hw5

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1 Let's check out how well our numerical methods work with a conserved quantity of the pedulum equation!

```
In [1]: %matplotlib inline
        import matplotlib.pyplot as plt
        import numpy as np
        eps = np.finfo(float).eps
        tiny = 100*eps
```

First we'll define some functions for RK4 and RK2

```
In [2]: '''
        @param x: Initial values for the IVP
        @param fx: System of equations in order of decreasing (derivative) order
                   This should be the same shape as @param x.
        @param hs: Step size. (Note, since this is only a single step calculation,
                              adaptive step calculation doesn't happen in here)
        @return: new values, same shape as x
                 The return value can be used for next iteration, as a new IVP
        '''
        def RK4(x, fx, h, debug=False):
            n = len(fx)
            x = x.copy()
            k1, k2, k3, k4, xk = [], [], [], [], []

            k1 = [fx[i](x)*h for i in range(n)]

            xk = [x[i] + k1[i]/2 for i in range(n)]
            if debug: print("XK1: {}".format(xk))
            k2 = [fx[i](xk)*h for i in range(n)]

            xk = [x[i] + k2[i]/2 for i in range(n)]
            if debug: print("XK2: {}".format(xk))
            k3 = [fx[i](xk)*h for i in range(n)]
```

```
        xk = [xk[i] + k3[i] for i in range(n)]
        if debug: print("XK3: {}".format(xk))
        k4 = [fx[i](xk)*h for i in range(n)]

        x = [x[i] + (k1[i] + 2*(k2[i] + k3[i]) + k4[i])/6 for i in range(n)]
        if debug: print("X: {}\n".format(x))
        return x
```

In [3]:
```
#AKA Midpoint Method
def RK2(x, fx, h, debug=False):
    n = len(fx)
    x = x.copy()
    k1, k2, xk = [], [], []

    k1 = [fx[i](x)*h for i in range(n)]
    xk = [x[i] + k1[i]/2 for i in range(n)]
    if debug: print("XK1: {}".format(xk))
    k2 = [fx[i](xk)*h for i in range(n)]

    x = [x[i] + k2[i] for i in range(n)]
    return x
```

1.1 Setting up the experiment...

In [4]:
```
x = [2, 1, 0]                     #[', , t]
fx = [lambda x: -np.sin(x[1]),
      lambda x: x[0],
      lambda x: 1]                #['', ', t'==1]
```

In [5]:
```
def pendulum(solver, x, fx, numsteps, stepsize, debug=False):
    res = [x.copy()]
    for i in range(numsteps):
        res.append(solver(res[-1], fx, stepsize))
    return res
```

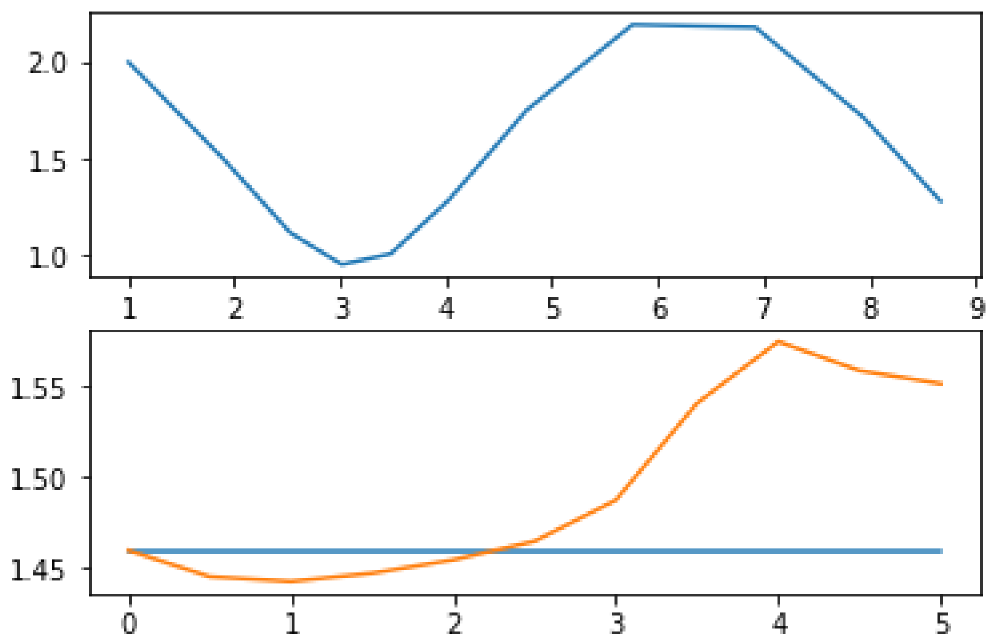
In [6]:
```
#Experiment always starts at t=0 due to IVP information.
def run_exper(steps, solver, t_end=5, debug=False):
    stepsize = t_end/steps
    out = "t={3:6.3f}\t(t)={2:6.3f}\t'(t)={1:6.3f}\t''(t)={0:6.3f}"
    ts, ys, vs = [], [], []
    conserved_true = [2 - np.cos(1)]
    conserved_actual = []
    for e in pendulum(solver, x, fx, steps, stepsize, debug=debug):
        ts.append(e[2])
        ys.append(e[1])
        vs.append(e[0])
        if debug: print(out.format(-np.sin(e[1]), *e))
        conserved_actual.append((e[0]**2)/2 - np.cos(e[1]))
```

