RE-ENTRY SYSTEMS AE4870B

EXAMINATION

June 21, 2011

Delft University of Technology
Faculty of Aerospace Engineering
Space Engineering Division

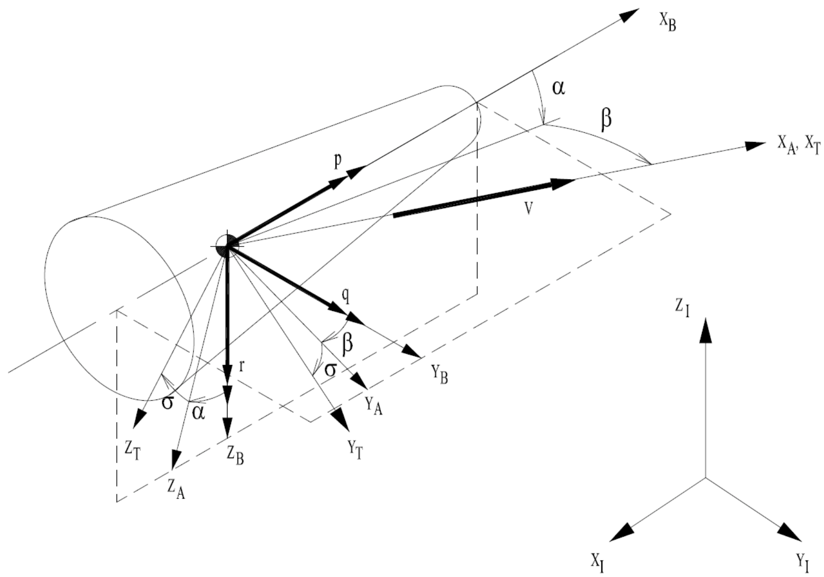
This exam contains 4 questions.

PLEASE NOTE

Always write down the correct units for each computed parameter value. Be mindful for any required conversion before making any computations. **Always** write down the derivations of your answers.

Question 1 (15 points)

In the accompanying figure three reference frames are indicated in relation with a schematic representation of a vehicle. These frames are the body frame (index B), the aerodynamic frame (index A) and the trajectory frame (index T). The relation between these frames can be described by the indicated angles (angle of attack α, angle of sideslip β and bank angle σ). In the figure, all angles are positive.

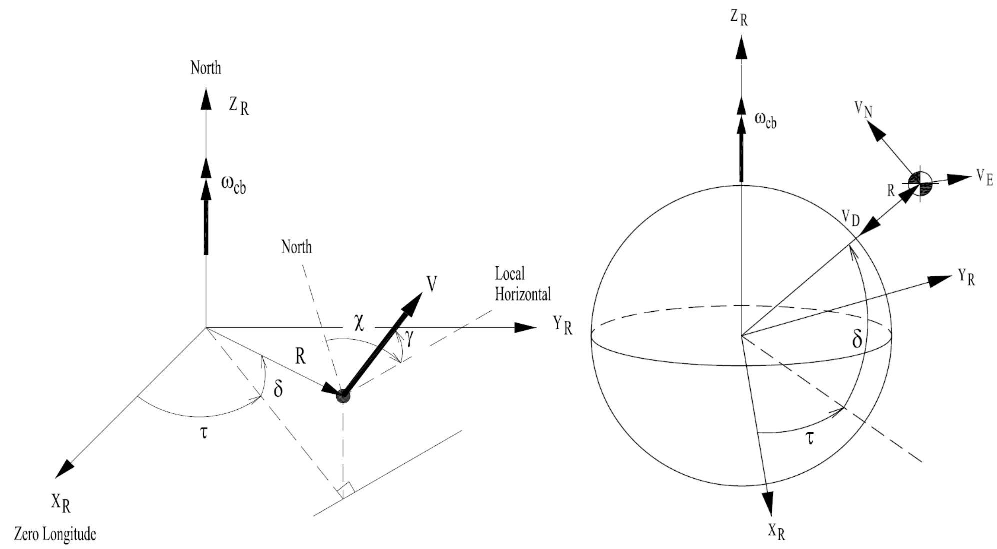


(a) **12 points** Derive the transformation matrix $\mathbf{C_{BT}}$ (from trajectory to body frame). Before composing the final transformation matrix $\mathbf{C_{BT}}$, specify the transformation matrices for each of the unit-axis rotations.

(b) **3 points** Suppose we also need the inverse transformation matrix. Explain (in words) how we can derive this matrix *from* $\mathbf{C_{BT}}$ *found above.* Carefully state the mathematical principle involved.

Question 2 (25 points)

For the analysis of re-entry problems it is common to express the position and velocity in spherical components, whereas for aircraft studies usually Cartesian velocity components are used. In the accompanying figure the two definitions of the state variables are given.



(a) Definition of spherical position and velocity state variables. (b) Definition of spherical position and Cartesian velocity.

(a) **6 points** Derive expressions for the spherical velocity components (velocity modulus V, flight-path angle γ and heading χ) as a function of the north, east and down velocity (V_N, V_E and V_D).

(b) **14 points** A simplified expressions for the velocity derivative is given by

$$\dot{V} = -\frac{D}{m} - g \sin\gamma \tag{1}$$

Linearize this equation, where you can assume that the gravitational acceleration g is constant, i.e., $g = g_0$ and the drag is a function of angle of attack and Mach number. Express the resulting equation in state perturbations (note: the Mach number is *not* a state).

