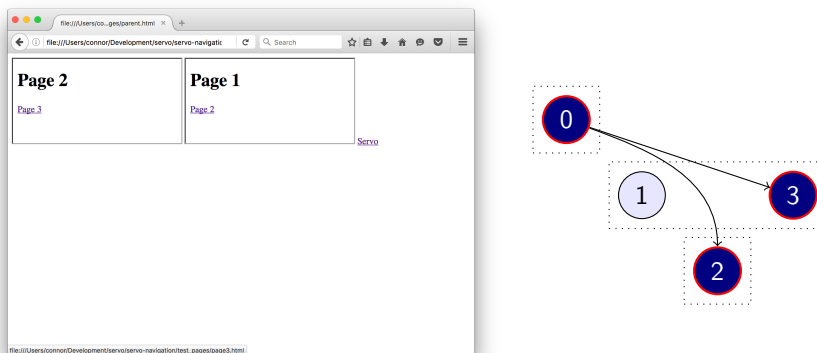


Figure 2: Navigate document 1 to Page 2.

Navigate document 3 to Page 3:



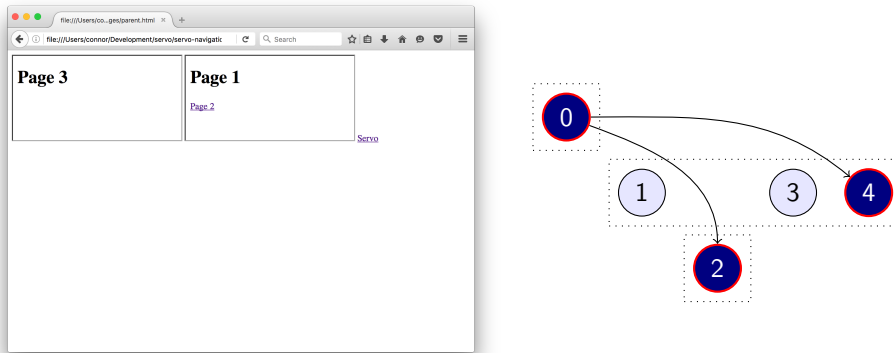


Figure 3: Navigate document 3 to Page 3.

Navigate document 2 to Page 2:

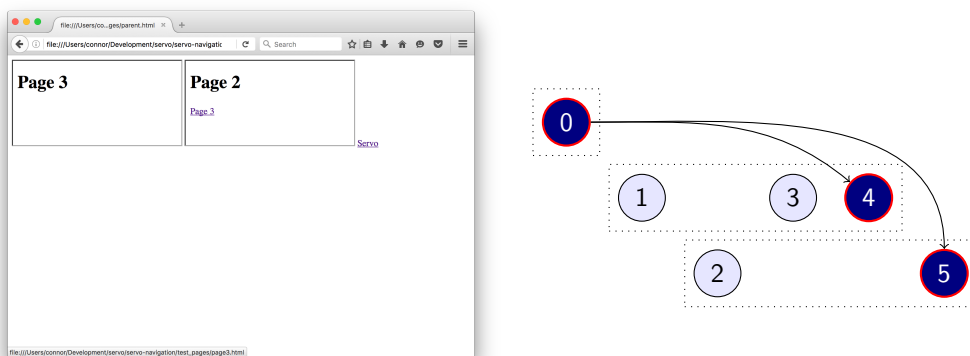


Figure 4: Navigate document 2 to Page 2.

Navigate document 5 to Page 3:

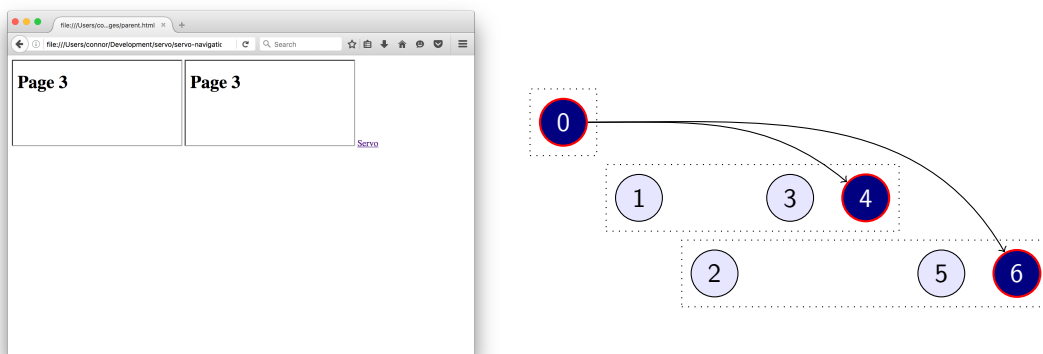
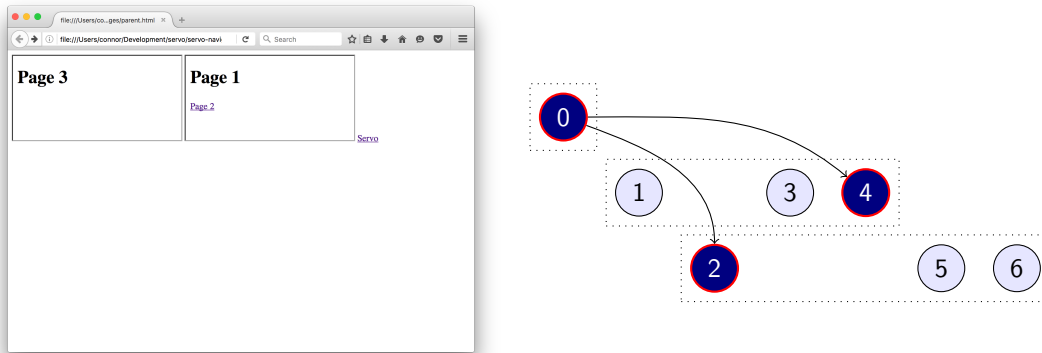


Figure 5: Navigate document 5 to Page 3.