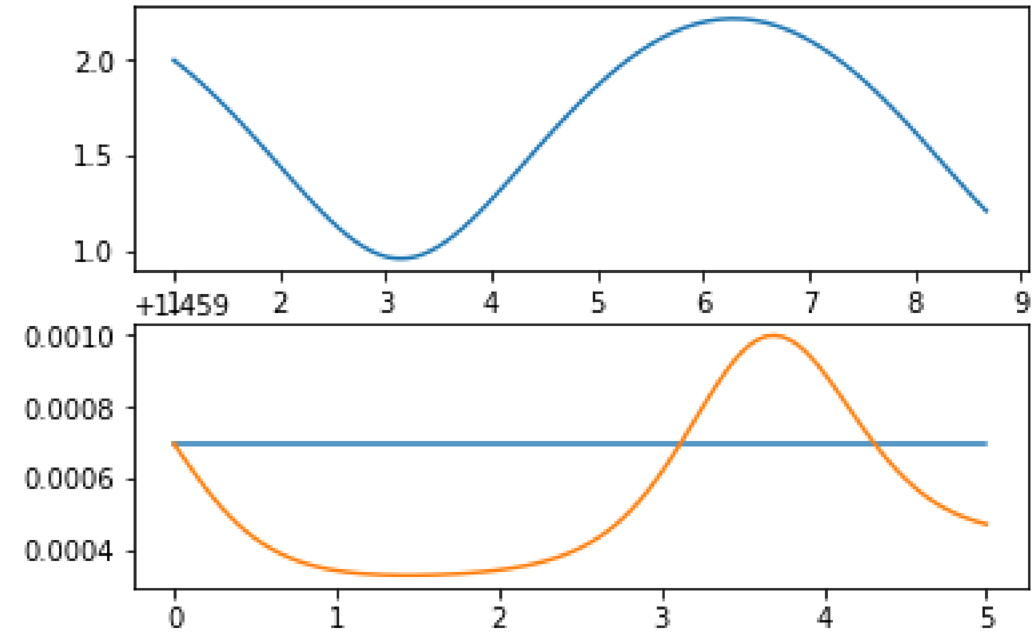
```
fig, axarr = plt.subplots(2)
axarr[0].plot(ys,vs)
#We gotta fix the length of conserved_true
conserved_true *= len(conserved_actual)
axarr[1].plot(ts, conserved_true, ts, conserved_actual)
print("Final Deviation from Conservation for h={:5.5f}: {:12.10f}".format(
        stepsize, conserved_true[-1] - conserved_actual[-1]))
```

