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**1: Question 1**

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Consider the data *q.clus.xlsx*. Do a k-nearest neighbour, *k*-NN, **density** based clustering on the variable *X*, using mode seeking. Use the following algorithm in answering the question:

- Estimate the density using the *k*-NN density estimator below.

$$\hat{f}_{knn}(x_i) = \frac{1}{||x_i - x_k||^2}$$

Thus the density at point  $x_i$ ,  $\hat{f}_{knn}(x_i)$ , is the reciprocal of the squared distance to the  $k$ -th nearest neighbour  $x_k$ .

- For each of the observations  $x_i$ 
  - Define a pointer to the observation within the  $k$ -nearest neighbours of  $x_i$  with the highest  $k$ -NN-density.
  - Repeat the process by following pointers from the initial pointer until a pointer that points to itself is found. This will be taken as a local mode of  $\hat{f}_{knn}$ .
- Assign each point,  $x_i$ , that converged to the same mode to the same cluster.

**HINT: The attached R code contains two functions, 1) a  $k$ -nn density estimation function and 2) a mode seeking function**

1. Give the  $k$ -NN density estimate for the observed data. Overlay the graph of the density estimate on a histogram of the data.
2. Use the algorithm above to determine the relevant mode(s), comparing different values of  $k$ . How many clusters, **c**, does your cluster solution suggest. Motivate your answer.
3. Graphically illustrate the effect of different choices of  $k$  on the cluster solution.