$H$  traverses the history by  $-4$  to  $H'$ :

Figure 6: Traversal by  $-4$ .

This state is obviously wrong, as document 4 should have traversed to document 1. This is similar to counterexample 1.

$H'$  traverses the history by 4 to  $H''$ :

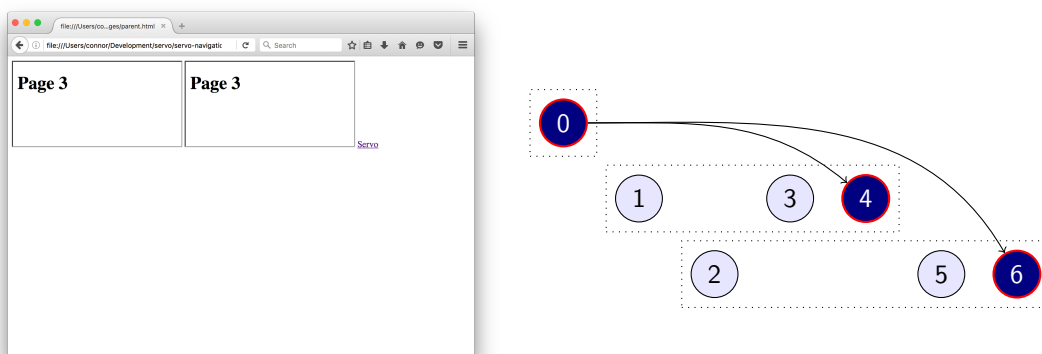


Figure 7: Traversal by 4.

While this result does satisfy Goal ??, there are still some issues:

- Figure 6 yields an incorrect traversal. [CGB: I believe this actually does break Goal ?? as navigating by  $-1$  four times should yield the correct state.]
- It is impossible to get back to Page 1 on both Frames. [CGB: Looks to be a bug in FF, when holding down the back button, the list of pages to traverse to shows up. Clicking on the oldest item on the list does nothing and does not activate that item.]

Safari:

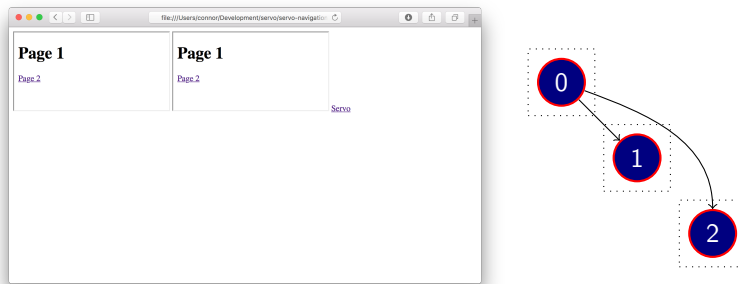


Figure 8: Initial State

Navigate document 1 to Page 2:

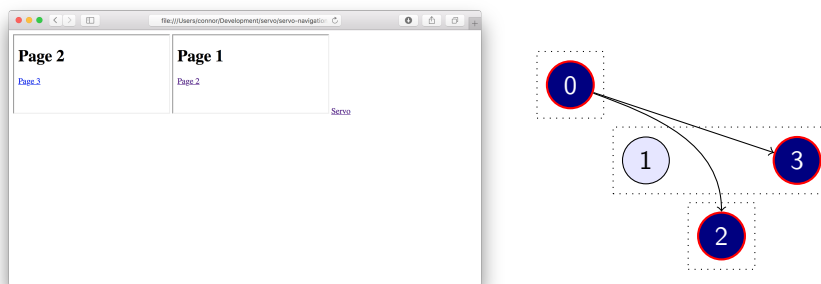


Figure 9: Navigate document 1 to Page 2.

Navigate document 3 to Page 3:

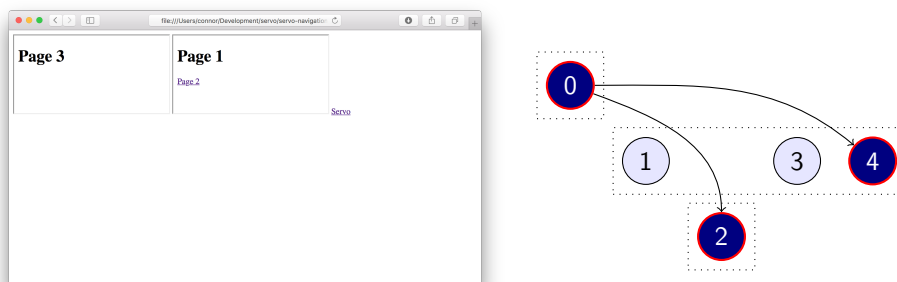


Figure 10: Navigate document 3 to Page 3.

Navigate document 2 to Page 2:

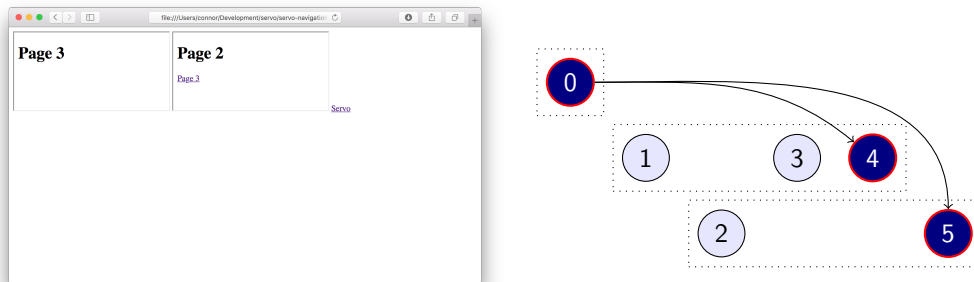


Figure 11: Navigate document 2 to Page 2.

