Chapter X - Failure and Fatigue

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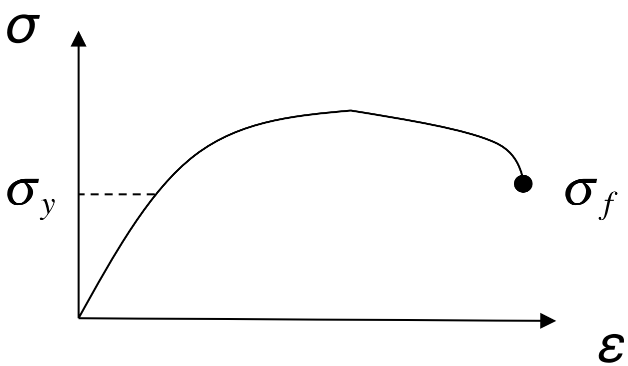
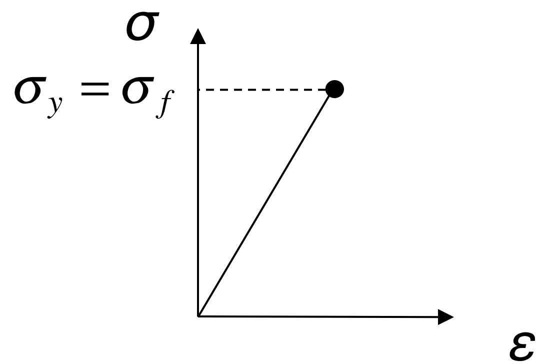
Failure Criteria

- Recall:



Ductile



Brittle

- To decide failure compare **stress field** to a **material property**.

- Need:
  - <u>3D information</u> $\left[\sigma_{ij}\right]=\begin{bmatrix}\sigma_{11} & \sigma_{12} & \sigma_{13}\\ \sigma_{12} & \sigma_{22} & \sigma_{23}\\ \sigma_{13} & \sigma_{23} & \sigma_{33}\end{bmatrix}$
  - <u>Objective</u> (i.e. frame independent) quantities

$\Rightarrow$ **use principal stresses**

$$\sigma_1, \sigma_2, \sigma_3$$

Maximum (tensile) Principal Stress

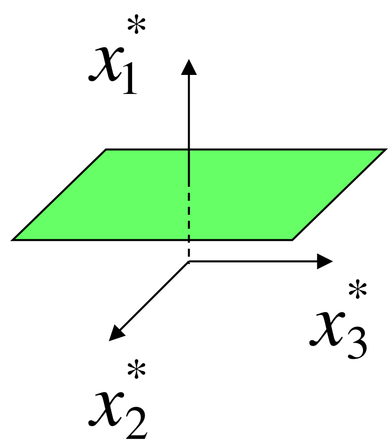
- $\sigma_1 > \sigma_2 > \sigma_3$

- Assume that failure occurs when:

$$\boxed{\sigma_1 \geq \sigma^*}$$

applied stress — material strength

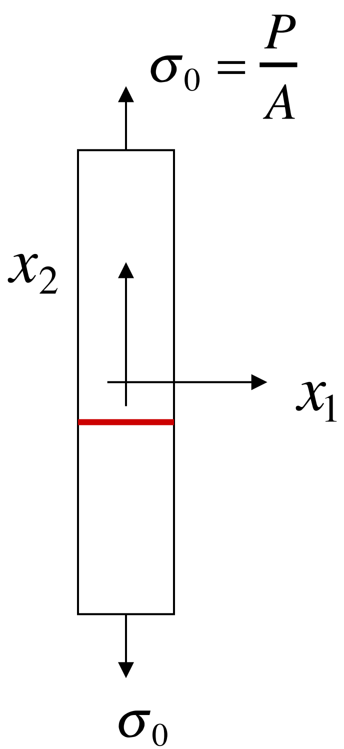
- Location of failure:



$\left(x_1^*, x_2^*, x_3^*\right)$: **Principal frame**

Maximum (tensile) Principal Stress (cont.)

