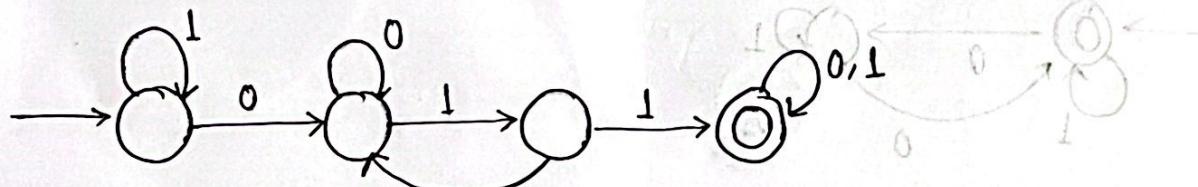
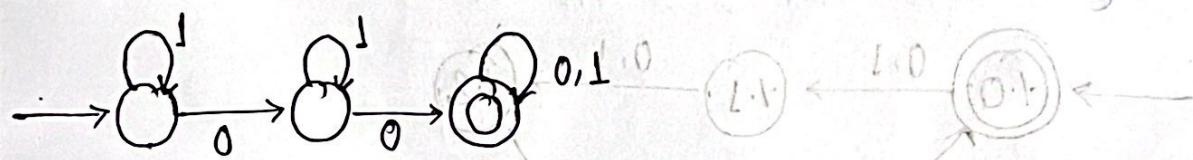


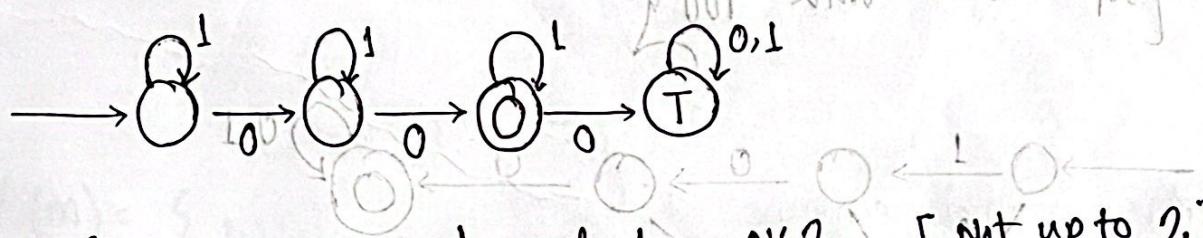
Type 1

 $L = \{ w \in \{0,1\}^*: w \text{ contains } 011 \}$


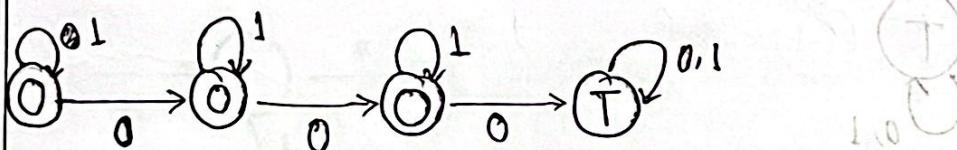
Type 2

 $L = \{ w | w \text{ contains at least two } 0's \}$ [two or more zeros]


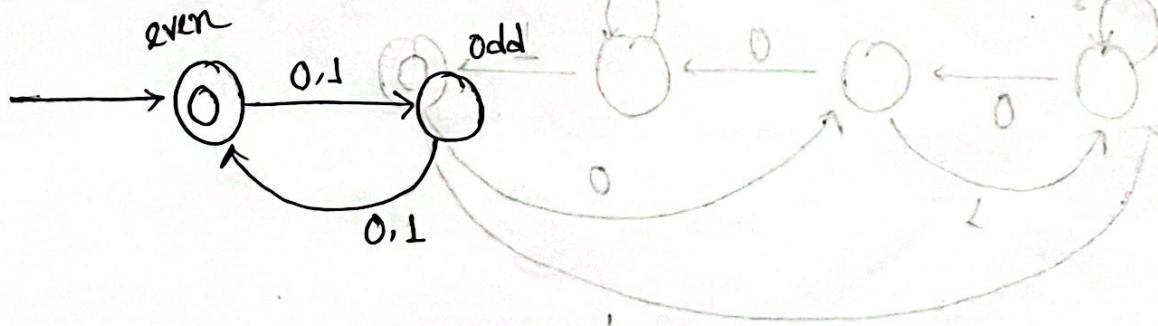
Type 3

 $L = \{ w | w \text{ contains exactly two } 0's \}$ [neither 1 or 3]


Type 4

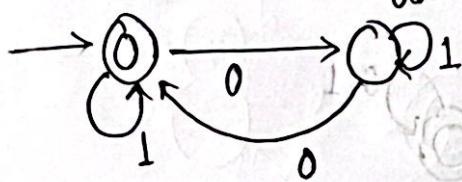
 $L = \{ w | w \text{ contains at most two } 0's \}$ [Not up to 2]


Type 5

 $L = \{ w | w \text{ length of } w \text{ is even} \}$


Type 6

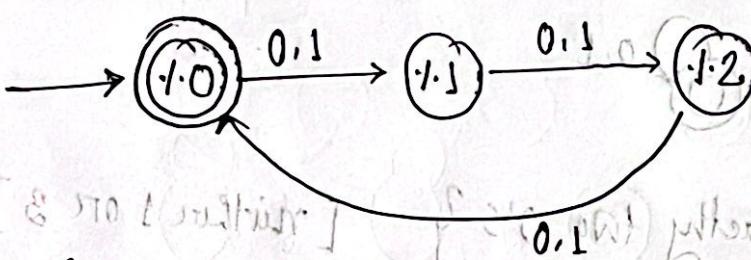
$L = \{ w \mid w \text{ count of 0's is even} \}$



$w : \{ 1, 0 \} \rightarrow \{ 0 \} = \{ \}$

Type 7

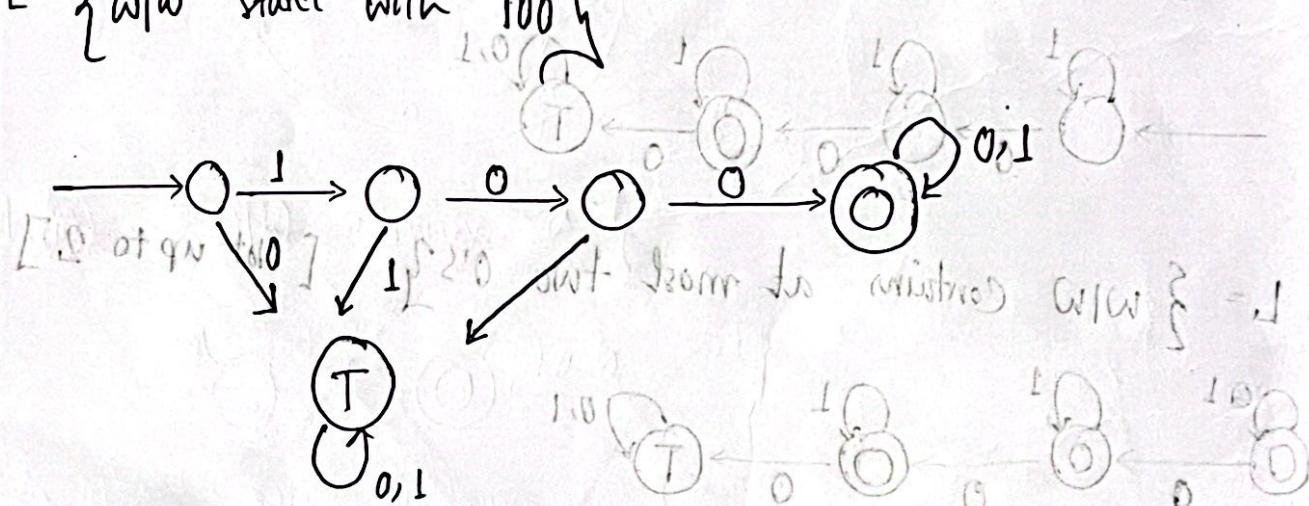
$L = \{ w \mid w \text{ multiple of 3} \}$



$w : \{ 0, 1 \} \rightarrow \{ 0 \} = \{ \}$

Type 8

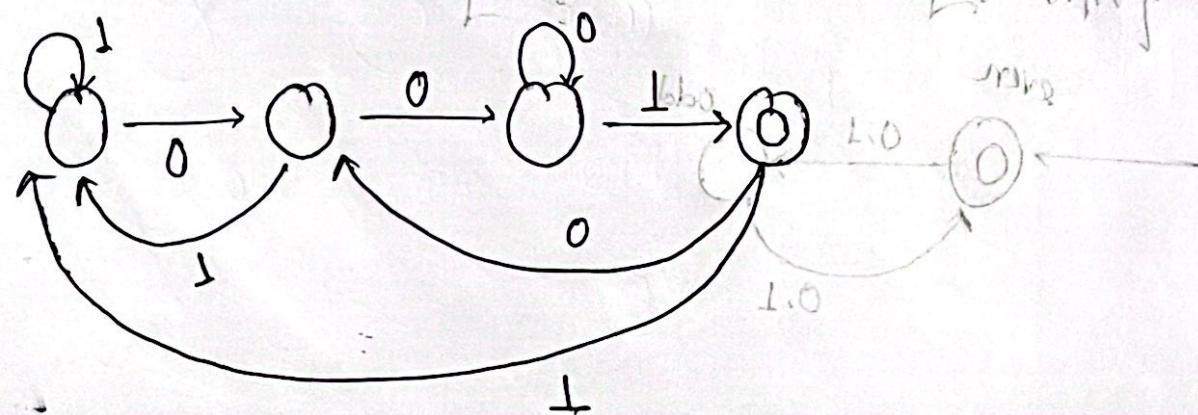
$L = \{ w \mid w \text{ start with 100} \}$



$w : \{ 0, 1 \} \rightarrow \{ 0 \} = \{ \}$

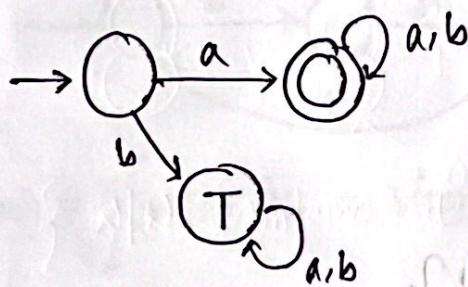
Type 9

$L = \{ w \mid w \text{ ends with 001} \}$

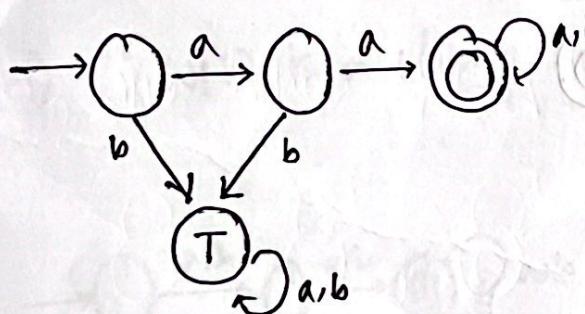


$w : \{ 0, 1 \} \rightarrow \{ 0 \} = \{ \}$

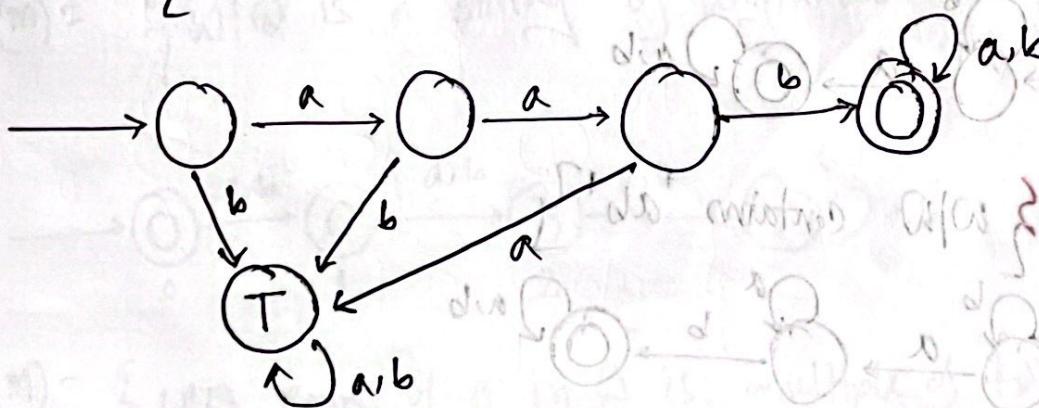
* $L(m) = \{ w \mid w \text{ starts with an 'a'} \}$



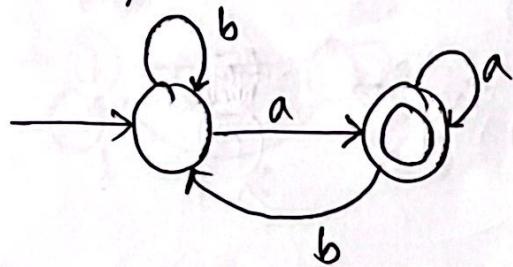
* $L(m) = \{ w \mid w \text{ starts with 'aa'} \}$



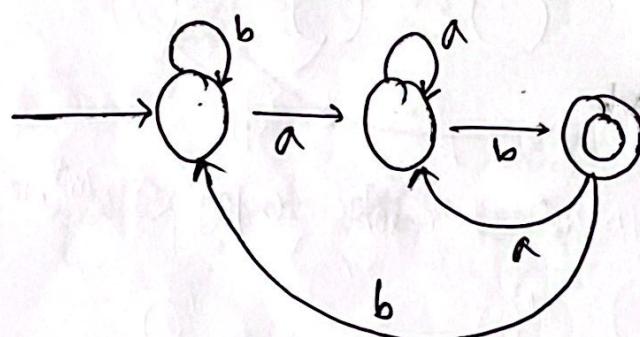
* $L(m) = \{ w \mid w \text{ starts with 'aab'} \}$



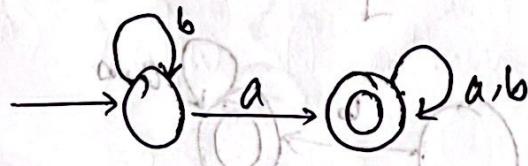
* $L(M) = \{ w | w \text{ ends with an 'a'} \}$



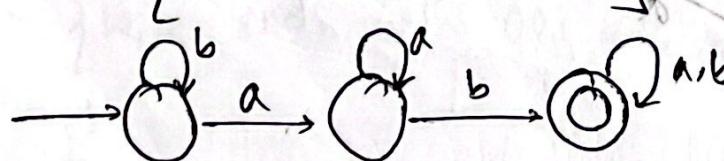
* $L(M) = \{ w | w \text{ ends with an 'ab'} \}$



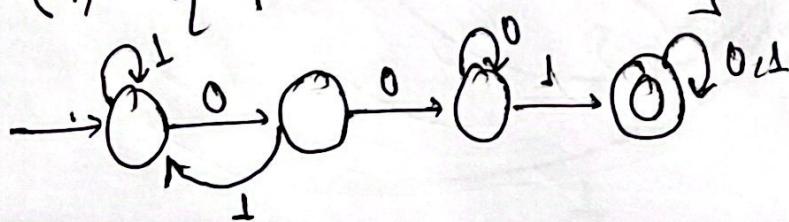
* $L(M) = \{ w | w \text{ contains 'a'} \}$



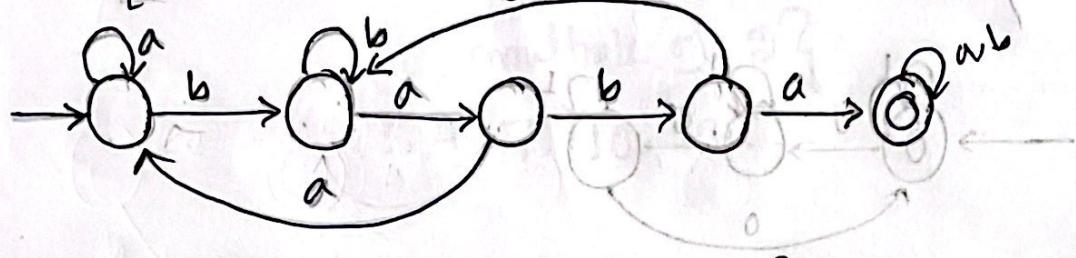
* $L(M) = \{ w | w \text{ contains 'ab'} \}$



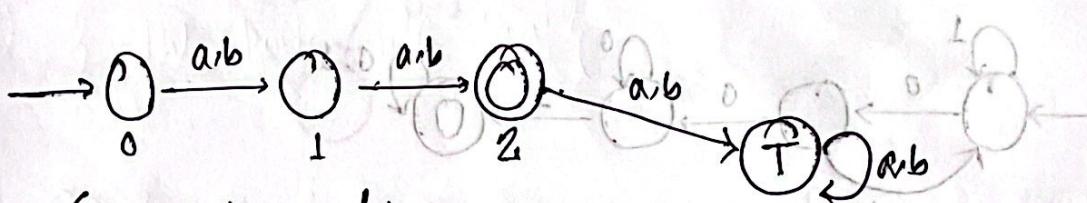
* $L(M) = \{ w | w \text{ contains '001'} \}$



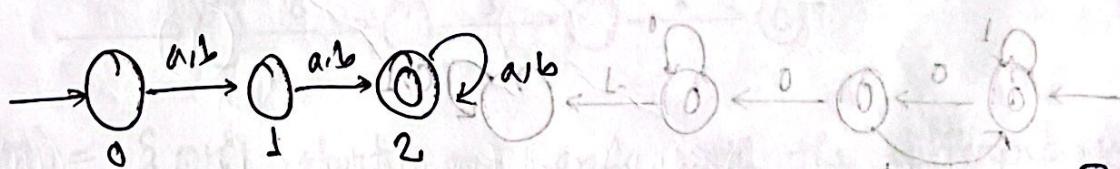
- * $L(m) = \{w \mid w \text{ contains substring 'babab'}\}$



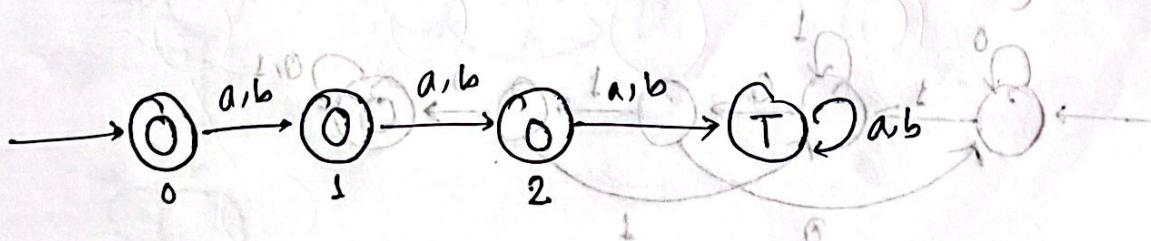
- * $L(m) = \{w \mid w \text{ is a string of length } 2\}$



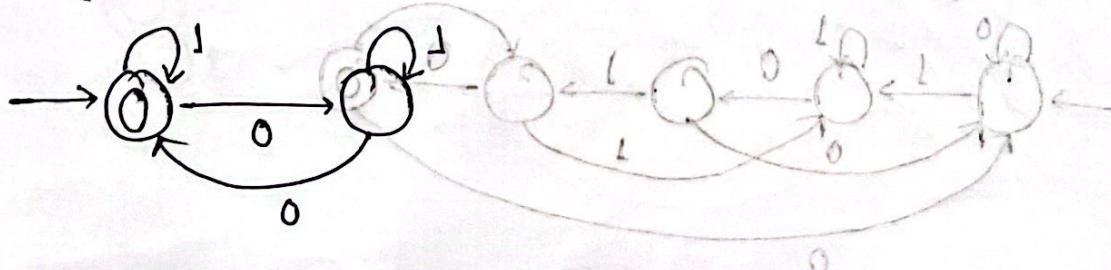
- * $L(m) = \{w \mid w \text{ is a string of length at least } 2\}$



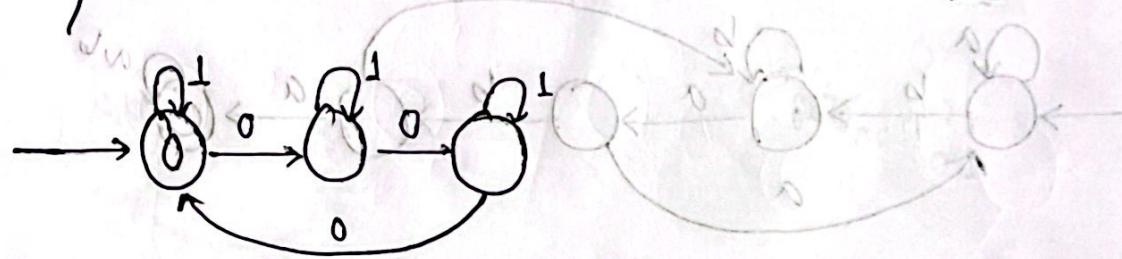
- * $L(m) = \{w \mid w \text{ is a string of length at most } 2\}$



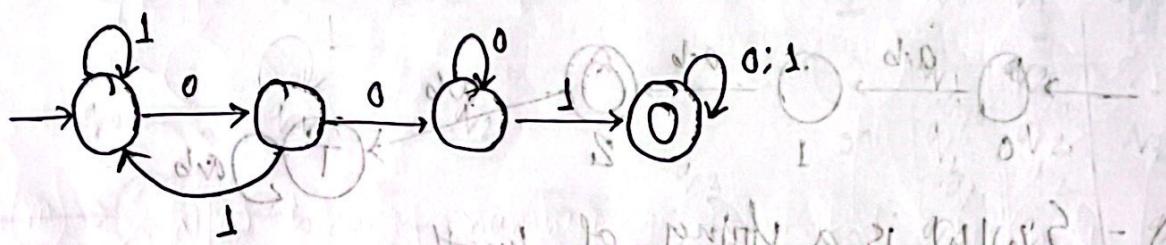
- * $L(m) = \{w \mid \text{count of '0' in } w \text{ is a multiple of two}\}$



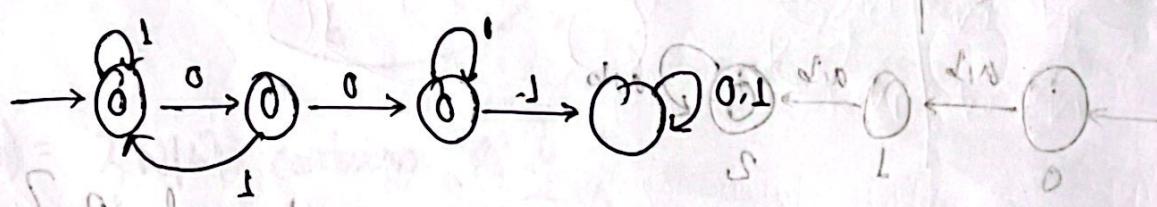
* $L(m) = \{ \omega | \omega \text{ count of } 000s \text{ is multiple of } 3 \}$



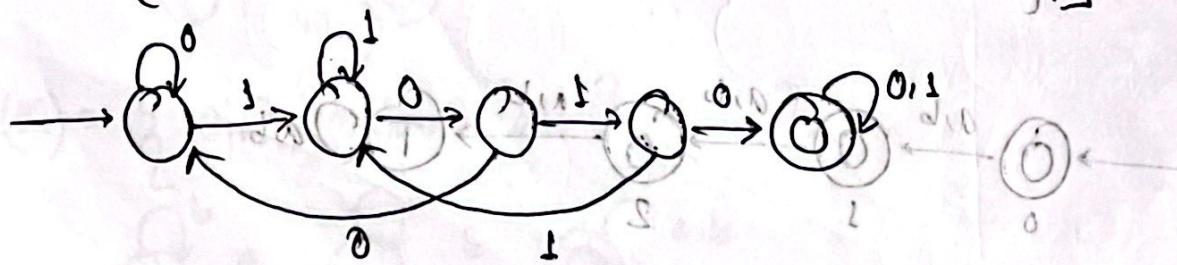
* $L(m) = \{ \omega | \omega \text{ contains } 001 \}$



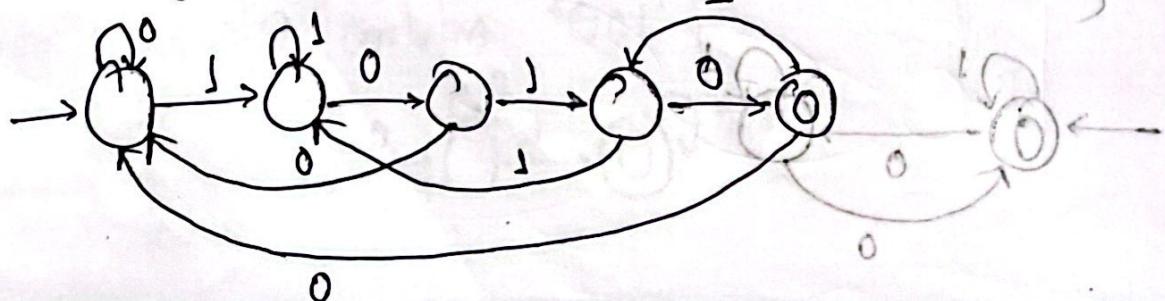
* $L(m) = \{ \omega | \omega \text{ does not contain } 001 \}$



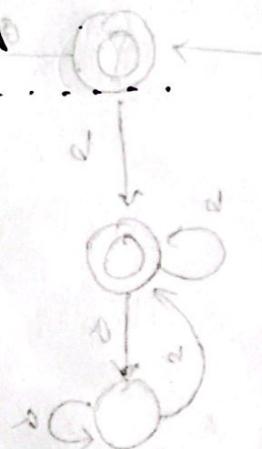
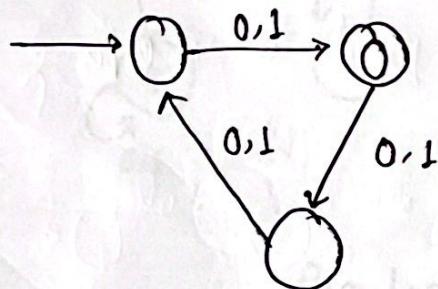
* $L(m) = \{ \omega | \omega \text{ contains } 1010 \text{ as substring} \}$



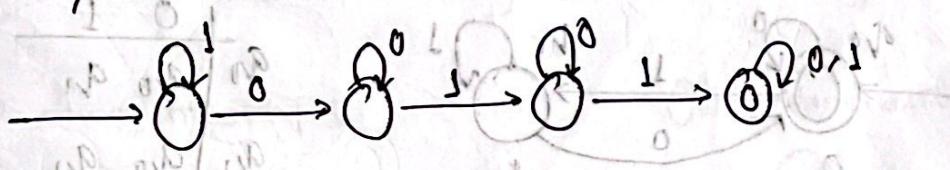
* $L(m) = \{ \omega | \omega \text{ ends with } 1010 \}$



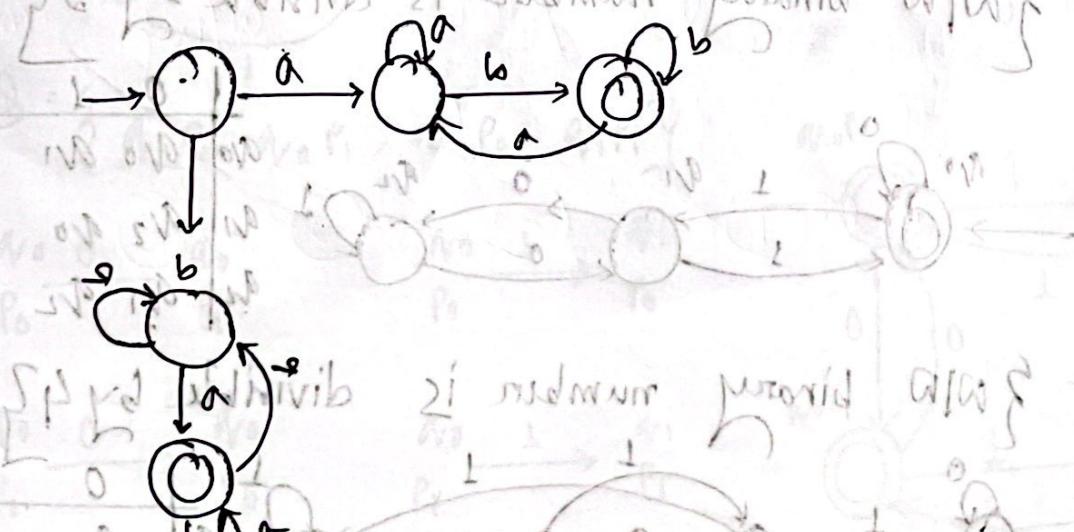
- $L(m) = \{w | w \text{ length of the string is one more than multiple of 3}\}$
 $1, 4, 7, 10, 13, \dots$



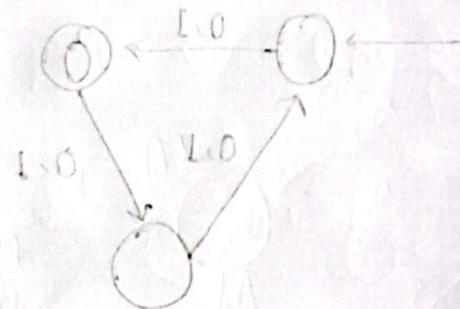
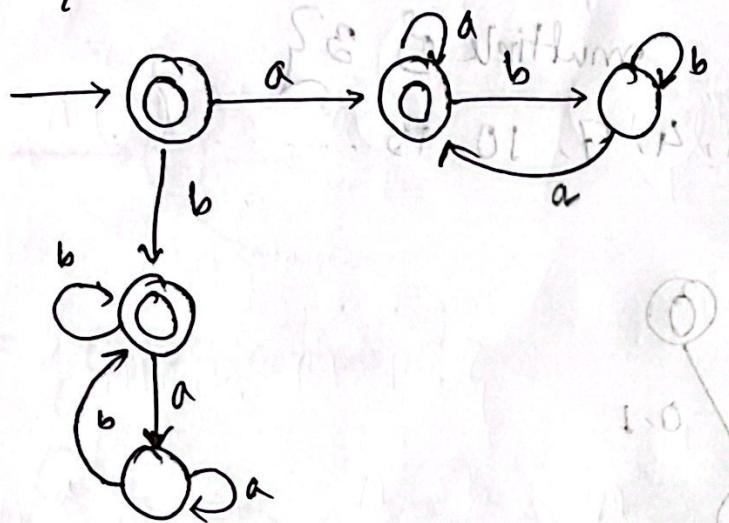
- $L(m) = \{w | w \text{ have 011 as subsequence}\}$



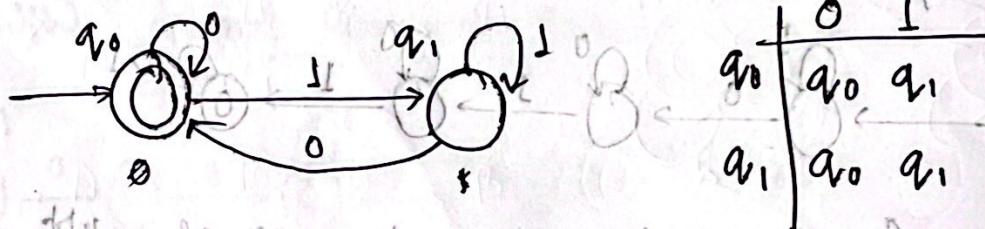
- $L(m) = \{w | w \text{ starts and ends with the different symbol}\}$



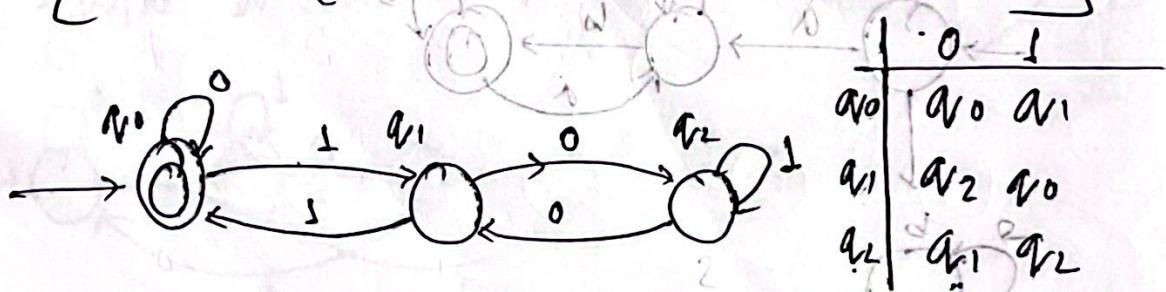
* $L(m) = \{w | w \text{ starts and ends with same symbol}\}$



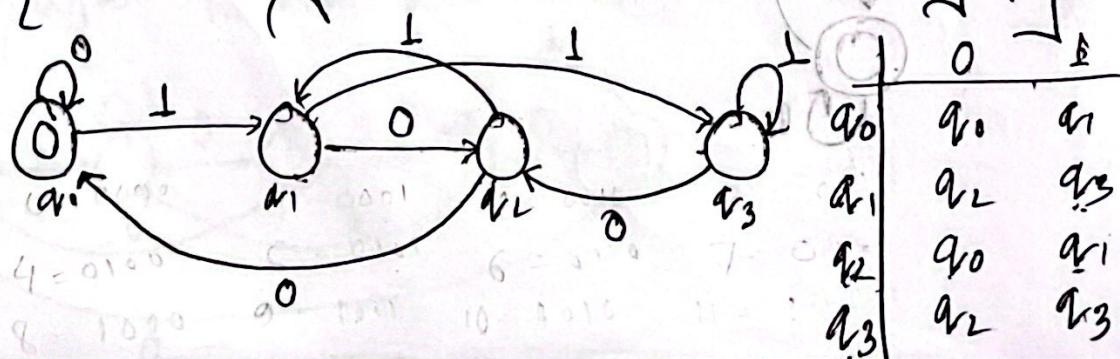
* $L(m) = \{w | w \text{ binary number is divisible by } 2\}$



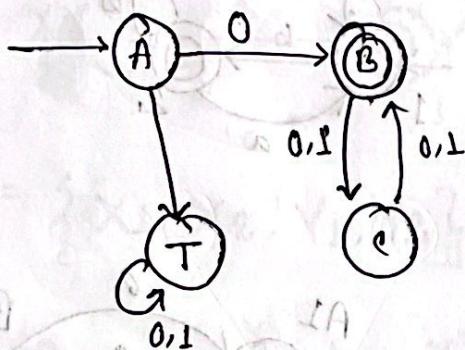
* $L(m) = \{w | w \text{ binary number is divisible by } 3\}$



* $L(m) = \{w | w \text{ binary number is divisible by } 4\}$



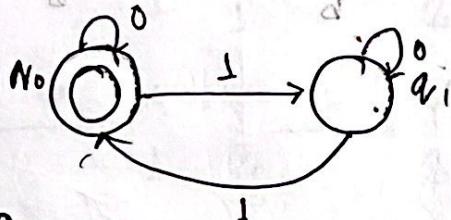
$L = \{w | w \text{ starts with } 0 \text{ and has odd length}\}$



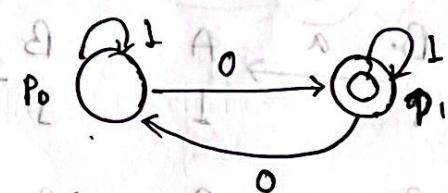
Regular Operation

$L = \{w | w \text{ has even number of 1's or odd number of 0's}\}$

Even number of 1's



Odd number of 0's



$Q = \{q_0p_0, q_0p_1, q_1p_0, q_1p_1\}$

$$q_0 \xrightarrow{0} q_0 \\ p_0$$

$$q_0 \xrightarrow{1} q_1 \\ p_0$$

$$q_0 \xrightarrow{0} q_0 \\ p_1$$

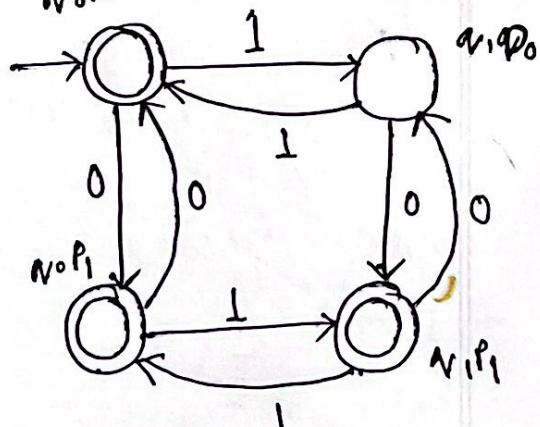
$$q_0 \xrightarrow{1} q_1 \\ p_1$$

$$q_1 \xrightarrow{0} q_1 \\ p_0$$

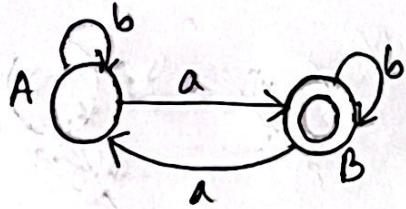
$$q_1 \xrightarrow{1} q_0 \\ p_0$$

$$q_1 \xrightarrow{0} q_0 \\ p_1$$

$$q_1 \xrightarrow{1} q_1 \\ p_1$$

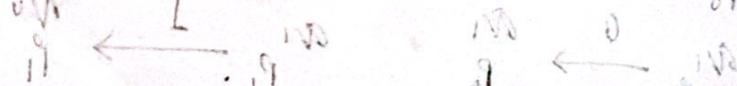
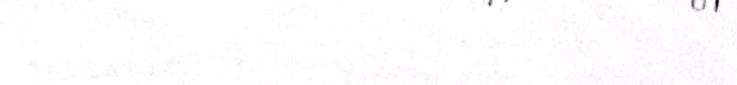
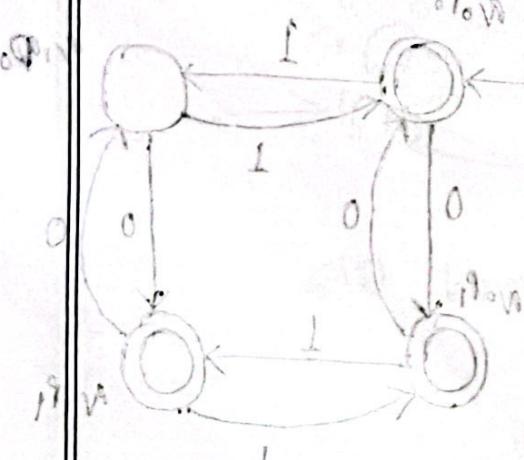
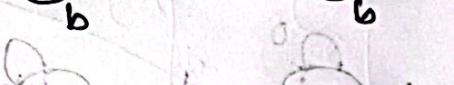
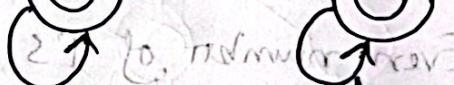
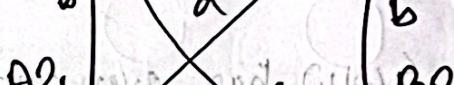
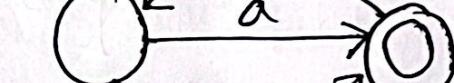
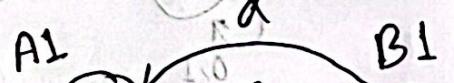
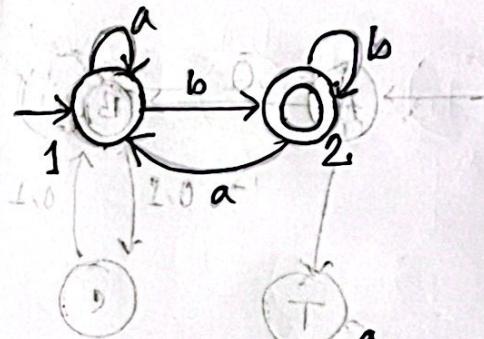
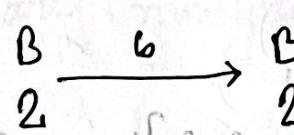
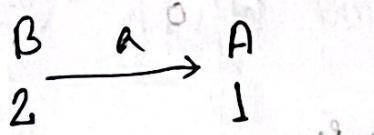
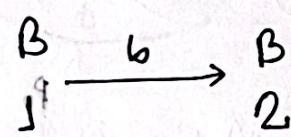
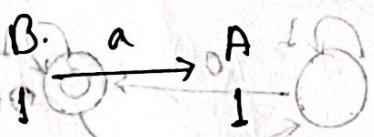
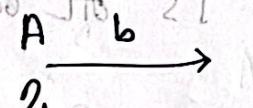
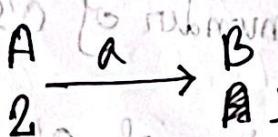
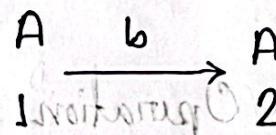
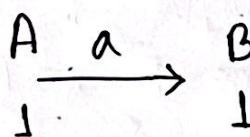