Navigate document 5 to Page 3:

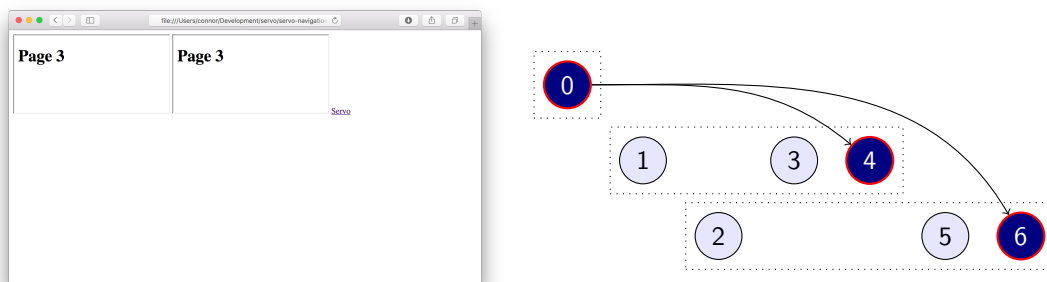
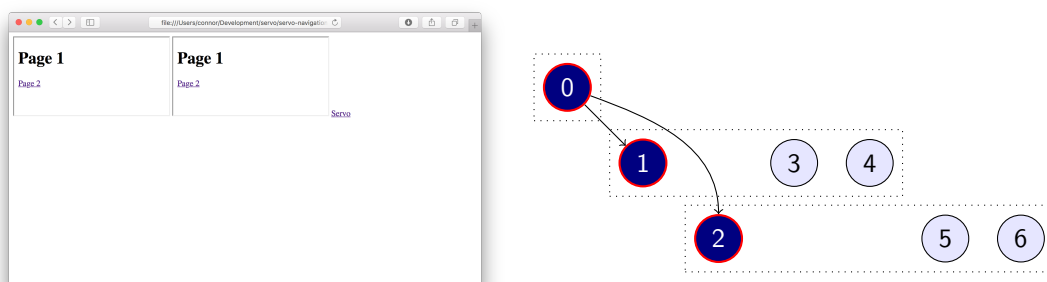


Figure 12: Navigate document 5 to Page 3.

$H$  traverses the history by  $-4$  to  $H'$ :

Figure 13: Traversal by  $-4$ .

$H'$  traverses the history by  $4$  to  $H''$ :

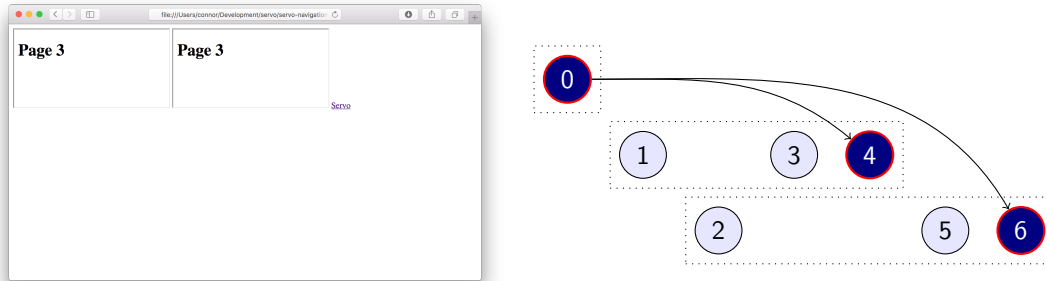


Figure 14: Traversal by 4.

These results in Safari satisfy Goal ??.  
Chrome:

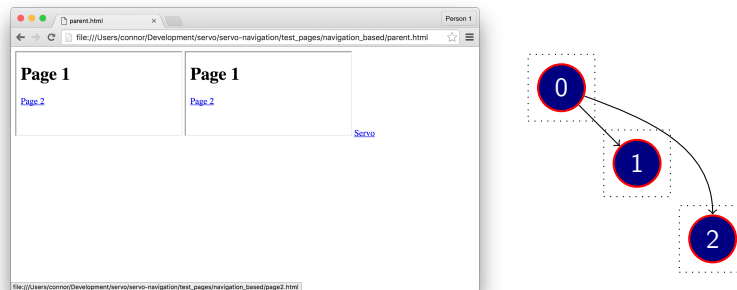


Figure 15: Initial State

Navigate document 1 to Page 2:

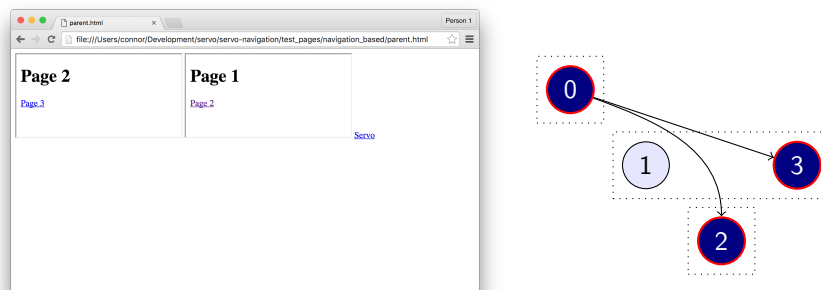


Figure 16: Navigate document 1 to Page 2.

Navigate document 3 to Page 3:

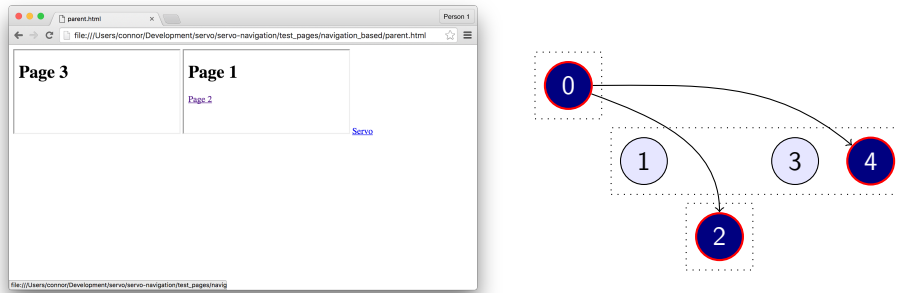


Figure 17: Navigate document 3 to Page 3.

