Computational Lab 4: Flow Over Thick Airfoils

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Introduction

Numerically determining the effects of flow over aerodynamic bodies is a vital component of aircraft design, and especially airfoil design. Using a vortex sheet is one such way to model flow. However, a single vortex sheet can only approximate the body as a thin, airfoil. A more accurate and robust method is the vortex panel method. This method can accommodate any streamlined aerodynamic body, allowing for analysis regarding airfoil properties such as camber and thickness. This lab applies the vortex panel method to several different NACA airfoils in an effort to predict the pressure distribution and lift slope associated with each airfoil. Additionally, the lab will explore the error associated with this method, and how the error varies with an increased number of vortex panels used in the simulation.

Theory

The vortex panel models potential flow over an airfoil using the combination of n discrete vortex panels distributed over the surface of the airfoil. Each panel is then associated with linearly varying vortex strength. The main concern of the vortex panel method is to assign the vortex strengths at the boundary points of each panel. These strengths are assigned such that they satisfy two conditions. The first of which is that the flow may not penetrate the airfoil. This is described mathematically as $\partial\phi/\partial\hat{n} = 0$ at every point on the airfoil. The second is the Kutta condition, which ensures that the flow leaves the body smoothly. This condition is met only when the vortex strength at the trailing edge is equal to zero. These two conditions boil down to the system of equations shown in Eq.(1), wherein subscripts i and j indicate values corresponding to different locations around the airfoil. A more in-depth explanation of the calculations and geometry involved in these equations can be found in Chapter 5.10 of "Foundations of Aerodynamics" by Kuethe and Chow.

$$\begin{aligned} \sum_{j=1}^{n+1} A_{n_{i,j}}\gamma_j &= sin(\theta_i - \alpha) \qquad && i = 1, 2, \dots n \\ \gamma_1 + \gamma_{j+1} &= 0 && i = n+1 \end{aligned} \tag{1}$$

Once the vortex strength at each boundary point has been determined, the flow velocity due to each vortex panel at the *midpoints* of each vortex panel can be determined. This allows for the calculation of the pressure distribution about the airfoil using Bernoulli's equation. Additionally, the velocity is used to approximate the total circulation about the airfoil using Eq.(2). Where V_i and S_i are the velocity over and length of the i^{th} panel.

$$\Gamma = \oint_c \vec{V} \cdot \vec{ds} \approx \sum_{i=1}^{n} V_i S_i \tag{2}$$

With the total circulation, and the Kutta-Joukowski theorem relating lift per unit span and circulation, the sectional lift coefficient can be determined. The results of both the pressure distribution and sectional lift coefficient calculations can be found in the Results section.

Results

After implementing the vortex panel method using MATLAB, the following results were gathered for various different airfoil shapes and angles of attack. Fig.(4) shows a typical pressure coefficient distribution for an airfoil. Fig.(5) indicates how camber and thickness can effect sectional coefficient of lift at varying angle of attack. Table 1 then gives estimated. lift slope and zero-lift angles of attack for each airfoil.

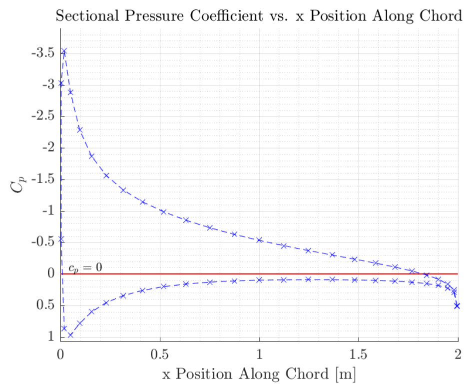

Figure 1: *Pressure coefficient plot for NACA 0012 airfoil at $\alpha = 8°$*

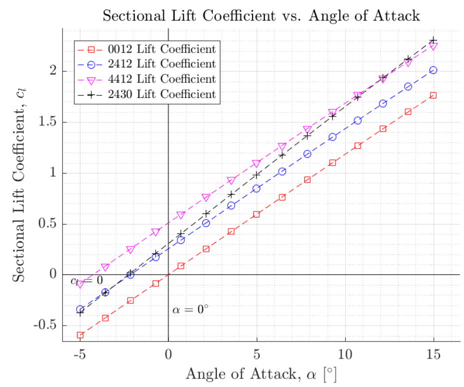

Figure 2: *Sectional lift coefficient for various airfoils at varying angles of attack.*

Table 1: $dc_l/d\alpha$ and $\alpha_{L=0}$ for various NACA airfoils.

NACA Code	$dc_L/d\alpha$ $[rad^{-1}]$	$\alpha_{L=0}$ [°]
0012	6.76	-8.828×10^{-4}
2412	6.74	-2.16
4412	6.72	-4.33
2430	7.68	-2.27

Fig.(3) gives a qualitative representation of the error trend for increasing the number of panels used. It compares both cosine spacing error and linear spacing error. The cosine spacing adjusts panel boundaries to place more panels towards the leading and trailing edges of the airfoil where greater pressure and velocity gradients are found. In this study, pressure coefficient distributions were compared to that of trial using 1×10^4 panels. Note that cosine spacing achieves much greater accuracies at the same n values when compared to linearly spaced panels.

