# AE 370 — Homework #2

1. **Probing the pitfalls of the monomial basis**. We discussed in class that when performing polynomial interpolation, a linear system $\boldsymbol{A}\boldsymbol{c} = \boldsymbol{f}$ arises to solve for coefficients $\boldsymbol{c}$ that express the interpolant in the particular basis used to build $\boldsymbol{A}$.

   We also discussed how the Lagrange basis is vastly superior to the monomial basis. You will demonstrate that for yourself in this problem. For all parts of this problem, use uniformly spaced points defined by $x_j = j/n$ for $j = 0, ..., n$, where $n$ is the degree of the polynomial being considered.

   (a) In Matlab (or your software of choice), build the matrix $\boldsymbol{A}$ for both the monomial basis and the Lagrange basis for $n = 3, 6, 12, 24, 48$, and 96. For each $n$, compute the condition number of $\boldsymbol{A}$, $\kappa(\boldsymbol{A})$, for both bases.

   (b) For $n = 5$, plot the first six basis functions for both the monomial basis and the Lagrange basis. These basis functions should not be plotted on the coarse grid defined by the $x_j$, but on a sufficiently fine grid that you can resolve the behavior of the basis functions. That is, build $x$ using $linspace(0, 1, m)$, where $m = 100$ or 1000 and plot each basis function on this fine grid. Use these plots to explain why the condition number is so much more poorly behaved for the monomial basis.

   (c) Assuming $\boldsymbol{A}$ was built using the monomial basis, would the poor conditioning of $\boldsymbol{A}$ be resolved if we used Chebyshev points instead of uniformly distributed points? Why or why not?

2. **Strengths and weaknesses of polynomial interpolation**. Runge's problem is a famous case of when polynomial interpolation can go dramatically wrong. In fact, it is one of the primary reasons why many experts believe that polynomial interpolation is doomed to fail, when this is in fact *not true* provided that one uses the right set of interpolation points!

   We will investigate behavior of polynomial interpolants on Runge's problem now: consider interpolating $f(x) = \frac{1}{1+x^2}$ over the interval $-5 \leq x \leq 5$.

   (a) Interpolate $f(x)$ in equispaced points using the Lagrange basis for polynomials of order $n = 5, 10, 15, 20$. Plot the results for the various $n$.

   (b) Interpolate $f(x)$ in Chebyshev points using the Lagrange basis for polynomials of order $n = 5, 10, 15, 20$. Plot the results for the various $n$.

   (c) Using the error bound from class, why do Chebyshev points lead to convergence whereas equispaced points do not?