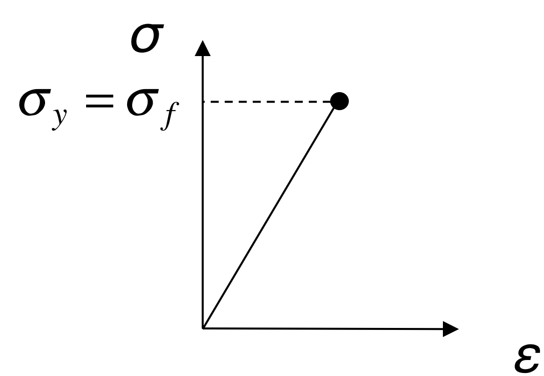
- Example: Simple tension



$$[\sigma] = \begin{bmatrix} 0 & 0 & 0 \\ & \sigma_0 & 0 \\ & & 0 \end{bmatrix} \quad \therefore$$

$\Longrightarrow$ Failure occurs when

$$\sigma_0 = \sigma^* = \sigma_y$$



Maximum Shear Stress (Tresca criterion)

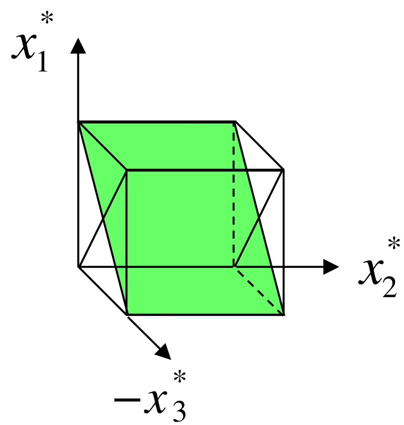
- Recall: $$\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_3)$$

- Assume that failure (yield) occurs when: $$\boxed{\tau_{\max} \geq \tau^*}$$

applied stress

material strength

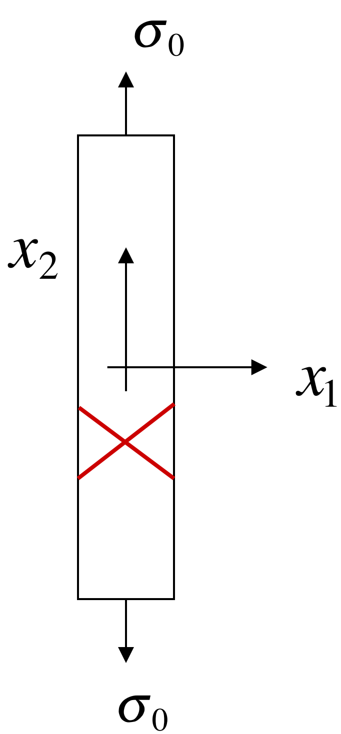
- Location of failure: On plane where τ_{max} acts!



$\left(x_1^*, x_2^*, x_3^*\right)$: **Principal frame**

Maximum Shear Stress (cont.)

- Example: Simple tension

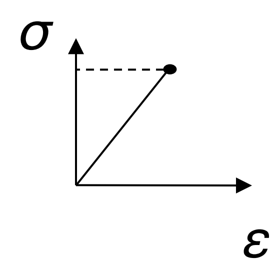


$$[\sigma]=\begin{bmatrix}0 & 0 & 0\\ & \sigma_0 & 0\\ & & 0\end{bmatrix} \Rightarrow \quad \sigma_1=\sigma_0 \quad \sigma_3=0$$

$$\Rightarrow \quad \tau_{\max}=\frac{1}{2}(\sigma_0-0)=\frac{\sigma_0}{2}$$

$$\Rightarrow \quad \text{Failure} \quad \tau_{\max}=\sigma_0/2=\tau^*$$

- From σ-ε curve: $\sigma_0=\sigma_y$ $\quad \boxed{\Rightarrow \quad \tau^*=\sigma_y/2}$



