RE-ENTRY SYSTEMS AE4870B

EXAMINATION

August 26, 2011

Delft University of Technology
Faculty of Aerospace Engineering
Space Engineering Division

This exam contains 4 questions.

PLEASE NOTE

Always write down the correct units for each computed parameter value. Be mindful for any required conversion before making any computations. **Always** write down the derivations of your answers.

Question 1 (25 points)

A common definition of aerodynamic forces is that of the drag, side and lift force, expressed in the aerodynamic A-frame (positive in negative axis direction). When the forces are not acting in the centre of mass, a moment due to these forces is the result. To calculate this additional moment the aerodynamic force in the body B-frame is required. In addition, the aerodynamic force should also be available in the inertial I-frame to propagate position and velocity with the equations of motion. You are required to derive the following.

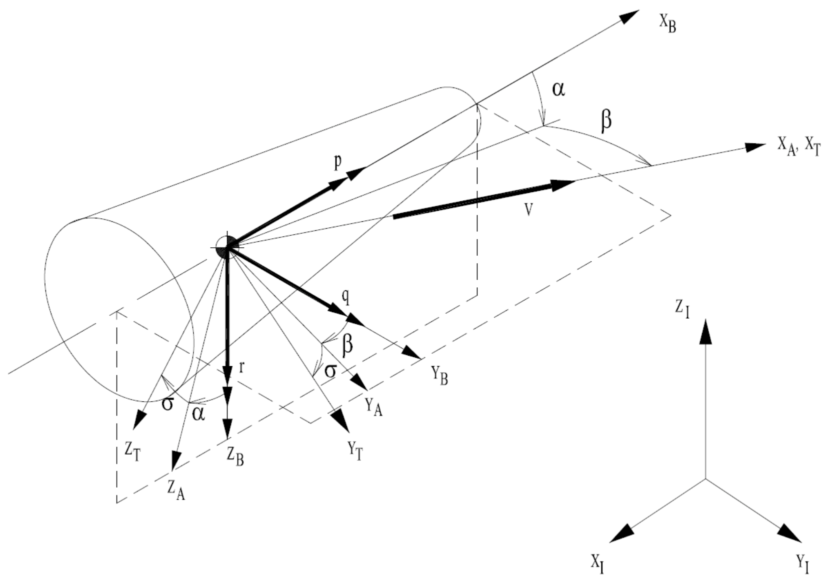


Figure 1. Aerodynamic-angle definition, including T-, B- and A-frames

(a) **7 points** Derive the transformation matrix $\mathbf{C_{BA}}$ (from aerodynamic to body frame, see also Fig. 1). Before calculating the final transformation matrix $\mathbf{C_{BA}}$, specify the transformation matrices for each of the unit-axis rotations.

(b) **5 points** Derive the expressions of the 3 components of the aerodynamic force in the body frame, $\mathbf{F_{A,B}}$.

(c) **6 points** Calculate the resulting moment due to $\mathbf{F_{A,B}} = (X, Y, Z)^T$, see also Fig. 2. Give the individual expressions for the x, y and z components.

(d) **7 points** Set up the sequence of rotations to go from body to inertial frame, $\mathbf{C_{I,B}}$, see also Figs. 1 and 3. Pay attention to the related angle, sign of the rotation, and the axis. Note: only the right sequence of unit-axis transformations should be given (not the final transformation matrix).