Navigate document 2 to Page 2:

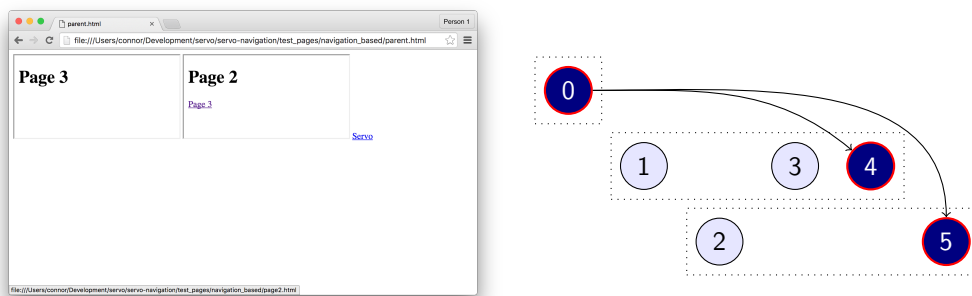


Figure 18: Navigate document 2 to Page 2.

Navigate document 5 to Page 3:

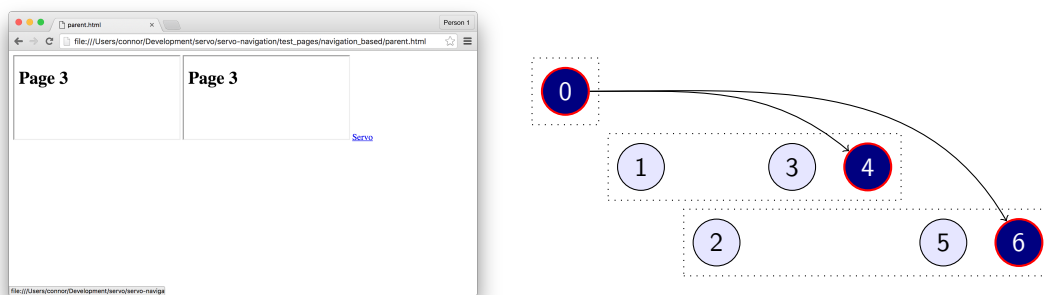
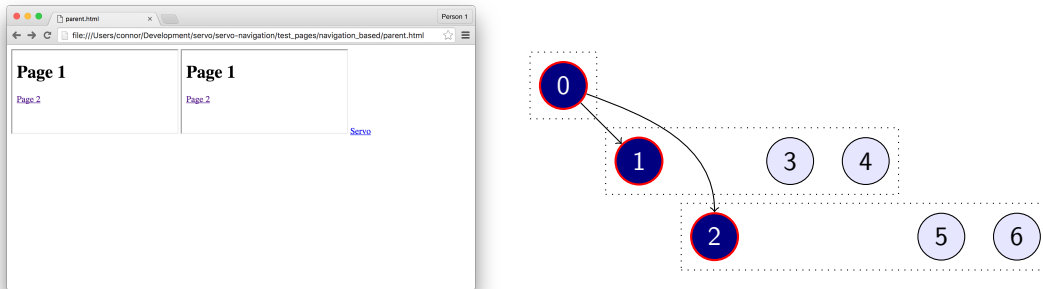


Figure 19: Navigate document 5 to Page 3.

$H$  traverses the history by  $-4$  to  $H'$ :

Figure 20: Traversal by  $-4$ .

$H'$  traverses the history by 4 to  $H''$ :

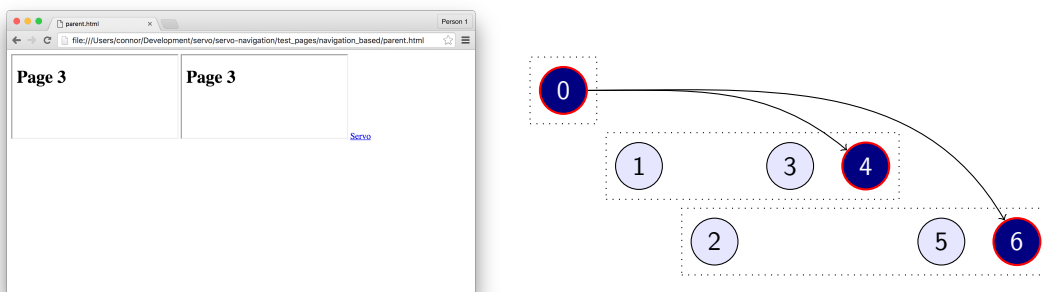


Figure 21: Traversal by 4.

These results in Chrome satisfy Goal ??.