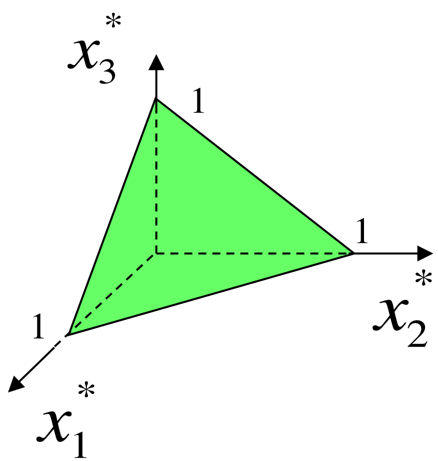
Octahedral Shear Stress (Von Mises criterion)

- Recall: $\sigma_{ns}^{oct} \equiv \tau^{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$

- Assume that failure (yield) occurs when: $\boxed{\sigma_{ns}^{oct} \geq \sigma^{cr}}$

applied stress — material strength

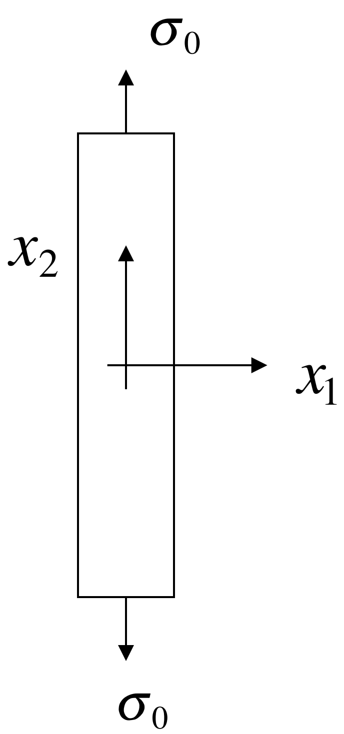
- Location of failure: On octahedral planes!



(x_1^*, x_2^*, x_3^*): **Principal frame**

Octahedral Shear Stress (cont.)

- Example: Simple tension



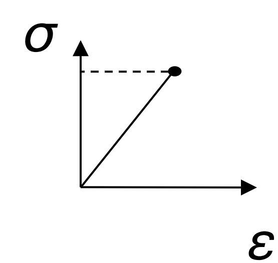
$$[\sigma] = \begin{bmatrix} 0 & 0 & 0 \\ & \sigma_0 & 0 \\ & & 0 \end{bmatrix} \Rightarrow \sigma_1 = \sigma_0, \sigma_2 = \sigma_3 = 0$$

$$\Rightarrow \quad \sigma_{ns}^{oct} = \frac{1}{3}\sqrt{(\sigma_0 - 0)^2 + (0 - \sigma_0)^2} = \frac{\sqrt{2}}{3}\sigma_0$$

$\Rightarrow$ Failure occurs when

$$\sigma_{ns}^{oct} = \frac{\sqrt{2}}{3}\sigma_0 = \sigma^{cr}$$

- From σ-ε curve: $\sigma_0 = \sigma_y$ $\boxed{\Rightarrow \sigma^{cr} = \frac{\sqrt{2}}{3}\sigma_y}$



Chapter X - Failure and Fatigue

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Failure criteria

- Maximum (tensile) principal stress
- Maximum shear stress (Tresca criterion)
- Octahedral shear stress (Von Mises criterion)

