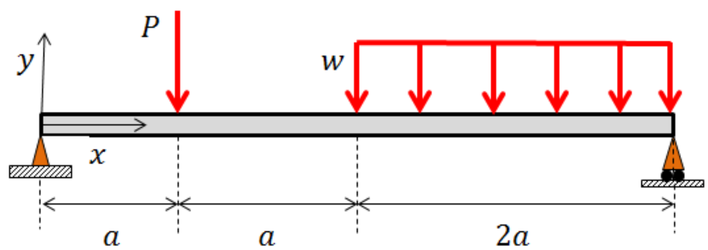
AE323 – Homework Assignment #2 – Spring 2019
Wednesday, Jan. 30, 2019
Due on Friday, Feb. 8 at class time

Topics: Internal Forces and Moments in Statically Determinate Structures

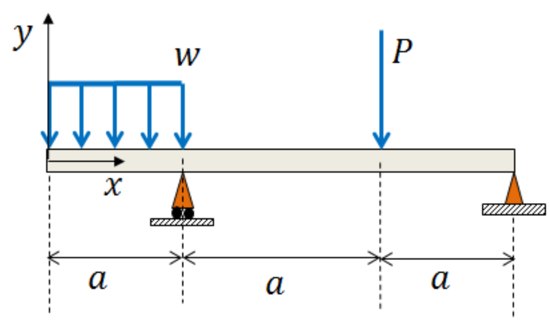
For the three structural problems shown below, answer the following three questions:

a) Is the problem statically determinate or indeterminate, and why?
b) Compute the reactions at the supports
c) Find the expression of the resultant shear force $V_y(x)$ and bending moment $M_z(x)$ in the beam.

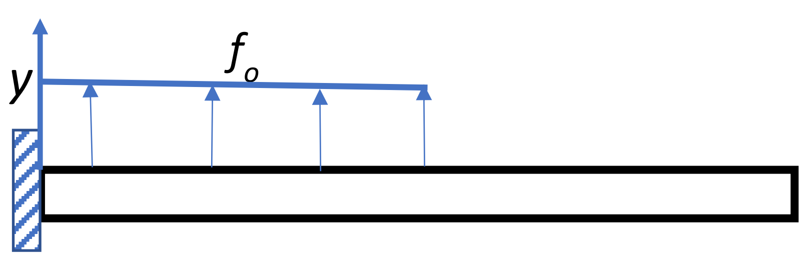
Problem 1. (Note: *P* is in *N*, *w* is in *N/m*)



Problem 2.



Problem 3. (Taken from the Fall 2017 final exam): The cantilever beam of length L is subjected to a transverse load f_o (given in N/m) applied over the first half of the beam.



<u>Problem 4.</u> Assuming that the lift distribution over a wing of length L is quadratic, as in

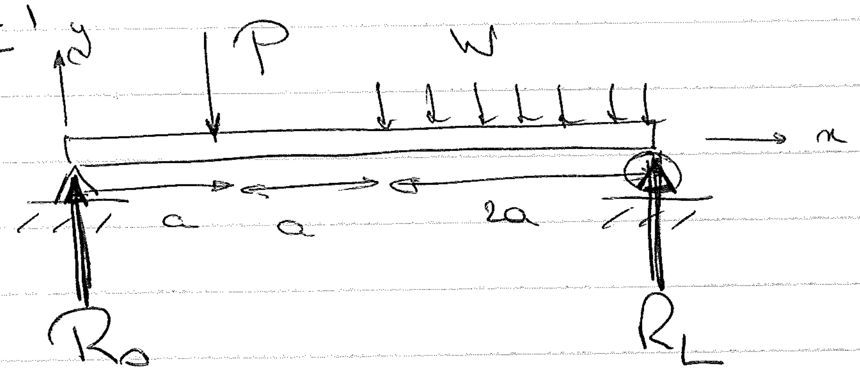
$$l(x) = l_0\left(1 - \left(\frac{x}{L}\right)^2\right),$$

with l_0 given in N/m, and assuming that the wing is cantilever to the fuselage,

a) Compute the reactions at the fuselage
b) Compute the distribution of the resultant shear force ($V_z(x)$) and resultant bending moment ($M_y(x)$). Put your solution in a non-dimensional form (i.e., as a function of (x/L)), and plot the two non-dimensional solutions for $0 \leq x/L \leq 1$.
c) Check that your solution found in b) matches the reactions found in a)
d) Check also that your solution found in b) corresponds to your expectation at the end of the wing (i.e., at $x = L$)