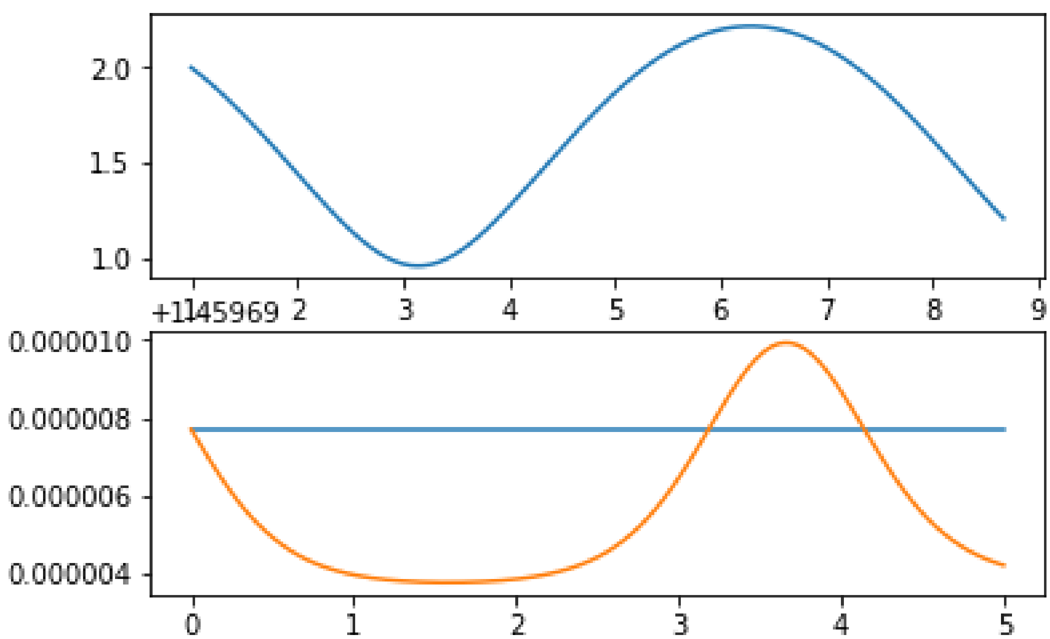
1.2 Let's run with RK2...

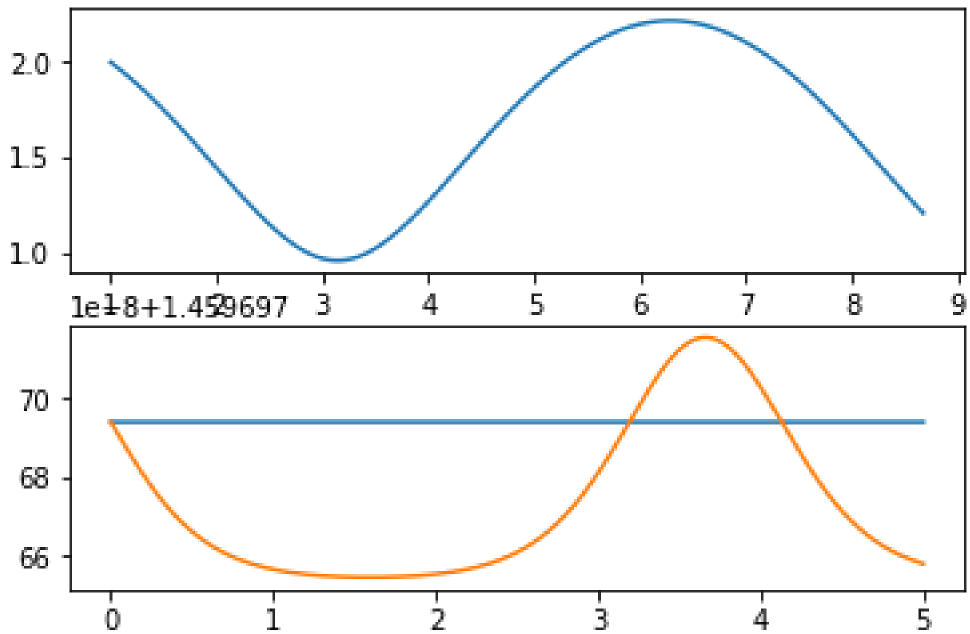
```
In [7]: run_exper(10,    RK2)
        run_exper(100,   RK2)
        run_exper(1000,  RK2)
        run_exper(10000, RK2)
```

```
Final Deviation from Conservation for h=0.50000: -0.0920798157
Final Deviation from Conservation for h=0.05000: 0.0002242213
Final Deviation from Conservation for h=0.00500: 0.0000034744
Final Deviation from Conservation for h=0.00050: 0.0000000360
```