$L = \{ \text{W} \in \Sigma^* \text{ has an odd number of 'a's or ends with 'b'} \}$

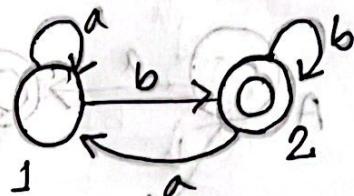
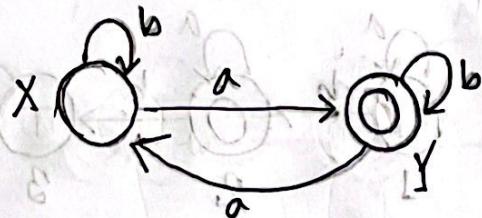


$ab \rightarrow \text{Initial stage}$

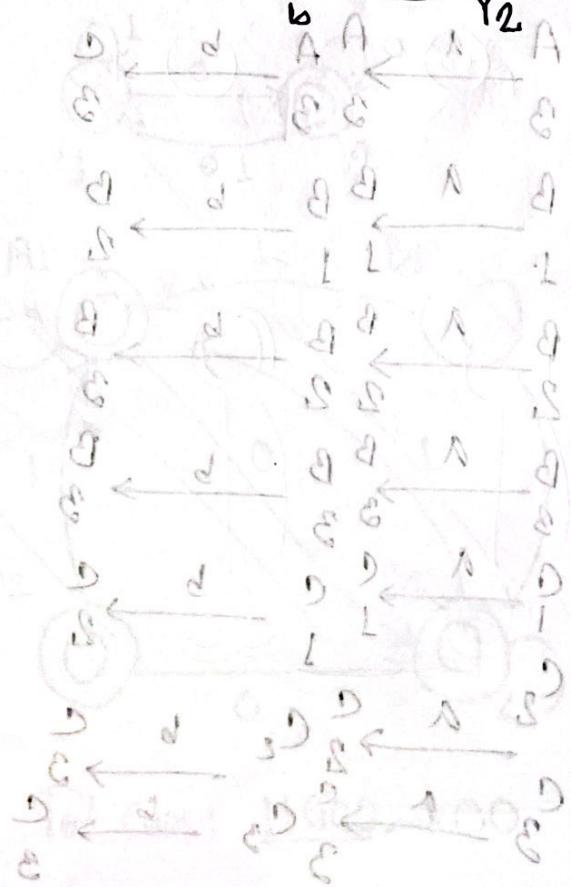
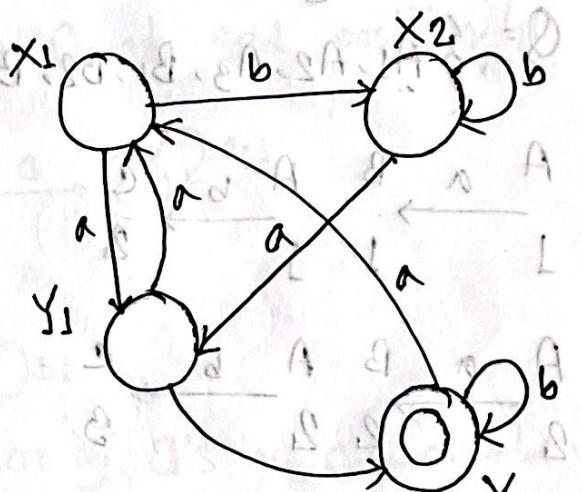
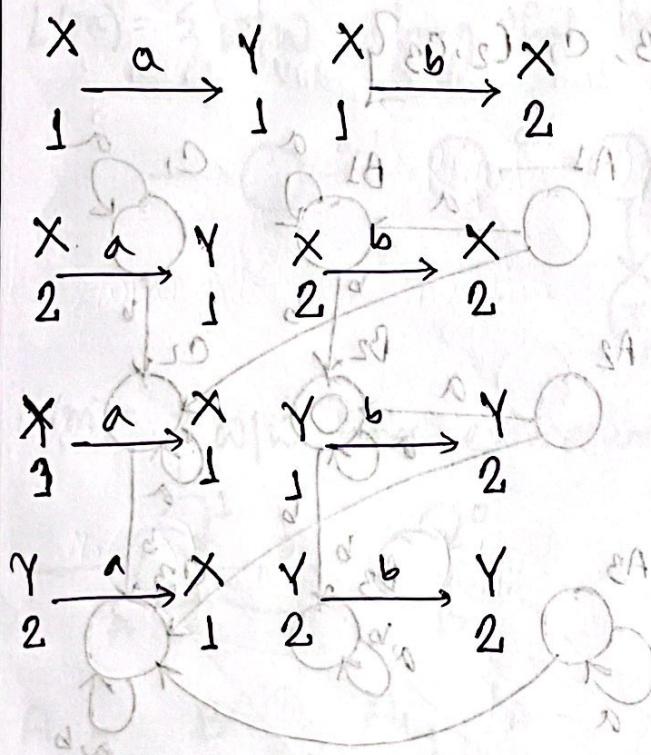
$$Q = \{ A_1, A_2, B_1, B_2 \}$$



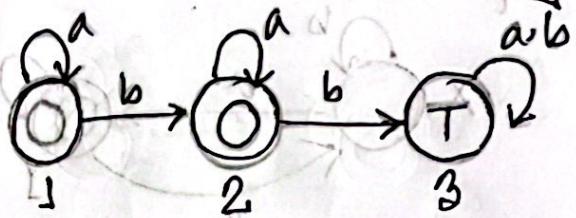
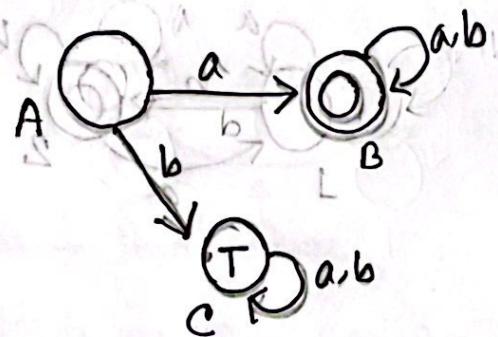
$L = \{w \mid w \text{ has an odd number of } a's \text{ and ends with } b\}$



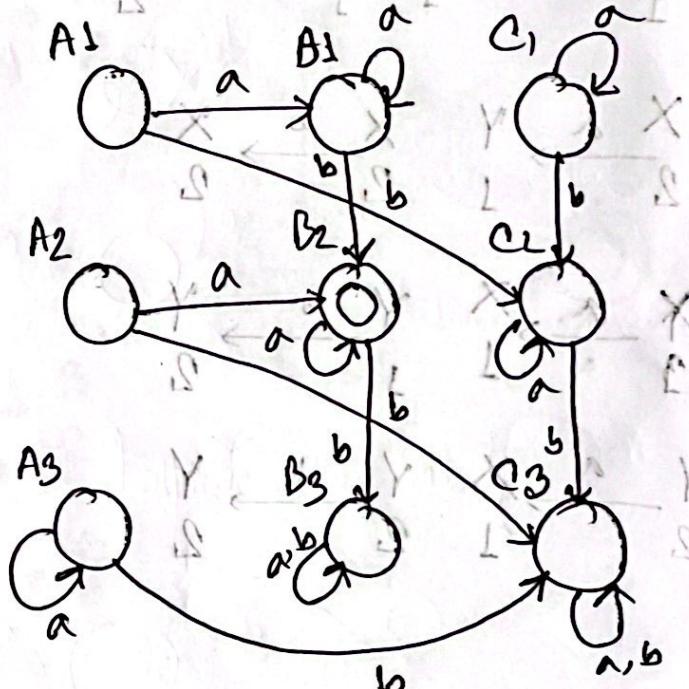
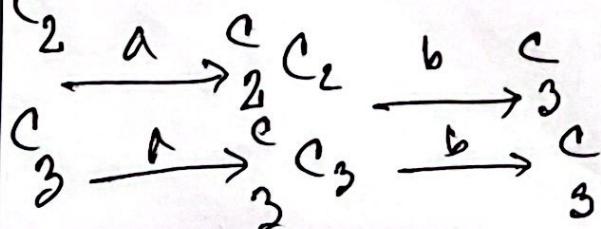
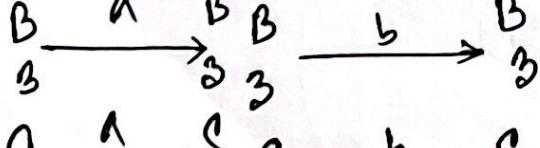
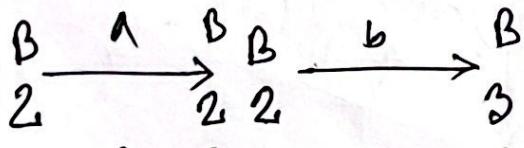
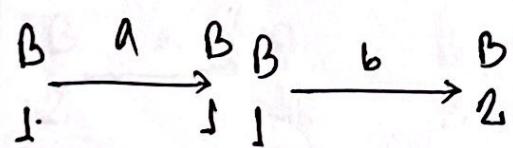
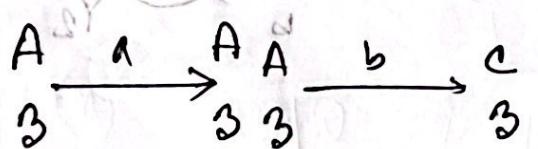
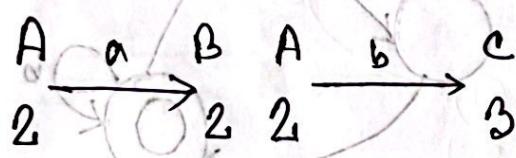
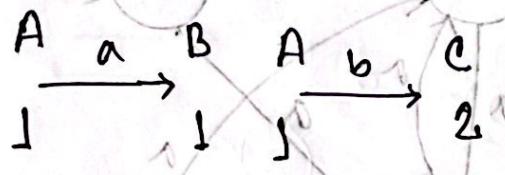
$$Q = \{x_1, x_2, y_1, y_2\}$$



$L = \{ \text{w} \mid \text{w} \text{ starts with an } a \text{ and has at most one } b \}$



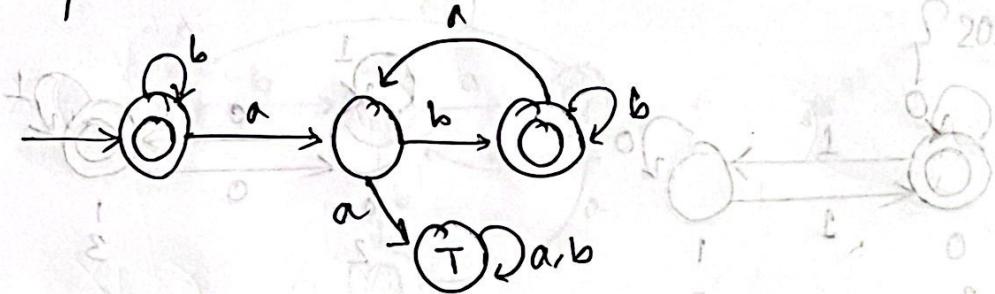
Q = $\{ A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3 \}$



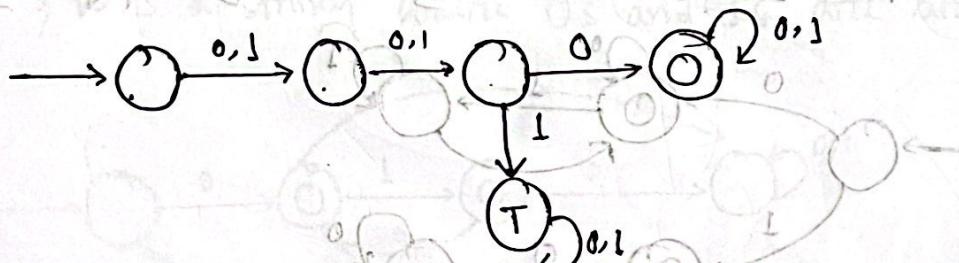
Subject :

Date :

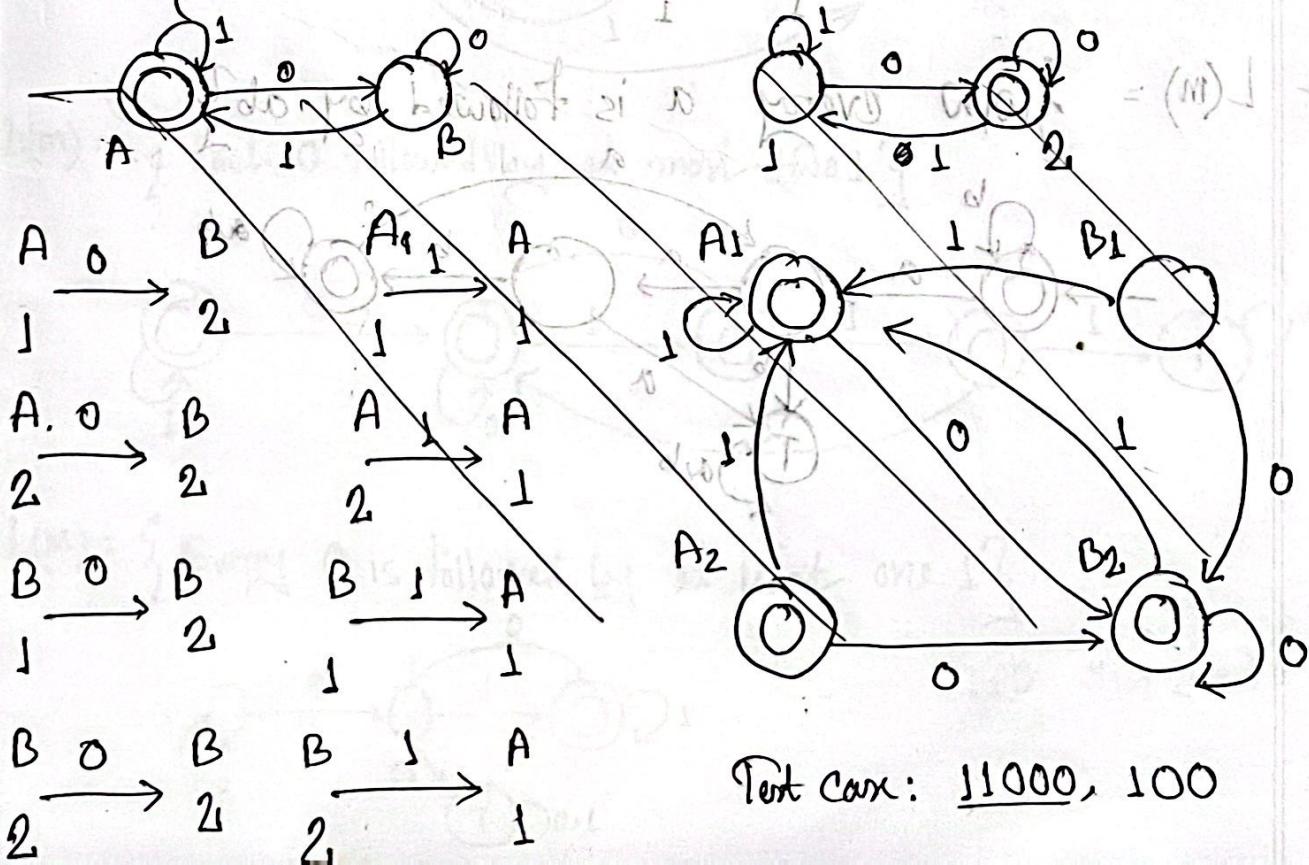
- * $L(M) = \{w \mid w \text{ every } a \text{ in } w \text{ is followed by one } b\}$



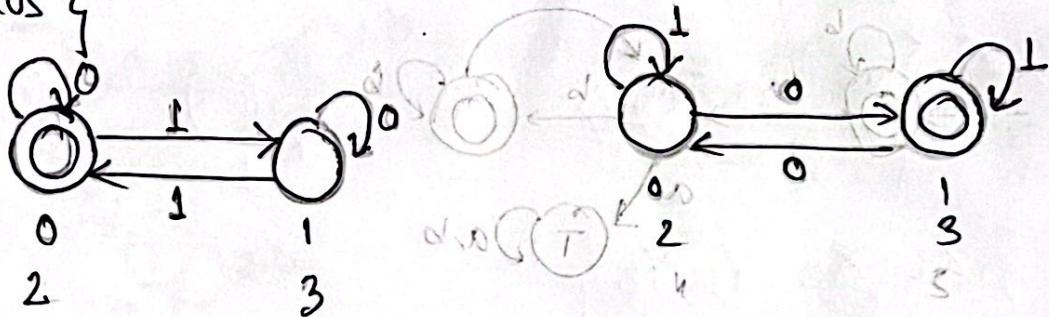
- * $L(M) = \{w \mid w \text{ string that has } 0 \text{ at 3rd position}\}$



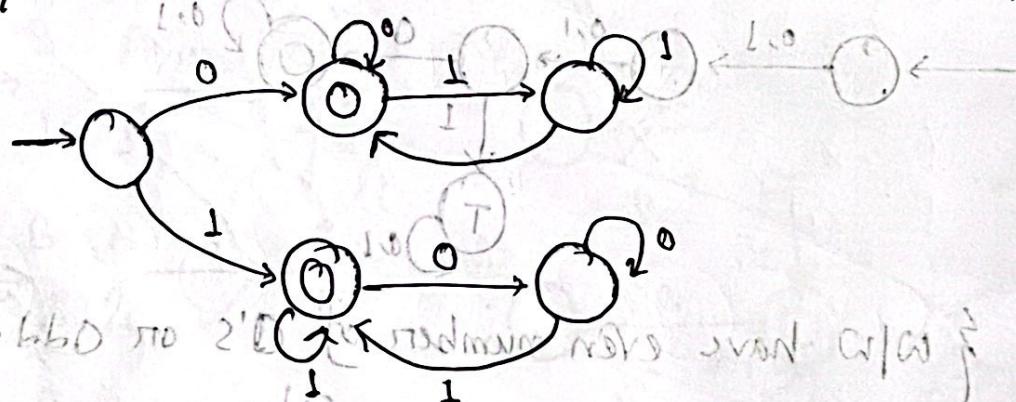
- * $L(M) = \{w \mid w \text{ have even number of } 0's \text{ on odd numbers of } 0's\}$



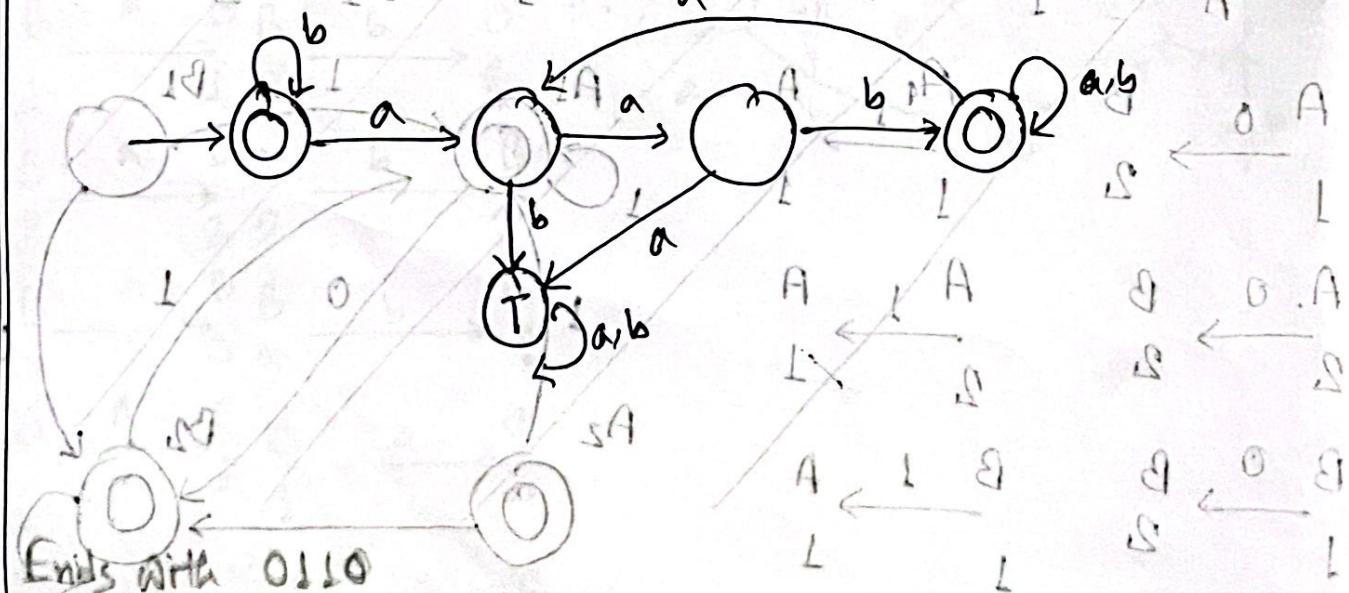
SWT have an even number of 1's or odd number of zeros?



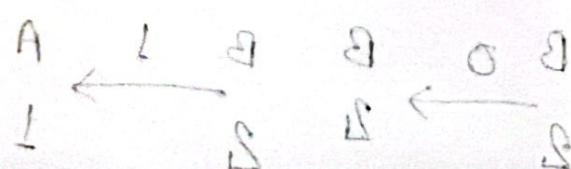
* $L(m) \{ w | w \text{ starts and ends with same symbol} \}$



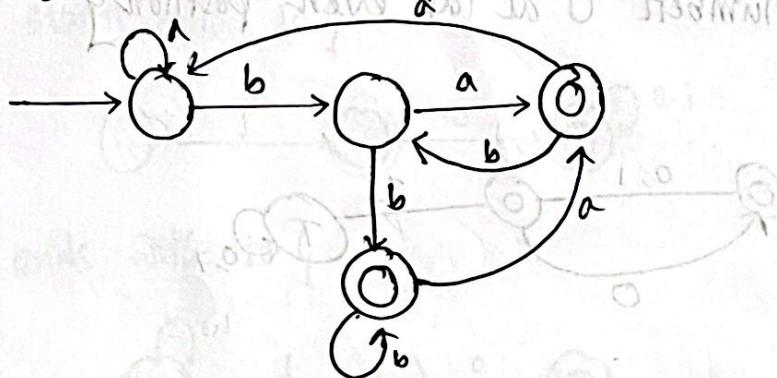
* $L(m) = \{ w | w \text{ every } a \text{ is followed by } ab \}$



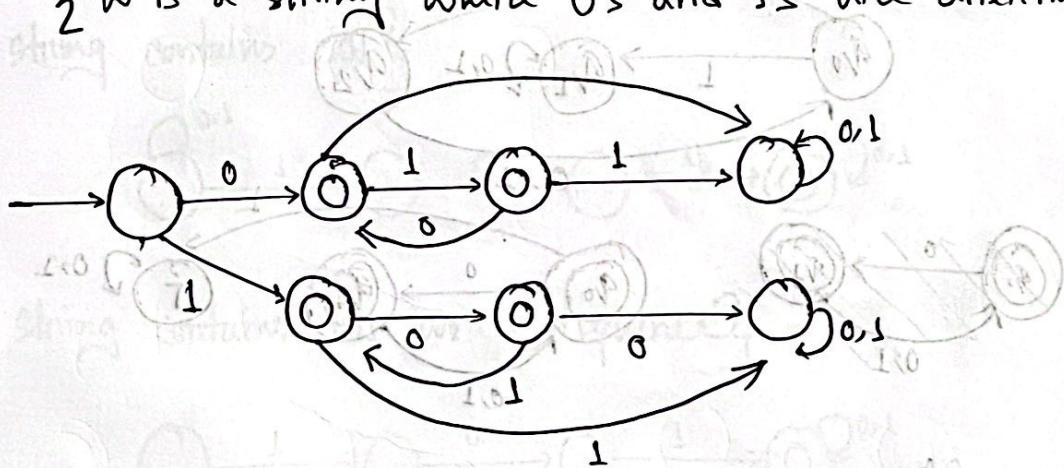
001, 00011, 110101



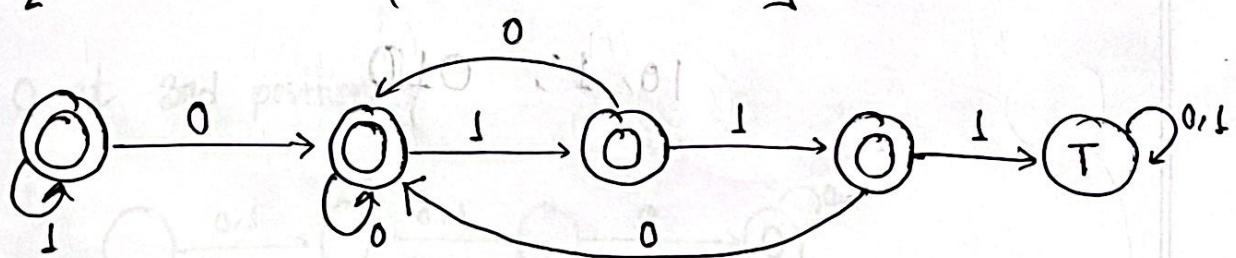
- 28 $L(m) = \{ \text{second last symbol is } b \}$



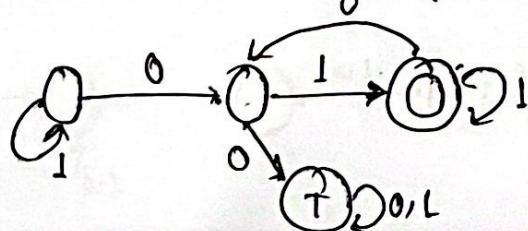
- * $L(m) = \{w \text{ is a string where } 0's \text{ and } 1's \text{ are alternate}\}$



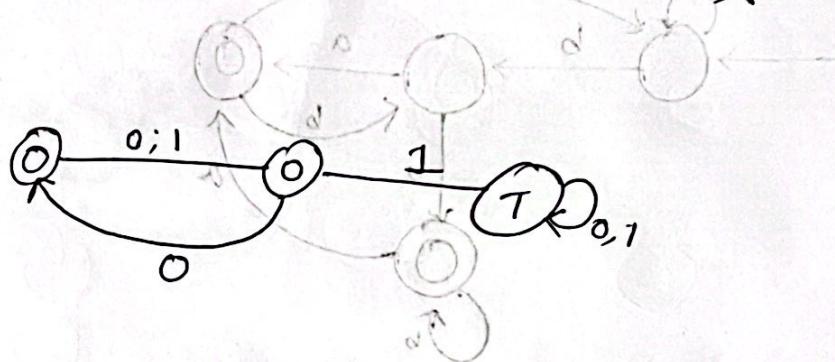
- $$L(m) = \{ \text{each } 0 \text{ followed by at most two } 1 \}$$



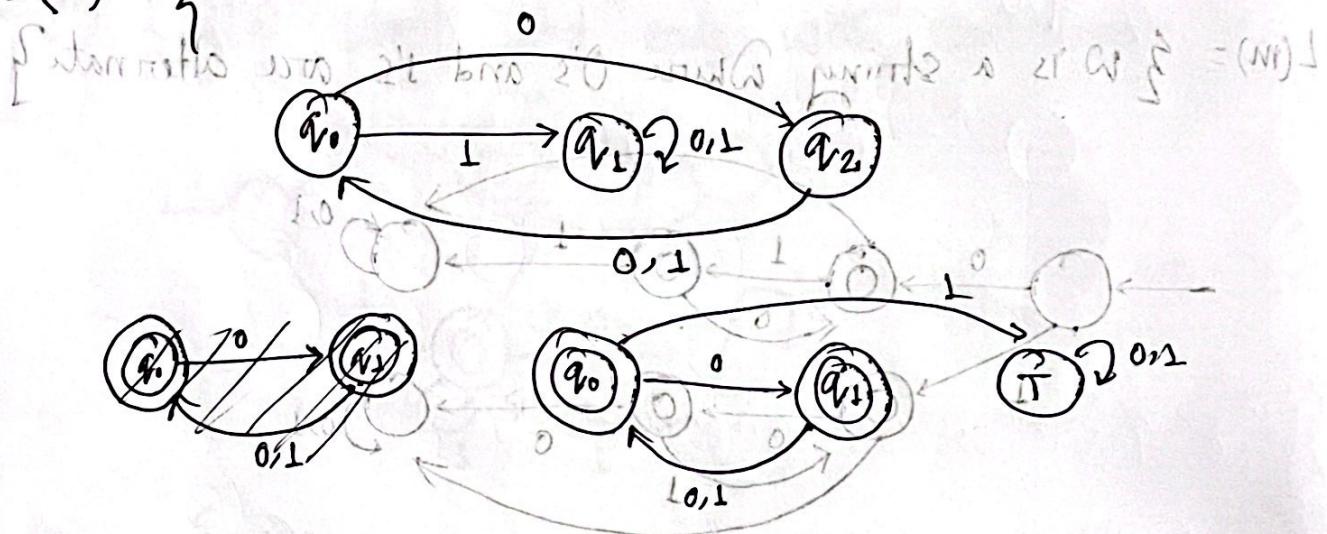
- $$L(M) = \{ \text{Every 0 is followed by at least one 1} \}$$



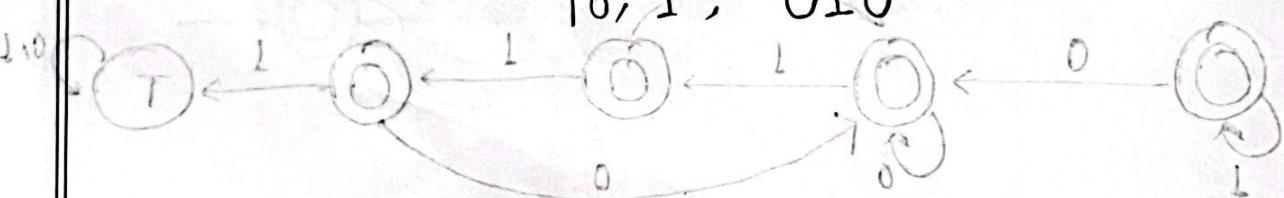
$L(M) = \{ \text{binary number } 0 \text{ at all even position} \}$



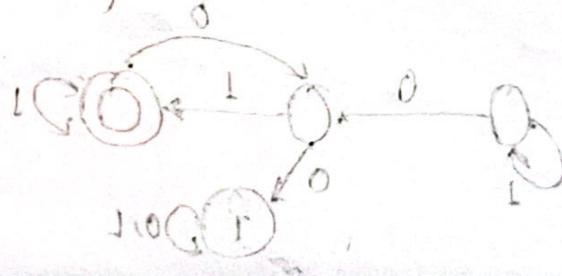
$L(M) = \{ \}$



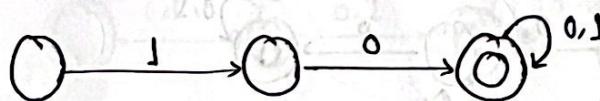
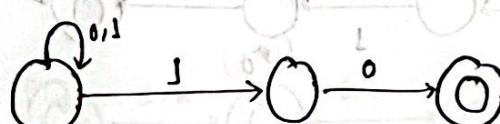
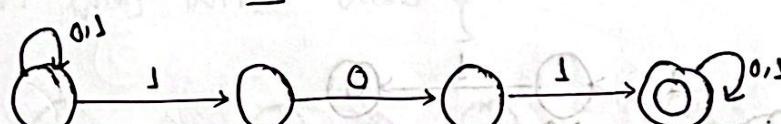
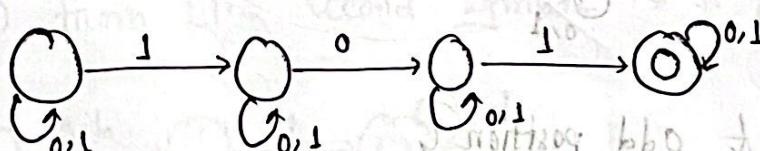
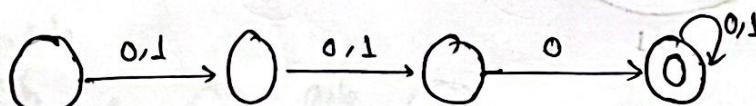
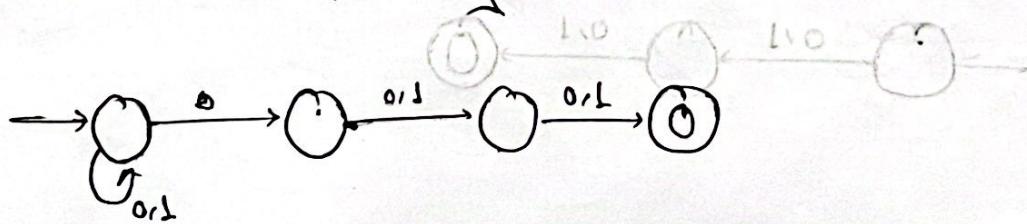
010
001
01



$\{ \text{1 zero need to be inserted in 0 binary} \} = (M)$

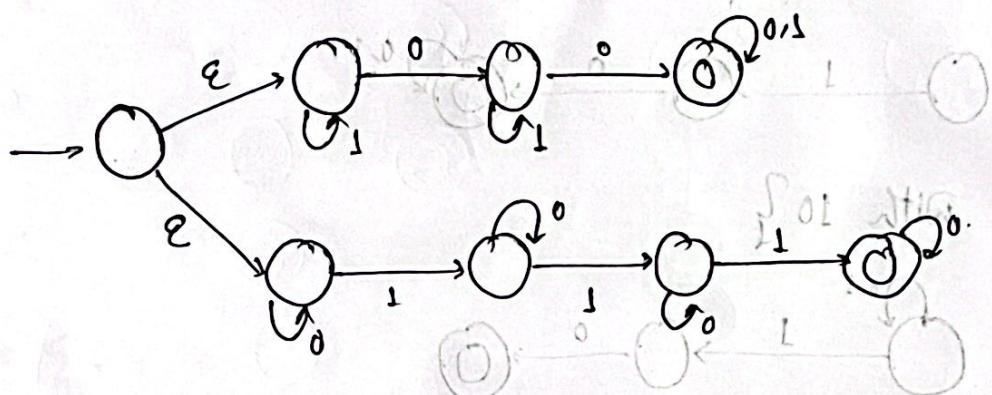


NFA

① $L = \{ \text{ starts with } 10 \}$ ② $L = \{ \text{ ends with } 10 \}$ ③ $L = \{ \text{ string contains } 101 \}$ ④ $L = \{ \text{ string contains } 101 \text{ as subsequence} \}$ ⑤ $L = \{ 0 \text{ at 3rd position} \}$ ⑥ $L = \{ 0 \text{ at 3rd last position} \}$ 

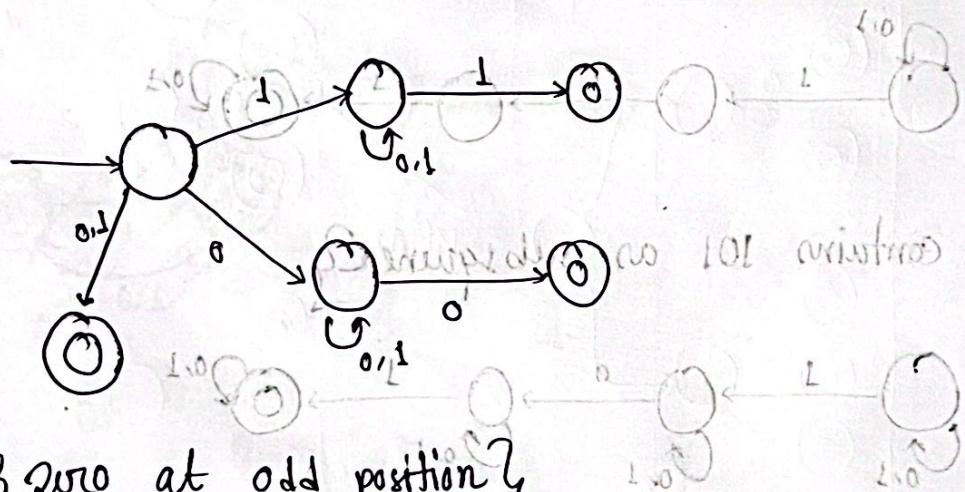
(7)

$L = \{ \text{At least two 0's on exactly 3 1's} \}$



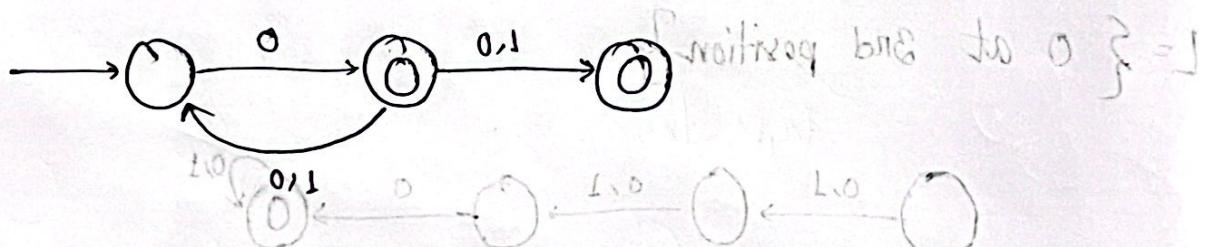
(8)