(c) **5 points** For a vehicle entering the atmosphere the following eigenvalue λ and eigenvector $\boldsymbol{\mu}$ are calculated: $\lambda = 0.872310^{-3} \pm 0.3257j$ and $\boldsymbol{\mu} = (0, 0, 0, 0.4007, 0, 0.0291, 0, 0.7225, 1.0)^T$. The non-zero elements in $\boldsymbol{\mu}$ are related to $\Delta p, \Delta r, \Delta\beta$ and $\Delta\sigma$.
(i) Identify the eigenmode.
(ii) Calculate the period P and damping ratio ζ of this eigenmode.
(iii) Is this mode stable? Explain why.

Question 3 (40 points)

Consider the entry and descent of the Huygens probe in the atmosphere of Titan. The nominal entry velocity is $V_E = 5.8$ km/s with a corresponding flight-path angle of $\gamma_E = 60°$ (entry altitude is $h_E = 1200$ km). Huygens will follow a ballistic path until parachute deployment at an altitude of 200 km. Further information about Huygens: diameter $d = 2.7$ m, mass $m = 318$ kg, reference area: $S_{ref} = 5.7$ m^2 and hypersonic drag coefficient $C_D = 1.54$ ($M > 4$).

Titan data: g_0= 1.354 m/s^2 and $R_e = 2575$ km.
Mars data: $g_0 = 3.727$ m/s^2 and $R_e = 3396$ km.

(a) **4 points.** Set up the general equations of 2D (in-plane) atmospheric motion of a vehicle with respect to a spherical, non-rotating Earth with a fixed inertial reference frame. These equations should represent the motion in the directions parallel with and perpendicular to the velocity vector. Also provide the equation for the time rate of change of the altitude.

(b) **8 points.** Given the relation between velocity and flight-path angle (positive when the velocity is below the local horizon), $\frac{V}{V_E} = \exp\left(-\frac{1}{2}\frac{g\rho}{K\beta\sin\gamma_E}\right)$, show that the maximum deceleration that Huygens experiences is $\bar{a}_{\max} = +\frac{\beta\ \sin\gamma_E}{2e}\ V_E^2$, starting from the equations under a). Clearly state the assumptions that you make.

(c) **3 points.** State the definition of the ballistic parameter, K. Given the data above, calculate K for Huygens. How does the ballistic parameter influence the maximum deceleration?

(d) **2 points.** The part of the Titan atmosphere from 140 km upwards can be approximated by an exponential model. The scale height and reference density are given by $H = 1/\beta = 50$ km and $\rho_0 = 0.0665$ kg/m^3. Calculate the maximum deceleration for $h > 140$ km.

(e) **2 points.** If a probe of similar shape and dimensions would enter the Martian atmosphere, what would be the maximum deceleration in that case? Assume an exponential atmosphere with $H = 1/\beta = 8.8$ km and ρ_0 = 0.0168 kg/m^3. Typical Mars entry conditions are $V_E = 6.3$ km/s and $\gamma_E = 20°$.

(f) **7 points.** After the ballistic entry, a parachute is deployed and Huygens continues its descent towards the surface. Close to the surface, the probe-parachute combination is in a so-called stationary descent. 1) Draw a sketch with all related forces for this situation. 2) Set up the differential equation of vertical motion. 3) For a given parachute drag area of $C_D S_{ref}$ of 1.757 m^2, what is the corresponding touch-down velocity? The drag of the vehicle can be neglected compared to the parachute drag. Surface-level atmospheric density ρ_0 = 5.44 kg/m^3.

(g) **2 points.** Suppose we want to fly a similar parachute descent in the Martian atmosphere. Assume again a stationary flight close to the surface. For a given touch-down velocity of 10 m/s, what would be the parachute drag area? How does this compare with the parachute for the Titan mission?

(h) **5 points.** As the computed parachute drag area (sub-question g) is too large to be feasible, it is decided to equip the Martian Huygens with so-called retro rockets to reduce the velocity in the final phase, after releasing the parachute. Set up the differential equation of vertical motion.

(i) **7 points.** The aforementioned retro rockets provide a constant thrust. For the sake of the exercise we assume that the vehicle mass remains constant. Calculate the required thrust to land with a velocity of 10 m/s. The powered descent is initiated at $h = 1400$ m right after parachute release (parachute drag area $C_D S = 41.15$ m^2). A constant gravity and average density may be assumed ($g_0 = 3.71$ m/s^2 and $\rho_0 = 0.016$ kg/m^3). Furthermore, on average during the final descent the drag is 10% of the thrust.

Question 4 (20 points)

At the end of the hypersonic descent, a winged re-entry vehicle enters the so-called Terminal Area. Here, a dedicated guidance system, the so-called Terminal Area Energy Management system takes over to control the flight range to-go. As a first approximation to design such a system we assume that the vehicle is flying in a stationary glide.

(a) **4 points.** Set-up the dynamic equations of motion for a stationary glide. State your assumptions. The flight-path angle is negative when the velocity is below the local horizon.

(b) **5 points.** From the dynamic equations as found under a), derive expressions for i) the (equilibrium) velocity, ii) the flight-path angle, iii) the rate of descent, and iv) the horizontal range.

(c) **7 points.** Sketch the performance curve of the vehicle, and indicate i) velocity, ii) flight-path angle, iii) minimum flight-path angle, iv) minimum rate of descent, and v) the stall speed. Pay attention to axis labels and positive axis directions.

(d) **2 points.** Given the stationary-flight condition, when will the vehicle obtain a maximum range?

(e) **2 points.** Suppose that during the glide, the vehicle would fly a turn with constant bank angle. How would the expressions for equilibrium velocity and flight-path angle change?