AE 323 - HWK 2 - Solution

Problem 1



(a) 2 reactions : R_o, R_L

2 equs : $\sum F_y = 0 \quad \sum M_z\big|_0 = 0$

$\longrightarrow$ statically determinate

(b) $\sum F_y = 0 \Longrightarrow R_o + R_L = P + 2aw$

$$\sum M_z\big|_0 = 0 \Longrightarrow 4aR_L = Pa + \int_{2a}^{4a} w\, x \, dx$$

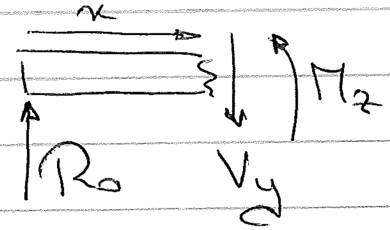
$$= Pa + w(2a)(3a)$$

$$\rightarrow R_L = \frac{P}{4} + \frac{3aw}{2}$$

$$R_o = P + 2aw - R_L = \frac{3P}{4} + \frac{aw}{2}$$

(c) We need to "cut" the beam at 3 locations

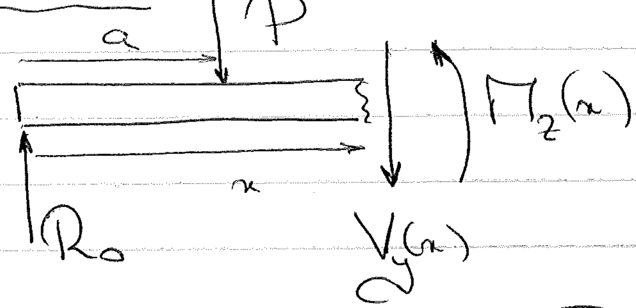
$0 \leq x \leq a$



$$\begin{cases} V_y(x) = R_o \\ M_z(x) = R_o x \end{cases}$$

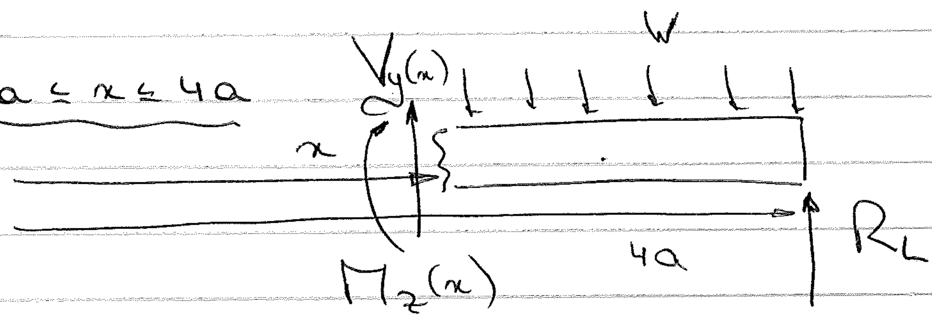
S2

$a \leq x \leq 2a$



$$\begin{cases} V_y(x) = R_0 - P = -\dfrac{P}{4} + \dfrac{aw}{2} \\ M_z(x) = R_0 x - P(x-a) \\ \quad = -\dfrac{P}{4}x + \dfrac{aw}{2}x + Pa \end{cases}$$

