ASEN 5050
SPACEFLIGHT DYNAMICS
Multi-Body Dynamics, Part 2

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Objectives:

- Introduce various dynamical structures, computational techniques
- Discuss value in trajectory design

Equilibrium Points

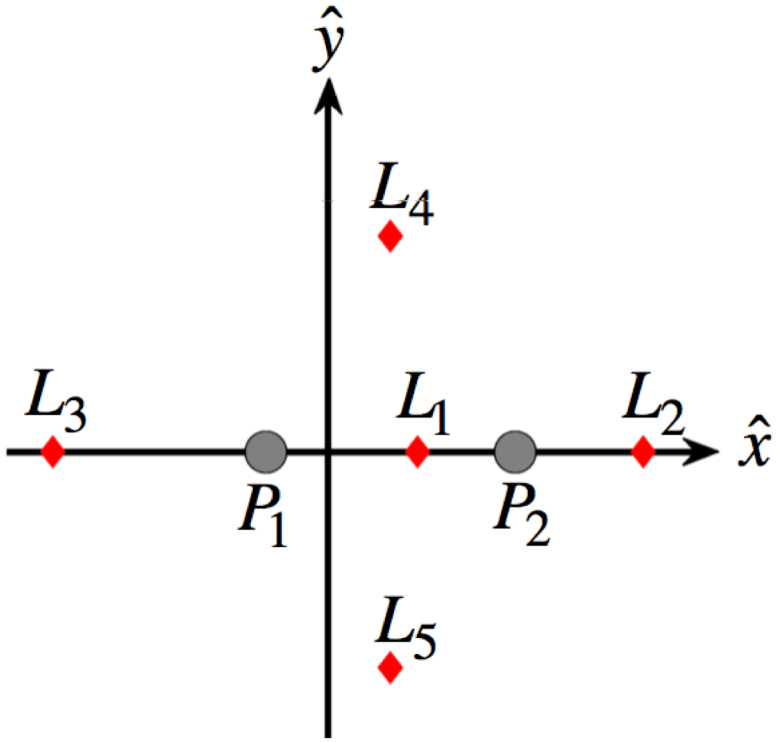
Locations where $\dot{x} = \dot{y} = \dot{z} = 0$ $\ddot{x} = \ddot{y} = \ddot{z} = 0$

$$\ddot{x} - 2\dot{y} = \frac{\partial U^*}{\partial x}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial U^*}{\partial y}$$

