

# Notes on navigation history

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## Abstract

Some notes on a model of navigation history.

## 1 Introduction

[These are rough notes, working towards a model of navigation history for the web.]

## 2 Preliminaries

[Define forest, tree, root, total order, equivalence.]

## 3 Model

A *navigation history*  $H = (D, A, \rightarrow, \leq, \sim)$  consists of:

- a set  $D$  (the *documents*),
- a subset  $A \subseteq D$  (the *active* documents),
- a forest  $(D, \rightarrow)$  (the *document hierarchy*),
- a total order  $(D, \leq)$  (the *chronological order*), and
- an equivalence relation  $(D, \sim)$  (the *same-session equivalence*).

such that:

- for every  $d$  there is a unique  $d' \in A$  such that  $d \sim d'$ ,
- for every  $d \rightarrow e \sim e'$  we have  $d \rightarrow e'$ , and
- for every  $d \rightarrow e$ , we have  $d \leq e$ .

Define:

- $d_0$  is the unique active root document,

- $d \rightarrow e$  when  $d \rightarrow e$  and  $e \in A$ ,
- $FA = \{d \mid d_0 \rightarrow^* d\}$  (the *fully active* documents),
- $d \lesssim e$  whenever  $d \sim e$  and  $d < e$ ,
- the *session future* of  $d$  is  $\{e \mid d \lesssim e\}$ ,
- the *session past* of  $d$  is  $\{e \mid d \gtrsim e\}$ ,
- the *joint session future* is  $\{e \mid \exists d \in FA . d \lesssim e\}$ ,
- the *joint session past* is  $\{e \mid \exists d \in FA . d \gtrsim e\}$ ,

Define *deleting  $d$  from  $H$* , when  $d \notin FA$ , to be  $H'$  where:

- $D' = D \setminus \{e \mid d \rightarrow^* e\}$ ,
- $e \in A'$  whenever  $e \in A$ ,
- $e \leq' f$  whenever  $e \leq f$ ,
- $e \rightarrow' f$  whenever  $e \rightarrow f$ , and
- $e \sim' f$  whenever  $e \sim f$ .

Define *replacing  $d$  by  $d'$  in  $H$* , where  $d \in A$  and  $d' \notin D$ , to be  $H'$  where:

- $D' = D \cup \{d'\}$ ,
- $e \in A'$  whenever  $e \in A$  and  $e \neq d$ , or  $e = d'$ ,
- $e \leq' f$  whenever  $e \leq f$ , or  $f = d'$ ,
- $e \rightarrow' f$  whenever  $e \rightarrow f$ , or  $e \rightarrow d$  and  $f = d'$ , and
- $e \sim' f$  whenever  $e \sim f$ , or  $e \sim d$  and  $f = d'$ , or  $d \sim f$  and  $e = d'$ .

Define *navigating from  $d$  to  $d'$  in  $H$*  to be the result of:

- deleting the session future of  $d$ , and
- replacing  $d$  by  $d'$ .

Define *traversing the history to  $d$  in  $H$*  to be  $H'$  where:

- $D'$  is  $D$ ,
- $e \in A'$  whenever  $d \not\prec e \in A$ , or  $d = e$ ,
- $e \leq' f$  whenever  $e \leq f$ ,
- $e \rightarrow' f$  whenever  $e \rightarrow f$ , and
- $e \sim' f$  whenever  $e \sim f$ .

Define  $H$  traverses the history by  $+\delta$  to  $H'$  when:

- the joint session past of  $H$  is  $d_1 > \dots > d_\delta > \dots$ ,
- $H$  traverses the history to  $d_\delta$  in  $H'$

Define  $H$  traverses the history by  $-\delta$  to  $H'$  when:

- the joint session future of  $H$  is  $d_1 < \dots < d_\delta < \dots$ ,
- $H$  traverses the history to  $d_\delta$  in  $H'$

Define  $H$  traverses the history by 0 to  $H'$  when  $H = H'$ .

[This defin is meant to align with the spec.]

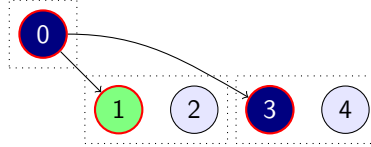
## 4 Properties

[State some goals, e.g.  $\text{go}(\delta);\text{go}(\delta')$  is the same as  $\text{go}(\delta + \delta')$ ,  $\text{navigate};\text{go}(-1)$  has the same fully active documents as doing nothing, session history can be implemented effeciently in memory...]

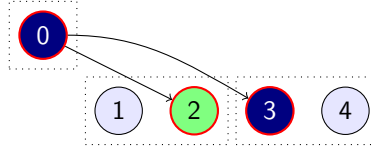
[I suspect none of these are true of the current spec, can we find a model in which they are true?]

**Goal 1** *If  $H$  traverses the history by  $\delta$  to  $H'$  and  $H'$  traverses the history by  $\delta'$  to  $H''$  then  $H$  traverses the history by  $\delta + \delta'$  to  $H''$ .*

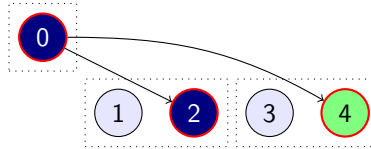
**Counterexample 1** *Let  $H$  be:*



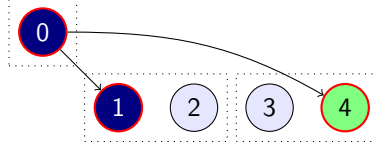
*which traverses the history by 1 to:*



*which traverses the history by 1 to:*



but  $H$  traverses the history 2 to:



This counterexample is caused by the definition of ‘traverses the history by  $\delta$ ’ which only traverses one document’s session history. Instead, we should traverse the history of all  $\delta$  documents.