★ **Yield surfaces**

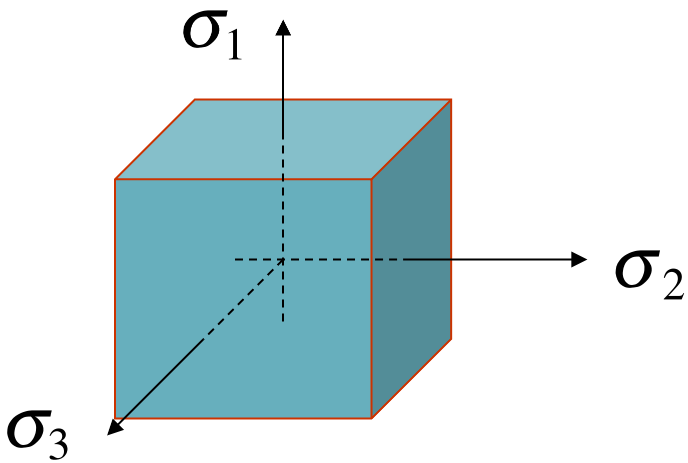
Fatigue

- S-N curves
- Miner’s rule

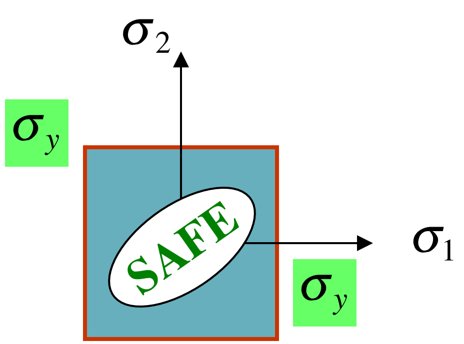
Yield Surfaces

- <u>Maximum (tensile) principal stress</u> $\sigma_1 \geq \sigma^*$

- Failure envelope (**yield surface**) of all possible 3D stress states:



- Assume 2D stress state:



Yield Surfaces (cont.)

- Assume a 2D situation, i.e. $\sigma_3 = 0$
- **Tresca** Condition: $\tau_{\max} = \tau^* = \sigma_y / 2$

Case

① $\sigma_1 > 0, \quad \sigma_2 < 0 \quad \Rightarrow \quad \tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \sigma_y / 2 \quad \Rightarrow \quad \sigma_1 - \sigma_2 = \sigma_y$

② $\sigma_1 < 0, \quad \sigma_2 > 0 \quad \Rightarrow \quad \tau_{\max} = \frac{1}{2}(\sigma_2 - \sigma_1) = \sigma_y / 2 \quad \Rightarrow \quad \sigma_2 - \sigma_1 = \sigma_y$

