Chapter 8 - Plane (2D) Problems

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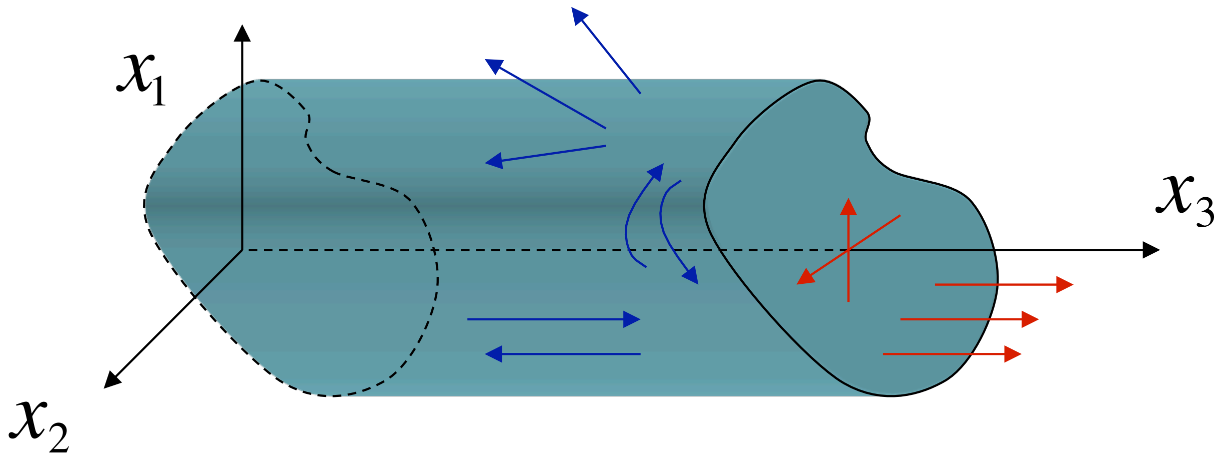
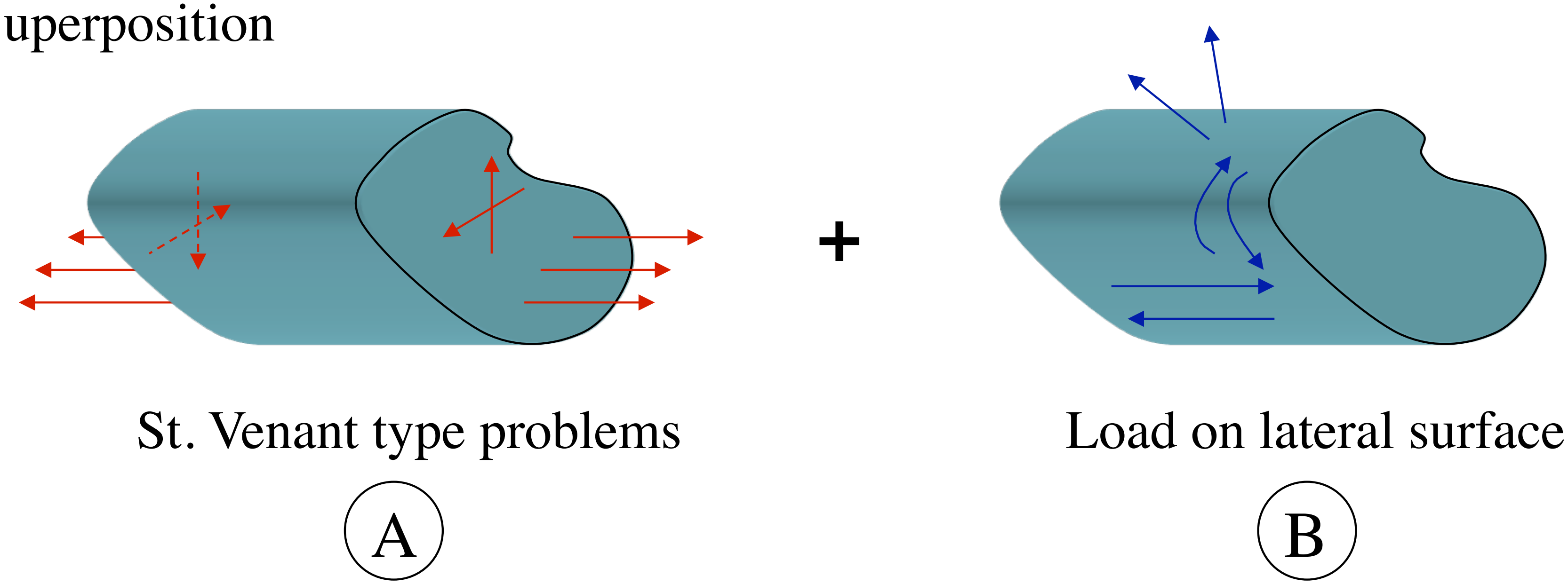
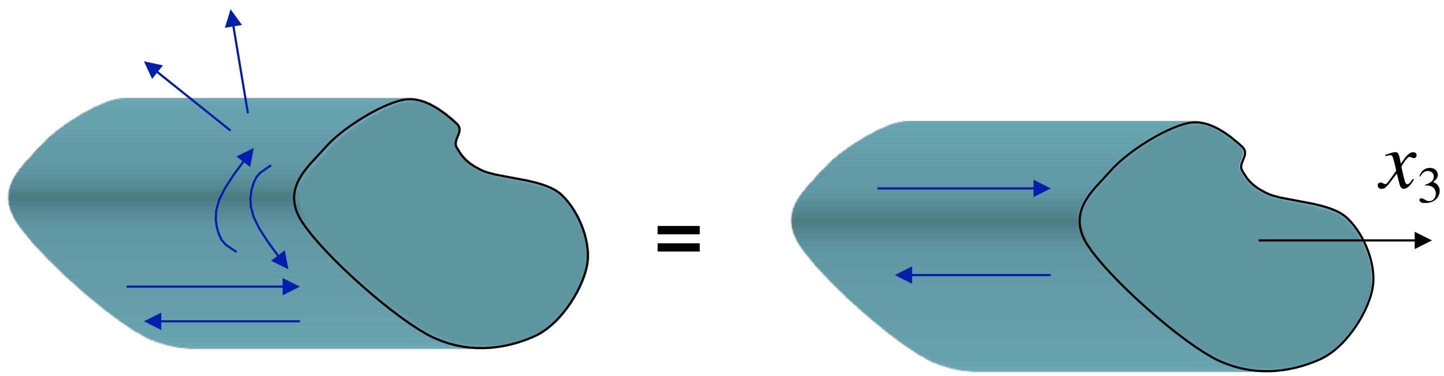
- Recall:



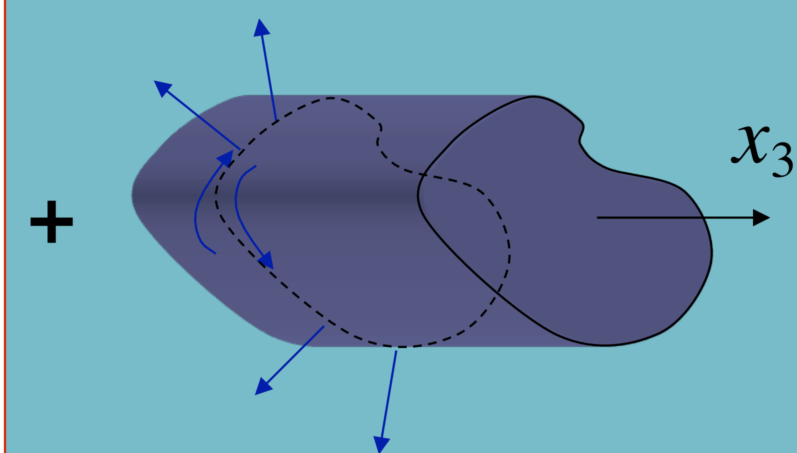
- Superposition



- Problem B :



