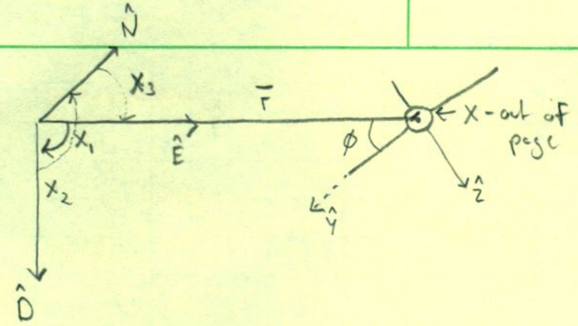



$|\bar r| = 1 \text{ km} = 1000 \text{ m}$

ground speed $= |\bar v| = 100 \text{ m/s} = \bar v^E$

1 $\boxed{\bar r}$: is a position vector from the center of the $F_E$ frame to the location 1 km due east aka reference from the origin to the center of gravity (CG) of the aircraft

$$\boxed{\bar r = (0\hat N + 1\hat E + 0\hat D)(\text{km})}$$

$\boxed{\bar r_E}$: coordinate representation of $\bar r$ in the E, earth fixed inertial frame

$$\boxed{\bar r_E = [0 \quad 1 \quad 0]^T (\text{km})}$$

"$\bar r$ in E"

Steps to make transformation matrix from E to B
- we start with the E axis frame + B axis frame aligned with each others relative axis
- rotate about the $\hat D$ axis by $+180°$
- rotate about the $\hat X$ axis by $+\phi$

$$\begin{array}{c} \text{about } \hat D \\ \bar L_3(X_3) = \begin{bmatrix} \cos 180° & \sin 180° & 0 \\ -\sin 180° & \cos 180° & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \text{about } \hat z \end{array} \quad \begin{array}{c} \text{about } \hat N \\ \bar L_1(X_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos X_1 & \sin X_1 \\ 0 & -\sin X_1 & \cos X_1 \end{bmatrix} \\ \text{about } \hat x,\ \phi \end{array} \quad \begin{array}{c} \text{about } \hat E \\ \bar L_2(X_2) = \begin{bmatrix} \cos X_2 & 0 & -\sin X_2 \\ 0 & 1 & 0 \\ \sin X_2 & 0 & \cos X_2 \end{bmatrix} \\ \text{about } \hat y \end{array}$$

$$\bar L_{BE} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & -s\phi & c\phi \end{bmatrix} \begin{bmatrix} c\,180 & s\,180 & 0 \\ -s\,180 & c\,180 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -c\phi & s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$\boxed{\bar r_B}$: coordinate representation of $\bar r$ in the B, body fixed frame

"$\bar r$ in B"

$$\bar r_B = \bar L_{BE}\, \bar r_E = \begin{bmatrix} \bar L_{BE} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \boxed{\bar r_B = \begin{bmatrix} 0 \\ -\cos\phi \\ \sin\phi \end{bmatrix} (\text{km})}$$

$\boxed{2}$ $\boxed{\overline{V}^E = \frac{d}{dt}^E \overline{r}}$ : the time rate of change of the position vector $\overline{r}$ as observed from the E frame aka the inertial velocity vector of the aircraft

$$\boxed{\overline{V}^E = \frac{d}{dt}^E \overline{r} = (-100 \hat{N} + \emptyset \hat{E} + \emptyset \hat{D}) [m/s]}$$

$\boxed{\overline{V}_E^E}$ : the inertial velocity vector expressed/represented in the inertial coordinate frame

$$\boxed{\overline{V}_E^E = \begin{bmatrix} -100 \\ 0 \\ 0 \end{bmatrix} [m/s]}$$

$\boxed{\overline{V}_B^E}$ : the inertial velocity vector expressed/represented in the body fixed coordinate frame

$$\boxed{\overline{V}_B^E = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} [m/s]}$$

$\boxed{3}$ $\boxed{\overline{\omega}^{EB}}$ : the inertial angular velocity vector of the aircraft relative to the inertial frame

$$\boxed{\overline{\omega}^{EB} = \frac{V}{r} = \frac{100 \ m/s}{1000 \ m} \hat{D} = 0.1 \hat{D} \ [rad/s]}$$

$\boxed{\overline{\omega}_E^{EB}}$ : the inertial angular velocity vector expressed/represented in the inertial Earth fixed coordinate frame, E.