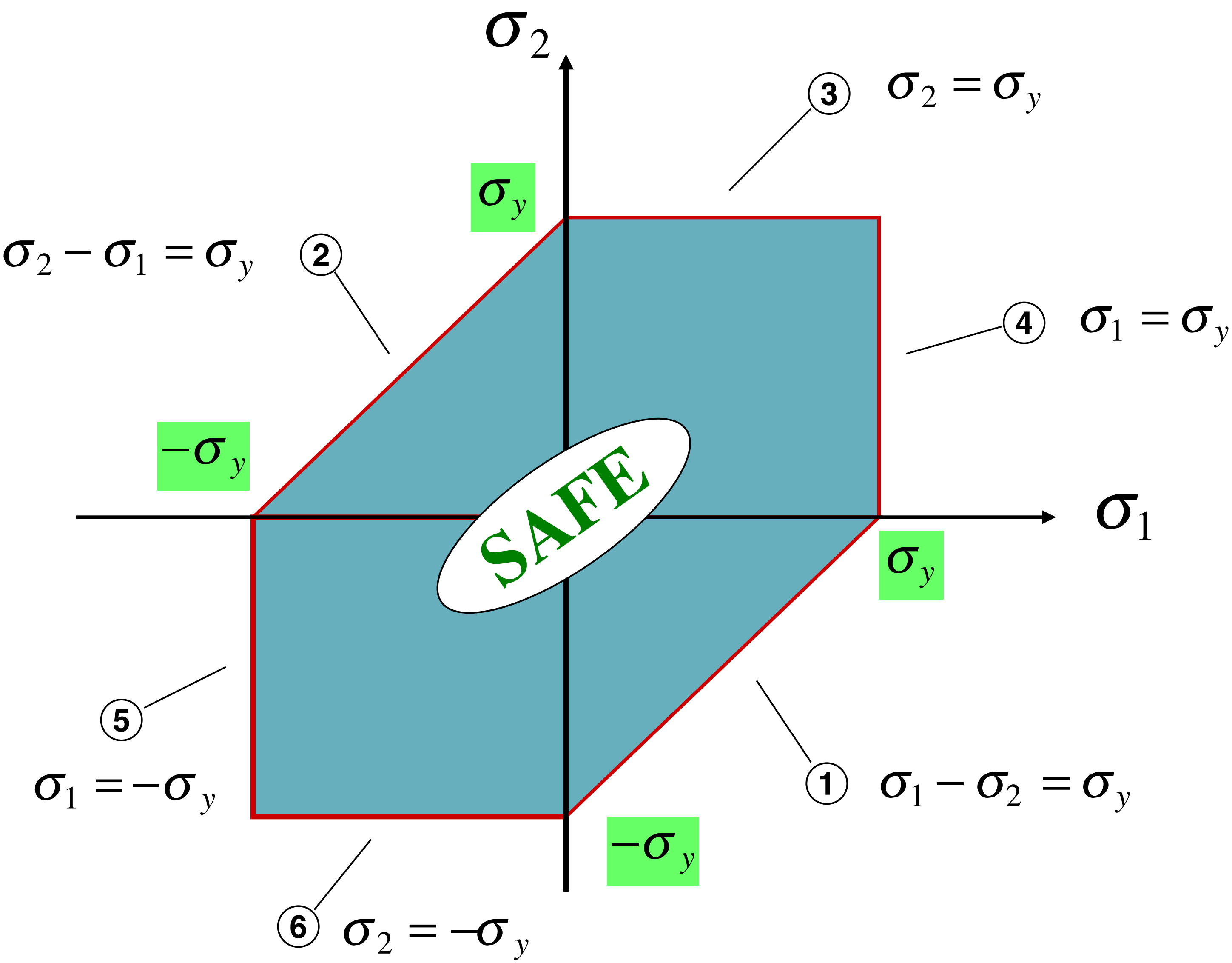
③ $\sigma_2 > \sigma_1 > 0 \quad \Rightarrow \quad \tau_{\max} = \frac{1}{2}(\sigma_2 - 0) = \sigma_y / 2 \quad \Rightarrow \quad \sigma_2 = \sigma_y$

④ $\sigma_1 > \sigma_2 > 0 \quad \Rightarrow \quad \tau_{\max} = \frac{1}{2}(\sigma_1 - 0) = \sigma_y / 2 \quad \Rightarrow \quad \sigma_1 = \sigma_y$

⑤ $\sigma_1 < \sigma_2 < 0 \quad \Rightarrow \quad \tau_{\max} = \frac{1}{2}(0 - \sigma_1) = \sigma_y / 2 \quad \Rightarrow \quad \sigma_1 = -\sigma_y$

⑥ $\sigma_2 < \sigma_1 < 0 \quad \Rightarrow \quad \tau_{\max} = \frac{1}{2}(0 - \sigma_2) = \sigma_y / 2 \quad \Rightarrow \quad \sigma_2 = -\sigma_y$

Yield Surfaces (cont.)



