AE323 – Spring 2019 – Homework assignment #5
Wednesday February 20, 2019
Due on Friday March 1, 2019 at class time

Note: Please make sure to write the solution of each problem on a separate sheet (making sure to write your name on every sheet!)

Problem 1

Consider the following homogeneous beam with triangular cross-section. The beam is of length L, and is made of a material of stiffness E, density ρ and Poisson's ratio ν. It is subjected to the effect of gravity (use g for the acceleration of gravity acting in the negative z-direction) and a varying tangential loading applied on its **top** surface (Figure 1)

$$q(x) = q_o \frac{x}{L}\left(1 - \frac{x}{L}\right)$$

where q_o is given in **Pa**. The torsional and linear springs have a stiffness of K and k, respectively.

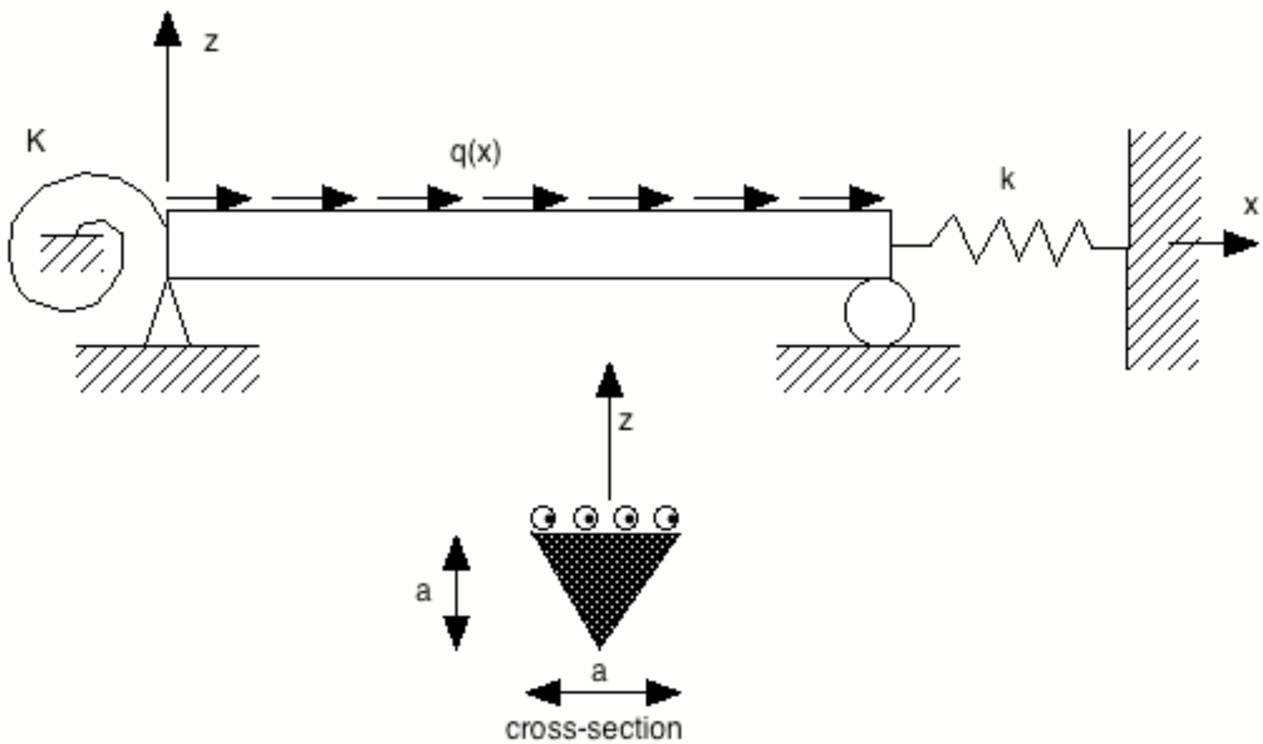


Figure 1

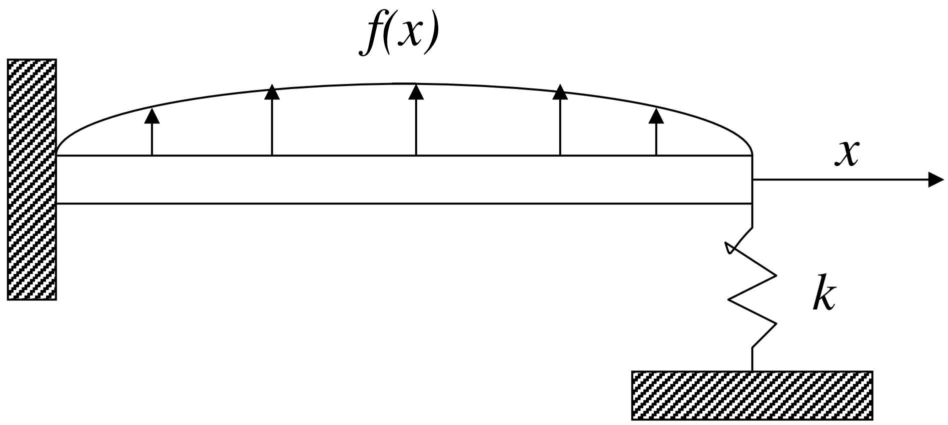
a) Write down the BVP describing the axial displacement $u(x)$, and the transverse deflections $v(x)$ and $w(x)$ for this problem. Write only the relevant equations (i.e., the equations for the non-zero displacements) and explain why the other displacements are zero. You do not need to compute the geometrical quantities such as moments of inertia, ... (unless they are obviously zero), but you need to explain how you would compute them.
b) Solve the equation for the axial displacement u and put your solution in a non-dimensional form.
c) Explain what to expect for the axial displacement of the end of the beam when the stiffness k of the linear spring (at $x=L$) goes to infinity. Then show that your solution found in b) matches your expectation.

