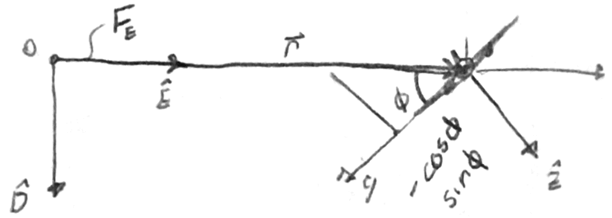
Assignment #1 | ASEN 3128 | Connor Ott

Problem 1



$\boxed{\vec{r} = (0\hat{N} + 1\hat{E} + 0\hat{D})\ km}$ This is a position vector connecting the origin of F_E & the C.G. the body of the plane

$\vec{r}_E$ is $\vec{r}$ written in F_E coordinates $F_E = (O, \hat{N}, \hat{E}, \hat{D})$. It is a coordinate representation of $\vec{r}$

$$\boxed{\vec{r}_E = \begin{bmatrix} 0 \\ 1\,km \\ 0 \end{bmatrix}}$$

$\vec{r}_B$ is a coordinate representation of $\vec{r}$ in $F_B = (\hat{x}, \hat{y}, \hat{z})$

$$\boxed{\vec{r}_B = \begin{bmatrix} 0 \\ -\cos\phi \\ \sin\phi \end{bmatrix} km}$$

Problem 2

$\vec{V}^E$ is the inertial velocity vector of the plane

$\boxed{\vec{V}^E = -100\hat{N} + 0\hat{E} + 0\hat{D}\ m/s}$

$\vec{V}^E_E$ is $\vec{V}^E$ represented in the F_E coordinate frame $F_E = (\hat{N}, \hat{E}, \hat{D})$

$$\boxed{\vec{V}^E_E = \begin{bmatrix} -100 \\ 0 \\ 0 \end{bmatrix} m/s}$$

$\vec{V}^E_B$ is $\vec{V}^E$ represented in the F_B coordinate frame, $F_B = (\hat{x}, \hat{y}, \hat{z})$

$$\boxed{\vec{V}^E_B = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} m/s}$$

$\Longrightarrow$

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Problem 3

$\vec{\omega}^{EB}$ is the angular velocity vector of the B frame as seen from the E frame. This is the inertial angular velocity of the B frame

$$\vec{\omega}^{EB}_{EN} = \cancel{\begin{bmatrix} 0 \\ 0 \\ \frac{v}{r} \end{bmatrix}} = \cancel{\begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}} \qquad \boxed{\vec{\omega}^{EB} = (0\hat{N} + 0\hat{E} + 0.1\hat{D})\ \text{rad/s}}$$

$\vec{\omega}^{EB}_{E}$ is the angular velocity of the plane represented in F_E coordinates. $F_E = (\hat{N}, \hat{E}, \hat{D})$

$$\boxed{\vec{\omega}^{EB}_{E} = \begin{bmatrix} 0 \\ 0 \\ 0.1\ \text{rad/s} \end{bmatrix}}$$

$\vec{\omega}^{EB}_{B}$ is the coordinate representation of $\vec{\omega}^{EB}$ in F_B coords

$F_B = (\hat{x}, \hat{y}, \hat{z})$

$$\boxed{\vec{\omega}^{EB}_{B} = \begin{bmatrix} 0 \\ 0.1 \sin\phi\ \text{rad/s} \\ 0.1 \cos\phi\ \text{rad/s} \end{bmatrix}}$$

Problem 4

$\frac{d^B}{dt}\vec{r}$ is the time rate of change of the $\vec{r}$ vector as seen by the body-fixed frame

$$\boxed{\frac{d^B}{dt}\vec{r} = (0\hat{x} + 0\hat{y} + 0\hat{z})\ \text{m/s}}$$

$\left(\frac{d^B}{dt}\vec{r}\right)_E$ is the body-fixed derivate of $\vec{r}$ represented in F_E coordinates

$$\boxed{\left(\frac{d^B}{dt}\vec{r}\right)_E = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\ \text{m/s}}$$