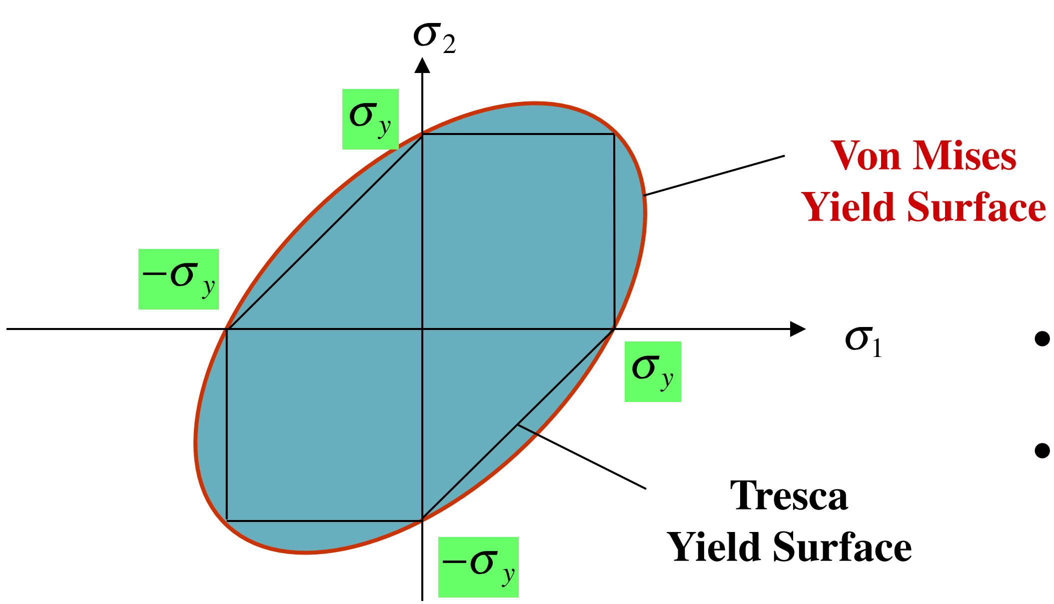
Tresca Yield Surface

Yield Surfaces (cont.)

- Assume a 2D situation, i.e. $\sigma_3 = 0$
- **Von Mises** Condition: $\sigma_{ns}^{oct} = \sigma^{cr} = \sqrt{2}\sigma_y / 3$

$$\Rightarrow \quad \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} = \frac{\sqrt{2}}{3}\sigma_y$$

$$\Rightarrow \quad \sigma_1^2 + \sigma_2^2 + (\sigma_1 - \sigma_2)^2 = 2\sigma_y^2 \qquad \boxed{\Rightarrow \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_y^2}$$



- Tresca: more conservative
- Which is “correct”?

$\Rightarrow$ EXPERIMENTS

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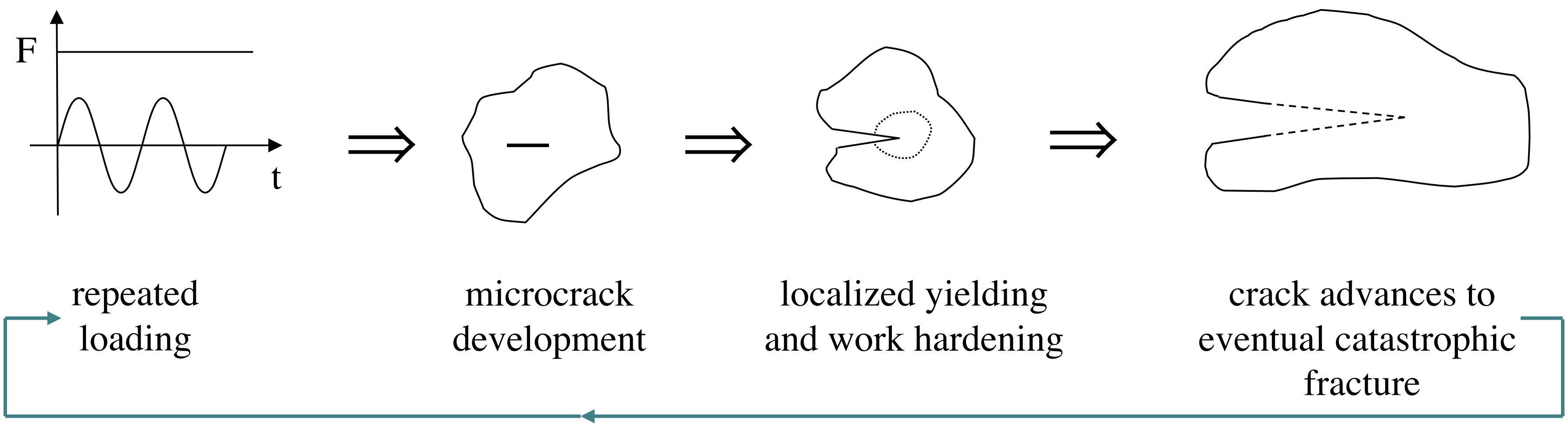
Fatigue