$$\ddot{z} = \frac{\partial U^*}{\partial z}$$

$$U^* = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$



Zero Velocity Curves

At a given energy level, or value of CJ, there are a set of points with

$$\dot{x} = \dot{y} = \dot{z} = 0$$

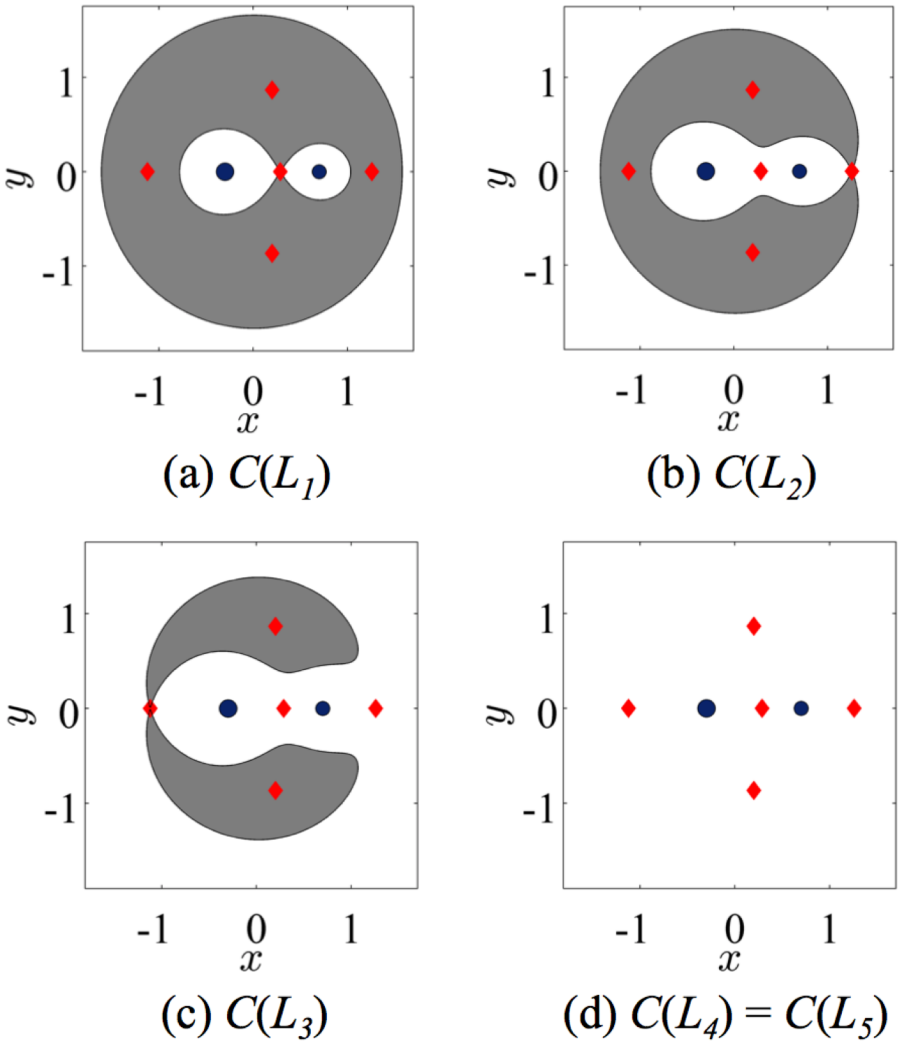
where solving

$$C_J = 2U^* - v^2$$

defines the boundary between real and imaginary speeds.

These points define zero velocity curves (ZVCs)

- Separate “allowable” and ”forbidden regions”

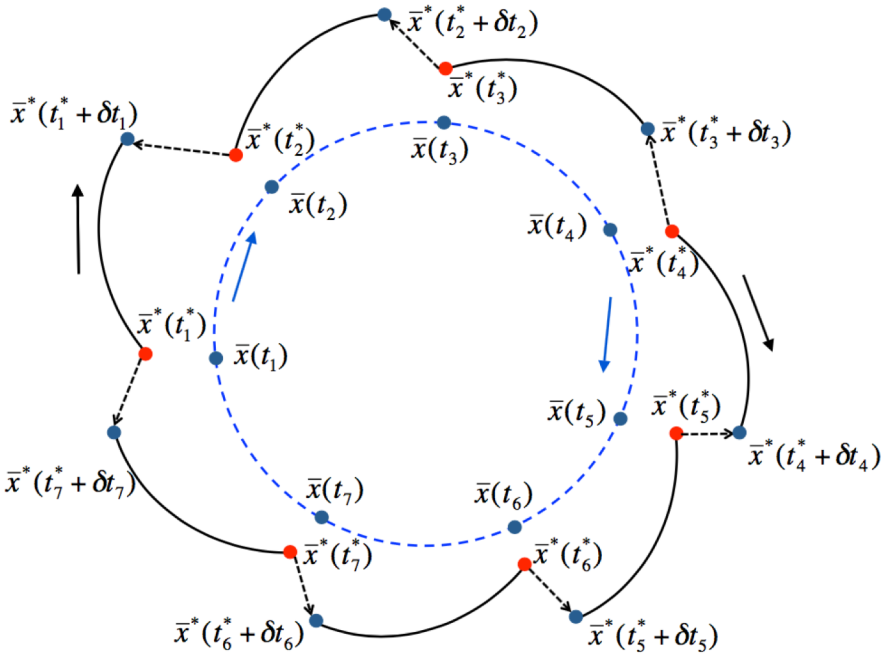


(a) $C(L_1)$ (b) $C(L_2)$

(c) $C(L_3)$ (d) $C(L_4) = C(L_5)$

Periodic Orbits

Since there are no analytical solutions, it is nontrivial to compute periodic orbits. Use corrections algorithm to iteratively refine an initial guess – various methods to achieve this goal.