$2a \leq x \leq 4a$



$$V_y(x) = (4a-x)w - R_L$$

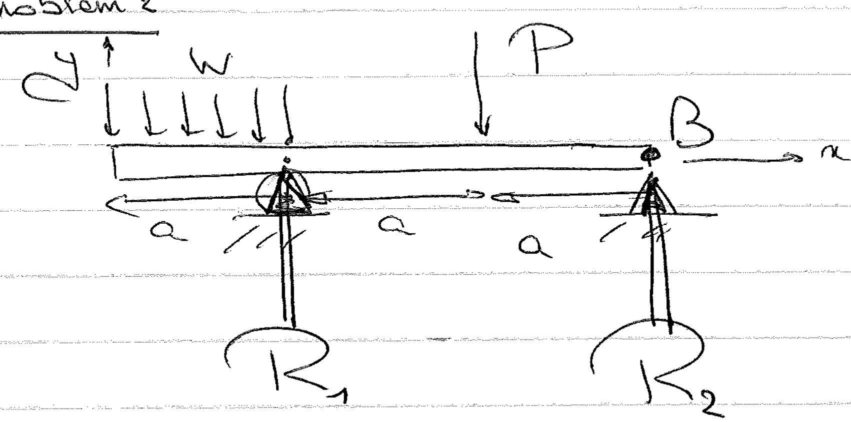
$$= \frac{5aw}{2} - wx - \frac{P}{4}$$

$$M_z(x) = R_L(4a-x) - \int_x^{4a} w(x'-x)\,dx'$$

$$= R_L(4a-x) - w\frac{(4a-x)^2}{2}$$

$$= \left(\frac{P}{4} + \frac{3aw}{2}\right)(4a-x) - \frac{w}{2}(4a-x)^2$$

Problem 2



(a) 2 reactions : R_1, R_2

2 eqns : $\sum F_y = 0$

$\sum M_z = 0$

$\Longleftrightarrow$ statically determinate

(b) $\sum F_y = 0 \Rightarrow R_1 + R_2 = P + aw$

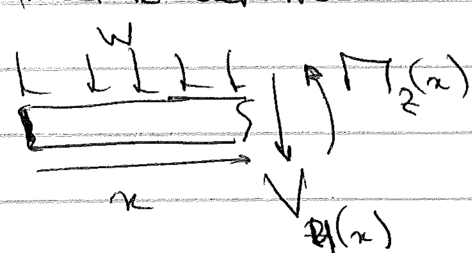
$\sum M_z\big|_B = 0 \Rightarrow Pa + wa\,\frac{5a}{2} = 2a R_1$