$L = \{ \text{start and ends with same symbol} \}$



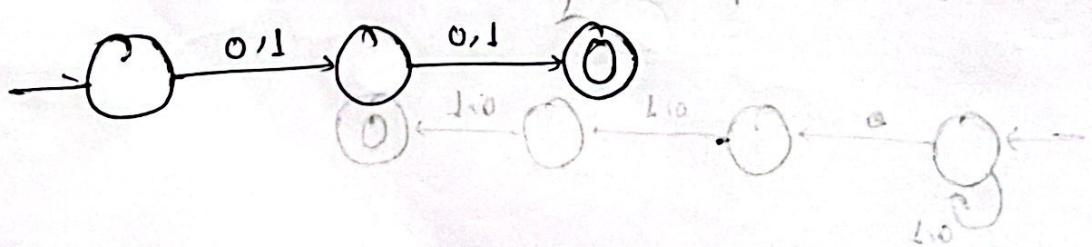
(9)

$L = \{ \text{0000 at odd position} \}$

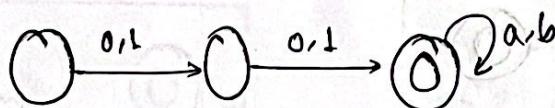


(10)

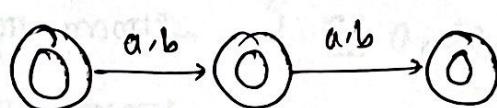
$L = \{ w \mid \text{the length of the string is exactly 2} \}$



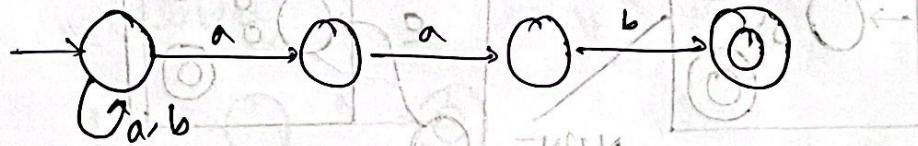
11) $L = \{ w \mid \text{the length of the string is at least 2} \}$



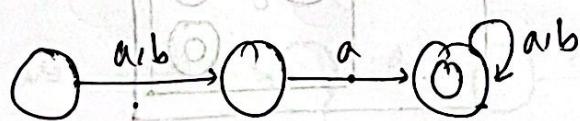
12) $L = \{ w \mid \text{the length of the string is at most 2} \}$



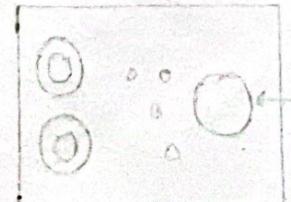
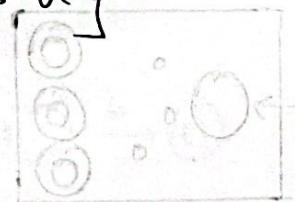
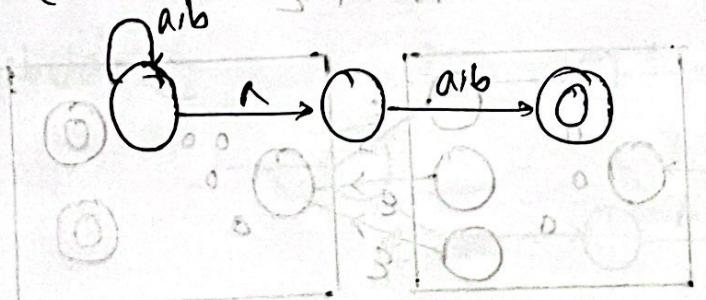
13) $L = \{ w \mid w \text{ ends with } aab \}$



14) $L = \{ w \mid w \text{ from LHS, second symbol is } a \}$



15) $L = \{ w \mid w \text{ from RHS, second symbol is } a \}$



Regular Expression to NFA

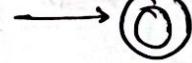
(a) $R = a$



(b) $R = b$



(c) $R = \epsilon$

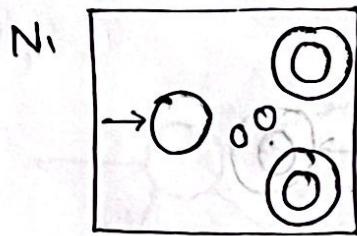


(d) $R = \emptyset$

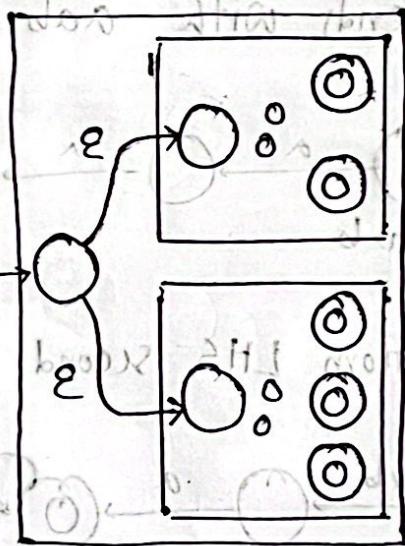
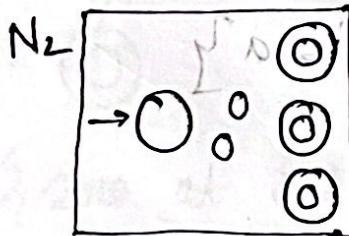


(e) $R = R_1 \cup R_2 = R_1 / R_2$

$N = N_1 \cup N_2$

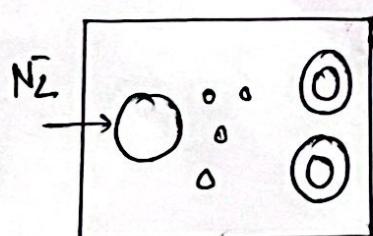
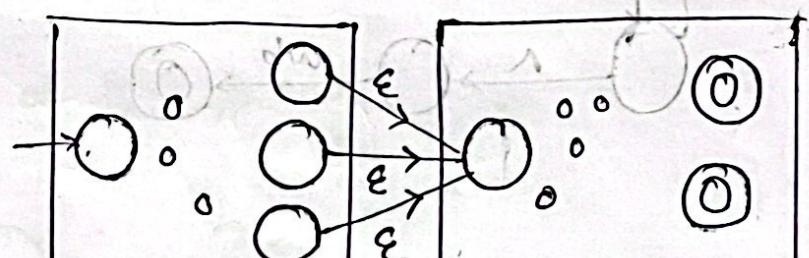
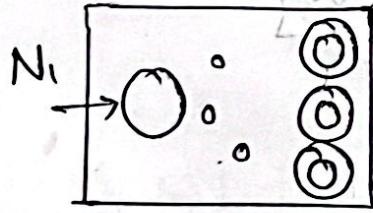


$N_1 \cup N_2$



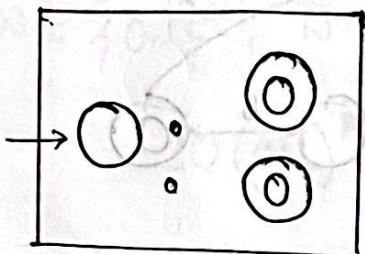
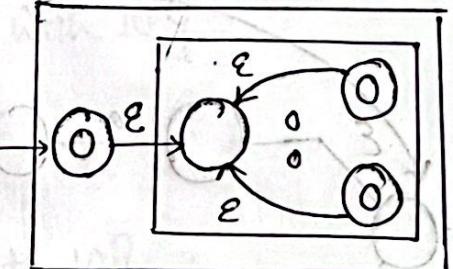
(f) $R = R_1 \circ R_2 = R_1 R_2$

$N = N_1 N_2$

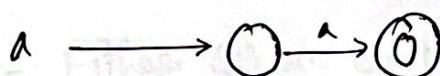
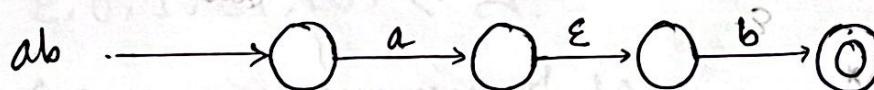
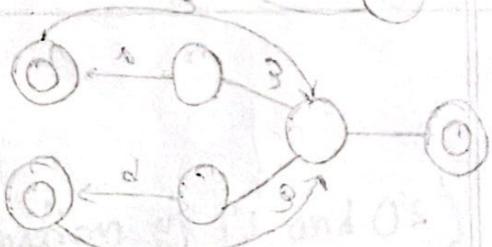
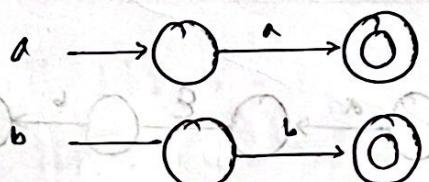
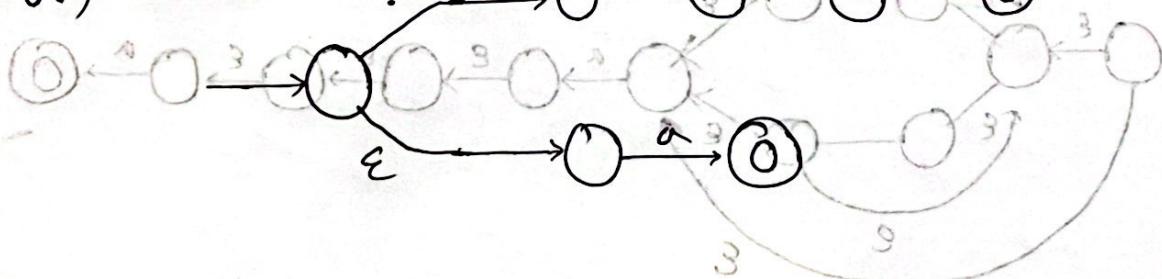


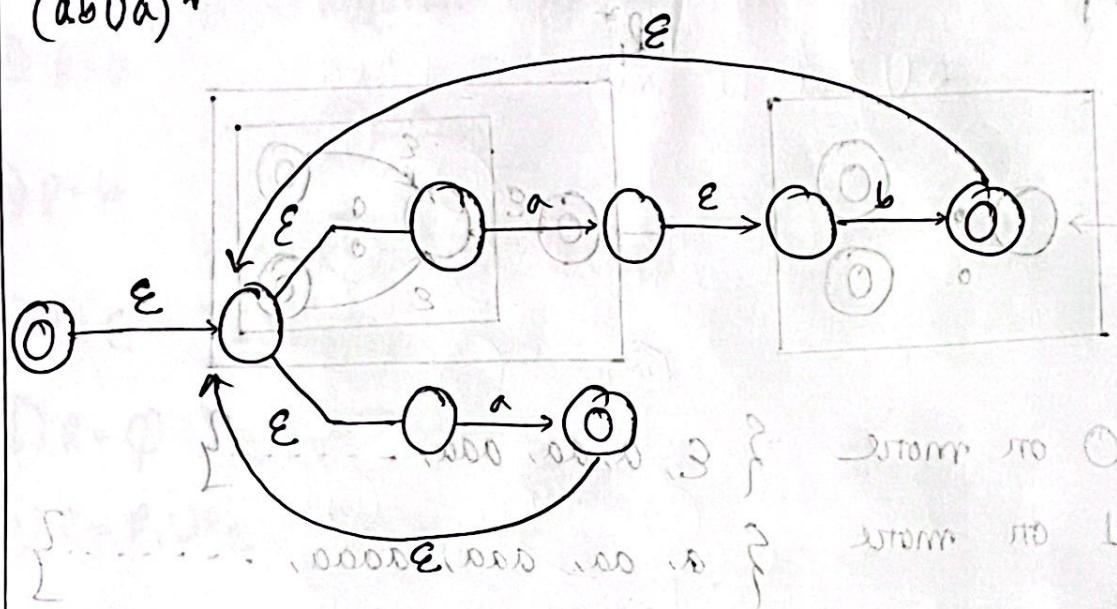
(29) $R = R_1^*$

N

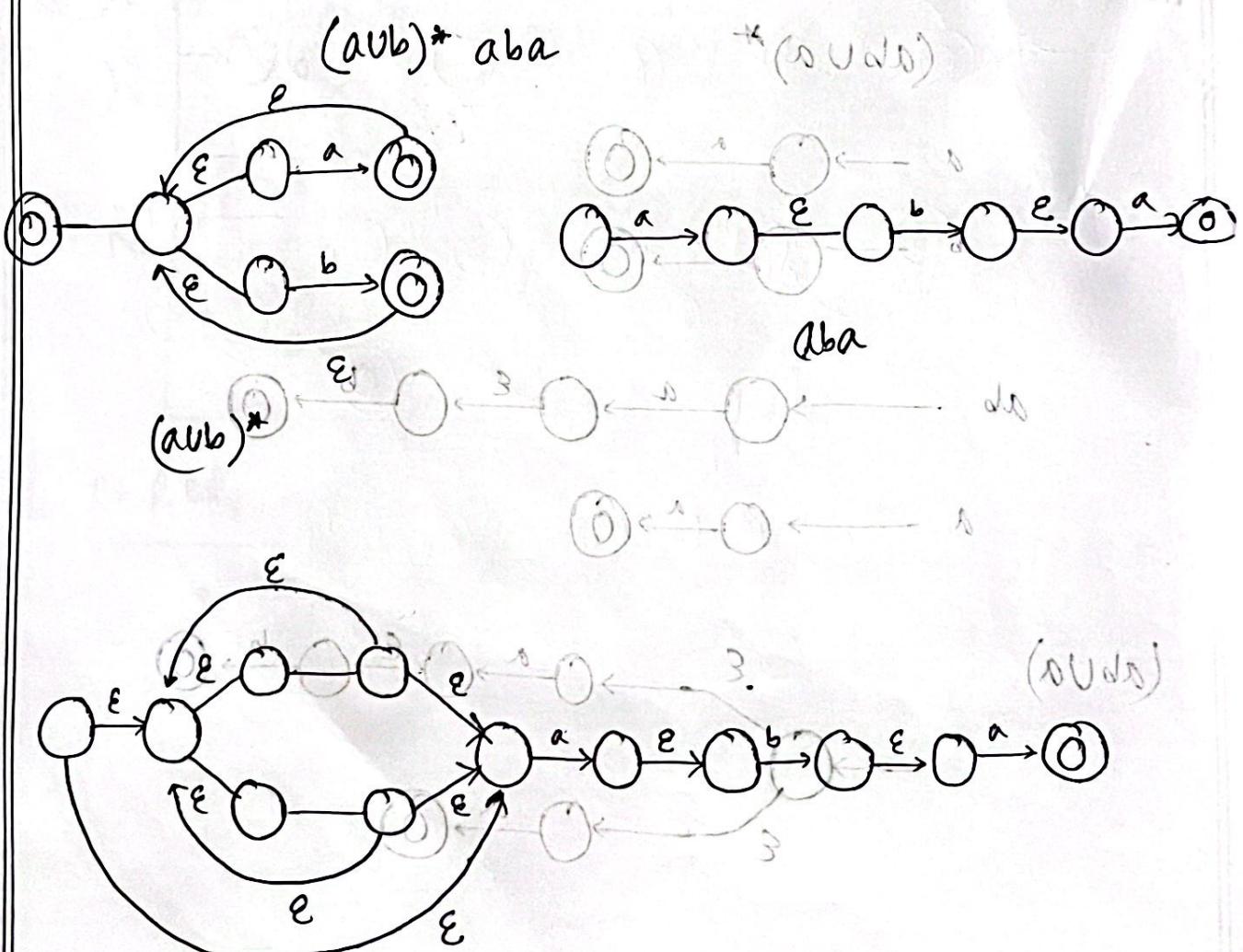
 R_1^*  (00010)
 $a^* = 0 \text{ or more}$ $\{ \epsilon, 0, 00, 000, \dots \}$
 $a^+ = 1 \text{ or more}$ $\{ 0, 00, 000, 0000, \dots \}$

Problem 1

 $(ab \cup a)^*$  $(ab \cup a)$ 

$(ab \cup a)^*$ 

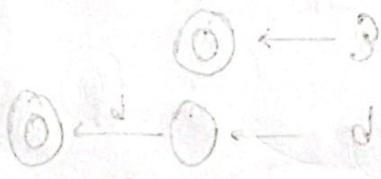
Problem 2:



Problem 3:

① $L = \{ w \in \{0,1\}^* : w \text{ starts with } 10 \}$

$$10(011)^*$$



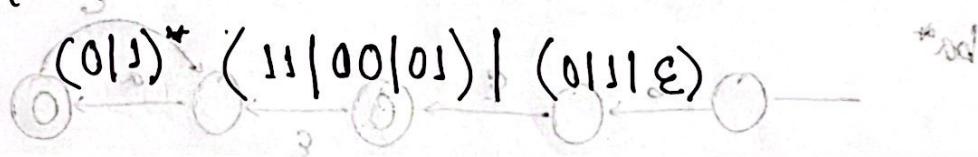
② $L = \{ w \in \{0,1\}^* : w \text{ ends with } 10 \}$

$$(011)^* 10$$



③ $L = \{ w \in \{0,1\}^* : w \text{ does not end with } 10 \}$

$$(011)^* (11|00|01) | (0111|)$$



$$1^* = "1, 11, 111, \dots"$$

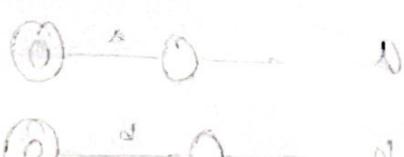
$$1^t = 1, 11, 111, \dots$$

$$(110)^* = \varepsilon, 0, 1, 01, 10, \text{ (Any combinations of 1's and 0's)}$$

$$(10)^t = \text{One or more occurrence of 10}$$

$$(00|01|10) = \text{Either } 00 \text{ or } 01 \text{ or } 10$$

$$(00|10) = \text{Either } 00 \text{ or } 10$$



Problem 3:

$$ba^* + \epsilon \text{ Same as } ba^* \cup \epsilon$$

$$\epsilon \rightarrow \textcircled{0}$$

$$b \rightarrow \textcircled{0} \xrightarrow{b} \textcircled{0}$$

$$a \rightarrow \textcircled{0} \xrightarrow{a} \textcircled{0}$$

$$a^* \textcircled{0} \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{a} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0}$$

$$ba^* \xrightarrow{b} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{a} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0}$$

$$ba^* + \epsilon \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{b} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{a} \textcircled{0}$$

Problem 4:

$$a^* \cup (ab)^* = aa^* \cup ab(ab)^* = (01 \mid 10 \mid 00)$$

$$a \rightarrow \textcircled{0} \xrightarrow{a} \textcircled{0}$$

$$b \rightarrow \textcircled{0} \xrightarrow{b} \textcircled{0}$$

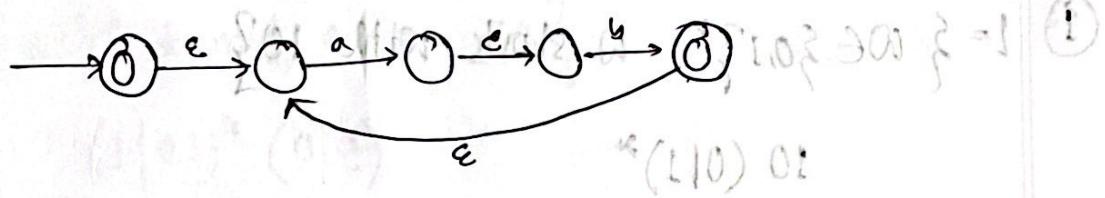
$$ab \rightarrow \textcircled{0} \xrightarrow{a} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{b} \textcircled{0}$$

$$aa^* \rightarrow \textcircled{0} \xrightarrow{a} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{a} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0}$$

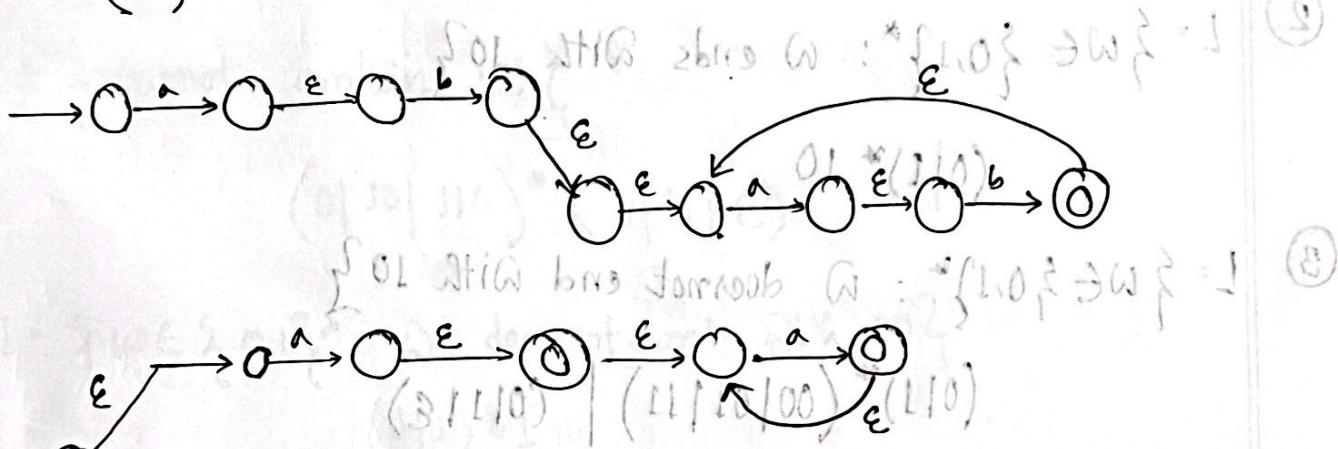
Subject :

Date :

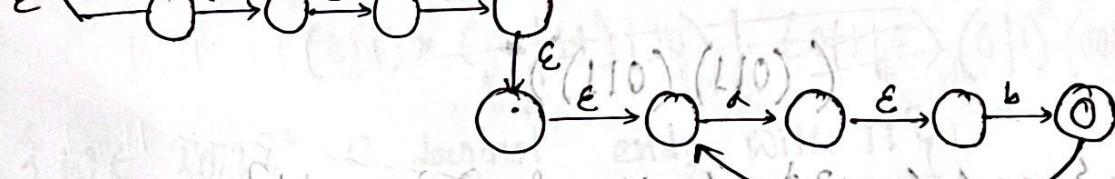
ab*



ab(ab)*



$(S1110) | (111000) (L10)$



$\{bb0\} \{ab\} \{aabb\} = 1$

$(L10) * ((L10) (L10))$

$\{2pinted\} \{ab\} \{aabb\} = 1$

$* (L10) 101 * (L10)$

$\{11\} \{aabb\} = 1$

$(S11) * (0110)$

① $L = \{ \omega \in \{0,1\}^*: \omega \text{ starts with } 10 \}$

$$10 (011)^*$$

② $L = \{ \omega \in \{0,1\}^*: \omega \text{ ends with } 10 \}$

$$(011)^* 10$$

③ $L = \{ \omega \in \{0,1\}^*: \omega \text{ does not end with } 10 \}$

$$(011)^* (00|01111) \mid (0111\epsilon)$$

④ $L = \{ \omega \in \{0,1\}^*: \text{length of } \omega \text{ is even} \}$

$$((011)(011))^*$$

⑤ $L = \{ \omega \in \{0,1\}^*: \text{length of } \omega \text{ is odd} \}$

$$((011)(011))^* (011)$$

⑥ $L = \{ \omega \in \{0,1\}^*: \omega \text{ contains } 101 \text{ as substrings} \}$

$$(011)^* 101 (011)^*$$

⑦ $L = \{ \omega \in \{0,1\}^*: \text{does not contain } 11 \}$

$$(0110)^* (1|\epsilon)$$

- ⑧ $L = \{ w \in \{0,1\}^* \text{ does not contain } 00 \}$ (8)
 $(1|01)^* (0|1)^* ((1|0) (1|0) (1|0))$
- ⑨ $L = \{ \text{does not contain } 111 \}$ (9)
 $(01|10|110)^* (1|11|1)^* ((1|0) (1|0) (1|0))$
- ⑩ $L = \{ w \in \{0,1\}^* : w \text{ does not ends with } 00 \}$ (10)
 $(1|(0|1) 1^*)^* (1|0) 00^* (1|0)$
- ⑪ $L = \{ w \in \{0,1\}^* : w \text{ does not ends with } 01 \}$ (11)
 ~~$(0|1)^* (00|11^*|10) | (0|1|1)^* (0|1)^* (00|11|10|0)$~~
- ⑫ $L = \{ w \in \{0,1\}^* : w \text{ does not ends with } 11 \}$ (12)
 ~~$(0|1)^* (0|1|0)^* 0 | (0|1)^* 0$~~
- ⑬ $L = \{ w \in \{0,1\}^* \text{ does not contains } 10 \}$ (13)
 ~~$0^* 1^*$~~
- ⑭ $L = \{ w \in \{0,1\}^* \text{ does not contains } 01 \}$ (14)
 ~~$1^* 0^*$~~
- ⑮ $L = \{ w \in \{0,1\}^* \text{ does not contains } 00 \}$ (15)
 $(1|01)^* (1|0)^* 10^* (1|0) 10^* (1|0)$

(16)

$$L = \{ \text{length expressed as } 3k+2 \}$$

$$((0|1)(0|1)(0|1))^* (0|1)(0|1)1$$

(17)

$$L = \{ \text{length expressed as } 2k+1 \}$$

$$((0|1)(0|1))^* (0|1)$$

(18)

$$L = \{ \omega \text{ contains } 00 \text{ or } 11 \}$$

$$(0|1)^* 00 (0|1)^* \mid (0|1)^* 11 (0|1)^*$$

(19)

$$L = \{ \omega \text{ contains at least two 1's} \}$$

$$(0|1) | 11|00 (1|0)(0|1) | (0|1)00 (1|0)$$

$$(0|1)^* \mid (0|1)^* 1 (0|1)^*$$

(20)

$$L = \{ \omega \text{ contains at least two 0's} \}$$

$$(0|1)^* 0 (0|1)^* 0 (0|1)^*$$

(21)

$$L = \{ \omega \text{ contains at least 1 zeros} \}$$

$$(0|1)^* 0 (0|1)^*$$

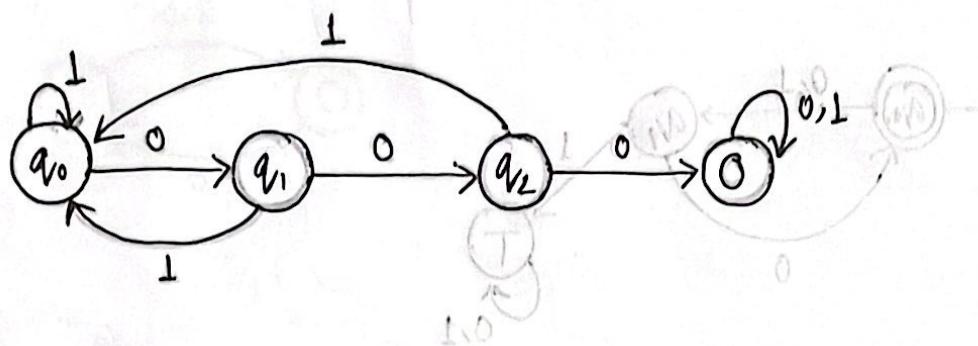
(22)

$$L = \{ \omega \text{ contains at least 2 '0's} \}$$

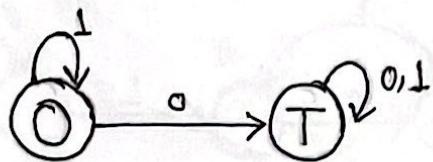
$$(0|1)^* 01 (0|1)^* 01 (0|1)^*$$

- 23) $L = \{ \omega \text{ contains exactly two 1's} \}$
 $0^* 1 0^* 1 0^*$
- 24) $L = \{ \omega \text{ contains exactly two 0's} \}$
 $1^* 0 1^* 0 1^*$
- 25) $L = \{ \omega \text{ contains exactly one 0} \}$
 $1^* 0 1^*$
- 26) $L = \{ \omega \text{ contains exactly three 0's} \}$
 $1^* 0 1^* 0 1^* 0 1^*$
- 27) $L = \{ \omega \text{ contains at most two 1's} \}$
 $0^* + 0^* 1 0^* + 0^* 1 0^* 1 0^*$
- 28) $L = \{ \omega \text{ contains at most three 0's} \}$
 $1^* + 1^* 0 1^* + 1^* 0 1^* 0 1^* + 1^* 0 1^* 0 1^* 0 1^*$
- 29) $L = \{ \omega \text{ contains at most one 1} \}$
 $0^* + 0^* 1 0^*$
- 30) $L = \{ \text{length of } \omega \text{ is not multiple of 3} \}$
 $((0|1)(0|1)(0|1))^* (0|1) (011|\epsilon)$
- 31) $L = \{ \text{length is multiple of 3} \}$
 $((011)(011)(011))^*$

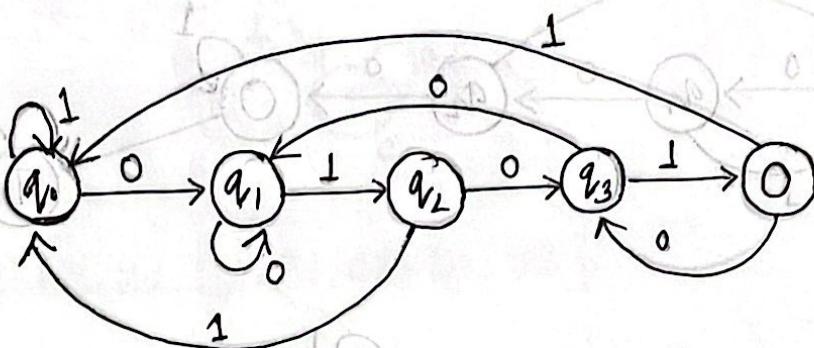
1. (a)



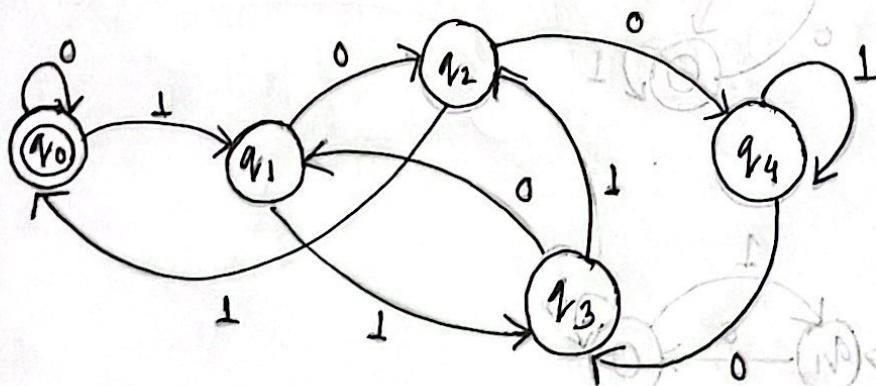
(b)



(c)

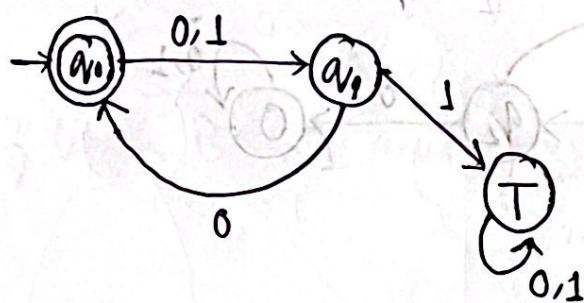


(d)



	0	1
0	q_0	q_1
1	q_2	q_3
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

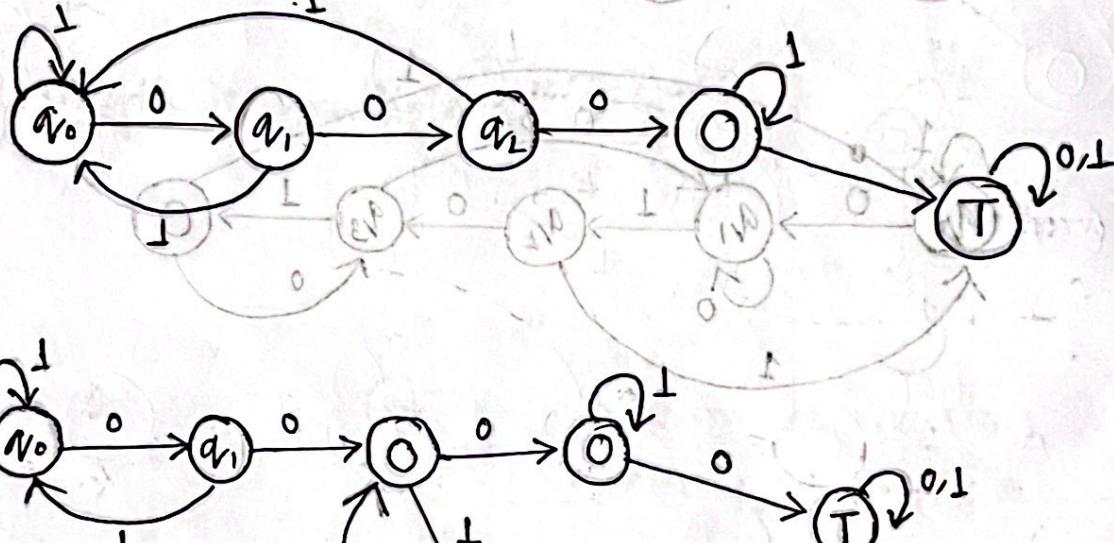
(e)



101 1010

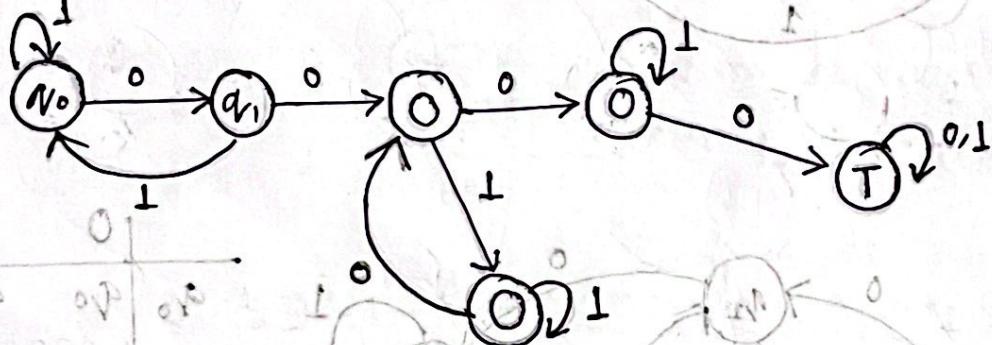
(b)

(f)

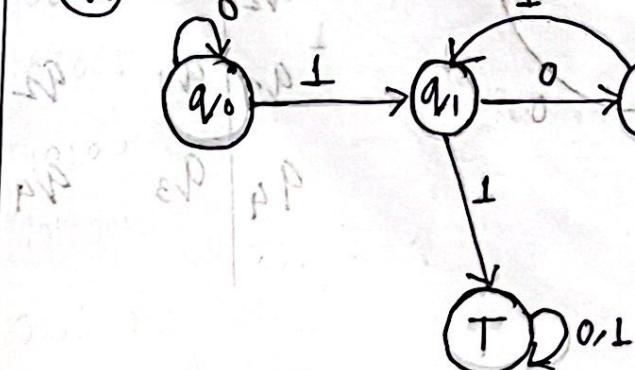


Hamid Book Binding

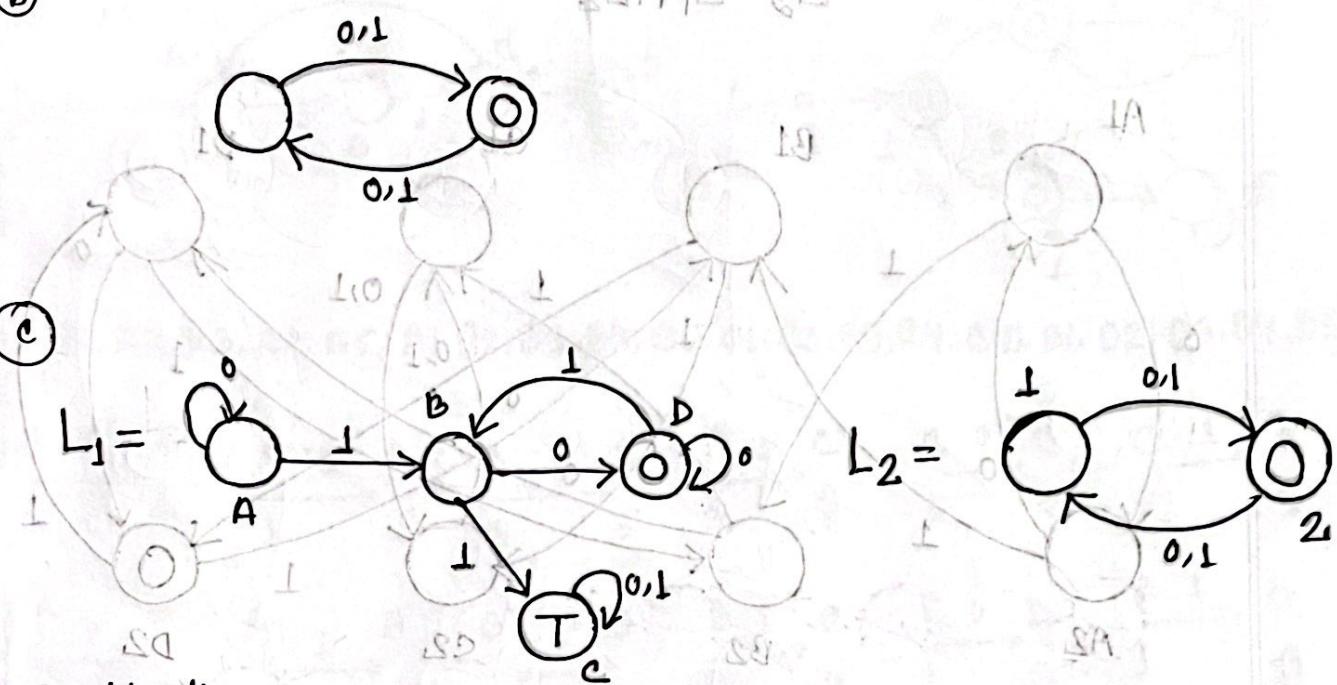
(g)



(h)

