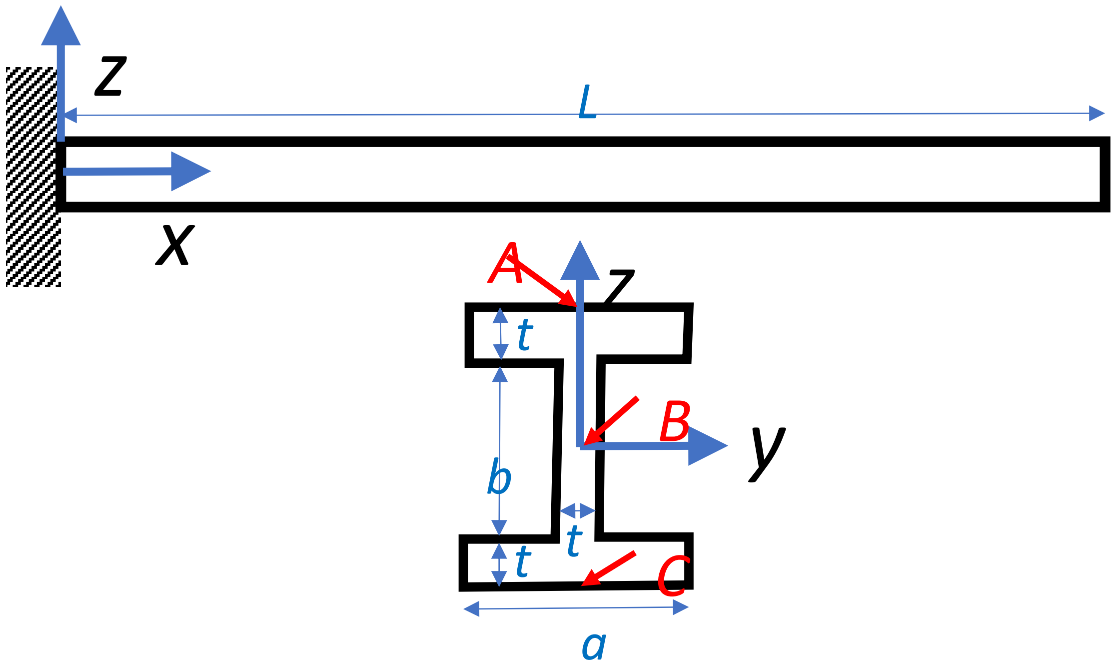
AE 323 – Spring 2019 – Homework #3
Wednesday Feb. 6, 2019
Due on Friday Feb. 15, 2019

Problem 1

Consider the cantilever I-beam shown in the figure below.



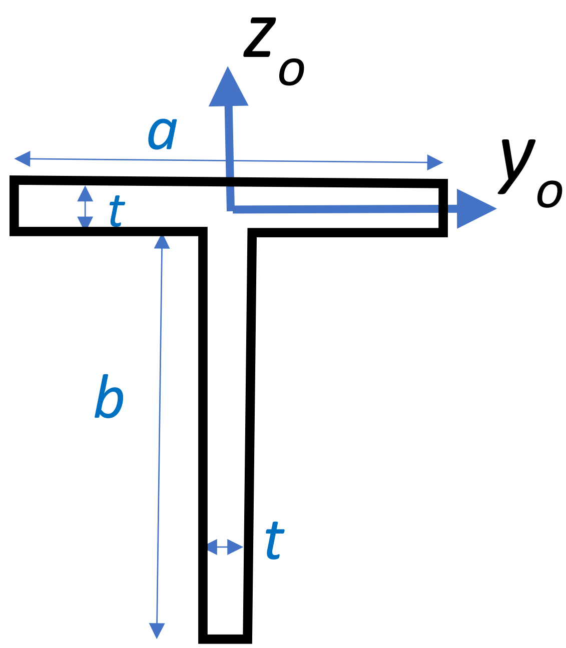
The cantilever beam is of length L and is made of a linearly elastic material with Young's modulus E, density ρ, and yield stress σ_y. It only supports its own weight (use g for the acceleration of gravity acting in the negative z direction).

1) What is the distributed transverse load f_z (in N/m) for this problem?
2) Compute the internal shear force $V_z(x)$ (in N) and internal bending moment $M_y(x)$ (in Nm).
3) Knowing that, by symmetry, the neutral axis of the beam goes through the center of the cross-section, compute the moments of inertia I_{yy}, I_{zz}, and I_{yz} of the cross-section (use the simplified relations assuming that $t << a$ and b).
4) Compute the axial stress σ_{xx} in the middle of the beam (i.e., $x = L/2$) at the top (point A in the figure), middle (point B) and bottom (point C) of the cross-section.
5) At what length L of the beam does the beam start to yield under its own weight, for $b=2a=12t$, and for ρ=7800 kg/m^3, g=10 m/s^2, t = 5 mm, and σ_y=200 MPa. Check the units of your solution.

