Slope Convexity Momentum-Based Indicator and Analysis

By Connor Russell

Introduction

Moving averages are a staple indicator used predominantly to forecast the trend of an underlying instrument and have a variety of applications. The usage of moving averages traces back to the 18th century when Japanese rice traders would utilize moving averages to analyze trends. Since then, a plethora of strategies have emerged, mostly relating to the relativity of the price to moving averages and relating moving averages with others of varying lookback windows. I believe the strategies that use “Golden Crosses” and “MACD” to forecast price movements are dislocated from what a moving average is in totality. A moving average is a summation of the prices of the underlying for n periods previously divided by n and, therefore, is an indicator of price movement based on inputs of price movement. By extrapolating the relative price of a moving average to other moving averages, the information obtained through this analysis is skewing away from the fundamentals of this indicator. Enough information is already embedded in each moving average as a stand-alone vehicle to make informed decisions. Specifically, moving averages, especially longer-term ones, have far less noise than the underlying price movements and, therefore, are more differentiable functions than the underlying assets. With that, moving averages contain information regarding the direction and magnitude of the underlying in a far easier format than the underlying. The strategy proposed in this document is meant to provide insight into a possible method to uncover the information embedded in 200-period moving averages.

Indicator Overview

As stated above, longer-duration moving averages create a more differentiable function that contains information on the trend and strength of the underlying. Using this differentiability, to get a better gauge on the actual strength of the trend in the underlying and understand the place where reversals in the trend may be more likely, the first and second derivatives of the 200 periods moving average with respect to price can be taken and standardized for applicability for all highly liquid and actively traded assets.

Mathematical Background

First Derivative

Understanding that moving averages have smoother differentiations compared to the underlying asset, it is still necessary to develop creative approaches to unlock and standardize the insights of these derivatives across all assets. To obtain the first derivative of the moving average, a second line is initially drawn from the current period i back to $i - n$, with n being the number of periods in the lookback. The slope of the second line is fairly intuitive, calculated as:

slope = (MA[i] - MA[i-n]) / n

Each point along the secant is the starting point MA[i-n] + slope. This slope, as defined above, also represents the first derivative. To make this first derivative value more applicable to all assets, the final equation for the first derivative relative value at index i is:

relative slope = (slope / closing_price[i]) * 100 * n

This equation includes scaling by the n value to ensure the percentage is scaled up for longer-term lookbacks. For longer time frames, the slope as a percentage of the underlying price for the first derivative equation becomes larger, so there needs to be an adjustment to the slope equation by dividing the final value inside the tanh function by a constant to ensure the value oscillates appropriately for very large time frames. This constant is indexed to the time frame described in the Pine Script for application across multiple time frames in TradingView.

Second Derivative

The convexity, using this stochastic approach, is calculated as a trapezoidal Riemann sum of the secant line minus the moving average value for every period along the lookback. This convexity value, in the context of an indicator, is ambiguous both when comparing it to previous time series data and other assets. However, by taking the area and dividing it by the moving average value at that point, the area is standardized relative to the price.

Oscillating Indicator

The standardized slope and area have little intrinsic meaning relative to price forecasting and even less value from a trading perspective since they are essentially percentages. To create an oscillating indicator that provides more robust backtesting and trading capabilities, adjustments are made within the tanh function. Tanh is the ratio of sinh and cosh functions, which equates to:

$\tanh(x) = (e^{(2x)} - 1) / (e^{(2x)} + 1)$

It is differentiable across its entire domain and has limits of +1 and -1 as *x* approaches infinity and negative infinity, respectively. The final equation for the slope indicator is:

Slope Indicator = $100 \times \tanh((\text{slope} / MA[i]) \times 100 \times n)$

For the convexity indicator, the tanh function is used to adapt the preceding area with the following equation:

Convexity Indicator = $100 \times \tanh((\text{area} / MA[i]) \times n)$

By multiplying the tanh slope and tanh convexity area by 100, the function is translated to a -100 to 100 oscillator. The tanh value is sensitive to small changes in convexity direction, which is common in trending markets. To help limit this whipsaw, a five-period moving average on the convexity values can be implemented. Additionally, a similar constant can be implemented to limit the whipsaw that occurs when using this indicator on different time frames.

Strength Indicator

Both the tanh slope and convexity indicators have value to traders and from the perspective of systematic investing. However, the convexity indicator is very sensitive to slight changes in convexity, so combining both indicators into one through the simple adaptation of averaging the tanh slope and convexity indicators creates a strength indicator. This provides a smooth way of viewing the actions of the derivatives of the underlying moving average (MA). The final equation for this strength indicator value is:

Strength Indicator = (Convexity Indicator Value + Slope Indicator Value) / 2

This indicator is very sensitive to the constant used for both the slope and convexity indicators and, as such, will not be used in the implementation below.