2.0
1.5
1.0
2
3
4
5
6
7
8
9
0.0010
0.0008
0.0006
0.0004
0
1
2
3
4
5


2.0
1.5
1.0
2
3
4
5
6
7
8
9
0.000010
0.000008
0.000006
0.000004
0
1
2
3
4
5

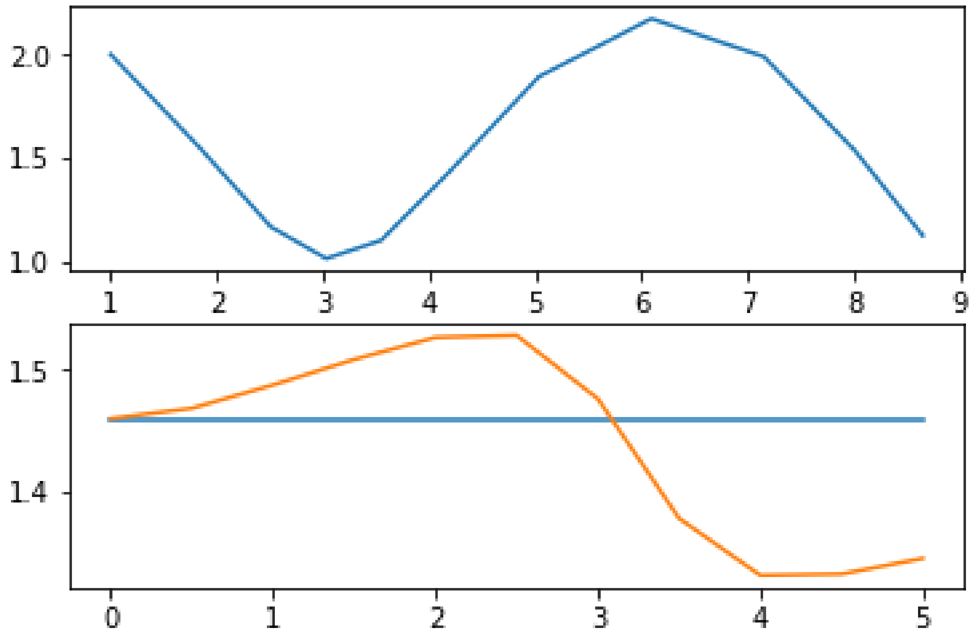


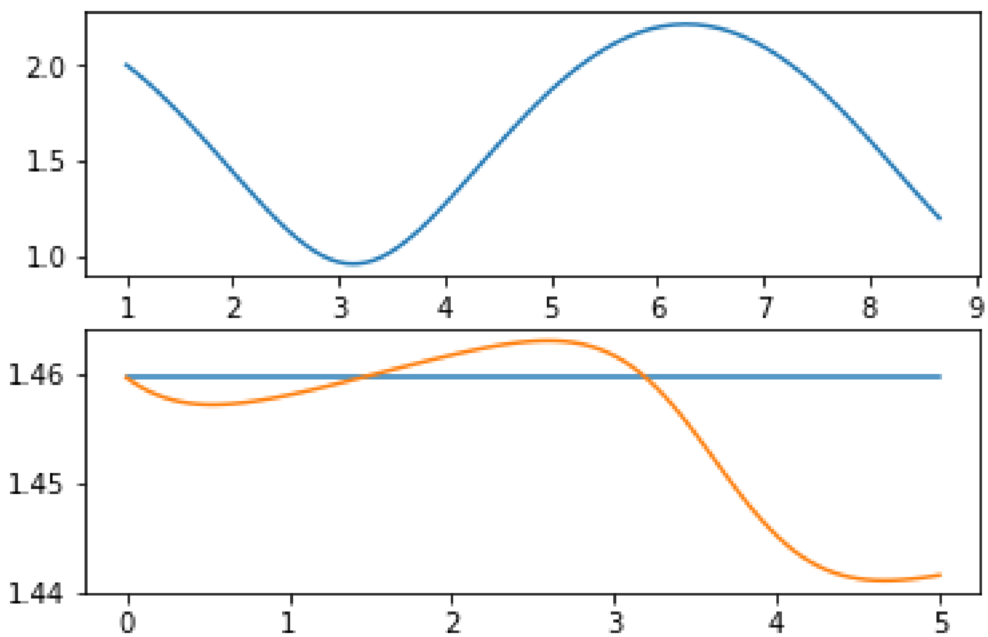
1.3 Allright, the error in the conserved quantity is definitely decreasing with smaller step sizes!