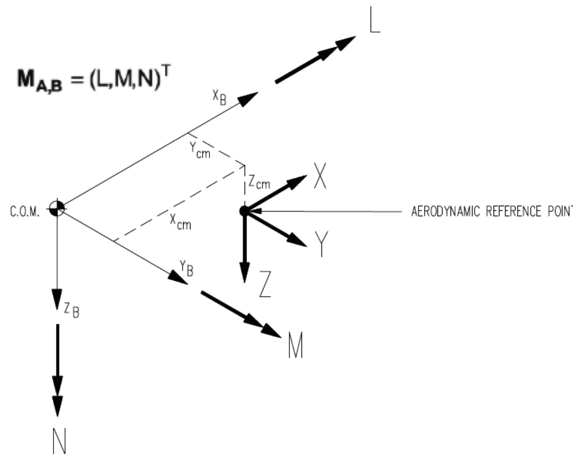
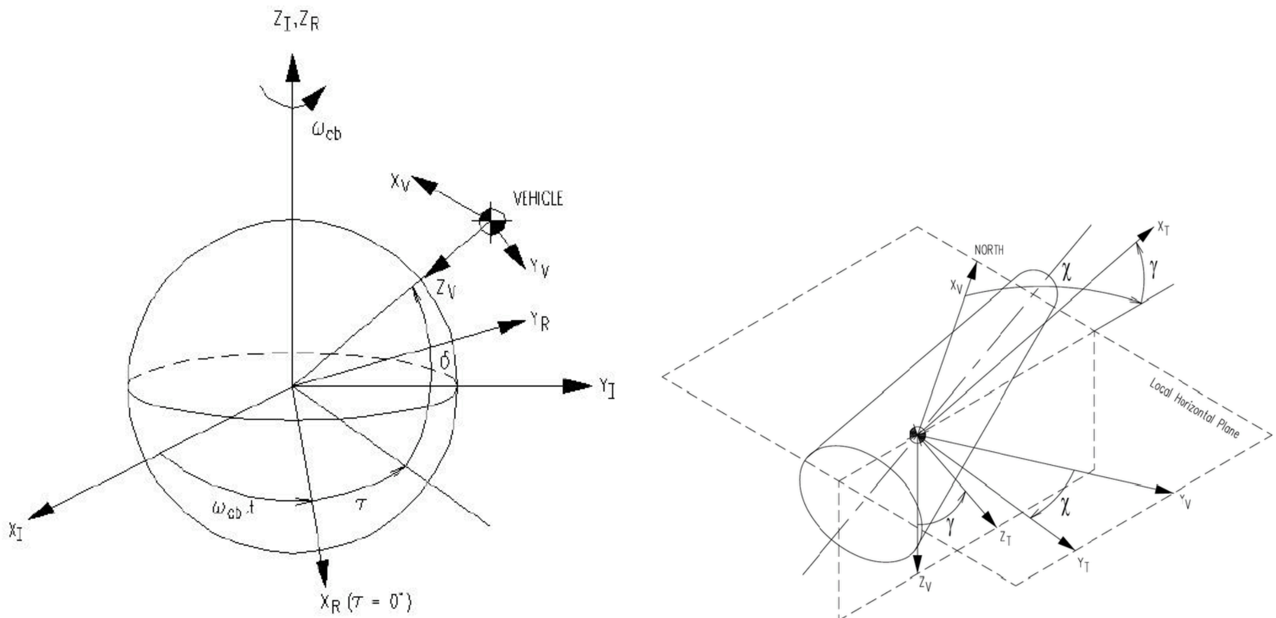


Figure 2. Aerodynamic moments due to forces acting in a point different than the c.o.m.



(a) *I*-, *R*- and *V*-frames (b) *V* and *T*-frames

Figure 3. Reference frames involved in the transformation between *I* and *V*-frames.

.

Question 2 (20 points)

For the linearization of the re-entry equations of motion, it is common to express the position and velocity in spherical components. The kinematic equations are in that case given by:

$$\dot{R} = V \sin\gamma \tag{1}$$

$$\dot{\tau} = \frac{V \cos\gamma \sin\chi}{R \cos\delta} \tag{2}$$

$$\dot{\delta} = \frac{V}{R} \cos\gamma \cos\chi \tag{3}$$

(a) **15 points** Linearize the above kinematic equations. Note: one can NOT assume that the nominal flight-path angle is small; only perturbations are considered to be small.