Problem 2

Consider a <u>simply supported</u> beam of length L subjected to a uniform pressure p (in Pa) applied along its top surface in the negative z-direction. The cross-section has a T-shape (see figure below), with $t \ll a$ and b. The material has a stiffness E.

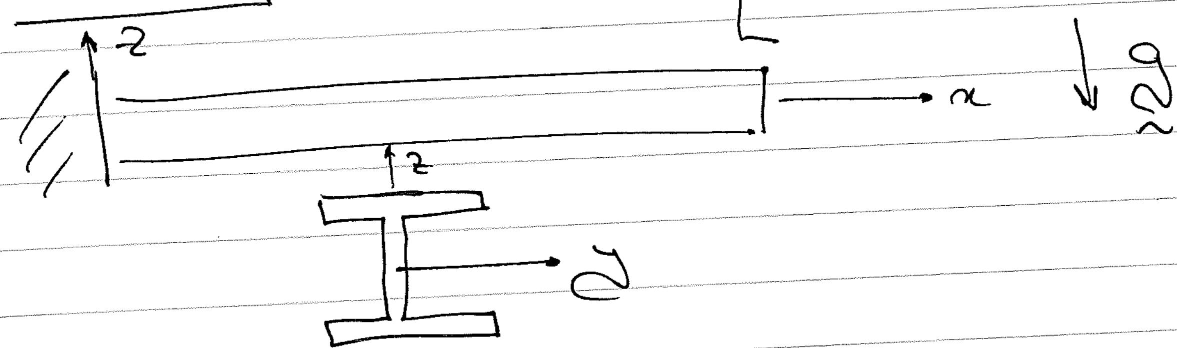
1) Find the location of the neutral axis (with respect to the (y_o,z_o) axis system shown in the figure) for $a = 12\,t$ and $b = 10\,t$.
2) Compute the moment of inertia I_{yy} and I_{yz} with respect to the (y,z) axes passing through the center of gravity of the cross-section (use the exact expressions).
3) What is the distributed transverse load f_z (in N/m) for this problem?
4) Compute the resultant bending moment $M_y(x)$ in the beam
5) Compute the maximum tensile and compressive axial stresses in the beam. Indicate where it is located.



AE 323 _ Homework assignment # 3

Solution

Problem 1



1) ? f_z associated with gravity load?

$$f_z = -\rho A g$$

where $A = (2a+b)t$ = cross-section

g = acceleration of gravity

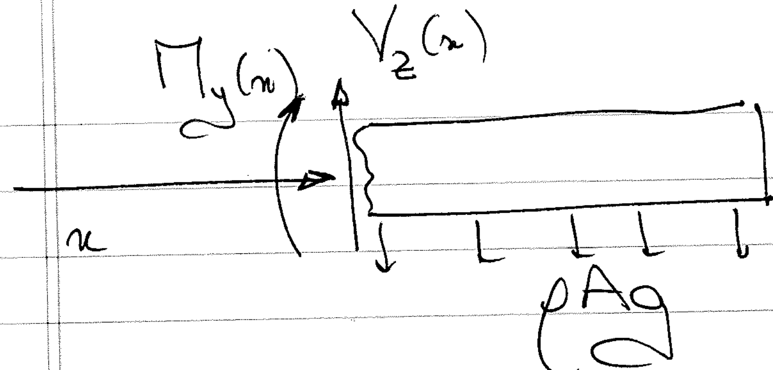
ρ = material density

Check units: $\left[f_z\right] = \frac{kg}{m^3} \times m^2 \times \frac{m}{s^2}$ N

$$= N \frac{m^2}{m^3} = \frac{N}{m} \checkmark$$

2) ? $V_z(x)$ and $M_y(x)$

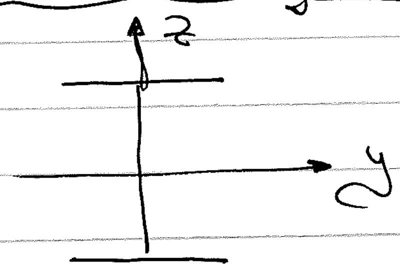
Let us cut the beam @ x and use a FBD for the equilibrium of the right side