$\Longleftrightarrow R_1 = \frac{P}{2} + \frac{5aw}{4}$

$R_2 = P + aw - R_1 = \frac{P}{2} - \frac{aw}{4}$

(c) We again need to cut the beam at 3 locations

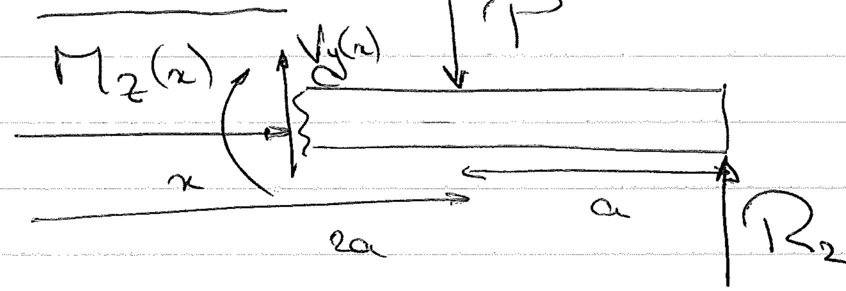
$0 \leq x \leq a$



$V_y(x) = -wx$

$M_z(x) = -wx\,\frac{x}{2}$

$a \leq x \leq 2a$



$$V_y(x) = P - R_2 = \frac{P}{2} + \frac{aw}{4}$$

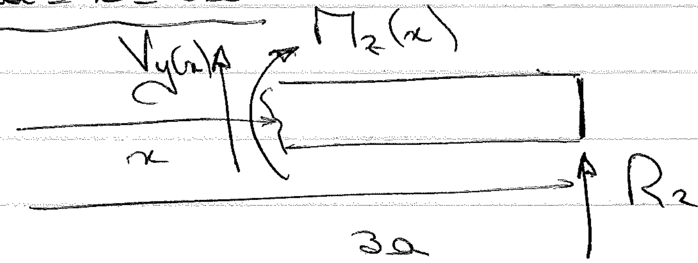
$$M_z(x) = -P(2a - x) + R_2(3a - x)$$

$$= -P(2a - x) + \left(\frac{P}{2} - \frac{aw}{4}\right)(3a - x)$$

$$= -P2a + xP + \frac{3aP}{2} - \frac{P}{2}x - \frac{3a^2w}{4} + \frac{awx}{4}$$

$$= -\frac{Pa}{2} + \frac{Px}{2} - \frac{3a^2w}{4} + \frac{awx}{4}$$

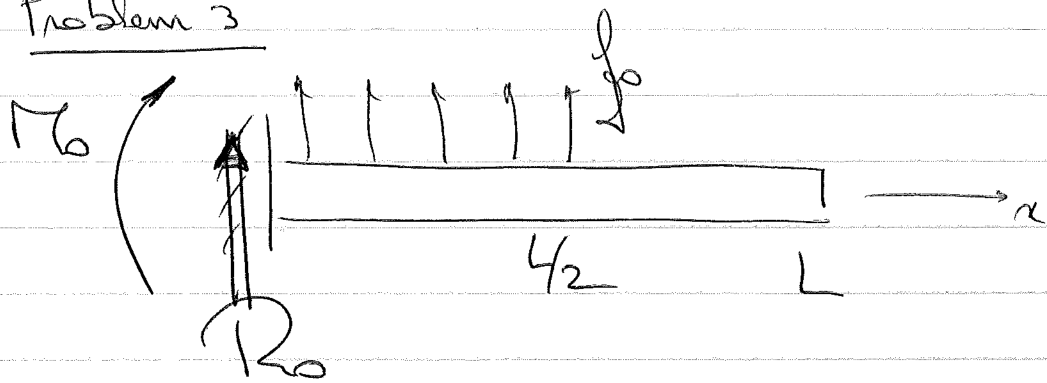
$2a \leq x \leq 3a$



$$V_y(x) = -R_2 = -\frac{P}{2} + \frac{aw}{4}$$

$$M_z(x) = R_2(3a - x) = \left(-\frac{P}{2} + \frac{aw}{4}\right)(3a - x)$$

Problem 3



(a) 2 reactions R_0, M_0

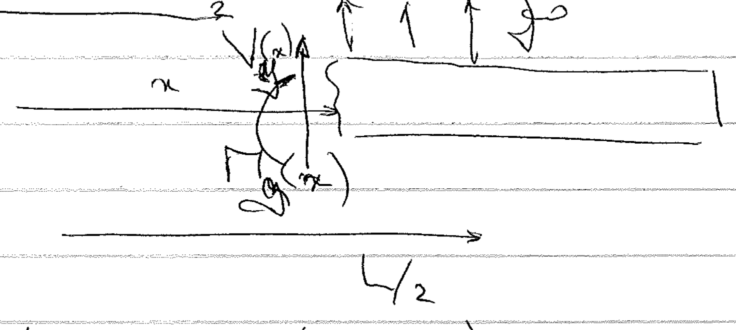
2 eqns $\Sigma F_y = 0$ $\Sigma M_z = 0$!

$\hookrightarrow$ statically determinate

(b) $\Sigma F_y = 0 \Rightarrow R_0 + f_0 \frac{L}{2} = 0 \Rightarrow R_0 = -f_0 \frac{L}{2}$

$\Sigma M_z|_0 = 0 \Rightarrow M_0 = \int_0^{L/2} f_0 x \, dx \Rightarrow M_0 = f_0 \frac{L^2}{8}$

(c) For $0 < x < \frac{L}{2}$



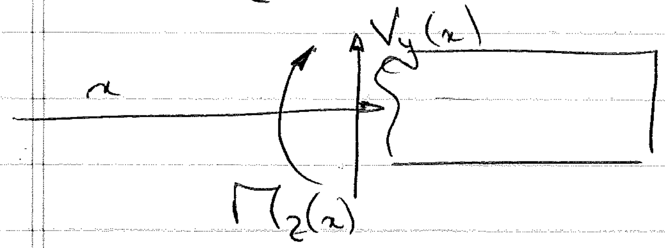
$$V_y(x) = -f_0 \left(\frac{L}{2} - x \right)$$

$$M_z(x) = \int_x^{L/2} f_0 (x' - x) \, dx' = \left[f_0 \left(\frac{(x')^2}{2} - x x' \right) \right]_x^{L/2}$$

$$= f_0 \left[\frac{L^2}{8} - x \frac{L}{2} - \frac{x^2}{2} + x^2 \right] = f_0 \left[\frac{L^2}{8} - x \frac{L}{2} + \frac{x^2}{2} \right]$$