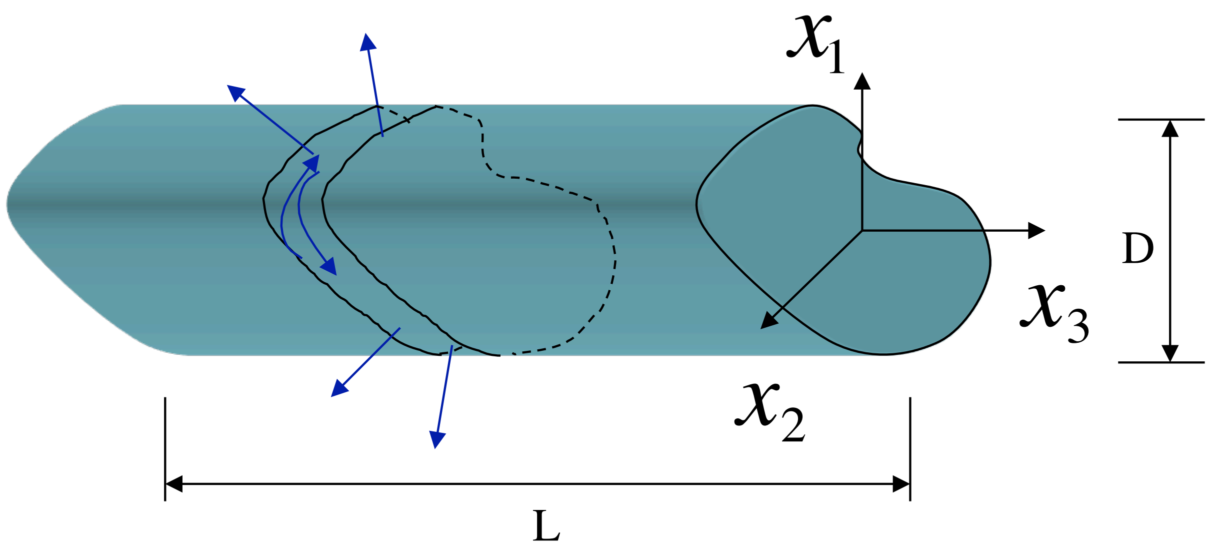
anti-plane shear
(i.e. T_3 only)



plane strain
or plane stress
(i.e. T_1, T_2 only)

Plane Strain Formulation

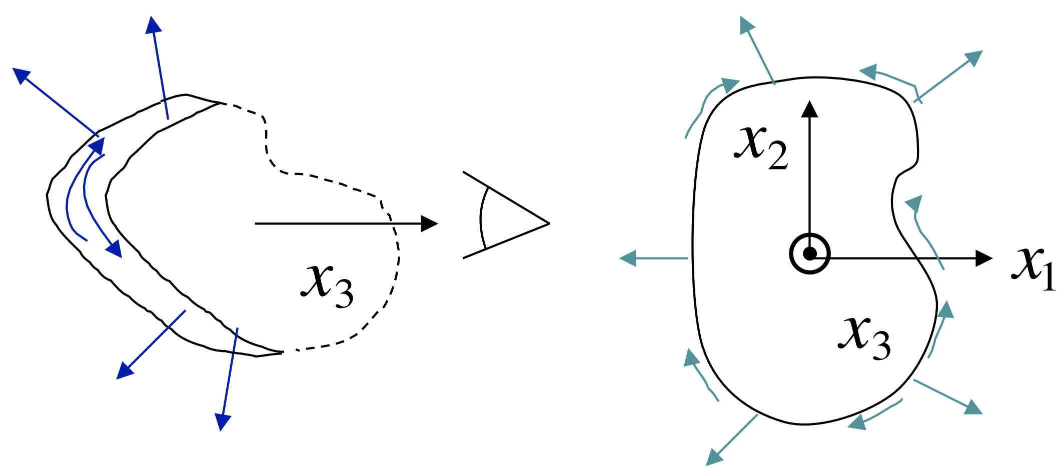
- Assume L>>D



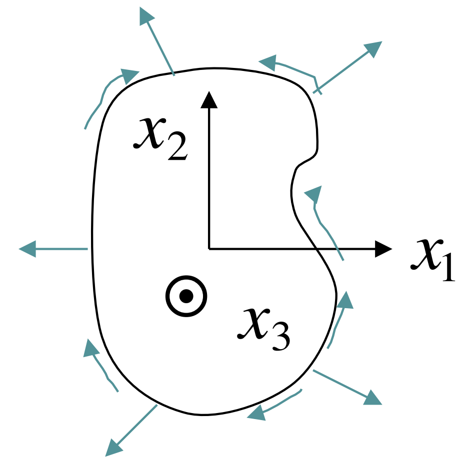
- Assume no x_3 dependence of tractions

$$T_\alpha = T_\alpha(x_1, x_2) \quad \alpha = 1,2$$

- Consider one "slice" of cylinder



Plane Strain Formulation (cont.)



- Guess:

$$u_1 = u_1(x_1, x_2)$$
$$u_2 = u_2(x_1, x_2)$$
$$u_3 = 0$$

(i.e. no axial strain since L>>D)

- Strains:

$$\varepsilon_{3i} = 0$$

$$\left.\begin{aligned} \varepsilon_{11} &= \varepsilon_{11}(x_1, x_2) \\ \varepsilon_{22} &= \varepsilon_{22}(x_1, x_2) \\ \varepsilon_{12} &= \varepsilon_{12}(x_1, x_2) \end{aligned}\right\} \quad \text{(i.e.2D only)}$$

- Stresses:

$$\left.\begin{aligned} E\varepsilon_{11} &= \sigma_{11} - \nu(\sigma_{22} + \sigma_{33}) \\ E\varepsilon_{22} &= \sigma_{22} - \nu(\sigma_{11} + \sigma_{33}) \\ E\varepsilon_{33} = 0 &= \sigma_{33} - \nu(\sigma_{11} + \sigma_{22}) \quad \boxed{\Rightarrow \quad \sigma_{33} = \nu(\sigma_{11} + \sigma_{22})} \end{aligned}\right\}$$

(i.e.axial stress present in pl-ε)

Plane Strain Formulation (cont.)

$$E\varepsilon_{11} = \sigma_{11} - \nu\left(\sigma_{22} + \nu\left(\sigma_{11} + \sigma_{22}\right)\right)$$

$$\Rightarrow \quad E\varepsilon_{11} = \left(1 - \nu^2\right)\sigma_{11} - \nu(1+\nu)\sigma_{22}$$

$$\therefore \quad \left.\begin{aligned} \frac{E}{1-\nu^2}\varepsilon_{11} &= \sigma_{11} - \frac{\nu}{1-\nu}\sigma_{22} \\ \frac{E}{1-\nu^2}\varepsilon_{22} &= \sigma_{22} - \frac{\nu}{1-\nu}\sigma_{11} \end{aligned}\right\}$$

Setting: $\left.\begin{aligned} E' &= \frac{E}{1-\nu^2} \\ \nu' &= \frac{\nu}{1-\nu} \end{aligned}\right\} \Rightarrow$

$$\boxed{\begin{aligned} E'\varepsilon_{11} &= \sigma_{11} - \nu'\sigma_{22} \\ E'\varepsilon_{22} &= \sigma_{22} - \nu'\sigma_{11} \end{aligned}}$$

Also, $\sigma_{13} = \sigma_{23} = 0$ and $\boxed{\sigma_{12} = 2\mu\varepsilon_{12}}$

Plane Strain Formulation (cont.)

- Equilibrium
(no body forces)

$$\boxed{\sigma_{11,1}+\sigma_{12,2}}+\overset{0}{\cancel{\sigma_{13,3}}}=0$$
$$\boxed{\sigma_{12,1}+\sigma_{22,2}}+\overset{0}{\cancel{\sigma_{23,3}}}=0$$
$$\overset{0}{\cancel{\sigma_{13,1}}}+\overset{0}{\cancel{\sigma_{23,2}}}+\underset{0}{\cancel{\sigma_{33,3}}}=0$$

0 since $\sigma_{33}=\nu(\sigma_{11}+\sigma_{22})=\sigma_{33}(x_1,x_2)$

- Thus, in plane strain:

<u>Displacements</u>

$$u_1=u_1(x_1,x_2)$$
$$u_2=u_2(x_1,x_2)$$
$$u_3=0$$

<u>Stress-strain</u>

$$E'\varepsilon_{11}=\sigma_{11}-\nu'\sigma_{22}$$
$$E'\varepsilon_{22}=\sigma_{22}-\nu'\sigma_{11}$$
$$\sigma_{12}=2\mu\varepsilon_{12}$$
$$\sigma_{33}=\nu(\sigma_{11}+\sigma_{22})$$

<u>Equilibrium</u>

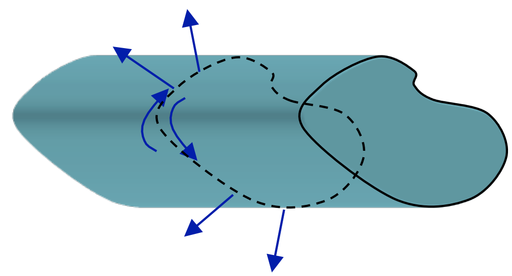
$$\sigma_{11,1}+\sigma_{12,2}=0$$
$$\sigma_{12,1}+\sigma_{22,2}=0$$

➔ i.e. we have now a 2D class of problems

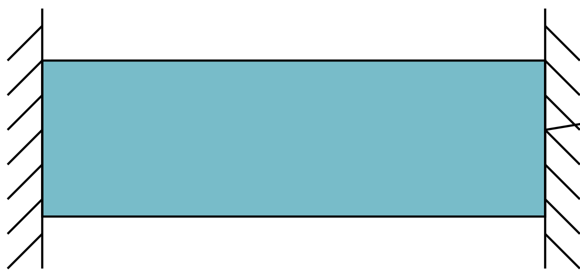
Plane Strain Formulation (cont.)