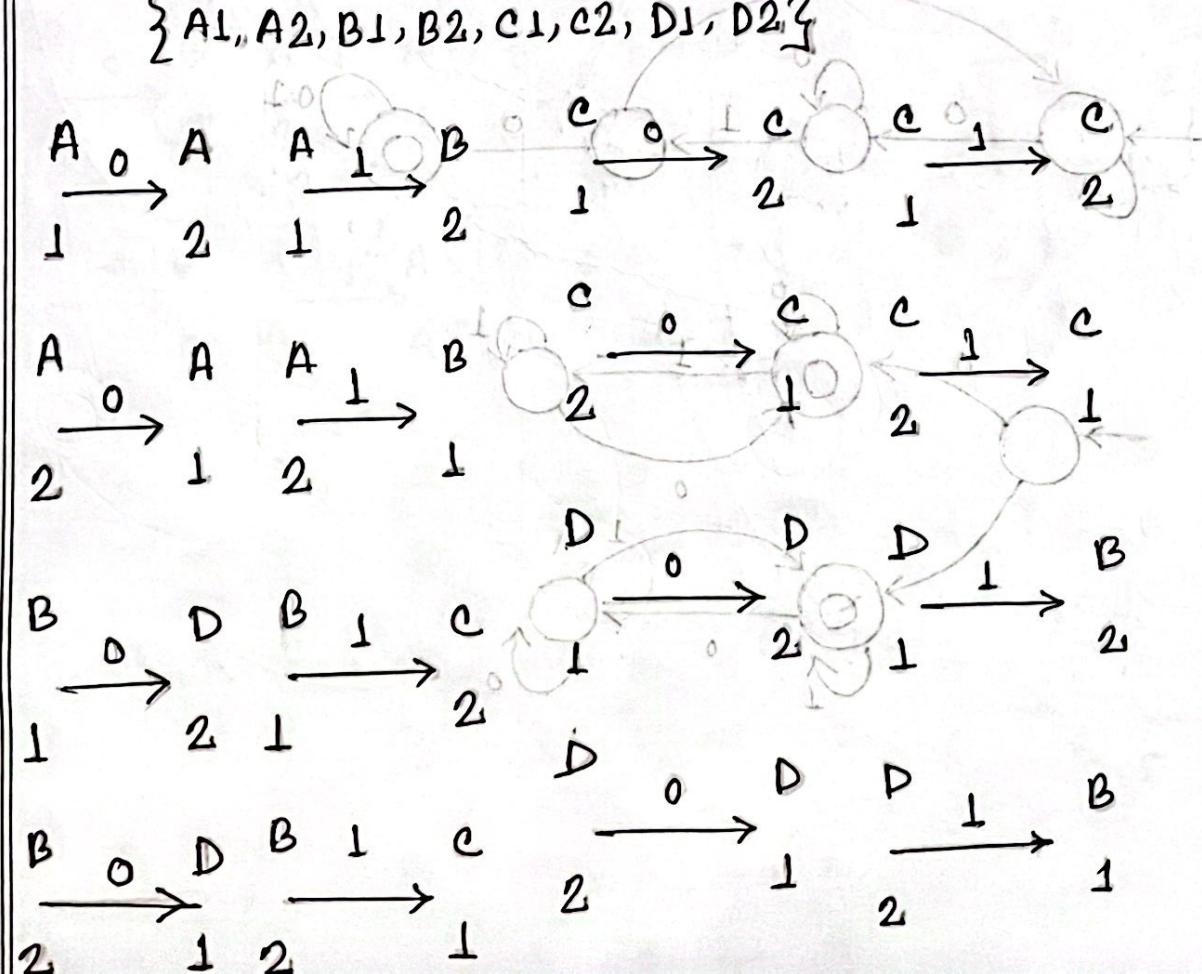


(b)



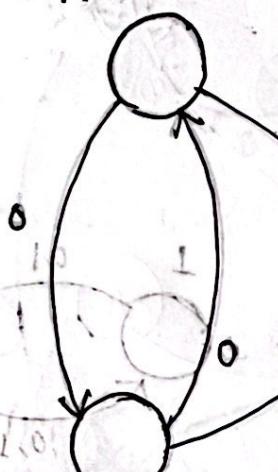
Combinations =

$$\{A1, A2, B1, B2, C1, C2, D1, D2\}$$



$$L_3 = L_1 \cap L_2$$

A1



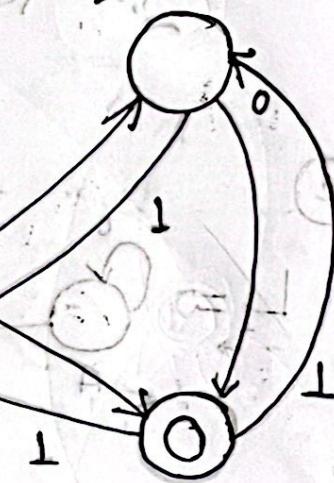
B1



C1



D1

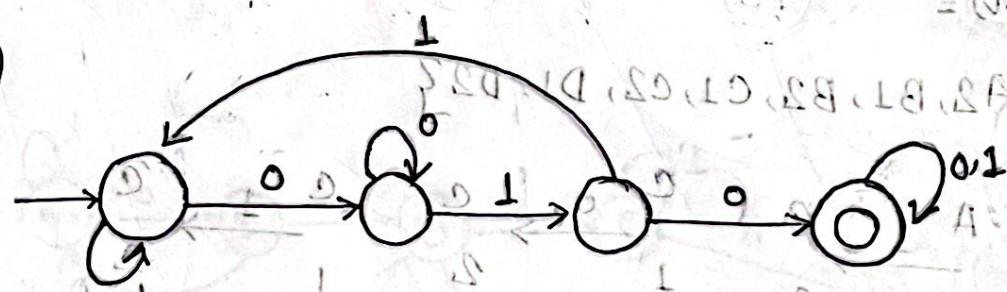


A2

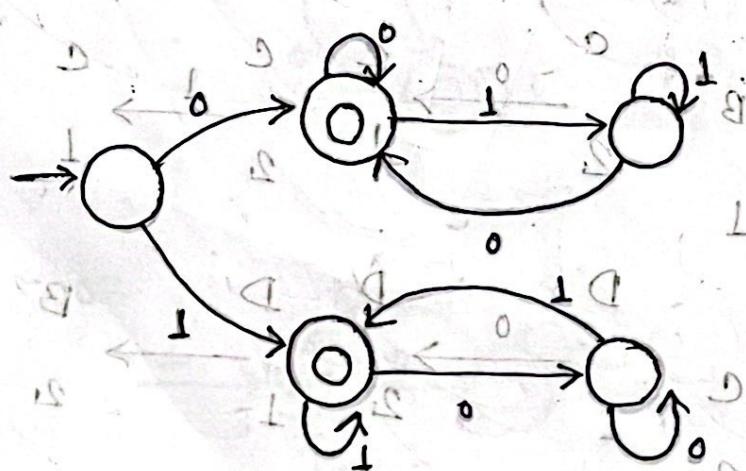
B2

C2

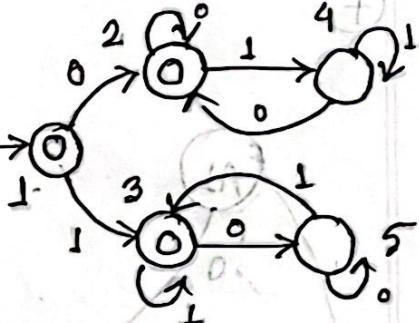
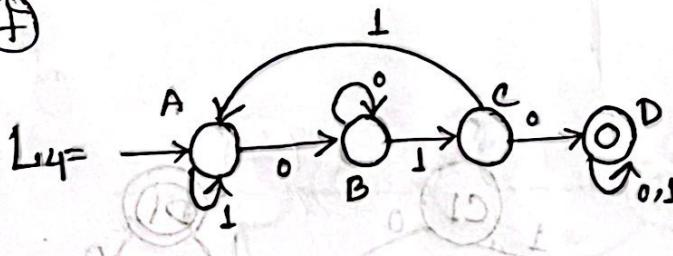
D2



e

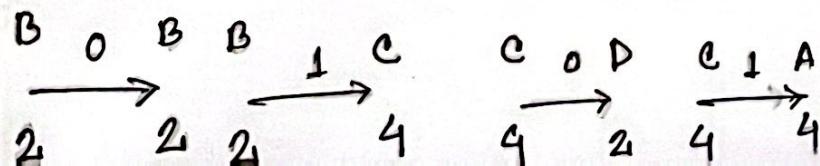
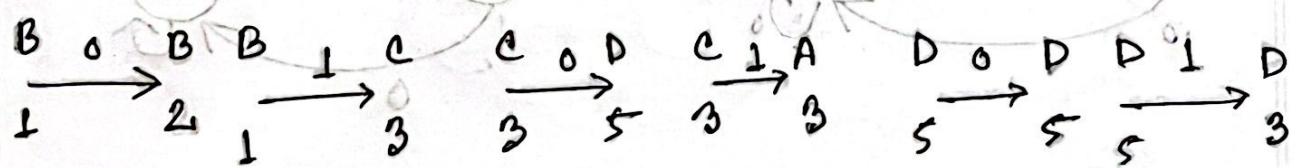
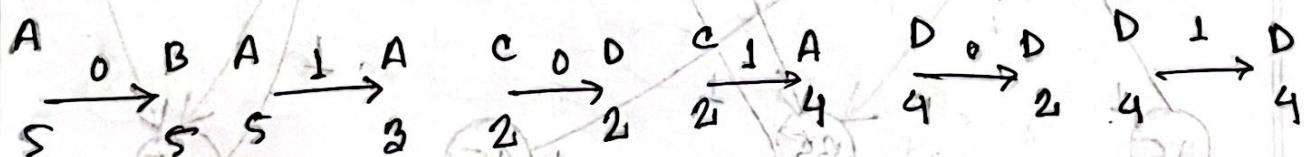
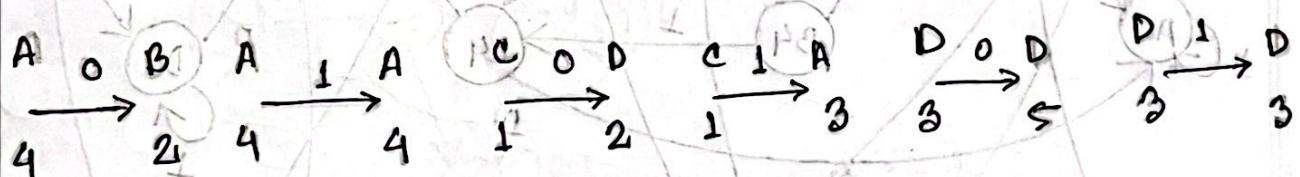
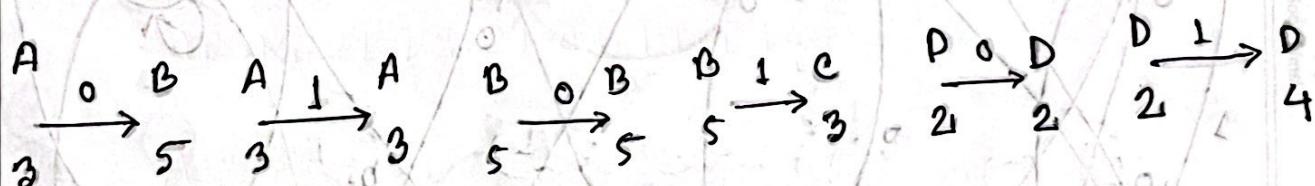
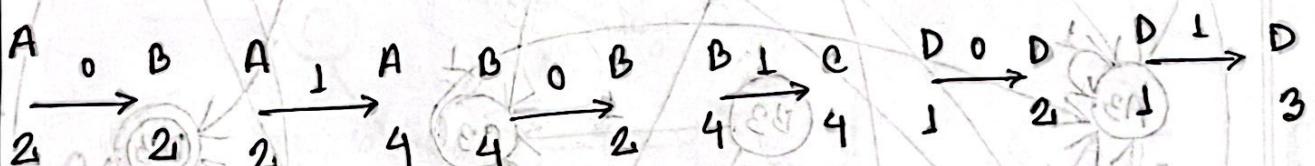
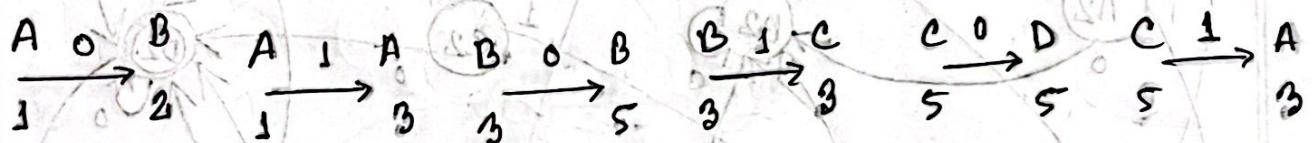


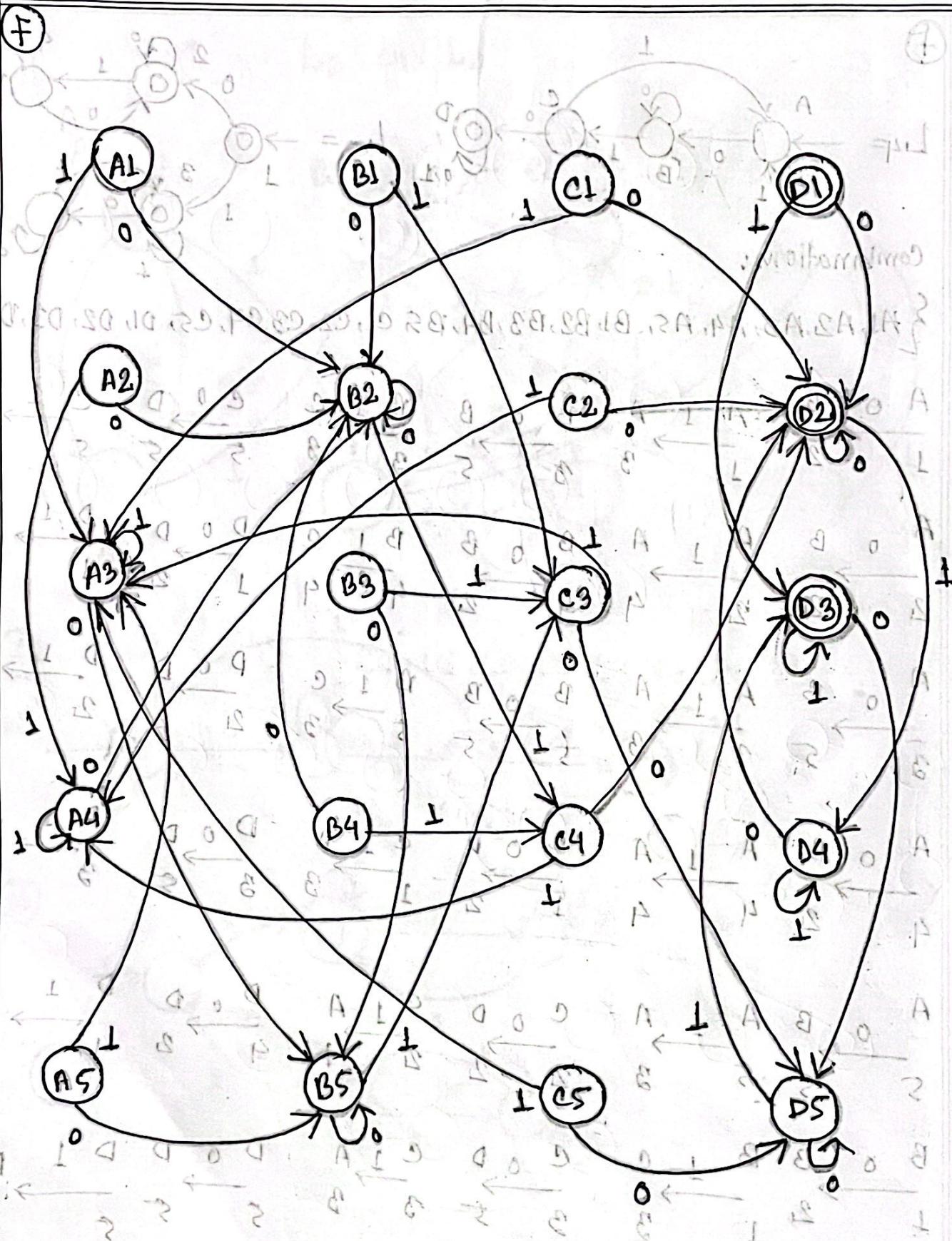
(F)



Combinations:

$\{A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4, B_5, C_1, C_2, C_3, C_4, C_5, D_1, D_2, D_3, D_4, D_5\}$



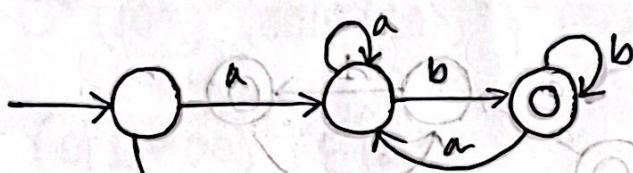


Subject :

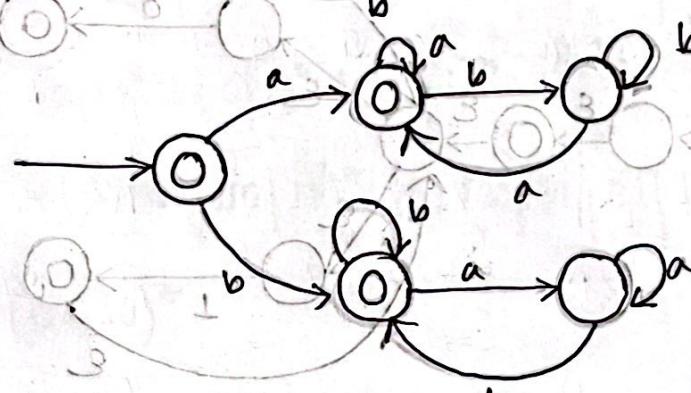
Date :

3.

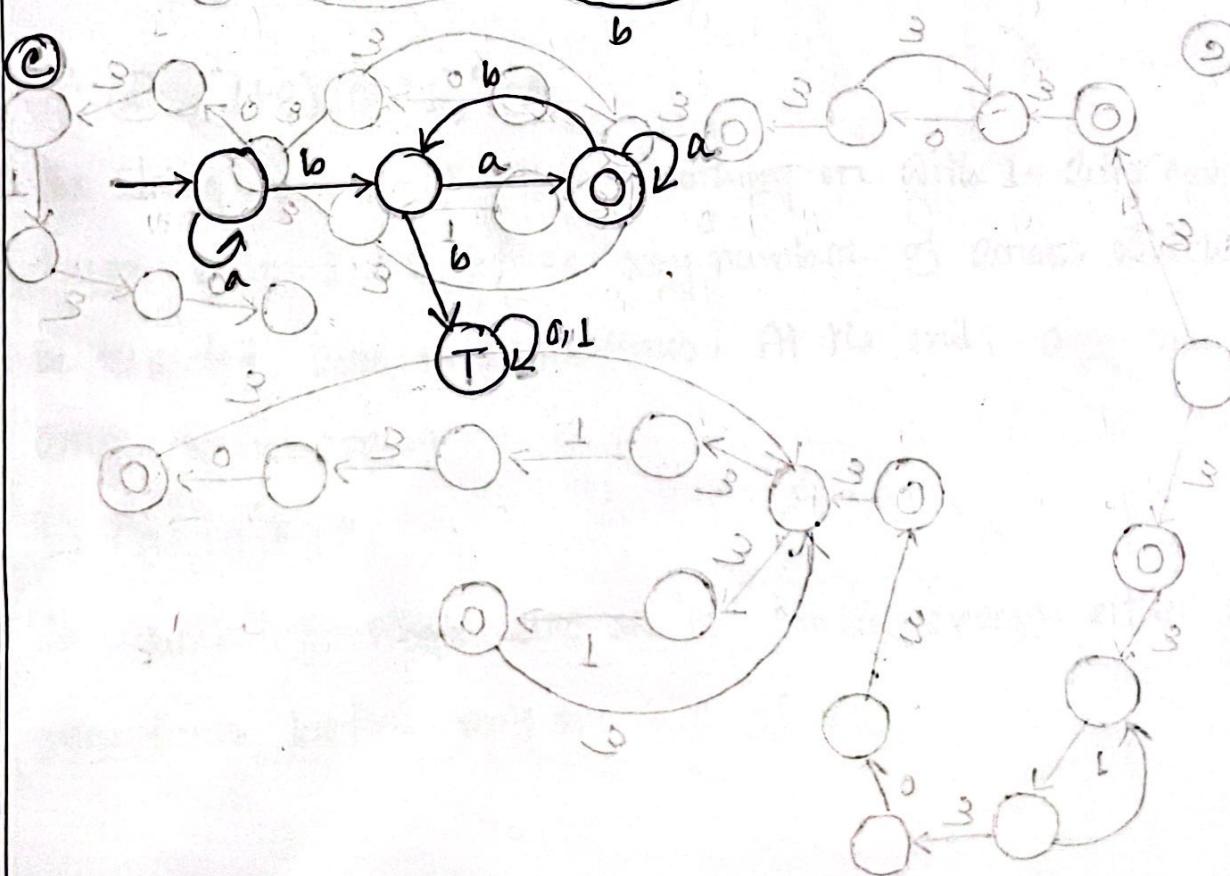
(a)



(b)



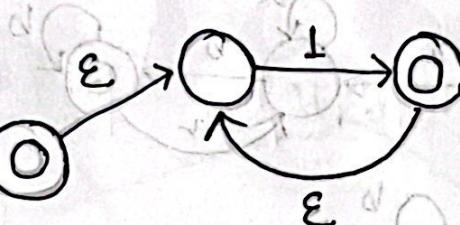
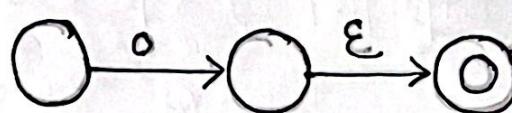
(c)



Hamld Book Binding

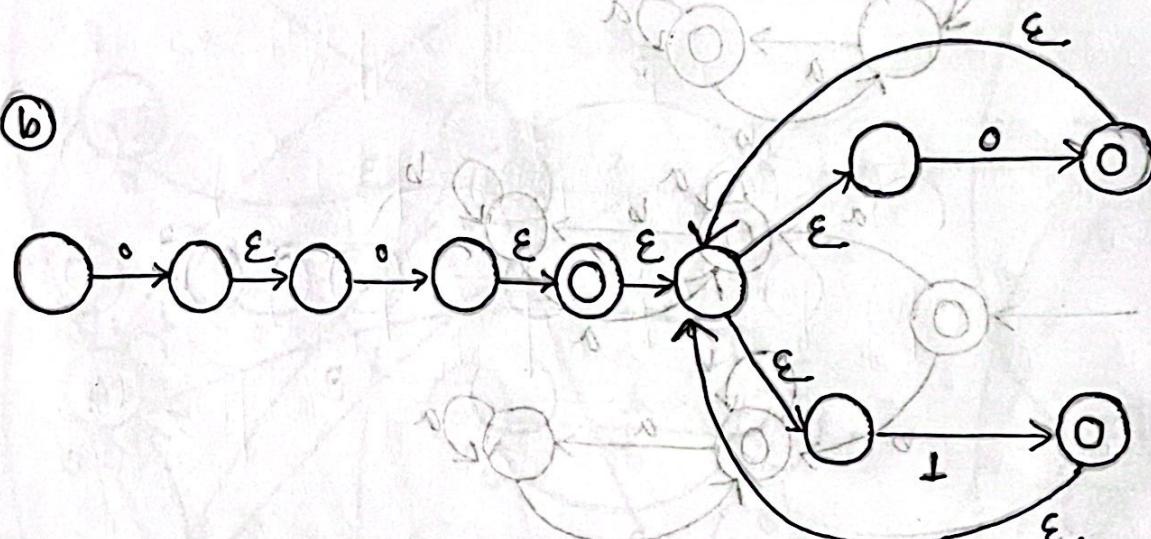
6

②



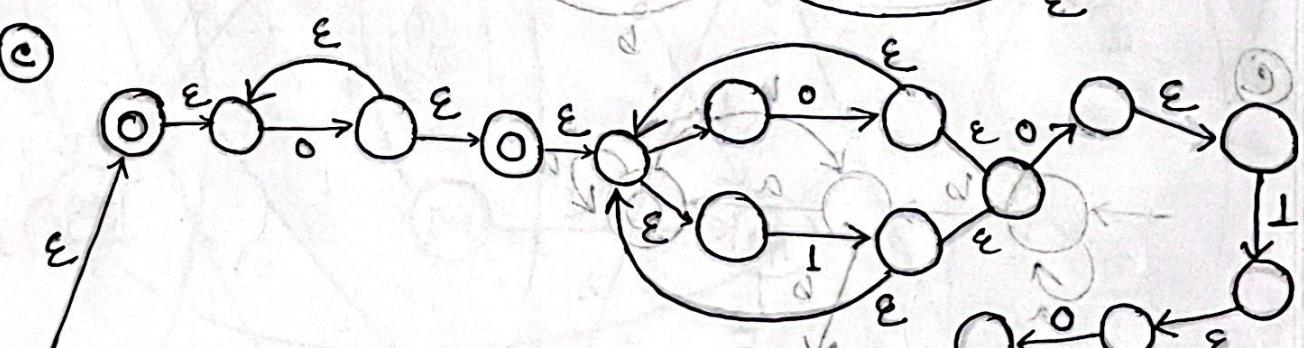
6

11



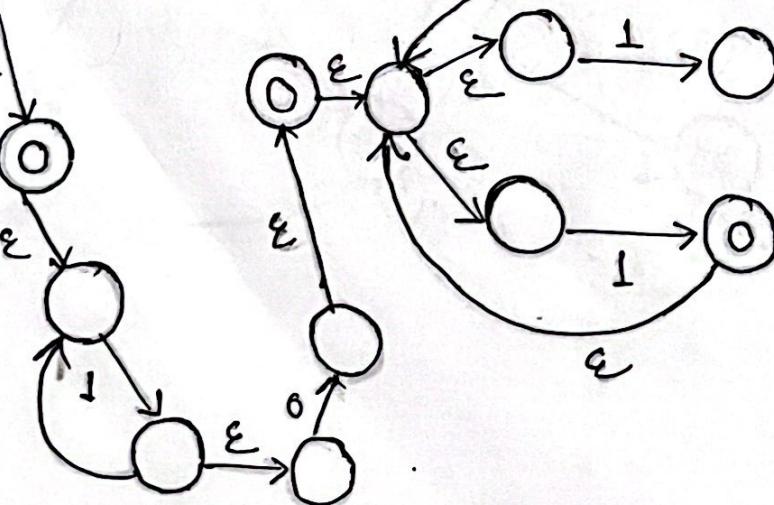
2

8

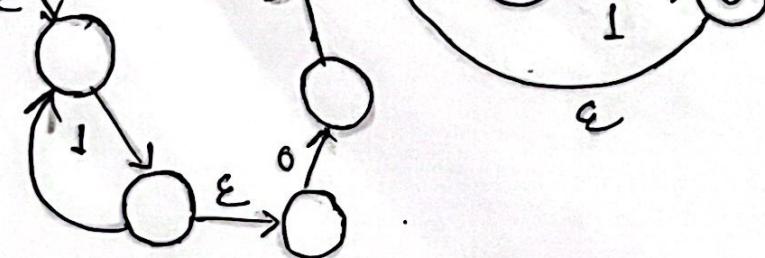


8

8



1



4.

- (a) $0^* (10^*)^* (000^* (10^*)^*)^* (111^*)^*$
- (b) $(0111^* 00)^* (\epsilon | 11^*)$
- (c) $(b^* (ab)^* b^*)^*$
- (d) $b^* (ab^* | \epsilon)$
- (e) $(0|1)^* \sqcup (0|1)^* \sqcup (0|1)^*$
- (f) $0(00|01|10|11)^* \sqcup (00|11|01|10)^*$
- (g) $(01|10)^*$

5.

- (a) $(1+\epsilon)(00^* 1)^* 0^*$

The string can start either nothing or with 1. Zero can be assign before 1 or before any number of zeros, which might be repeated zero or more times. At the end, any number of zeros can be appear.

- (b) $(0+10)^* 1^*$

A sequence of single zero or '10' can be appear either zero or more times before any number of 1's.

4. a) $(0+10)^* (1+01)^* (0+\epsilon)$ (0) $(111)^* (01)^* (000)^* (01)^* 0$
- b) $(0|1)^* ((00|01|10|11) (0|1)^* (00|01110|11))^*$ (6) $(1|3)^* (00^* 11|0)$
- c) $(b^* (ab)^* b^*)^*$ $(a^* b^* (a^* b^*)^*)^*$ (5) $(a^* b^*)^* a$
- d) $b^* (a^* b^* | \epsilon)$ (3) $(b^* a^*)^* a$
- e) $(0|1)^* \perp (0|1)^* \perp (0|1)^*$ (3) $(1|0)^* \perp (1|0)^* \perp (1|0)^*$
- f) $0((0|1) (0|1))^*$ $\perp (0|1) ((0|1) (0|1))^*$ (7) $(01|10)^* \perp (11|01|10|00) 0$
- g) $(01|10)^*$ $\perp (01|10)$ (2)

1. $L = \{ w \in \{0,1\}^* \mid w \text{ starts with } 10 \}$
- $10 (0|1)^*$
- Dont start with 00
 $(0+1)(0+1)^* + 0(1+0(0+1)^*)$
2. $L = \{ w \in \{0,1\}^* \mid w \text{ ends with } 10 \}$
- $(0|1)^* 10 (1|0)^* ((1|0))$
- Dont start with 01
 $(0+1)(0+1)^* + 1(0+0(0+1)^*)$
3. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 00 \}$
- $((0|1)^* 1 (1|0)^* ((1|0) (1|0)))$
- Dont start with 10
 $(0+1)(0+1)^* + 1(1+0(0+1)^*)$
4. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 01 \}$
- $(0|1)^* (00|10|11) | (1|0|\epsilon)$
- Dont start with 11
 $(0+1)(0+1)^* + 1(1+0(0+1)^*)$
5. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 10 \}$
- $(0|1)^* (00|01|11) | (1|0|\epsilon)$
- Dont start with 01
 $(0+1)(0+1)^* + 1(0+0(0+1)^*)$
6. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 11 \}$
- $(0|1)^* 0$
- Dont start with 10
 $(0+1)(0+1)^* + 0(1+0(0+1)^*)$
7. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 000 \}$
- $(0|1)^* (001|010|011|100|101|110|111|0|1)$
- Dont start with 100
 $(0+1)(0+1)^* + 1(0+0(0+1)^*)$
8. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 001 \}$
- $(0|1)^* (000|010|011|100|101|110|111|0|1)$
- Dont start with 101
 $(0+1)(0+1)^* + 1(0+0(0+1)^*)$

9. $L = \{ w \in \{0,1\}^* \mid \text{length of } w \text{ is even} \}$
- $$((0|1)(0|1))^*$$
10. $L = \{ w \in \{0,1\}^* \mid \text{length of } w \text{ is odd} \}$
- $$((0|1)(0|1))^* (0|1)$$
11. $L = \{ \text{length expressed as } 3k+2 \}$
- $$((0|1)(0|1)(0|1))^* (0|1)(0|1)$$
12. $L = \{ \text{length expressed as } 2k+1 \}$
- $$((0|1)(0|1))^* (0|1)$$
13. $L = \{ \text{count of 1's/0's is even} \}$
- $$(0^* 1 0^* 1)^*$$
14. $L = \{ \text{count of 0's is odd} \}$
- $$(1^* 0 1^* 0)^* (1^* 0 1^* 0)$$
15. $L = \{ w \in \{0,1\}^* \mid w \text{ contains } 101 \text{ as a substring} \}$
- $$(0|1)^* 101 (0|1)^*$$
16. $L = \{ w \in \{0,1\}^* \mid w \text{ contains } 11 \text{ as a substring} \}$
- $$(0|1)^* 11 (0|1)^*$$

17. $L = \{ w \in \{0,1\}^* \text{ does not contain } 00 \text{ as substring} \}$

$$\underline{(0|1)^* 0 (0|1)^* 1 (0|1)^*} \quad (1|01)^* (0|1)^*$$
18. $L = \{ w \in \{0,1\}^* \text{ does not contain } 01 \text{ as substring} \}$

$$\underline{(0|1)^* 0^* (0|1)^*} / 1^* 0^* 1^* 0^* 1^* 0^* 1^* 1^*$$
19. $L = \{ w \in \{0,1\}^* \text{ does not contain } 10 \text{ as substring} \}$

$$\underline{(0|1)^* 1^* (0|1)^*} / 0^* 1^*$$
20. $L = \{ w \in \{0,1\}^* \text{ does not contain } 11 \text{ as substring} \}$

$$\underline{(0|1)^* 0^* (0|1)^*} \quad (0|10)^* (1|1)^*$$
21. $L = \{ \text{contains } 00 \text{ or } 11 \text{ as substrings} \}$

$$(0|1)^* 00 (0|1)^* \quad | \quad (0|1)^* 11 (0|1)^*$$
22. $L = \{ w \text{ contains at least two } 0's \}$

$$(0|1)^* 0 (0|1)^* 0 (0|1)^* (1|0) (1|0) (1|0)$$
23. $L = \{ w \text{ contains at least one } 0 \}$

$$(0|1)^* 0 (0|1)^* (1|0) (1|0) (1|0) (1|0)$$
24. $L = \{ w \text{ contains at least two } 01 \}$

$$(3 (0|1)^* 01, (0|1)^* (1|0) (0|1)^* (1|0) (1|0) (1|0))$$

17. $L = \{ w \in \{0,1\}^* \text{ does not contain } 00 \text{ as substring} \}$

$$\underline{(0|1)^*} \ 0 \underline{(0|1)^*} \ 1 \underline{(0|1)^*} \ (1|01)^* (0|1^* \epsilon)$$
18. $L = \{ w \in \{0,1\}^* \text{ does not contain } 01 \text{ as substring} \}$

$$\underline{(0|1)^*} 0^* \underline{(0|1)^*} \ / \ 1^* 0^* \ / \ 10^* 1 + 10^* 1 + 1^* 1$$
19. $L = \{ w \in \{0,1\}^* \text{ does not contain } 10 \text{ as substring} \}$

$$\underline{(0|1)^*} 1^* \underline{(0|1)^*} \ / \ 0^* 1^*$$
20. $L = \{ w \in \{0,1\}^* \text{ does not contain } 11 \text{ as substring} \}$

$$\underline{(0|1)^*} 0^* \underline{(0|1)^*} \ / \ (0|10)^* (1| \epsilon)$$
21. $L = \{ w \text{ contains } 00 \text{ or } 11 \text{ as substrings} \}$

$$(0|1)^* 00 \underline{(0|1)^*} \ / \ (0|1)^* 11 \underline{(0|1)^*} (1|0))$$
22. $L = \{ w \text{ contains at least two } 0's \}$

$$(0|1)^* 0 \ (0|1)^* 0 \ (0|1)^* 0 \ (0|1)^* 0 \ (1|0) (2|0))$$
23. $L = \{ w \text{ contains at least one } 0 \}$

$$(0|1)^* 0 \ (0|1)^* 1|0 \ (1|0) (2|0) (3|0) (4|0))$$
24. $L = \{ w \text{ contains at least one } 01 \}$

$$(0|1)^* 01 \ (0|1)^* 1 (0|1) (0|1) 1^* (1|0) (1|0) (1|0))$$

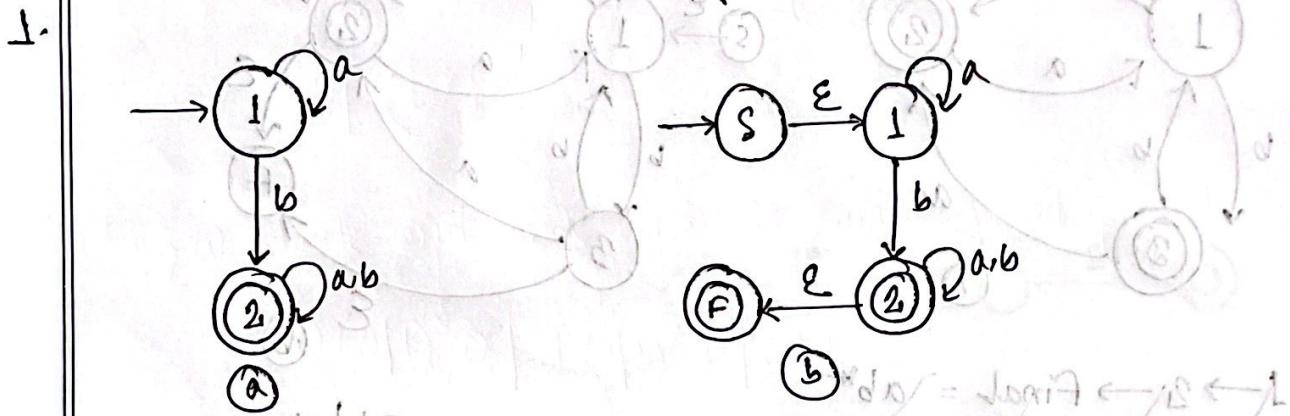
25. $L = \{ \omega \text{ contains at most two 1's} \}$ forwards $\{1, 0\}^* \{0, 1\}^* = \{1, 0\}$
- $$(0^* 1^* 0^* 1^* 0^* + 0^* 1^* 0^* 1^* 0^* (1/0) 0^* (1/0))$$
26. $L = \{ \omega \text{ contains at most three 0's} \}$ forwards $\{1, 0\}^* \{0, 1\}^* = \{1, 0\}$
- $$1^* + 1^* 0 1^* + 1^* 0 1^* 0 1^* + 1^* 0 1^* 0 1^* 0 1^*$$
27. $L = \{ \omega \text{ contains exactly two 1's} \}$ forwards $\{1, 0\}^* \{0, 1\}^* = \{1, 0\}$
- $$0^* 1 0^* 1 0^*$$
28. $L = \{ \omega \text{ contains exactly two 0's} \}$ forwards $\{1, 0\}^* \{0, 1\}^* = \{1, 0\}$
- $$1^* 0 1^* 0 1^*$$
29. $L = \{ \text{length of } \omega \text{ is multiple of 3} \}$ forwards $\{1, 0\}^* \{0, 1\}^* = \{1, 0\}$
- $$((0|1) (0|1) (0|1))^*$$
30. $L = \{ \text{length of } \omega \text{ is not multiple of 3} \}$ forwards $\{1, 0\}^* \{0, 1\}^* = \{1, 0\}$
- $$((0|1) (0|1) (0|1))^* (0|1)^* (0|1|1|ε)$$
31. $L = \{ \text{length of } \omega \text{ is multiple of 5} \}$ forwards $\{1, 0\}^* \{0, 1\}^* = \{1, 0\}$
- $$((0|1) (0|1) (0|1) (0|1) (0|1))^*$$
32. $L = \{ \text{length of } \omega \text{ is not multiple of 5} \}$ forwards $\{1, 0\}^* \{0, 1\}^* = \{1, 0\}$
- $$((0|1) (0|1) (0|1) (0|1) (0|1))^* (0|1)^* (0|1|1|ε)$$

33. $L = \{ \text{count of } 1 \text{ is multiple of 3} \}$
- $$0^* \mid (0^* 1 0^* 1 0^*)^*$$
34. $L = \{ w \text{ starts and ends with the same symbol} \}$
- $$0 (0|1)^* 0 \mid 1 (0|1)^* 1 \mid 0 | 1$$
35. $L = \{ w \text{ starts and ends with different symbols} \}$
- $$0 (0|1)^* 1 \mid 1 (0|1)^* 0$$
36. $L = \{ w \text{ contains 0 at every 3rd position} \}$
- $$((0|1) (0|1) 0)^* (0|1) (0|1) \mid ((0|1) (0|1) 0)^* (0|1) \mid ((0|1) (0|1) 0)^*$$
37. $L = \{ w \text{ have 0's and 1's in alternates} \}$
- $$(1|\varepsilon) (0|1)^* (0|\varepsilon) \mid (0|\varepsilon) (10)^* (1|\varepsilon)$$
38. $L = \{ w \text{ divisible by 3} \}$
- $$((0|1) (0|1) (0|1))^*$$
39. $L = \{ \text{Every second letter is zero} \}$
- $$((0|1) 0)^* (0|1|\varepsilon)$$

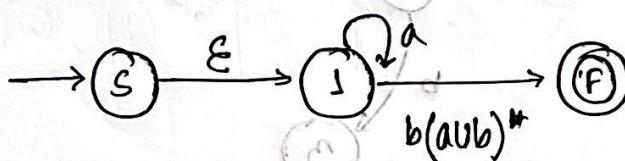
40. $L = \{ \text{string that have neither consecutive 1's, nor consecutive 0's} \}$
- $$(0|\epsilon) (10)^* (1|\epsilon)$$
41. $L = \{ \text{string that might have consecutive 1's, or consecutive 0's but not both} \}$
- $$(1|01)^* (0|\epsilon)$$
42. $L = \{ 1 \text{ does not appear at any even position in } \omega \}$
43. $L = \{ \text{every 1 is followed by at least one 0} \}$
44. $L = \{ \text{every 1 is followed by at most one 0} \}$
45. $L = \{ \text{every 1 is followed by exactly one 0} \}$
46. $L = \{ \text{every second letter is zero} \}$
47. $L = \{ 01 \text{ appears even a number of times} \}$

Subject:

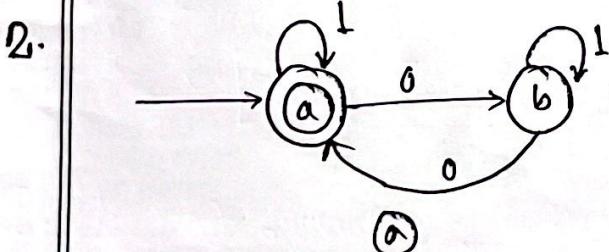
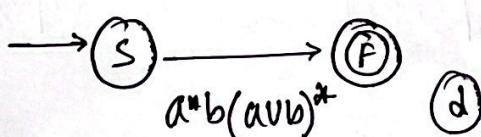
Date:

DFA \rightarrow Regular Expression

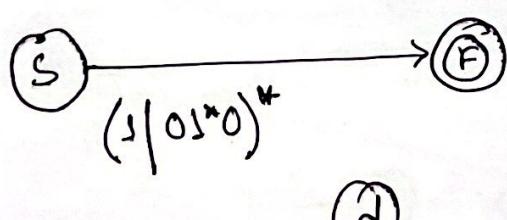
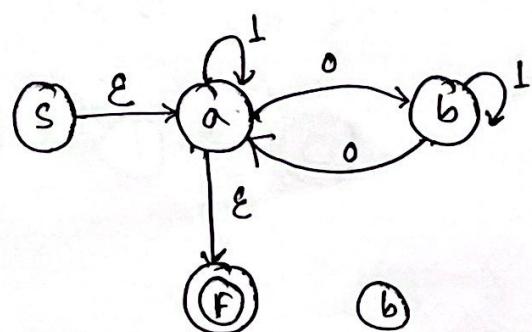
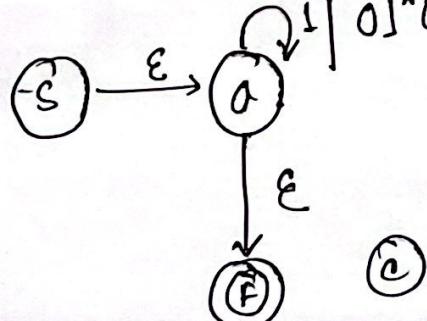
$$1 \rightarrow 2 \rightarrow \text{final} = a b (a \cup b)^*$$



$$\text{Start} \rightarrow 1 \rightarrow \text{finish} = a^* b (a \cup b)^*$$

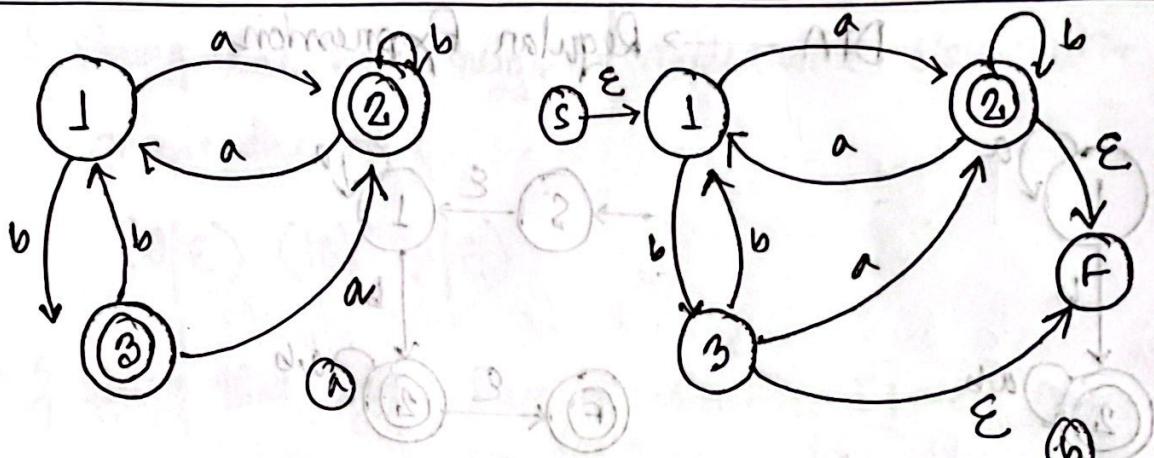


$$a \rightarrow b \rightarrow a = 01^* 0$$



(d)

3.



