# Systems of Non-interacting Particles

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In this report we produce a graph of the average energy of a system of 10 particles against temperature. In our system, each particle can be in one of two discreet states, with energy levels of $+1$ and $-1$, shown in the diagram:

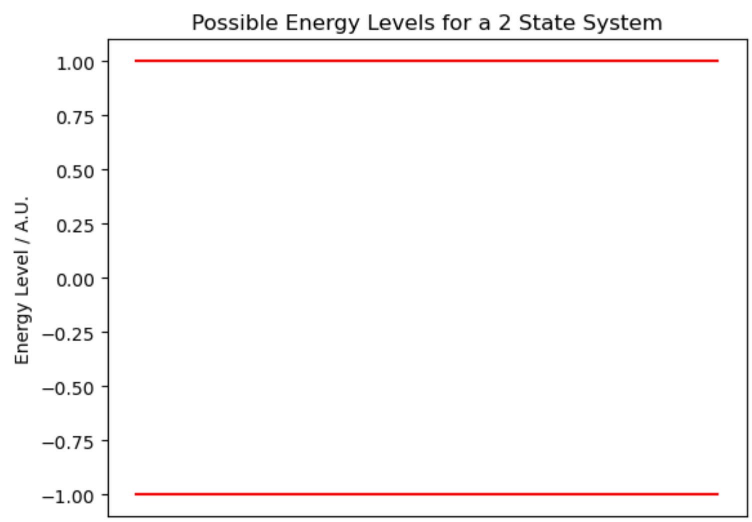


Figure 1: Available energy levels for particles in our system. The energy values are +1 and -1.

Enumerating over every possible microstate for a given temperature can allow us to compute the average energy as:

$$\langle E \rangle = \frac{1}{Z} \sum_j E_j e^{-\beta E_j} \qquad\qquad Z = \sum_j e^{-\beta E_j} \tag{1}$$

where $E_j$ is the index of the $j^{th}$ microstate and $\beta$ is defined as $1/k_bT$, where $k_b$ is Boltzman's constant, which we set to 1, and $T$ the temperature. Applying this for various values of $T$ allows us to produce a graph showing the relationship between the average energy and temperature.

It is possible to derive an analytic solution for this problem, which gives the average energy for a system of $n$ particles as:

$$\langle E \rangle = -n \cdot \tanh(\beta) \tag{2}$$

where again $\beta$ is equal to $1/k_bT$.

Plotting the enumerated points and the function on the same graph shows that the methods agree:

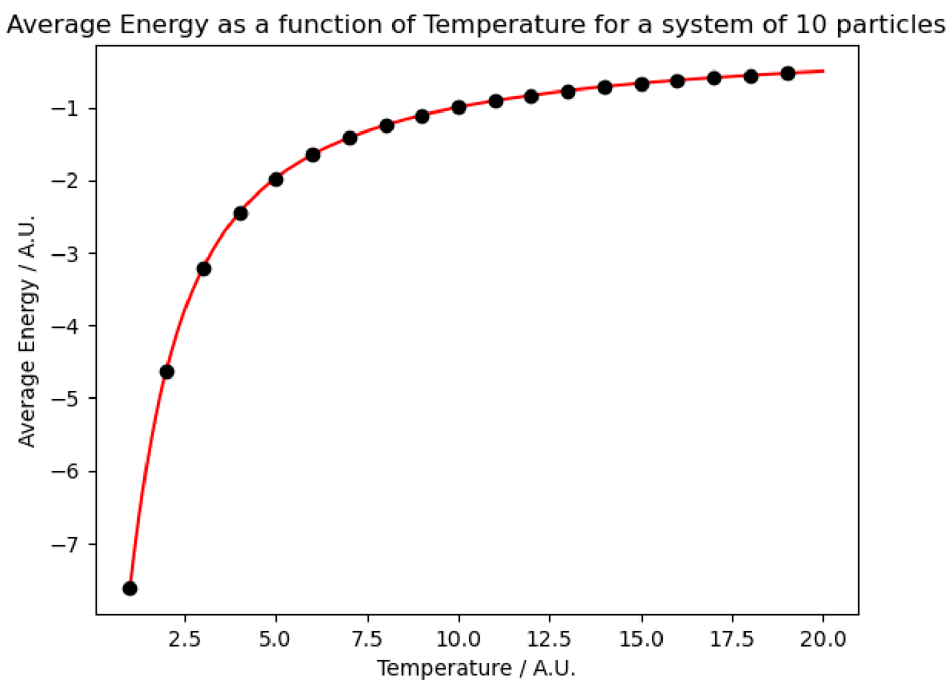


Figure 2: Average energy against temperature for a system of 10 particles which can exist in discreet states with energies of $+1$ and $-1$.