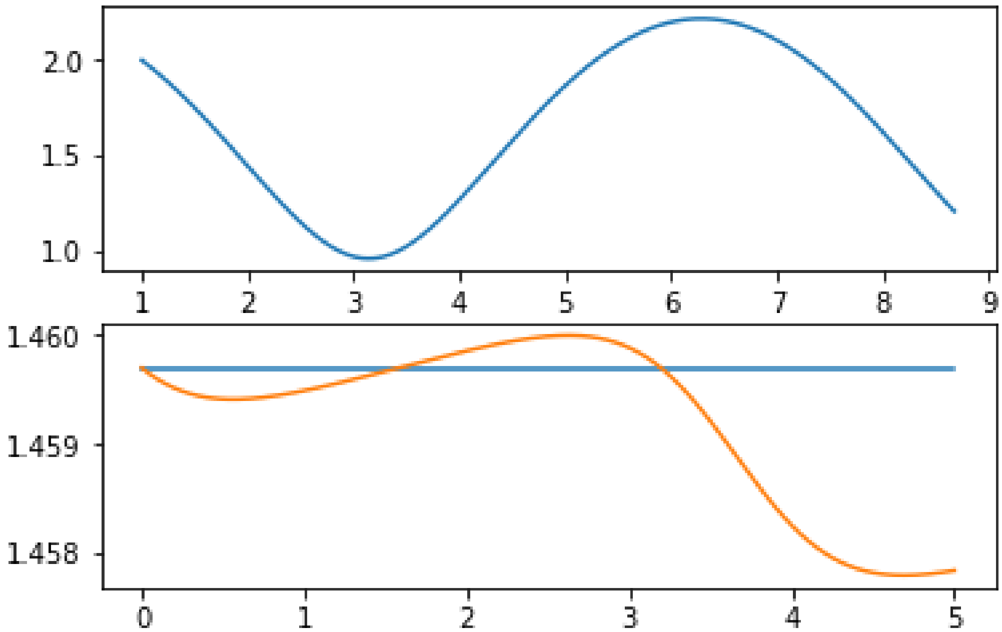
1.4 Let's see if the same holds for our RK4 implementation.

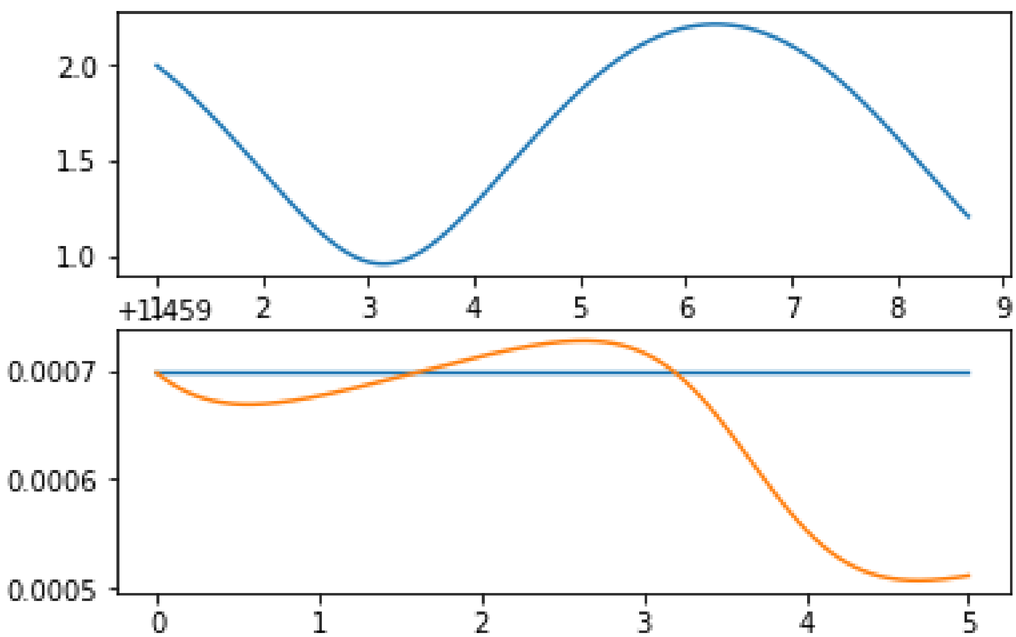
```
In [8]: run_exper(10,    RK4)
        run_exper(100,   RK4)
        run_exper(1000,  RK4)
        run_exper(10000, RK4)
```

```
Final Deviation from Conservation for h=0.50000: 0.1146286733
Final Deviation from Conservation for h=0.05000: 0.0180889035
Final Deviation from Conservation for h=0.00500: 0.0018566014
Final Deviation from Conservation for h=0.00050: 0.0001861126
```