Implementation

The three indicators described in this paper open up a variety of strategies and real-world implementation and strategy development. Using the math described above to calculate the indicators and lookback window n of 30 periods and a 200-period moving average. In further testing, the lookback period and different moving averages can be optimized to specific strategies, timeframes, and asset classes.

Strategy Overview

Using knowledge from introductory calculus, the position of a particle is moving in the positive direction and accelerating in that direction over time if it has a positive first and second derivative and the opposite is true under the opposite conditions. Applying that same theory to the movement of the underlying asset with positive movements being indicative of a higher price and under the assumption that the moving average is a good proxy to the movement of the underlying, then if the slope and convexity are positive then the price should accelerate up and the opposite assumption is made under opposite conditions. With that, the underlying can be forecasted with some confidence level by forecasting the direction and acceleration of the moving average. The basis of the strategy is that when the slope of the moving average goes from negative to positive, and the convexity indicator is also positive with a value over some predetermined level (0 in this case), the asset would be bullish and would be bearish when the opposite conditions are met.

Data Description

The data used to derive the following analysis is 15-minute historical S&P 500 closing price data pulled from backtestmarket.com, a low-cost data provider. The data was then filtered for just the periods within the normal hours of a trading day and eliminated open, high, low, and volume information for easier computation and more robust applicability. Closing prices from 9:30 am on 1/14/2014 until 3:45 pm on 10/7/2022 are utilized. This data was then pulled into a Jupyter Notebook for a 200-period moving average and a 400-period moving average to be initiated. From there, the slope indicators were constructed for the 200 and 400-period moving averages, and the 5-period rolling convexity indicator for the 200-period moving average was initiated using the computations described earlier. From there, the final pandas data frame was constructed.

Functionality

After the final data frame was constructed, it was iterated through to find situations in which the 200-period moving average slope value went from negative to positive (slope[i-1]<0 and slope[i]>0). Furthermore, the convexity value had to be positive and the 400-period moving average slope had to be positive. The reasoning behind this later piece is that the 400-period moving average is a very close gauge of the 200-period moving average using 30-minute bars. Essentially, having the validation of bullish trends on longer-term moving averages using the same methodology helps mitigate false positives and provide more robust signals. Lastly, the price has to be greater than the 200-period moving average to combat false signals. After the indexes of the bullish signals were appended to the signals list, some data was gathered about the movements of the underlying. The statistics and histograms for this data were created. The exact opposite criteria were used to get the statistics from the perspective of a bearish signal.

