A model of national-level food system security

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Abstract. Global food security is threatened by various endogeneous and exogeneous, biotic and abiotic factors, including a rising human population, higher population densities, price volatility and climate change. Perturbations to the global food system have a direct effect on the security and resilience food systems. These effects are felt, in different ways, by both producers, processors and consumers. Various mathematical and computational models exist to understand food systems' responses to shocks and stresses, but most models are tailored to making predictions of specific food systems and contexts. Here, we present and analyse a stylised mathematical model of a national-level food system that incorporates dynamic interactions between domestic production of a food commodity, international trade, domestic demand and consumption, and food commodity price. The model exhibits two dominant modes of behaviour, unsustainable and sustainable domestic production, the stabilty of which depends on a balance between the strength of internationa trade and local production costs. As an example, we fit our dynamic systems model to data from the UK pig industry using Bayesian estimation.

Contents

1 Introduction

Food security is defined as "when all people at all times have physical and economic access to sufficient, safe and nutritous food to meet their dietary needs and preferences for an active and healthy life" (FAO. 1996). The realisation of food security depends on the three pillars of access, utilisation and availability (Maxwell 1996; Barrett 2010), and therefore is an outcome of coupled agricultural, ecological and sociological systems (Hammond and Dubé 2012; Ericksen 2008; Ingram 2011). In recent years, the resilience of food security has become a priority area of research (e.g. Nyström et al. 2019; Tendall et al. 2015; Béné et al. 2016; Seekell et al. 2017) as biotic and abiotic, endogeneous and exogeneous demands on food systems grow, including the deleterious effects food systems have on their own environments (Springmann et al. 2018; Strzepek and Boehlert 2010). The challenge of food security is for food systems to expand their production capacities while both remaining resilient to unpredictable perturbations and limiting their effects on the environment (Ericksen et al. 2010).

Food system research is inherently transdisciplinary (Drimie and McLachlan 2013; Hammond and Dubé 2012), of which one strand is computational and mathematical modelling. The utility of quantitative modelling is the ability to build and perturb realistic 'systems models' of food systems to project important outcomes, such as food production levels, farmer profitability, envirnomental degradation, food waste, and consumer behaviour (e.g. Springmann et al. 2018; Marchand et al. 2016; Sampedro et al. 2020; Suweis et al. 2015; Scalco et al. 2019; Allen and Prosperi 2016). The difficulty in modelling food systems is their complexity, frequently resulting in large models with tens to hundreds of parameters and variables, which are challenging to analyse and even more challenging to statistically estimate from noisy real-world data. In contrast, a handful of authors have used relatively

simple, theoretical models that are more amenable to formal analysis, and have fewer pa-
rameters to estimate from data sources, to understand the activities of food systems. For
example, Suweis et al. (2015) link population dynamics to food availability and international
trade using a generalised logistic model. Tu, Suweis, and D'Odorico (2019) recently reported
that the global food system is approaching a critical point deliminating collapse into an
unsustainable regime by condensing the multi-dimensional global food trade network into a
bistable one-dimensional system (using the framework of Gao, Barzel, and Barabási 2016).
Simple models of coupled ecological, economic and agricultural processes have also been in-
vestigated. For instance, Ngonghala et al. (2017) explain the emergence of poverty traps,
which have direct effects on individual's access to food, by coupling differential equations of
human poverty, human disease, and economic growth.

Simplified models of complex systems elicit causal explanations and generate hypotheses
of how systems work (Smaldino 2017; Smaldino 2019). Such *stylised* models are staples
of scientific disciplines such as ecology (May 1973), evolutionary biology (Boyd et al. 2003),
epidemiology (Kermack and McKendrick 1927), economics (Nerlove 1958), and physics (Stro-
gatz 1994). Food systems research, however, lacks such foundational models. A potential
candidate is research on commodity production cycles that has developed models to couple
agricultural production, supply chains, consumer demand, price and human decision-making,
such as Meadows (1971). These approaches have inspired systems dynamics modelling of
general commodity and business cycles (Sterman 2000). However, applications of systems
dynamics models of commodity cycles are frequently high dimensional, encoding multiple
modes of behaviour that are not easily amenable to standard mathematical analysis. Gain-
ing greater theoretical insight into the dynamics of food systems and food security would be
aided by simpler models of coupled agri-food systems.

In this paper, we develop and analyse a stylised model of a food system inspired by the
systems dynamics modelling of Meadows (1971) and Sterman 2000, but simple enough to
offer general theoretical results. To limit our scope, we focus on modelling a national-level
food system, where the effects of international trade on domestic production is examined.
Like the stylised models used to understand the causal processes in evolution, epidemics, and
ecological interactions, our approach necessarily ignores many important features of real food
systems. Nonetheless, the simplicity of our model allows us to elucidate the precise conditions
under which different stable modes of behaviour important to food security emerge in our
system. As an example application, we apply our model to national-level data from the UK
pig industry using Bayesian estimation.