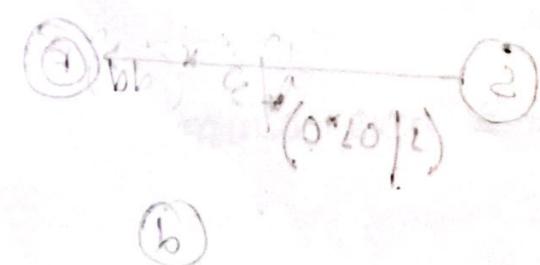
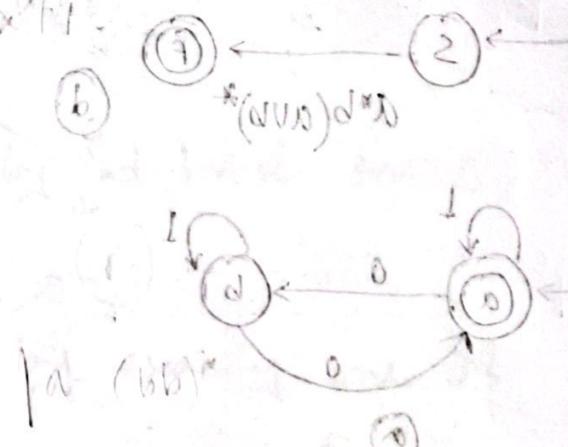
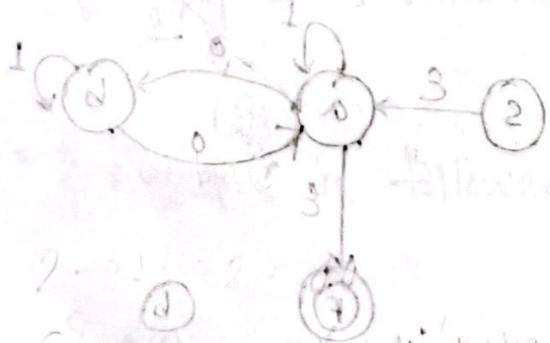
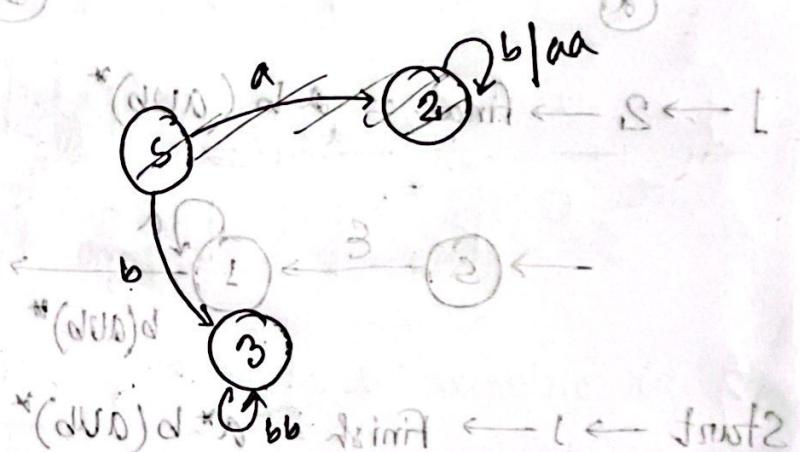
$$1 \rightarrow 2 \rightarrow \text{final} = ab^*$$

$$1 \rightarrow 2 \rightarrow 1 = ab^*a$$

$$1 \rightarrow 3 \rightarrow 1 = bb$$

$$2 \rightarrow 1 \rightarrow 2 = aa$$

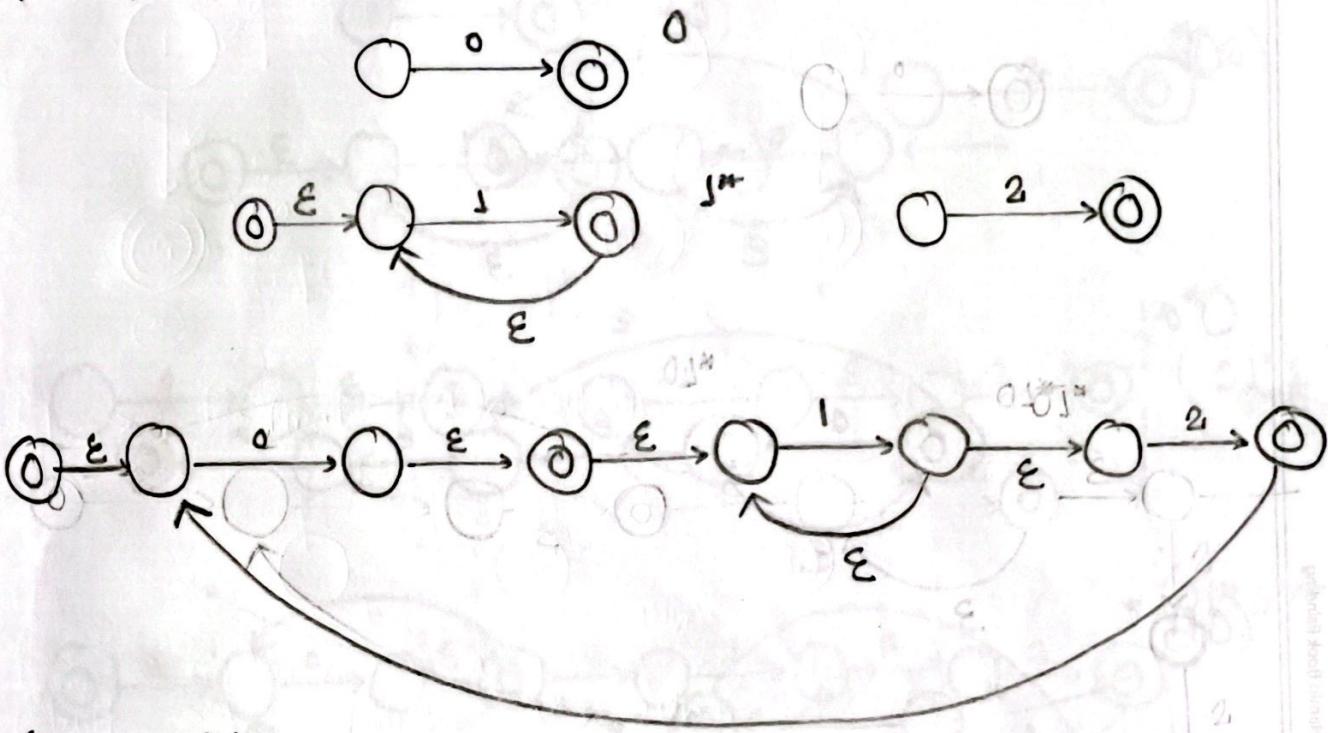
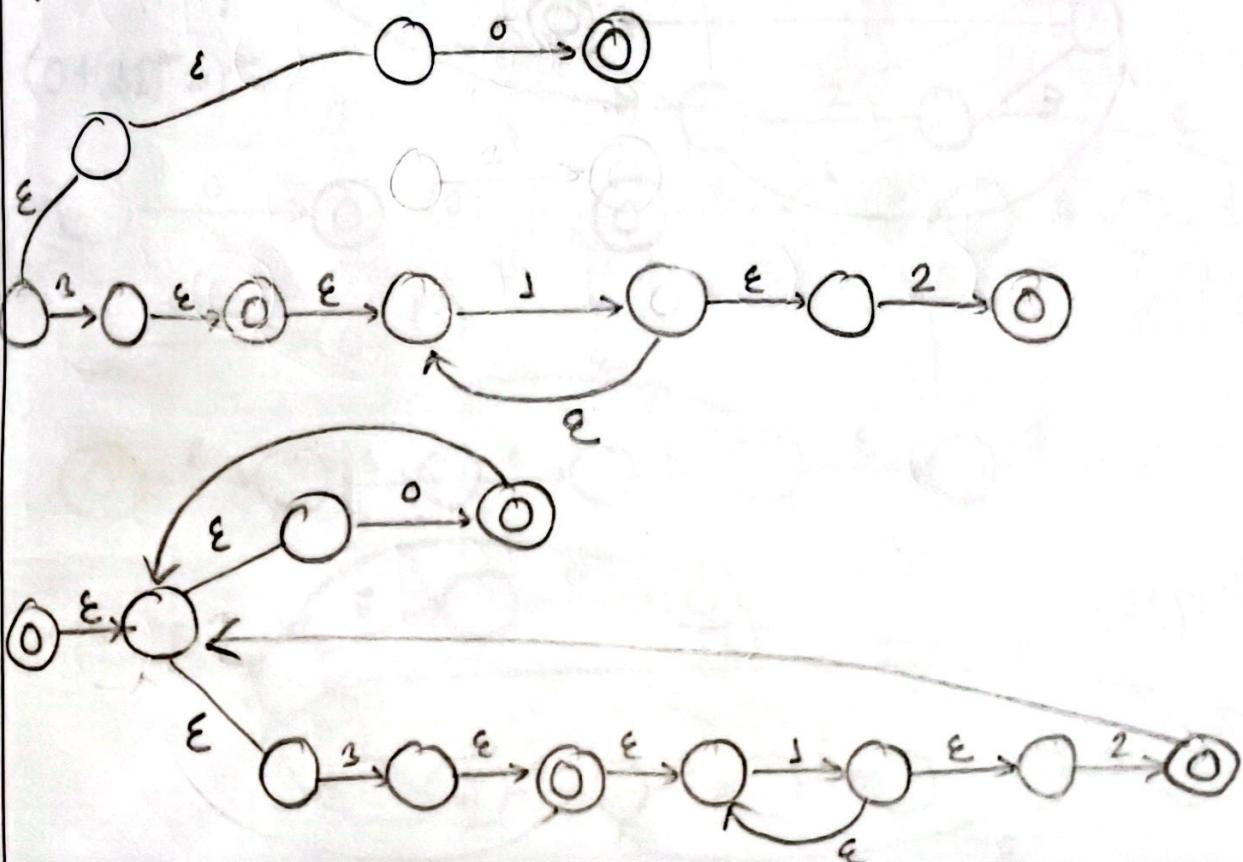
$$3 \rightarrow 1 \rightarrow 3 = bb$$

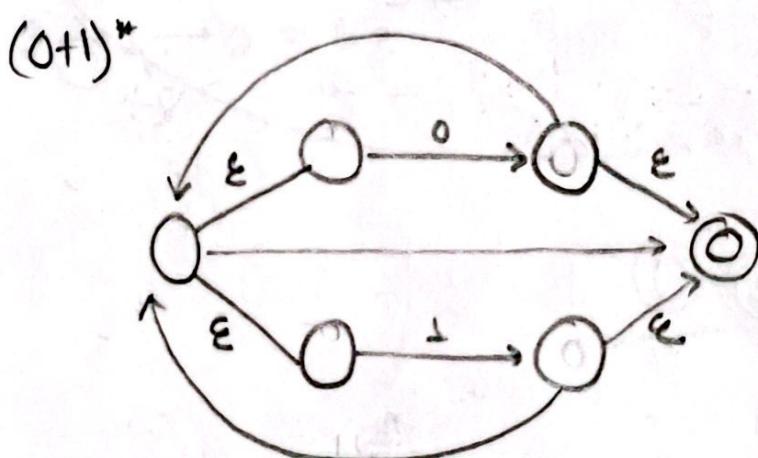
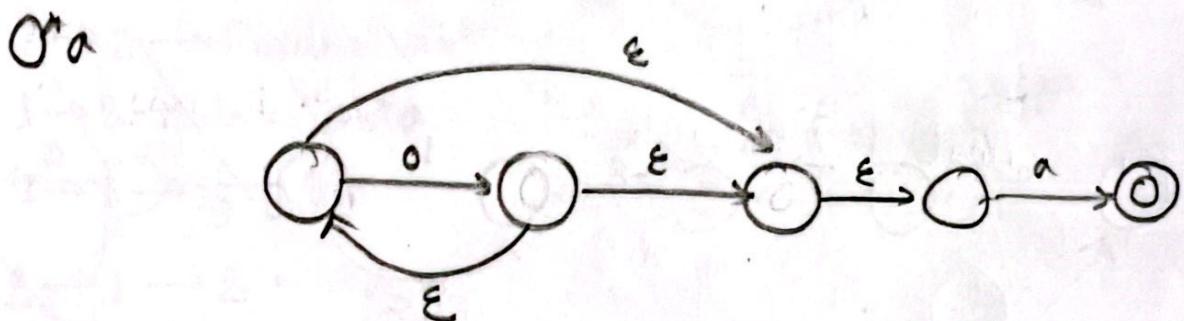
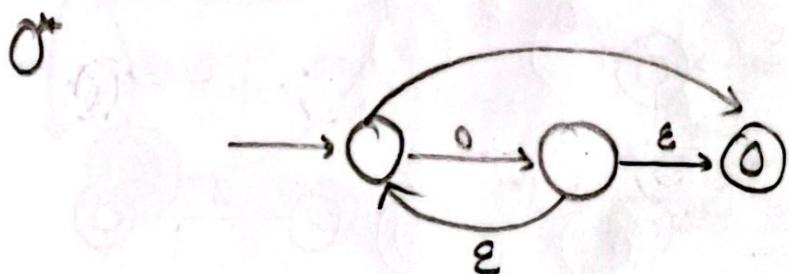


$$0^*10 = 0 \leftarrow 1 \leftarrow 0$$

$$0^*101 = 0 \leftarrow 1 \leftarrow 0$$

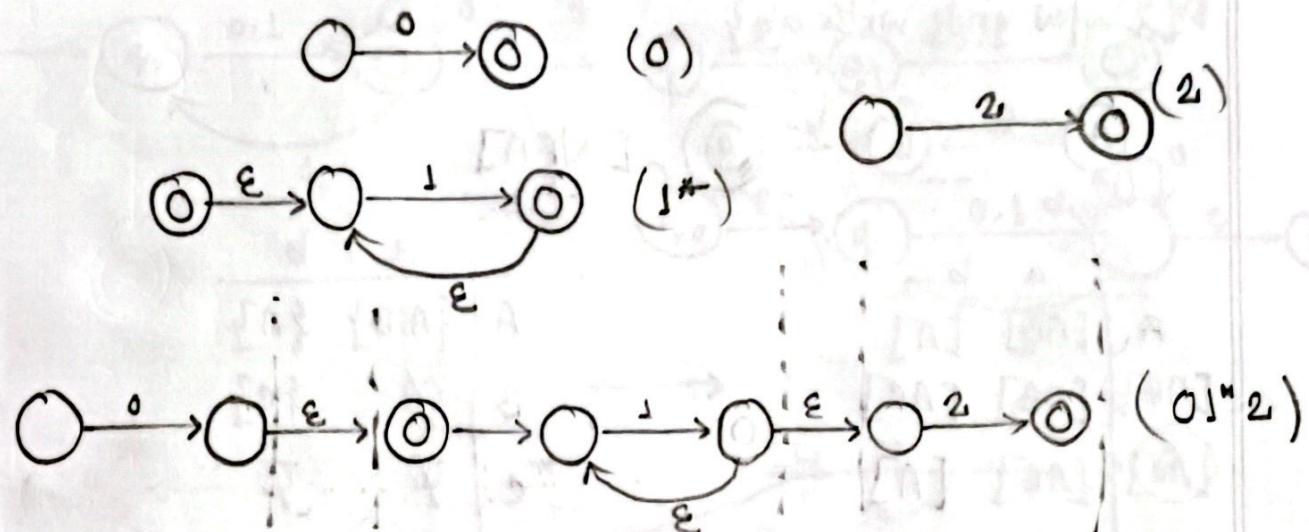
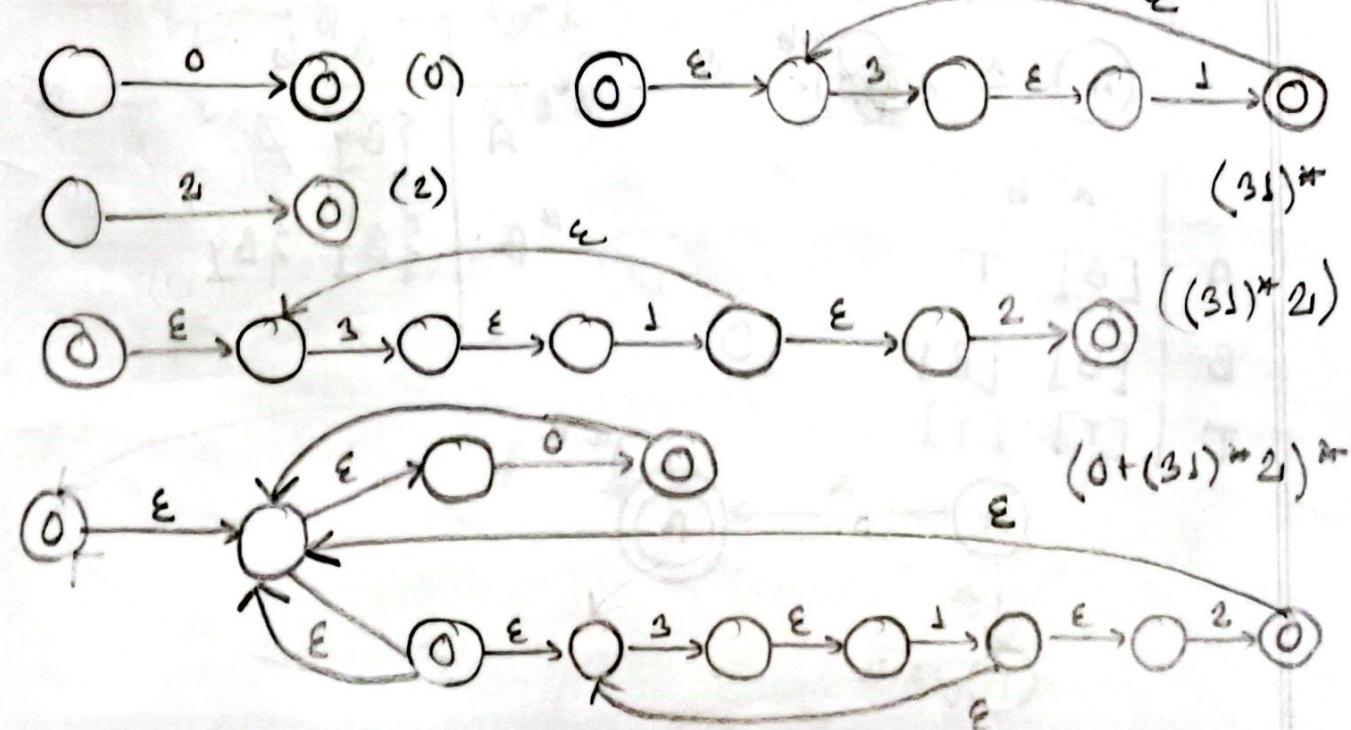
RE to NFA

 $(01^* 2)^*$  $(0+31^*2)^*$ 



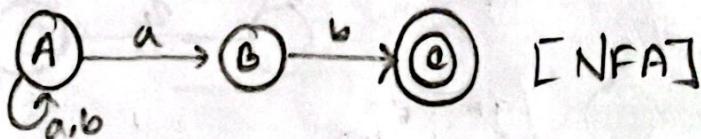
Subject:

Date:

 $(01^*2)^*$  $(0+(31)^*2)^*$ 

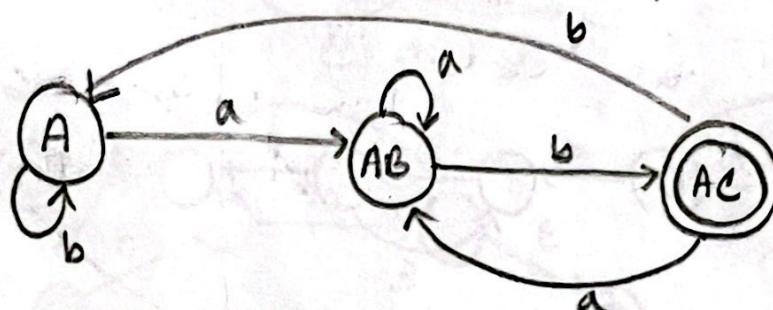
NFA TO DFA

$L = \{ w | w \text{ ends with } ab \}$

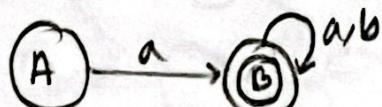


	a	b
A	[AB]	[A]
[AB]	[AB]	[AC]
[AC]	[AB]	[A]

	a + b	
A	{A, B}	{A}
B	\emptyset	{C}
*C	\emptyset	\emptyset

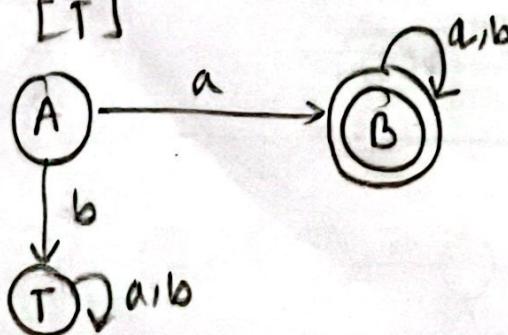


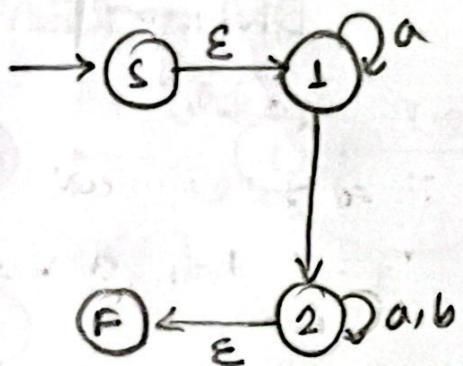
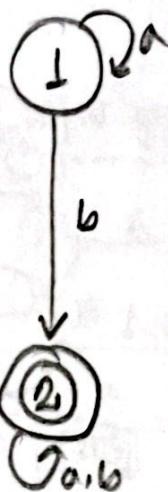
$L = \{ w | w \text{ starts with 'ab'} \}$



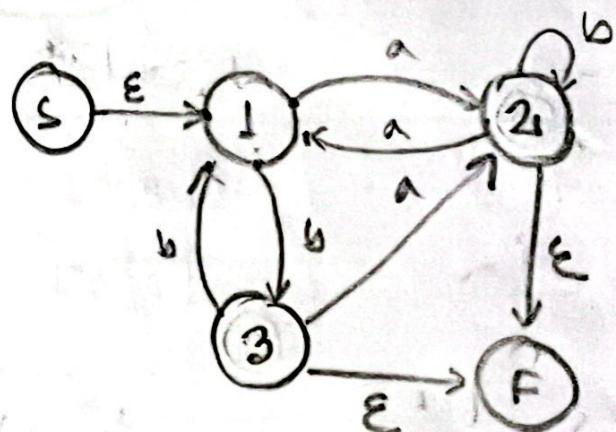
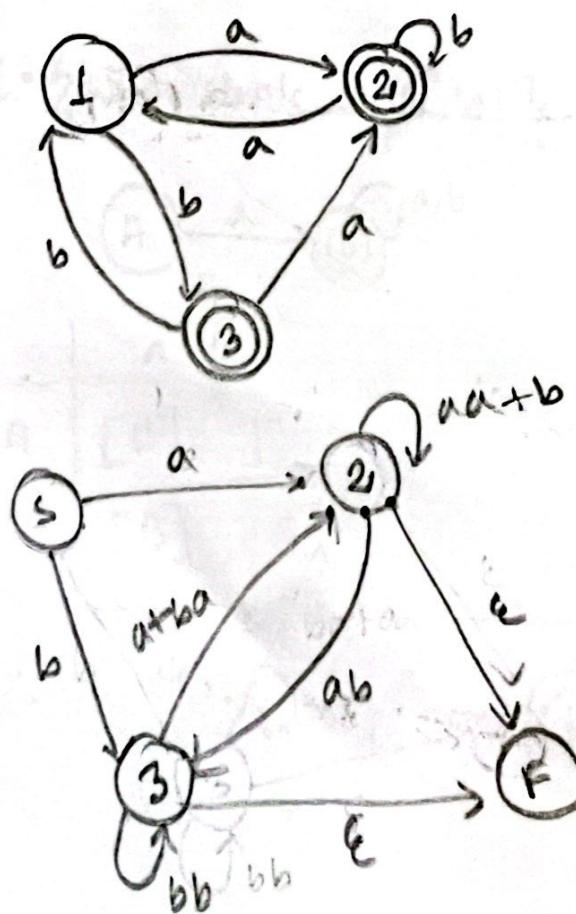
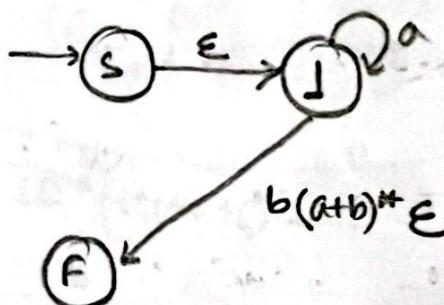
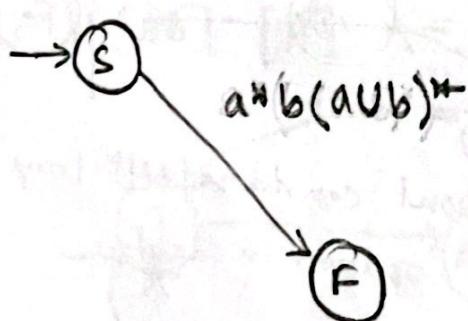
	a	b
A	[B]	T
B	[B]	[B]
T	[T]	[T]

	a	b
A	{B}	\emptyset
*B	{B}	{B}





$$1 \rightarrow 2 \rightarrow F \rightarrow b(a+b)^* \epsilon$$



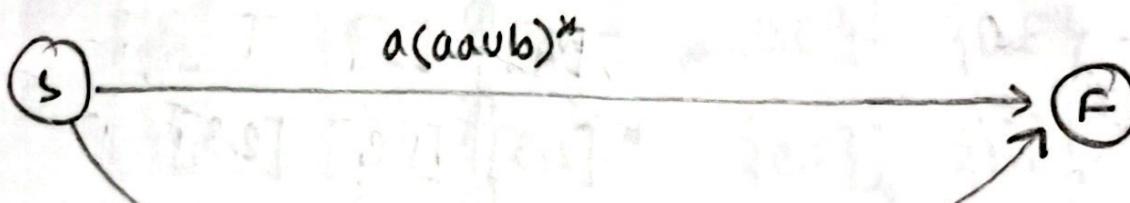
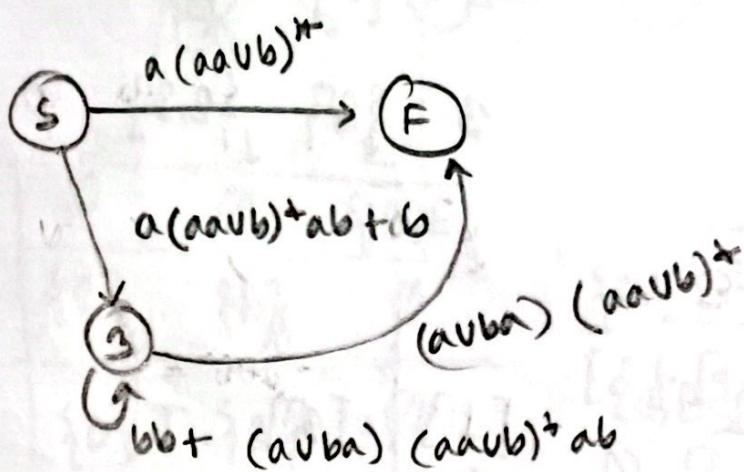
$$\begin{aligned}
 S \rightarrow 1 \rightarrow 2 &= a \checkmark \\
 S \rightarrow 1 \rightarrow 3 &= b \checkmark \\
 3 \rightarrow 1 \rightarrow 2 &= bba \checkmark \\
 3 \rightarrow 1 \rightarrow 3 &= bbb \checkmark \\
 2 \rightarrow 1 \rightarrow 2 &= aab \checkmark \\
 2 \rightarrow 1 \rightarrow 3 &= ab
 \end{aligned}$$

$$S \rightarrow 2 \rightarrow F = a(aa \cup b)^*$$

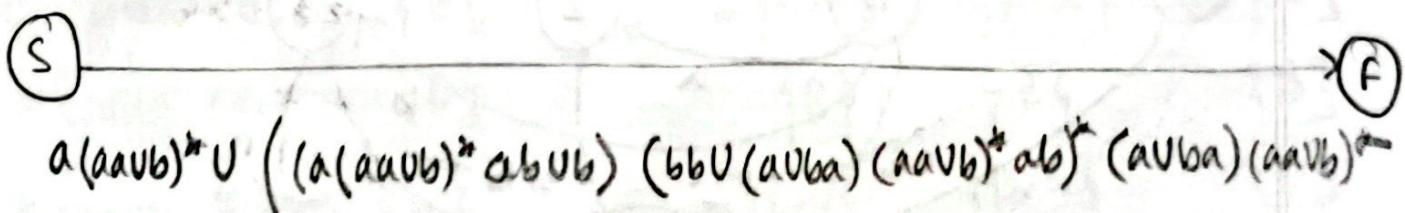
$$S \rightarrow 2 \rightarrow 3 \Leftrightarrow a(aa \cup b)^* ab$$

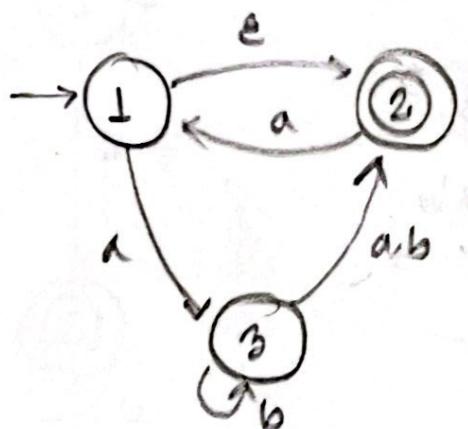
$$3 \rightarrow 2 \rightarrow F = (a \cup ba)(aa \cup b)^*$$

$$3 \rightarrow 2 \rightarrow 2 = (a \cup ba)(aa \cup b)^* ab$$



$$(a(aa \cup b)^* ab \cup b) (bb \cup (a \cup ba)(aa \cup b)^* ab)^* (a \cup ba)(aa \cup b)^*$$

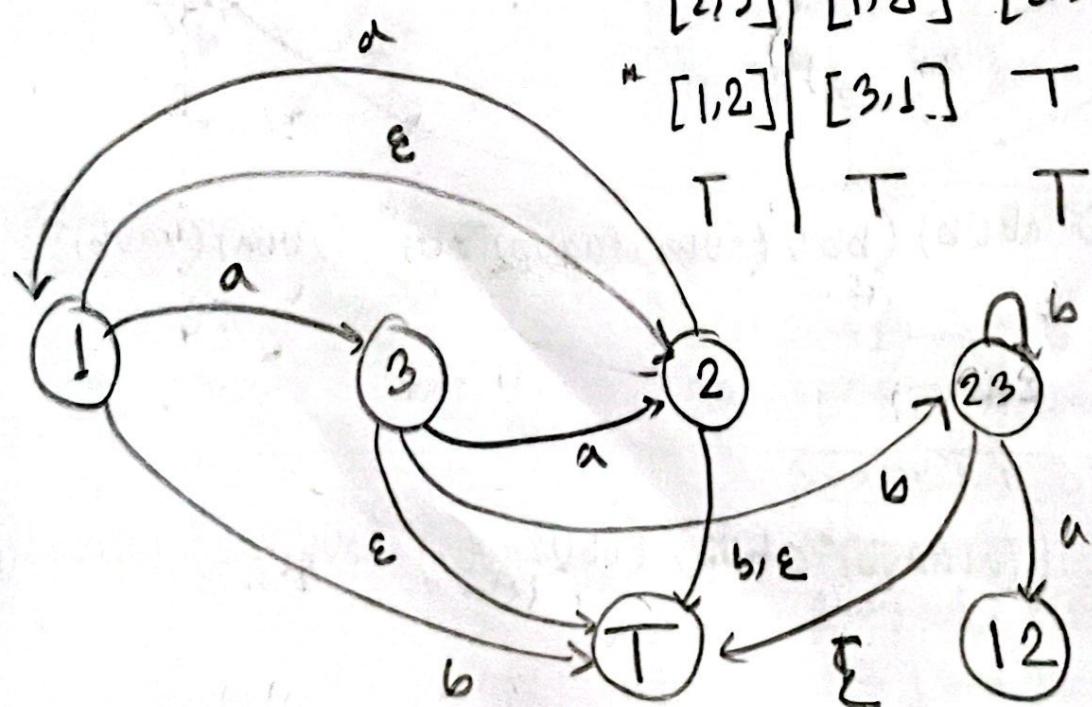


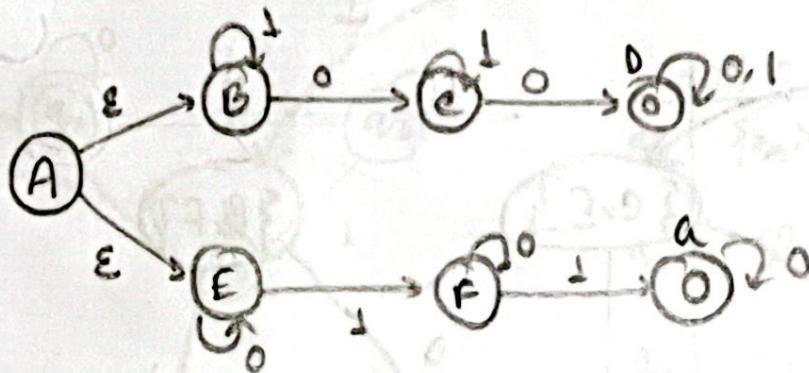
NFA \rightarrow DFA

	a	b	b
1	$\{3\}$	\emptyset	\emptyset
2	$\{1\}$	\emptyset	\emptyset
3	$\{2\}$	$\{2,3\}$	\emptyset

	a	b	ϵ
$[3,1]$	$[3,2]$	$[2,3]$	$[2]$
$[3,2]$	$[1,2]$	$[2,3]$	\emptyset

	a	b	ϵ
$[1]$	$[3]$	T	$[2]$
$[3]$	$[2]$	$[2,3]$	T
$^*[2]$	$[1]$	T	T
$^*[2,3]$	$[1,2]$	$[2,3]$	T
$^*[1,2]$	$[3,1]$	T	$[1]$
T	T	T	T