- <u>Note:</u> end faces should be traction free

  $\rightarrow$ problem, since we have $\sigma_{33} = \nu(\sigma_{11} + \sigma_{22})$!!!



- Plane strain would be an <u>exact</u> solution for:

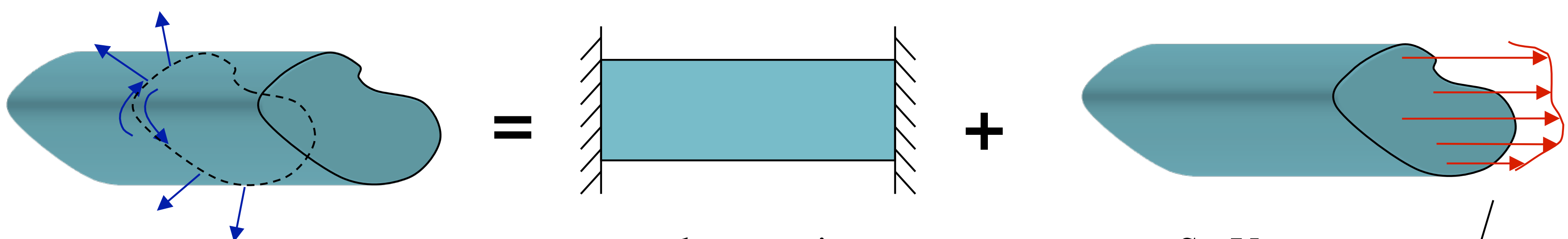


Lubricated wall $\sigma_{13} = \sigma_{23} = 0$

and $u_3 = 0 \Rightarrow \varepsilon_{33} = 0$

$$\Rightarrow \sigma_{33} = \nu(\sigma_{11} + \sigma_{22})$$

- In general we would need one more superposition:



plane strain

St. Venant type

$$\sigma_{33} = -\nu(\sigma_{11} + \sigma_{22})$$

Airy Stress Function

- Assume a function $\Phi(x_1, x_2)$: **Airy Stress Function**

$$\sigma_{11} = \frac{\partial^2 \Phi}{\partial x_2^2}$$

$$\sigma_{22} = \frac{\partial^2 \Phi}{\partial x_1^2}$$

$$\sigma_{12} = -\frac{\partial^2 \Phi}{\partial x_1 \partial x_2}$$

- Equilibrium:

$$\left.\begin{array}{l} \sigma_{11,1} + \sigma_{12,2} = 0 \\ \sigma_{12,1} + \sigma_{22,2} = 0 \end{array}\right\} \Rightarrow \begin{array}{l} \dfrac{\partial^3 \Phi}{\partial x_2^2 \partial x_1} + \left(-\dfrac{\partial^3 \Phi}{\partial x_1 \partial x_2^2}\right) = 0 \\ -\dfrac{\partial^3 \Phi}{\partial x_1^2 \partial x_2} + \dfrac{\partial^3 \Phi}{\partial x_1^2 \partial x_2} = 0 \end{array}$$

i.e. equilibrium identically satisfied!

Airy Stress Function (cont.)

- Strains: $E'\varepsilon_{11}=\sigma_{11}-\nu'\sigma_{22}$

$$E'\varepsilon_{22}=\sigma_{22}-\nu'\sigma_{11}$$

$$\sigma_{12}=2\mu\varepsilon_{12}$$

$$\sigma_{33}=\nu\left(\sigma_{11}+\sigma_{22}\right)$$

- Compatibility **<u>needed</u>**: $2\varepsilon_{12,12}=\varepsilon_{11,22}+\varepsilon_{22,11}$

$$\Rightarrow \quad \frac{\partial^4\Phi}{\partial x_1^4}+2\frac{\partial^4\Phi}{\partial x_1^2\partial x_2^2}+\frac{\partial^4\Phi}{\partial x_2^4}=0$$

$$\Rightarrow \quad \boxed{\nabla^4\Phi\left(x_1,x_2\right)=0}$$ **Bi-harmonic equation**

- Any Φ that satisfies this and gives σ as above is a possible pl-ε solution. We have solved the entire class of pl-ε problems. What distinguishes between problems are the b.c.s.

Airy Stress Function (cont.)

- If body forces are present, $f_i(x_1, x_2)$

$$\sigma_{11} = \frac{\partial^2 \Phi}{\partial x_2^2} + V$$

$$\sigma_{22} = \frac{\partial^2 \Phi}{\partial x_1^2} + V$$

$$\sigma_{12} = -\frac{\partial^2 \Phi}{\partial x_1 \partial x_2}$$

where

$$\frac{\partial V}{\partial x_1} = -f_1$$

$$\frac{\partial V}{\partial x_2} = -f_2$$

V: "potential" energy

- Equilibrium is again identically satisfied
- Compatibility becomes:

$$\nabla^4 \Phi(x_1, x_2) = -\left(\frac{1-2\nu}{1-\nu}\right)\nabla^2 V$$

Chapter 8 - Plane (2D) Problems

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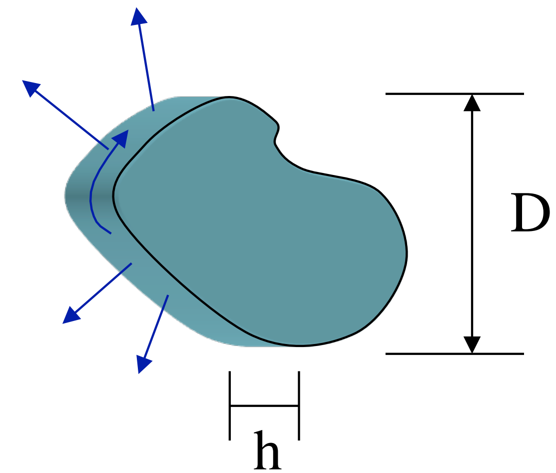
Plane Stress

- "Short" cylinder, i.e. thin plate!

h<<D



- Assume $\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$ and

$$\sigma_{11} = \sigma_{11}(x_1, x_2)$$
$$\sigma_{22} = \sigma_{22}(x_1, x_2)$$
$$\sigma_{12} = \sigma_{12}(x_1, x_2)$$

- Strains:

$$\boxed{\begin{aligned} E\varepsilon_{11} &= \sigma_{11} - \nu(\sigma_{22} + \overset{0}{\cancel{\sigma_{33}}}) \\ E\varepsilon_{22} &= \sigma_{22} - \nu(\sigma_{11} + \overset{0}{\cancel{\sigma_{33}}}) \end{aligned}}$$
$$E\varepsilon_{33} = \overset{0}{\cancel{\sigma_{33}}} - \nu(\sigma_{11} + \sigma_{22})$$

$$\sigma_{13} = \sigma_{23} = 0$$
$$\sigma_{12} = 2\mu\varepsilon_{12}$$

$\leftarrow$ Note: there is an ε_{33}

Plane Stress (cont.)

- Equilibrium (no body forces):

$$\begin{aligned} \sigma_{11,1} + \sigma_{12,2} + \cancelto{0}{\sigma_{13,3}} &= 0 \\ \sigma_{12,1} + \sigma_{22,2} + \cancelto{0}{\sigma_{23,3}} &= 0 \\ \cancelto{0}{\sigma_{13,1}} + \cancelto{0}{\sigma_{23,2}} + \cancelto{0}{\sigma_{33,3}} &= 0 \end{aligned}$$

- Plane stress:

<u>Stress-strain</u>

$$\begin{aligned} E\varepsilon_{11} &= \sigma_{11} - \nu\sigma_{22} \\ E\varepsilon_{22} &= \sigma_{22} - \nu\sigma_{11} \\ \sigma_{12} &= 2\mu\varepsilon_{12} \end{aligned}$$

<u>Equilibrium</u>

$$\begin{aligned} \sigma_{11,1} + \sigma_{12,2} &= 0 \\ \sigma_{12,1} + \sigma_{22,2} &= 0 \end{aligned}$$

- Thus plane stress is also a two dimensional problem with identical equations, but different "coefficients" in the stress-strain equations. Solve both problems using Airy stress function.