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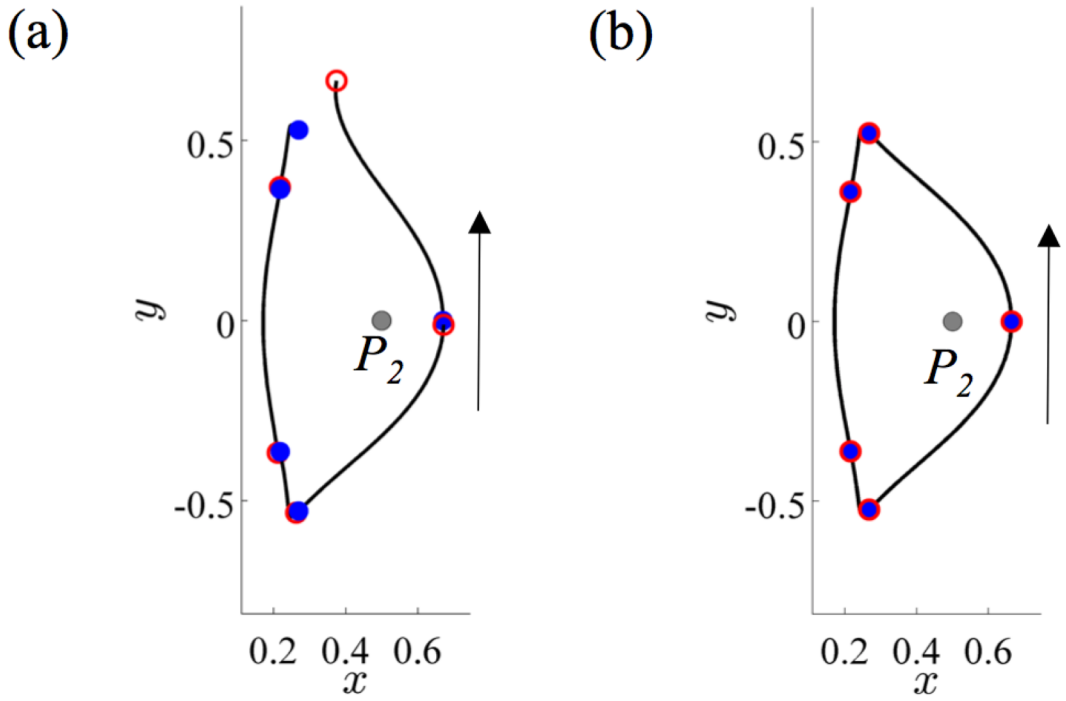
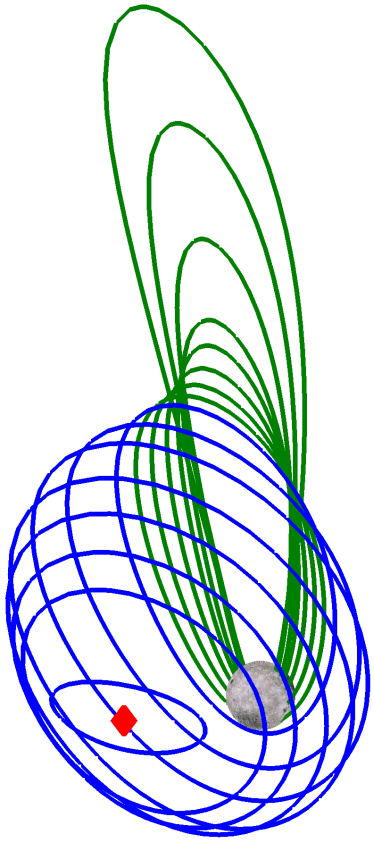
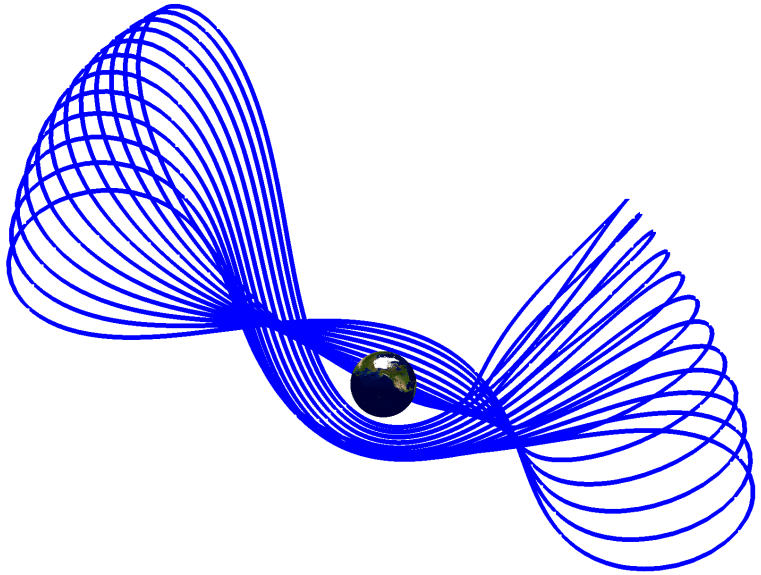