- $V_z(x) = \rho A g (L - x)$
- $M_y(x) = -\int_x^L \rho A g (x' - x)\, dx'$

$$= -\rho A g \frac{(L-x)^2}{2}$$

3) ? Moments of inertia ? (simplified relation assuming that $t \ll a$ & b)



$$I_{yy} \simeq \frac{b^3 t}{12} + 2 a t \left(\frac{b}{2}\right)^2$$

$$I_{zz} \simeq 2 \frac{a^3 t}{12}$$

$$I_{yz} = 0$$ (since we have (at least) one axis of symmetry)

4) ? $\sigma_{xx}\left(x = \frac{L}{2}, y = 0, z = \frac{b}{2} + t\right)$ pt A

$\sigma_{xx}\left(L/2, 0, 0\right)$ pt B

$\sigma_{xx}\left(\frac{L}{2}, 0, z = -\frac{b}{2} - t\right)$ pt C

Since $\begin{cases} \cdot\ I_{yz} = 0 \\ \cdot\ \text{no thermal load} \\ \cdot\ \text{no load in } x\text{-direction } (N=0) \end{cases}$

$$\hookrightarrow \sigma_{xx} = -\frac{z M_y(x)}{I_{yy}}$$

$$@\ x = \frac{L}{2}, \quad M_y = -\rho A g \frac{L^2}{8}$$

$$\Rightarrow \text{Point A:} \quad \sigma_{xx} = \rho A g \frac{L^2}{8}\left(\frac{b}{2} + t\right) \frac{1}{\frac{b^3 t}{12} + \frac{a b^2 t}{2}} > 0$$

$$\text{Point B} \quad \sigma_{xx} = 0$$

$$\text{Point C} \quad \sigma_{xx} = -\rho A g \frac{L^2}{8}\left(\frac{b}{2} + t\right) \frac{1}{\frac{b^3 t}{12} + \frac{a b^2 t}{2}} < 0$$

5) Let $b = 2a = 12t$

$$(M_y)_{max} = M_y(0) = -\rho A g \frac{L^2}{2}$$

$$\hookrightarrow (\sigma_{xx})_{max} = \sigma_{xx}\left(0, 0, \frac{b}{2} + t\right)$$

$$= \rho A g \frac{L^2}{2}\left(\frac{b}{2} + t\right) \frac{1}{\frac{b^3 t}{12} + \frac{a b^2 t}{2}}$$

$$= \rho A g \frac{L^2}{2}\, 7t \frac{1}{144 t^4 + \frac{6 * 144 t^4}{2}}$$

$$= \frac{\rho A g L^2\, 7t}{2 * 4 * 144\, t^4} = \frac{7 \rho A g L^2}{1152\, t^3}$$

$\hookrightarrow L$ s.t

$$(\sigma_{xx})_{max} = \sigma_y$$

$$\frac{7\rho A g L^2}{1152\, t^3} = \sigma_y$$

$$L = \sqrt{\frac{1152\, \sigma_y\, t^3}{7\rho A g}} = \sqrt{\frac{48\, \sigma_y\, t}{7\rho g}}$$

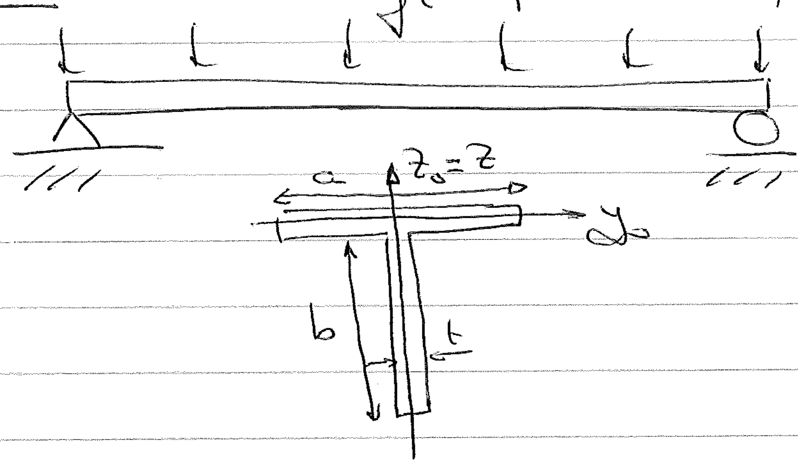
with $A = (2a+b)t = 24t^2$

Check units $\sqrt{\frac{\cancel{N}}{\cancel{m^2}} m \frac{\cancel{m^3}}{\cancel{kg}} \frac{\cancel{s^2}}{\cancel{m}}} = m$ ✓

With $\sigma_y = 200\ MPa$
$\rho = 7800\ kg/m^3$
$t = 5\ mm$
$g = 10\ m/s^2$

we get
$$L = 9.376\ m$$

Problem 2



1) ? Location of neutral axis? (i.e. of the center of gravity of the cross-section)

