Grab your broom

Friday, January 18, 2019 1:53 PM

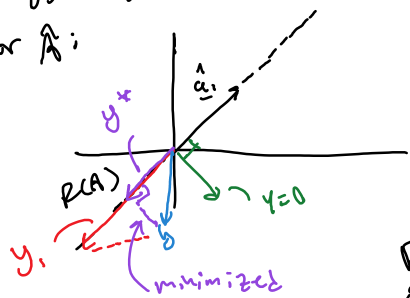
"Hide your kids, hide your wife"

"sprite cranberry"

"Aaaaah Ahhhh"

Ax will always belong to R(A) so looking for some vector y that is in R(A)

Consider for $\hat{A}$:



For $\min_{x \in \mathbb{R}^n} \|Ax-b\| = \min_{y \in R(A)} \|y-b\|$

So y_1 is NO GOOD

Instead y^* is BEST

RESULT: The minimizer, y^*, in R(A) satisfies $(y^*-b, r) = 0$ for any $r \in R(A)$ dot product

GREAT! But how to compute?

- Use a basis for the R(A)
- y^* can be written as $y^* = \sum_{j=1}^{k} \alpha_j s_j$ (basis vectors) but WHAT ARE α's?

$(y^*-b, r) = 0$ (1)

$y^* = \sum_{j=1}^{k} \alpha_j s_j$ (2) $\rightarrow \left(\sum_{j=1}^{k} \alpha_j s_j - b, s_i\right) = 0$ (3) can pick, true for any s's $i \to k$

$\sum_{j=1}^{k} \alpha_j (s_j, s_i) - (b, s_i) = 0$

$\sum_{j=1}^{k} \left[\alpha_j (s_j, s_i)\right] = (b, s_i) \quad i = 1, \dots, k \Rightarrow$

$$\begin{bmatrix} (s_1,s_1) & (s_1,s_2) & \dots & (s_1,s_k) \\ \vdots & \vdots & & \vdots \\ (s_k,s_1) & (s_k,s_2) & \dots & (s_k,s_k) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_k \end{bmatrix} = \begin{bmatrix} (b,s_1) \\ \vdots \\ (b,s_k) \end{bmatrix}$$

Best App.

$y^* = \sum_{j=1}^{k} \alpha_j s_j$

What About Numerical Sensitivity of Solutions

- Case studies

$A = \begin{bmatrix} 1 & 1 \\ 0 & \varepsilon \end{bmatrix} \quad \varepsilon > 0,\ \varepsilon \ll 1$ Ill-conditioned → columns are nearly collinear

$Ax = b$? $\rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ \varepsilon \end{bmatrix} x_2 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ Gotta get ε very right

So $x_2 = b_2/\varepsilon$, $x_1 = b_1 - x_2 = b_1 - b_2/\varepsilon$

- Now:

$A = \begin{bmatrix} 1 & \varepsilon \\ 0 & 1 \end{bmatrix} \rightarrow Ax = b \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} \varepsilon \\ 1 \end{bmatrix} x_2 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ — columns are nearly orthogonal

$x_2 = b_2$

$x_1 = b_1 - \varepsilon x_2 = b_1 - \varepsilon b_2$ — small changes in ε, small change in solution

Want to change basis as close to lin. ind. as possible

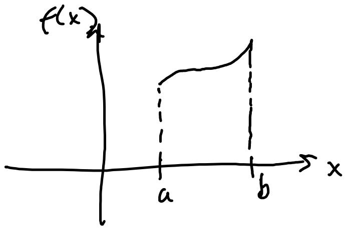
- The "condition #" of a matrix formalizes this idea
  - Just use MATLAB:

    $\text{cond}(A) = \text{condition \#}, \underset{\text{kappa}}{\kappa}(A)$ (depends on your problem)

  - Large kappa, poorly conditioned

<u>Function Interpolation & Approximation</u>

Goal:



C means space of continuous functions on $a \le x \le b$

$f(x) \in C_{[a,b]}$