Periodic Orbit Families

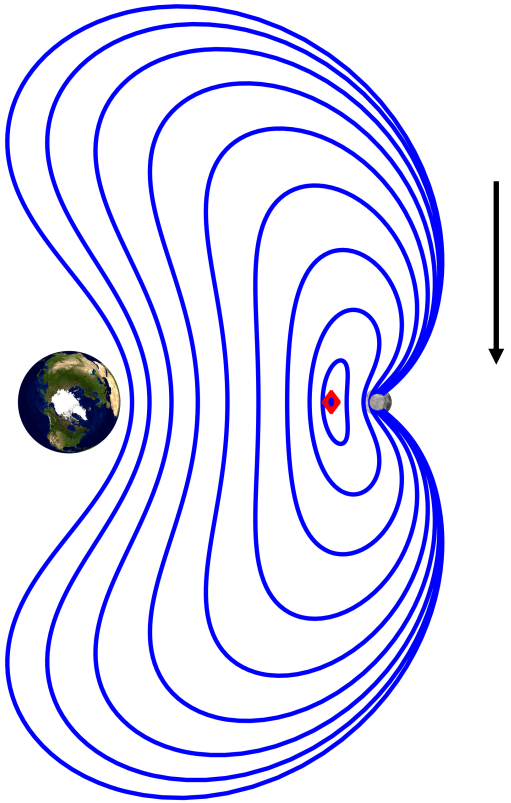
Once we have one orbit, can find others in the family using a technique called continuation.