- <u>Note:</u> In pl-σ bi-harmonic (i.e. compatibility) we neglect the effect of $\varepsilon_{33}(\neq 0)$. This is small and as h $\longrightarrow$ 0 it becomes smaller.

Plane Stress (cont.)

- BVP for two-dimensional plane problems (both pl-σ and pl-ε):

Compatibility

$$\nabla^4 \Phi(x_1, x_2) = 0$$

Stress-strain

$$E' \varepsilon_{11} = \sigma_{11} - \nu' \sigma_{22}$$
$$E' \varepsilon_{22} = \sigma_{22} - \nu' \sigma_{11}$$
$$\sigma_{12} = 2\mu \varepsilon_{12}$$

$$E' = \begin{cases} E & for \quad pl-\sigma \\ E/(1-\nu^2) & for \quad pl-\varepsilon \end{cases}$$

$$\nu' = \begin{cases} \nu & for \quad pl-\sigma \\ \dfrac{\nu}{1-\nu} & for \quad pl-\varepsilon \end{cases}$$

Equilibrium (identically satisfied)

$$\sigma_{11} = \frac{\partial^2 \Phi}{\partial x_2^2}$$

$$\sigma_{22} = \frac{\partial^2 \Phi}{\partial x_1^2}$$

$$\sigma_{12} = -\frac{\partial^2 \Phi}{\partial x_1 \partial x_2}$$

$$\sigma_{33} = \begin{cases} 0 & for \quad pl-\sigma \\ \nu(\sigma_{11} + \sigma_{22}) & for \quad pl-\varepsilon \end{cases}$$

- Solution methods:
  - — Trial and error
  - — Separation of variables
  - — Transforms (e.g. Fourier)
  - — Green's functions

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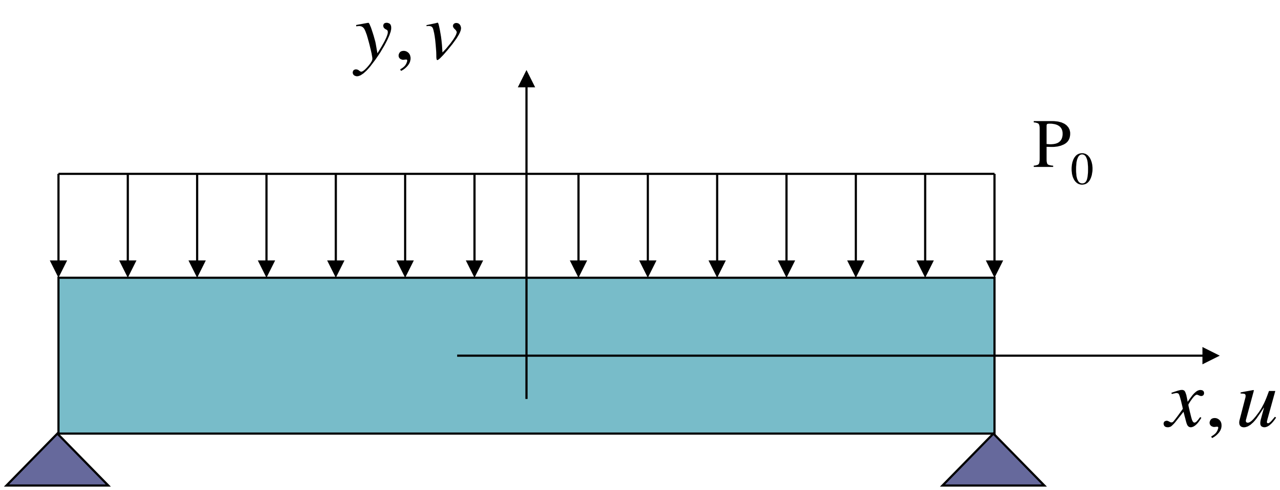
Rectangular Domains



- Simply supported beam
- No body forces
- Plane strain

- Solution (Done in class... **Example 8-3** from textbook)
- See also examples 8-1, 8-2

Rectangular Domains (cont.)

- Solution

$$\sigma_{xx} = -\frac{3P_0}{2}\left\{\left(\frac{l}{h}\right)^2 - 4\left(\frac{x}{l}\right)^2\right\}\frac{y}{l} + P_0\left\{\frac{3}{5} - 4\left(\frac{y}{h}\right)^2\right\}\frac{y}{l}$$

$$\sigma_{xy} = \frac{3P_0}{2}\left\{1 - 4\left(\frac{y}{h}\right)^2\right\}\frac{x}{h}$$

$$\sigma_{yy} = -\frac{P_0}{2} - \frac{3P_0}{2}\frac{y}{h} + 2\left(\frac{y}{h}\right)^3$$

Strength of Materials solution

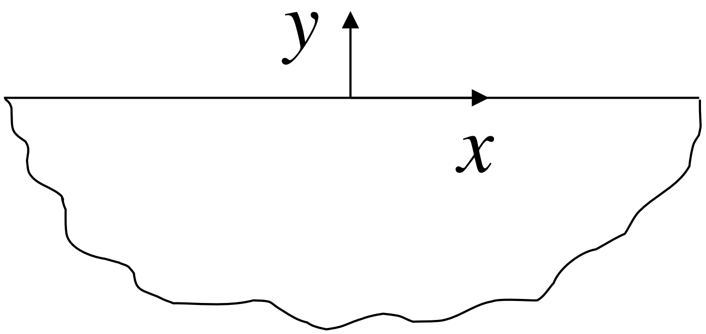
- <u>Note:</u> Reaction forces come in through a <u>net</u> shear force S^* at supports

$$S^* = \int_{-h/2}^{h/2} \sigma_{xy}\, dy = \frac{3P_0}{2}\int_{-h/2}^{h/2}\left\{1 - 4\left(\frac{y}{h}\right)^2\right\}\frac{l}{2h}\, dy$$

$$\boxed{\Rightarrow \quad S^* = \frac{1}{2}P_0 l}$$

Rectangular Domains (cont.)

- Other rectangular domains:

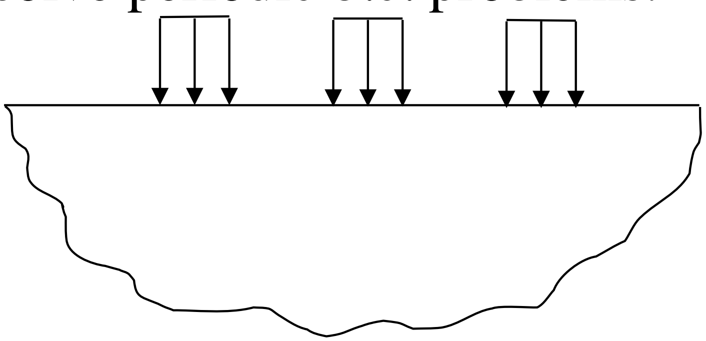


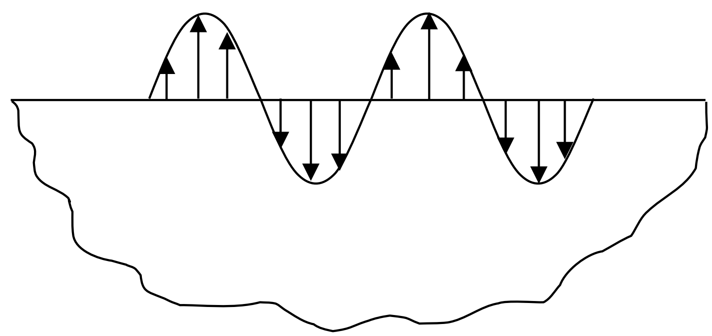
Half Space problems

- Fourier transform or series
- Separation of variables

$$\Phi(x,y)=\sum_{n=1}^{\infty}\underbrace{\left(a_n\cos\alpha_n x+b_n\sin\alpha_n x\right)}_{f(x)}\underbrace{\left(A_n e^{\alpha_n y}+B_n e^{-\alpha_n y}\right)}_{f(y)}$$

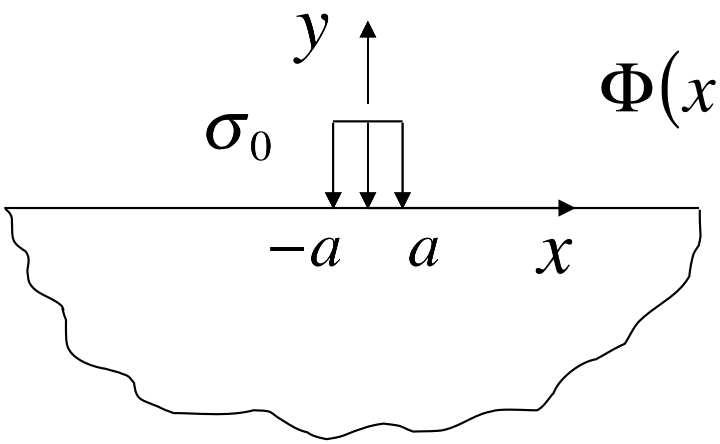
- Can solve periodic b.c. problems:





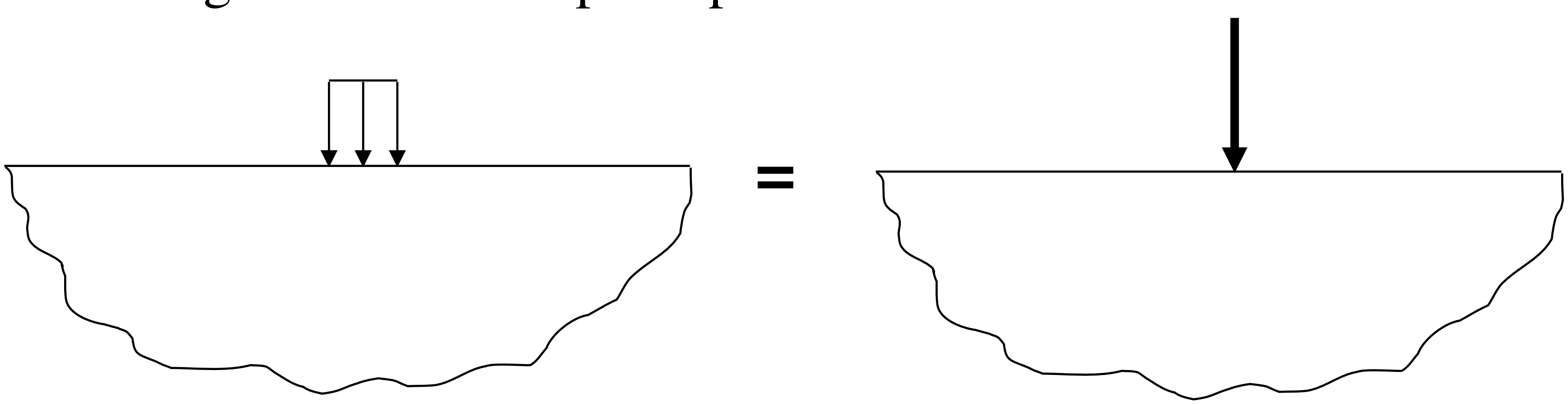
Rectangular Domains (cont.)

- For non-periodic b.c.s use Fourier Transform



$$\Phi(x, y) = -\frac{\sigma_0}{\pi}\int_{-\infty}^{\infty}\frac{\sin \alpha a}{\alpha^3}\cos \alpha x(1-\alpha y)e^{\alpha y}\,d\alpha$$

- Note: According to St Venant's principle



Axisymmetric Domains

- Equilibrium in cylindrical coordinates:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + R = 0$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r} + \Theta = 0$$

$$\frac{\partial \tau_{zr}}{\partial z} + \frac{\partial \tau_{\theta z}}{r\partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{zr}}{r} + Z = 0$$

- Reduce to 2D pl-σ and pl-ε as before and use:

$$\sigma_{rr} = \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta_2^2} + V$$

$$\sigma_{\theta\theta} = \frac{\partial^2 \Phi}{\partial r^2} + V$$

$$\sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Phi}{\partial \theta}\right)$$

With:

$$\{R, \Theta\} = \left\{-\frac{\partial V}{\partial r}, -\frac{1}{r}\frac{\partial V}{\partial \theta}\right\}$$

Axisymmetric Domains (cont.)

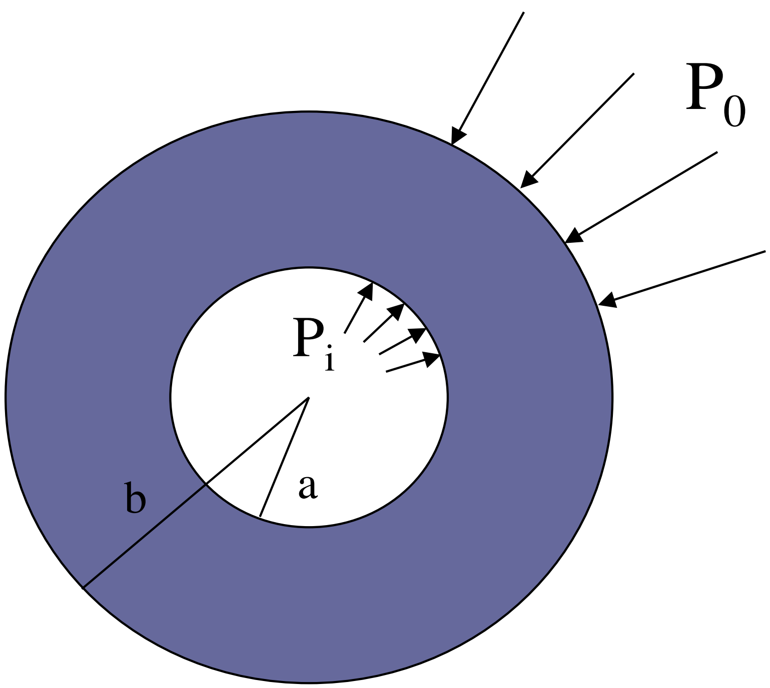
- Compatibility: $\boxed{\nabla^4\Phi(r,\theta)=0}$ $\qquad \nabla^2 \equiv \dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}$

- General solution:

$$\begin{aligned}\Phi(r,\theta) = {} & a_0\log r + b_0 r^2 + c_0 r^2\log r + d_0 r^2\theta + a_0'\theta \\ & + \frac{a_1}{2} r\theta\sin\theta + \left(b_1 r^3 + a_1' r^{-1} + b_1' r\log r\right)\cos\theta \\ & - \frac{c_1}{2} r\theta\cos\theta + \left(d_1 r^3 + c_1' r^{-1} + d_1' r\log r\right)\sin\theta \\ & + \sum_{n=2}^{\infty}\left(a_n r^n + b_n r^{n+2} + a_n' r^{-n} + b_n' r^{-n+2}\right)\cos n\theta \\ & + \sum_{n=2}^{\infty}\left(c_n r^n + d_n r^{n+2} + c_n' r^{-n} + d_n' r^{-n+2}\right)\sin n\theta\end{aligned}$$

Axisymmetric Domains (cont.)

- Thick walled cylinder with internal and external pressure:



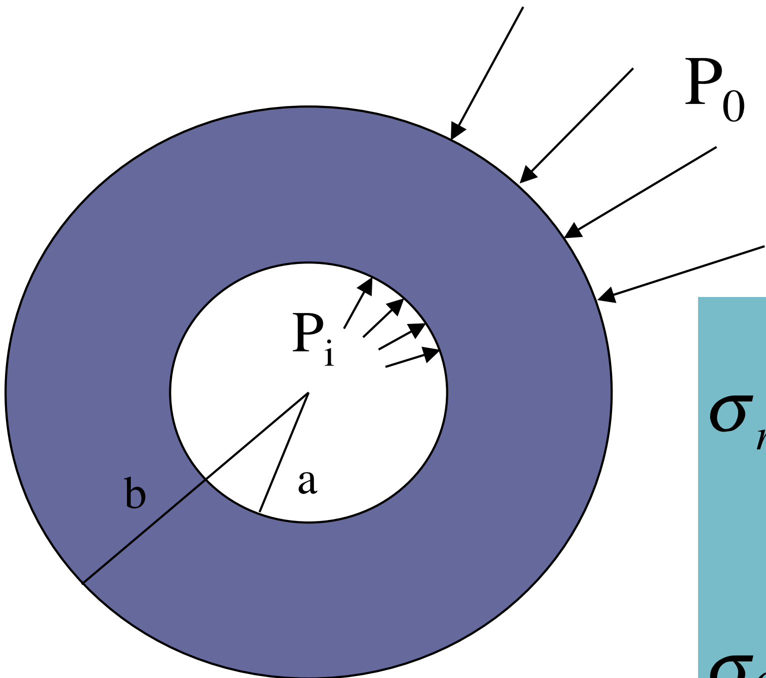
- **Axisymmetry**, i.e. no θ dependence

$$\boxed{\frac{\partial}{\partial \theta} \equiv 0}$$

- Solution (Done in class…**Example 8-6**)

Axisymmetric Domains (cont.)