2.0
1.5
1.0
1 2 3 4 5 6 7 8 9
1.460
1.459
1.458
0 1 2 3 4 5


2.0
1.5
1.0
1 2 3 4 5 6 7 8 9
+1.459
0.0007
0.0006
0.0005
0 1 2 3 4 5

1.5 Yep! Now let's move on to some other code.

2 Tridiagonal Gaussian Elimination:

2.0.1 Let's setup the values we'll use. Random value in a determined form!

In [9]:
```
from random import randrange as rand

N = 10
tridiag = [[rand(1,10) if i-2<j<i+2 else 0 for j in range(N)] for i in range(N)]
d = [rand(1,10) for i in range(N)]

tridiag, d
```

Out[9]:
```
([[7, 7, 0, 0, 0, 0, 0, 0, 0, 0],
  [9, 6, 9, 0, 0, 0, 0, 0, 0, 0],
  [0, 3, 5, 2, 0, 0, 0, 0, 0, 0],
  [0, 0, 9, 2, 1, 0, 0, 0, 0, 0],
  [0, 0, 0, 3, 9, 6, 0, 0, 0, 0],
  [0, 0, 0, 0, 8, 4, 7, 0, 0, 0],
  [0, 0, 0, 0, 0, 3, 7, 9, 0, 0],
  [0, 0, 0, 0, 0, 0, 1, 6, 4, 0],
  [0, 0, 0, 0, 0, 0, 0, 6, 3, 7],
  [0, 0, 0, 0, 0, 0, 0, 0, 9, 7]],
 [3, 6, 9, 7, 2, 3, 4, 4, 6, 3])
```

2.0.2 On second thought, let's pull those diagonals out...

In [10]:
```
a = [tridiag[i][i-1] if i>0 else 0 for i in range(N)]
b = [tridiag[i][i] for i in range(N)]
c = [tridiag[i][i+1] if i<N-1 else 0 for i in range(N)]
a, b, c
```

Out[10]:
```
([0, 9, 3, 9, 3, 8, 3, 1, 6, 9],
 [7, 6, 5, 2, 9, 4, 7, 6, 3, 7],
 [7, 9, 2, 1, 6, 7, 9, 4, 7, 0])
```

2.1 We now calculate the modified coefficients

In [11]:
```
modif_c = [c[0]/b[0]]
for i in range(1,N):
    modif_c.append(c[i] / (b[i] - a[i]*modif_c[i-1]))
modif_c
```

Out[11]:
```
[1.0,
 -3.0,
 0.14285714285714285,
 1.3999999999999997,
 1.2499999999999998,
```

```
 -1.166666666666667,
 0.8571428571428571,
 0.7777777777777777,
 -4.200000000000001,
 0.0]
```

```
In [12]: #modif_d = [d[i]/b[i] if i==1 \
         #            else (d[i] - a[i]*modif_d[i-1])/(b[i] - a[i]*modif_c[i-1]) for i in range(N
         modif_d = [d[0]/b[0]]
         for i in range(1,N):
             modif_d.append((d[i] - a[i]*modif_d[i-1]) / (b[i] - a[i]*modif_c[i-1]))
         modif_d
```

```
Out[12]: [0.42857142857142855,
          -0.7142857142857144,
          0.7959183673469388,
          -0.2285714285714288,
          0.5595238095238095,
          0.24603174603174613,
          0.31065759637188206,
          0.7173721340388006,
          -1.017460317460318,
          0.2713647959183674]
```