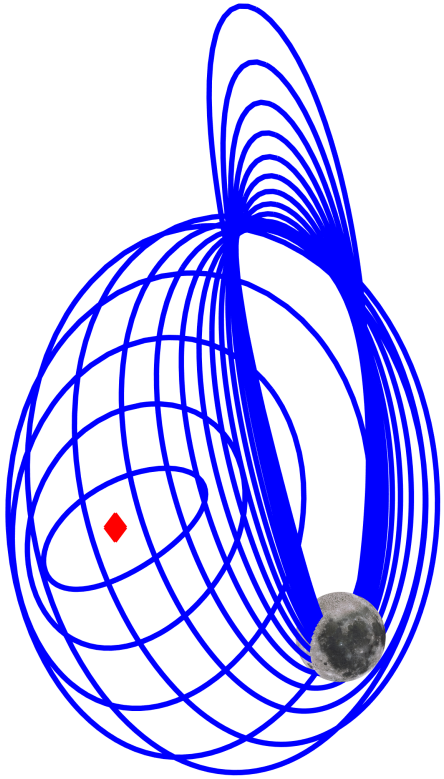
Earth-Moon Libration Point Orbits

L1 Lyapunov Orbits



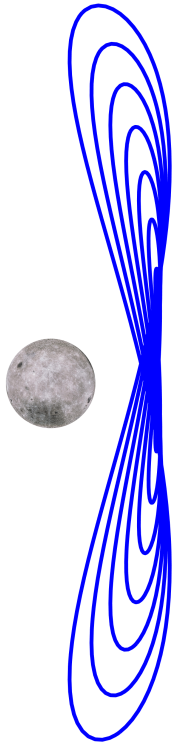
Earth-Moon 10x scale

L1 Halo orbits



Moon 5x scale

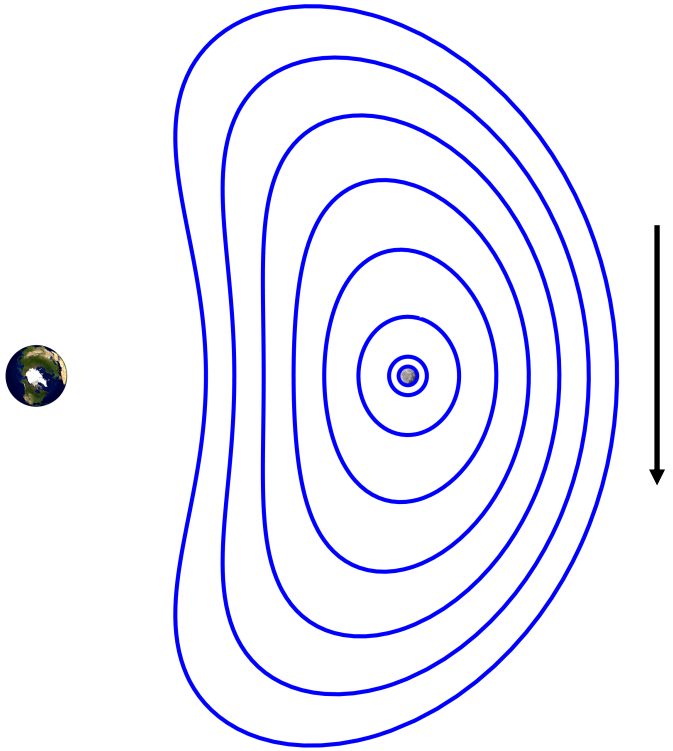
L2 Vertical Orbits



Moon 5x scale

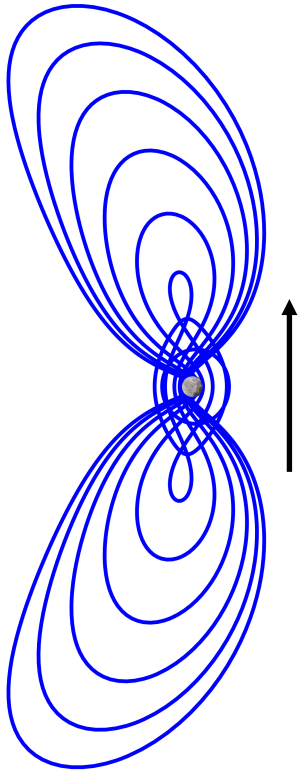
Moon-Centered Orbits