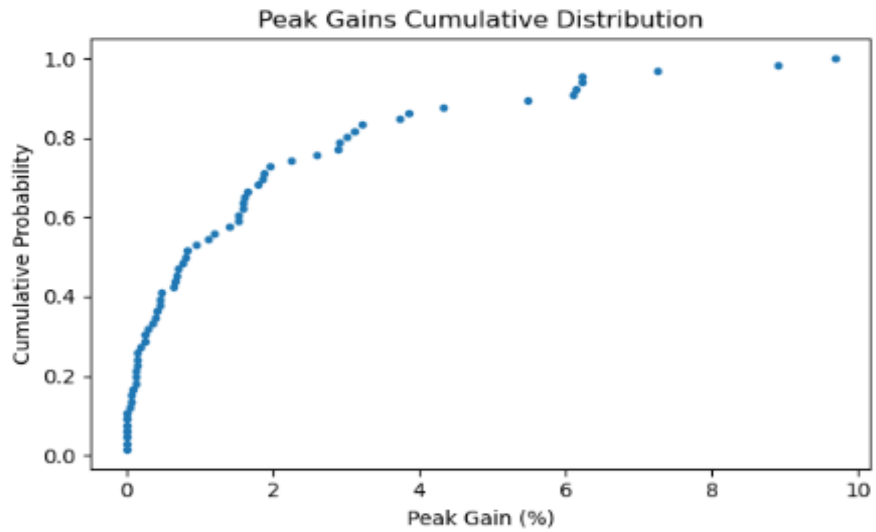
Statistics Discussion

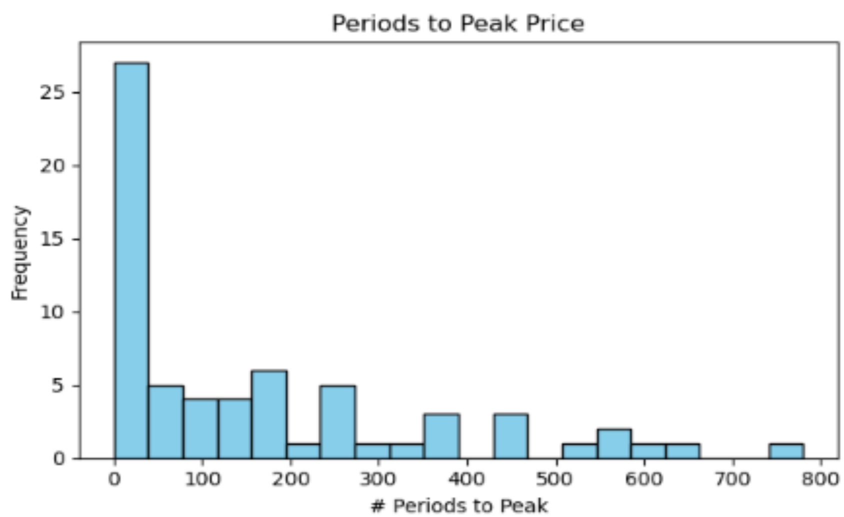
Under the conditions described above, there were a total of 66 indications for the bullish trends and 36 indications for the bearish trend across the 3200 trading days studied. The discrepancy between the number of bullish and bearish indications is most likely related to the long-run equity premium in the S&P 500 index. It can be reasonably assumed that this discrepancy would be less pronounced if the strategy was subjected to the index components. The frequency of trades could hypothetically be increased through usage across the components of the S&P 500 as well, but this could not be tested due to a lack of data for the components in this specific timeframe.

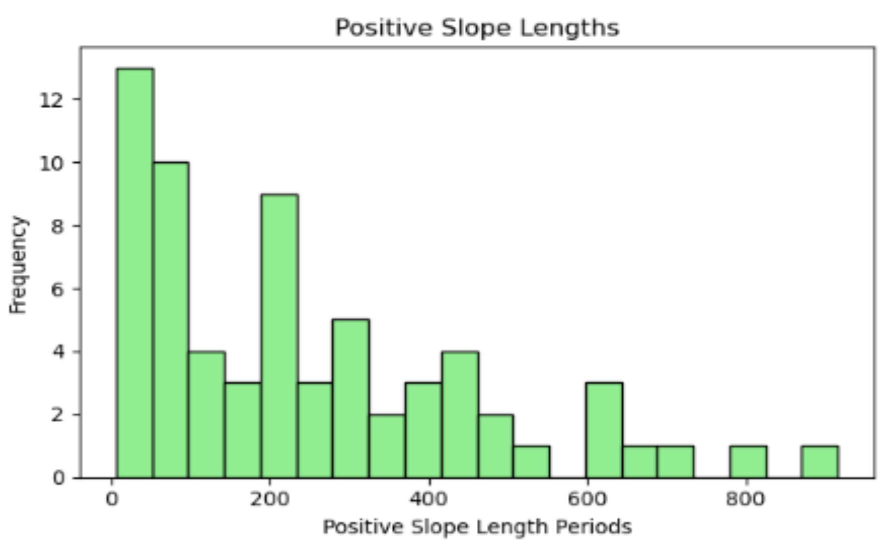
In analyzing the data presented through this initial search, it is clear that the bullish and bearish conditions have a strong ability to forecast the change in trends but are also susceptible to general market noise. This is evident through the high STD in the duration of the trends, with the number of periods in the 200-period moving average increased (decreased) consecutively. Also, there is a wide discrepancy in the ranges in which the peak (trough) price was reached after initiating the indicator. When coupled with the mediocre success rate for both bullish and bearish signals, this specific indicator is best used in utilizing the non-linear payoff, such as equity options. Under the assumption that each indicator will have a success rate of near 50% (defined as peak gain over 1% and vice versa for bearish signals), while there is an outsized probability of being on the correct side of a swift and outsized move in that direction makes the non-linear payoff of an option benefit from the distribution of expected returns given the signal.