	0	1	ϵ
A	\emptyset	\emptyset	$\{B, E\}$
B	$\{C\}$	$\{B\}$	\emptyset
C	$\{D\}$	$\{C\}$	\emptyset
D	$\{D\}$	$\{D\}$	\emptyset
E	$\{E\}$	$\{F\}$	\emptyset
F	$\{F\}$	$\{G\}$	\emptyset
G	$\{G\}$	\emptyset	\emptyset

	0	1
$\{A, B, E\}$	$\{C, E\}$	$\{B, F\}$
$\{C, E\}$	$\{D, E\}$	$\{C, F\}$
$\{B, F\}$	$\{C, F\}$	$\{B, G\}$
$\{D, E\}$	$\{D, E\}$	$\{D, F\}$
$\{C, F\}$	$\{D, F\}$	$\{C, G\}$
$\{B, G\}$	$\{C, G\}$	$\{B\}$

ϵ closure (A) = $\{A, B, E\}$

$\rightarrow \{D, F\}$ $\{D, F\}$ $\{D, G\}$

ϵ closure (B) = $\{B\}$

$\rightarrow \{C, G\}$ $\{D, G\}$ $\{D\}$

ϵ closure (C) = $\{C\}$

$\rightarrow \{B\}$ $\{C\}$ $\{D\}$

ϵ closure (D) = $\{D\}$

$\rightarrow \{D, G\}$ $\{D, G\}$ $\{D\}$

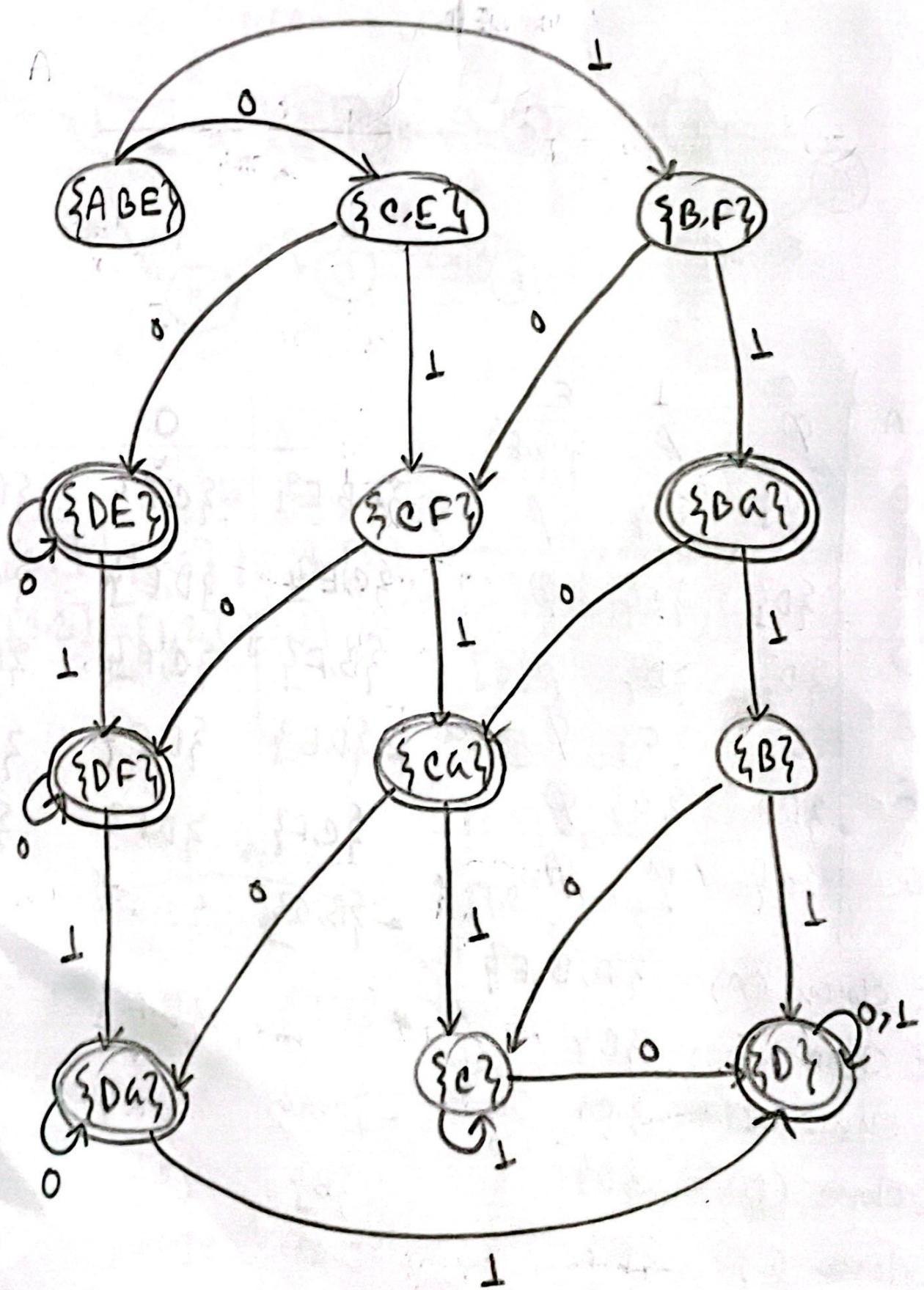
ϵ closure (E) = $\{E\}$

$\rightarrow \{C\}$ $\{D\}$ $\{D\}$

ϵ closure (F) = $\{F\}$

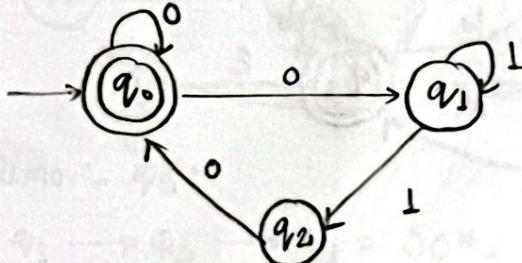
$\rightarrow \{D\}$ $\{D\}$ $\{D\}$

ϵ closure (G) = $\{G\}$

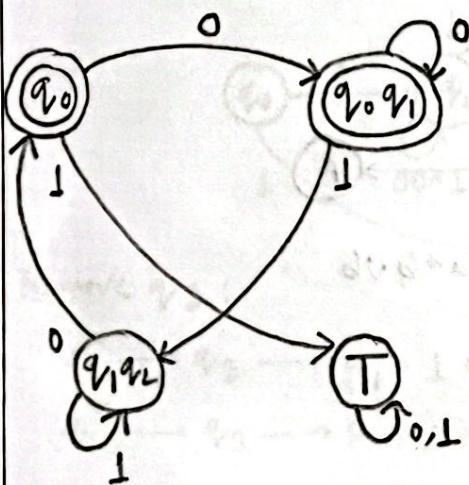


1.

NFA TO DFA

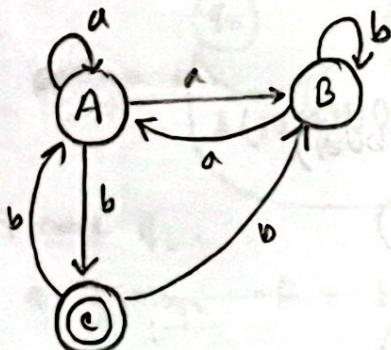


	0	1
q_0	$\{q_0, q_1\}$	\emptyset
q_1	\emptyset	$\{q_1, q_2\}$
q_2	q_0	\emptyset



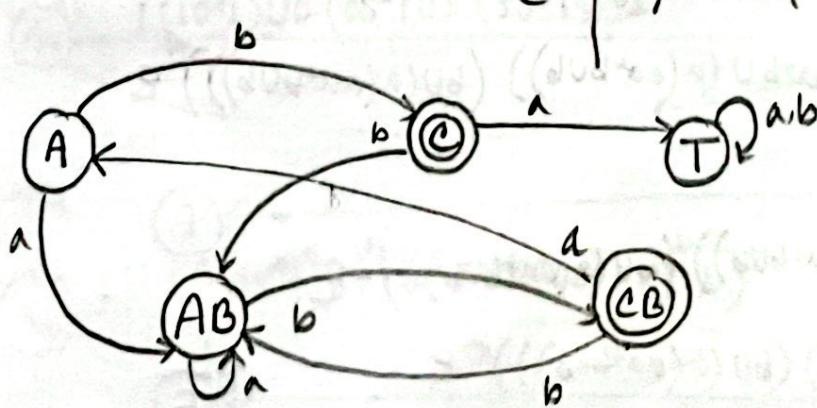
	0	1
$\{q_0\}$	$\{q_0, q_1\}$	T
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_0\}$	$\{q_1, q_2\}$
T	T	T

2.



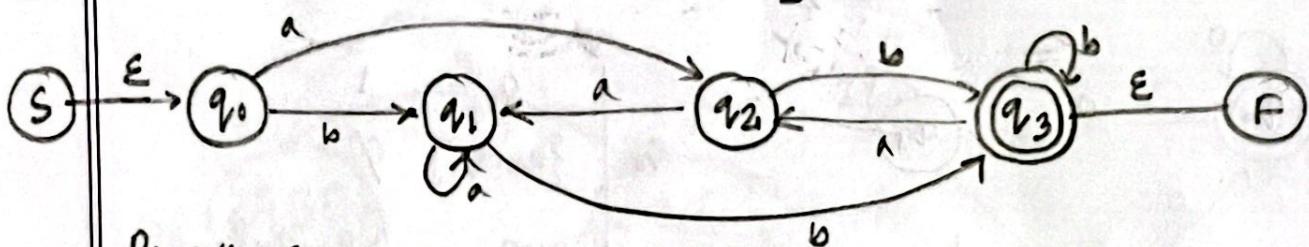
	a	b
A	$\{A, B\}$	$\{C\}$
B	$\{A\}$	$\{B\}$
C	\emptyset	$\{A, B\}$

	a	b
$\{A\}$	$\{A, B\}$	$\{C\}$
$\{A, B\}$	$\{A, B\}$	$\{C, B\}$
$\{C\}$	T	$\{A, B\}$
$\{C, B\}$	$\{A\}$	$\{A, B\}$
T	T	T



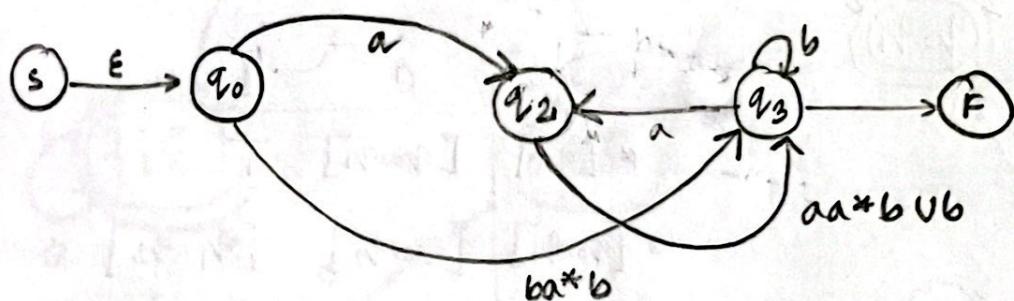
1.

DFA to RE (Mid Question)

Remove q_1

$$q_0 \rightarrow q_1 \rightarrow q_3 = b a^* b$$

$$q_2 \rightarrow q_1 \rightarrow q_3 = a a^* b$$

Remove q_2

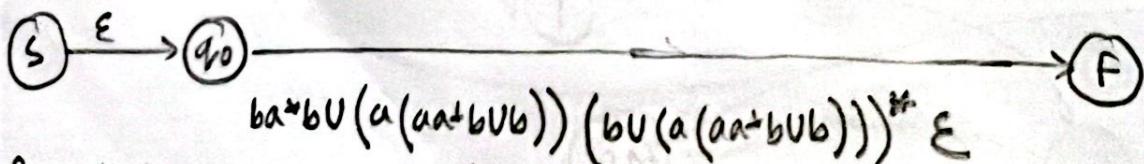
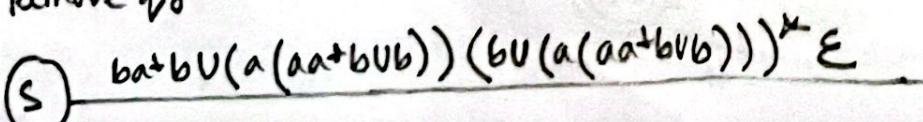
$$q_0 \rightarrow q_2 \rightarrow q_3 = a (a a^* b \cup b)$$

$$q_3 \rightarrow q_2 \rightarrow q_3 = a (a a^* b \cup b)$$

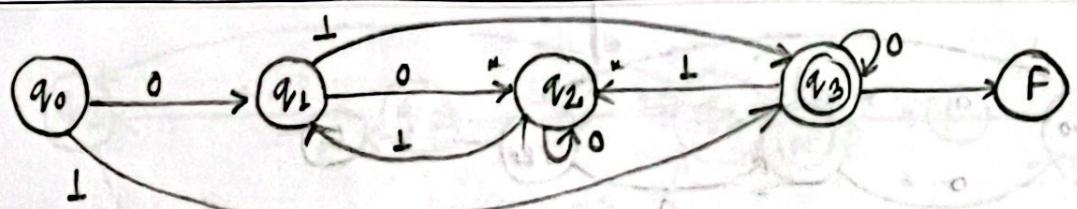
Remove q_3

$$b a^* b \cup (a (a a^* b \cup b))$$

$$q_0 \rightarrow q_3 \rightarrow F = b a^* b \cup (a (a a^* b \cup b)) (b \cup (a (a a^* b \cup b)))^* \epsilon$$

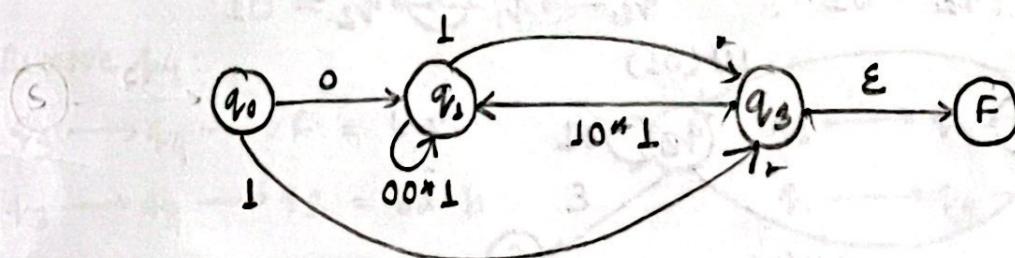
Remove q_0 

2.

Remove q_2 :

$$q_1 \rightarrow q_2 \rightarrow q_1 = 00^*1$$

$$q_3 \rightarrow q_2 \rightarrow q_1 = 10^*1$$

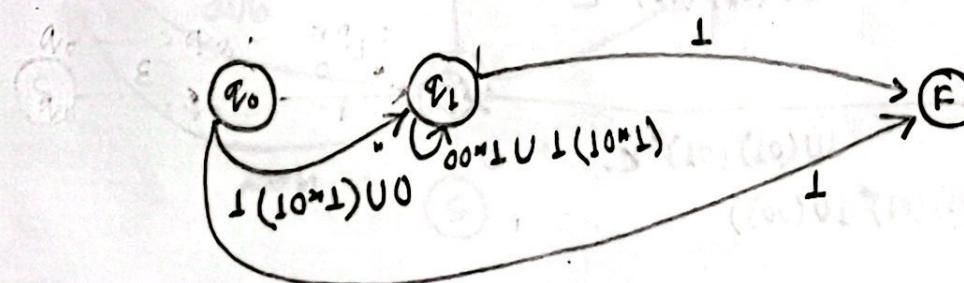
Remove q_3 :

$$q_1 \rightarrow q_3 \rightarrow q_1 = 1(10^*1)$$

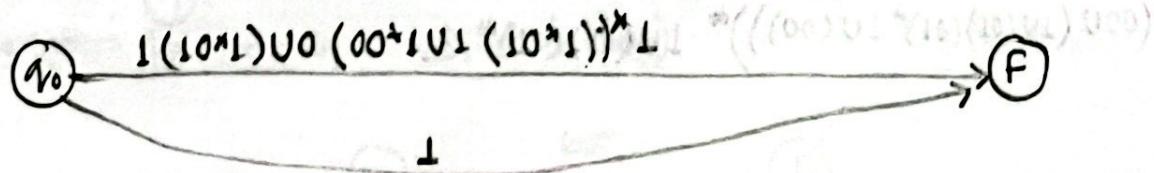
$$q_1 \rightarrow q_3 \rightarrow F = 1$$

$$q_0 \rightarrow q_3 \rightarrow q_1 = 1(10^*1)$$

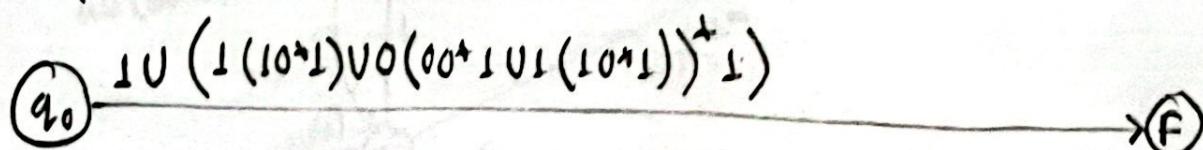
$$q_0 \rightarrow q_3 \rightarrow F = 1$$

Remove q_1 :

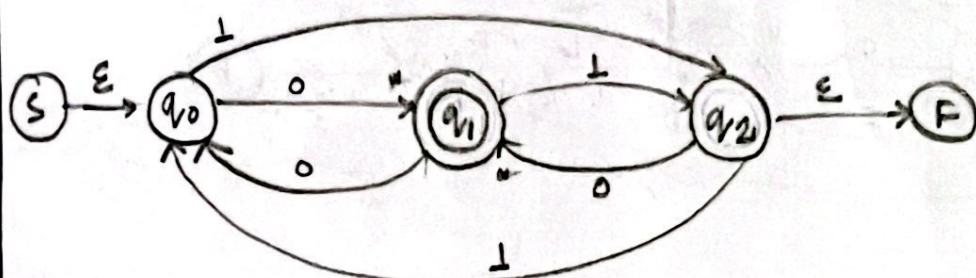
$$q_0 \rightarrow q_1 \rightarrow F = 1(10^*1) U 0 (00^*1 U 1(10^*1))^* 1$$



finally



3.

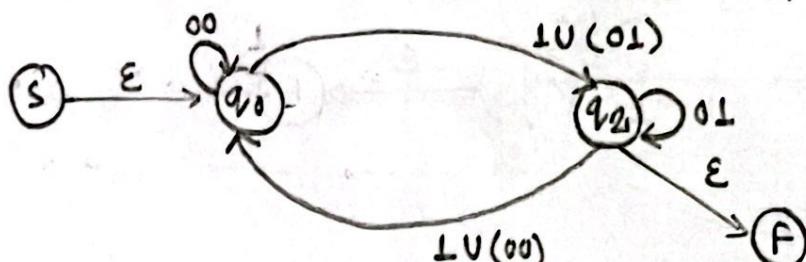
Remove q_1 :

$$q_0 \rightarrow q_1 \rightarrow q_2 = 00$$

$$q_2 \rightarrow q_1 \rightarrow q_0 = 00$$

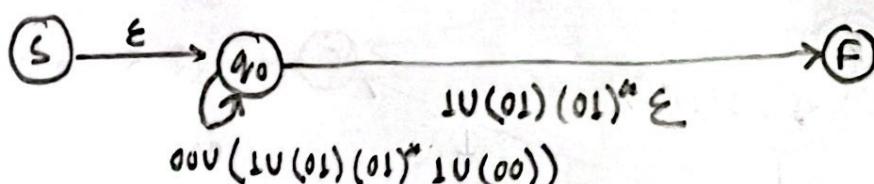
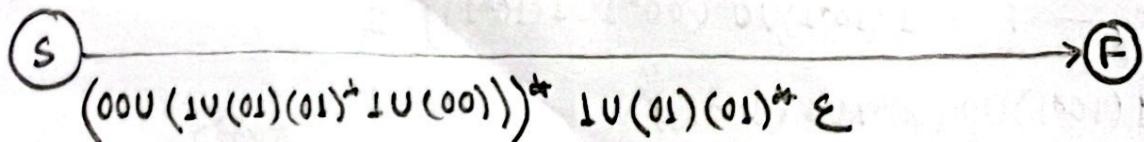
$$q_0 \rightarrow q_1 \rightarrow q_2 = 01$$

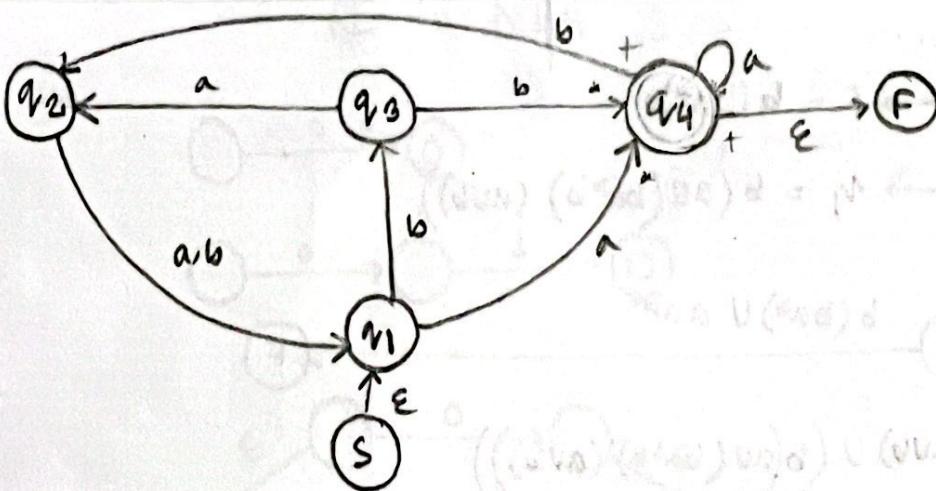
$$q_2 \rightarrow q_1 \rightarrow q_2 = 01$$

Remove q_2 :

$$q_0 \rightarrow q_2 \rightarrow q_0 = 1U(01)(01)^* 1U(00)$$

$$q_0 \rightarrow q_2 \rightarrow F = 1U(01)(01)^* \epsilon$$

Remove q_0 :



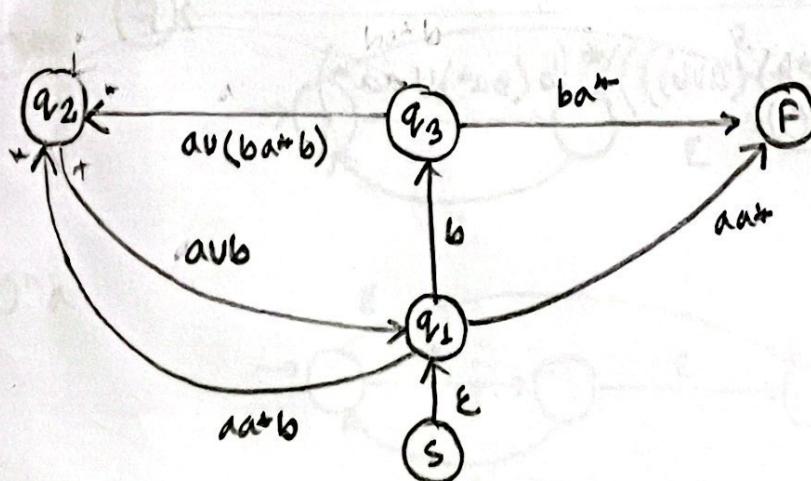
Remove q_4 :

$$q_3 \rightarrow q_4 \rightarrow F = ba^*$$

$$q_3 \rightarrow q_4 \rightarrow q_2 = ba^*b$$

$$q_1 \rightarrow q_4 \rightarrow F = aa^*$$

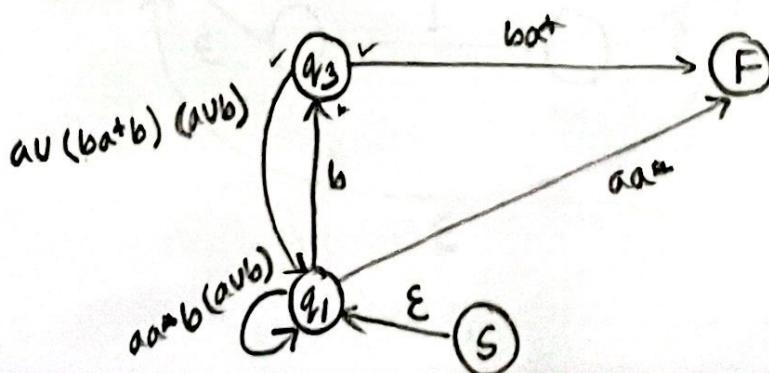
$$q_1 \rightarrow q_4 \rightarrow q_2 = aa^*b$$



Remove q_2 :

$$q_1 \rightarrow q_2 \rightarrow q_1 = aa^*b (a|b)$$

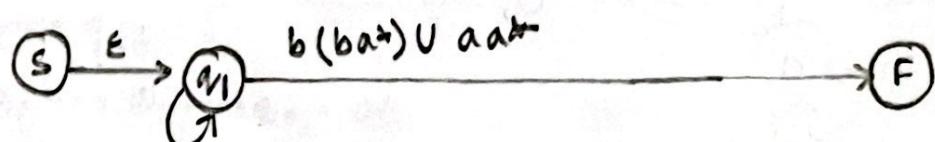
$$q_3 \rightarrow q_2 \rightarrow q_1 = aU(ba^*b) (a|b)$$



Remove q_3 :

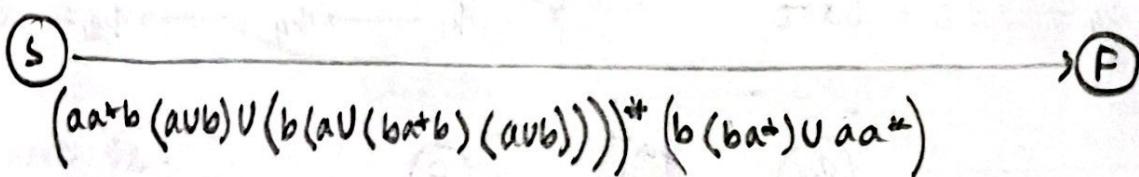
$$q_1 \rightarrow q_3 \rightarrow f = b(ba^*)$$

$$q_1 \rightarrow q_3 \rightarrow q_1 = b(a \cup (ba^*b) (a \cup b))$$



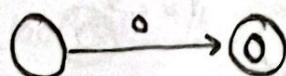
$$aa^*b(a \cup b) \cup (b(a \cup (ba^*b) (a \cup b)))$$

Remove q_1 :



RE to NFA

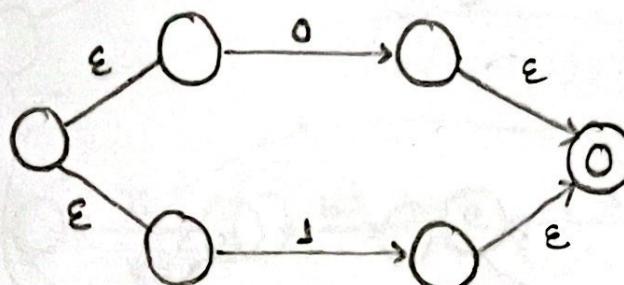
0



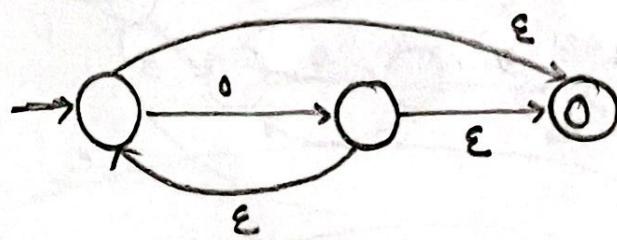
10



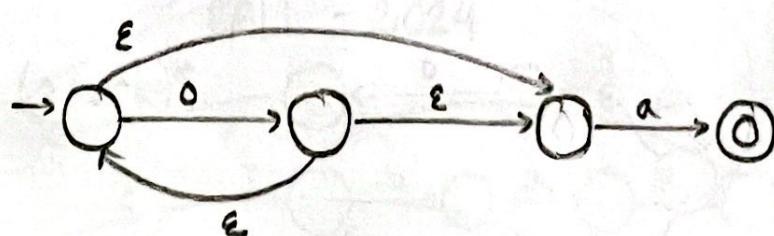
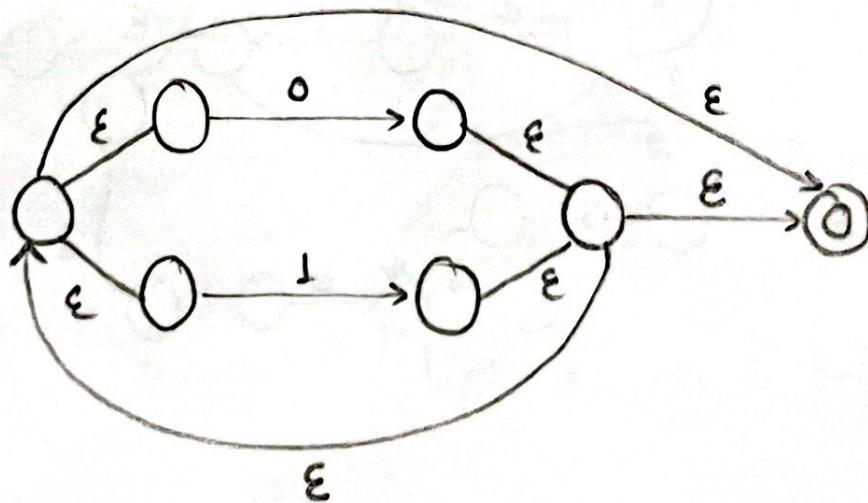
0+1

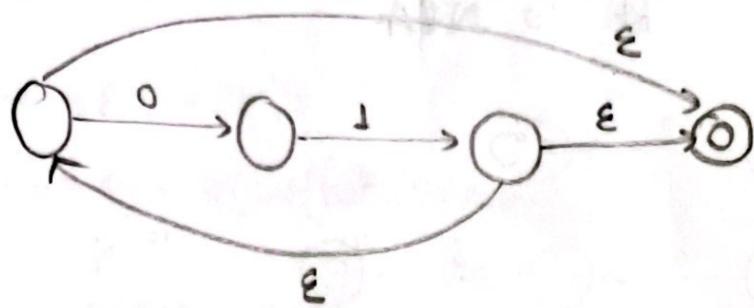
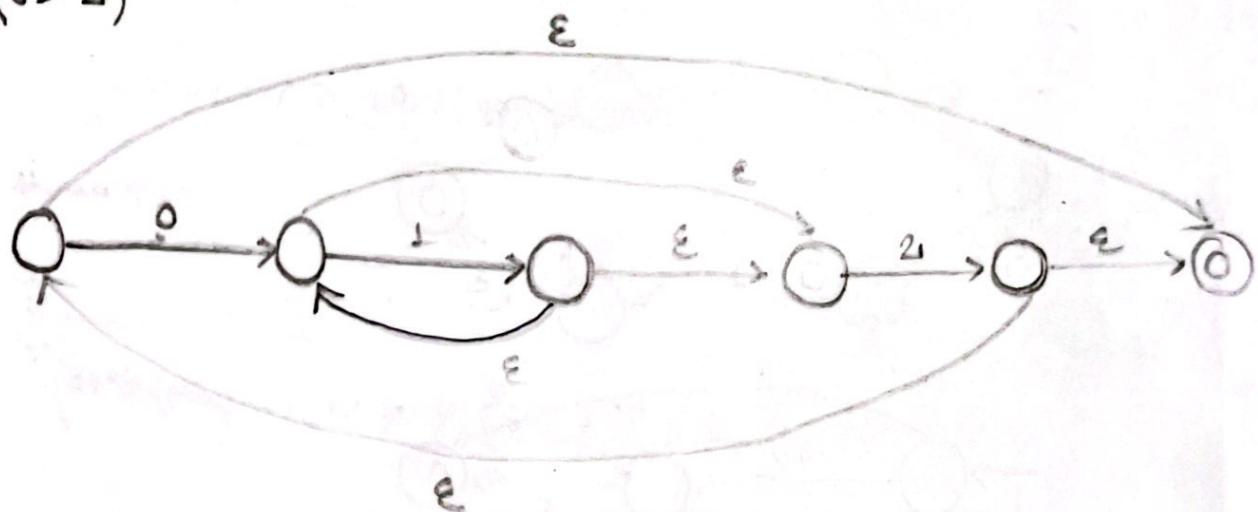
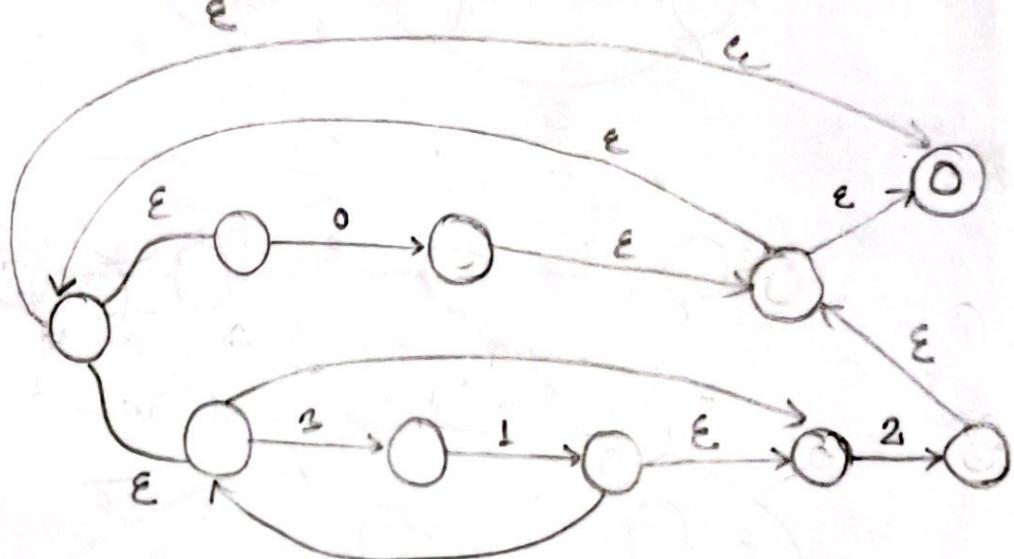