- Thick walled cylinder with internal and external pressure:



$$\sigma_r = \frac{(P_0 - P_i)a^2b^2}{(b^2 - a^2)}\frac{1}{r^2} - \frac{(P_0 b^2 - P_i a^2)}{(b^2 - a^2)}$$

$$\sigma_\theta = -\frac{(P_0 - P_i)a^2b^2}{(b^2 - a^2)}\frac{1}{r^2} - \frac{(P_0 b^2 - P_i a^2)}{(b^2 - a^2)}$$

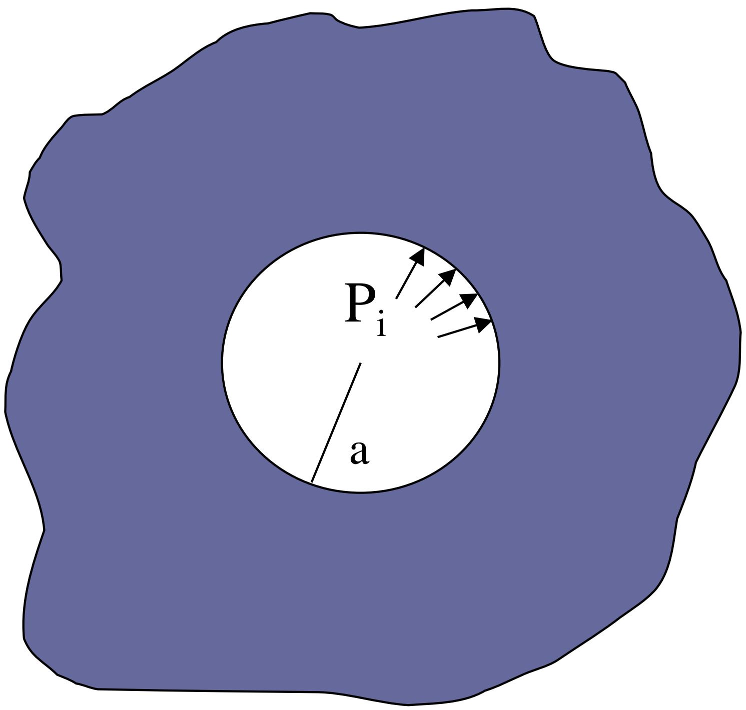
$$\sigma_{r\theta} = 0$$

- Note: in the thin walled cylinder σ_θ does NOT depend on r.

Axisymmetric Domains (cont.)

Special Cases:

- "Infinite cylinder" with internal pressure: $P_0 = 0 \quad \text{and} \quad b \to \infty$



$$\left.\begin{aligned}\sigma_r &= \frac{-P_i a^2 b^2}{\left(b^2 - a^2\right)} \frac{1}{r^2} - \frac{-P_i a^2}{\left(b^2 - a^2\right)} \\ \sigma_\theta &= -\frac{-P_i a^2 b^2}{\left(b^2 - a^2\right)} \frac{1}{r^2} - \frac{-P_i a^2}{\left(b^2 - a^2\right)}\end{aligned}\right\}$$

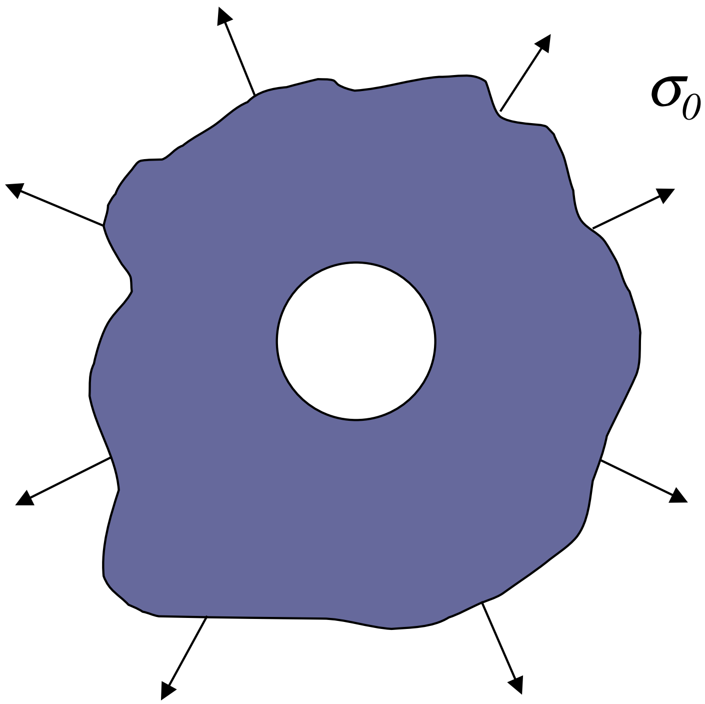
$$\Rightarrow \boxed{\begin{aligned}\sigma_r &= -P_i \left(\frac{a}{r}\right)^2 \\ \sigma_\theta &= P_i \left(\frac{a}{r}\right)^2\end{aligned}}$$

- Stresses decay to 0 far away from the hole since load is self equilibrating

$\Rightarrow$ St. Venant!

Axisymmetric Domains (cont.)

- "Infinite cylinder" with external tension: $P_i = 0,\ \ P_0 = -\sigma_0\,, \quad \text{and} \quad b \to \infty$



$$\left.\begin{aligned}\sigma_r &= -\frac{\sigma_0 a^2 b^2}{\left(b^2 - a^2\right)} \frac{1}{r^2} + \frac{\sigma_0 b^2}{\left(b^2 - a^2\right)} \\ \sigma_\theta &= \frac{\sigma_0 a^2 b^2}{\left(b^2 - a^2\right)} \frac{1}{r^2} + \frac{\sigma_0 b^2}{\left(b^2 - a^2\right)}\end{aligned}\right\}$$

$$\Rightarrow \quad \boxed{\begin{aligned}\sigma_r &= \sigma_0 \left\{1 - \frac{a^2}{r^2}\right\} \\ \sigma_\theta &= \sigma_0 \left\{1 + \frac{a^2}{r^2}\right\}\end{aligned}}$$

- As $r \longrightarrow \infty$ $\quad \sigma_r = \sigma_0$, $\quad \sigma_\theta = \sigma_0$

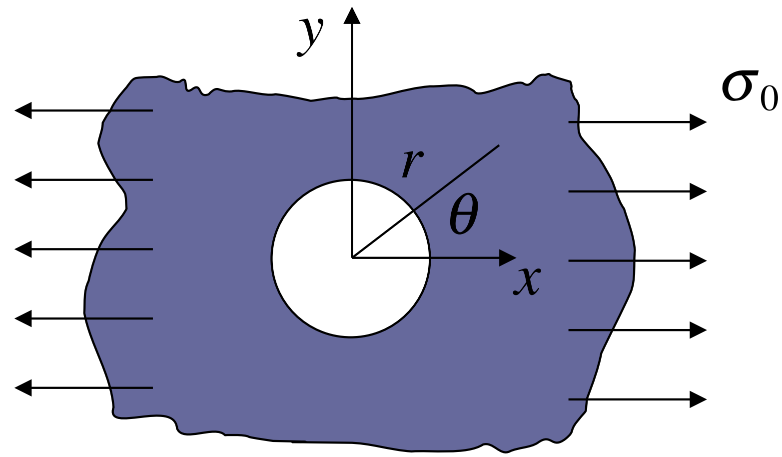
- At $r=a$ $\quad \sigma_r = 0$, $\quad \sigma_\theta = 2\sigma_0$ $\quad \Longrightarrow$ **Stress concentration**

Axisymmetric Domains (cont.)

- Hole in uniaxial tension

  Example 8-7 in textbook

NOT an axisymmetric problem!