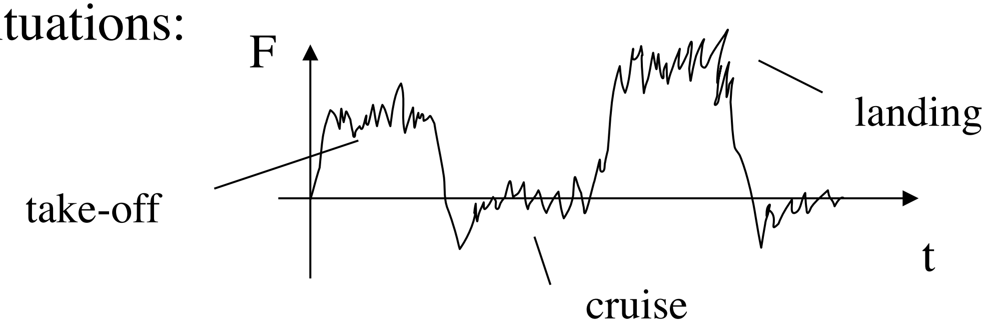
- Repeated loading at loads levels well below critical still leads to failure!

$\Rightarrow$ **FATIGUE**

- Mechanism of fatigue:



- In real situations:



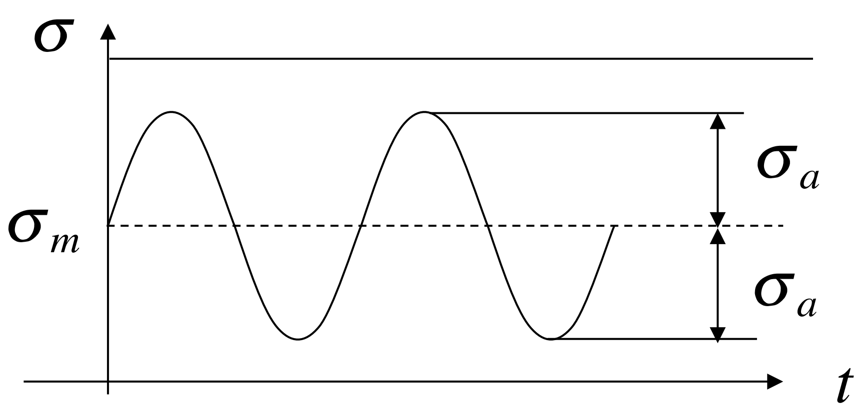
Fatigue (cont.)

- <u>Example:</u> Aloha Airlines 243, April 28, 1988:

Fatigue crack generated from repeated (de)pressurization of the fuselage.

S-N Curves

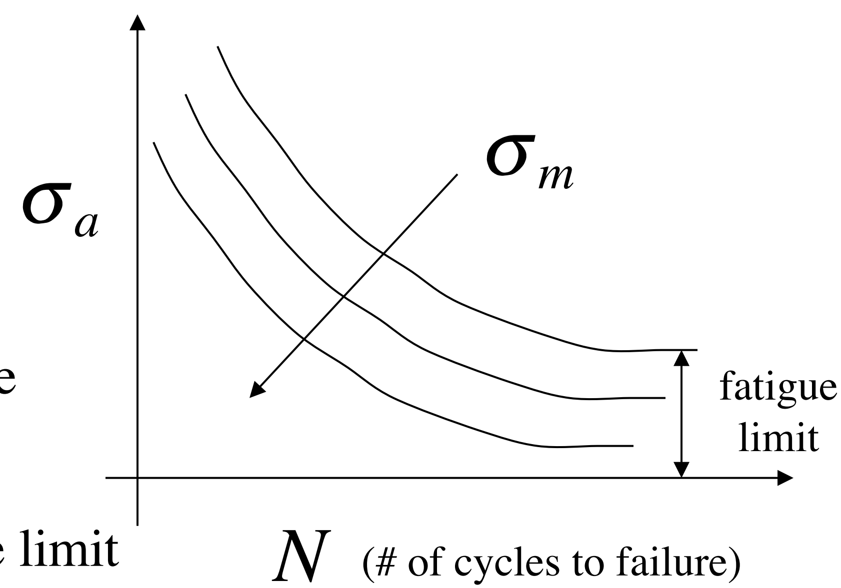
- Since exact analysis of actual loading is very difficult
  → use "fracture mechanics approach"
- Consider periodic loading:



σ_m : mean stress

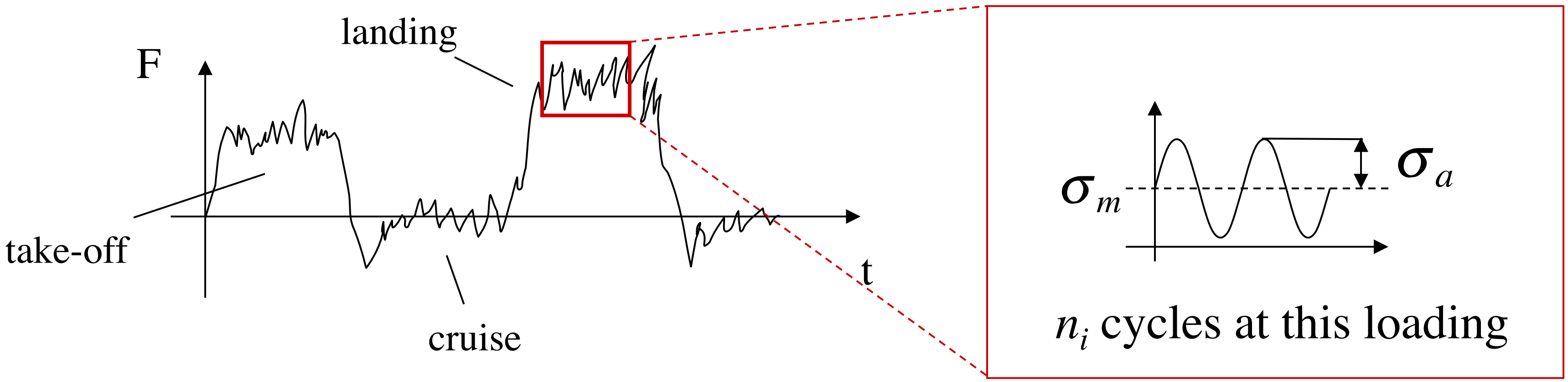
σ_a : alternating stress

- Count number of cycle to failure, N:

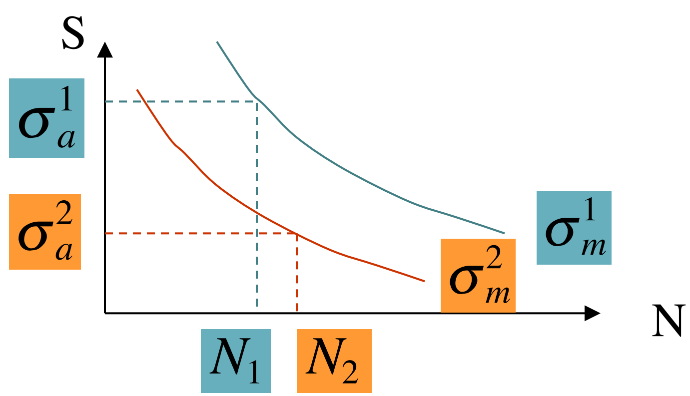


- **S-N curve**: stress level vs. cycles to failure
- <u>Note:</u> Aluminum does NOT have a fatigue limit

Miner's Rule



- From S-N curve obtain # of cycles to failure



- Miner's rule of accumulated damage: Failure occurs when,

$$\sum_i \frac{n_i}{N_i} = 1$$

"i": different load conditions

END OF CHAPTER X