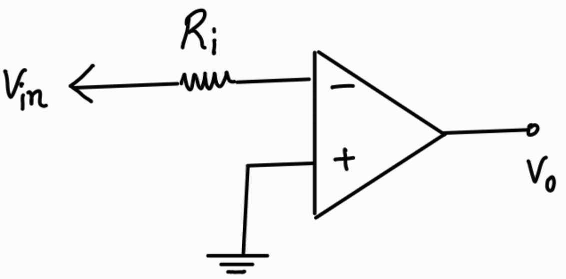
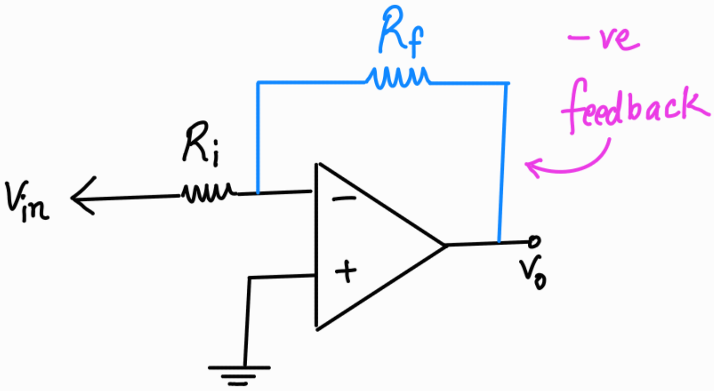
Since ideal op-amps have ∞ gain, we cannot use op-amps directly for amplification.

We have to add a negative feedback loop to manage the gain —

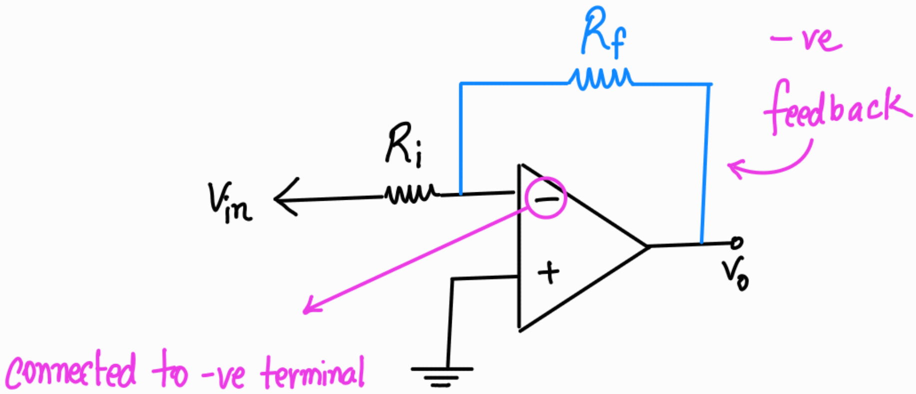


Open loop comparator, gain $= -\infty$



Closed loop amplifier, gain $= -\dfrac{R_f}{R_i}$

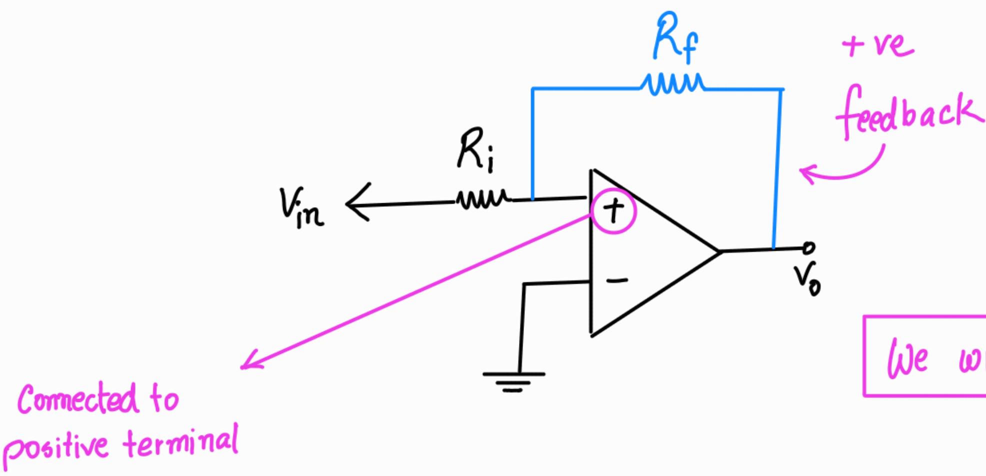
So what happens, when you connect a −ve feedback in an op-amp ckt?



We know, $V_o = A(v^+ - v^-)$

the open loop gain (A) cannot be changed. So to make the output small we minimize the effect of $(v^+ - v^-)$ by increasing v^- using the negative feedback

Note — Positive feedback is also possible —



We will not use this in 251

So when -ve feedback is used, something cool happens in the ckt.

-ve feedback gives you a finite gain by controlling v_d in the formula, $v_o = Av_d$. Now, lets assume $v_o = 10V$ and see how v_d has to be adjusted for changing open-loop gain (A).

$\because v_o = Av_d$, if $v_o = 10V$, we have $10 = Av_d$

Lets assume $A = 10 \longrightarrow V_d = 1V$

for $A = 100 \longrightarrow V_d = 0.1V$

for $A = 1000 \longrightarrow V_d = 0.01V$

$\vdots \qquad \vdots$

$\boxed{\text{for } A = \infty \longrightarrow V_d = 0V}$ → Since the open-loop gain for an ideal op-amp is ∞, connecting a -ve feedback must make $V_d = 0V$ to maintain finite gain.

$\because V_d = v^+ - v^-$, if $V_d = 0$, it implies $v^+ = v^-$. This phenomenon is called virtual short.

Note — This only happens when there is a -ve feedback

— In reality $v^+ \neq v^-$ as you will get $v_o = 0$ then. Instead they are almost equal, $v^+ \approx v^-$.

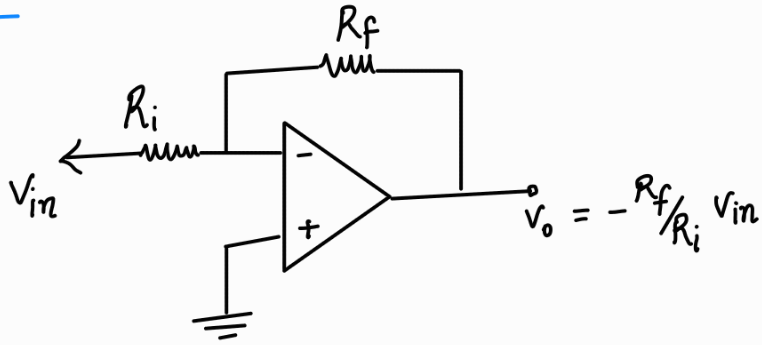
For practical op-amps, $A = 10^6$ (around), so V_d is in the μV range. But for our calculations, we will use $v^+ = v^-$ as it will make the maths easier.

For closed loop configs, we will mostly use a few combinations. I will encourage you to memorize these configs and their gains. Also, you should understand their derivations as you may need these concepts for more complicated questions.

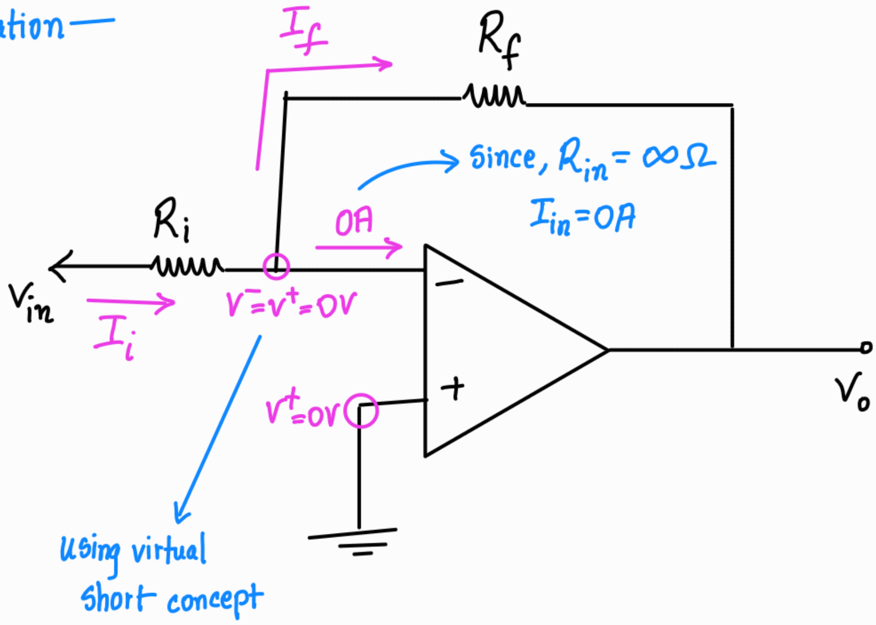
1. Inverting amplifier —

Inverts the input signal and amplifies it by a factor R_f/R_i. Effective gain $= -R_f/R_i$

Ckt —



Derivation —



KCL @ the −ve terminal —

$I_i = 0 + I_f$

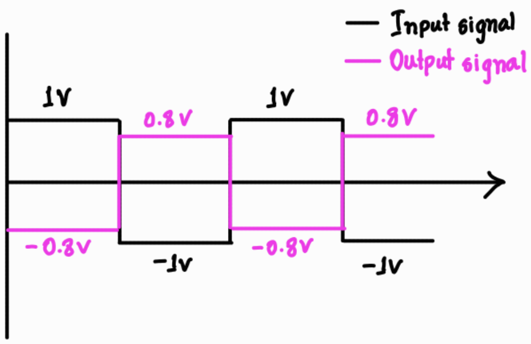
$\Rightarrow I_i = I_f$

$\Rightarrow \dfrac{V_{in} - V^-}{R_i} = \dfrac{V^- - V_o}{R_f}$

$\Rightarrow V_o = -\dfrac{R_f}{R_i} V_{in} \quad [\because V^- = 0V]$

Terminology — In inverting amplifier, since the positive terminal is grounded, and −ve feedback is connected, by the concept of virtual short, $V^+ = V^- = 0V$. So it appears that the −ve terminal is virtually (not physically) grounded. This concept is called virtual ground.

Example —



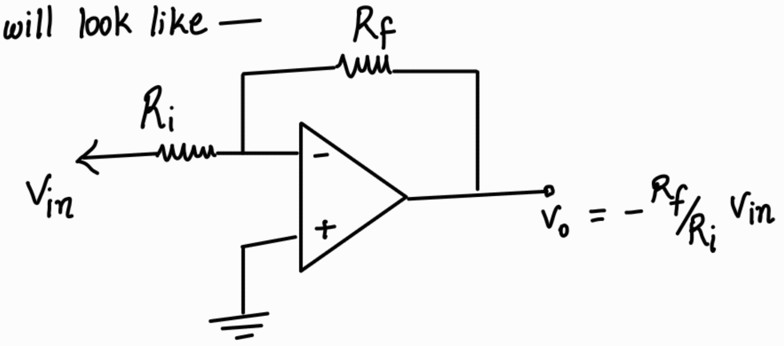
Design an op-amp ckt to get the output signal for the given input signal.

Answer —

Here, $\dfrac{V_o}{V_i} = \dfrac{-0.8}{1} = -0.8$ ⟵ −ve finite gain

can be achieved by inverting amplifier

the ckt will look like —



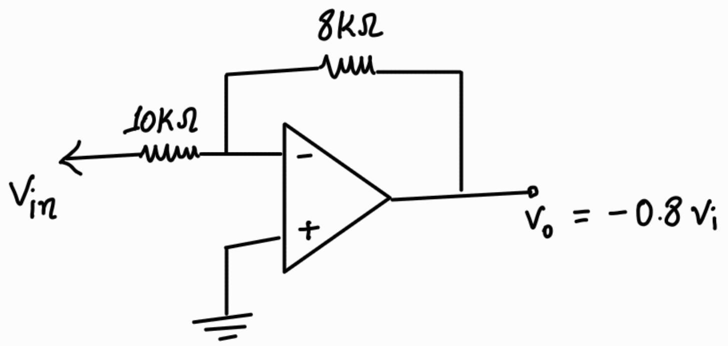
You can choose any values for R_i & R_f as long as the gain is achieved.

Here, $-\frac{R_f}{R_i} V_{in} = -0.8 V_{in}$

$\Rightarrow R_f = 0.8 R_i$

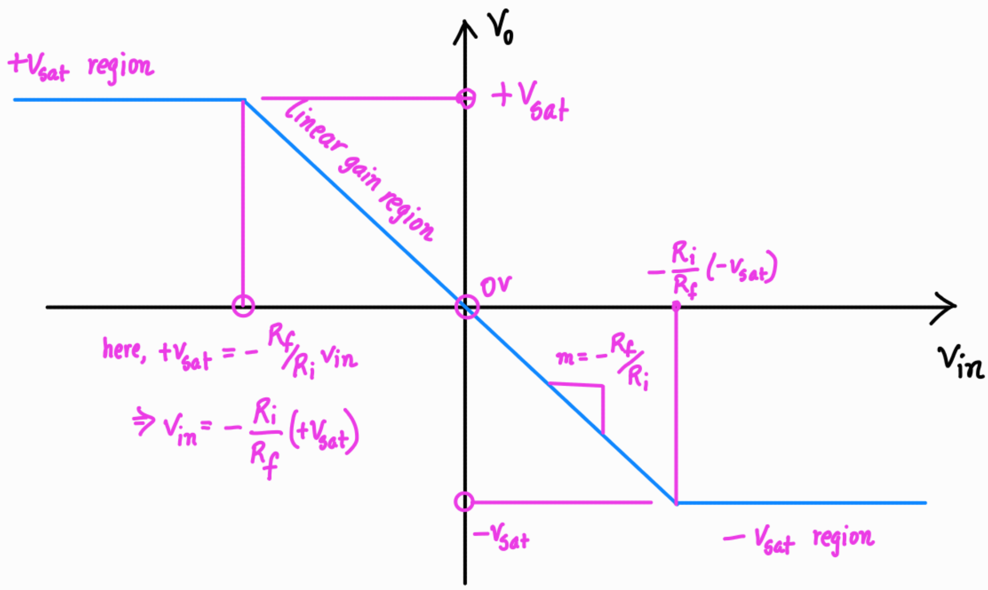
Let $R_i = 10K\Omega$, then $R_f = 8K\Omega$

$\therefore$ final ckt —



VTC —

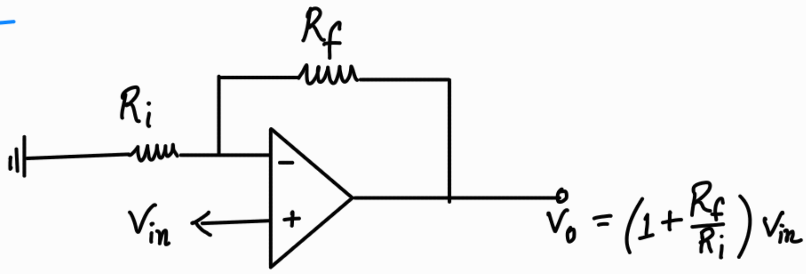
$\because V_o = -\frac{R_f}{R_i} V_{in}$ between the limits $[-V_{sat}, +V_{sat}]$, the VTC —



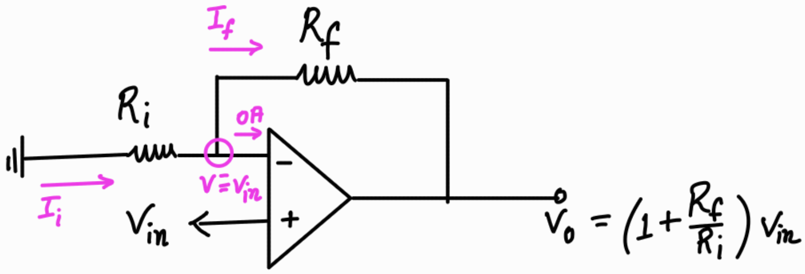
2. Non-inverting amplifier —

Amplifies the input signal at v^+ by a gain of $1+\frac{R_f}{R_i}$.

Ckt —



Derivation —



The derivation is similar —

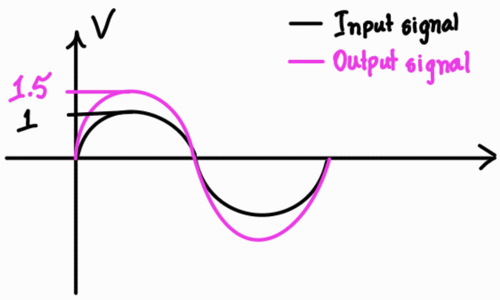
KCL @ −ve terminal —

$$I_i = 0 + I_f$$

$$\Rightarrow \frac{0-V_{in}}{R_i} = \frac{V_{in}-V_o}{R_f}$$

$$\Rightarrow V_o = \left(1+\frac{R_f}{R_i}\right)V_{in}$$

Example —



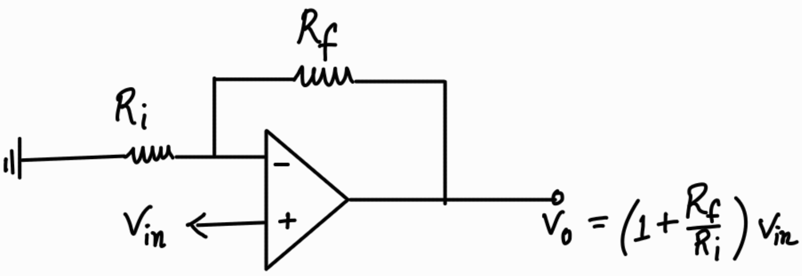
Design an op-amp ckt to get the output signal for the given input signal.

Answer —

gain required $= \frac{1.5}{1} = 1.5$ ⟵ +ve finite gain

can be implemented using non-inverting amplifiers.

the ckt will look like —

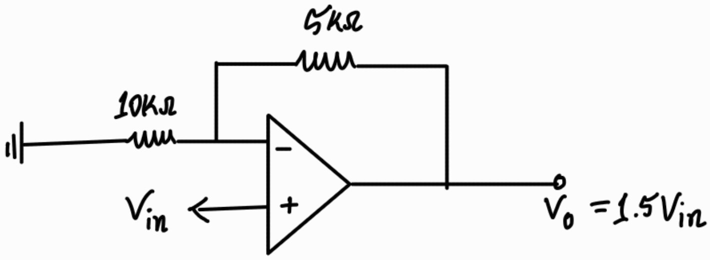


Here, gain $\left(1+\frac{R_f}{R_i}\right) = 1.5$

$\Rightarrow R_f = 0.5\,R_i$

Let $R_i = 10K\Omega$, then $R_f = 5K\Omega$

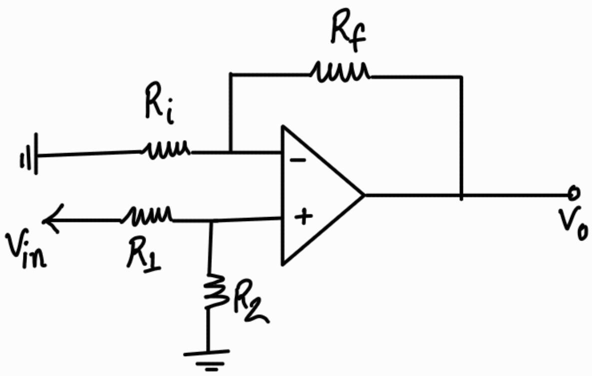
∴ the ckt becomes —



Limitation —

Since the gain $= 1+\frac{R_f}{R_i}$, it can never be reduced below 1.

Modification —



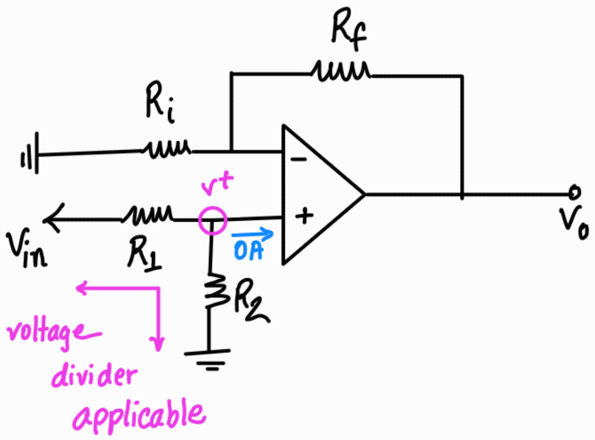
although, we have derived $V_o = \left(1+ R_f/R_i\right) V_{in}$, a more generalized version is —

$$V_o = \left(1+ R_f/R_i\right) v^+$$

∵ earlier $v^+ = v_{in}$, we got the old formula.

for this modified case, we will use $V_o = \left(1+ R_f/R_i\right) v^+$ to derive the final relationship between V_o & V_{in}

Derivation —



applying voltage divider,

$$V_{R_2} = \frac{R_2}{R_1+R_2} V_{in}$$

$\therefore v^+ = 0 + V_{R2}$ [transferring node voltage]

$$= V_{R2}$$

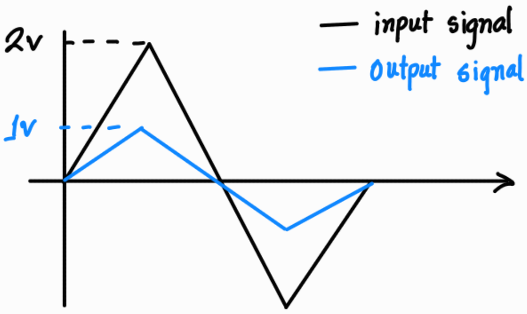
$$\therefore v^+ = \frac{R_2}{R_1+R_2} v_{in}$$

Now using the formula — $V_o = \left(1+ R_f/R_i\right) v^+$

$$= \underbrace{\frac{R_2}{R_1+R_2}\left(1+ R_f/R_i\right)}_{\text{gain can now be reduced below 1}} v_{in}$$

Example —

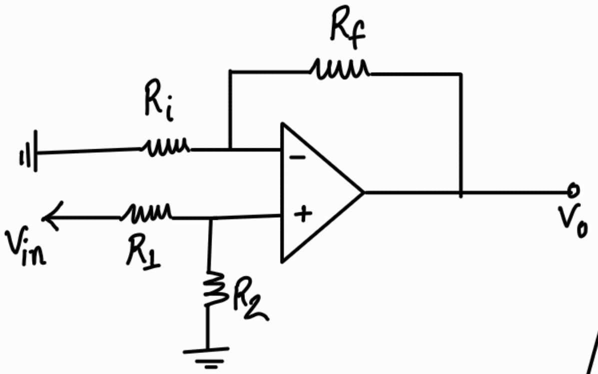
Design an op-amp ckt to get the output signal for the given input signal.



Answer —

$gain = \frac{1}{2} = 0.5 \longrightarrow$ positive finite gain (gain < 1)

We can implement this gain using this ckt —



$$gain = 0.5 = \frac{R_2}{R_1+R_2}\left(1+\frac{R_f}{R_i}\right)$$

Let $R_f = R_i = 10K\Omega$, then $gain = 0.5 = \frac{2R_2}{R_1+R_2}$

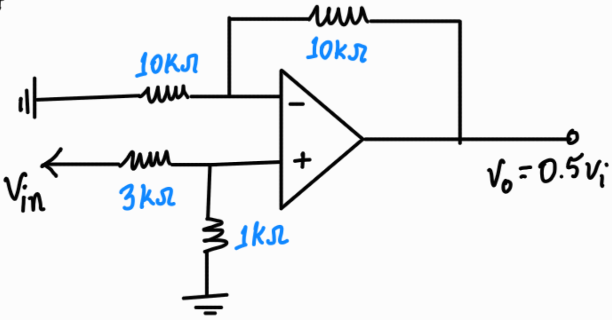
$$\frac{R_2}{R_1+R_2} = 0.25 \Rightarrow R_2 = 0.25R_1 + 0.25R_2$$

$$\Rightarrow 0.75R_2 = 0.25R_1$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{0.75}{0.25} = 3$$

Let $R_2 = 1K\Omega$. Then $R_1 = 3K\Omega$

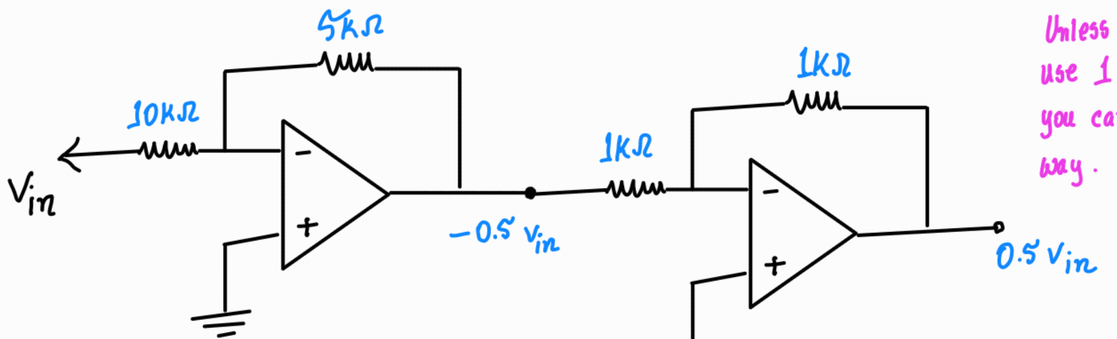
$\therefore$ final ckt —



Question — Could we have reached the same goal using 2 inverting amplifier?

Answer — You surely could have.

The ckt —



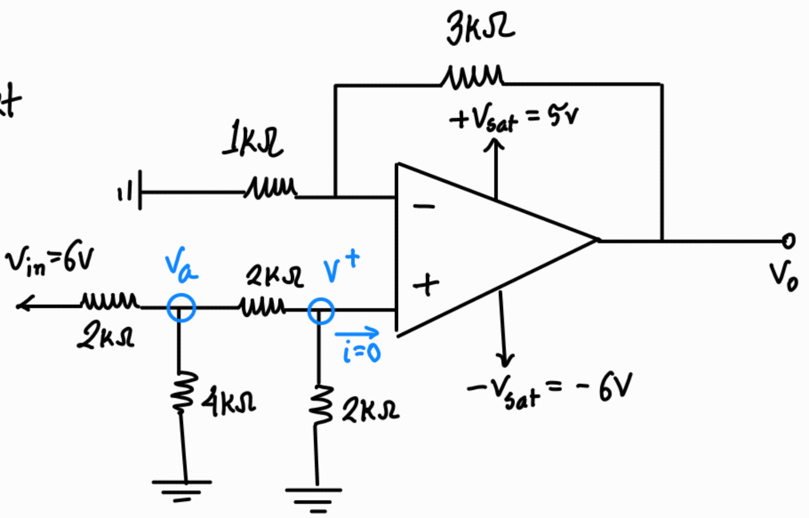
Inverting ckt (gain = -1)

This is easier to design, although it requires 2 op-amps.

Unless the question tells you to strictly use 1 op-amp (generally not imposed), you can always solve such problems this way.

Example —

Find the value of v_0 for the given ckt



Answer —

KCL @ node a —

$$\frac{V_a-6}{2}+\frac{V_a}{4}+\frac{V_a-v^+}{2}=0$$

$$\Rightarrow V_a(2^{-1}+2^{-1}+4^{-1})-v^+(2^{-1})=3 \quad (i)$$

KCL @ the +ve terminal

$$\frac{v^+-V_a}{2}+\frac{v^+}{2}=0$$

$$\Rightarrow v^+(2^{-1}+2^{-1})-V_a(2^{-1})=0 \quad (ii)$$

Solving (i) & (ii), we get —

$v^+ = 1.5V$

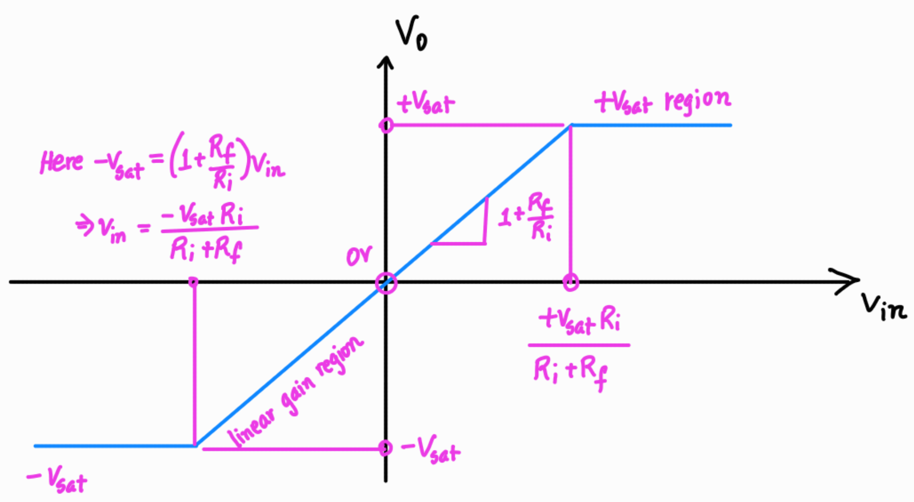
$\therefore v_0=(1+R_f/R_i)v^+=(1+3)\times 1.5=6V$

$\because v_0=6V > +V_{sat}$

$\therefore v_0=+V_{sat}=5V$

VTC —

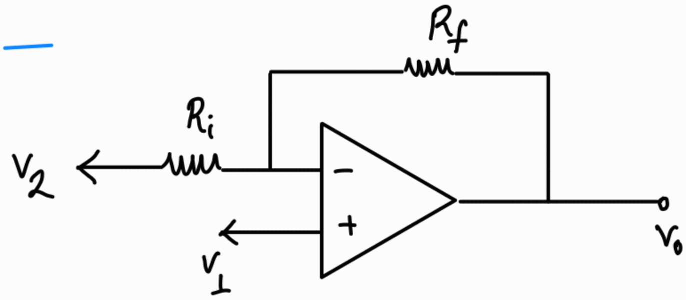
for the simpler non-inverting case, $v_0=(1+R_f/R_i)V_{in}$. So the VTC —



Do the VTC for the modified version yourself.

3. Difference amplifier (Subtractor) —

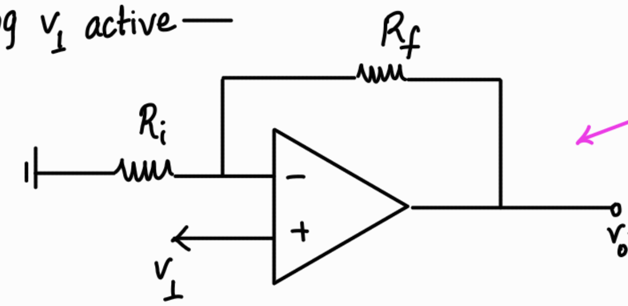
Ckt —



$$V_0 = \left(1 + \frac{R_f}{R_i}\right) V_2 - \frac{R_f}{R_i} V_1$$

Derivation —

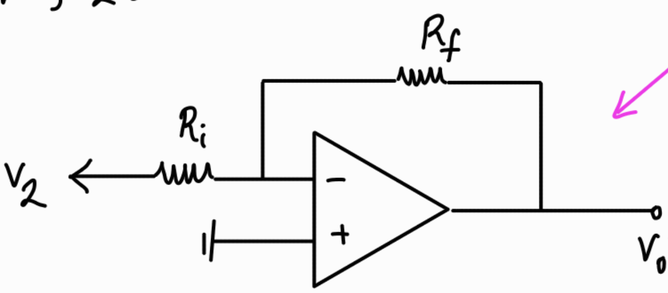
Keeping V_1 active —



basically a non-inverting amplifier

$$V_0^1 = \left(1 + \frac{R_f}{R_i}\right) V_1$$

Keeping V_2 active —

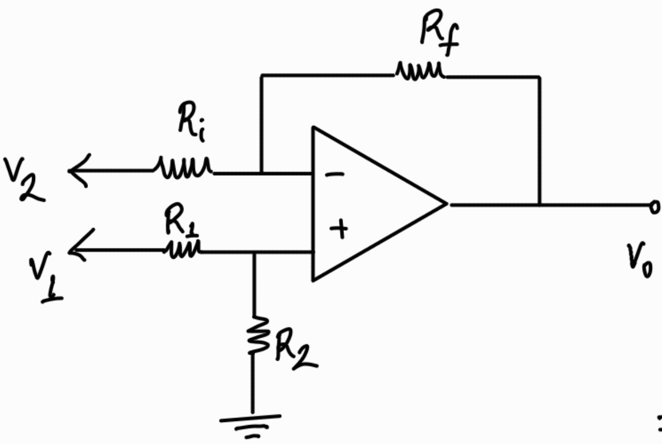


basically an inverting amplifier

$$V_0^2 = -R_f/R_i \, V_2$$

Combining effects of V_1 & V_2 $\longrightarrow$ $V_0 = V_0^1 + V_0^2 = \left(1 + R_f/R_i\right) V_1 - R_f/R_i \, V_2$

Making it more manageable ——

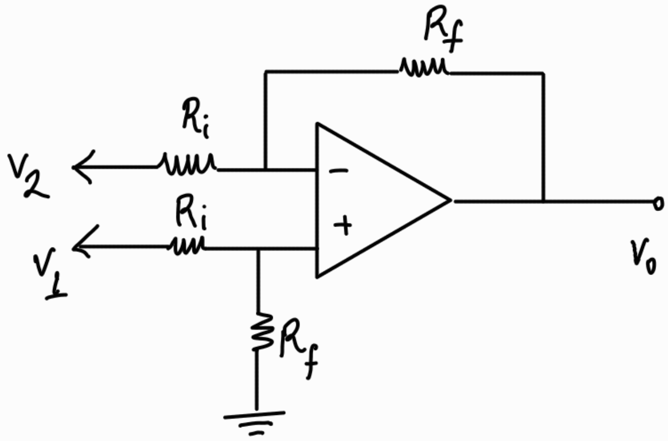


Similar derivation

$$V_0 = \frac{R_2}{R_1 + R_2} \cdot \left(1 + \frac{R_f}{R_i}\right) V_1 - \frac{R_f}{R_i} V_2$$

$$= \frac{R_2}{R_1 + R_2} \cdot \frac{R_i + R_f}{R_i} \cdot V_1 - \frac{R_f}{R_i} V_2$$

if we set $R_2 = R_f$ & $R_1 = R_i$ —



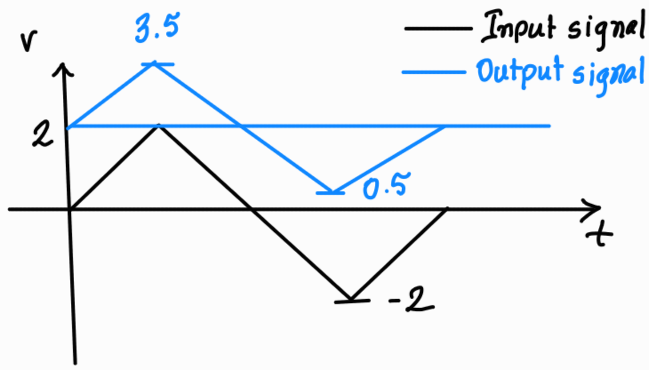
$$V_0 = \frac{R_f}{\cancel{R_i+R_f}} \cdot \frac{\cancel{R_i+R_f}}{R_i} \cdot V_1 - \frac{R_f}{R_i} V_2$$

$$= \frac{R_f}{R_i} V_1 - \frac{R_f}{R_i} V_2$$

$$= \frac{R_f}{R_i} \left[V_1 - V_2 \right]$$

Example —

Design an op-amp ckt to convert the input signal to the output signal



Answer —

So we have 2 things going on here. Input signal's amplitude is 2v and the output signal's amplitude is 1.5

So, gain $= \frac{1.5}{2} = 0.75$ ← positive gain

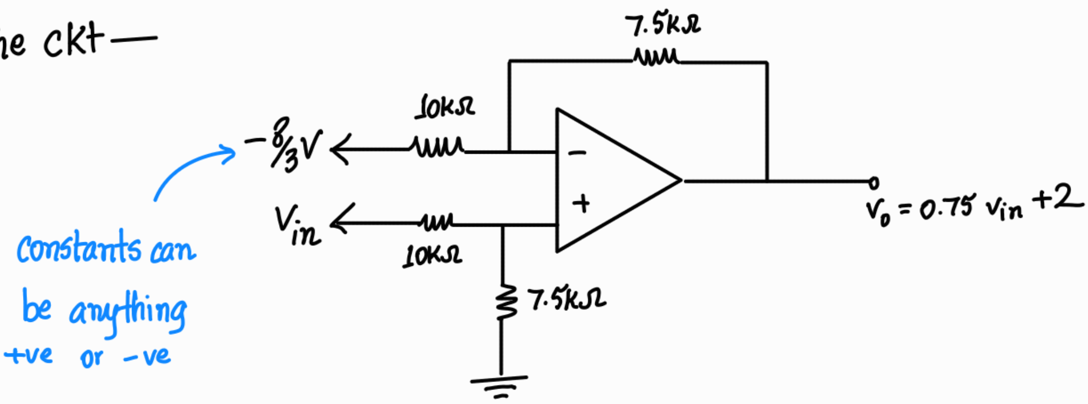
Also the whole signal is getting offsetted by 2V. This can be implemented using the subtractor.

So, $V_0 = 0.75\, V_{in} + 2$ ← lets rearrange it a bit to fit the subtractor better

$$= 0.75 \left[V_{in} + \frac{8}{3} \right]$$

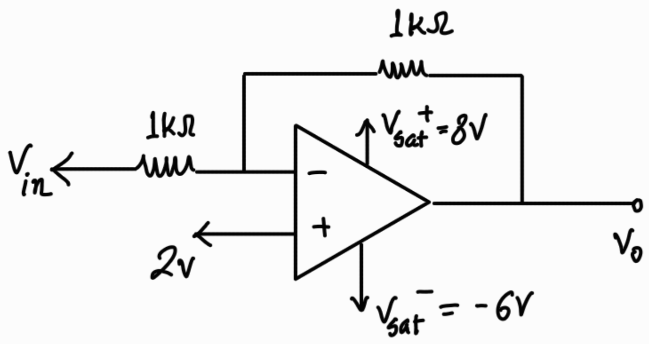
$$= 0.75 \left[V_{in} - \left(-\tfrac{8}{3}\right) \right] \longrightarrow \therefore \; R_f/R_i = 0.75 \longrightarrow \text{if } R_i = 10\,k\Omega \; ; \; R_f = 7.5\,k\Omega$$

∴ the ckt —



Example —

Draw the VTC for the given ckt —



Answer —

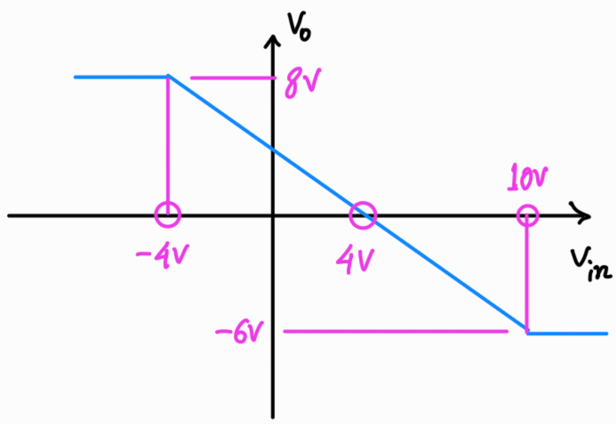
here, $v_o = \left(1+\frac{1}{1}\right)2 - \frac{1}{1}v_{in}$

$= 4 - v_{in}$

when $v_o = 0V \longrightarrow v_{in} = 4V$

when $v_o = 8V \longrightarrow v_{in} = -4V$

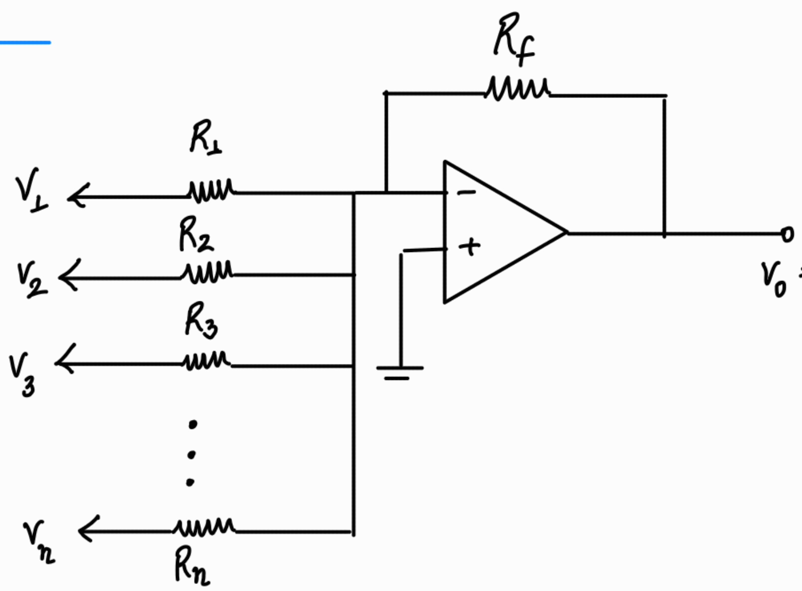
when $v_o = -6 \longrightarrow v_{in} = 10V$



Try out the reverse problem — take a random VTC and design the ckt

4. Inverting summing amplifier (adder) —

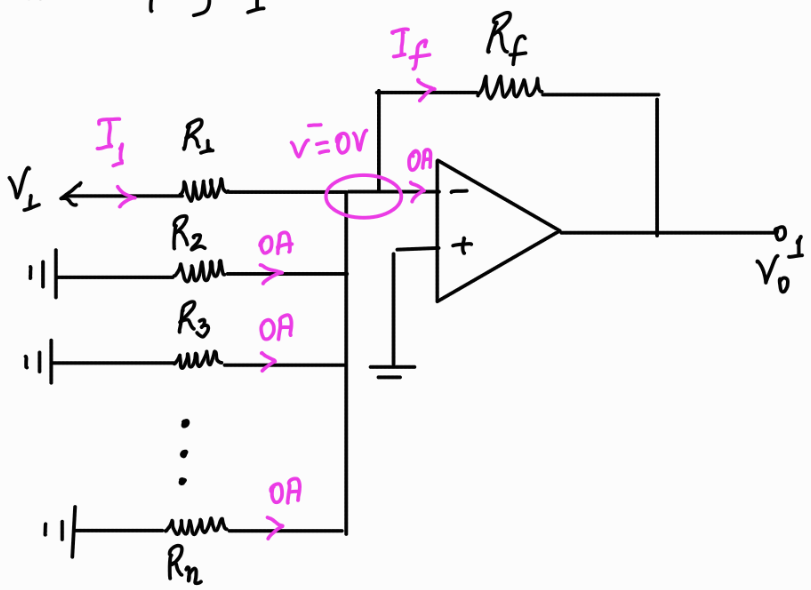
Ckt —



$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \dots + \frac{R_f}{R_n}V_n\right)$$

Derivation —

Using superposition, keeping V_1 ON —



KCL @ -ve terminal —

$$I_1 = I_f$$

$$\Rightarrow \frac{V_1}{R_1} = \frac{-V_0^1}{R_f}$$

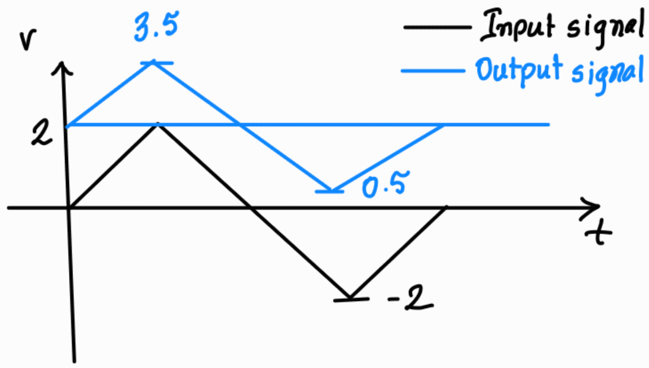
$$\Rightarrow V_0^1 = -\frac{R_f}{R_1} V_1$$

Continuing in the same pattern, $V_0 = V_0^1 + V_0^2 + V_0^3 + \dots + V_0^n$

$$= -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 + \dots + \frac{R_f}{R_n} V_n\right)$$

Example —

Design an op-amp ckt to convert the input signal to the output signal using an adder.



Same problem as the subtractor but using an adder —

$$V_0 = 0.75 V_{in} + 2$$

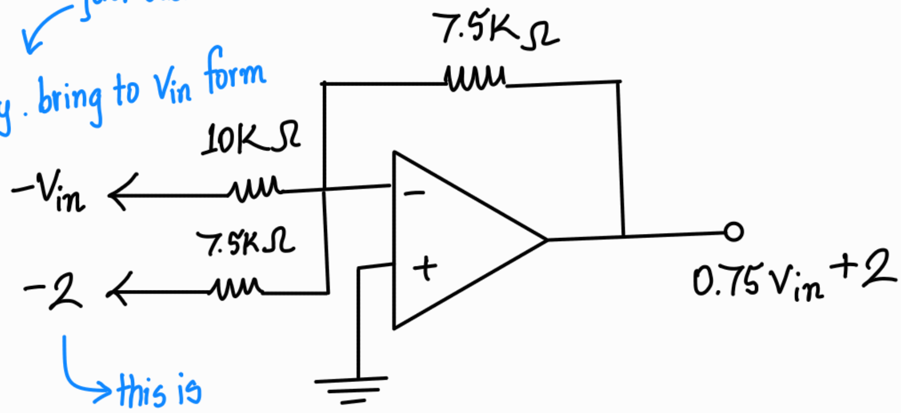
$$= -0.75(-V_{in}) - (-2)$$

$$= -\frac{7.5K}{10K}(-V_{in}) - \frac{7.5K}{7.5K}(-2)$$ [in the adder format $\rightarrow V_0 = -\frac{R_f}{R_1} V_1 - \frac{R_f}{R_2} V_2$]

∴ the ckt —

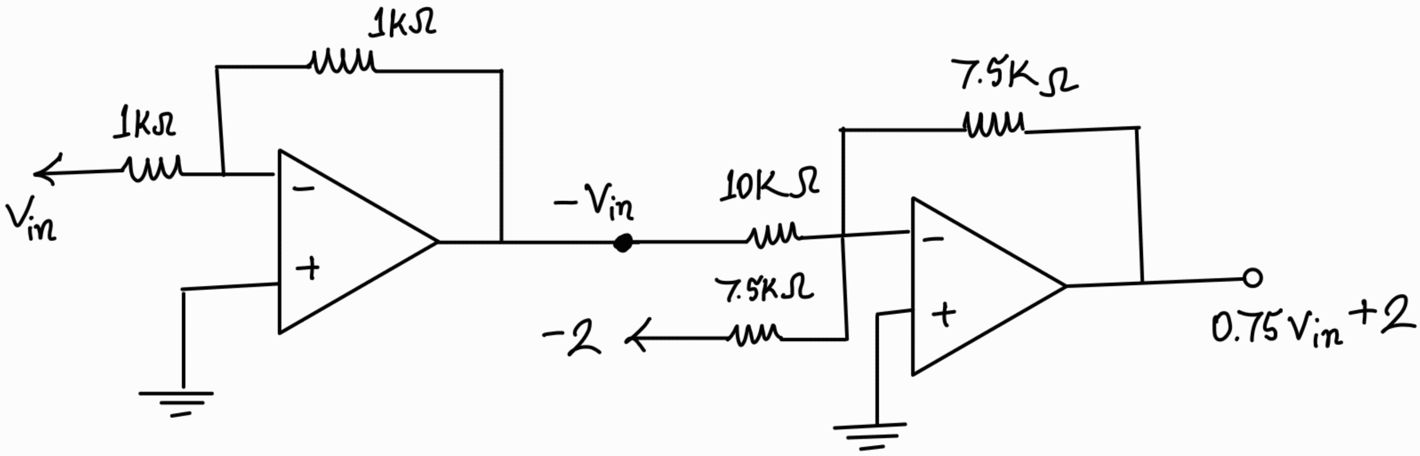
just add an inverter

this isn't okay. bring to Vin form



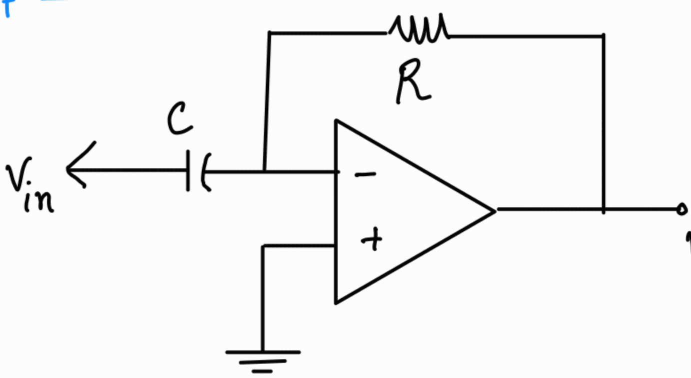
this is okay

after adding the inverter —



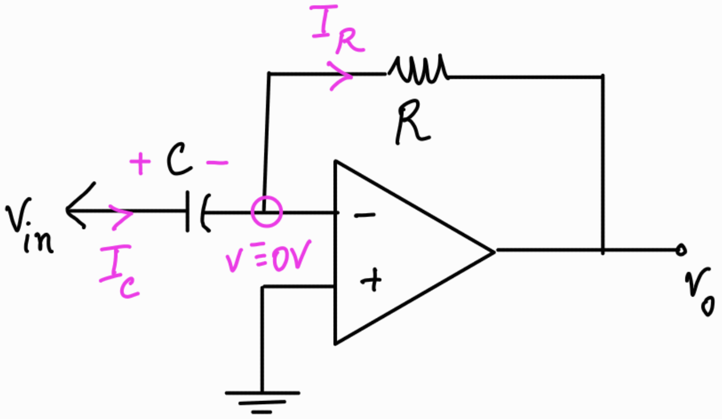
5. Differentiator —

ckt —



$$v_0 = -RC\frac{dV_{in}}{dt}$$

Derivation —



KCL @ −ve terminal —

$$I_c = I_R$$

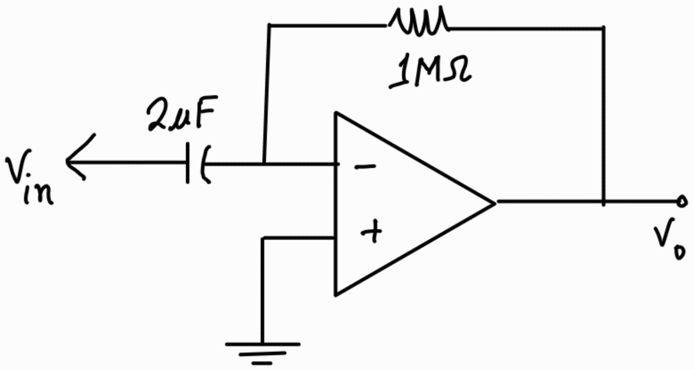
$$\Rightarrow C\frac{d(V_{in}-0)}{dt} = \frac{0-V_0}{R}$$

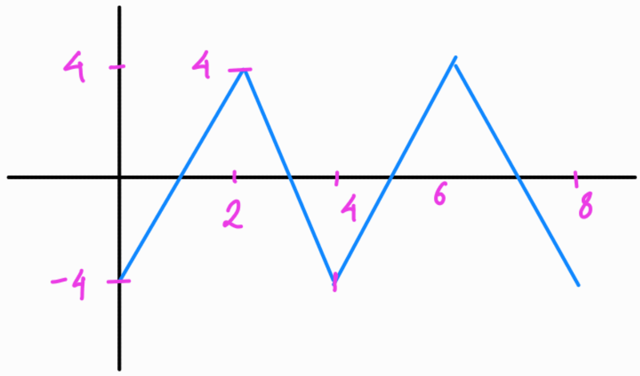
$$\Rightarrow V_0 = -RC\frac{dV_{in}}{dt}$$

You won't need VTC for this.

Example —

for the given ckt and the input signal, draw the output signal —





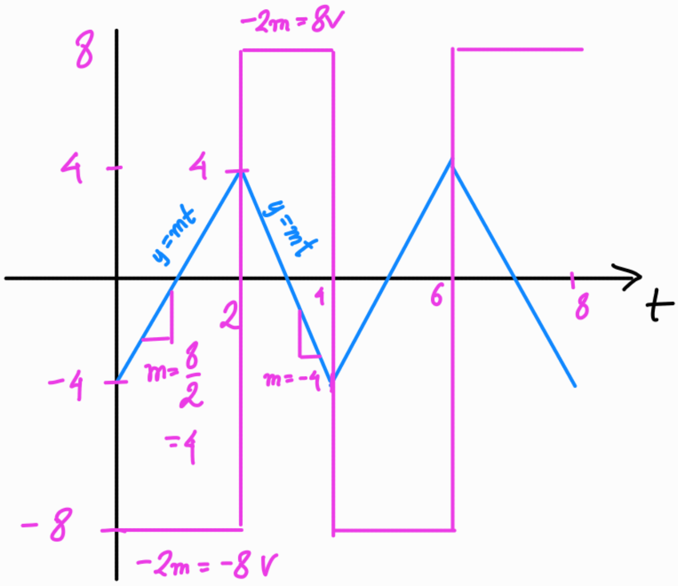
Answer —

for all the segments, $V_{in} = mt$

$$V_0 = -RC\frac{dV_{in}}{dt}$$

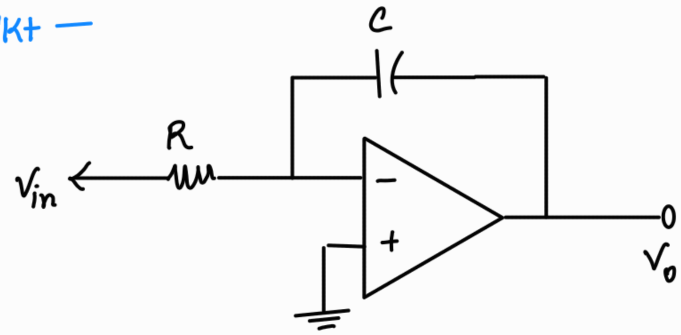
$$= -10^6 \times 2 \times 10^{-6} \times \frac{d}{dt}(mt)$$

$$= -2m$$



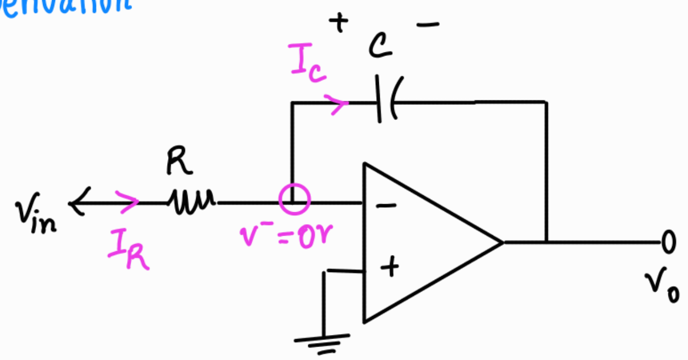
6. Integrator —

Ckt —



$$V_o = -\frac{1}{RC}\int V_{in}\,dt$$

Derivation —



KCL @ −ve terminal —

$$I_R = I_C$$

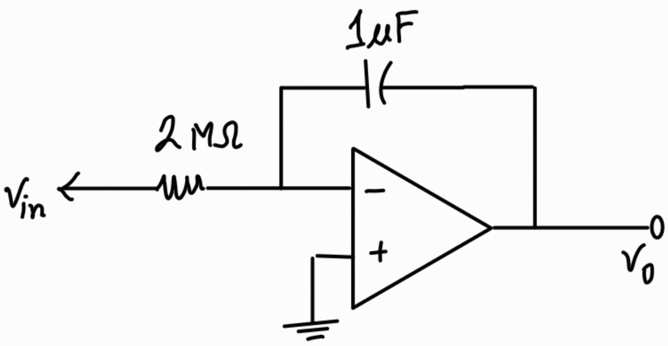
$$\Rightarrow \frac{V_{in} - 0}{R} = C\frac{d(0 - V_o)}{dt}$$

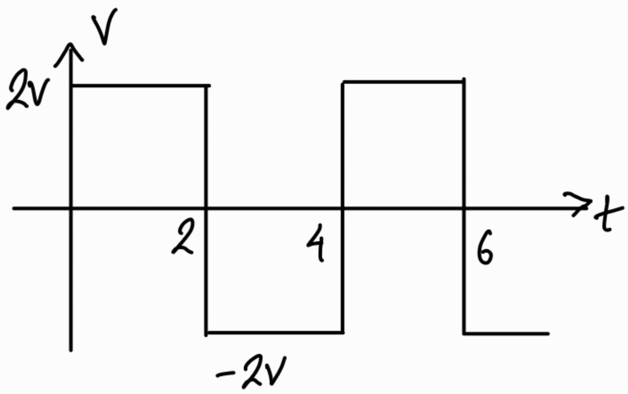
$$\Rightarrow V_o = -\frac{1}{RC}\int V_{in}\,dt$$

Once again, no VTC.

Example —

For the given input signal and the ckt draw the output signal —





Answer—

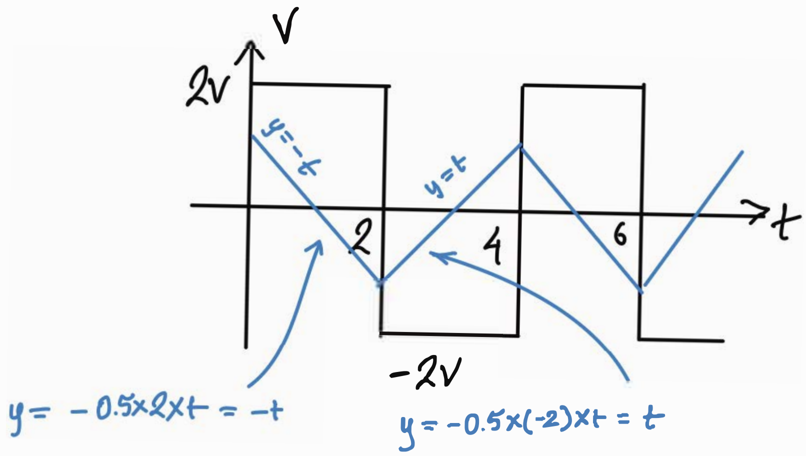
Since the ckt is an integrator — $V_o = \frac{-1}{RC}\int V_{in}\,dt$

$$= -0.5\int V_{in}\,dt$$

for the input signal all the segments are constant values, so $V_o = -0.5\int c\,dt$

$$= -0.5ct + \text{const.}$$

so the output will look something like this—



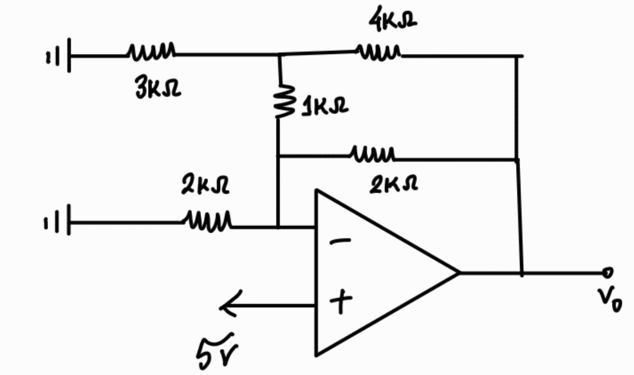
now the problem in the $V_o = -0.5ct + \text{const}$ this const.
this adds arbitray initial offset ⟶ you can ignore this const. if you understand that an integrated square wave is a triangle wave, you will get the marks.

→ Try out cosine & sine.

→ For function implementation please check the video I provided. It may seem daunting at first but it gets easier with practice ⟶ https://youtu.be/CTi9f0USdg8

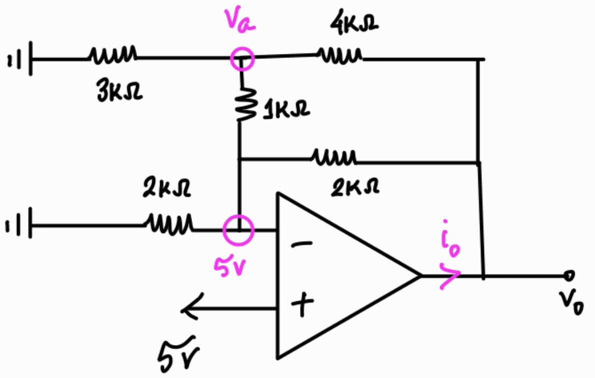
Solved problems —

①



Find V_o

Answer —



KCL @ −ve terminal —

$$\frac{5}{2}+\frac{5-V_o}{2}+\frac{5-V_a}{1}=0$$

$$\Rightarrow 5-\frac{V_o}{2}+5-V_a=0$$

$$\Rightarrow V_o+V_a=20 \quad \text{— (i)}$$

KCL @ node a —

$$\frac{V_a}{3}+\frac{V_a-V_o}{4}+\frac{V_a-5}{1}=0$$

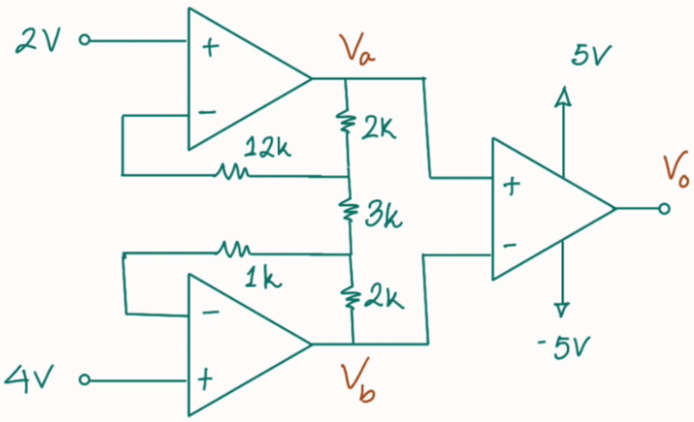
$$\Rightarrow 4V_a+3V_a-3V_o+12V_a-60=0$$

$$\Rightarrow -3V_o+19V_a=60 \quad \text{— (ii)}$$

Solve (i) and (ii) to get V_a & V_o

Note — according to nodal you would have applied KCL at the output node. But as you do not know the value of i_o, you cant. Hence, we had to improvise.

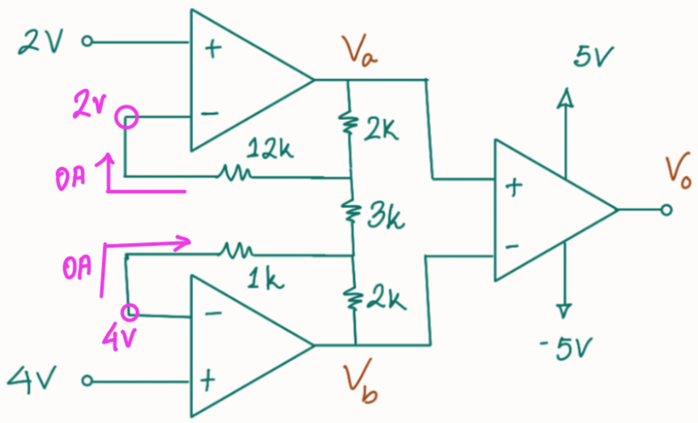
②



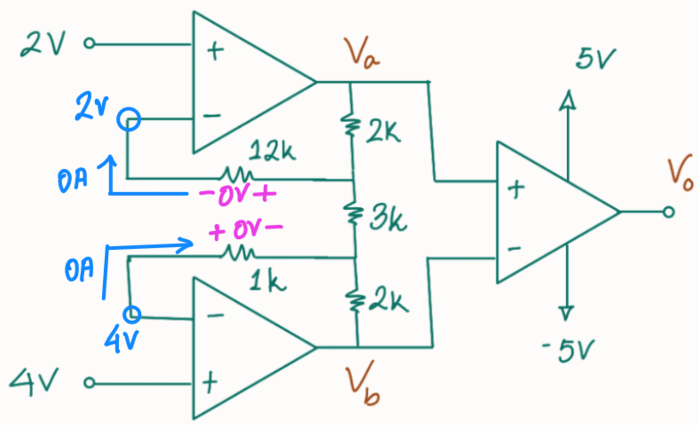
Find V_o

Answer —

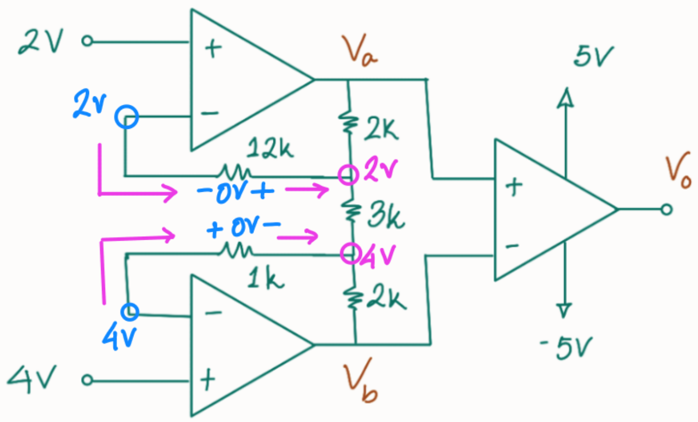
Step 1 —



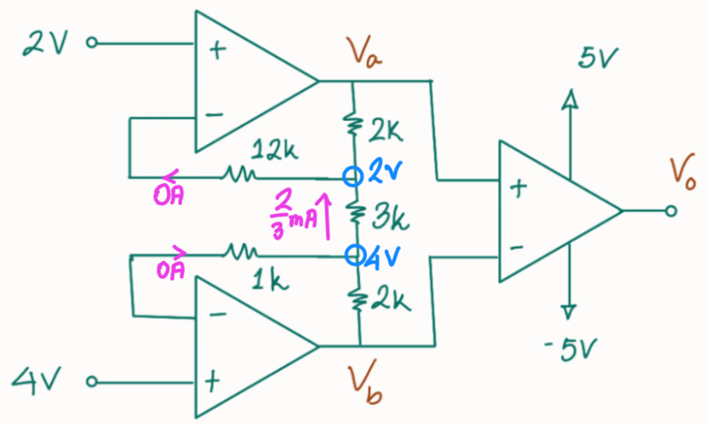
Step 2 —



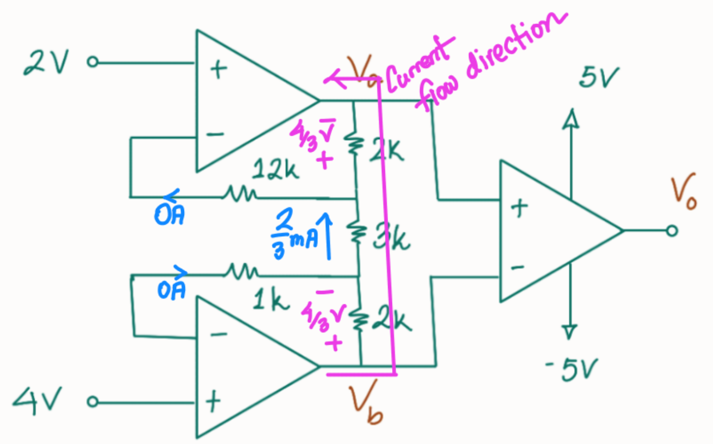
Step 3 —



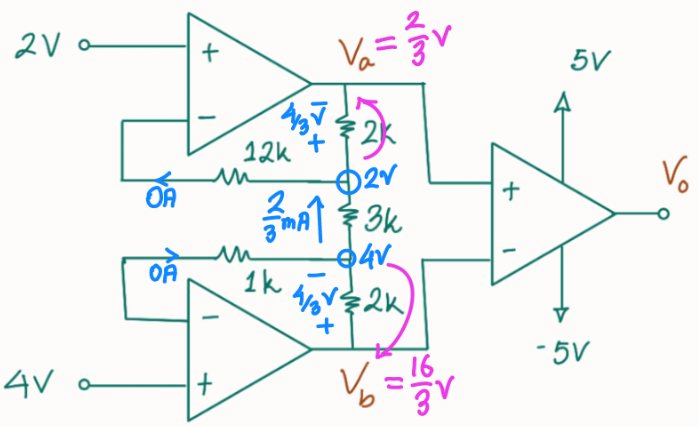
Step 4 —



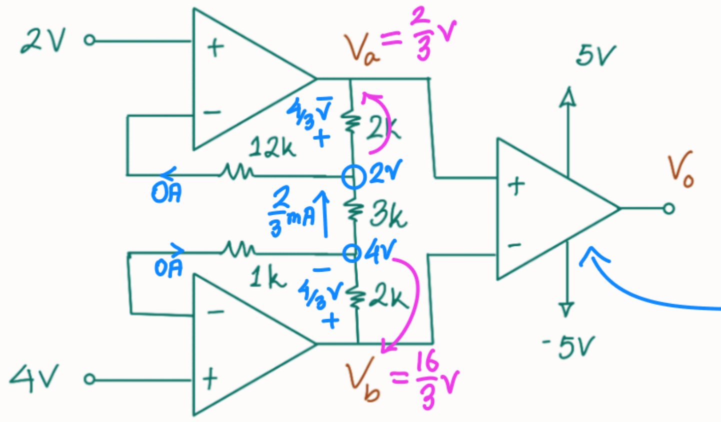
Step 5 —



Step 6 —



Step 7—



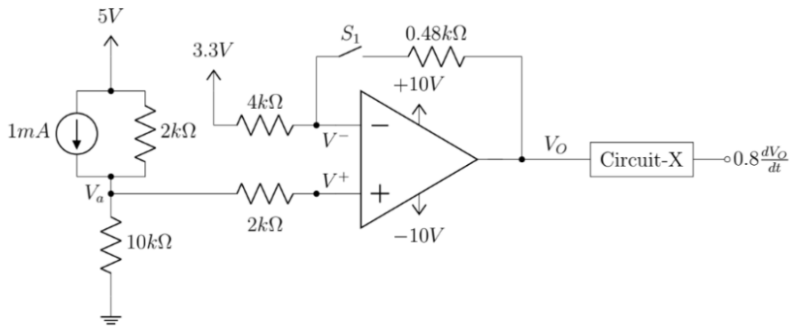
comparator ckt

$v^- > v^+$

$\therefore V_o = -V_{sat} = -5V$

③

The circuit diagram has a switch S_1 which is shown to be 'open' in the figure. The output V_O is passed through an unknown block of 'Circuit-X' and a differentiated result is generated.



(a) [1 mark] **State** the equation of gain of a non-inverting amplifier.

(b) [3 marks] **Calculate** the values of V_a and $V+$.

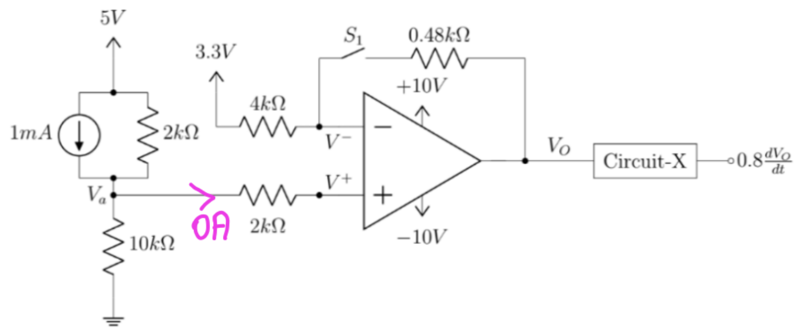
(c) [2 marks] **Determine** V_O when the switch S_1 is closed.

(d) [2 marks] **Determine** V_O when the switch S_1 is open.

Answers —

ⓐ gain, $K = -\frac{R_f}{R_i}$

ⓑ



KCL @ node a—

$$\frac{V_a}{10} - 1 + \frac{V_a - 5}{2} = 0$$

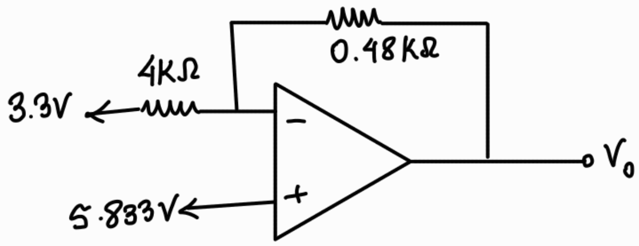
$$\Rightarrow V_a - 10 + 5V_a - 25 = 0$$

$$\Rightarrow 6V_a = 35$$

$$\Rightarrow V_a = 5.833V$$

$$\therefore V^+ = V_a + 0 = 5.833V$$

(c) Omitting the complexity on the +ve side and redrawing the ckt with S_1 closed—

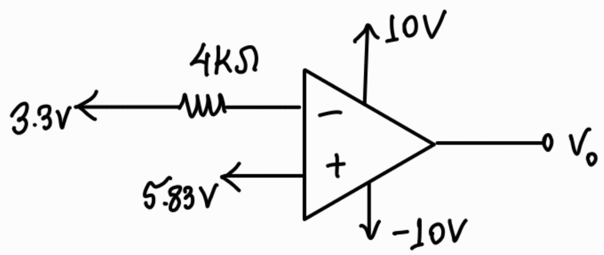


This is a subtractor, so, $v_o = \left(1+\frac{R_f}{R_i}\right)5.833 - \frac{R_f}{R_i}3.3$

$$= \left(1+\frac{0.48}{4}\right)\times 5.833 - \frac{0.48}{4}\times 3.3$$

$$= 6.137\ V$$

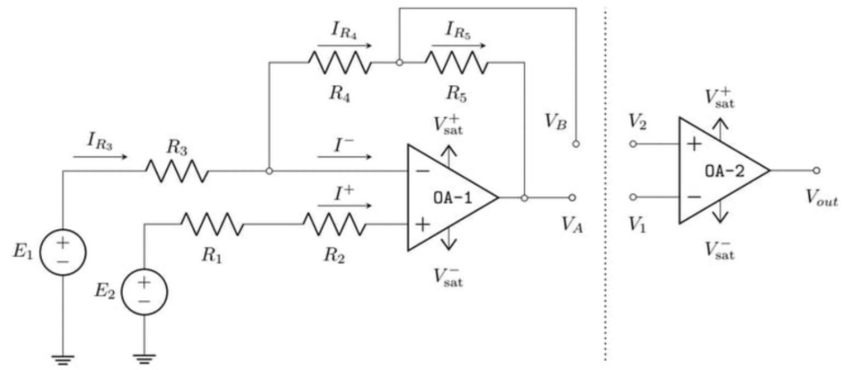
(d) When the switch S_1 is open, the ckt boils down to —



Its a comparator ckt & $v^+ > v^-$. $\therefore v_o = 10V$

4

The 'ideal' operational amplifiers (Op-Amp) below have been connected to saturation voltages $V_{\text{sat}}^{+} = +8\,\text{V}$ and $V_{\text{sat}}^{-} = -8\,\text{V}$. The resistor values are given as: $R_1 = R_2 = 1\text{ k}\Omega$, and $4R_4 = 10R_5 = 20\text{ k}\Omega$.



(a) [1 mark] **State** the current values of I^- and I^+.

(b) [4 marks] If $E_1 = 5\,\text{V}$, $E_2 = 0\,\text{V}$, and $R_3 = 10\,\text{k}\Omega$, **determine** V_A and V_B.

(c) [2 marks] **Find** the value of V_{out} if $V_1 = V_A$ and $V_2 = V_B$.

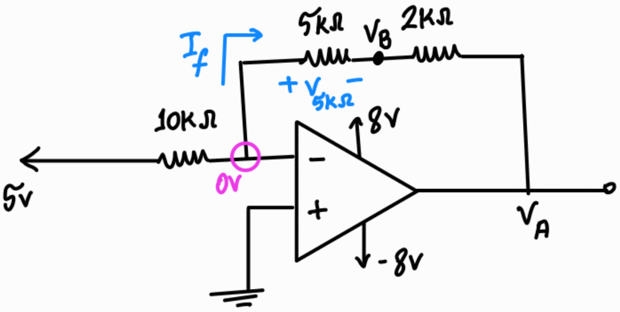
(d) [3 marks] For $E_1 = 0\,\text{V}$ and $E_2 = 2.2\,\text{V}$, we measure V_A to be 5.13 V. Showing necessary calculations, **select** what value of R_3 will make this possible.

(e) [2 marks (bonus)] After obtaining R_3 in question (d), find the value of V_B.

Answers —

a $I^- = I^+ = 0$ [ideal op-amp]

b $E_1 = 5v$ & $E_2 = 0v$ & $R_3 = 10k\Omega$. The ckt becomes —



R_1 & R_2 are completely unnecessary

$4R_4 = 10R_5 = 20k\Omega$

$\therefore R_4 = 5k\Omega$ & $R_5 = 2k\Omega$

This is an inverting amplifier. So, $V_a = -\frac{R_f}{R_i}V_{in} = \frac{-7}{10}5 = -3.5v$

$\therefore I_f = \frac{0-(-3.5)}{5+2} = 0.5V$

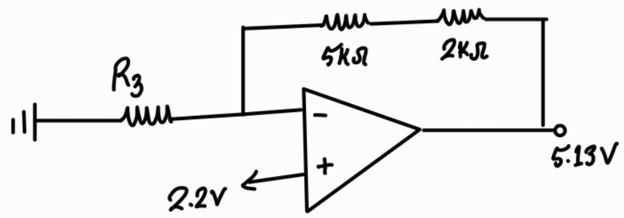
$\therefore V_{5k\Omega} = 0.5 \times 5 = 2.5V$

$\therefore V_B = -2.5V$

c Assuming c as an extension of b, $v^- = v_1 = v_A = -3.5V$ & $v^+ = v_2 = v_B = -2.5V$

$\therefore V_{out} = V_{sat}^{+} = 8v$ [$\because$ the RHS ckt is a comparator and $v^+ > v^-$]

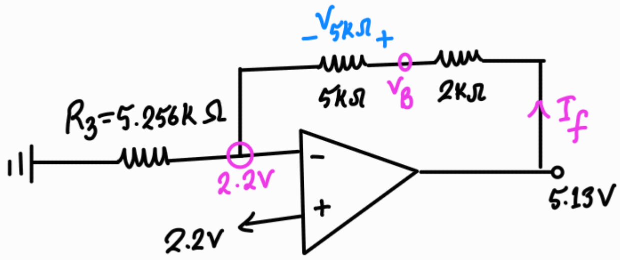
(d) for $E_1 = 0V$ & $E_2 = 2.2V$ and $V_A = 5.13V$, redrawing the ckt —



This is a non-inverting amplifier. So, $V_{out} = 5.13 = \left(1 + \frac{5+2}{R_3}\right) \times 2.2$

$\Rightarrow R_3 = 5.256 k\Omega$

(e)



$$I_f = \frac{5.13 - 2.2}{7} = 0.4186 \, mA$$

note how the value of R_3 isn't even required

$\therefore V_{5k\Omega} = 2.093V$

$\therefore V_B = 4.186V$