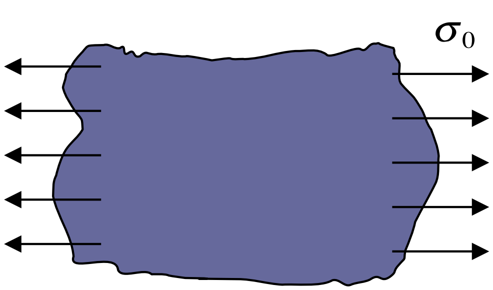


- At far field we get uniaxial tension:

$$\sigma_x = \sigma_0 \qquad \sigma_y = \sigma_{xy} = 0$$

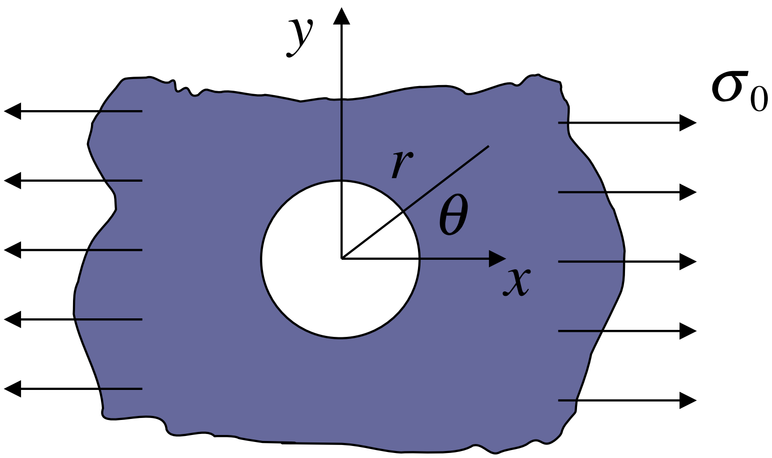
or

$$\sigma_r = \frac{\sigma_0}{2}(1+\cos 2\theta)$$
$$\sigma_\theta = \frac{\sigma_0}{2}(1-\cos 2\theta)$$
$$\sigma_{r\theta} = -\frac{\sigma_0}{2}\sin 2\theta$$



$$\Rightarrow \quad \Phi_I(x,y) = \frac{\sigma_0}{2}y^2 = \frac{\sigma_0}{2}r^2\sin^2\theta = \frac{\sigma_0}{4}r^2(1-\cos 2\theta)$$

Axisymmetric Domains (cont.)



- Try:

$$\Phi = f(r)\cos 2\theta + axisymmetry$$

$$\Rightarrow \Phi(r,\theta) = f(r)\cos 2\theta + \underbrace{\cancel{Ar^2\ln r}}_{\mathbf{0}} + B\ln r + Cr^2 + D$$

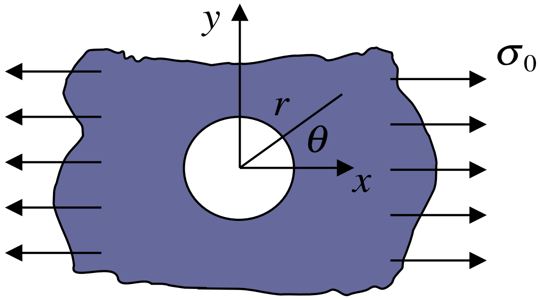
- Solution ...

Axisymmetric Domains (cont.)

$$\sigma_r = \frac{\sigma_0}{2}\left[1-\underline{\left(\frac{a}{r}\right)^2}\right]+\frac{\sigma_0}{2}\left[1-4\underline{\left(\frac{a}{r}\right)^2}+3\underline{\left(\frac{a}{r}\right)^4}\right]\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma_0}{2}\left[1+\underline{\left(\frac{a}{r}\right)^2}\right]-\frac{\sigma_0}{2}\left[1+3\underline{\left(\frac{a}{r}\right)^4}\right]\cos 2\theta$$

$$\sigma_{r\theta} = -\frac{\sigma_0}{2}\left[1+2\underline{\left(\frac{a}{r}\right)^2}-3\underline{\left(\frac{a}{r}\right)^4}\right]\sin 2\theta$$



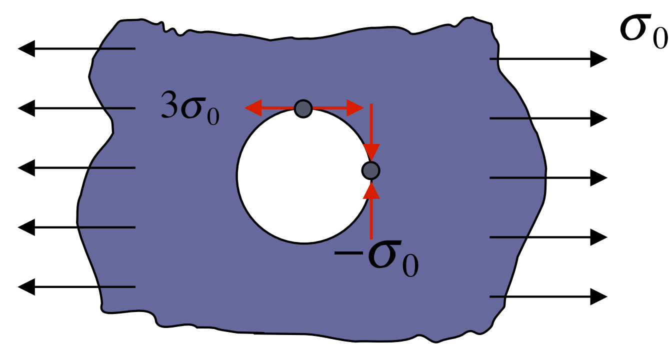
- As $r \longrightarrow \infty$ we recover the uniaxial stress state

- But at the hole, $r=a$:

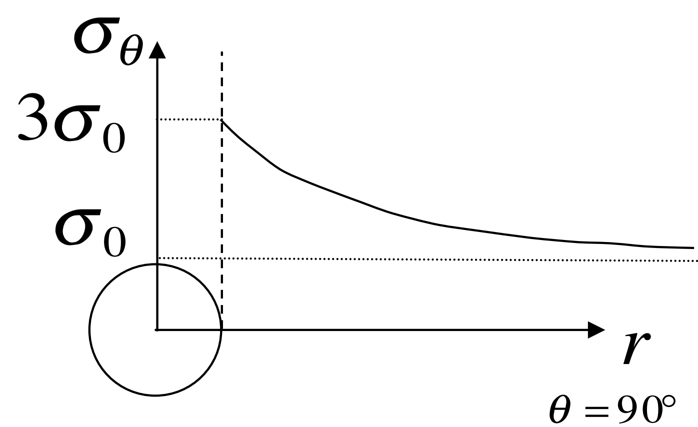
$$\left.\begin{aligned}\sigma_r &= 0\\ \sigma_\theta &= \sigma_0 - 2\sigma_0\cos 2\theta\\ \sigma_{r\theta} &= 0\end{aligned}\right\} \Rightarrow \left.\begin{aligned}\sigma_\theta^{\max}\left(a,\frac{\pi}{2}\right) &= 3\sigma_0\\ \sigma_\theta^{\min}(a,0) &= -\sigma_0\end{aligned}\right\} \Rightarrow$$

Axisymmetric Domains (cont.)

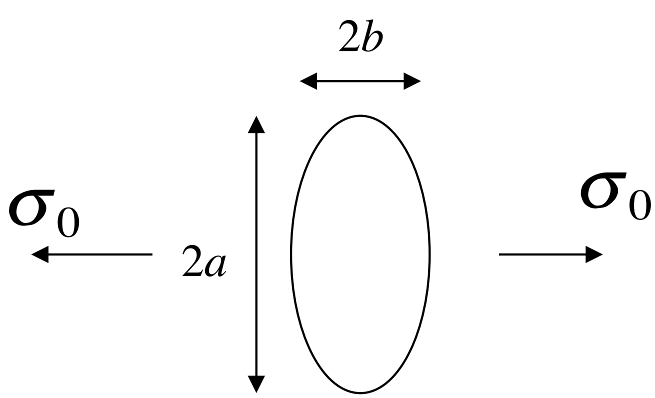
- Define **Stress Concentration Factor**, k



$$k = \frac{\sigma_\theta^{\max}}{\sigma_0} = 3$$



- For an elliptical hole



$$k = \frac{\sigma_{\max}}{\sigma_0} = 1 + 2\frac{a}{b}$$

<u>Note:</u> As $b \to 0$

$k \to \infty$

Axisymmetric Domains (cont.)

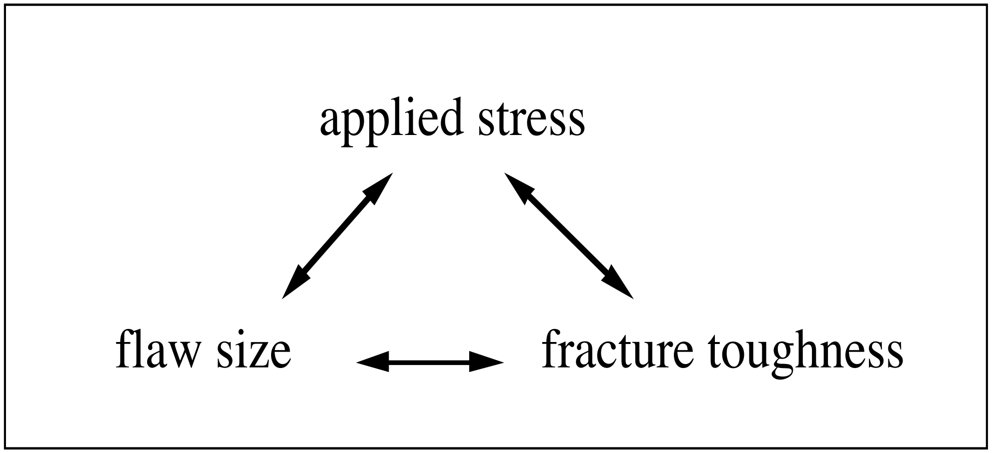
De Havilland DH-106 Comet - First commercial jet airliner

