AE 370 — Homework #3

1. **Re-visiting Runge's problem**. Last week, we established that when globally interpolating Runge's function with polynomials, it was crucial to use a nonuniform point distribution. We will investigate whether this restriction applies to cubic splines.

   Re-consider interpolating $f(x) = \frac{1}{1+x^2}$ over the interval $-5 \leq x \leq 5$. This time, locally interpolate Runge's function using *natural* cubic splines with equispaced points for $n = 5, 10, 25, 50$, and plot the result for these different values of n. You may **not** use Matlab's spline command. You may, however, use the lecture notes on splines posted to Piazza. These notes contain a significant chunk of the code that you can draw from, with a few lines left empty and waiting to be filled in by you.

   Do splines suffer from the same dramatic issues in equispaced points as global polynomial interpolants?

2. **Strengths and weaknesses of trigonometric interpolation**. Consider the following trigonometric interpolation problems over the domain $0 \leq x \leq 2\pi$.

   (a) Interpolate $f(x) = e^{\cos(x)+\sin(3x)}$ in equispaced points using an FFT for $n = 5, 25, 50$. Plot the results for the various n.

   (b) Interpolate $f(x) = x$ in equispaced points using an FFT for $n = 5, 25, 50$. Plot the results for the various n.

   (c) Why does trigonometric interpolation work so well in part (a) but so poorly in part (b)? *Hint*: recall that in trigonometric interpolation we represent the function using a basis of periodic functions.