Geodesics for Dionic Particles in Dionic Spacetimes

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Abstract.

We calculated the geodesics for dionic particles in a dionic kerrspacetime.

1. Kerr Equations of Motion

Start with the Kerr equations of motion

$$\begin{aligned}
\left(\frac{dr}{d\lambda}\right)^2 &= (\mathcal{E}(r^2+a^2)-a\mathcal{L})^2 - \Delta(r^2+(a\mathcal{E}-\mathcal{L})^2+Q) \\
&= (1-\mathcal{E}^2)(r_1-r)(r_2-r)(r_3-r)(r-r_4) \\
&= R(r),
\end{aligned} \tag{1}$$

$$\begin{aligned}
\left(\frac{dz}{d\lambda}\right)^2 &= Q - z^2(a^2(1-\mathcal{E}^2)(1-z^2)+\mathcal{L}^2+Q) \\
&= (z^2-z_1^2)(a^2(1-\mathcal{E}^2)z^2-z_2^2) \\
&= Z(z),
\end{aligned} \tag{2}$$

$$\frac{dt}{d\lambda} = \frac{(r^2+a^2)}{\Delta}(\mathcal{E}(r^2+a^2)-a\mathcal{L}) - a^2\mathcal{E}(1-z^2)+a\mathcal{L}, \quad \text{and} \tag{3}$$

$$\frac{d\phi}{d\lambda} = \frac{a}{\Delta}(\mathcal{E}(r^2+a^2)-a\mathcal{L}) + \frac{\mathcal{L}}{1-z^2} - a\mathcal{E}, \tag{4}$$

Now we know everywhere a Δ shows up we don't care as this will be substituted everywhere for the modified form with an electric and magnetic charge say $\tilde{\Delta}$. ow for Kerr our conserved quantities are given by

$$\mathcal{E} := -u^\nu g_{\mu\nu}\left(\frac{\partial}{\partial t}\right)^\nu = -u^\nu g_{\mu\nu}(\Delta)\left(\frac{\partial}{\partial t}\right)^\nu \tag{5}$$

$$\mathcal{L} := u^\nu g_{\mu\nu}\left(\frac{\partial}{\partial \phi}\right)^\nu = u^\nu g_{\mu\nu}(\Delta)\left(\frac{\partial}{\partial \phi}\right)^\nu \tag{6}$$

$$Q := u^\mu u^\nu \mathcal{K}_{\mu\nu} - (\mathcal{L}-a\mathcal{E})^2 = u^\mu u^\nu \mathcal{K}_{\mu\nu}(\Delta) - (\mathcal{L}-a\mathcal{E})^2 \tag{7}$$

$$-1 := u^\mu u^\nu g_{\mu\nu} = u^\mu u^\nu g_{\mu\nu}(\Delta) \tag{8}$$

To make things easier to deal with lets work instead with the always-positive carter constant

$$C := Q + (\mathcal{L}-a\mathcal{E})^2 \tag{9}$$

and rewrite the kerr equations of motion in terms of C

2. Dionic Equations of Motion

Now the conserved quantities in the dionic case are given by

$$\mathcal{E}_D := -u^\nu g_{\mu\nu}(\tilde{\Delta}) \left(\frac{\partial}{\partial t}\right)^\nu - \mathcal{P}_t(r, z) \tag{10}$$

$$\mathcal{L}_D := u^\nu g_{\mu\nu}(\tilde{\Delta}) \left(\frac{\partial}{\partial \phi}\right)^\nu - \mathcal{P}_\phi(r, z) \tag{11}$$

$$C_D := u^\mu u^\nu \mathcal{K}_{\mu\nu}(\tilde{\Delta}) \tag{12}$$

$$-1 := u^\mu u^\nu g_{\mu\nu}(\tilde{\Delta}) \tag{13}$$

Hence by taking the equations of motion for Kerr and making the transformations

$$\Delta \to \tilde{\Delta} \tag{14}$$

$$\mathcal{E} \to \mathcal{E}_D - \mathcal{P}_t(r, z) \tag{15}$$

$$\mathcal{L} \to \mathcal{L}_D + \mathcal{P}_\phi(r, z) \tag{16}$$

$$C \to C_D : \tag{17}$$

$$\tag{18}$$

Then what we are left with should be a fully consistent set of equations for the Dionic motion as all the algebra of kerr carries through, ive attached the notebook with the first form of the solutions but preface it with that I still need to check it again. But doing this substitution im currently funding that all of the equations decouple nice as in the regular kerr case.