(b) **5 points** For a vehicle entering the atmosphere the following eigenvalue λ and eigenvector $\boldsymbol{\mu}$ are calculated: $\lambda = -0.7302 \cdot 10^{-7} \pm 0.7415 \cdot 10^{-2} j$ and $\boldsymbol{\mu} = (0.0129, 0, 1, 0, 0.1678 \cdot 10^{-3}, 0, 0.0226, 0, 0)^T$. The non-zero elements in $\boldsymbol{\mu}$ are related to $\Delta V, \Delta R, \Delta q$ and $\Delta\alpha$.
(i) Identify the eigenmode.
(ii) Calculate the natural frequency ω_n and halving time $T_{1/2}$ of this eigenmode.
(iii) Is this mode stable? Explain why.

Question 3 (30 points)

Consider the motion of an arbitrary re-entry vehicle in the atmosphere of the Earth

(a) **5 points.** Draw a clear sketch of the 2D (in-plane) motion of this vehicle with respect to a spherical, non-rotating Earth with a fixed inertial reference frame. Indicate the state variables velocity, altitude and flight-path angle, as well as the external forces of aerodynamic and gravitational origin. Note to pay attention to the correct orientation of forces and state variables.

(b) **5 points.** Starting with the sketch obtained under a), set up the general equations of motion for a re-entry flight in the directions parallel with and perpendicular to the velocity vector. Also provide the equation for the time rate of change of the altitude.

(c) **12 points.** Assume now that the vehicle is flying a gliding trajectory. Show that the flight range, starting with the equations obtained under b), is given by $\frac{R}{R_e} = -\frac{1}{2}\frac{L}{D}\ln\left(1 - \frac{V_E^2}{V_c^2}\right)$. What is the main assumption with respect to the flight path that you can make to simplify the derivation?

(d) **4 points.** For the gliding re-entry vehicle, the requirement is that it has to cover a range of $R = 6000$ km to reach its landing site. What should be the minimum lift-to-drag ratio (L/D) of the vehicle such that this can be achieved? Assume that the vehicle enters the atmosphere with a velocity equal to 80% of the circular velocity, and that the Earth's equatorial radius is $R_e = 6378$ km.

(e) **4 points.** What can you say about the flight range when the entry velocity V_E approaches the circular velocity V_c? What is the limit value for $V_E = V_c$? What does this physically mean?

Question 4 (25 points)

A re-entry vehicle returns in the atmosphere using a repeating skipping trajectory, entering the atmosphere twice. At first contact with the atmosphere, the entry angle $\gamma_E = 15°$ and the entry velocity $V_E = 8$ km/s. The lift-to-drag ratio of the vehicle is: $L/D = 1$ (constant).

(a) **9 points.** For the skipping trajectory, starting with these equations, show that the relationship between velocity and flight path angle is given by $\frac{V}{V_E} = e^{\frac{\gamma - \gamma_E}{L/D}}$, and the relationship between flight path angle and air density is given by $\cos\gamma - \cos\gamma_E = \frac{g}{2\beta} \frac{1}{\frac{W/S}{C_L}} \rho$, the latter under the assumption of an exponential atmosphere. Indicate which simplifying assumption(s) is/are made in this derivation.

(b) **16 points.** Next, draw a clear sketch of the variation of the velocity as a function of the flight path angle from the moment of first contact with the atmosphere until the second exit. In order to make this sketch, both parameters have to be computed in the following characteristic points of the trajectory:

- lowest point of the first skipping trajectory;
- exit of the atmosphere at the end of the first skipping trajectory;
- highest point of the (ballistic) flight after the first skipping trajectory;
- atmospheric entry point of the second skipping trajectory;
- lowest point of the second skipping trajectory;
- exit of the atmosphere at the end of the second skipping trajectory.

If you have problems to compute these quantities, then at least try to make a qualitative sketch and motivate your ideas about the variation of the velocity as a function of the flight path angle.