S6

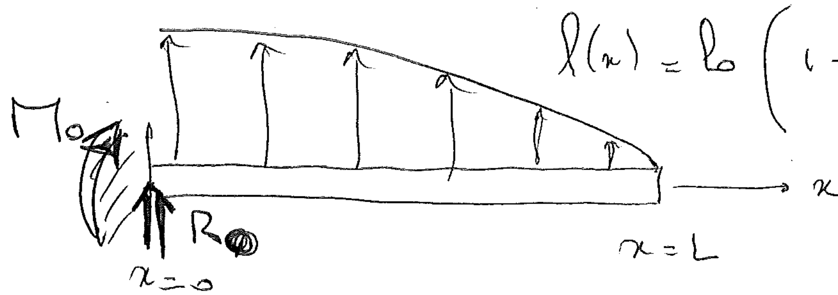
For $\frac{L}{2} \le x \le L$



$$\begin{cases} V_y(x) = 0 \\ M_z(x) = 0 \end{cases}$$

Problem 4.

$$\ell(x) = \ell_0 \left(1 - \left(\frac{x}{L}\right)^2\right) \quad (N/m)$$



. Reactions at fuselage

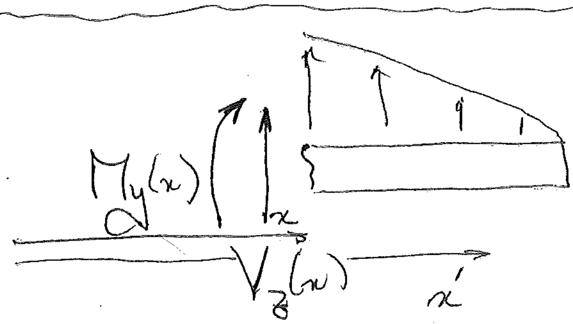
$$R_0 = -\int_0^L \ell(x')\,dx' = -\int_0^L \ell_0 \left[1 - \left(\frac{x'}{L}\right)^2\right] dx'$$

$$= -\ell_0 \left[L - \frac{1}{L^2}\left(\frac{L^3}{3}\right)\right] = -\frac{2\ell_0 L}{3}$$

$$M_0 = \int_0^L x'\ell(x')\,dx' = \ell_0 \int_0^L \left[x' - \frac{1}{L^2}(x')^3\right] dx'$$

$$= \ell_0 \left[\frac{L^2}{2} - \frac{1}{L^2}\frac{L^4}{4}\right] = \frac{\ell_0 L^2}{4}$$

. Shear and moment diagrams



$$V_z(x) = -\int_x^L p(x')\,dx' = -p_0 \int_x^L \left[1 - \left(\frac{x'}{L}\right)^2\right] dx'$$

$$= -p_0 \left[x' - \frac{1}{L^2}\frac{(x')^3}{3}\right]_x^L$$

$$= -p_0 \left[L - \frac{L}{3} - x + \frac{x^3}{3L^2}\right]$$

$$= -p_0 L \left[\frac{2}{3} - \left(\frac{x}{L}\right) + \frac{1}{3}\left(\frac{x}{L}\right)^3\right]$$