0*

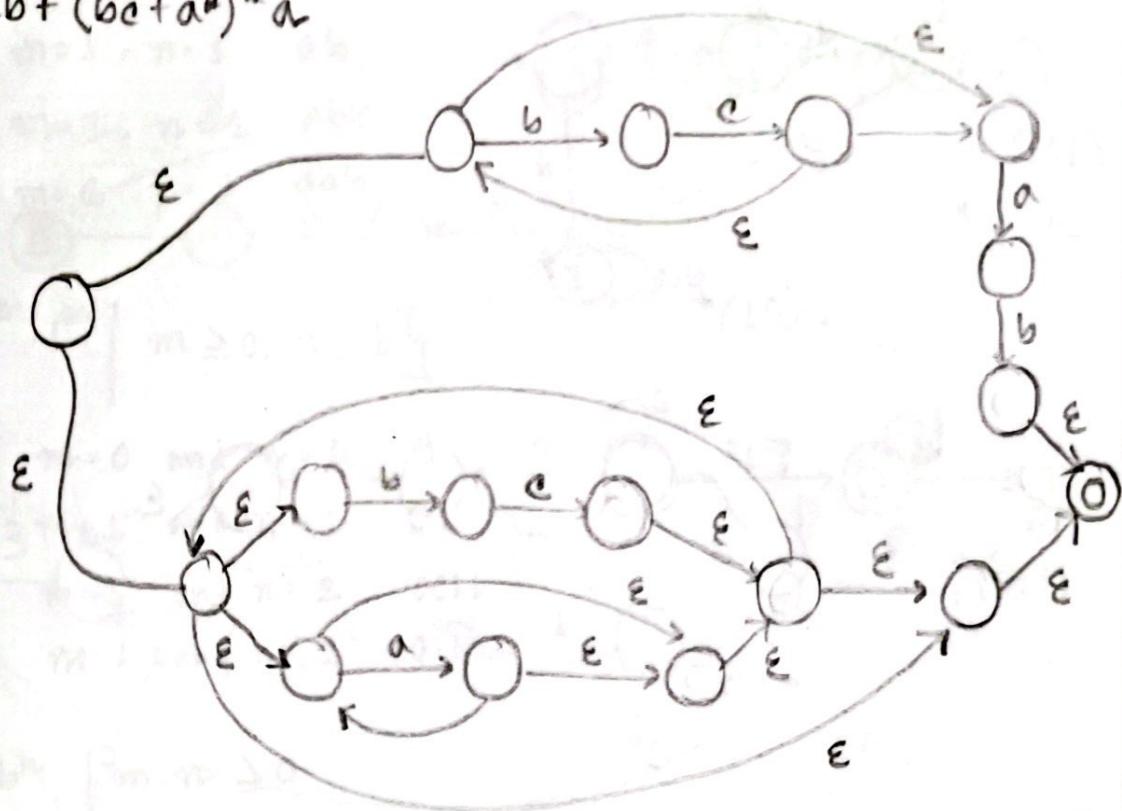


0*a

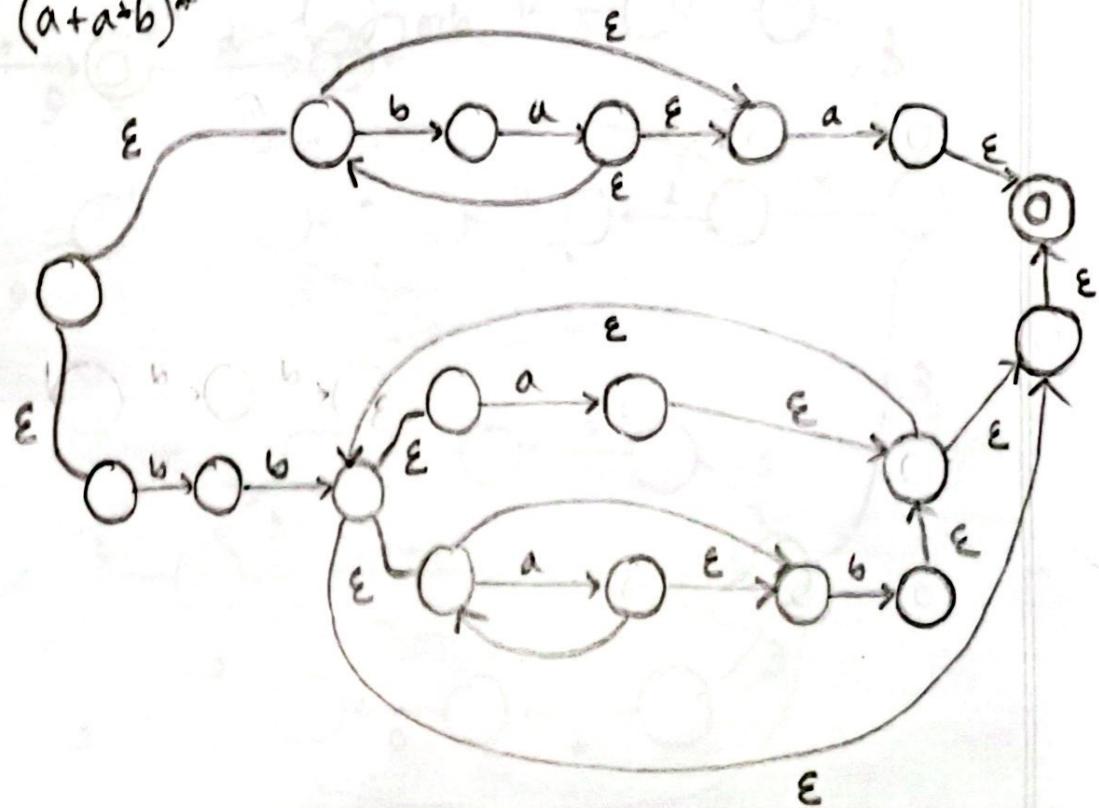
 $(0+1)^*$ 

$(01)^*$  $(01^*2)^*$  $(0 + (31^*2))^*$ 

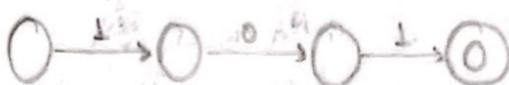
1

RE \rightarrow NFA $(bc)^* ab + (bc + a^*)^* a$ 

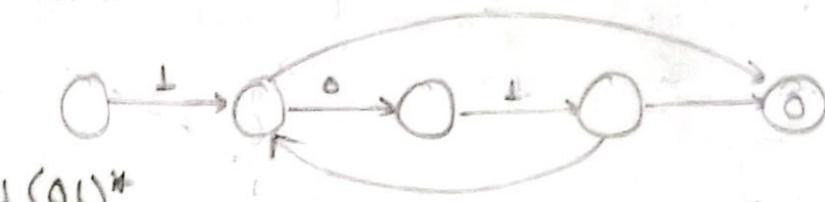
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 $(ba)^* a + bba (a + a^* b)^*$ 

$$101 + \perp (01)^* + (01 + 10)^*$$

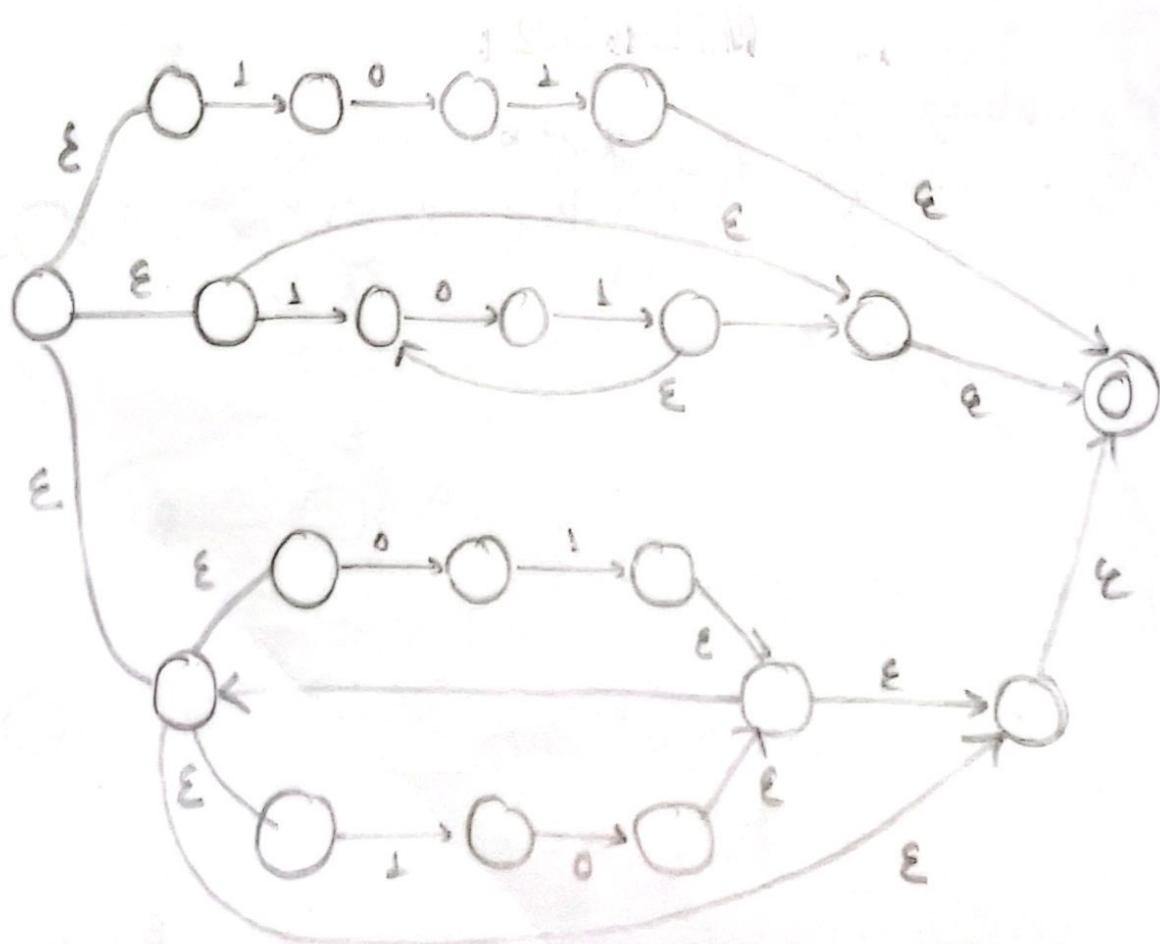
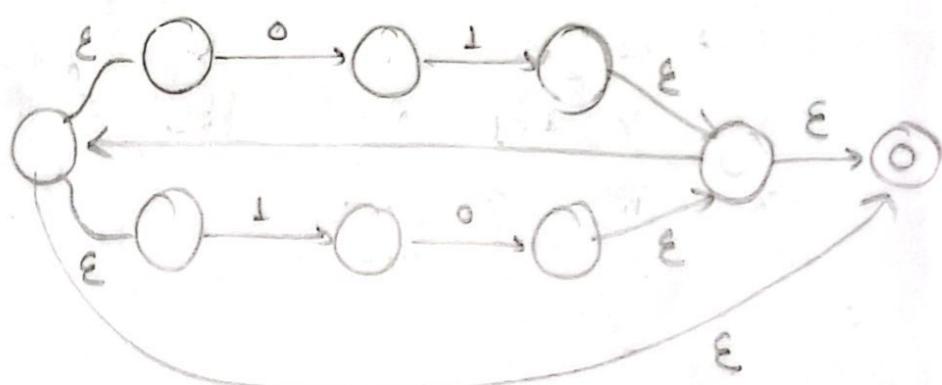


(101)



$\perp (01)^*$

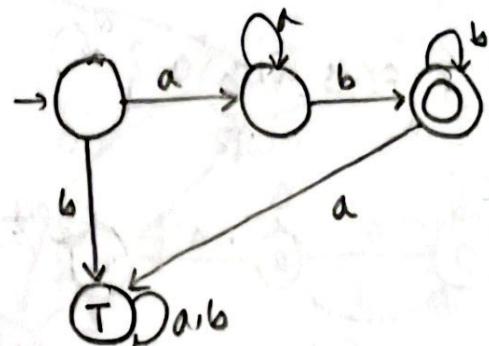
$(01 + 10)^*$



DFA

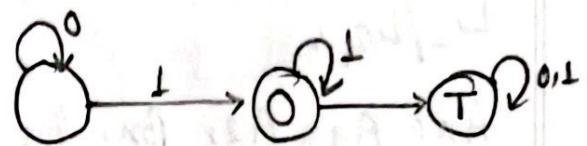
$$L = a^m b^n \mid m, n \geq 1$$

- When, $m=1, n=1$ ab
 $m=1, n=2$ abb
 $m=2, n=1$ aab



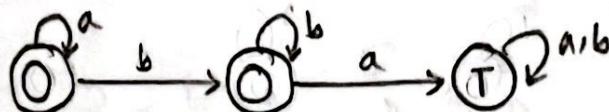
$$L = \{0^m, 1^m \mid m \geq 0, n \geq 1\}$$

- When, $m=0$ and $n=1$ 1
 $m=1$ and $n=1$ 01
 $m=2$ and $n=2$ 0011
 $m=1$ and $n=2$ 011



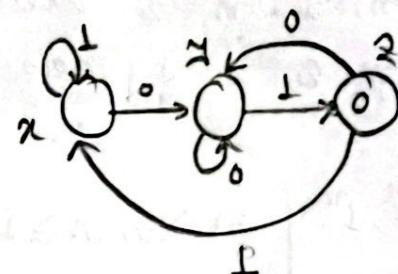
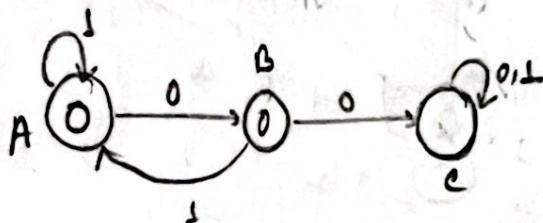
$$L = a^m b^n \mid m, n \geq 0$$

$$\{\epsilon, a, b, ab, aab, abb, aabb, \dots\}$$



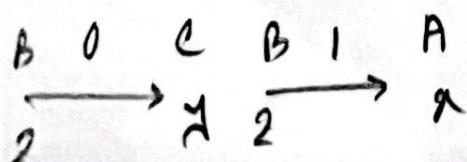
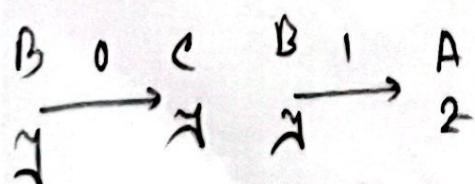
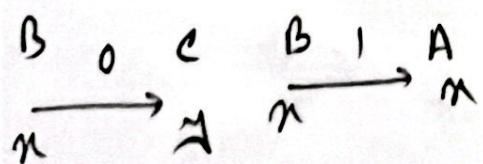
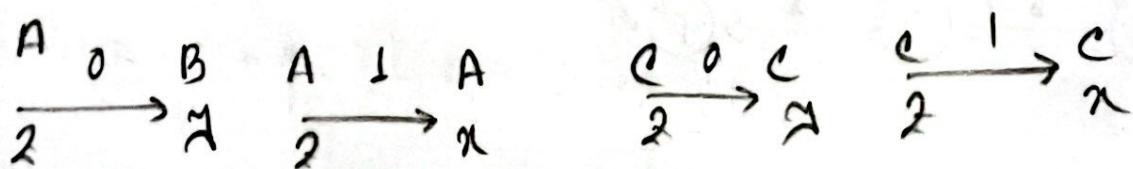
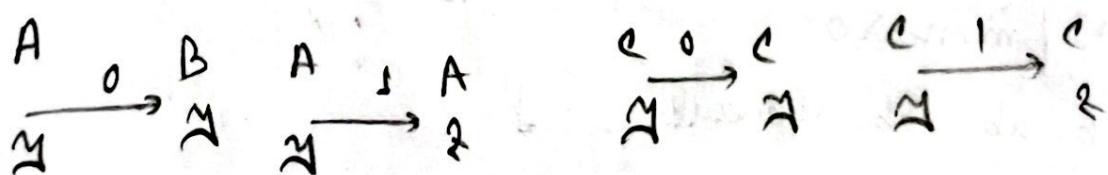
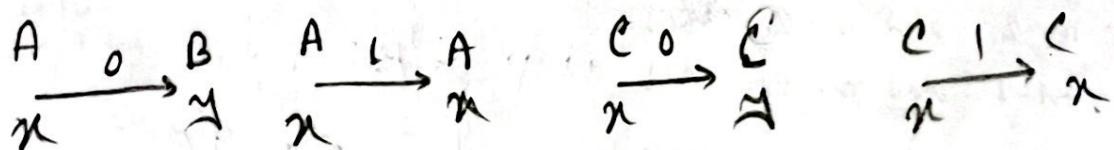
$L_1 = \{ \omega, \omega \text{ is not a starting of } \omega \}$

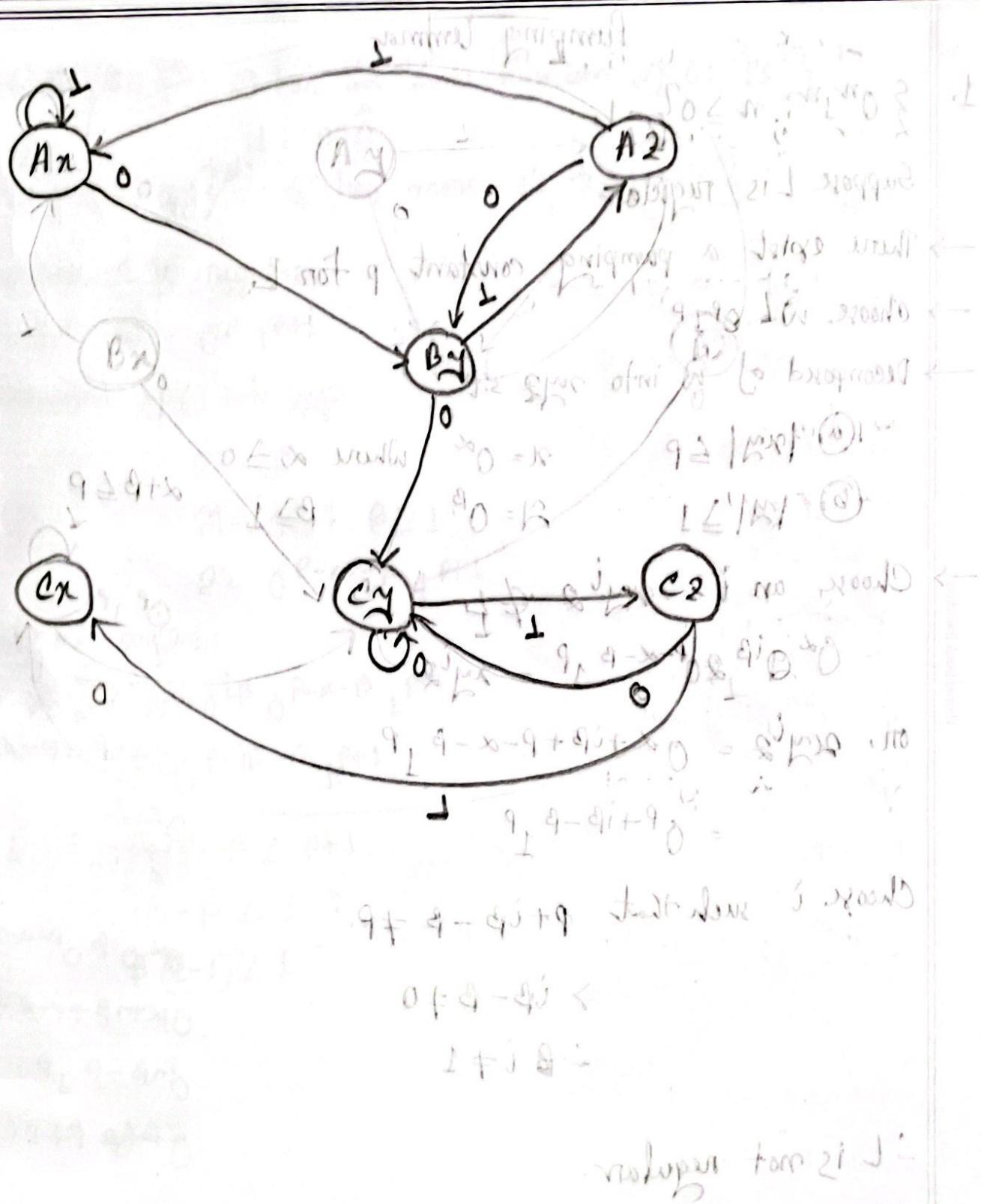
$L_2 = \{ \text{ends with } 01 \}$



$L_1 \cap L_2 :$

$A_x, A_y, A_z, B_x, B_y, B_z, C_x, C_y, C_z$





Pumping Lemma

$$\{0^n 1^n : n \geq 0\} = L$$

$$\frac{000111}{p}$$

Suppose L is regular

→ There exist a pumping constant p for L

→ Choose $w = 0^p 1^p$

→ Decomposed of w into xyz s.t.

$$(a) |xy| \leq p \quad x = 0^\alpha \quad \text{where } \alpha \geq 0$$

$$(b) |y| \geq 1 \quad y = 0^\beta \quad \beta \geq 1 \quad \alpha + \beta \leq p$$

→ Choose an i $xy^i z \notin L$

$$0^\alpha 0^i 0^{p-\alpha-\beta} 1^p = xy^i z$$

$$\text{or, } xy^i z = 0^{\alpha+i\beta+p-\alpha-\beta} 1^p$$

$$= 0^{p+i\beta-\beta} 1^p$$

Choose i such that $p+i\beta-\beta \neq p$

$$\Rightarrow i\beta - \beta \neq 0$$

$$\therefore \beta \neq 1$$

∴ L is not regular.

$$0^p 1^p$$

$$0^\alpha 0^\beta 0^{p-\alpha-\beta} 1^p$$

$$0^{\alpha+\beta+p-\alpha-\beta} 1^p$$

$$0^p 1^p$$

$$xy^i z$$

$$0^\alpha 0^i 0^{p-\alpha-\beta} 1^p$$

$$0^{\alpha+i\beta+p-\alpha-\beta} 1^p$$

$$0^{p+i\beta-\beta} 1^p$$

$$0^{\beta(p-p+i\beta)} 1^p$$

$$p + \beta(p-i\beta) \neq p$$

2. $\{w \in \{0,1\}^*: w \text{ has the same number of 0's, 1's}\}$

3. $L = \{w \in \{0,1\}^*: w \text{ has more 1's than 0's}\}$

Suppose L is regular

Choose $w = 0^p 1^{p+1}$

Decomposed of w into $\alpha\beta\gamma$:

$$\alpha = 0^\alpha$$

$$\beta = 0^\beta \quad \beta \geq 1$$

$$\gamma = 0^{p-\alpha-\beta} 1^{p+1}$$

Look at $\alpha\beta\gamma$:

$$\alpha\beta\gamma = 0^\alpha 0^{i\beta} 0^{p-\alpha-\beta} 1^{p+1}$$

$$= 0^{p-i\beta-\beta} 1^{p+1}$$

in L iff $p+i\beta-\beta \leq p+1$

$$i\beta - \beta \leq 1$$

$$\beta(i-1) \leq 1$$

4. $L = \{0^{2n}1^n : n \geq 0\}$

Choose $\omega = 0^{2p}1^p$

Decomposed: $\omega\bar{\omega}^2 : x = 0^\alpha, \bar{x} = 0^\beta, Q = 0^{2p-\alpha-\beta}1^p$

$$\begin{aligned}\omega\bar{\omega}^2 &= 0^\alpha 0^{ip} 0^{2p-\alpha-\beta}1^p \\ &= 0^{2p-\beta+ip}1^p\end{aligned}$$

$$2p+ip-\beta = 2p$$

$$ip-\beta = 0$$

$$\begin{aligned}(i-1) &= 0 \\ i &= 1\end{aligned}$$

5.

$L = \{0^n10^n : n \geq 0\}$

Choose $\omega = 0^p10^p$

Decomposed: $\omega\bar{\omega}^2 : x = 0^\alpha, \bar{x} = 0^{\beta+q}1^q, Q = 0^{p-\alpha-\beta}1^p$

$$\begin{aligned}\omega\bar{\omega}^2 &= 0^\alpha 0^{ip} 0^{p-\alpha-\beta}1^p \\ &= 0^{ip+\beta-p}1^p\end{aligned}$$

$$p+ip-\beta = p$$

$$ip-\beta = 0$$

$$i = 1$$

6. $L = \{0^n 1^n : n \in \mathbb{N}\}$

Suppose L were regular.

$$W = 0^p 1^{p+1}$$

$$x = 0^\alpha, y = 0^\beta, z = 0^{p-\alpha-\beta} 1^{p+1}$$

$$xy^iz = 0^\alpha 0^{i\beta} 0^{p-\alpha-\beta} 1^{p+1}$$

$$= 0^{\alpha+i\beta+p-\alpha-\beta} 1^{p+1}$$

$$= 0^{p+i\beta-\beta} 1^{p+1}$$

$$p+i\beta-\beta \neq p+1$$

$$\beta(i-1) \neq p+1$$

$$i \neq \frac{p}{\beta} + 1$$

$$\begin{matrix} x & 0^\alpha \\ y & 0^\beta \\ z & 0^{p-\alpha-\beta} \end{matrix}$$

7. $L = \{0^n 1^m : n \leq 3m\}$

Suppose L were regular

$$W = 0^p 1^{p-1} 1^p$$

$$x = 0^\alpha, y = 0^\beta, z = 0^{3p-1-\alpha-\beta}$$

$$\begin{aligned} xy^iz &= 0^\alpha 0^{i\beta} 0^{3p-1-\alpha-\beta} 1^p \\ &= 0^{3p+i\beta-1-\beta} 1^p \end{aligned}$$

$$3p+i\beta-1-\beta \leq 3p$$

$$i\beta-1-\beta \leq 0$$

$$\cdot \beta(i-1) \leq 1$$

9. $L = \{0^{2^n} : n \geq 0\}$

$$W = 0^{2^p}$$

$$x = 0^\alpha, y = 0^\beta, z =$$

$$\begin{aligned} xy^iz &= 0^\alpha 0^{i\beta} 0^{2^p-\alpha-\beta} \\ &= 0^{2^p+(i-1)\beta} \end{aligned}$$

$$\text{choose } i = 2$$

$$2^p \leq 2^p + \beta \leq 2 + p \leq 2^{p+2}$$

$$1120 \leftarrow 2$$

$$3 \leftarrow 2$$

8. $L = \{0^{n^2} : n \geq 0\}$

$$W = 0^{p^2}$$

$$x = 0^\alpha, y = 0^\beta, z = 0^{p^2-\alpha-\beta}$$

$$xy^iz = 0^\alpha 0^{i\beta} 0^{p^2-\alpha-\beta}$$

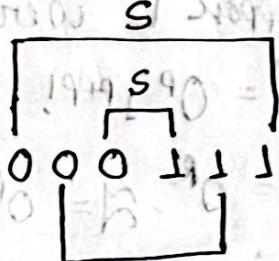
$$= 0^{p^2+\beta}$$

$$p^2 \leq p^2 + \beta \leq p^2 + p \leq (p+1)^2$$

CFA

Examples:

① $0^n 1^n$ $S \rightarrow 0S1$
 $S \rightarrow \epsilon$

$$\begin{aligned}
 S &\Rightarrow 0S1 \\
 &\Rightarrow 00S11 \\
 &\Rightarrow 000S111 \\
 &\Rightarrow 000111
 \end{aligned}$$
② $0^n 1^{2n}$ $S \rightarrow 0S11$
 $S \rightarrow \epsilon$

$$\begin{aligned}
 S &\Rightarrow 0S11 \\
 &\Rightarrow 00S111 \\
 &\Rightarrow 000S1111
 \end{aligned}$$
③ $0^{3n} 1^{5n}$ $S \rightarrow 000S11111$ $S \rightarrow \epsilon$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$ $0 = 0 \quad 0 = 1 \quad 0 = 0$

Subject:

Date:

1. $\{w | w \text{ contains at least three } 1's\}$

$$(0|1)^* 1 (0|1)^* 1 (0|1)^* 1 (0|1)^*$$

$$S \rightarrow A 1 A 1 A 1 A$$

$$A \rightarrow 0A | 1A | \epsilon$$

2. $\{w | w \text{ starts and ends with same symbol}\}$

$$0 (0|1)^* 0 | 1 (0|1)^* 1$$

$$S \rightarrow 0 A 0 | 1 A 1$$

$$A \rightarrow 0A | 1A | \epsilon$$

3. $\{w | w \text{ the length of } w \text{ is odd}\}$

$$((0|1)^* (0|1)^* (0|1))$$

$$S \rightarrow A B$$

$$A \rightarrow B B A | \epsilon$$

$$B \rightarrow 01 | 10 | 00 | 01 | 00 | 01 | 00 | 01$$

4. $\{w | \text{the length of } w \text{ is odd and its middle symbol is } 0\}$

$$S \rightarrow 0 S 0 | 1 S 1 | 0 S 1 | 1 S 0 | 0$$

5. $\{w | w = w^R, w \text{ is a palindrome}\}$

$$S \rightarrow 0 S 0 | 1 S 1 | 1 | 0 | \epsilon$$

$$S \Rightarrow$$

$$\Rightarrow 1 S 1$$

$$\Rightarrow 1 0 S 0 1$$

$$\Rightarrow 1 0 0 0 1$$

6. $\{w \mid 0^i 1^j 2^k, i=j \text{ for } i \neq k\}$ $\rightarrow \{0^i 1^j 2^k \mid i=j \text{ or } i \neq k\}$

for $i=i$

$\hookrightarrow A|B$

$\frac{0^i 1^j}{P} \frac{2^k}{Q} \frac{i=j}{A} \frac{i \neq k}{B}$

$A \rightarrow PQ$

$Q \rightarrow 2Q|\epsilon$

$P \rightarrow OP|1|\epsilon$

$1^i(1|0) \mid 0^j(1|0)$

for $i \neq k$

$\frac{0^i 1^j 2^k}{W} \frac{i=j}{X} \frac{i \neq k}{A} \frac{i \neq k}{B}$

$W \rightarrow \omega X$

$\omega \rightarrow 0\omega|\epsilon$

$\{ \text{bbb si } \omega \text{ si } \omega \text{ si } \omega \}$

$X \rightarrow Y|2$

$(1|0) \mid (1|0) \mid (1|0)$

$Y \rightarrow 1Y2|M$

number of 0's in $w_1 = \text{no. of } 1's \text{ in } w_2$

$M \rightarrow 2M|\epsilon$

$S \rightarrow 1S|S0|0S1|\#1|0$

$2 \rightarrow 122|N$

8. $A = \{w \in \{0,1\}^*: w \text{ contains odd no. of } 1's\}$

$N \rightarrow 1N|\epsilon$

$(0^* 1 0^* 1)^* 0^* 1 0^* 1 0 20 \leftarrow 2$

$\leftarrow 2$
 $131 \leftarrow$

$10201 \leftarrow$

$10001 \leftarrow$

$A \rightarrow BC|C$

$C \rightarrow 0C|\epsilon$

$B \rightarrow DB|\epsilon$

$D \rightarrow C1C1$

$3101121020 \leftarrow 2$

Subject: _____

Date: _____

$$9. L_1 = \{ 1^i 0^j 2^k \mid i, j, k \geq 0, i = k \}$$

$$S \rightarrow 1S1 \mid A \mid \epsilon$$

$$A \rightarrow 0B$$

$$B \rightarrow 2B \mid \epsilon$$

$$10. \{ 1^i 0^j 2^k \mid i, j, k \geq 0, k = i+2j \}$$

$$S \rightarrow S \mid 1S1 \mid C$$

$$C \rightarrow 0Cn$$

$$a \rightarrow 2a11 \mid \epsilon$$

$$11. L_3 = \{ 1^i 0^j 2^k \mid i, j, k \geq 0, k = 2i + 3j \}$$

$$S \rightarrow 1S22 \mid 0S222 \mid \epsilon$$

$$12. L_4 = \{ 1^i 0^j 2^k \mid i, j, k \geq 0, k = 3i + j \}$$

$$S \rightarrow 1S222 \mid 0S2 \mid \epsilon$$

$$13. L_5 = \{ 0^i 2^j 1^k \mid i, j, k \geq 0, k = 2i + 2j \}$$

$$S \rightarrow 0S21 \mid 2S11 \mid \epsilon$$

$$14. L_6 = \{ 2^i 0^j 1^k \mid i, j, k \geq 0, k = 3i + 2j \}$$

$$S \rightarrow 2S111 \mid 0S11 \mid \epsilon$$

15. Starts with 10

$$S \rightarrow 10A$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

16. Ends with 10

$$S \rightarrow A10$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

17. Contains 101 as substring

$$S \rightarrow A101B$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

$$B \rightarrow 0B \mid 1B \mid \epsilon$$

19. Even number of zeros

$$S \rightarrow 0A \mid 1S \mid \epsilon$$

$$A \rightarrow 0S \mid 1A$$

20. Odd number of ones

$$S \rightarrow 1E \mid 0S$$

$$E \rightarrow 1S \mid 0E$$

21. Length is Even

$$S \rightarrow 0A \mid 1A \mid \epsilon$$

$$A \rightarrow 0S \mid 1S$$

22. Length is odd

$$S \rightarrow 0A \mid 1A$$

$$A \rightarrow 0S \mid 1S \mid \epsilon$$

23. At most 2 zeros

$$S \rightarrow A \mid 0A \mid 0AOA$$

$$A \rightarrow 1A \mid \epsilon$$

24. Exactly 3 ones

$$S \rightarrow A1A1A1A$$

$$A \rightarrow 0A \mid \epsilon$$

25. At least 2 zeros

$$S \rightarrow 00AO$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

26. Number of zeros is multiple of 2

$$S \rightarrow$$

$$S \mid 1120 \mid 11122 \leftarrow 2$$

27. $a^n b^n, L = \{ \}$ No regular recognizable

$$S \rightarrow aAb$$

$$A \rightarrow aAb | \epsilon$$

28. $A = \{ w \in \{0,1\}^*: w \text{ contains at least two } 0s \}$

$$S \rightarrow 0A0$$

$$A \rightarrow 0A | 1A | \epsilon$$

$L = \{ w \in \{0,1\}^*: w = 0^{3i} A 1^{2i} \text{ and } i \geq 0 \}$

$$S \rightarrow 000 S 11 | A$$

$$A \rightarrow 0B0$$

$$B \rightarrow 0B | 1B | \epsilon$$

29. $L = \{ w \in \Sigma^*: w \text{ is an odd length palindrome} \}$

$$S \rightarrow asa | bsb | a | b$$

30. $L = \{ w \in \Sigma^*: w \text{ is an even length palindrome} \}$

$$S \rightarrow asa | bsb | \epsilon$$

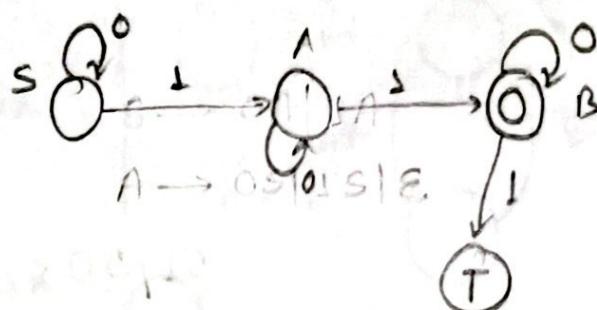
31. $S \rightarrow A \perp \perp B$

$$A \rightarrow 0C | 1C$$

$$C \rightarrow 0A | 1A | \epsilon$$

$$B \rightarrow 0D | 1D$$

$$D \rightarrow 0B | 1B | \epsilon$$



$$S \rightarrow 0S | 1A$$

$$A \rightarrow 0A | 1B$$

$$B \rightarrow 0B | \epsilon$$

1. ① $L = \{w \in \{a, b, c\}^* \mid w = aibick, n = 2i + k, 1k = 2i + 2j, i, j, k \geq 0\}$

$a^i b^{2i+k} c^k$

$a^i b^{2i} b^k c^k$

$S \rightarrow XY$

$X \rightarrow aXbb|\epsilon$

$Y \rightarrow bYc$

$a^i b^j c^{2i+2j}$

$a^i b^j c^{2i} c^{2j}$

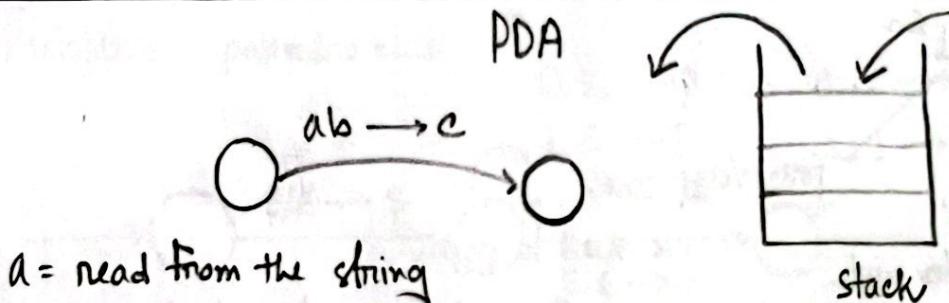
$S \rightarrow XY$

$X \rightarrow aXcc|Y$

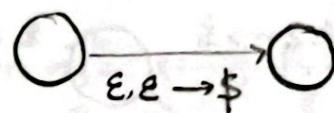
$Y \rightarrow bYcc|\epsilon$

Subject:

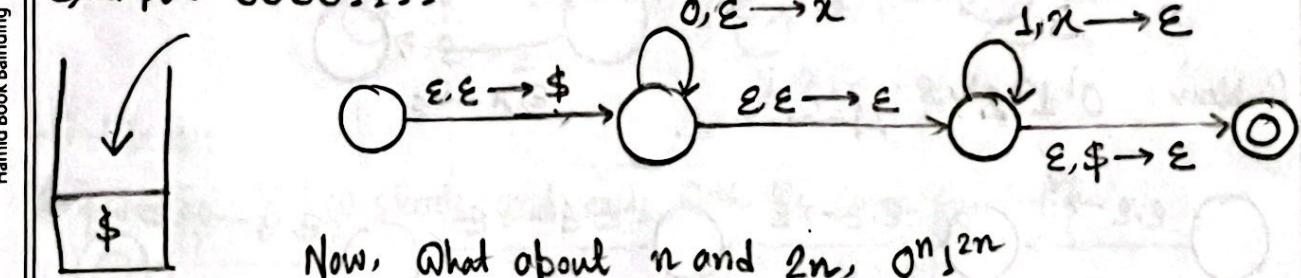
Date:



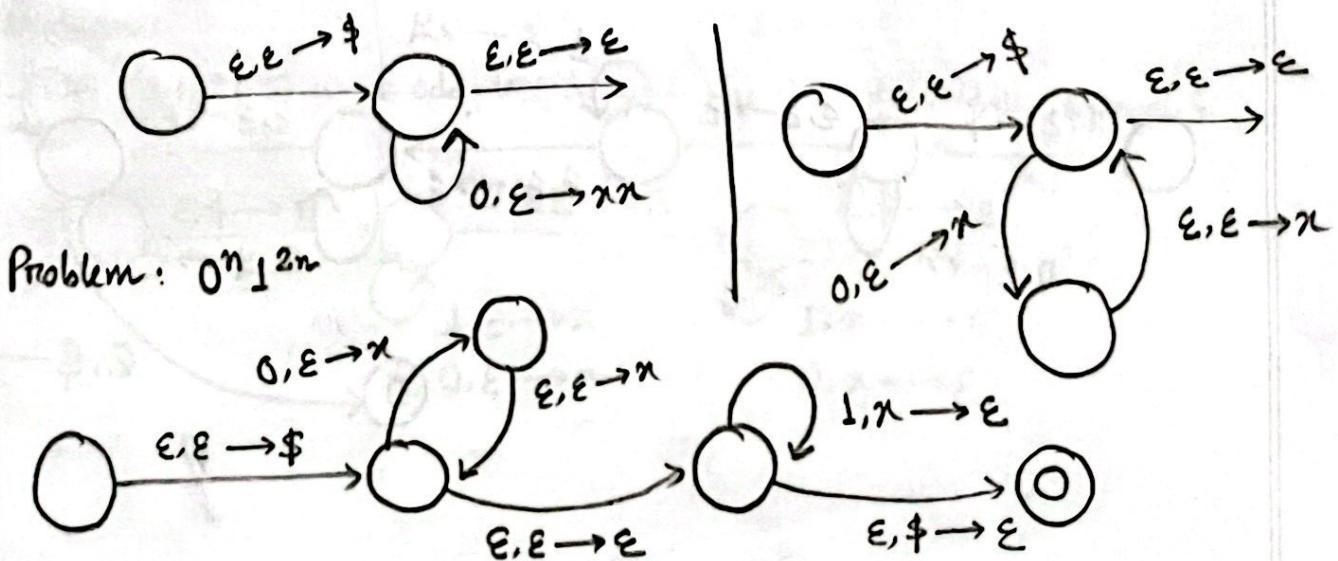
Default state

 $\$$ = when to stopProblem: $0^n 1^n$ (equal 0 and 1)

Example: 00001111



Now, What about n and $2n$, $0^n 1^{2n}$
 for every zero we need to push two x in the stack
 and for every one we need to pop only one x .