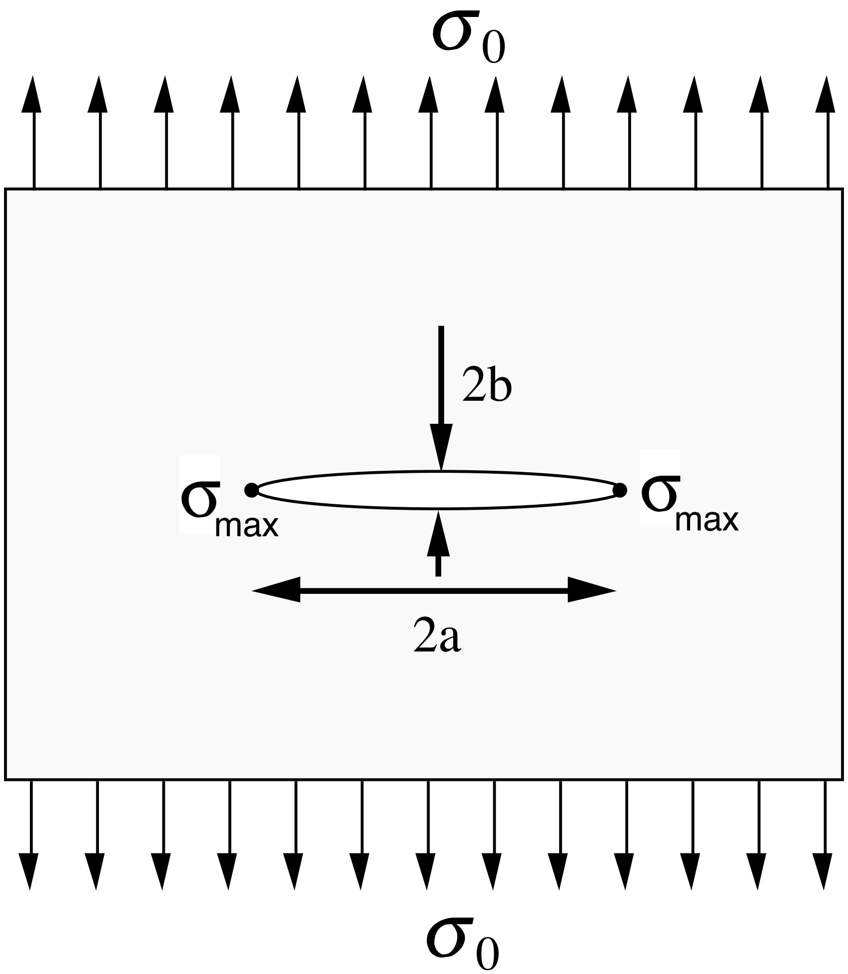
- 3/3/53 Canadian-Pacific Airlines, Karachi, Pakistan
- 5/2/53 British Overseas Airways, Jalalogori, India
- 1/10/54 British Overseas Airways, Elba, Italy
- 4/8/54 British Overseas Airways, Stromboli, Italy

Axisymmetric Domains (cont.)

Fracture Mechanics based approach in design

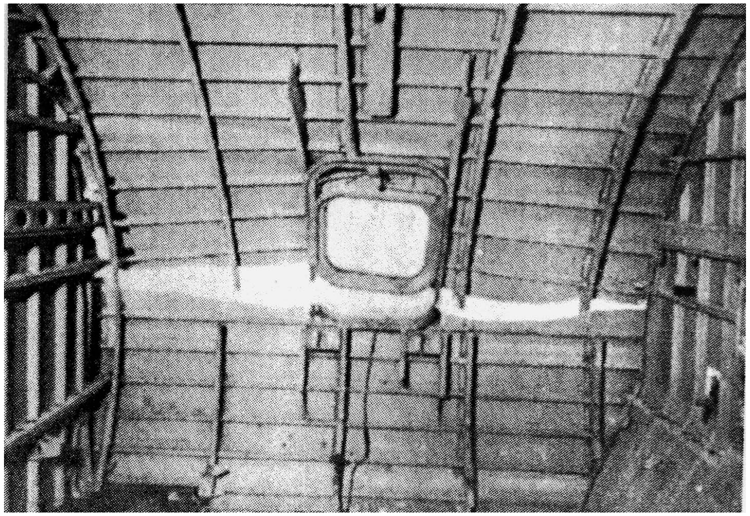
$$\frac{\sigma_{max}}{\sigma_0} = 1 + 2\frac{a}{b}$$





Axisymmetric Domains (cont.)

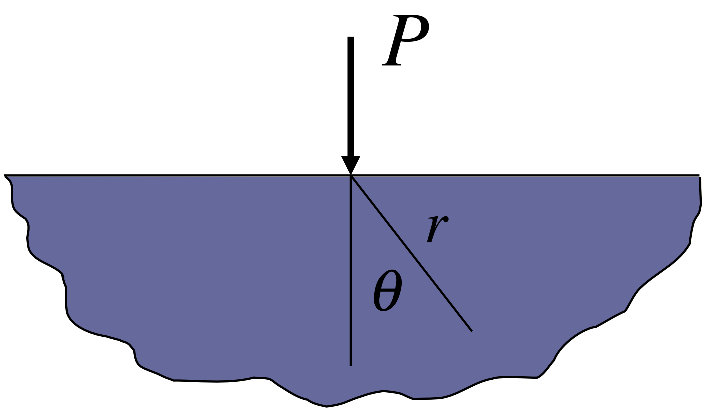
Old design



New design

Half-space Problems

- Point load on half-space:



$$\Phi(r,\theta) = -\frac{P}{\pi} r\theta \sin\theta$$

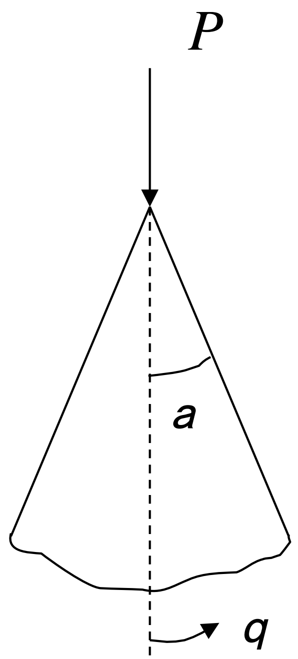
$$\Rightarrow \left.\begin{aligned} \sigma_r &= -\frac{2P}{\pi}\frac{\cos\theta}{r} \\ \sigma_\theta &= \sigma_{r\theta} = 0 \end{aligned}\right\}$$

$$u_r = -\frac{2P}{\pi E'} \cos\theta \ln r - \left(\frac{1-\nu'}{\nu'}\right)\frac{P}{\pi}\theta \sin\theta + R.B.$$

$$u_\theta = \frac{2P}{\pi E'}(\nu' + \ln r)\sin\theta - \left(\frac{1-\nu'}{E'}\right)\frac{P}{\pi}\theta\cos\theta + \left(\frac{1-\nu'}{E'}\right)\frac{P}{\pi}\sin\theta + R.B.$$

Half-space Problems (cont.)

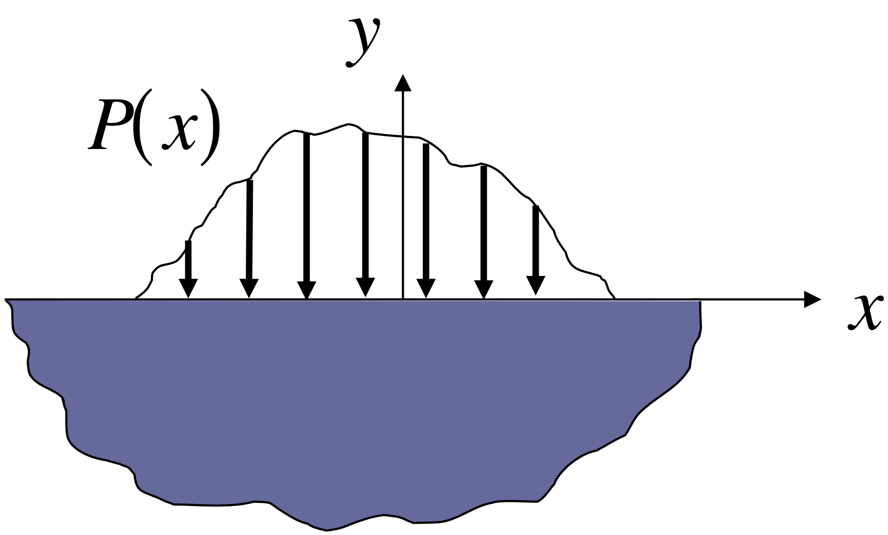
- Wedge with load at tip (Example 8-8):



- Solution...

Half-space Problems (cont.)

- Arbitrary pressure distribution:



- Solution...

END OF CHAPTER 8