2 Theoretical model

Our food system model is composed as a system of coupled ordinary differential equations,
with the state variables of capital, inventory, consumer demand, and price. The general
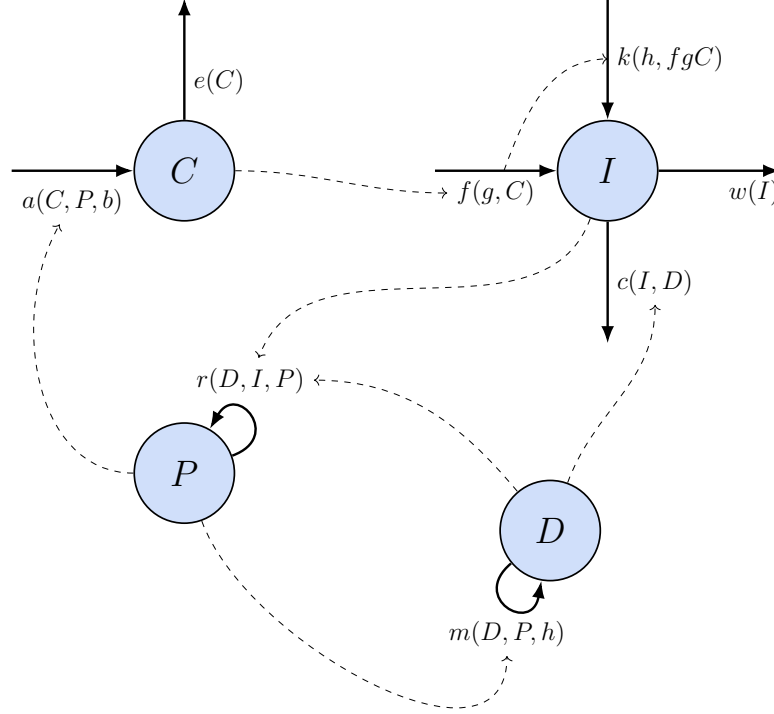
Figure 1: *The general structure of the theoretical complex food system model. Blue circles denote the rates of change of the four state variables (capital, inventory, demand and price). Solid arrows indicate the different flows into and out of each state variable as generic functions of the model state variables and parameters. Dashed arrows show dependencies between different state variables and flows.*

structure of our model is presented in Figure 1, and Table 1 provides the exact variable parameter definitions. Capital represents a raw component used to gauge the viability of the domestic industry, which could represent, for instance, the number of animals in the breeding herd for meat industries (e.g. Meadows 1971), the number of paddy fields in rice supply chains (e.g. Chung 2018), or infrastructure or equipment in neoclassical economic growth models (Ngonghala et al. 2017). Inventory is the stock of processed food commodity being investigated. Customer demand represents the amount of inventory demanded per time unit by the population of consumers. The commodity price represents the price received by producers per unit of commodity produced, and is taken to be directly proportional to the price presented to consumers (i.e. we do not account for retail mark-ups on price). While many of the mechanisms in supply chain functioning may be represented as a disrecte-time system, we assume the aggregate behaviour of the national-level food system is adequately approximated in continuous time by a system of differential equations.

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Capital changes according to the equation:

$$\frac{dC}{dt} = aC\left(\frac{P}{b} - 1\right) - eC \quad , \quad C(0) = C_0 \tag{1}$$

with initial condition at $t = 0$, C_0. The parameter a is the rate of capital change (increase or decrease) depending on the price to production cost (b) ratio. Captial depreciates at a per-capital rate e, which can be interpreted as the inverse of the average life-time of capital (see supplementary materials for derivation).

Inventory changes according to:

$$\frac{dI}{dt} = fgC - wI - \frac{I}{sD + I}D + k(h - fgC) \quad , \quad I(0) = I_0 \tag{2}$$

The first term represents the amount of inventory generated by domestic capital per time unit (i.e. domestic production), where f is a production rate, and g is a conversion factor representing the amount of inventory units produced per unit of capital. Inventory is wasted at rate w. The third term denotes the rate of inventory consumption by consumers, which is a non-linear Holling type-II/Michaelis-Menten function asymptoting at I for $D >> I$. The function $I/(sD + I)$ can be interpreted as the proportion of the demand that can be satisfied with the current inventory level. The parameter s is the 'reference coverage': the number of time units-worth of inventory processors desire to have in-stock. Perishable food commodities (e.g. meat) will have a lower reference coverage, whereas less perishable items (e.g. rice, flour) can be stored for longer periods and, therefore, processors can control stock levels by increasing s, lowering the proportion of demanded units satisfied.