$$M_y(x) = \int_x^L (x' - x)\, p(x')\,dx'$$

$$= p_0 \int_x^L (x' - x)\left[1 - \left(\frac{x'}{L}\right)^2\right] dx'$$

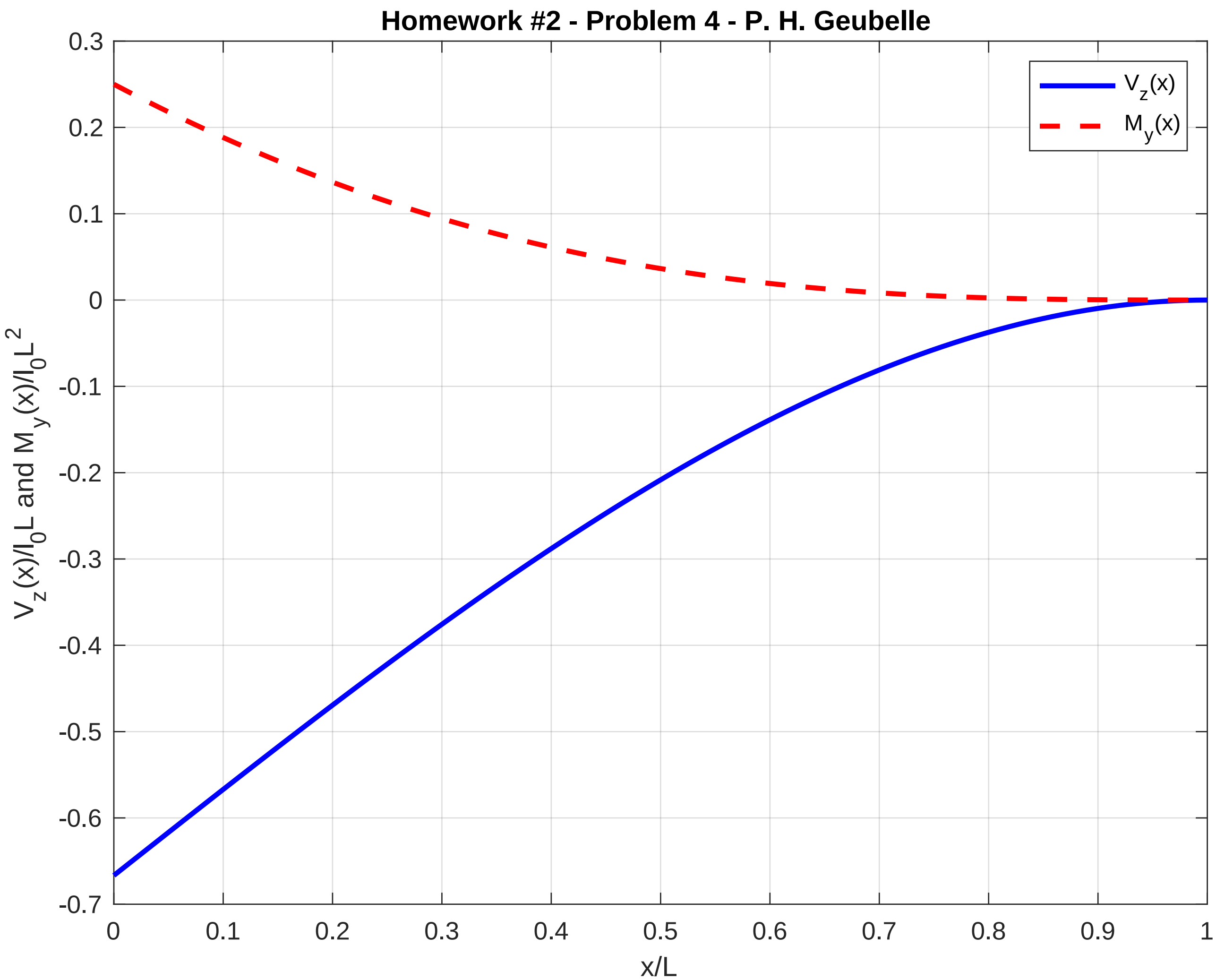
$$= p_0 \int_x^L \left[x' - x - \frac{1}{L^2}(x')^3 + \frac{x}{L^2}(x')^2\right] dx'$$

$$= p_0 \left[\frac{(x')^2}{2} - x x' - \frac{1}{L^2}\frac{(x')^4}{4} + \frac{x}{L^2}\frac{(x')^3}{3}\right]_x^L$$

$$= p_0 \left[\frac{L^2}{2} - xL - \frac{L^2}{4} + \frac{xL}{3} - \frac{x^2}{2} + x^2 + \frac{x^4}{4L^2} - \frac{x^4}{3L^2}\right]$$

$$= p_0 \left[\frac{L^2}{4} - \frac{2xL}{3} + \frac{x^2}{2} - \frac{x^4}{12L^2}\right]$$

$$= p_0 L^2 \left[\frac{1}{4} - \frac{2}{3}\left(\frac{x}{L}\right) + \frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{1}{12}\left(\frac{x}{L}\right)^4\right]$$


Homework #2 - Problem 4 - P. H. Geubelle
$V_z(x)$
$M_y(x)$
$V_z(x)/l_0L$ and $M_y(x)/l_0L^2$
x/L
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

- At $x=0$, we find from the solution in b)

$$\left.\begin{aligned} V_z(0) &= -\frac{2}{3} p_0 L \\ M_y(0) &= \frac{p_0 L^2}{4} \end{aligned}\right\} \quad \text{same as part a)} \quad \checkmark$$

- At $x=L$, we expect to find $V_z(L) = M_y(L) = 0$

Indeed, that's what the solution b) evaluated at $x=L$ is. ✓