Bullish Signals

Number of Signals	66
Average # of periods slope was positive after signal	246.17
Median # of periods slope was positive after signal	215.5
STD of # of periods slope was positive after signal	215.37
Q1 and Q3 for # of periods slope was positive after signal	55.25, 366
Average # of periods until peak price after signal	162.83
Median # of periods until peak price after signal	96.5
STD # of periods until peak price after signal	189.78
Q1 and Q3 for # of periods until peak price after signal	8.75, 252.25
Average peak gain	1.81%
STD of peak gain	2.26%
Median Peak Gain	0.82%
Q1 and Q3 of peak gain	0.16%, 2.52%
Success rate (peak gain over 1%)	46.97%
Max peak gain	9.68%
Average drawdown after signal	-0.94%
Median drawdown after signal	-0.57%
STD of drawdown after signal	0.99%

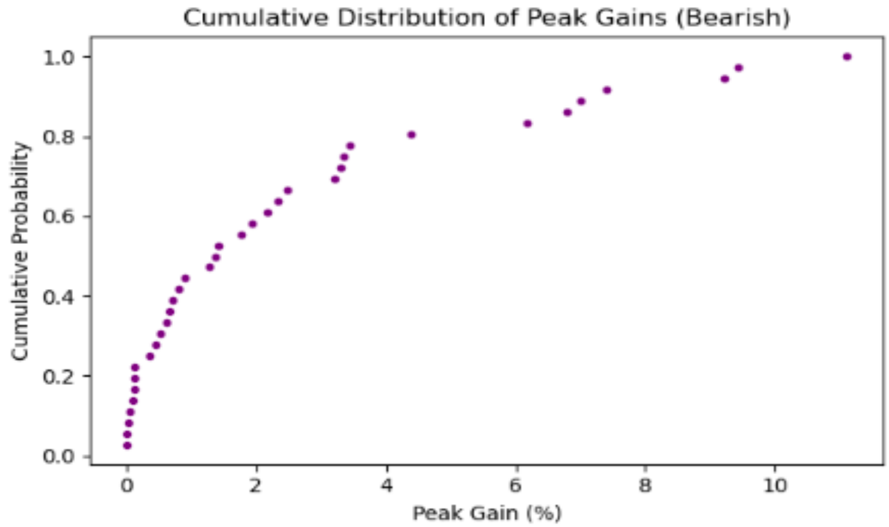

Peak Gains Cumulative Distribution
Cumulative Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
Peak Gain (%)

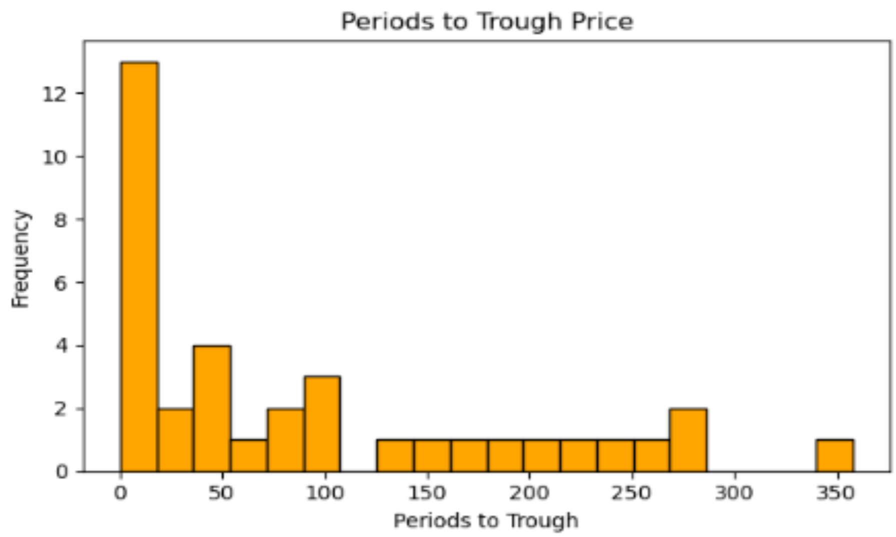

Periods to Peak Price
Frequency
0
5
10
15
20
25
0
100
200
300
400
500
600
700
800
Periods to Peak