The final term in equation 2 represents international trade, and its formulation can communicate different dynamics between domestic producers, processors and retailers. We retain simplicity by assuming that trade is proportional to the difference between reference demand (h) and current domestic production (fgC). When $h > fgC$, inventory is imported, and when $h < fgC$, inventory is exported. Because actual demand is a function of both h and price, processors adapt their trade levels to the demand expected if all else were equal (e.g. the current commodity is just as attractive as competing substitute items). In some industries, such as the UK pig industry, cheaper international imports lower the domestic commodity price (AHDB 2015), and thus processors can import more items when demand drops below the reference demand level and domestic production is insufficient. The parameter k controls the proportion of the difference between the reference demand and current domestic production that can be imported/exported. This is a placeholder for more complex mechanisms that could influence the volumes of trade (e.g. see Tu, Suweis, and D'Odorico 2019), such as the effects of trade tariffs, or the costs of production of commodities in trade partner countries.

The instantaneous rate of change in demand is modelled as a simple function of reference

Symbol	Definition	Units
Variables		
C	Capital	$[C]$
I	Inventory	$[I]$
D	Demand	$[It^{-1}]$
P	Price	$[PI^{-1}]$
t	Time	$[t]$
Parameters		
a	Capital growth rate	$[t^{-1}]$
b	Cost of capital production	$[PI-1]$
e	Capital depreciation rate	$[t^{-1}]$
f	Capital production rate	$[t^{-1}]$
g	Capital conversion factor	$[IC^{-1}]$
w	Inventory waste rate	$[t^{-1}]$
s	Reference coverage	$[t]$
k	Maximum trade proportion	$[-]$
h	Reference demand	$[It^{-1}]$
m	Demand response rate	$[t^{-1}]$
q	Reference price	$[PI^{-1}]$
r	Price growth rate	$[t^{-1}]$

Table 1: *Symbols, definitions and their units for the complex food system model.*

demand and the commodity price to reference price ratio:

$$\frac{dD}{dt} = m\left(h\frac{q}{p} - D\right) \quad , \quad D(0) = D_0 \tag{3}$$

The parameter m controls the time-responsiveness of demand. The reference price q is typically interpreted as the price of substitute items (Sterman 2000) or could represent customer's willingness to pay. In any case, when the current price exceeds the reference price, demand falls, and vice versa.

Many models exist for describing the price of commodities (e.g. see Legrand 2019 or De Goede, Gremmen, and Blom-Zandstra 2013 for some examples), and we adopt a relatively simple formulation here that has the rate of change of price depend only on the coverage:

$$\frac{dP}{dt} = rP\left(\frac{sD}{I} - 1\right) \quad , \quad P(0) = P_0 \tag{4}$$

The coverage is a dimensionless quantity representing the amount of inventory needed to sustain current demand for s time periods divided by the current inventory level. At rate given by r, the price increases when the coverage exceeds one, $sD/I > 1$, and decreases when coverage falls below 1.

To make our model more generalisable, we non-dimensionalise the system of equations above (see supplmentary materials for non-dimensionalisation) using the dimensionless quan-

Symbol	Definition	Description
Variables		
v	$\frac{C}{C_0}$	Rescaled capital
x	$\frac{I}{hs}$	Rescaled inventory
y	$\frac{D}{h}$	Rescaled demand
z	$\frac{P}{q}$	Rescaled price
τ	$\frac{t}{1/a}$	Rescaled time
Parameters		
α	q/b	Reference price compared to cost of production
β	e/a	Capital depreciation rate compared to its growth rate
δ	$fgC_0/(ahs)$	Initial capital production compared to expected demand per capital growth rate
ω	w/a	Inventory waste rate compared to capital growth rate
γ	$1/(as)$	Inverse of captial growth rate over reference coverage
κ	k	Maximum import proportion of reference demand
μ	m/a	Demand change rate compared to capital growth rate
ρ	r/a	Price change rate compared to capital growth rate

Table 2: *Symbols and definitions for the dimensionless complex food system model.*

tities in Table 2, which reduces the number of parameters from 12 to 8. The non-dimensionalised system of equations is:

$$\frac{dv}{d\tau} = v\Big(\alpha z - 1\Big) - \beta v \tag{5}$$

$$\frac{dx}{d\tau} = \delta v - \omega x - \gamma\frac{xy}{y + x} + \kappa(\gamma - \delta v) \tag{6}$$

$$\frac{dy}{d\tau} = \mu\Big(z^{-1} - y\Big) \tag{7}$$

$$\frac{dz}{d\tau} = \rho z\Big(\frac{y}{x} - 1\Big) \tag{8}$$

where $\{v, x, y, z\}$ now represent the dimensionless state variables, τ is rescaled time, and the dimensionless parameter groups are denoted by Greek letters.