. By symmetry, $y_0^* = 0 \Rightarrow$ z-axis $\equiv$ z_0-axis

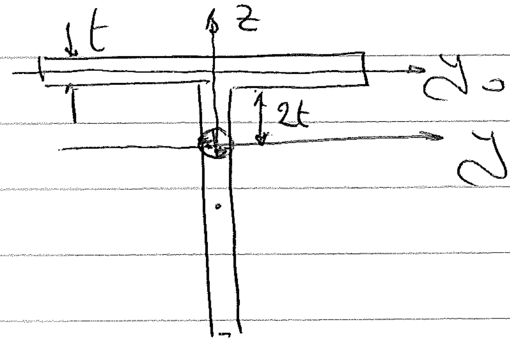
. $$z_0^* = \frac{1}{A}\sum_{i=1}^{2} \bar{z}_i A_i$$

$$= \frac{1}{(a+b)t}\left(0 * at - \left(\frac{b+t}{2}\right) bt\right)$$

$$= \frac{-(b+t)b\cancel{t}}{2(a+b)\cancel{t}} = \frac{-11t * \cancel{10}^{5}t}{2 * \cancel{22}\cancel{t}} = -\frac{5t}{2}$$

$a = 12t$
$b = 10t$

We thus have



2) Moment of inertia ?

$$I_{yy} = \frac{at^3}{12} + \left(\frac{5t}{2}\right)^2 at + \frac{b^3 t}{12} + (3t)^2 bt$$

$$= at^3\left(\frac{1}{12} + \frac{25}{4}\right) + \frac{b^3 t}{12} + 9bt^3$$

$$= 12t^4 \frac{76}{12} + \frac{1000t^4}{12} + 90t^4$$

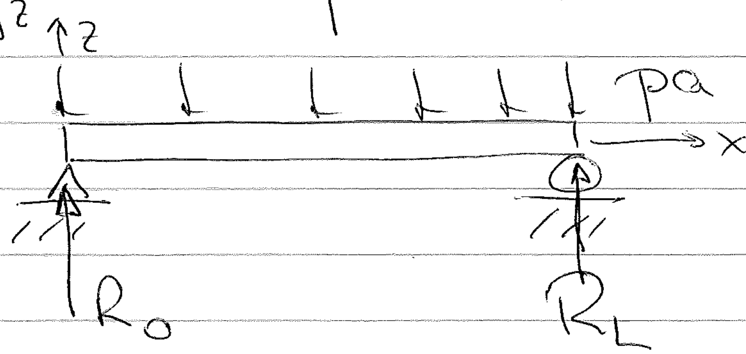
$a = 12t$
$b = 10t$

$$= \left(76 + \frac{250}{3} + 90\right)t^4$$

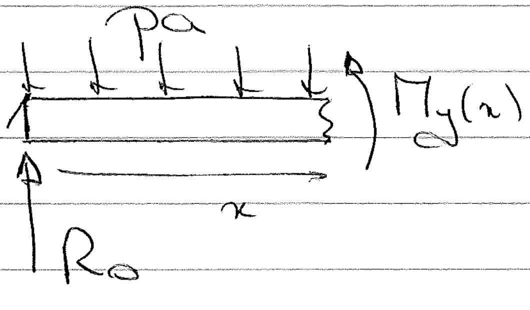
Since the z axis is an axis of symmetry, $I_{yz} = 0$

3) $f_z(x) = -pa$ (in N/m)

4)



By symmetry, $R_0 = R_L = \frac{1}{2}paL$



$$M_y(x) = R_0 x - pa\frac{x^2}{2}$$

$$M_y(x) = \frac{1}{2} p a L x - \frac{1}{2} p a x^2$$

$$= \frac{1}{2} p a x (L - x) > 0$$ since the beam deflects as 

Note: as expected $M_y(0) = M_y(L) = 0$ since the beam is simply supported

5) ? Maximum tensile & compressive stress?

$$M_y\Big|_{max} = M_y(L/2) = \frac{1}{2} p a \left(\frac{L}{2}\right)^2 = \frac{p a L^2}{8}$$

$$\sigma_{xx}\left(\frac{L}{2}, y, z\right) = -\frac{M_y\left(\frac{L}{2}\right) z}{I_{yy}}$$

Maximum compressive stress @ $z = 3t$ (upper surface)

$$\sigma_{xx}\left(\frac{L}{2}, y, 3t\right) = -\frac{paL^2/8 * 3t}{I_{yy}}$$

Maximum tensile stress @ $z = -8t$ (bottom surface)

$$\sigma_{xx}\left(\frac{L}{2}, y, -8t\right) = \frac{paL^2/8 * 8t}{I_{yy}}$$