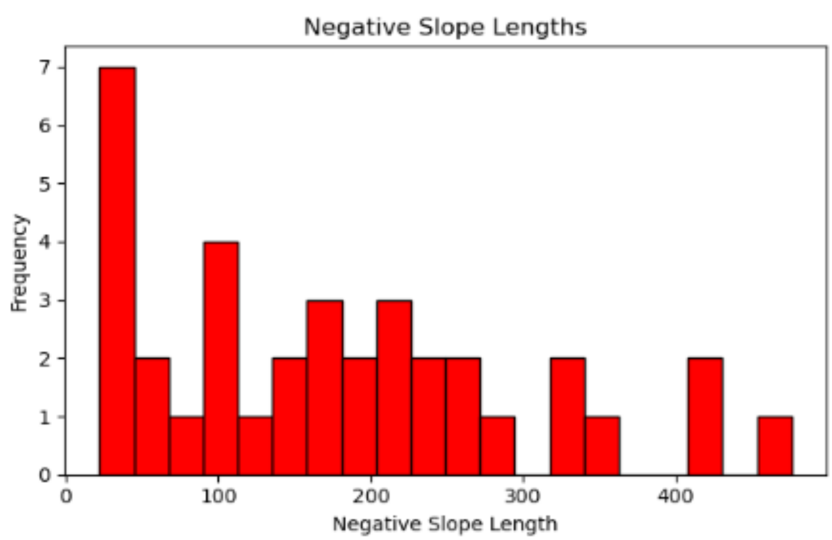

Positive Slope Lengths
Frequency
0
2
4
6
8
10
12
0
200
400
600
800
Positive Slope Length Periods

Bearish Signals

Number of Signals	36
Average # of periods slope was negative after signal	176.22
Median # of periods slope was negative after signal	165
STD of # of periods slope was negative after signal	123.47
Q1 and Q3 for # of periods slope was negative after signal	81.5, 243
Average # of periods until lowest price after signal	92.86
Median # of periods until lowest price after signal	47.5
STD # of periods until lowest price after signal	98.27
Q1 and Q3 for # of periods until lowest price after signal	12.75, 153.5
Average peak gain (short)	2.64%
Median peak gain (short)	1.39%
STD of peak gain	3.03%
Q1 and Q3 of peak gain	0.43%, 3.38%
Success rate (peak gain over 1%)	55.56%
Max Peak Gain	11.11%
Average drawdown after signal	1.29%
Median drawdown after signal	1.1%
STD of drawdown after signal	0.98%


Cumulative Distribution of Peak Gains (Bearish)
Cumulative Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
Peak Gain (%)


Periods to Trough Price
Frequency
0
2
4
6
8
10
12
0
50
100
150
200
250
300
350
Periods to Trough


Negative Slope Lengths
Frequency
0
1
2
3
4
5
6
7
0
100
200
300
400
Negative Slope Length

Backtest

Description

The conditions for bullish and bearish entries are the same as described above for this backtest. As suggested earlier, the conditions for bearish and bullish signals lend themselves best to utilizing nonlinear payoffs to capture the skewed distribution of outsized trends in length and magnitude. With this, a backtest was conducted to estimate the historical returns that would have been achieved through trading puts and calls utilizing the bullish and bearish criteria described. For each bullish (bearish) signal, a 30-day at-the-money (ATM) call (put) option will be initiated and will be held for 211 15-minute bars (8.1 trading day), and then the return in dollar terms will be calculated and added to the value of the portfolio before the trade was initiated. The 211 period is the average number of periods for a trend weighed by the number of bullish and bearish signals. The portfolio will begin with a simulated $10,000, and the backtest will run from 1/14/2014 to 10/7/2022. Each trade will be the purchase of 80 contracts and if an opposing signal is hit while a trade is currently active, the initial trade will be closed, and the new signal trade will commence. For example, if a call is initiated but then a bearish signal is hit, the return of the call option at the bearish signal will be calculated, and then the put will be initiated. Having a fixed purchase of 80 contracts per simulated trade is not the most optimal way of dynamically adjusting portfolio allocation, but through previous tests, this strategy is profitable. Since this strategy is modeled on the S&P 500, which has had strong returns over the testing period, the size of each position in dollar terms is increasing but, due to its profitability, the size of each position as a percentage of the portfolio remains around 20%. If 20% of the portfolio was allocated to each trade, this would mean that during downturns in the portfolio, dollar allocation would drop, and during periods of outperformance, the opposite would occur. This would make this backtest very susceptible to changes in portfolio value from a few consecutive good or bad trades. With a fixed allocation and positive returns on average for both the S&P 500 and the strategy, the size of each position remains relatively constant across the test while mitigating the impact of position sizing based on the portfolio value.

Options Pricing Methodology

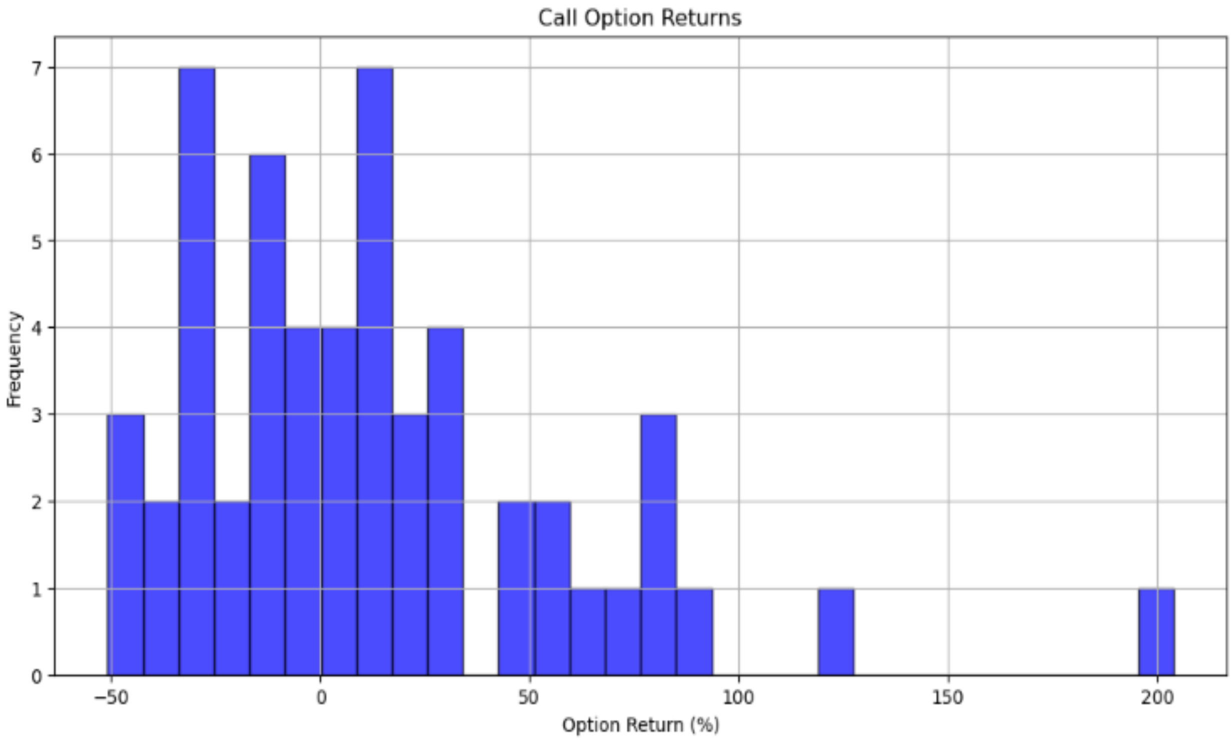
Due to a lack of data capabilities, there was no direct way to pull in the options surface at each individual signal. Instead, daily VIX data was downloaded from the CBOE, and the value of the VIX at the given signal date would be the input for the Black Scholes options pricing algorithm. The closing value of the VIX on the day aligned with the entry or exit of a contract will be used for simplicity reasons. With the constraints of Black Scholes and the absence of information regarding the skew for any individual expiration, a hypothetical 30-day ATM options value would be calculated for each trade. Using the VIX as a proxy for the implied volatility input for the given contract constrains the applicability of the backtest to contracts that are not ATM and at different expirations far outside of one month since the VIX is the strike-weighted average implied volatility for the options for 23 to 37 days in advance. The VIX also trades at a premium to the implied volatility for ATM options on the underlying since it incorporates the IVs of OTM options and, thus, the skew of the implied volatilities for the strikes at a given expiration. This means that the IVs used in this backest are systematically higher than those found in the market and also makes this backtest a conservative approximation for the real returns that would have been achieved. This is not an exact way of calculating the return of a portfolio trading this strategy, but it provides a decent, dynamic estimate to gauge the strengths of the strategy. After a position is closed, the Black Scholes algorithm will use the VIX value at that time while adjusting the time to expiration input to value the new contract to calculate the return. At the closing of the position, the value of the exit contract does have less than 30 days until expiration, which means the IV input using the VIX is an approximation, but it will adjust for general changes in the volatility term structure between entry and exit which is the goal of this adjustment. This means that if the number of periods held is extended from 211 periods, the IV input will become less realistic unless all contracts are held until expiration because there would not be an IV input at expiration. Since ATM options have the lowest IV compared to those on the wings, if the option goes deep ITM or OTM, the volatility input at the close of the contract underestimates the more realistic IV, meaning that this backtest is more conservative than reality.

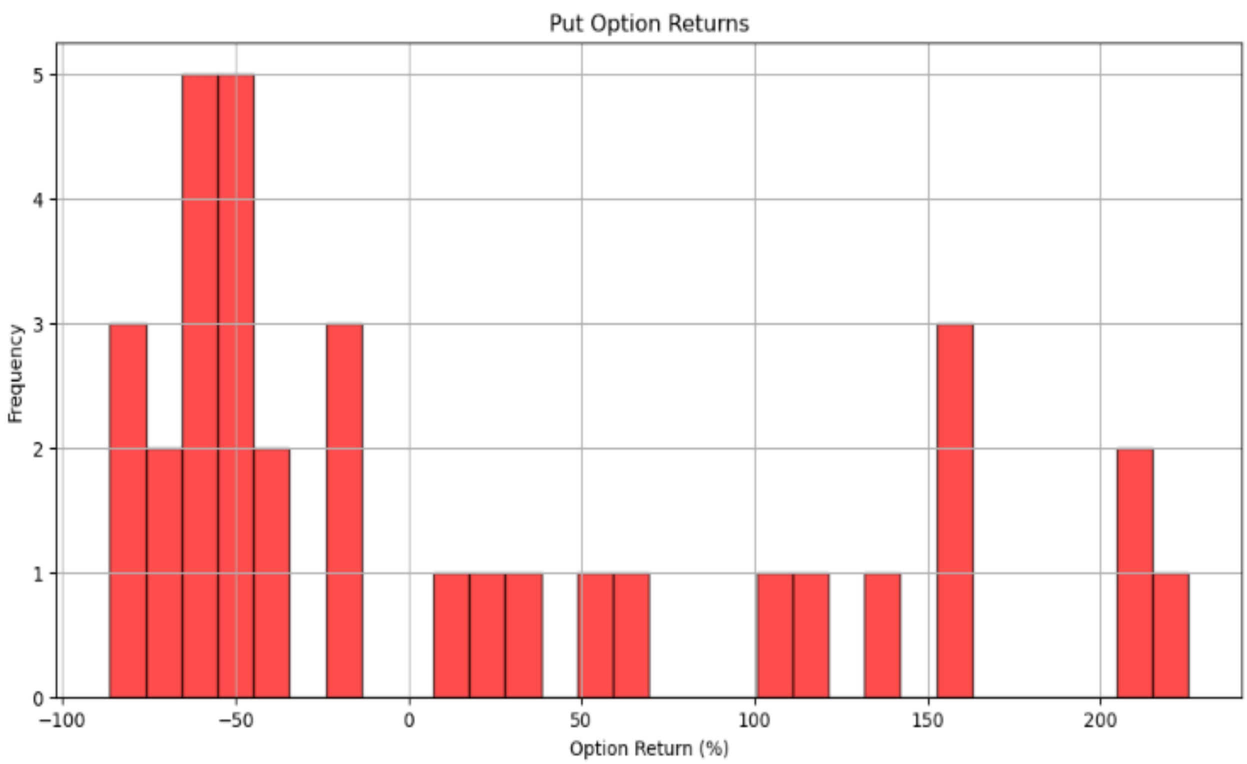
The hypothetical contracts constructed for this backtest have a fixed risk-free rate (r) assumption of 3% annualized. For the input of 30 days until expiration and the input of 211 periods or 8.11 trading days, the impact of interest rates on the value of the options is minimal. Adding in a dynamic adjustment for r would make the model more realistic but it would have a minimal impact on the portfolio while just adding unnecessary complications.

Results Discussion

As expected, this backtest returned alpha of 437.7% by returning 540.11% compared to a buy-and-hold strategy that returned 102.3% over the same period. Throughout the backtest, the initial investment of $10,000 turned into $64,010.90. The portfolio had a max drawdown of 33.61% between the start of 2016 and the third quarter of 2017. Slippage and trading costs were not considered for this backtest, and no leverage was implemented. The percentage of calls and puts that had a positive return was 55.56% and 41.18%, respectively. The total rate of positive return for this strategy was exactly 50%. The average gain for the calls and puts was 15.56% and 15.06%, respectively. The largest gain for a single contract for calls was 223.41% and 214.54%

for puts. Regarding dollar return, the calls yielded a total increase of $33685.10, and the puts yielded $20325.80. The skew of the S&P 500 towards the upside over longer time horizons is likely attributed to the larger gain and number of trades for the calls compared to puts.


Call Option Returns
Frequency
Option Return (%)
0
1
2
3
4
5
6
7
−50
0
50
100
150
200


Put Option Returns
Frequency
Option Return (%)
0
1
2
3
4
5
−100
−50
0
50
100
150
200

Further Research and Optimization

The indicator and strategy described in this document are a first of its kind, so this document is essentially a proof of concept for alternative application of moving averages. As such, there are a variety of ways that this strategy can be improved and optimized in the future.