**Experiment 2:** In this experiment *pushState* and *replaceState* traversals are tested against Goal ??.

Firefox:

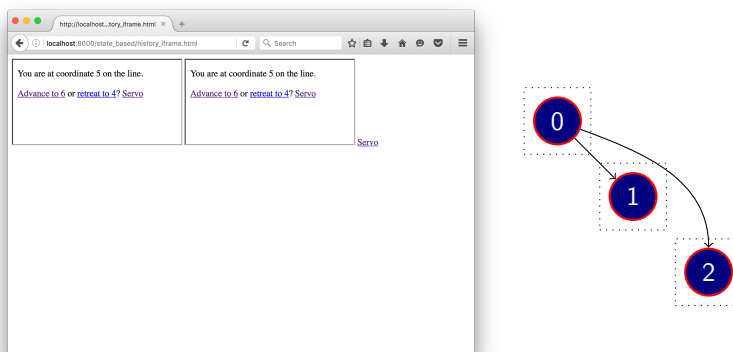


Figure 22: Initial State

Advance document 1 to 6:

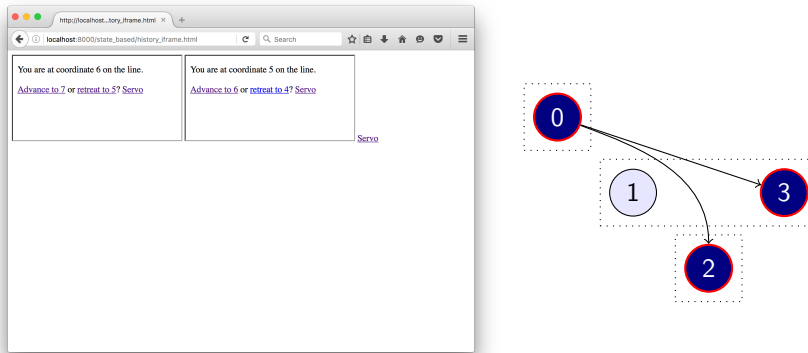


Figure 23: Advance document 1 to 6

Advance document 3 to 7:

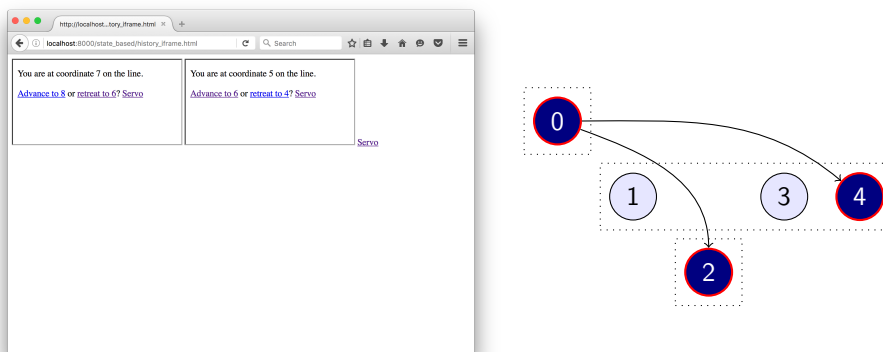


Figure 24: Advance document 3 to 7

Advance document 2 to 6:

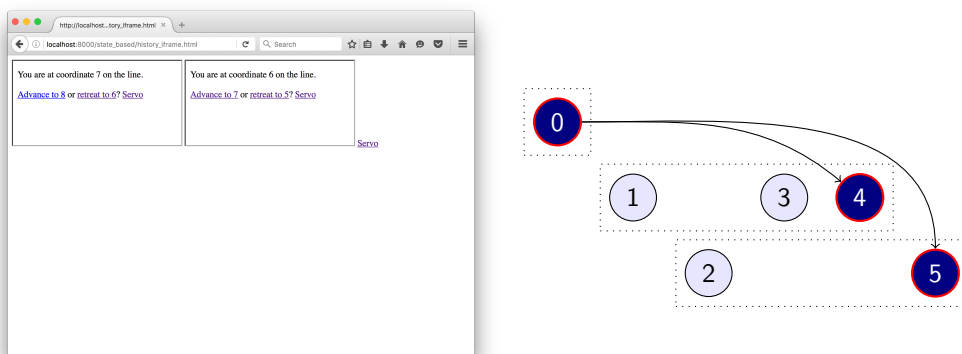


Figure 25: Advance document 2 to 6

Advance document 2 to 7:



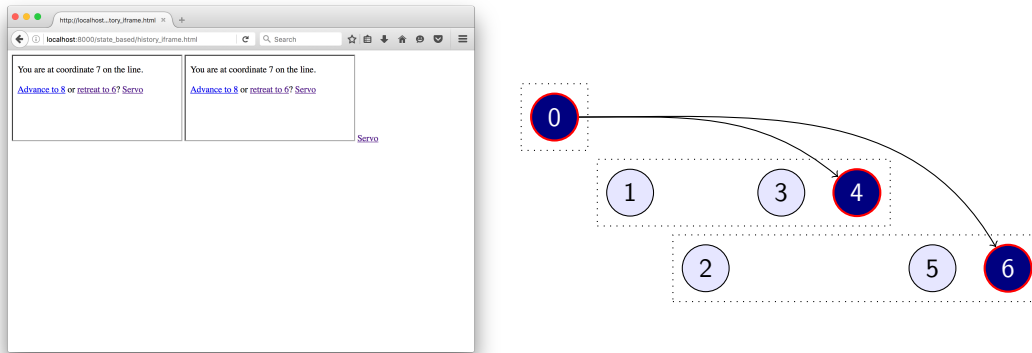
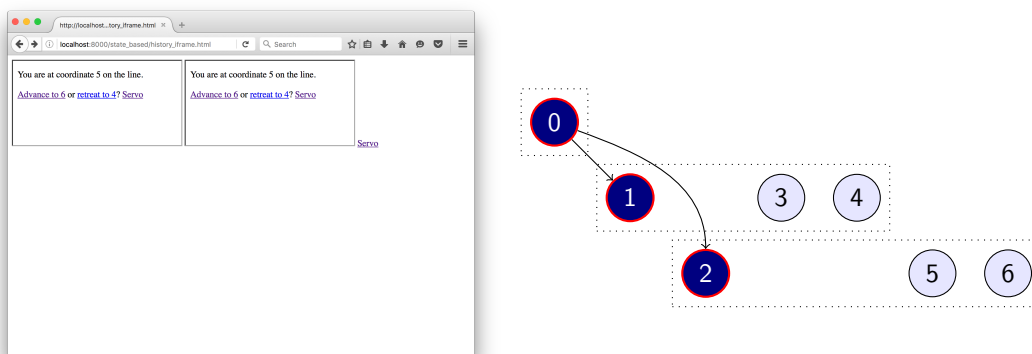
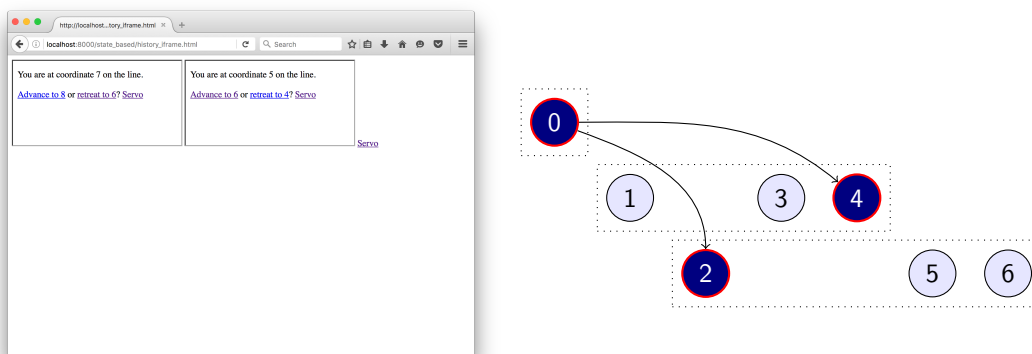