3 Model analysis

While the mechanisms represented by the model are simple, it still comprises a four dimensional system of non-linear equations, of which there are no explict solutions. Nontheless, its dynamics can be summarised by investigating its stable modes of behaviour. To investigate stability, we conduct linear stability analyses (Strogatz 1994). Linear stability analysis is based on a Taylor series expansion in multiple variables around the fixed points, where asymptotic (i.e. $t \to \infty$) stability to small perturbations can be inferred when the real part of the eigenvalues of the system's matrix of partial derivatives (the Jacobian matrix representing the linearisation around the fixed points) evaluated at each equilibrium are negative. Notably, the inverse of the leading eigenvalue of the Jacobian matrix determines the 'characteristic return time' of the system, with more resilient systems returning more quickly to their equilibria following a disturbance (Pimm 1984).

3.1 Without international trade

When international trade is not present (i.e. $\kappa = 0$), there is one stable fixed point of the system given by the state variable values $\left\{ \frac{\alpha(2\omega+\gamma)}{2\delta(1+\beta)}, \frac{\alpha}{1+\beta}, \frac{\alpha}{1+\beta}, \frac{1+\beta}{\alpha} \right\}$. Another fixed point is where all state values are 0 (i.e. no industry). Conducting a linear stability analysis around the the latter fixed point, the eigenvalues of the Jacobian matrix at this equilibria are $\{(-1-\beta), -\omega, -\mu, -\rho)\}$. Because all parameters are defined to be positive (except κ, which is 0 here), there are no conditions where the 'no industry' equilibria will be stable when international trade is absent. In other words, the domestic industry is always viable.

3.2 With international trade

When $\kappa > 0$, international trade is possible, opening the possibility of competition between domestic and international products, or of an export market for domestic production. An unstable equilibria still exists where all state variables are 0. However, in addition, there is an unsustainable domestic production equilibria, where the food system is viable overall through complete reliance on international imports.

The unsustainable domestic production equilibria is given by the set of fixed points $\left\{ 0, \frac{\kappa\gamma}{\omega+\frac{\gamma}{2}}, \frac{\kappa\gamma}{\omega+\frac{\gamma}{2}}, \frac{\omega+\frac{\gamma}{2}}{\kappa\gamma} \right\}$. The Jacobian matrix ($\boldsymbol{J}$) at this equilibria evalutes to:

$$\boldsymbol{J}\Big|_{\left\{0, \frac{\kappa\gamma}{\omega+\frac{\gamma}{2}}, \frac{\kappa\gamma}{\omega+\frac{\gamma}{2}}, \frac{\omega+\frac{\gamma}{2}}{\kappa\gamma}\right\}} = \begin{pmatrix} \frac{\alpha(\omega+\frac{\gamma}{2})}{\kappa\gamma} - 1 - \beta & 0 & 0 & 0 \\ \delta(1-\kappa) & -\omega - \frac{\gamma}{4} & -\frac{\gamma}{4} & 0 \\ 0 & 0 & -\mu & -\mu(\frac{\kappa\gamma}{\omega+\frac{\gamma}{2}})^2 \\ 0 & -\rho(\frac{\omega+\frac{\gamma}{2}}{\kappa\gamma})^2 & \rho(\frac{\omega+\frac{\gamma}{2}}{\kappa\gamma})^2 & 0 \end{pmatrix} \tag{9}$$

and its eigenvalues ($\boldsymbol{\lambda}$) are the roots of the fourth-degree characteristic polynomial:

$$\left(\frac{\alpha(\omega + \frac{\gamma}{2})}{\kappa\gamma} - 1 - \beta - \lambda\right)\left[-\lambda^3 + \left(-\omega - \frac{\gamma}{4} - \mu\right)\lambda^2 + \left(-\mu(\omega + \rho) - \frac{\gamma}{4}\right)\lambda - \frac{\gamma}{2}\mu\rho\right] = 0 \quad (10)$$

The first eigenvalue is easy to determine:

$$\lambda_1 = \frac{\alpha(\omega + \frac{\gamma}{2})}{\kappa\gamma} - 1 - \beta \quad (11)$$

and by using the Routh-Hurwitz conditions, we can assess the sign of the remaning eigenvalues' real parts (Otto and Day 2011). As it turns out, all other eigenvalues will always be negative (see the supplementary materials), and the non-sustainable domestic production mode will be stable if:

$$1 < \frac{\kappa\gamma(1 + \beta)}{\alpha(\omega + \frac{\gamma}{2})} \quad (12)$$

The previous term is not the easiest to discern, but by perturbing the individual parameters it depends on (see Figure S1), the most influential parameters are α, κ and β. Figure 1a illustrates the bifurcation diagram of the system, where when the previous term is greater than 1, domestic production of capital is sustainable. Figures 1a-c demonstrate the two stable modes of behaviour in (κ, α) space for differing values of β.

4 Application: the UK pig industry

4.1 Model estimation

5 Discussion

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