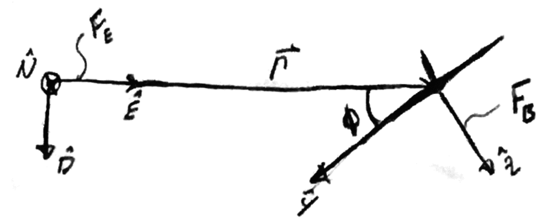
Assignment 2 | 3128 | Connor Ott

Problem 1

ψ - Azimuth - rotation about $\hat{D}$, measured from $\hat{N}$

$\boxed{\psi = 180^\circ}$



θ - Elevation - rotation about $\hat{E}$, $\boxed{\theta = 0^\circ}$

ϕ - Bank - rotation about $\hat{N}$, $\boxed{\phi = -\phi^\circ}$

Problem 2

$\dot{\psi}$ - rotation <u>rate</u> about $\hat{D}$

1 rotation in 1 period $\Rightarrow 360^\circ / \tau$

$$\tau = \frac{2\pi}{0.1 \text{ rad/s}} = 62.83 \text{ s}$$

$$\dot{\psi} = \frac{360^\circ}{62.83} = \boxed{5.73^\circ/\text{s} = 0.1 \text{ rad/s}}$$

$\dot{\theta}$ - rotation <u>rate</u> about $\hat{E}$

$\boxed{\dot{\theta} = 0 \text{ deg/s}}$

$\dot{\phi}$ - rotation rate about $\hat{N}$

$$\dot{\phi} = \frac{d}{dt}\phi = \boxed{0 \text{ deg/s} = \dot{\phi}}$$

Problem 3

p - roll rate, the rate at which the plane rotates about its body-fixed $\hat{x}$ axis

$\boxed{p = 0 \text{ deg/s}}$

q - Pitch rate, the rate at which the plane rotates about its body-fixed $\hat{y}$ axis

$\boxed{q = \cancel{0 \text{ deg/s}}}$

r - Yaw rate, the rate at which the plane rotates about its body-fixed $\hat{z}$ axis $\boxed{r = \cancel{0 \text{ deg/s}}}$

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Problem 4

$\dot{p}$ - $\hat{x}$ component of $\frac{d^B}{dt}\vec{\omega}^{EB} = \frac{d}{dt}(0) = 0 \ rad/s^2$

$\dot{q}$ - $\hat{y}$ component of $\frac{d^B}{dt}\vec{\omega}^{EB} = \frac{d}{dt}(0.1 \sim) = 0 \ rad/s^2$

$\dot{r}$ - $\hat{z}$ component of $\frac{d^B}{dt}\vec{\omega}^{EB} = \frac{d}{dt}(0.1 \sim) = 0 \ rad/s^2$

Problem 5

$\vec{G} = \frac{d^E}{dt}\vec{h}^E$, the net moment about the c.o.m. of the aircraft expressed in Body frame coordinates

$$\vec{G} = \frac{d^E}{dt} I\vec{\omega}^{EB} = \bar{I}_B \frac{d}{dt}\vec{\omega}_B^{EB} + \vec{\omega}_B^{EB} \times \vec{h}_B^E$$

~~angular velocity is constant~~

$$\vec{G} = -\begin{bmatrix}(I_y - I_z)qr \\ (x_z - I_x)\cancel{p}r \\ (I_x - I_y)\cancel{pq}\end{bmatrix} + \underbrace{\begin{bmatrix}\cancel{\dot{p}} \\ \cancel{\dot{q}} \\ \cancel{\dot{r}}\end{bmatrix}}_{\vec{0}}$$

$p = 0$

$$\vec{G} = -(I_y - I_z)qr$$

$$\boxed{\vec{G} = -0.1^2(I_y - I_z)\cos\phi\sin\phi \ \hat{x}}$$

- $\vec{G}$ is not fixed in the Inertial Frame since $\hat{x}$ is constantly changing direction
- $\vec{G}$ is fixed in the body frame since ϕ isn't changing

Problem 6

$\vec{h}$ is the Inertial Angular momentum vector of the plane

$$\vec{h} = \overline{I}\,\vec{\omega}^{EB}$$

$$= \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}$$

$$\boxed{\vec{h} = 0.1 I_z \hat{D}} = 0.1\, I_z (\sin\phi\, \hat{y} + \cos\phi\, \hat{z})$$

$\vec{h}$ is not fixed in the inertial frame since $\vec{\omega}^{EB}$ is non-zero

$\vec{h}$ is fixed in the body frame since ϕ is not changing

Problem 7

Any value dependant on mass or moment of inertia would change

These would be #7 ($\vec{f}, \vec{f}_B, \vec{f}_E$) from assignment 1, + problems 4 ($\dot{p}, \dot{q}, \dot{r}$), 5 ($\vec{G}, \vec{G}_E, \vec{G}_B$) + 6 ($\vec{h}, \vec{h}_E, \vec{h}_B$)

Problem 8

Attitude will definitely induce translation on a quad copter since it will introduce some horizontal force

Translation does not necessarily cause changes in attitude, however