This indicator is best optimized to look for changes in the direction of the current trend and the strength of that given trend. Generally, this will lend itself better to entering into positions as it provides an early indication of changes in the skew of the distribution of expected returns. In the future, analysis of this indicator regarding exit conditions would be highly informative.

In the practical application as a live indicator in TradingView, there need to be some adjustments made to the calculation of the slope and secant-moving average areas relative to volatility. On high-volatility tickers, the values for the slope and convexity vary a lot since the slope has a far stronger direction than a low-volatility asset and the area between the secant line and the moving average suffers from a similar effect and the opposite is true for those low volatility assets. In the functionality of this strategy, the actual values of the slope and convexity were only applicable in terms of their orientation (positive or negative), so there was no need for a volatility-adjusted slope or area. A constant multiple could be implemented in the area function that will adjust the slope relative to volatility for strategies that use the relative values of the indicators as inputs.

This strategy only tested the returns relative to the S&P 500 and as such further research is needed to show its capabilities across different asset classes. Due to a lack of data, it was not able to be tested through this document. A possible means of implementation could be using this indicator strategy on a basket of correlated stocks, and if a similar indicator is given across some or all the constituents a position would be placed. Similarly, if diverging signals were triggered on a correlated basket, then a pair trade could be implemented to capitalize on this divergence.

Similarly to the limited scope of assets studied, the timeframes studied were limited to just 15-minute time frames with an additional 30-minute condition. With the 15-minute dataset, any time frame larger could be constructed and tested, but for simplicity and ease of computing, these were not used. In the live TradingView application, constant multiples were included to adjust for different time frames. For larger time frames, the slope and secant-moving average values were much larger than smaller time frame values. This disparity is not as prevalent for the small differences in time frames (like between 15 minutes and 30) but it becomes very apparent when looking at one-minute versus one-day time frames.

Due to data limitations, the ability to test the different option strategies, strike prices or expirations was limited. Options strategies that profit from large, undisturbed trends, such as ratio back spreads, OTM options, or debit spreads, would be more optimal than ATM options but cannot be tested due to lacking data on the volatility skew at any given signal. Furthermore, the available implied volatility data limits the usage of varying strikes. With each position lasting just over a week, a 30-day option limits the gamma exposure of each position. Reducing the expiration should, hypothetically, improve the overall return but needs to be tested further.

Final Thoughts

This indicator and the following strategy are the culmination of dozens of iterations to explore the trading possibilities of a unique approach to a well-known indicator, the moving average. On my personal account, I have been trading a very similar strategy to that discussed in this document, and I actively utilize the screener and indicator in Trading View daily. There are numerous key differences between the one discussed in this document and the one used on my personal account. I screen for changes in slope and convexity across the 15-minute, 30-minute, and 1-hour bars for the desired tickers. When a ticker has bullish or bearish signals across numerous time frames, the magnitude and win rate of these trades tends to be greater, from my experience. Furthermore, this indicator is just an indicator and not a determinant so when this strategy is coupled with fundamental analysis, and both validate the same thesis, there tends to be a stronger-than-expected move in that given direction. For example, this strategy gave a very strong bullish indication for some of the Chinese stocks that I screened for (BIDU, BABA, and JD) concurrently with the Peoples Bank of China lowering their reverse repo rate from 1.7% to 1.5% on Thursday, September 24th, 2024. That following week, the PBOC released the first round of comprehensive monetary stimulus, which drove these Chinese stocks into a buying frenzy, with JD rising 70%+ in a month. I also screen across far more stocks and ETFs (including fixed-income ETFs) than just the S&P 500. By screening for more tickers that fit the given criteria, I can broaden the number of trades that I can make over a year and decrease the variability of the profitability of the strategy. Lastly, the skew of the distribution in expected paths described through the statistics gathered through this strategy lends itself better to betting in fatter tails than priced in the market in the direction of the signal generated. There is a variety of ways to capitalize on this through options like ratio backspreads, debit spreads, and possibly straddles. More generally, this means that OTM calls (puts) should hypothetically, and in practice, generate more outsized returns from bullish (bearish) signals while limiting downside potential than ATM options. However, limitations on data capabilities meant that just ATM options could be tested for the most accurate results.

Overall, this document and the supporting code have been a very informative and challenging experience. I hope that some readers will be able to apply pieces of this analysis to their own strategies and I encourage further exploration and adaptations to the theory and practical applications described in this document. If there are any questions or concerns, feel free to reach me at russell.1553@osu.edu or (440) 351-1615.