Problem 2

Consider the following cantilever beam bending problem. The beam has a symmetric cross-section, is homogeneous and linearly elastic (with stiffness E), has a moment of inertia I, and length L. It is subjected to a spatially varying transverse load

$$f(x) = f_0 \frac{x}{L}\left(1 - \frac{x}{L}\right) \quad \text{with } f_0 \text{ in } N/m$$

and to the effect of an elastic boundary condition at $x=L$ represented by a linear spring of stiffness k.

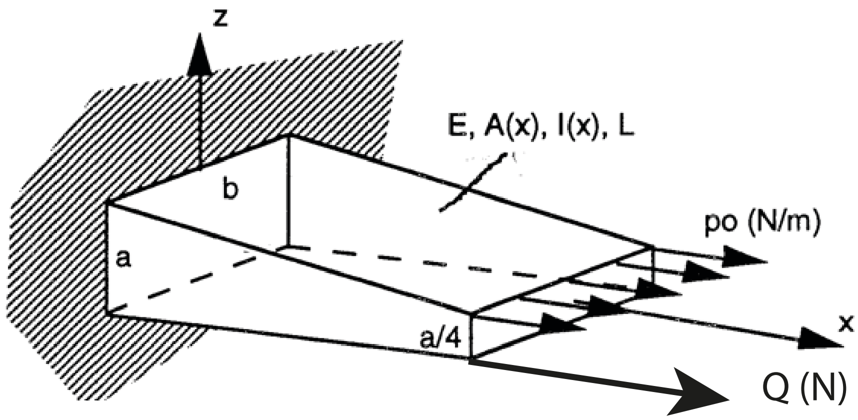


a) Is this problem statically determinate or indeterminate? Why?
b) Using the Euler-Bernouilli beam theory, write the boundary value problem (governing equation and boundary conditions) describing the equilibrium of the beam.
c) Solve for the transverse deflection $w(x)$ of the neutral axis of the beam and write your solution in a non-dimensional form. Check that the non-dimensional parameters you introduce are indeed dimensionless.
d) Write the (non-dimensional) deflection of the end of the beam (i.e., for $x=L$). What happens when the linear spring stiffness k tends to infinity? Check that your solution for the end deflection corresponds to your expectation.

Problem 3

Consider the problem shown in the figure below. The beam has a uniform stiffness E, a length L, and a spatially varying rectangular cross-section with a constant width b and a linearly varying thickness with value a at $x=0$ and value $a/4$ at $x=L$. Assume that both a and b are much smaller than L.

The beam is fixed at x=0 and is subjected to an axial line load of amplitude p_o applied along the upper edge of its end and an axial load Q applied at a corner of the end cross-section as shown in the figure.

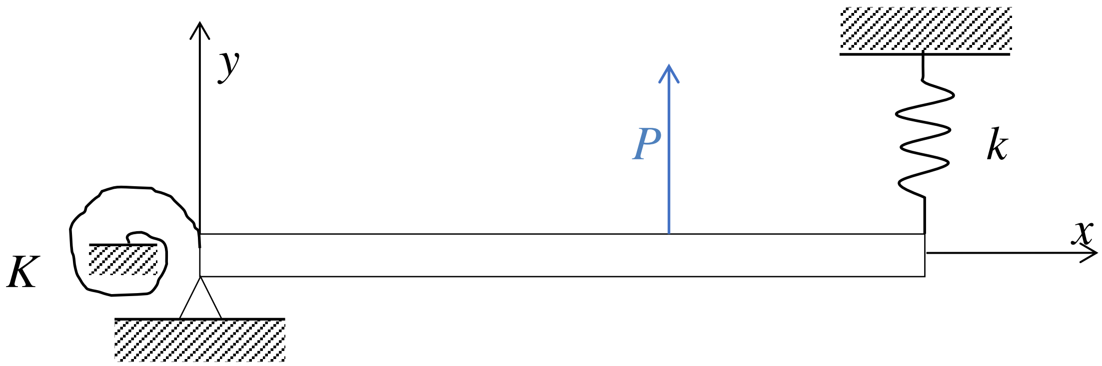


a) Write down the complete boundary value problem describing the equilibrium of the beam in terms of the three deflections u, v and w. Make sure to provide the expression of all the quantities entering the problem.

b) Solve for the axial displacement u.

Problem 4

Consider the beam bending problem shown below. The beam is of length L, stiffness E, and density ϱ. It has a rectangular cross-section of width $2a$ and thickness a (with the thickness defined as the dimension in the y-direction), and is subjected to the effect of gravity (with acceleration g in the negative y-direction), and a concentrated transverse load P applied at $x=2L/3$ as indicated in the figure.



The boundary conditions are described by a torsional spring of stiffness K at $x=0$ and by a linear spring of stiffness k at $x=L$.

(a) Provide the full boundary value problem (i.e., governing differential equations and boundary conditions) describing the equilibrium of the beam. Give the expression of the quantities (e.g., moment of inertia) entering these equations.

(b) Derive the solution for the deflection of the beam <u>in the absence of gravity</u> and <u>in the absence of the linear spring</u> (i.e., $k=0$). Put your solution in a non-dimensional form. Check that the non-dimensional parameter describing the torsional spring is indeed non-dimensional.

(c) Obtain <u>and discuss</u> the value of the end deflection $v(L)$ for the following two limiting cases

- (i) $K \to 0$,
- (ii) $K \to \infty$.