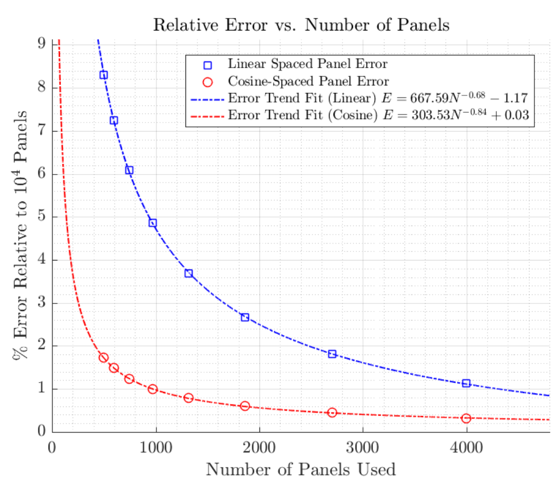

Figure 3: *Error trend for both linearly spaced and cosine spaced panels as number of panels is increased.*

Using the fitting curve shown in Fig.(3), it's estimated that 969 panels are required for 1% error using cosine spacing for the panels. This is a huge advantage over the ≈ 4303 panels required for the same error with linear spacing. This dramatically lowers run time for the vortex panel method. An example $\alpha - c_l$ and $x - c_p$ plots are shown below using the nominal 969 panels required for 1% accuracy using cosine spacing for $\alpha = -5°, 0°, 5°$ and $10°$ angles of attack.

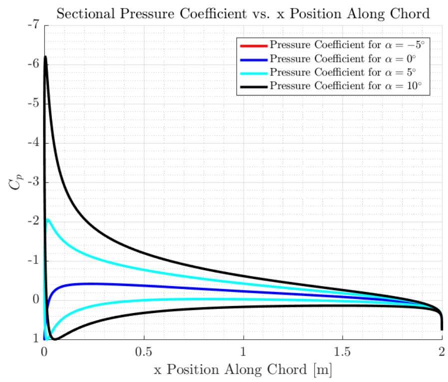

Figure 4: *Pressure coefficient plot for NACA 0012 airfoil at varying angles of attack using 969 panels.*

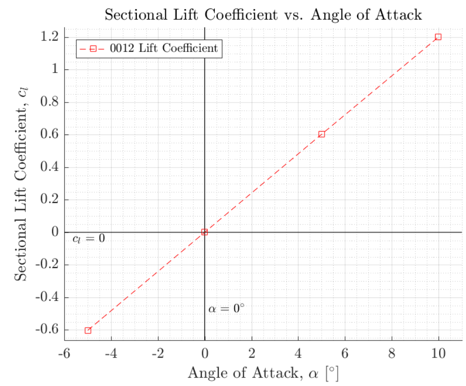

Figure 5: *Sectional lift coefficient for NACA 0012 at varying angles of attack using 969 panels.*

Discussion

Fig.(5) in conjunction with the data in Table 1 allows for some discussion concerning thin airfoil theory. To start, notice that the NACA 0012 has a lift slope of 6.76, or approximately 2.15π. This is comparable to the results of thin airfoil theory which dictate that the slope of 2π. This indicates that thin airfoil somewhat unsurprisingly diverges from the results found when analyzing a thick airfoil. Notice also that the NACA 0012, 2412, and 4412 have almost identical lift slopes, while the NACA 2430 has a greater lift slope. This implies that increased thickness increases the lift slope of an airfoil. Another thing to look at is the zero lift angle of attack. For a thin, symmetric airfoil, zero lift angle of attack will be at $0°$. This is consistent with the results found with the vortex panel method. Moreover, the zero-lift angle of attack decreases with increased camber, also consistent with thin airfoil theory.

Appendix

A. Bonus Problem

An interesting challenge involving the vortex panel method is applying it to a two-element system. Fig.(6) and Fig.(7) show how drastically adding a flap to an airfoil can alter effects the flow has on it. The implementation of the two-element vortex panel method can be found in the script *flapVnoflap.m*.

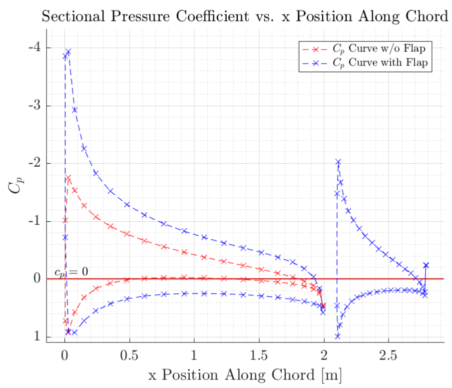

Figure 6: *Pressure coefficient plot comparison between airfoil with and without a trailing edge flap.*

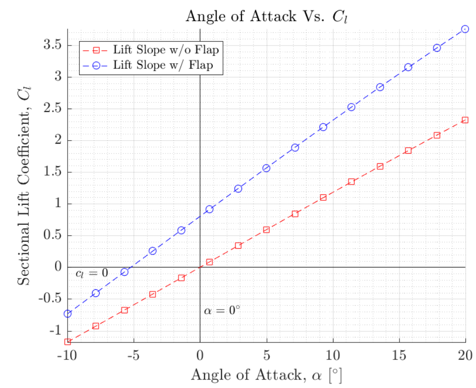

Figure 7: *Sectional lift coefficient plot comparison between airfoil with and without a trailing edge flap.*

References

[1]Cox, C. A. "Two Element Linear Strength Vortex Panel Method," March 2011.

[2]Kuethe, A. M., Chow, C.-Y., "Aerodynamic Characteristics of Airfoils", *Foundations of Aerodynamics*, 5th Ed. John Wiley and Sons, New York, 1998, pp. 156-164