$$\boxed{\overline{\omega}_E^{EB} = \begin{bmatrix} 0 \\ 0 \\ .1 \end{bmatrix} [rad/s]}$$

$\boxed{\overline{\omega}_B^{EB}}$ : the inertial angular velocity vector expressed/represented in the body fixed coordinate frame

$$\overline{\omega}_B^{EB} = \overline{L}_{BE} \ \overline{\omega}_E^{EB} = \begin{bmatrix} & & \\ & \overline{L}_{BE} & \\ & & \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ .1 \end{bmatrix} = \boxed{\overline{\omega}_B^{EB} = \begin{bmatrix} 0 \\ .1 \sin \phi \\ .1 \cos \phi \end{bmatrix} [rad/s]}$$

$\boxed{4}$ $\boxed{\frac{d}{dt}^B \bar{r}}$ : the body-fixed frame derivative of the position vector $\bar{r}$, looking out the window of the plane the $\bar{r}$ position vector never changes

$$\boxed{\frac{d}{dt}^B \bar{r} = (0\,\hat{x} + 0\,\hat{y} + 0\,\hat{z})\,[m/s]}$$

$$(\dot{x}_B\,\hat{x} + \dot{y}_B\,\hat{y} + \dot{z}_B\,\hat{z})$$

$\boxed{\left(\frac{d}{dt}^B \bar{r}\right)_E}$ : the body-fixed frame derivative of the position vector $\bar{r}$, expressed/represented in the inertial Earth fixed frame E

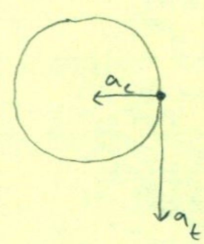
$$\boxed{\left(\frac{d}{dt}^B \bar{r}\right)_E = [0 \quad 0 \quad 0]^T\,[m/s]}$$

$\boxed{\left(\frac{d}{dt}^B \bar{r}\right)_B}$ : the body-fixed frame derivative of the position vector $\bar{r}$, expressed/represented in the body fixed frame, B

$$\boxed{\left(\frac{d}{dt}^B \bar{r}\right)_B = [0 \quad 0 \quad 0]^T\,[m/s]}$$

$\boxed{5}$ $\boxed{\left(\frac{d}{dt}^E \bar{V}^E\right)_E}$ : the inertial fixed frame derivative of the inertial velocity vector $\bar{V}^E$, of the aircraft expressed/represented in the inertial Earth Fixed frame E.

Acceleration has two components tangential + centripital

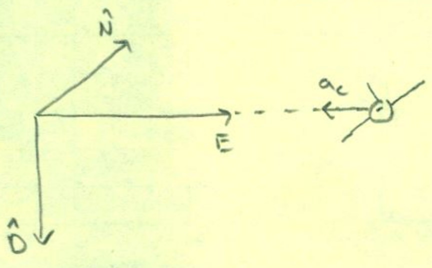



$$a_t = 0 \qquad a_c = \frac{V^2}{r}$$

$$\bar{V}^E = (-100\,\hat{N} + 0\,\hat{E} + 0\,\hat{D}) \qquad |\bar{V}^E| = \sqrt{(-100)^2 + 0^2 + 0^2} = 100$$

$$\bar{r} = (0\,\hat{N} + 1000\,\hat{E} + 0\,\hat{D}) \qquad |\bar{r}| = \sqrt{0^2 + 1000^2 + 0^2} = 1000$$

$$a_c = \frac{100^2}{1000} = 10\ m/s^2 \qquad \boxed{\left(\frac{d}{dt}^E \bar{V}^E\right)_E = [0 \quad -10 \quad 0]^T\,[m/s^2]}$$



$\boxed{5}$ $\boxed{\left(\frac{d}{dt}^B \bar{V}^E\right)_B}$ : the body-fixed frame derivative of the inertial velocity vector $\bar{V}^E$, of the aircraft expressed / represented in the body-fixed frame B

$$\boxed{\left(\frac{d}{dt}^B \bar{V}^E\right)_B = \left(\frac{d}{dt}^B (0\,\hat{N} + 1000\,\hat{E} + 0\,\hat{D})\right)_B = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T [m/s^2]}$$