Distant Retrograde Orbits



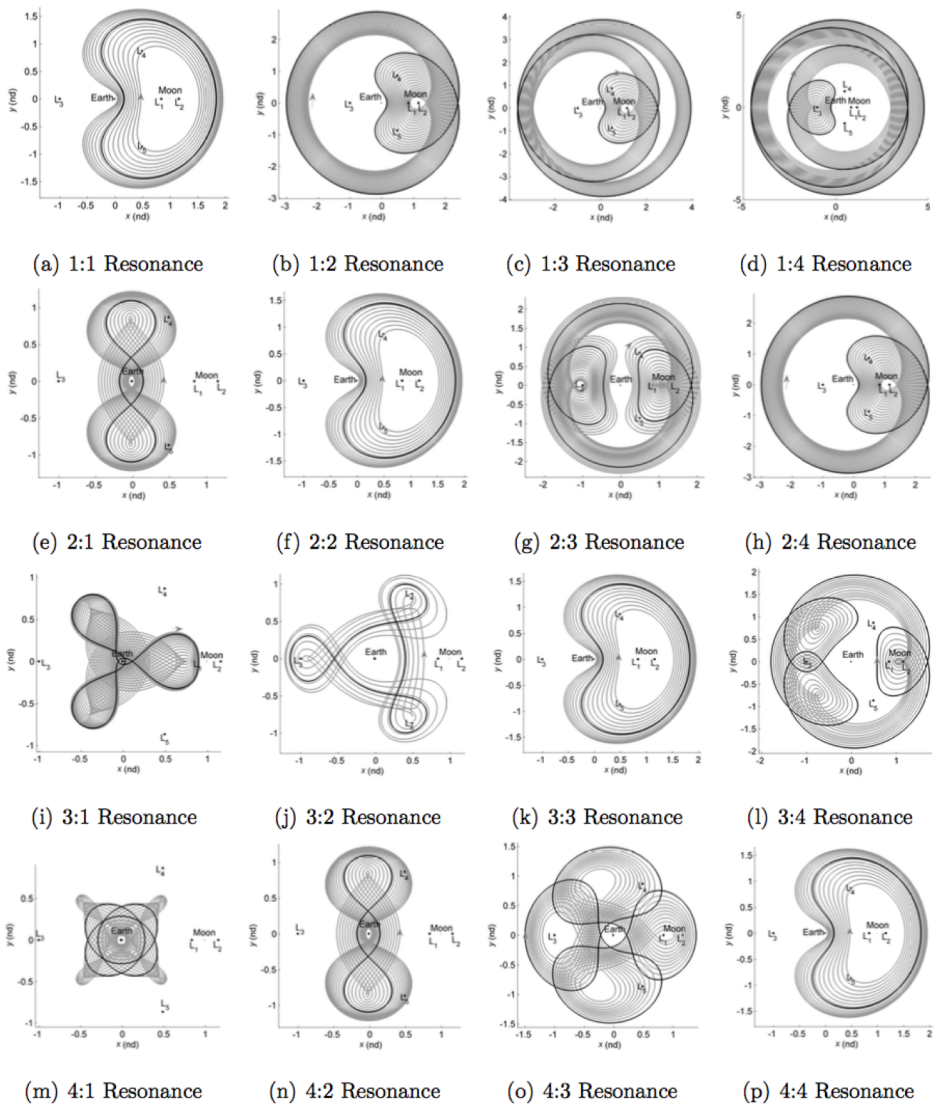
Earth-Moon 5x scale

Distant Prograde Orbits



Moon 5x scale

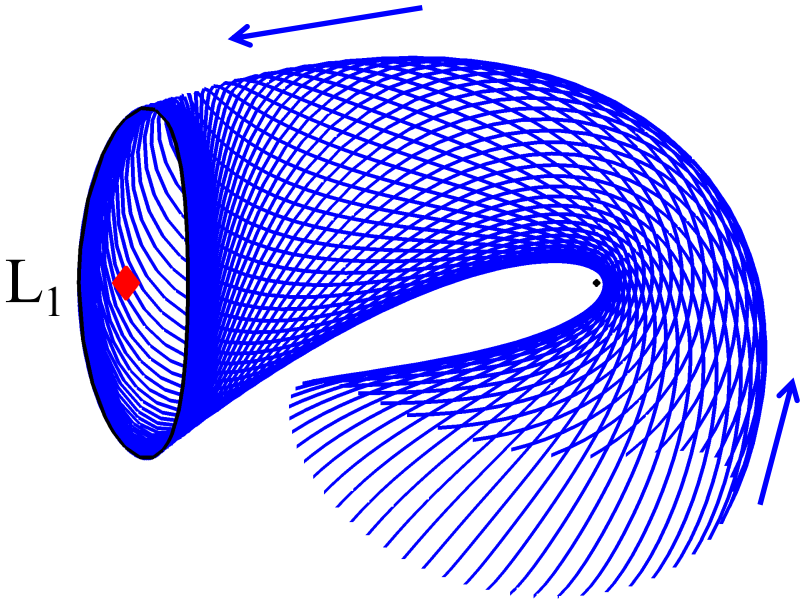
Resonant Orbits



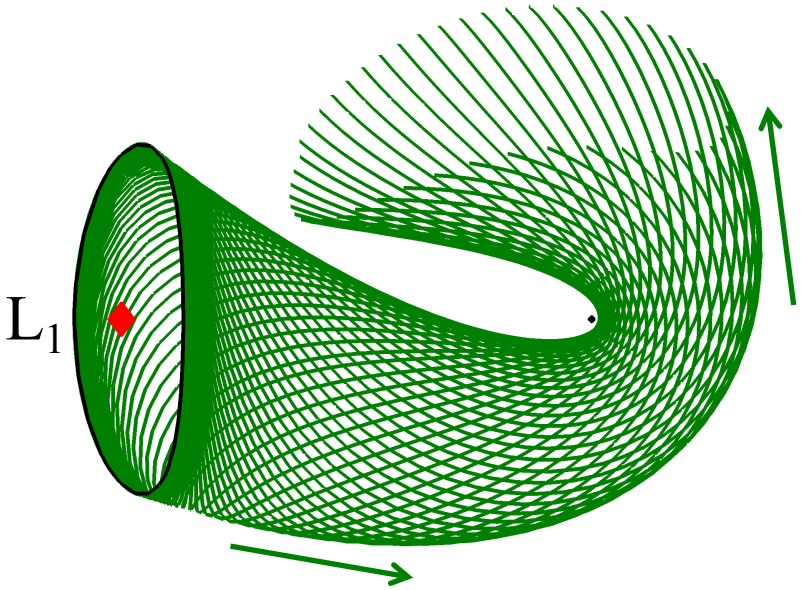
Credit: Vaquero, M, 2013, "Spacecraft Transfer Trajectory Design Exploiting Resonant Orbits in Multi-Body Environments"

