Computational Lab 1 - Computation of Lift and Drag

Connor Ott - 104690690

Introduction

Fluid dynamics, and in particular, aerodynamics, frequently runs into the problem of not having an analytical solution to describe flow over a body. One solution to this problem is to use computational approximations. This allows scientists and engineers to determine flow properties and the effects the flow has on bodies as it passes over them. This lab is concerned with determining lift and drag created by flow over two different bodies. For one body, a cylinder, an analytical solution is available. For the second, a NACA 0012 airfoil, data and computational methods must be employed, namely numerical integration methods. In addition to this, the lab explores how effective these methods are, and what is required of them to reach desired error tolerances.

Theory

Lift and Drag on a Cylinder

For the cylinder, an analytical description of the Coefficient of Pressure, C_P was available for determining lift and drag. This was evaluated using Simpson's rule for numerical integration, shown below in Eq.(1).

$$\int_c f ds \approx \frac{h}{3} R \sum_{k=1}^{N/2} \left[f(t_{2k-1}) + 4f(t_{2k}) + f(t_{2k+1}) \right] \tag{1}$$

Eq.(1) has been simplified for a line integral over a circle. Here, R is the circle's radius, $t_k = (k-1)h$, and $h = 2\pi/N$ where N is the number of subintervals used in the integration. From here, a function for pressure must be determined in order to find lift and drag on the upper and lower surfaces. The following equations aid in this.

$$C_p = 1 - 4\sin^2(\theta) \tag{2}$$

$$C_p = \frac{p - p_\infty}{q_\infty} \tag{3}$$

Eq.(3) describes the definition for C_p. This, in combination with Eq.(2) provides an analytical description for flow pressure. The diagram below, Fig.(1), shows how horizontal and vertical components of this pressure can be gathered from the geometry of the cylinder. With this, we can develop Eq.(4) and Eq.(5), which can then be integrated using Simpson's rule to find lift and drag. Note that the following equations were determined using only the pressure distribution over the cylinder. This is because the flow is assumed to be inviscid, so no shear forces act on the body.

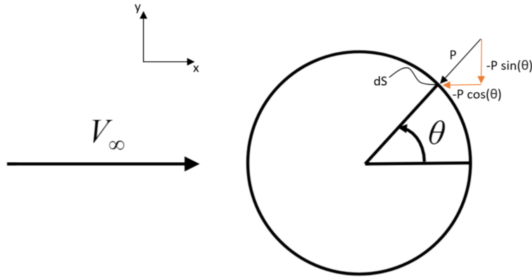

Figure 1. Diagram indicating components of pressure acting on a differential area dS.[1]

$$dL = -\big[q_\infty(1 - 4\sin^2(\theta)) + P_\infty\big] sin(\theta)dS \tag{4}$$

$$dD = -\big[q_\infty(1 - 4\sin^2(\theta)) + P_\infty\big] cos(\theta)dS \tag{5}$$

Lift and Drag on an Airfoil

A NACA 0012 airfoil was also analyzed using numerical integration. This time, with the Trapezoidal rule. This required a more general formula which integrates pressure to determine normal and axial forces on the airfoil. Both the Trapezoidal rule simplification and original equations are shown below in Eq.(6) and Eq.(7).

$$N'_u \approx \sum_{k=1}^{N} \Big[-\frac{1}{2}(P_u(x_k) + P_u(x_{k+1}))\Delta x_k \Big] \tag{6}$$

$$N'_u = \int_{LE}^{TE} P_u \cos(\theta) ds_u \tag{7}$$

Note that again, these equations neglect shear forces, as the flow is assumed to be inviscid. Also note that these only give the normal force for the upper surface of the airfoil. For the equations which yield normal and axial forces on both the upper and lower surfaces, look to the appendix. An important simplification between the two equations is the $\cos(\theta)$ which appears in Eq.(7) but not in Eq.(6). Fig.(2) shows how $cos(\theta)$ and $\sin(\theta)$ can be substituted for a simpler equation. In this figure, we see there is an associated Δx_k and Δy_k associated with a given length along an airfoil ΔS_k. These can be calculated using the equation describing the shape of the airfoil, found in the appendix.