$\boxed{6}$ Calculate $\frac{d}{dt}^E \bar{V}^E$ using the velocity rule

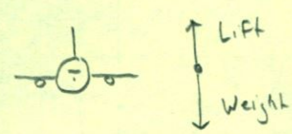
$$\frac{d}{dt}^E \bar{V}^E = \frac{d}{dt}^B \bar{V} + \bar{\omega}^{EB} \times \bar{V}$$

$$= 0 + 0.1\,\hat{D} \times (-100)\,\hat{N}$$

$$\boxed{\frac{d}{dt}^E \bar{V}^E = -10\ \hat{E}\ [m/s^2]}$$

$\boxed{7}$ $\boxed{\bar{F}}$ : is the resultant / net of all forces acting on the aircraft

$$\bar{F} = m\ddot{\bar{r}} = m\dot{\bar{V}} = m\,a_c \qquad F_{cent} = m\frac{v^2}{r}$$




cancel out

$$\boxed{\bar{F} = m(-10\ \hat{E})\ [N]}$$

$\boxed{\bar{F}_E}$ : the resultant force vector expressed in the inertial Earth fixed frame E.

$$\boxed{\bar{F}_E = m\begin{bmatrix} 0 & -10 & 0 \end{bmatrix}^T [N]}$$

$\boxed{\bar{F}_B}$ : the resultant force vector expressed in the body fixed frame B

$$\bar{F}_B = \bar{L}_{BE}\,\bar{F}_E = \begin{bmatrix} \bar{L}_{BE} \end{bmatrix}\begin{bmatrix} 0 \\ -10 \\ 0 \end{bmatrix} = \boxed{\bar{F}_B = m\begin{bmatrix} 0 \\ 10\cos\phi \\ -10\sin\phi \end{bmatrix}[N]}$$

8 Given: $\overline{W}_E = [10 \quad 20 \quad -5]^T \,[m/s] = [W_x \quad W_y \quad W_z]^T$

$$\overline{V}_E^E = [-100 \quad 0 \quad 0]^T \,[m/s] = [\dot{x}_E \quad \dot{y}_E \quad \dot{z}_E]^T$$

$$\underbrace{\overline{V}_E^E}_{\text{Inertial Velocity}} = \underbrace{\overline{V}_E}_{\text{Velocity of CG to air "relative velocity"}} + \underbrace{\overline{W}_E}_{\text{Wind velocity relative to } F_E}$$

Find: Relative wind vector $\overline{V}_E$

Which is $\overline{V}_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$

$$\overline{V}_E = \overline{V}_E^E - \overline{W}_E$$

$$\boxed{\overline{V}_E = \begin{bmatrix} -100 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 10 \\ 20 \\ -5 \end{bmatrix} = \begin{bmatrix} -110 \\ -20 \\ 5 \end{bmatrix} [m/s]} \quad \boxed{\overline{V} = (-110\,\hat{N} - 20\,\hat{E} + 5\,\hat{D})\,[m/s]}$$

$$\overline{V}_B = \overline{L}_{BE}\,\overline{V}_E = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -C\phi & S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} -110 \\ -20 \\ 5 \end{bmatrix} = \boxed{\begin{bmatrix} 110 \\ 20\cos\phi + 5\sin\phi \\ -20\sin\phi + 5\cos\phi \end{bmatrix} = \overline{V}_B}$$

If defined in E frame, transform to B frame $\overline{L}_{\underset{\text{to}}{B}\underset{\text{from}}{E}}$

$$\hat{x} = -\hat{N}$$

$$\hat{y} = -\cos\phi\,\hat{E} + \sin\phi\,\hat{D}$$

$$\hat{z} = \sin\phi\,\hat{E} + \cos\phi\,\hat{D}$$

$$\overline{r}_E,\ \overline{V}_E,\ \overline{\omega}_E^{EB} = \begin{bmatrix} \hat{N} \\ \hat{E} \\ \hat{D} \end{bmatrix}$$

If defined in B, transform to E $\quad L_{EB}$

$$\hat{N} = -\hat{x}$$

$$\hat{E} = -\cos\phi\,\hat{y} + \sin\phi\,\hat{z}$$

$$\hat{D} = \sin\phi\,\hat{y} + \cos\phi\,\hat{z}$$

$$\overline{r}_B,\ \overline{V}_E,\ \omega_B^{EB} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$$