Figure 26: Advance document 2 to 7

Traverse  $H$  by  $-4$ :

Figure 27: Traverse  $H$  by  $-4$ 

Traverse  $H$  by 4:

Figure 28: Traverse  $H$  by 4

The last traversal does not satisfy Goal ??.

Safari:

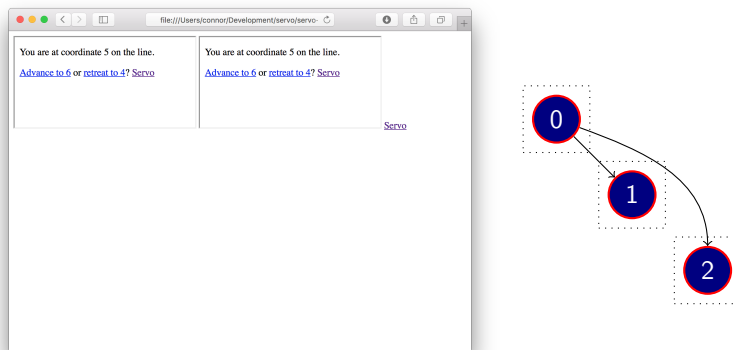


Figure 29: Initial State

Advance document 1 to 6:

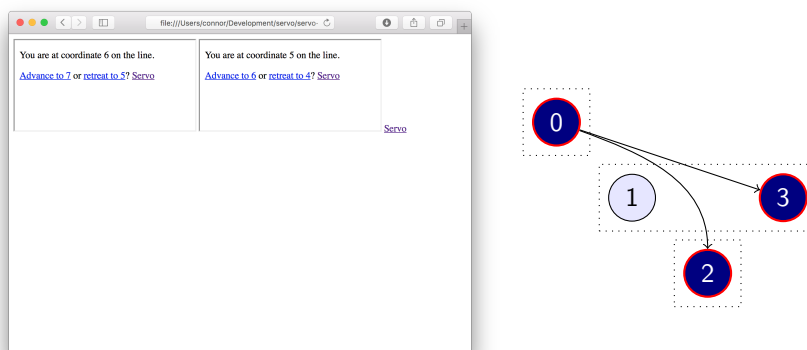


Figure 30: Advance document 1 to 6

Advance document 3 to 7:

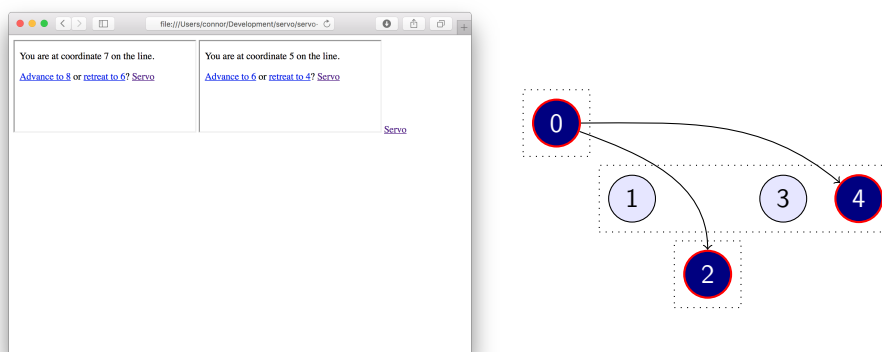


Figure 31: Advance document 3 to 7

Advance document 2 to 6:

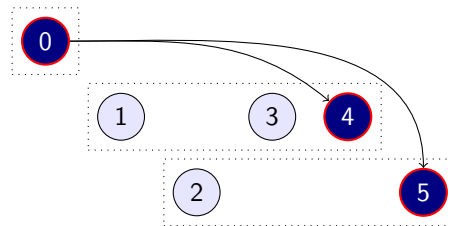
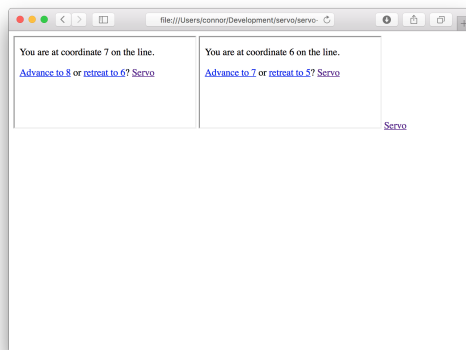


Figure 32: Advance document 2 to 6

Advance *document2* to 7:

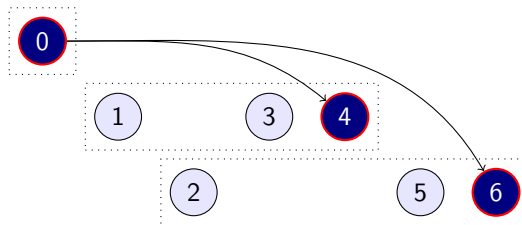
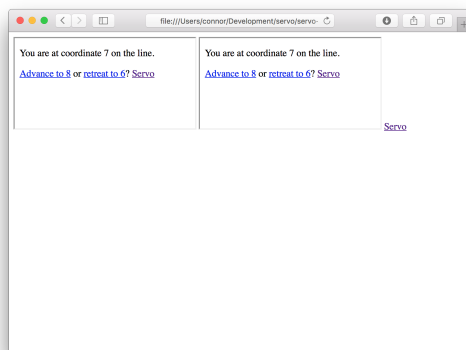


Figure 33: Advance document 2 to 7

Traverse  $H$  by  $-4$ :

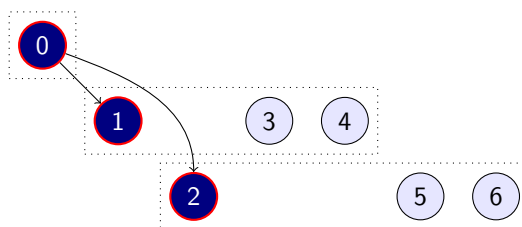
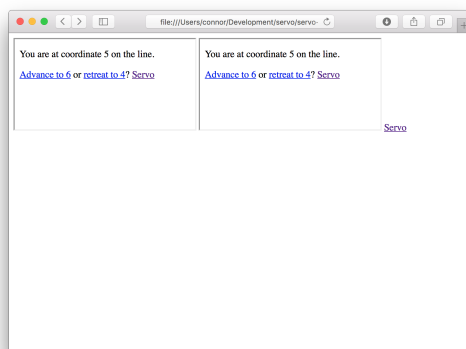


Figure 34: Traverse  $H$  by  $-4$

Traverse  $H$  by 4:

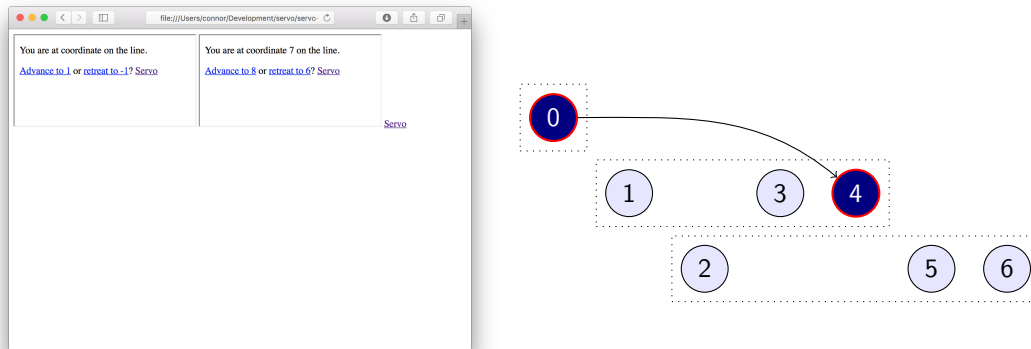


Figure 35: Traverse  $H$  by 4

In this traversal, we are unable to determine the active entry for first entry as the state is *null*. This traversal does not satisfy Goal ??.

Chrome:

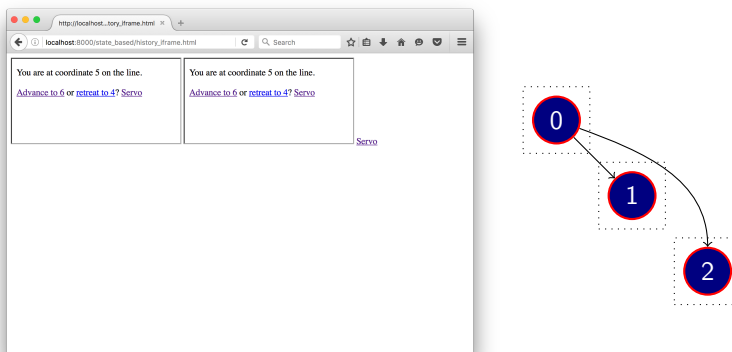


Figure 36: Initial State

Advance document 1 to 6:

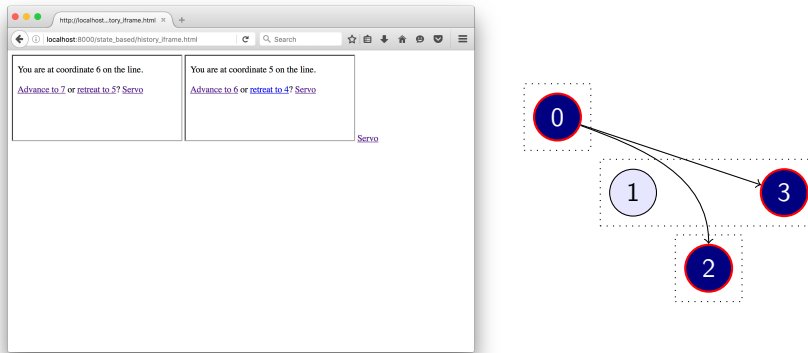


Figure 37: Advance document 1 to 6

Advance document 3 to 7:

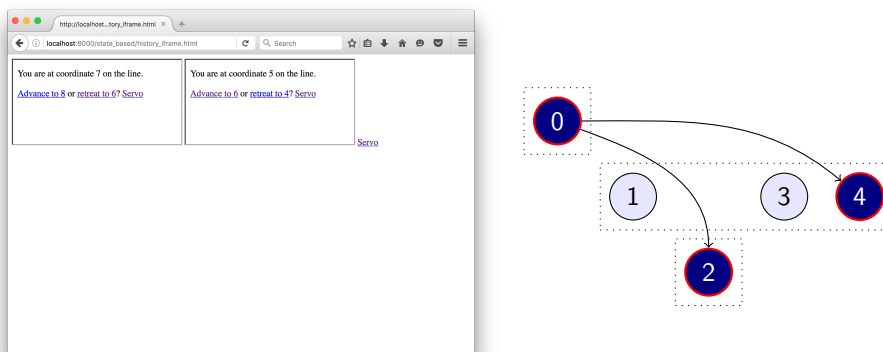


Figure 38: Advance document 3 to 7

Advance document 2 to 6:

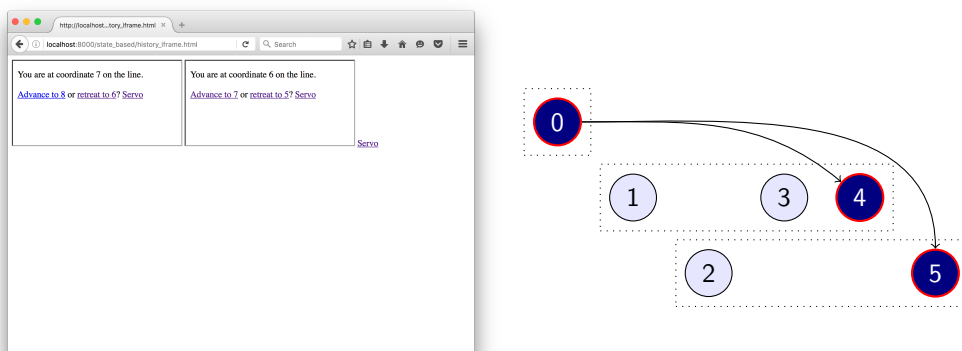


Figure 39: Advance document 2 to 6

Advance document 2 to 7:

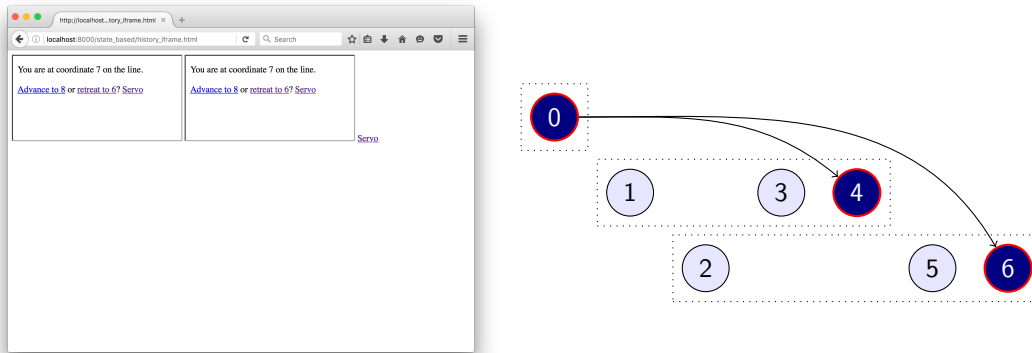
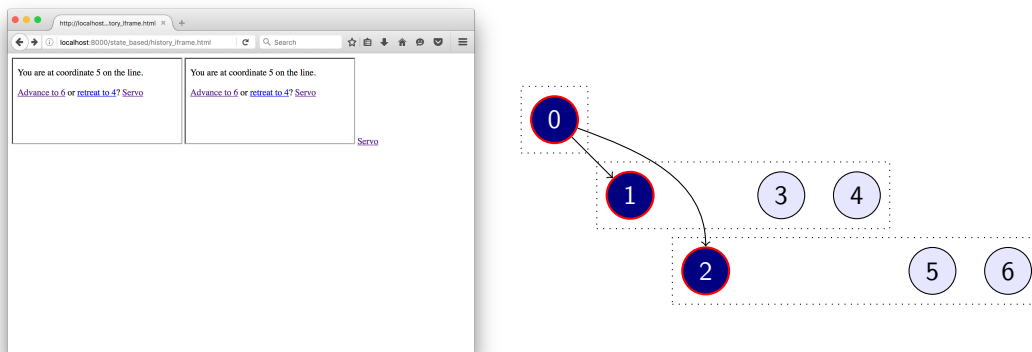
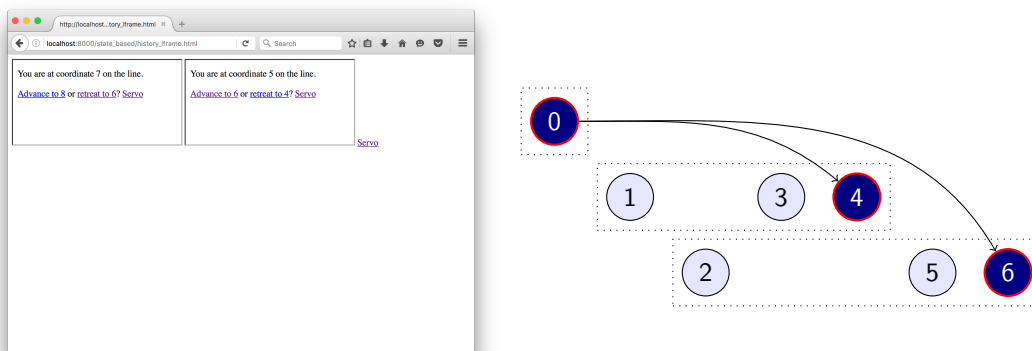


Figure 40: Advance document 2 to 7

Traverse  $H$  by  $-4$ :

Figure 41: Traverse  $H$  by  $-4$ 

Traverse  $H$  by 4:

Figure 42: Traverse  $H$  by 4

In this experiment, Chrome meets Goal ??.