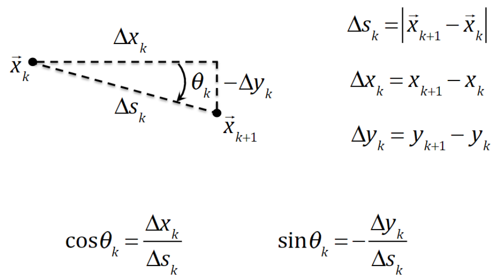

Figure 2. Simplifying substitution for normal and axial force calculations.[1]

For the airfoil, rather than having an analytical function describing the pressure distribution, discrete values for C_p were measured and interpolated for points along the airfoil. It's these that were plugged directly in to Eq.(6) (and its related summations) to solve for normal and axial forces. Finally, to calculate lift and drag, Eq.(8) and Eq.(9) were used in conjunction with the angle of attack, α.

$$L = N\cos(\alpha) - A\sin(\alpha) \tag{8}$$

$$D = N\sin(\alpha) + A\cos(\alpha) \tag{9}$$

Results

Lift and Drag on a Cylinder

For the airflow over the cylinder, the following parameters were used:

- Radius R = 1 m
- V_∞ = 25 m/s
- ρ_∞ = 0.9093 kg/m^3
- P_∞ = 7.01210 x 10^4 Pa

From the methods describe above, it was ascertained that the lift and drag acting on the cylinder is 0 N and 0 N respectively. More specifically, 1.806e-11 N and 3.048e-11 N according to the MATLAB script used to perform these calculations. This was achieved using N = 4 subintervals for the Simpson's Rule approximation.

Lift and Drag on an Airfoil

For the airfoil, these flow parameters were used:

- Chord Length c = 0.5 m
- Angle of Attack $\alpha = 9\,\text{deg}$
- V_∞ = 20 m/s
- ρ_∞ = 1.225 kg/m^3
- P_∞ = 10.13 x 10^4 Pa

In addition, the MATLAB file *cp.mat* held all C_p values used for the problem. With these parameters and the methods described above, a lift of 132.01 N and drag of 0.16 N was calculated. Table (1) indicates the number of subintervals needed to attain different tolerances for the accuracy of this number, with the accepted value obtained at 500,000 subintervals.

Tolerance	# of Subintervals
5%	47
1%	167
0.1%	963

Table 1. Subintervals required for different tolerances.

Discussion

Lift and Drag on a Cylinder

A lift and drag of 0 N makes sense for this problem because of the unique symmetry of the cylinder. Due to its shape, the pressure distribution above and below are identical. Additionally, the pressure distribution ahead of, and behind the cylinder are identical. This results in equal and opposite forces acting all around the cylinder, and a net zero force overall.

Lift and Drag on an Airfoil

The number of subintervals which yield acceptable tolerances gives insight into how aerodynamic coefficients should be determined in a wind tunnel. The number of subintervals corresponds to the number of discrete pressure measurements that would be taken around an airfoil. This means that an increased number of static port measurements taken on an airfoil will increase the accuracy of the measurement. It's then reasonable to say that a full size NACA 0012 airfoil in a wind tunnel could produce results accurate to 5% with ~ 50 static ports. Similarly, ~ 170 static ports could yield tolerances as low as 1%.

I. Appendix

A. More Equations

$$N' = -\int_{\mathrm{LE}}^{\mathrm{TE}} (p_u \cos\theta + \tau_u \sin\theta)\, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l \cos\theta - \tau_l \sin\theta)\, ds_l$$

$$A' = \int_{\mathrm{LE}}^{\mathrm{TE}} (-p_u \sin\theta + \tau_u \cos\theta)\, ds_u + \int_{\mathrm{LE}}^{\mathrm{TE}} (p_l \sin\theta + \tau_l \cos\theta)\, ds_l$$

These equations include both the upper and lower surfaces, denoted by subscripts u and l respectively.

$$y_t = \frac{t}{0.2} c \left[0.2969 \sqrt{\frac{x}{c}} - 0.1260 \left(\frac{x}{c}\right) - 0.3516 \left(\frac{x}{c}\right)^2 + 0.2843 \left(\frac{x}{c}\right)^3 - 0.1036 \left(\frac{x}{c}\right)^4 \right]$$

This equation describes the shape of the airfoil. The variables c, t, and x represent the chord length, percentage of thickness to chord, and horizontal distance along the chord, respectively.

References

[1] Evans, John. ASEN 3111. *Numerical Integration.* 15 Sept. 2017. Desire2Learn `https://learn.colorado.edu/d2l/le/content/217338/viewContent/3150737/View`