**Patch 1** Define  $H$  traverses the history by  $+\delta$  to  $H'$  when:

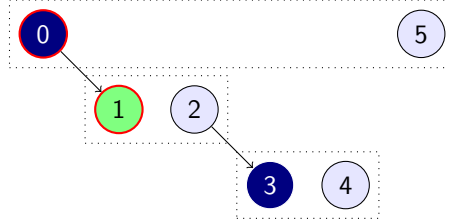
- the joint session future of  $H$  is  $d_1 < \dots < d_\delta < \dots$ ,
- there is some  $H = H_0, \dots, H_\delta = H'$ , such that
- $H_{i-1}$  traverses the history to  $d_i$  in  $H_i$  for each  $1 \leq i \leq \delta$ .

Define  $H$  traverses the history by  $-\delta$  to  $H'$  when:

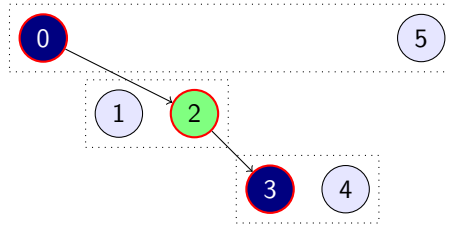
- the joint session past of  $H$  is  $d_1 > \dots > d_\delta > \dots$ ,
- there is some  $H = H_0, \dots, H_\delta = H'$ , such that
- $H_{i-1}$  traverses the history to  $d_i$  in  $H_i$  for each  $1 \leq i \leq \delta$ .

Unfortunately, Goal 1 is not satisfied, even with this patch.

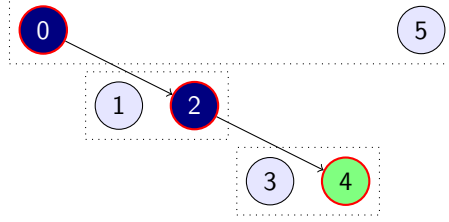
**Counterexample 2** Let  $H$  be:



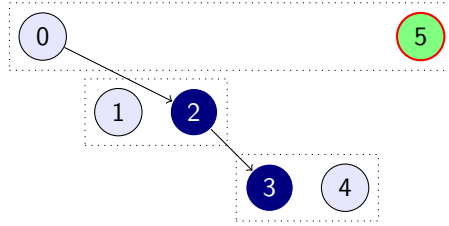
which moves forwards by 1 to:



which in turn moves forwards by 1 to:



but  $H$  goes forward by 2 to:



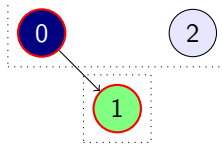
The problem this time is that the definition of ‘joint session history’ only includes the fully active documents, not all active documents.

**Patch 2** Define:

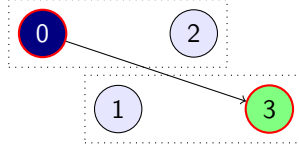
- the joint session future is  $\{e \mid \exists d \in A. d \lesssim e\}$ , and
- the joint session past is  $\{e \mid \exists d \in A. d \gtrsim e\}$ .

**Goal 2** If  $d$  in  $H$  navigates to  $d'$  in  $H'$ , and  $H'$  traverses the history by  $-1$  to  $H''$ , then  $FA = FA''$ .

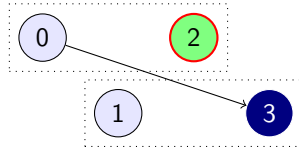
**Counterexample 3**



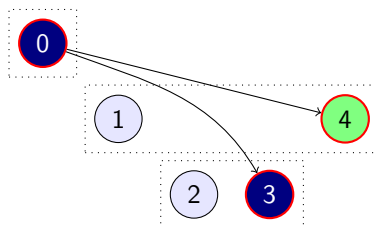
navigates 1 to 3 in:



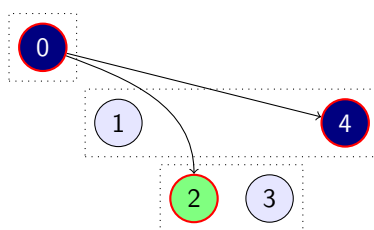
which traverses the history by  $-1$  to:



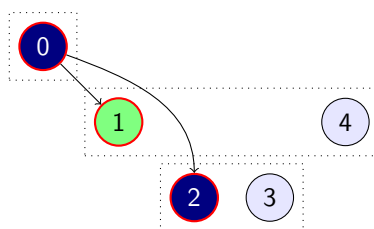
#### Counterexample 4



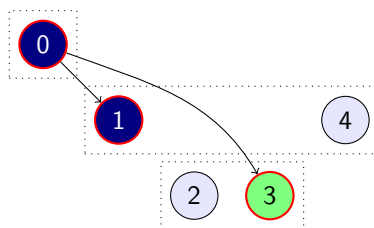
which traverses the history by  $-1$  to:



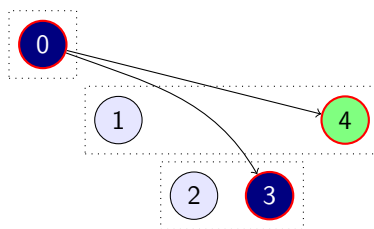
which traverses the history by  $-1$  to:



which traverses the history by  $1$  to:



which traverses the history by  $1$  to:



## 5 Experiments

[A write-up of CGB's experiments with how different browsers handle navigation.]

## 6 Specification

[Suggested edits to the spec: 1. traverse to each document, not just the selected one, 2. keep all documents in the session history, not just the fully active ones, 3. change the session history order.]

## 7 Conclusion

[We did stuff.]