Assignment 1 Samples

1. A sample question and solution for Question 3 on page 59.

$(n^3 + n^2 - 2n + 1)^5$
$(n^3 + n^2 - 2n + 1)^5 \approx (n^3)^5 = n^{15} \in \Theta(n^{15})$

Explanation: In determining the efficiency class, $(n^3 + n^2 - 2n + 1)^5$ can be approximated by $(n^3)^5$, and $(n^3)^5$ has efficiency class n^{15}

Proof:

$$\lim_{n\to\infty}(n^3+n^2-2n+1)^5/n^{15} = \lim_{n\to\infty}(n^3+n^2-2n+1)^5/(n^3)^5 = \lim_{n\to\infty}((n^3+n^2-2n+1)/(n^3))^5 =$$

$$\lim_{n\to\infty}(n^3/n^3 + n^2/n^3 - 2n/n^3 + 1/n^3)^5 = \lim_{n\to\infty}(1 + 1/n - 2/n^2 + 1/n^3)^5 = 1$$

Explanation: To prove that the efficiency class of $(n^3 + n^2 - 2n + 1)^5$ is n^{15}, we calculate $\lim_{n\to\infty}(n^3 + n^2 - 2n + 1)^5/n^{15}$. The calculation result is a constant. This indicates $(n^3 + n^2 - 2n + 1)^5$ and n^{15} have the same asymptotic order of growth, and thus n^{15} can be the efficiency class of $(n^3 + n^2 - 2n + 1)^5$.

2. A sample question and solution for Question 5 on page 60.
$(n^3 + n^2 - 2n + 1)^5$, $(n-2)!$, $5\lg(n + 100)^{10}$

Determine efficiency class for each term:
$(n^3 + n^2 - 2n + 1)^5 \in \Theta(n^{15})$, $(n-2)! \in \Theta(n!)$, $5\lg(n + 100)^{10} \in \Theta(\log n)$,

Explanation: We have discussed the first term. When n approaches infinity, $(n-2)! \approx n!$. $5\lg(n+100)^{10} = 50\lg(n+100)$. When n approaches infinity, $50\lg(n+100) \approx 50\lg n \in \Theta(\log n)$.

Order the terms by their efficiency classes:

$5\lg(n + 100)^{10}$, $(n^3 + n^2 - 2n + 1)^5$, $(n-2)!$

Explanation: Compare the asymptotic orders of growth of n^{15}, $\log n$, and $n!$. If necessary, use the method of limits, L'Hopital's rule, and Stirling's formula for comparing the efficiency classes.

3. Sample questions and solutions for Question 1 on page 67.

1. $1 + 3 + 5 + 7 + ... + 999 = \sum_{i=1}^{500}(2i - 1) = \sum ... + \sum ... = ...$

   This is an incomplete solution. Please complete it.

2. $\sum_{i=3}^{n+1} 1 = (n + 1) - 3 + 1 = n - 1$

   Use the first summation formula on page 476 in our text. In this sum, upper limit u is $n + 1$ and lower limit l is 3. $u - l + 1$ is $n + 1 - 3 + 1 = n - 1$.

4. A sample question and solution for Question 2 on page 67.

$\sum_{i=0}^{n-1}(i+1)^2 = \sum_{i=0}^{n-1}(i^2+2i+1) = \sum_{i=0}^{n-1} i^2+\sum_{i=0}^{n-1} 2i+\sum_{i=0}^{n-1} 1 \in \Theta(n^3)+\Theta(n^2)+\Theta(n) = \Theta(n^3)$

Explanation: Use the summation formulas 1, 2, and 3 to simplify the three sums. Determine the efficiency class for each of them. Then choose the efficiency class of the sum having greatest order of growth as the efficiency class of the original sum.

5. A sample question and solution for Question 3 on page 67:

Find the numbers of divisions, multiplications, additions/subtractions that are required to compute the formula:

$$\frac{\sum_{i=1}^{n}(x_i-2)^3}{n}$$

$D(n) = 1$, $M(n) = 2n$, $A(n) + S(n) = (n-1) + n = 2n - 1$.

Explanation: Calculating one cube needs 2 multiplications. Calculating the sum of n items needs $n-1$ additions. You may use a small n (like $n = 3$) to verify the result.