2.1.1 Those are probably none too fun looking. Let's keep going.

2.1.2 It's time for back substitution!

```
In [13]: #Got to have an array we are actually solving into. Let's initialize that.
         x = np.zeros(N)
         x[-1] = modif_d[-1]
         for i in range(N-2, -1, -1):
             x[i] = modif_d[i] - modif_c[i]*x[i+1]
```

2.2 We're going to want to check this against what numpy computes

```
In [14]: np_solution = np.linalg.solve(tridiag, d)
         diff = [np_solution[i] - x[i] for i in range(N)]
```

2.2.1 Moment of truth...

```
In [15]: print('{:20}{:20}{:20}'.format('My Solution', 'Numpy Solution', 'Diff'))
         for i in range(N):
             print('{: 2.5f}{: 20.5f}{: 20.5f}'.format(x[i],
                                                       float(np_solution[i]),
                                                       diff[i]))
```

```
My Solution         Numpy Solution      Diff
-1.68893            -1.68893             0.00000
```

```
 2.11750             2.11750            -0.00000
 0.94393             0.94393             0.00000
-1.03607            -1.03607             0.00000
 0.57678             0.57678            -0.00000
-0.01381            -0.01381             0.00000
-0.22272            -0.22272            -0.00000
 0.62227             0.62227            -0.00000
 0.12227             0.12227             0.00000
 0.27136             0.27136            -0.00000
```

2.3 Woo!

3 Now just a PLU Decomposition:

Note. This doesn't work.

In [16]:
```
M = np.random.randint(-30, 30, (9,9))
M[5,5] = 0
M
```

Out[16]:
```
array([[  2, -12, -21,   7, -22,  -5, -21, -26, -10],
       [-27, -18, -26, -17,  12,   2,  25,  26,  15],
       [ 22, -10,  16,  19,  -1,  -1,  26,  10,   5],
       [ -4,  -5,   3,  12, -21,  -8, -26,  -8,  25],
       [ 18,  24,  15,  18,   3, -17,   0, -22, -29],
       [-10,   0,  -9,  -7,  14,   0,   2, -13,  -4],
       [ 18, -29, -18,   4,   6, -11, -19, -15,  21],
       [ 14,  23,  15, -29,  22,   7, -30,  15,  13],
       [  8,   9, -22,  -4, -29,   1, -27,  -4,  -4]])
```

In [17]:
```
def pivot(i, M):
    N = M.shape[0]
    row_maxes = np.zeros(N)
    for j in range(i, N):
        row_maxes[j] = abs(M[j,j])
        for k in range(i+1, N):
            if row_maxes[j] < abs(M[j,k]):
                d[j] = abs(M[j,k])
    print(row_maxes)


    pivotrow = i
    pivot = abs(M[i,i])/row_maxes[i]
    for j in range(i+1, N):
        if pivot < abs(M[j,i])/row_maxes[j]:
            pivot = abs(M[j,i])/row_maxes[j]
            pivotrow = j
```

```
        return pivotrow
    pivot(0, M)
```

```
[  2.  18.  16.  12.   3.   0.  19.  15.   4.]
```

```
/usr/lib/python3.6/site-packages/ipykernel/__main__.py:15: RuntimeWarning: divide by zero encoun
/usr/lib/python3.6/site-packages/ipykernel/__main__.py:16: RuntimeWarning: divide by zero encoun
```

Out[17]: 5

```
In [18]: #@param: M, a square matrix
         #@returns: Three matrices
         #          P: A permutation matrix - keeps track of pivoting swaps we've done
         #          L: A lower triangular matrix
         #          U: An upper triangular matrix
         def PLU(M):
             M = np.array(M)
             P, L, U = np.zeros(M.shape), np.zeros(M.shape), np.zeros(M.shape)
             #Unfinished.
```

In [ ]: