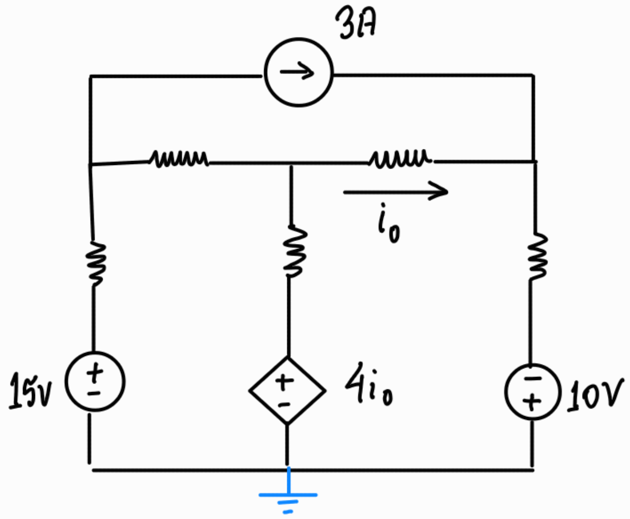
Alternate ckt diagram or line representation helps us to draw ckts in a compact form. Plus in IRL designs, this type of representation is more preferred.

To change a regular ckt representation to line representation —

Steps

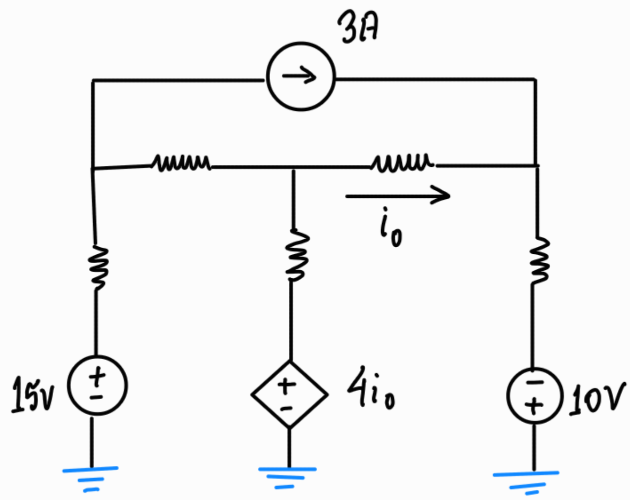
0. Place a ground in the ckt. Ignore this step if a ground is already present.

   To select the ground node just follow the rule of thumbs introduced in nodal analysis during your 250 course

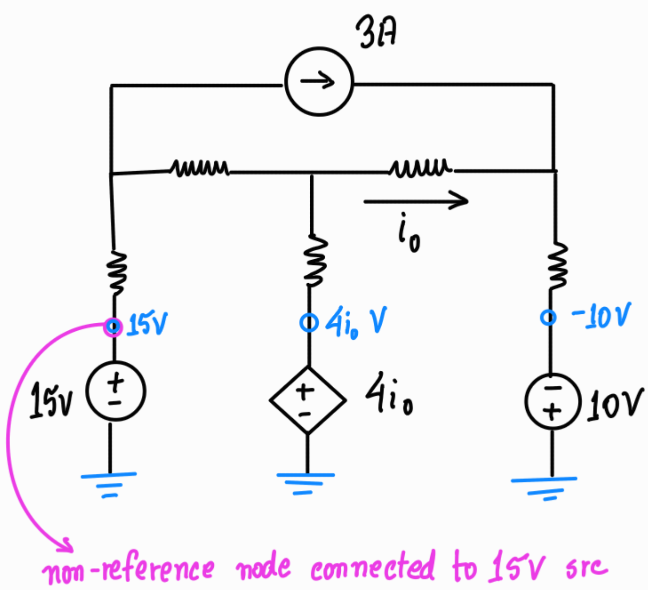


1. Seperate the grounded points.

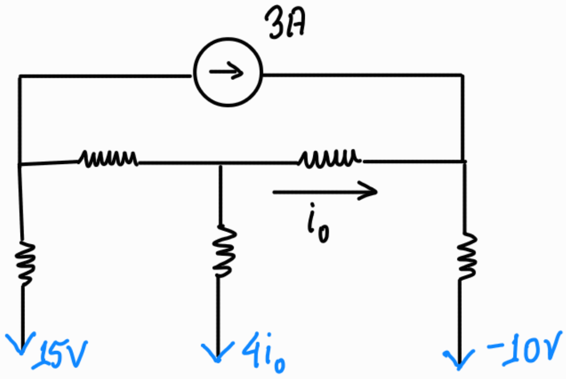
   Note that these points are still the same nodes



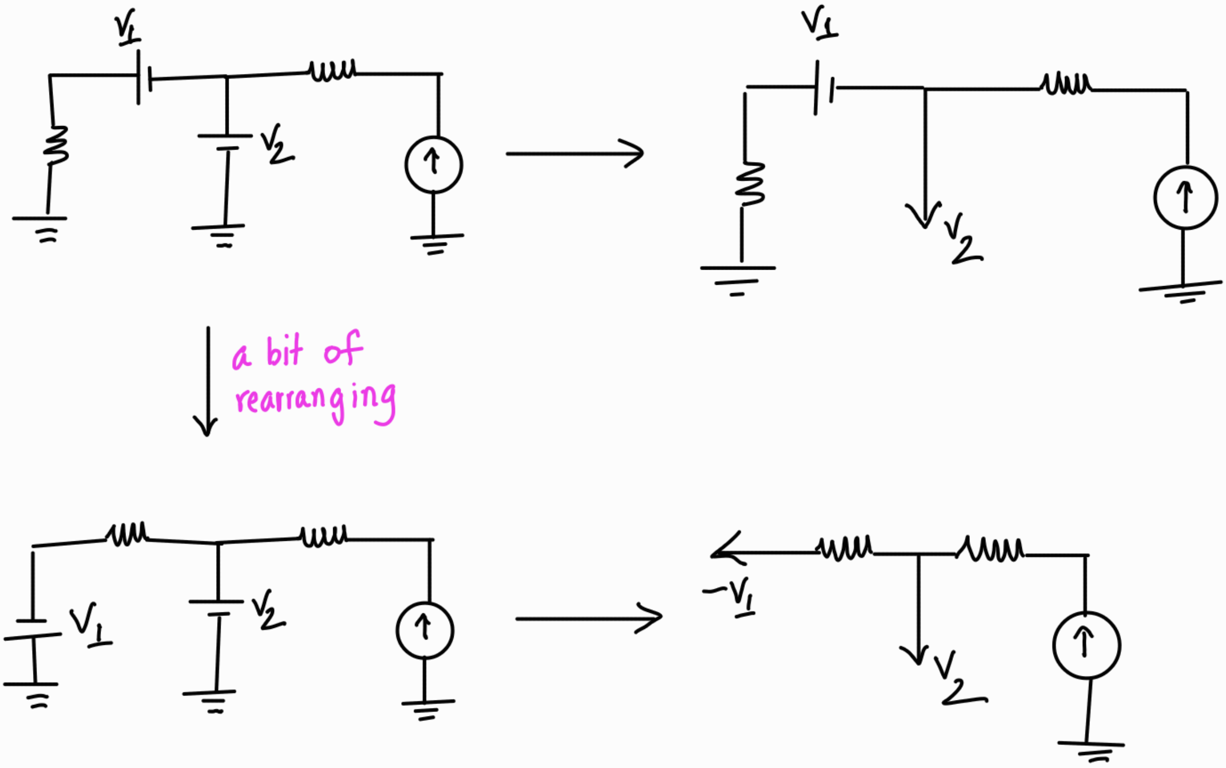
2. Replace all the voltage srcs connected to the ground (, ,) with an arrow ( ) or just a dot ( ). Assign the node voltage at the non-reference node connected with these srcs to the arrow or the dot.



Note — You cannot convert current srcs; neither can you convert non-grounded voltage srcs. So keep them as they are.



One exception —

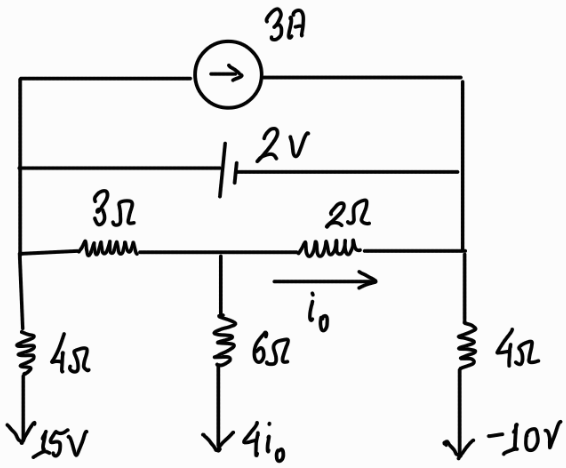


Sometimes a bit of ckt re-arrangment can allow to further simplify the ckt in alternate representation.

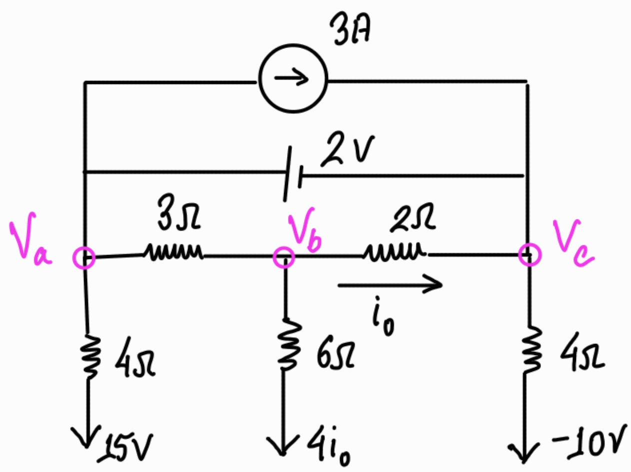
Should you do it? → Its upto you. Its fine even if you don't do it.

Nodal recap —

Find all node voltages in the ckt —



1. Place voltage variables at all essential nodes with unknown node voltages



2. Write KCL equations for all the marked nodes and supernodes. Supernodes will also give you voltage relationship equations —

Voltage relationship equation @ SN a,c —

$$V_a - V_c = 2 \quad \text{—— (i)}$$

KCL @ SN a,c —

$$\frac{V_a - 15}{4} + \frac{V_a - V_b}{3} + \frac{V_c - V_b}{2} + \frac{V_c - (-10)}{4} = 0$$

[3A src shorted between the SN]

$$\Rightarrow V_a(4^{-1}+3^{-1}) - V_b(3^{-1}+2^{-1}) + V_c(2^{-1}+4^{-1}) = \frac{15}{4} - \frac{10}{4} \quad \text{—— (ii)}$$

KCL @ b —

$$\frac{V_b - V_a}{3} + \frac{V_b - 4i_o}{6} + \frac{V_b - V_c}{2} = 0$$

in the ckt $i_o = \dfrac{V_b - V_c}{2}$

$$\therefore \frac{V_b - V_a}{3} + \frac{V_b - 4i_o}{6} + \frac{V_b - V_c}{2} = 0 \Rightarrow \frac{V_b - V_a}{3} + \frac{V_b - 2V_b + 2V_c}{6} + \frac{V_b - V_c}{2} = 0$$

$$\rightarrow -V_a(3^{-1}) + V_b(3^{-1} - 6^{-1} + 2^{-1}) - V_c(-3^{-1} + 2^{-1}) = 0 \quad \text{—— (iii)}$$

(3) Solve the equations to get the values of V_a, V_b & V_c

Some key points —

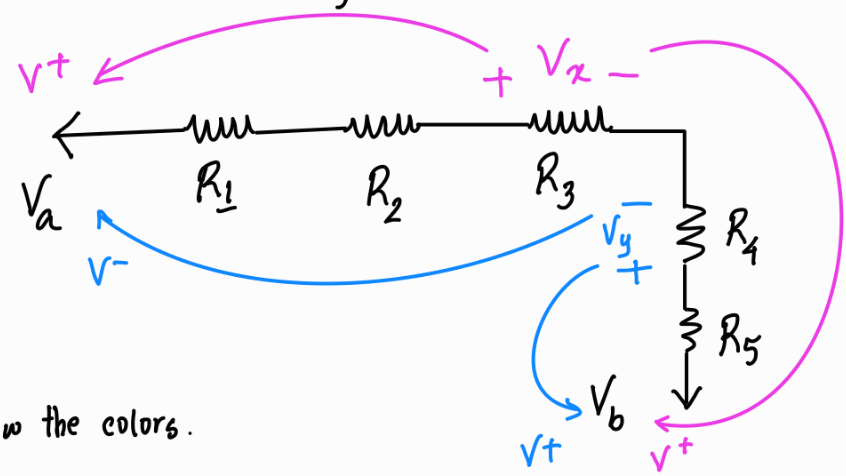
(i) If current enters the node ⟶ −ve term
current leaves the node ⟶ +ve term

(ii) for resistive branches, always consider outward current

(iii) Always replace extra variables in terms of node variables.

Voltage divider ——

for any series string of resistances ——



$$V_x = \frac{R_3}{\Sigma R} \times (V^+ - V^-)$$

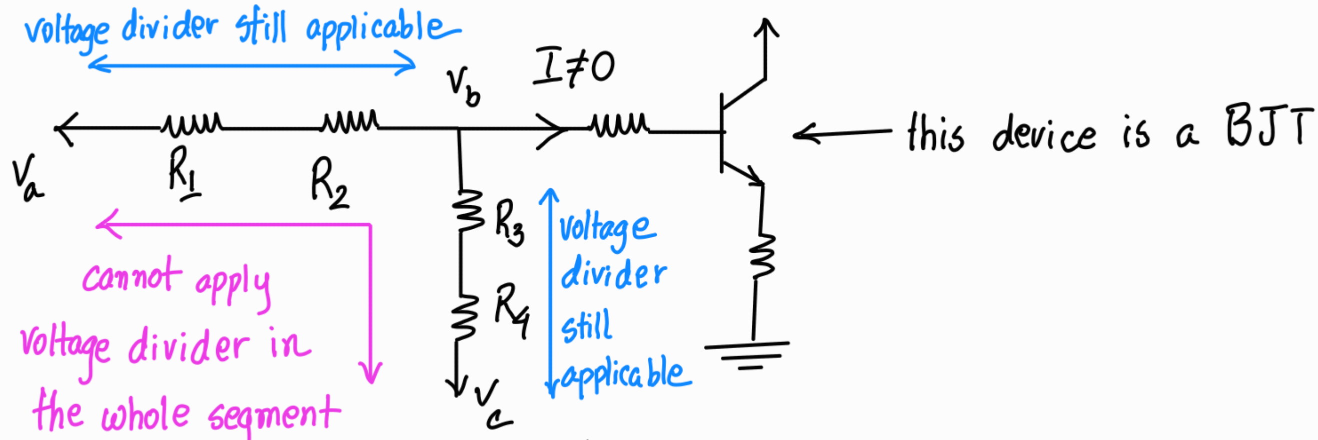
$$= \frac{R_3}{\Sigma R}(V_a - V_b)$$

$$V_y = \frac{R_4}{\Sigma R}(V^+ - V^-)$$

$$= \frac{R_4}{\Sigma R}(V_b - V_a)$$

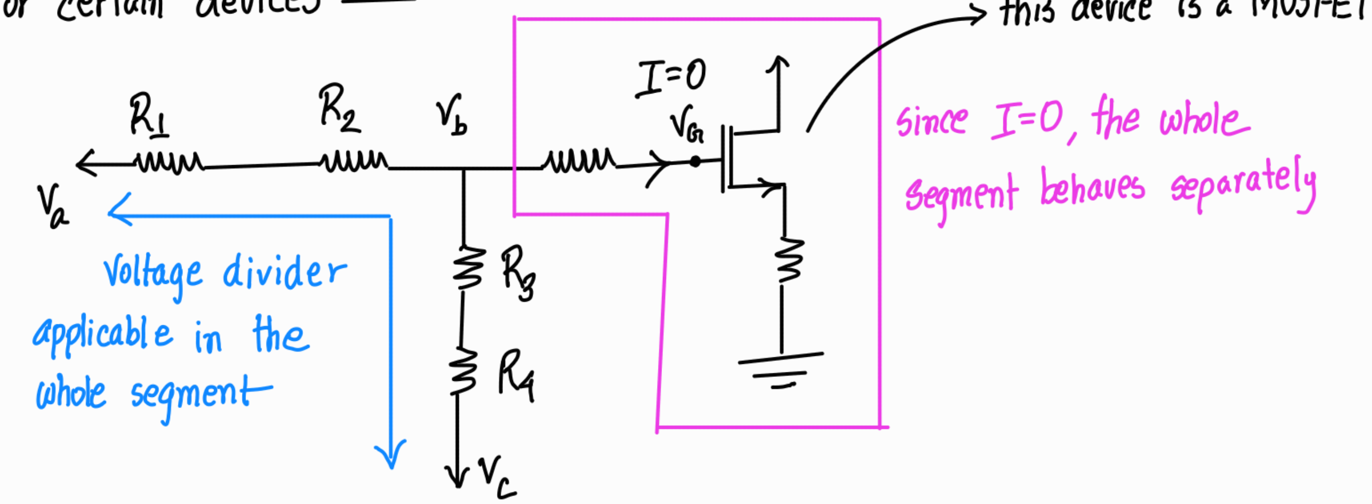
follow the colors.

In case there is a branch connected to the string of resistors like this —

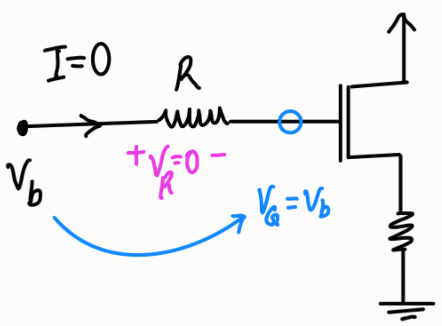


voltage divider still applicable in smaller segments

However for certain devices —



Since $I=0$ in the connecting branch, the node voltage V_b can be transferred inward —

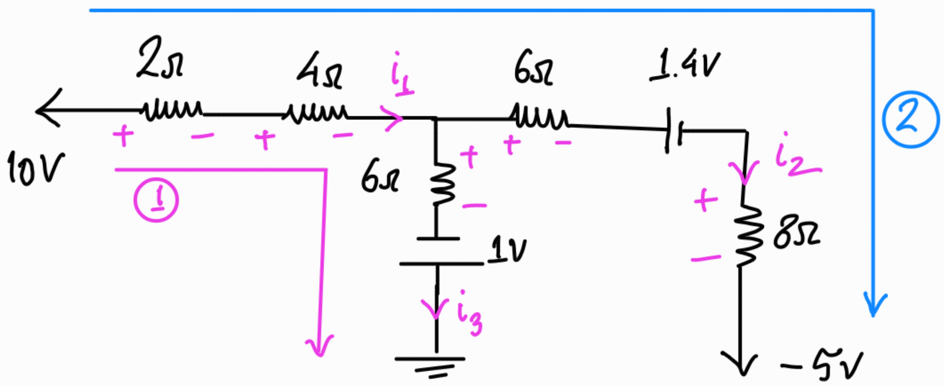


This concept will be tested multiple times in the course.

KVL in a line —

While voltage divider will help you out in some of the tricky case. KVL will remain a staple in the harder (read lengthy) but straight-forward problems.

Unlike 250 where we applied KVL in a loop, we will apply KVL in lines for 251.



polarity placed following passive sign convention.

KVL in ① —

$$2i_1 + 4i_1 + 6i_3 - 1 = V_{start} - V_{end}$$

$$\Rightarrow 6i_1 + 6i_3 = 10 - 0 + 1 \Rightarrow 6i_1 + 6i_3 = 11$$

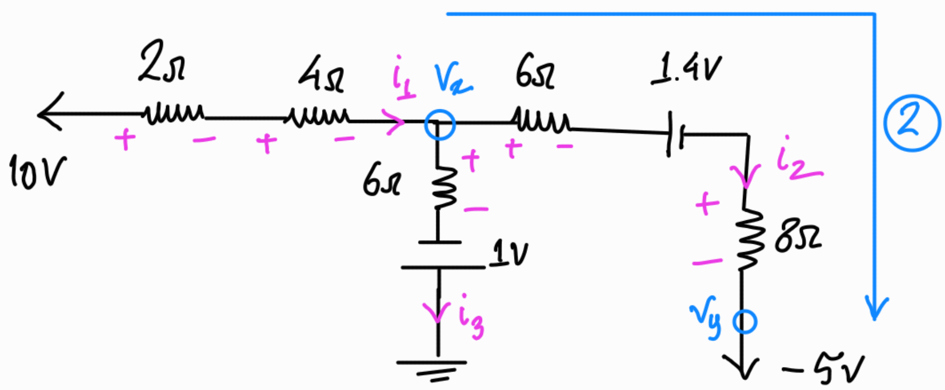
KVL in ② —

$$2i_1 + 4i_1 + 6i_2 + 1.4 + 8i_2 = V_{start} - V_{end}$$

$$\Rightarrow 6i_1 + 14i_2 = 10 - (-5) - 1.4 \Rightarrow 6i_1 + 14i_2 = 13.6$$

KVL in line is more flexible than regular KVL. Lets take the above example and update line 2.

We can update the span of KVL freely



KVL in line 2 $\underbrace{\text{starting from node } x \text{ to } y}$ —

Optionally you can specify the range → I generally do it.

Going ahead most ckts will have named nodes, so you will not have to name them all

$$6i_2 + 1.4 + 8i_2 = V_{start} - V_{end}$$

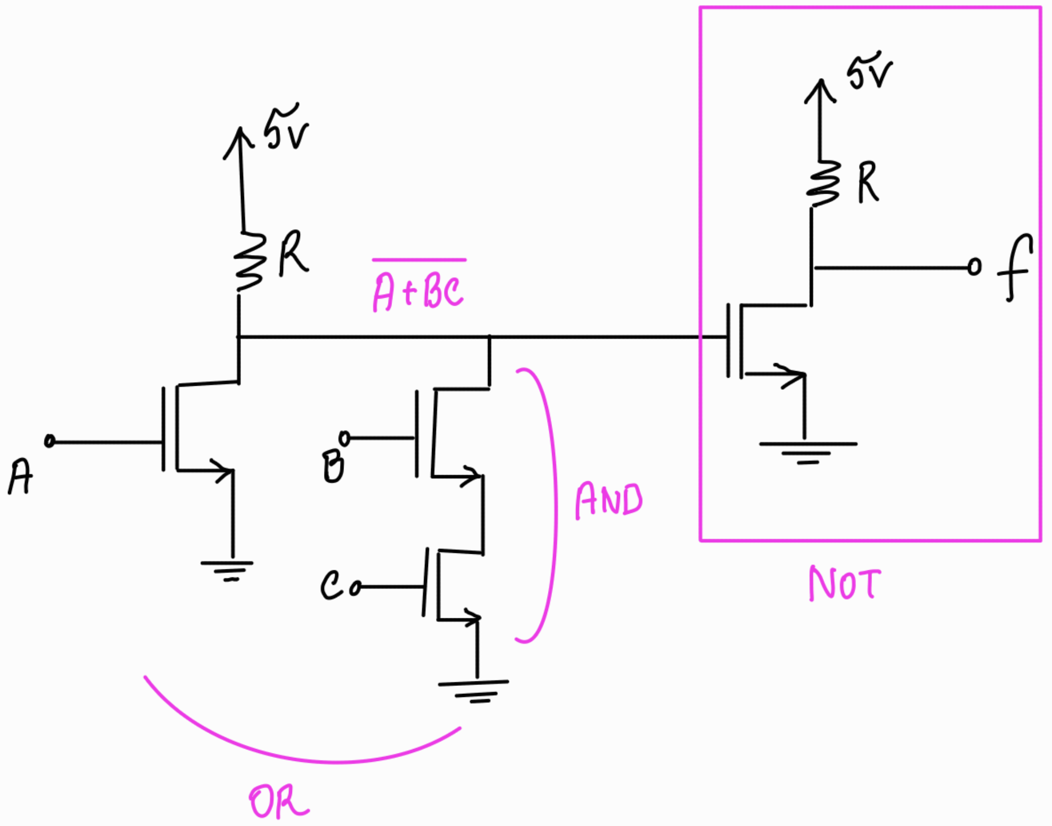
$$\Rightarrow 14i_2 = V_x + 5 - 1.4 \Rightarrow 14i_2 = V_x + 3.6$$

Some extra note —

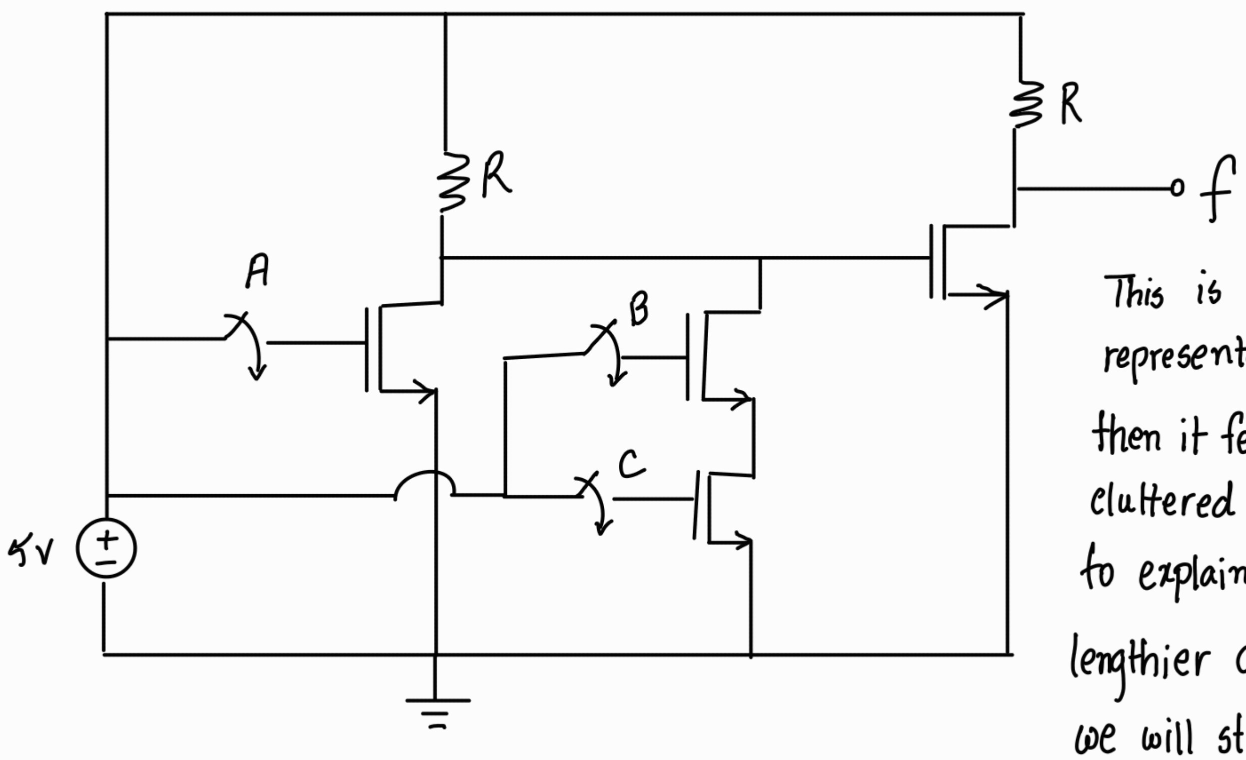
Why do we need alternate representation?

Let me present a small logic function implemented with ckt —

$f = A + BC$ [A OR (B AND C)]



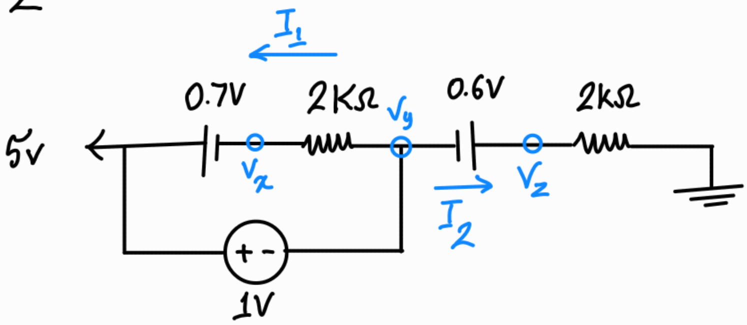
If you want to convert it to 250's representation it will look something like this —

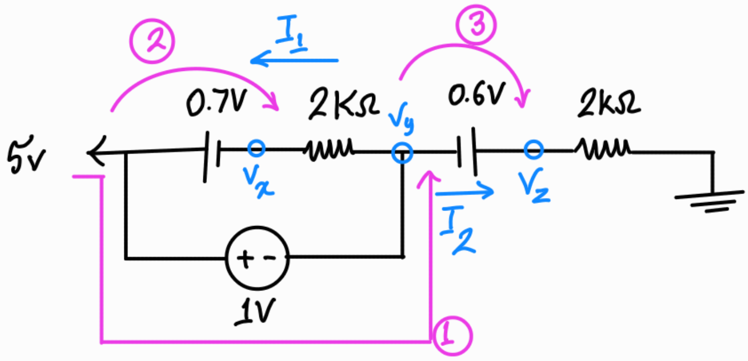


This is a very optimized representation and even then it feels a bit cluttered and is harder to explain. You can try lengthier ckt (later when we will study them) and you will appreciate line representation more.

One last note — I personally treat all 251 ckts like puzzles and generally try to solve them by node hopping, a bit of Ohm's law and a bit of KVL but I do realize most people find nodal analysis to be easier. I would suggest you to find your own comfort zone. I have left all the assignment problems open-ended except Q1, so feel free to go ham. If you are struggling with any of the maths, you can always consult me.

For the given ckt, find V_x, V_y, V_z, I_1 & I_2 —





① → $V_y = 5 - 1 = 4V$

② → $V_x = 5 - 0.7 = 4.3V$

③ → $V_z = V_y + 0.6 = 4.6V$

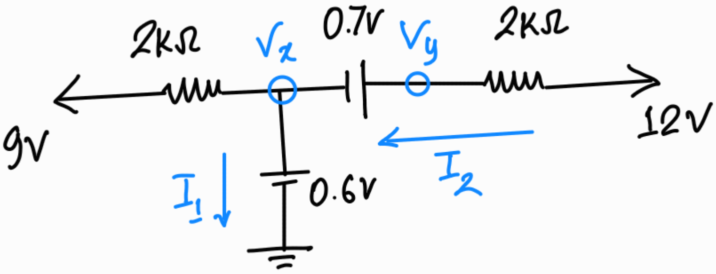
$$I_1 = \frac{V_y - V_x}{2} = \frac{4 - 4.3}{2} = -0.15mA$$

$$I_2 = \frac{V_z - 0}{2} = \frac{4.6}{2} = 2.3mA$$

the arrows represent node voltage transfer paths

You do NOT have to show the paths
I am showing them for you to understand

For the following ckt find V_x, V_y, I_1 & I_2 —



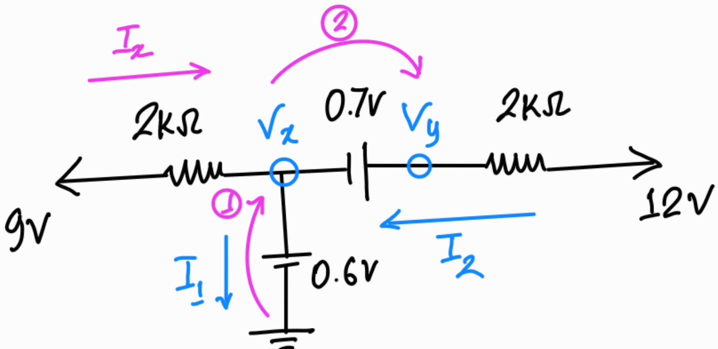
① $V_x = 0.6V$

② $V_y = V_x + 0.7 = 1.3V$

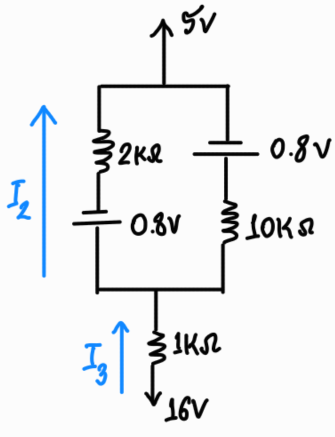
$$I_x = \frac{9 - V_x}{2} = \frac{8.4}{2} = 4.2mA$$

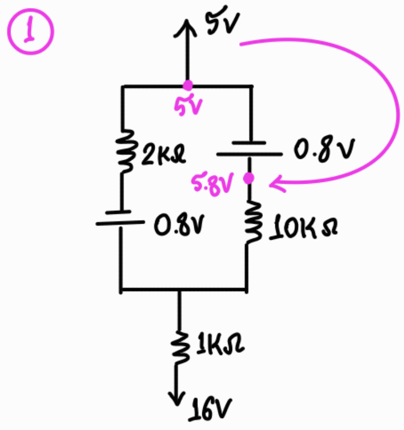
$$I_2 = \frac{12 - V_y}{2} = \frac{10.7}{2} = 5.35mA$$

$\therefore I_1 = I_x + I_2 = 9.55mA$

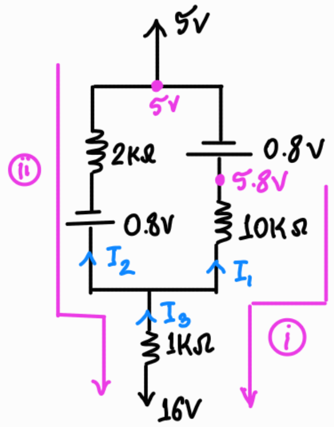


Find I_1, I_2, I_3 and all the node voltages





Transferring node voltages



KVL in line (i) —

$$-10I_1 - I_3 = 5.8 - 16$$

KVL in line (ii) —

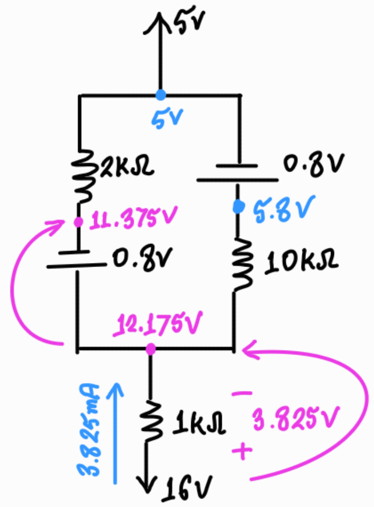
$$-2I_2 - 0.8 - I_3 = 5 - 16$$

Using KCL —

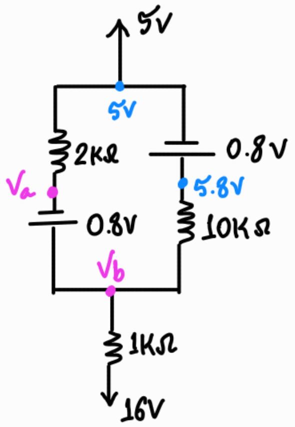
$$I_3 = I_1 + I_2$$

Solving the equations we get —

$$I_1 = 0.6375\,mA$$
$$I_2 = 3.1875\,mA$$
$$I_3 = 3.825\,mA$$



Using nodal analysis —



KCL @ SN a,b —

$$\frac{V_a-5}{2} + \frac{V_b-5.8}{10} + \frac{V_b-16}{1} = 0$$

$$\Rightarrow V_a(2^{-1}) + V_b(10^{-1}+1^{-1}) = 2.5+0.58+16$$

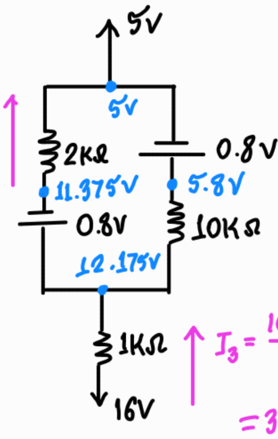
Voltage relation equation @ SN a,b —

$$-V_a + V_b = 0.8$$

Solving the equations —

$$V_a = 11.375V$$
$$V_b = 12.175V$$

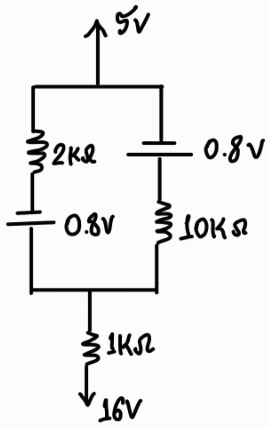
$$I_2 = \frac{11.375-5}{2} = 3.1875\,mA$$



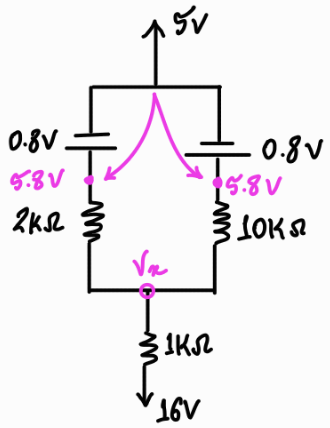
$$I_1 = \frac{12.175-5.8}{10} = 0.6375\,mA$$

$$I_3 = \frac{16-12.175}{1} = 3.825\,mA$$

You can make the nodal analysis even shorter —



Rearranging



KCL @ node x —

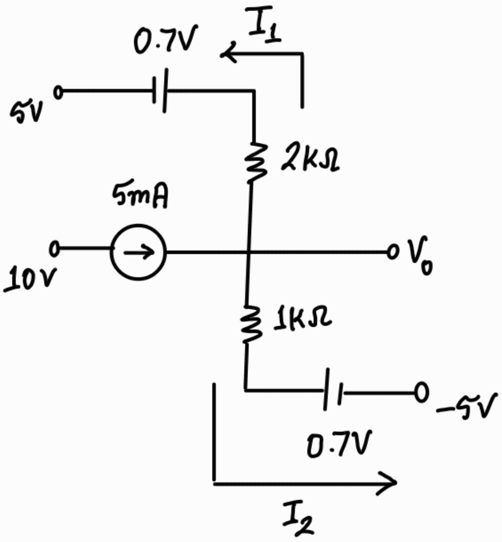
$$\frac{V_x - 5.8}{2} + \frac{V_x - 5.8}{10} + \frac{V_x - 16}{1} = 0$$

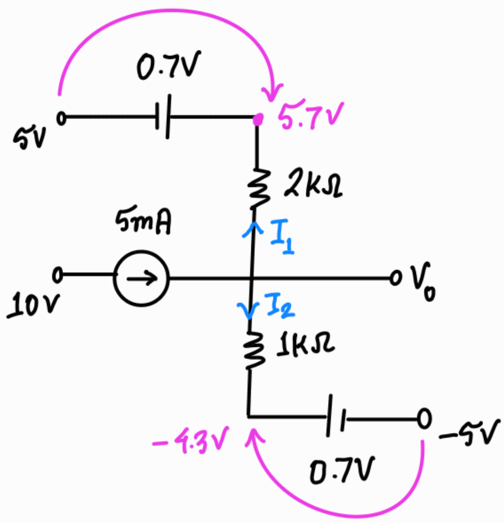
$\therefore V_x = 12.175$

Then just find I_1, I_2 & I_3 as usual.

Takeaway — There are multiple ways to solve the same problem.

Find I_1, I_2 & V_0





Using KCL @ the output node —

$$5 = I_1 + I_2 \rightarrow 5 = \frac{V_0 - 5.7}{2} + \frac{V_0 - (-4.3)}{1}$$

$\therefore V_0 = 2.367\text{ V}$

$$\therefore I_1 = \frac{2.367 - 5.7}{2} = -1.6665\text{ mA}$$

$$I_2 = \frac{2.367 - (-4.3)}{1} = 6.667\text{ mA}$$