# Systems of Non-interacting Particles

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## 1 Introduction

In this article we investigate the total energy levels of a system of non-interacting particles.

We consider the state of each particle to be a discreet energy level, with each state equally likely for each particle to be found in.

As the particles are independent, we can compute the total energy of any given microstate as a sum of each particle's energy. For a sysyem of $N$ particles, with energy states $s_i$, this summation is:

$$E_{total} = \sum_{i=1}^{N} s_i \tag{1}$$

## 2 Two-State System

Our first system we investigate is a system of 20 particles with 2 energy states, of $-1$ and $+1$ units. The energy levels are given below:

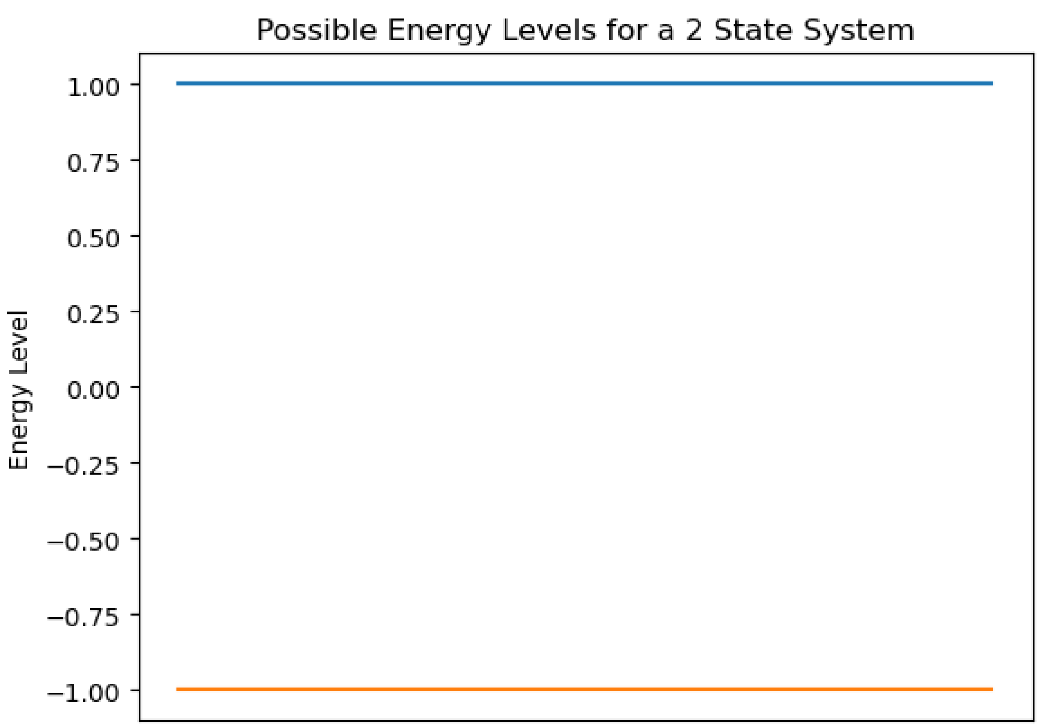


Figure 1: Possible energy levels for a particle in our system.

Given these energy levels, we can compute the minimum and maximum possible energy for the full system of particles.

The minimum levels has all particles at energy state $-1$, with total energy $-1 \times 20 = -20$. Similarly, the maximum total energy is $+1 \times 20 = -20$.

Using equation 1 we can compute the total energy for any given microstate. When we do this for every possible microstate of our system, we can find the energy density for our system. The result of this is given:

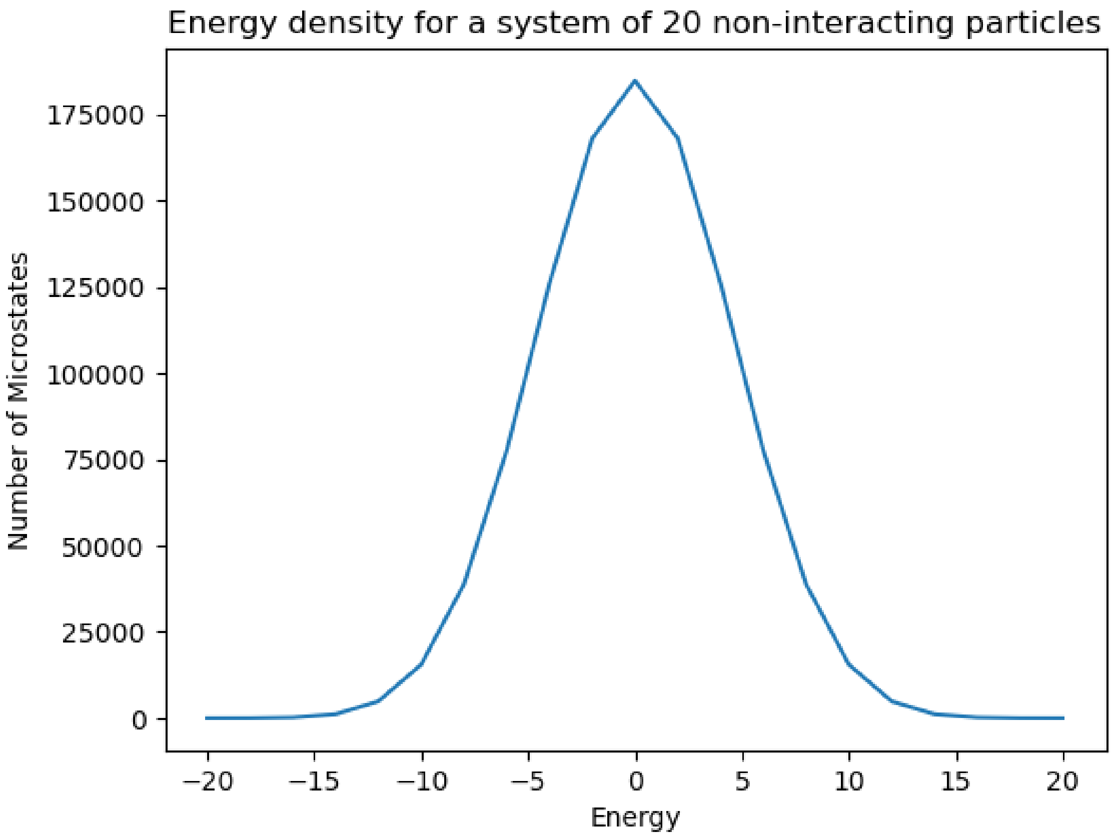


Figure 2: Energy density for all possible microstates of a system of 20 non-interacting particles. Each particle can exist in a state with energy $+1$ or with an energy of $-1$. There are $2^{20}$ possible microstates.

We can see in figure 2 that the energy is distributed symmetrically about zero, which is to be expected with our setup of equal opposing energy levels.

## 3 Three-State System

We add complexity to our system by including an extra possible state, which unbalances the energy and will skew the graph in our result.

The new energy levels are $-1$, $+1$ and $+2$, which is an unbalanced system with an average energy per particle of $2/3$.

We show this on the energy diagram below:

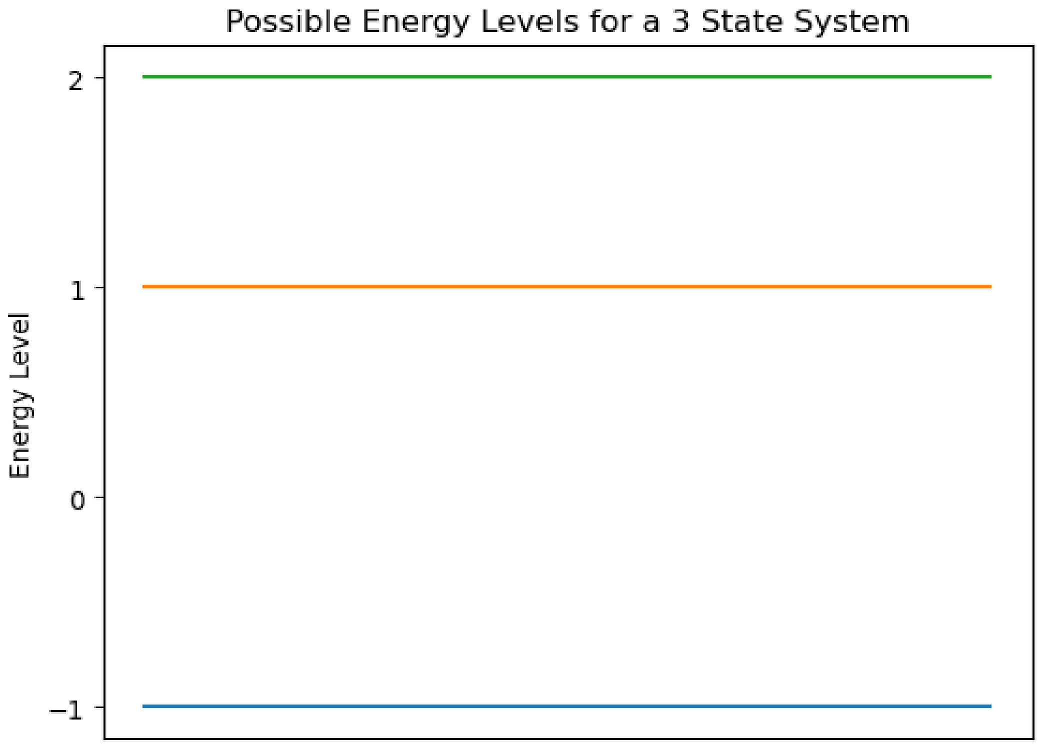


Figure 3: Possible energy levels for a particle in our system.

The minimum energy level again has all particles at energy state $-1$, with total energy $-1 \times 12 = -12$. This time, the maximum total energy is when each particle is in the state with an energy of $+2$, giving the maximum as $+2 \times 12 = 24$.

Again, we compute the energy for every possible microstate and produce a graph of the energy density of the system.

As seen in figure 4 below, the energy is no longer distributed about zero. The energy levels reach up to 24, and the modal energy is 8.

This modal energy can be computed as the product of the average energy ($\frac{-1+1+2}{3} = \frac{2}{3}$) and the number of particles (12).

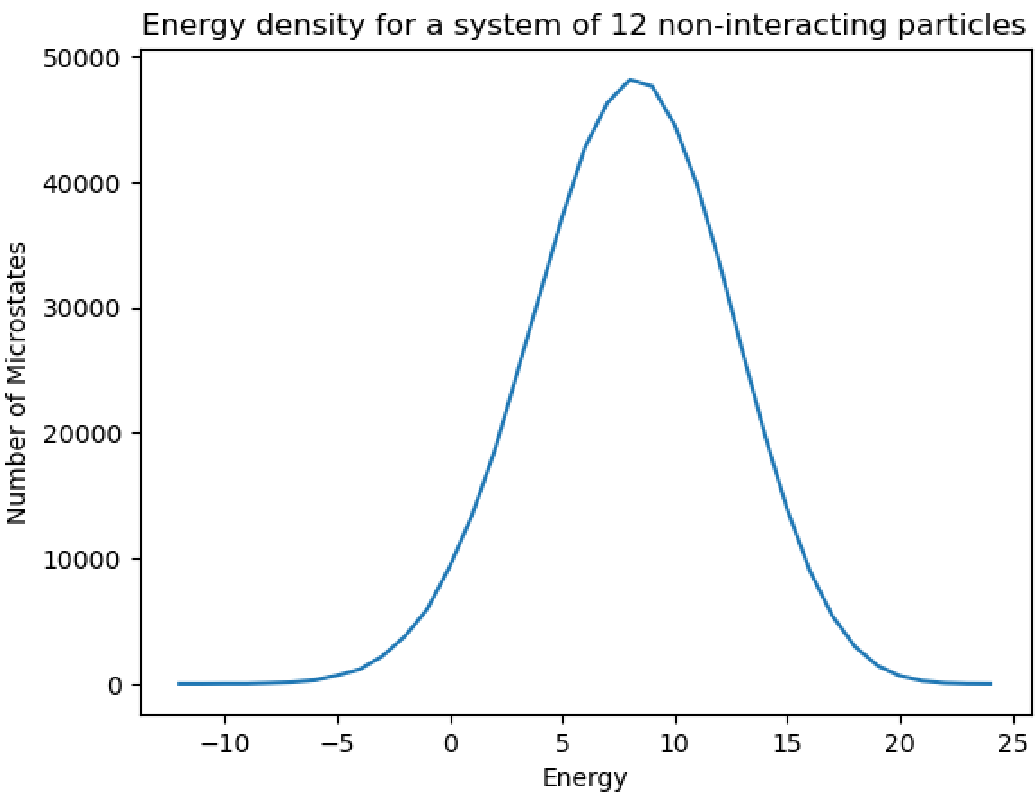


Figure 4: Energy density for all possible microstates of a system of 12 non-interacting particles. Each particle can exist in a state with energies of $-1$, $+1$ or $+2$. There are $3^{12}$ possible microstates.