Problem: $0^{3n-1} 1^{2n}$

for every 2010,

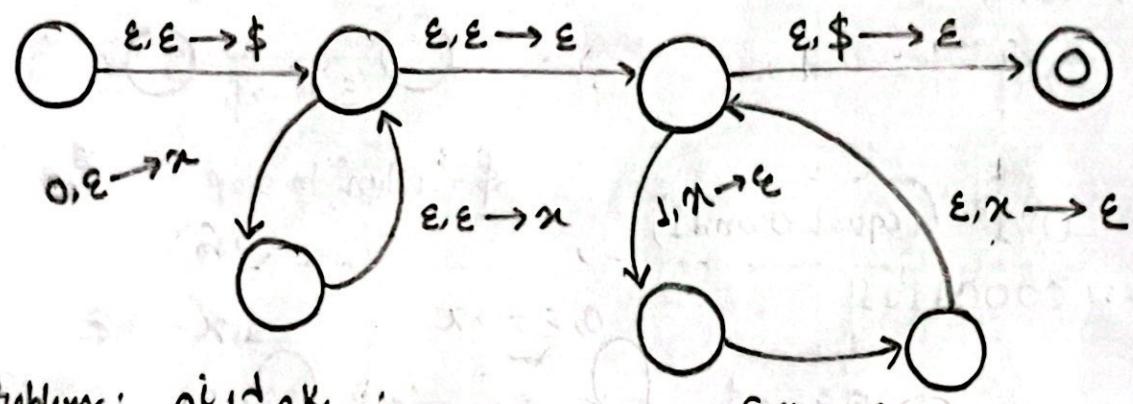
we need to push

2 one into the stack

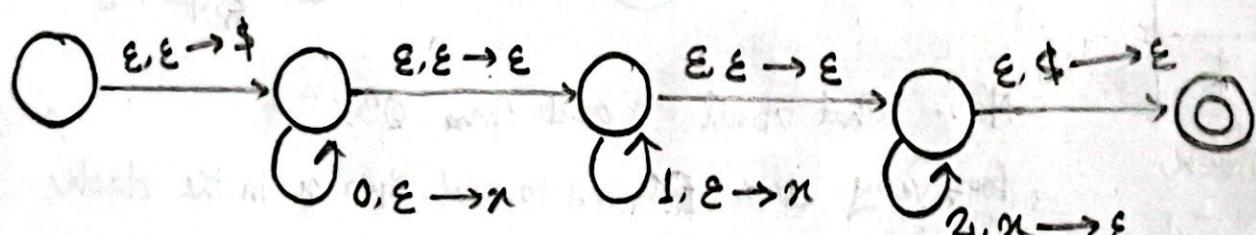
for every one,

we need to pop

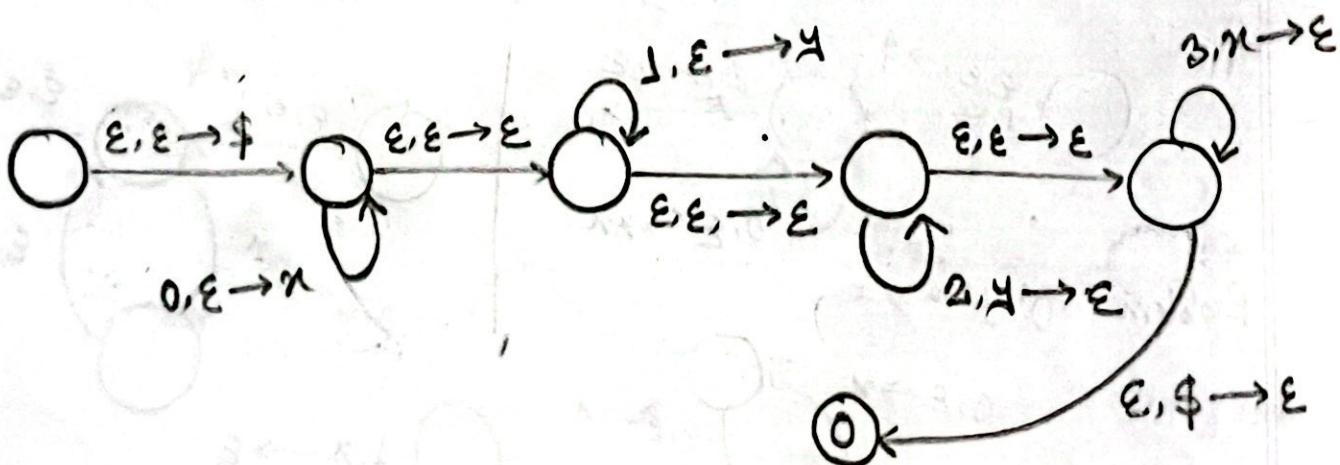
3 zeros from the stack



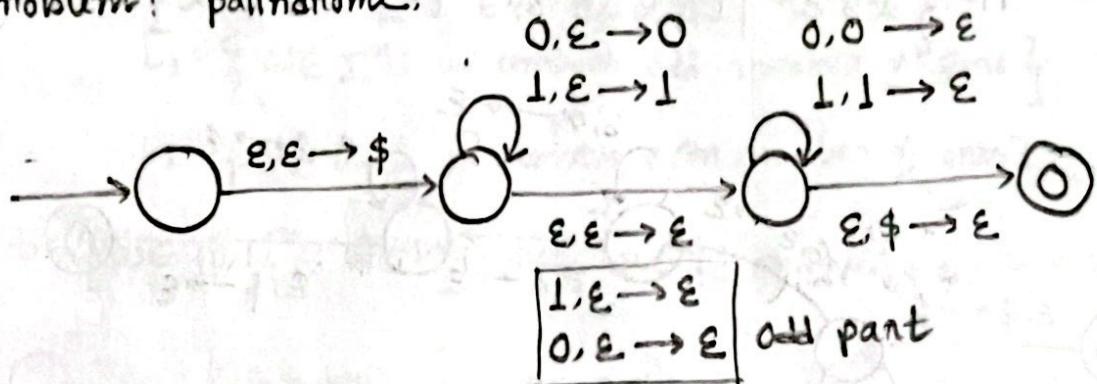
Problem: $0^i 1^j 2^k$, $i+j=k$



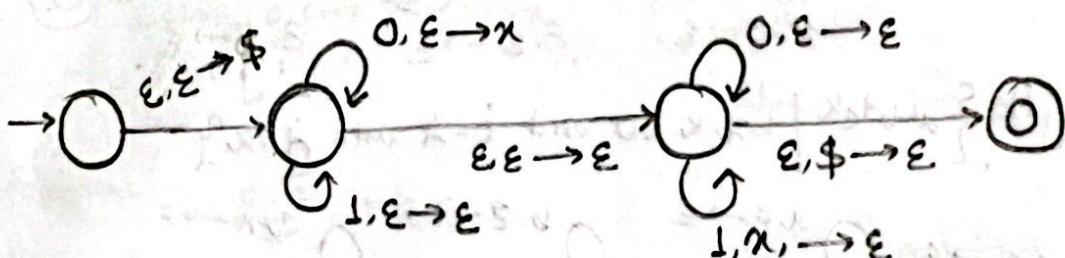
Problem: $0^i 1^j 2^k 3^l$



Problem: palindrome:

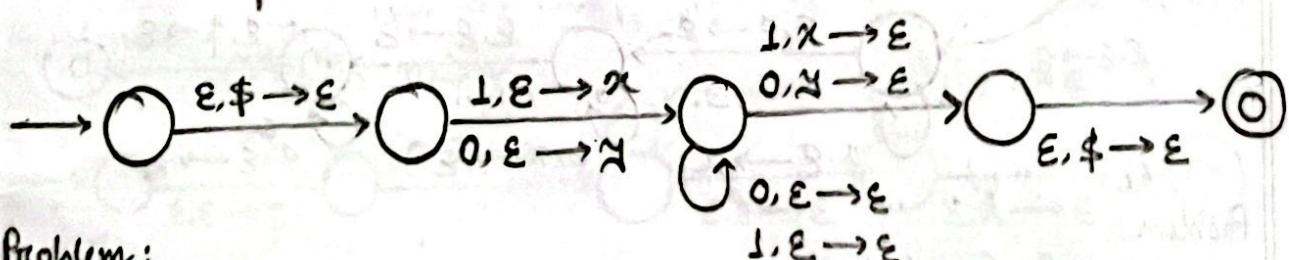


Problem: $W_1 \# W_2$, where, no. of 0's in W_1 = no. of 1's in W_2 .



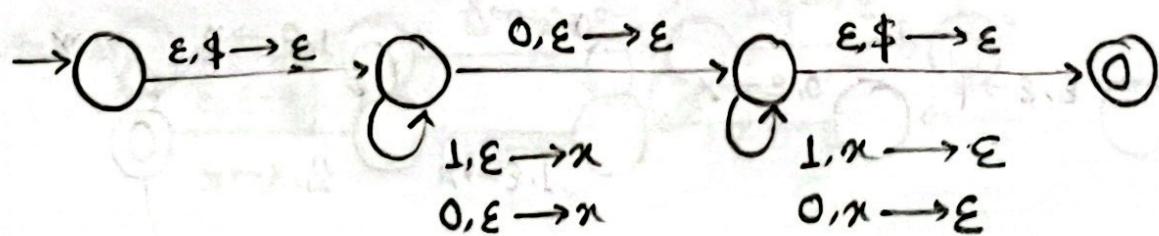
Problem:

$\{W \in \{0,1\}^* \mid W \text{ starts and ends with the same symbol}\}$

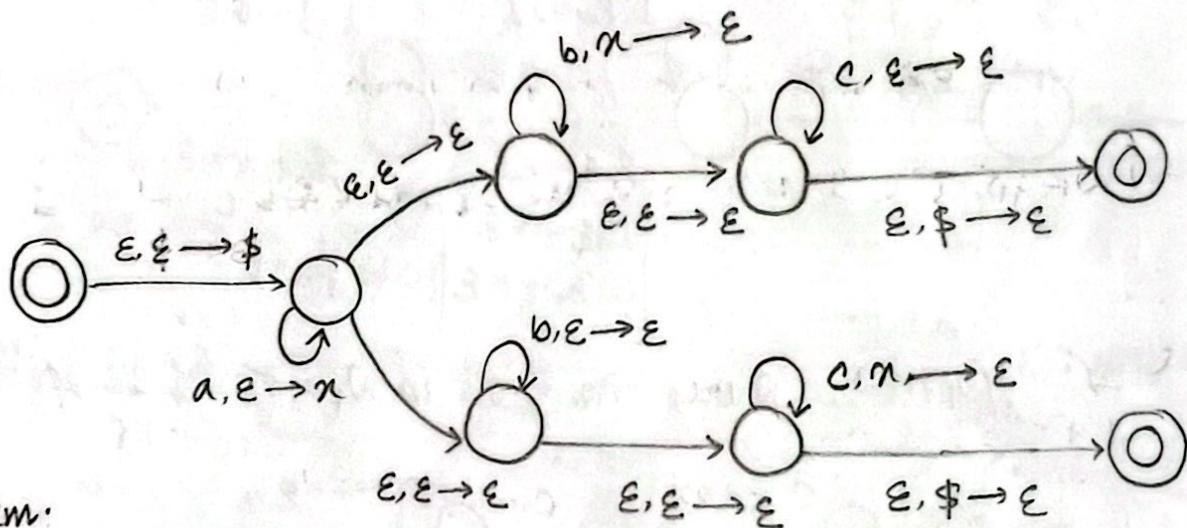


Problem:

$\{W \in \{0,1\}^* \mid W \text{ is of odd length and has } 0 \text{ as its middle symbol}\}$

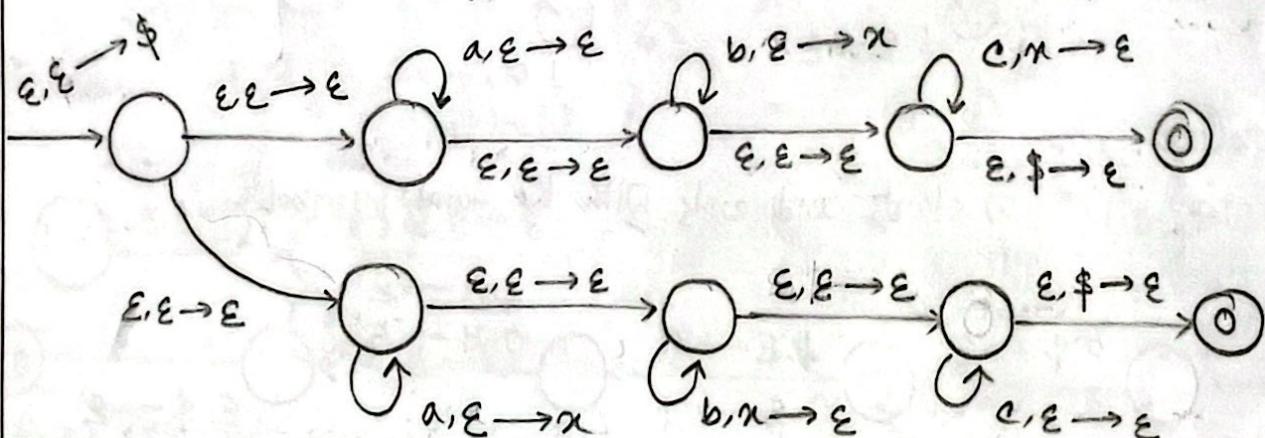


Problem: $A = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ and } i=k \}$



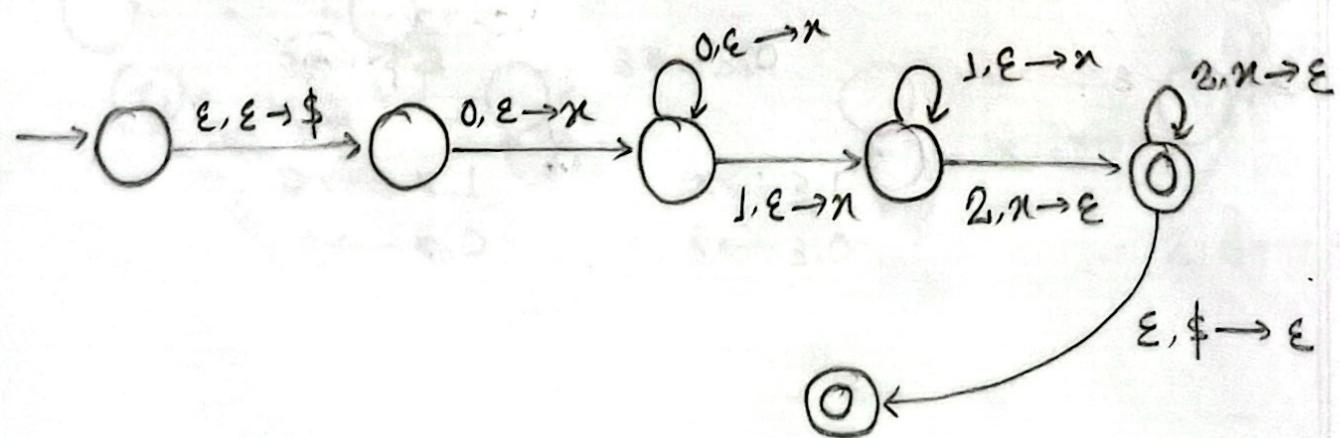
Problem:

$A = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ and } i=k \}$

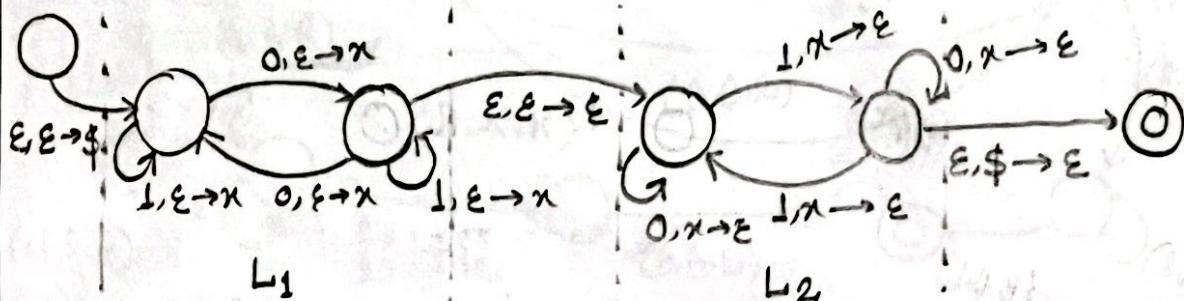


Problem:

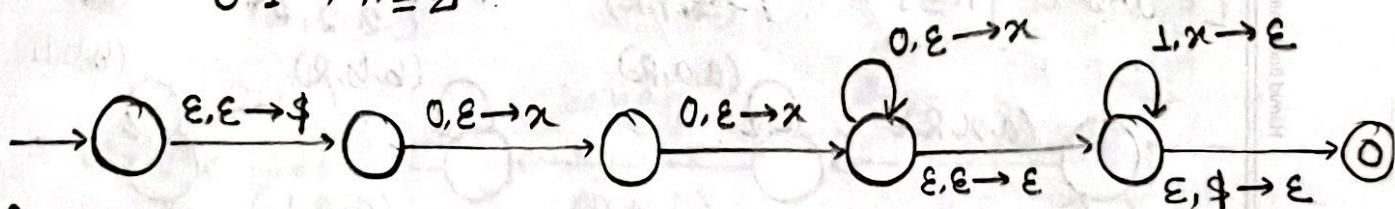
$A = \{ 0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and } i=j \text{ and } i=k \}$



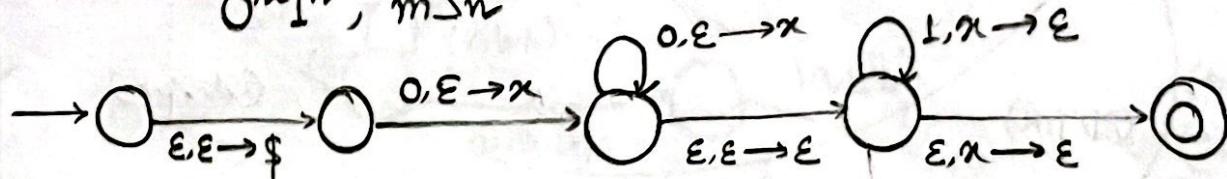
Problem:

 $L_1 = \{ \omega \in \Sigma^* : \omega \text{ contains odd number of zeros} \}$
 $L_2 = \{ \omega \in \Sigma^* : \omega \text{ contains even number of ones} \}$
 $L = \{ \omega \in \{0,1\}^* : \omega = uv, \text{ where } u \in L_1 \text{ and } v \in L_2, |u| = |v| \}$


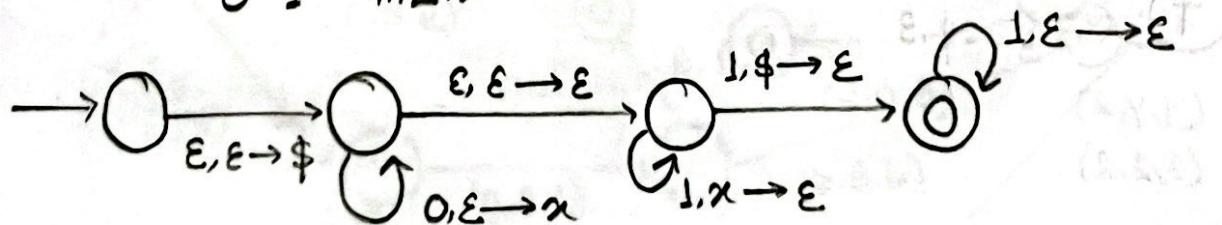
Problem:

 $0^n 1^n, n \geq 2$


Problem:

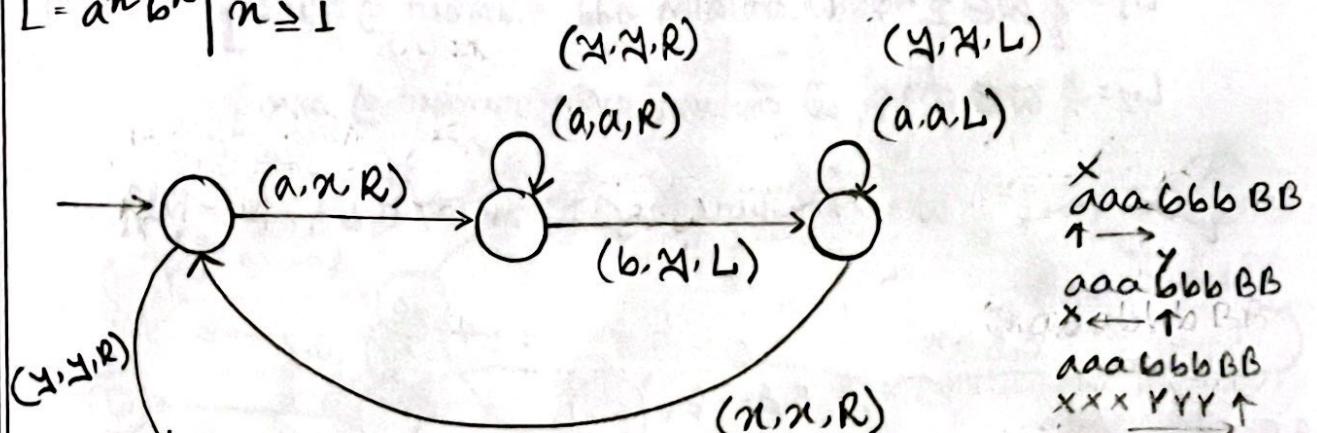
 $0^m 1^n, m \geq n$


Problem:

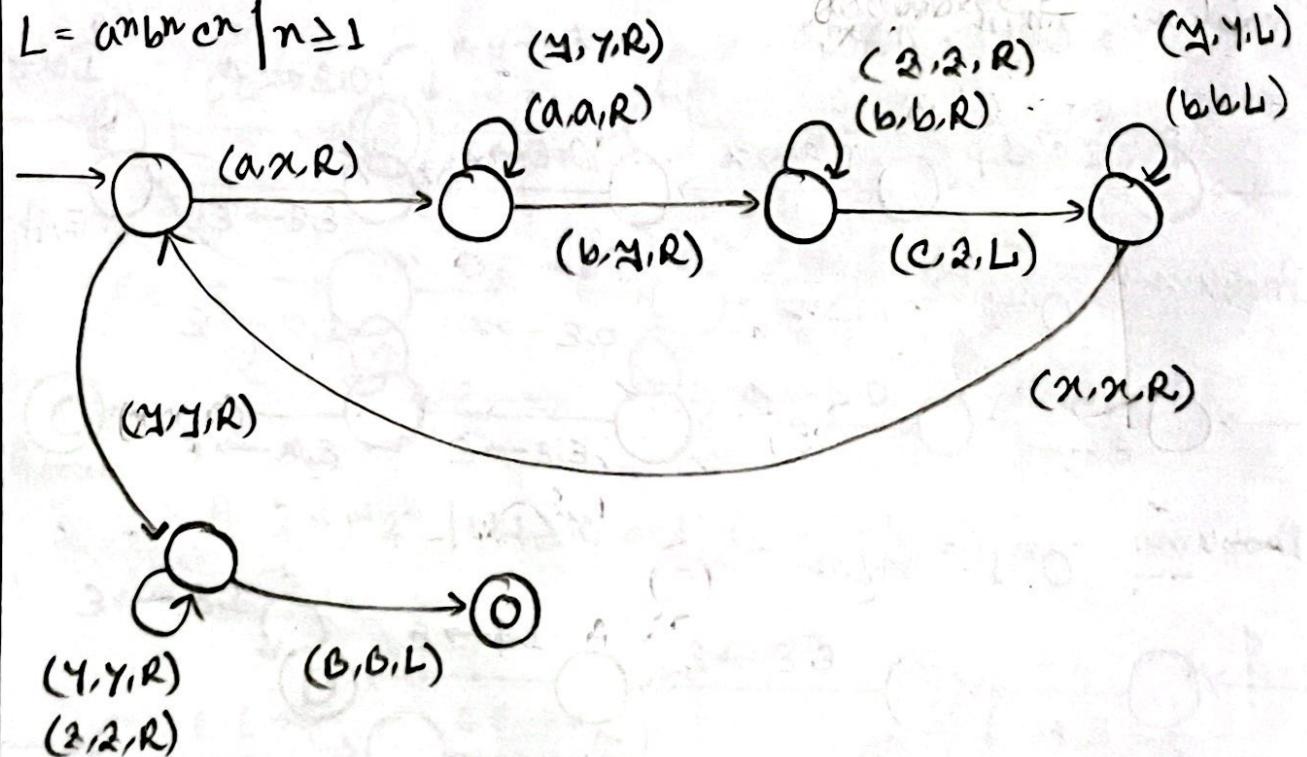
 $0^m 1^n, m < n$


Turing Machine

$$L = a^n b^n \mid n \geq 1$$



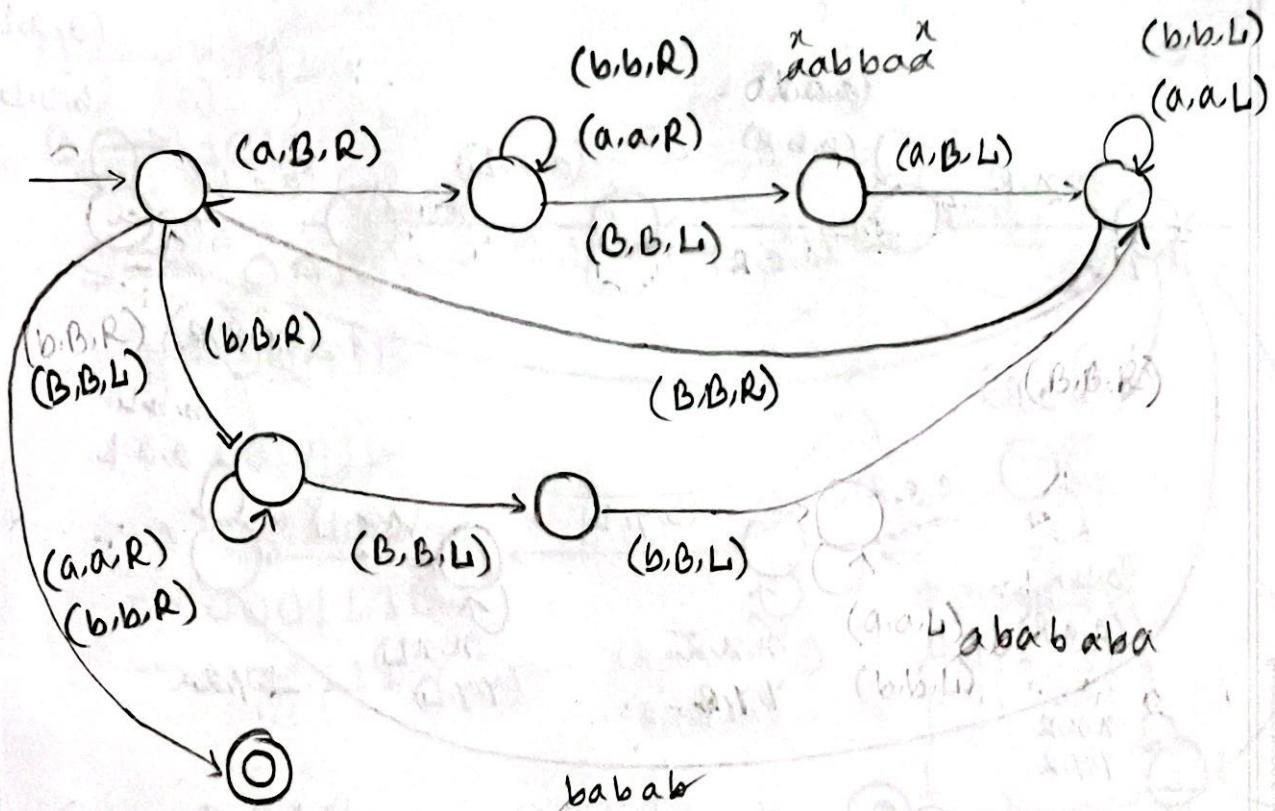
$$L = a^n b^n c^n \mid n \geq 1$$



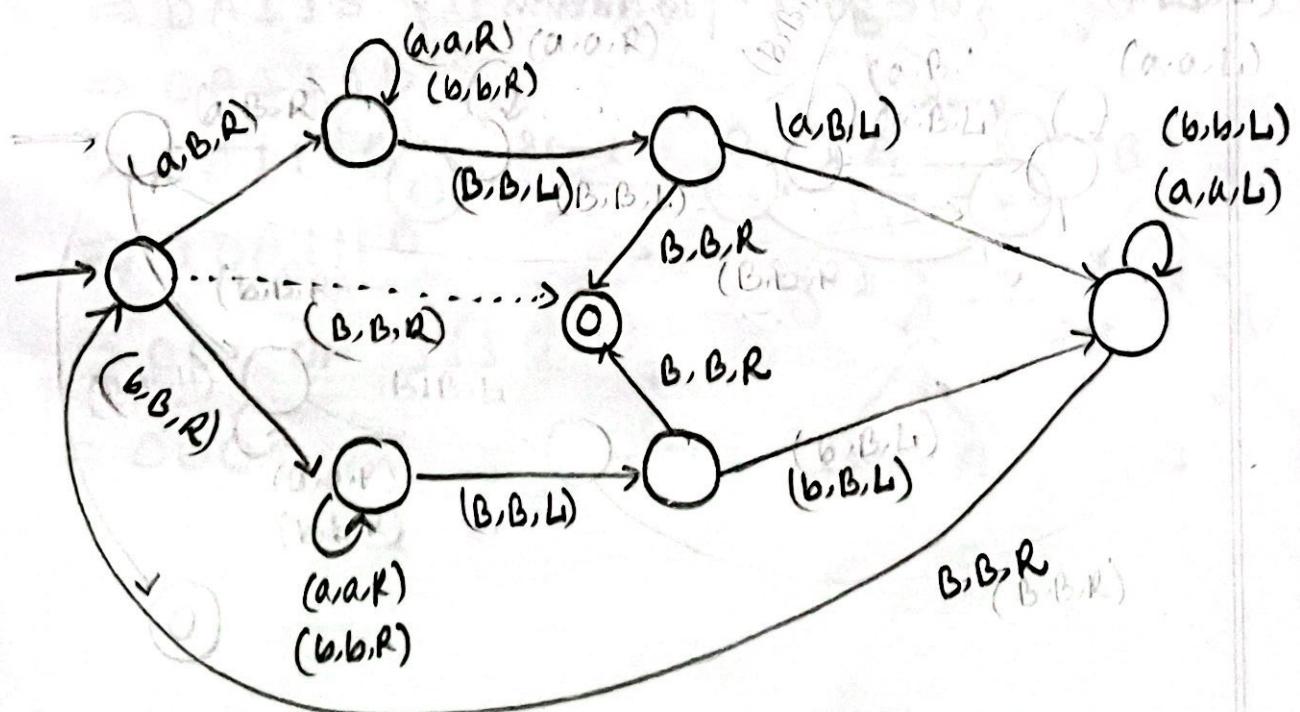
Subject :

Date :

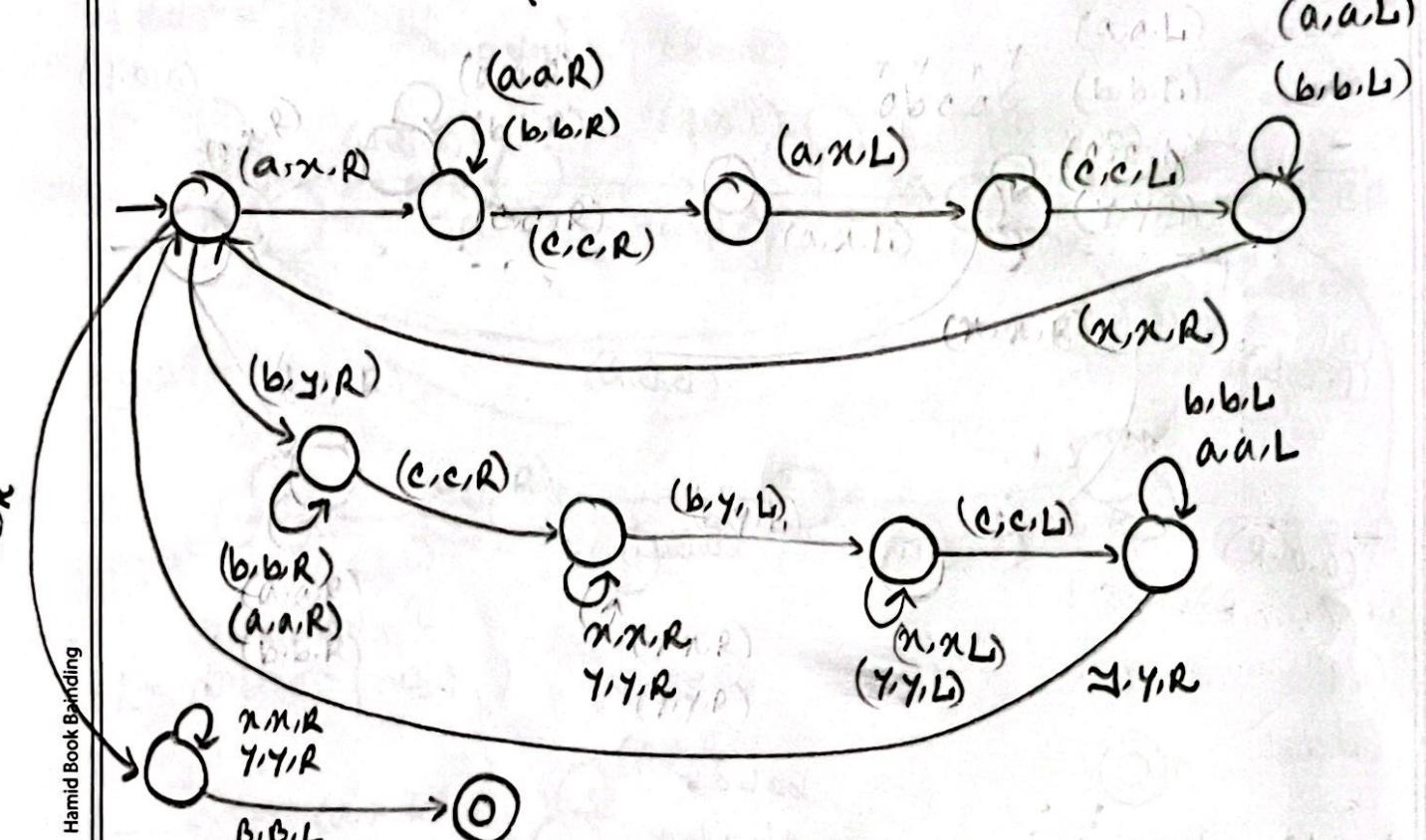
$L = WWR$ $W \in \{a, b\}$ Even



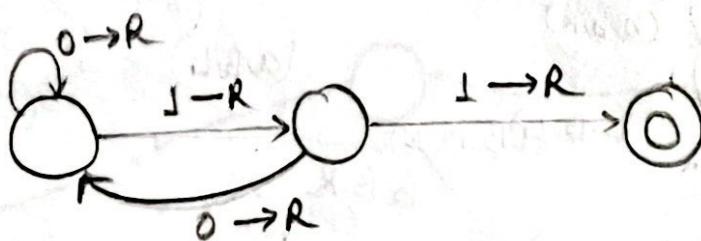
$L = W a W R$ [Odd]



$$L = w \in \mathcal{W} \mid w \in \{a, b\}^*$$



$$L(M) \rightarrow \{ \omega \in \{0,1\}^* \mid \omega \text{ contains } 11\}$$



$$S \rightarrow OAIB$$

$$A \rightarrow OA|I$$

$$B \rightarrow IB|0$$

$$S \rightarrow OAIB$$

$$\Rightarrow \underline{OOAIB}$$

$$\Rightarrow \underline{OOOAIB}$$

$$\Rightarrow \underline{OOOIB}$$

$$\Rightarrow \underline{OOOIIIB}$$

$$\Rightarrow \underline{OOOIIIO}$$

$$S \Rightarrow OAIB$$

$$OAIB$$

$$\Rightarrow OAIIIB$$

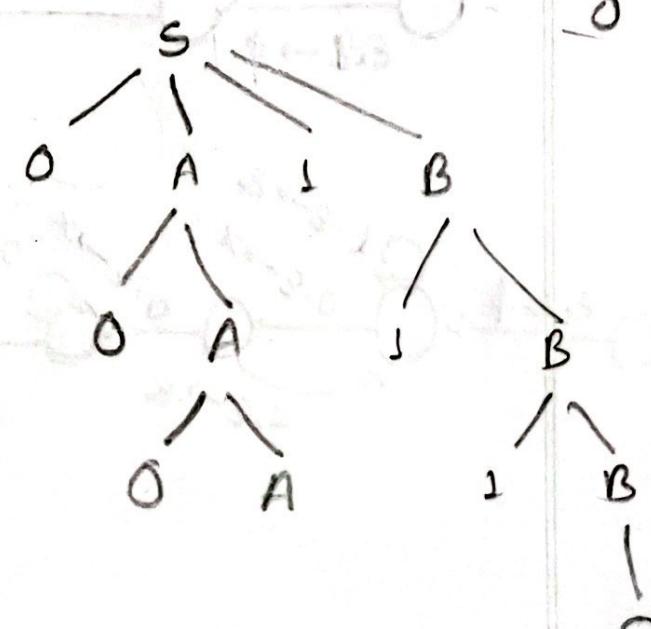
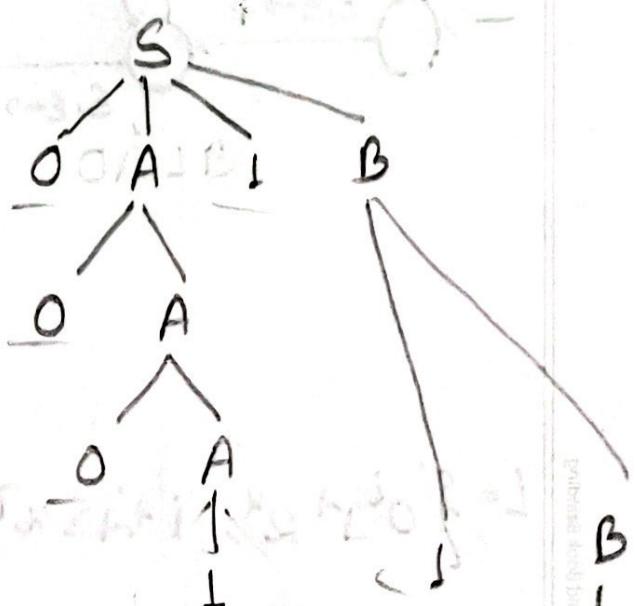
$$\Rightarrow OAIIIIIB$$

$$\Rightarrow OAIIIIIO$$

$$\Rightarrow OOAIIIIIO$$

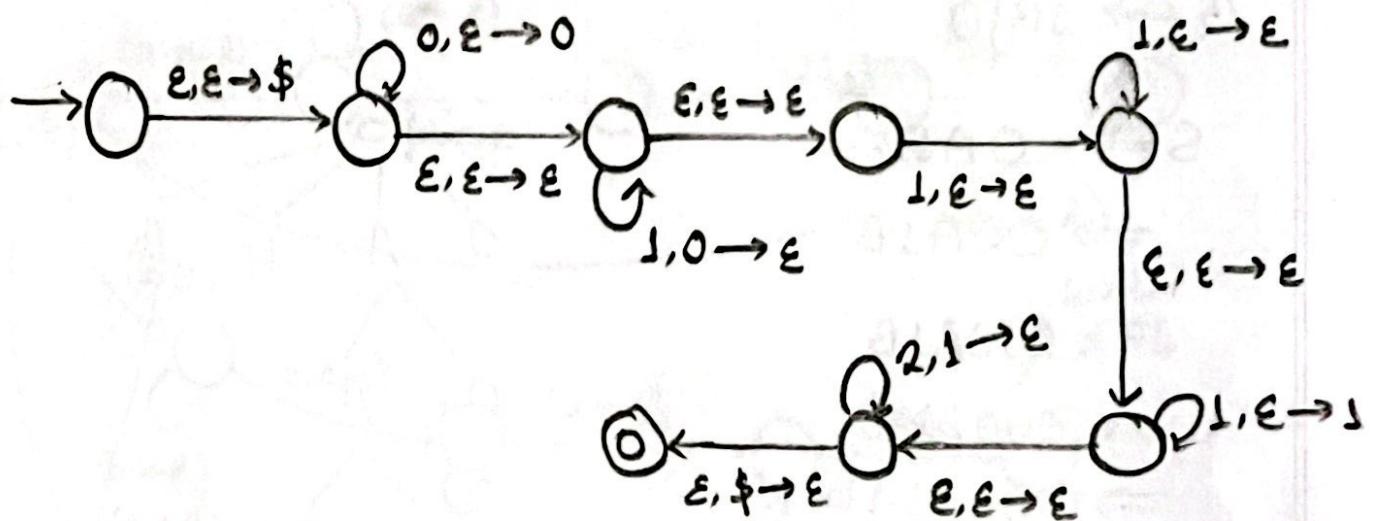
$$\Rightarrow OO OA IIIO$$

$$\Rightarrow OOO$$

$$0111000$$


PDA

$$L = \{0^i 1^j 2^k; i \leq j+k\}$$



$$L = \{0^i 1^i 2^k, i+k \geq k\}$$