Problem 4 cont

$\left(\frac{d^B}{dt}\vec{r}\right)_B$ is the body-fixed time derivative of $\vec{r}$ represented in F_B coordinates. $F_B = (\hat{x}, \hat{y}, \hat{z})$

$$\boxed{\left(\frac{d^B}{dt}\vec{r}\right)_B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} m/s}$$

Problem 5

$\left(\frac{d^E}{dt}\vec{V}^E\right)_E$ is the inertial Earth-fixed time derivative of the inertial velocity vector of the plane represented in F_E coordinates. $F_E = (\hat{N}, \hat{E}, \hat{D})$

$$\left(\frac{d^E}{dt}\vec{V}^E\right)_E = \begin{bmatrix} 0 \\ \frac{-|\vec{V}|^2}{r} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-100^2}{1000} \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ -10 \\ 0 \end{bmatrix} m/s^2 = \left(\frac{d^E}{dt}\vec{V}^E\right)_E}$$

$\left(\frac{d^B}{dt}\vec{V}^E\right)_B$ is the body-fixed time derivative of the inertial velocity vector of the plane expressed in F_B coordinates

$F_B = (\hat{x}, \hat{y}, \hat{z})$

$$\left(\frac{d^B}{dt}\vec{V}^E\right)_B = \frac{d}{dt}\underbrace{\begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}}_{\vec{V}^E_B} = \boxed{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} m/s^2 = \left(\frac{d^B}{dt}\vec{V}^E\right)_B}$$

Problem 6

$$\frac{d^E}{dt}\vec{V}^E = \dot{\vec{V}}^E = \underbrace{\dot{\vec{V}}^B}_{\vec{0}} + \vec{\omega}^{EB} \times \vec{V}^E$$

$$\dot{\vec{V}}^E = (0\hat{N} + 0\hat{E} + 0.1\hat{D}) \times (-100\hat{N} + 0\hat{E} + 0\hat{D})$$

$$\boxed{\dot{\vec{V}}^E = (0\hat{N} - 10\hat{E} + 0\hat{D})\, m/s^2}$$

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Problem 7

$\vec{f}$ is the inertial force vector acting on the plane

$$\vec{f} = m \frac{d^E}{dt} \vec{V}^E = \boxed{-10m\,\hat{E}\;N = \vec{f}} \quad \text{where } m = \text{mass of plane}$$

$\vec{f}_E$ is the coordinate representation of $\vec{f}$ in F_E coordinates

$F_E = (\hat{N}, \hat{E}, \hat{D})$

$$\boxed{\vec{f}_E = \begin{bmatrix} 0 \\ -10m \\ 0 \end{bmatrix} N}$$

$\vec{f}_B$ is the coordinate representation of $\vec{f}$ in F_B coordinates

$F_B = (\hat{x}, \hat{y}, \hat{z})$

$$\boxed{\vec{f}_B = \begin{bmatrix} 0 \\ 10\cos\phi\, m \\ -10\sin\phi\, m \end{bmatrix} N}$$

Problem 8

$$\vec{W} = (10\hat{N} + 20\hat{E} - 5\hat{D})\ m/s$$

$$\vec{V} = \vec{V}^E - \vec{W}$$

$$= -100\hat{N} - (10\hat{N} + 20\hat{E} - 5\hat{D})$$

$$\boxed{\vec{V} = (-110\hat{N} - 20\hat{E} + 5\hat{D})\ m/s \quad \text{relative wind } \vec{V}}$$

$\vec{V}_B$ is the relative wind $\vec{V}$ represented in F_B coordinates

$\hat{N} = -\hat{x} \quad \hat{E} = -\cos\phi\,\hat{y} + \sin\phi\,\hat{z} \quad \hat{D} = \sin\phi\,\hat{y} + \cos\phi\,\hat{z}$

$$\boxed{\vec{V}_B = \begin{bmatrix} 110 \\ 20\cos\phi + 5\sin\phi \\ -20\sin\phi + 5\cos\phi \end{bmatrix} m/s}$$