Manifolds

Stable manifolds emanate from orbits with stable and unstable modes

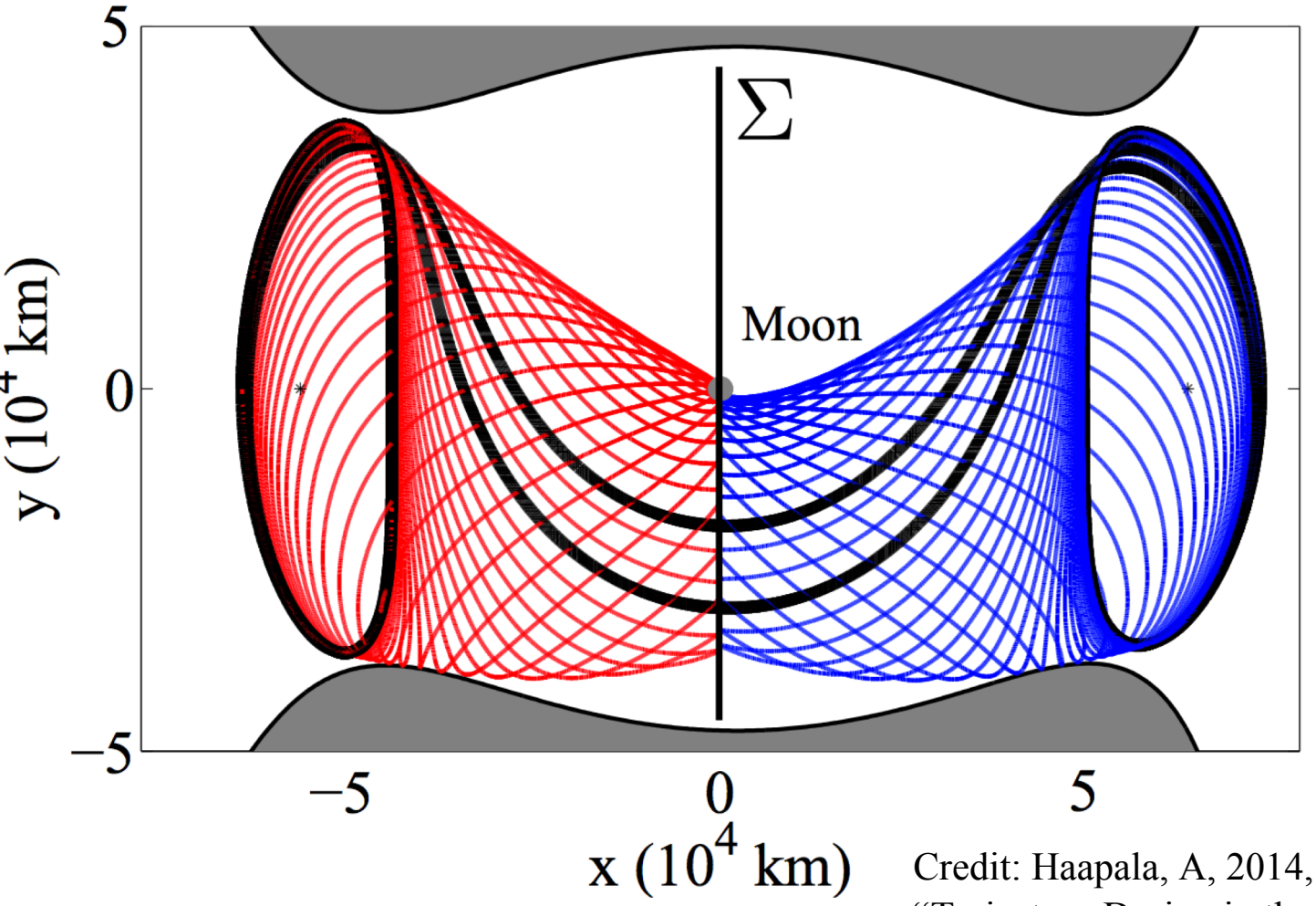


Stable Manifold



Unstable Manifold

Constructing Transfers Using Manifolds



Credit: Haapala, A, 2014, "Trajectory Design in the Spatial Circular Restricted Three-Body Problem Exploiting Higher-Dimensional Poincare Maps"

Using Manifolds to Reach Science Orbit